Networked Systems with Time Delay: Stability and Performance Issues with an Input-Output-Transformation Approach

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***Abstract*— Communication time delay in a networked control system (NCS) degrades the performance and may lead to instability. The time delay value depends on the network configuration, e.g. number of nodes and is not exactly known during the controller design stage. In this paper the problem of unknown constant time delay is addressed. A transformation of the plant and the controller input and output is considered, the transformed signals are sent through the communication channel. Performance issues are considered and a comparison with two standard control strategies is performed. Experimental validation shows that the input-output transformation approach is superior over two other approaches, the small gain theorem approach and the Lead-Lag controller.**

I. INTRODUCTION

Due to the increasing complexity of control systems and novel requirements, such as decentralized control, the traditional control architecture with point to point interconnection between the plant and the controller is increasingly replaced by Networked Control Systems (NCS). In NCS the components are spatially separated and the control loop is closed physically through a communication network, see Fig. 1. Significant advantages are reduced wiring and cost, increased modularity, easier maintenance and system diagnosis as well as high flexibility and reconfigurability, meaning that nodes can be added or removed without additional wiring effort. In NCS, however, the signal transmission between the plant an the controller can no longer be regarded ideal. New challenges arise, i.e. time delay, data drop out and limited communication resources in the closed loop. These challenges depend on the number of active nodes sharing the communication line and are consequently not exactly known during the controller design stage. Various approaches have appeared in NCS, for an overview on recent advances see [1], [2].

In this work the problem of unknown constant time delay is addressed. It is well-known that time delay in a closed control loop degrades the performance and can lead to instability. There is a rich literature on time delay systems, see [3], [4] for an overview. Most approaches consider a Lyapunov-like stability analysis (Lyapunov-Krasovskii, Razumikhin) [4] requiring a state-space representation of the system. Delay-dependent and delay-independent stability conditions can be derived from there. Input-output approaches achieve delay-independent stability by the application of the small gain theorem leading to a very conservative design. Input-output approaches for time delay systems are considered in [5], [6] where the time delay is assumed to be known.

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This work was supported in part by the Japanese Society for the Promotion of Science (JSPS) by a *Postdoctoral Fellowship for Foreign Researchers* granted to the second author.

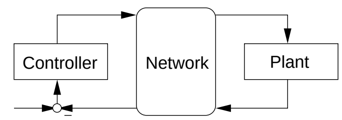

Fig. 1. Networked control system architecture.

Motivated by the fact, that the time delay is not exactly known we consider a delay-independent input-output approach where stability is guaranteed for arbitrarily large constant time delay. A linear memoryless input-output transformation is used [7], instead of the initial input and output the transformed signals are sent through the communication network. In [7] it is shown that not only delay-independent stability is guaranteed, but the performance is not sensitive to time delay variations as well. In case of passive subsystems the well known scattering transformation is recovered, widely used for stability in the presence of time delay in telepresence systems [8] [9]. In a similar manner to the scattering transformation, for stability the boundness of the L_2 gain of the time delay operator is exploited. Thus, well-known approaches for time-varying time delay [10], [11] and packet loss [12], [13] can directly be carried over.

The main focus of this paper is on performance issues and on the experimental validation of the aforementioned input-output transformation approach. The input-output transformation is compared with two typical control strategies, a Lead-Lag controller, and a controller designed with the standard delay-independent small gain approach. The results show that the input-output transformation outperforms the other typical approaches as far as stability performance and insensitivity with respect to time delay is concerned, in simulation and experiment.

This paper is organized as follows: In Section II. the necessary background is presented. The system description together with the input-output transformation is presented in Section III., followed by the stability considerations in Section IV and design performance and design issues in Section V. A performance comparison in simulations and experiments is presented in Section VI. Conclusions are given in section VII.

II. BACKGROUND

A. L_2 Stability

Let $\|u\|_{L_2}$ denote the L_2 norm of a piecewise square-integrable function $u(\cdot): \mathbb{R}_+ \to \mathbb{R}^m$ with $\mathbb{R}_+$ being the set of non-negative real numbers and $\mathbb{R}^m$ the Euclidean space of dimension m. The truncation of $u(\cdot)$ up to the time t is denoted by $u_t(\cdot)$, and the extended space of Lebesgue integrable functions by L_{2e}.

A system is said to be finite gain L_2 stable if there is a constant $\gamma \geq 0$ such that between the input $u(\cdot) \in L_{2e}$ the output $y(\cdot) \in L_{2e}$ of the system the following inequality holds [14]

$$\|y_t\|_{L_2} \leq \gamma \|u_t\|_{L_2} \quad \forall u \in L_{2e},\ t \in [0, \infty). \tag{1}$$

The smallest possible value γ, such that (1) is satisfied, is the L_2 gain of the system. The L_2 stability condition states, that each bounded L_2 norm input signal is mapped to a bounded L_2 norm output signal.

B. Input-Feedforward-Output-Feedback-Passivity

A system is said to be input-feedforward-output-feedback-passive (IF-OFP) if for each input signal $u(\cdot) \in L_{2e}$

$$\int_0^t u^T(\tau) y(\tau)\, \mathrm{d}\tau \geq \delta \|u_t\|_{L_2}^2 + \varepsilon \|y_t\|_{L_2}^2, \quad \forall u \in L_{2e},\ t \in [0, \infty) \tag{2}$$

holds, where $u^T(\tau)y(\tau)$ represents the instantaneous power input to the system and $\delta, \varepsilon \in \mathbb{R}$ are some constants. The above input-output description can be seen as a generalization of the passivity concept. If $\delta = \varepsilon = 0$ then the system is passive, which means that it does not generate energy. If $\delta = 0$ and $\varepsilon > 0$ the system is called output-feedback strictly passive and if $\delta > 0$ and $\varepsilon = 0$ input-feedforward strictly passive. In both these cases the system dissipates energy. If one or both of the values δ, ε are negative then there is a shortage of passivity in the system. In other words, the system can generate energy, but this energy is bounded by the squared L_2 norm of the input and/or the output signal.

For IF-OFP systems the next lemma holds:

Lemma 2.1: [7] Without loss of generality the domain of δ, ε (2) is considered to be $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\delta, \varepsilon \in \mathbb{R} | \delta\varepsilon < 1/4\}$ and $\Omega_2 = \{\delta, \varepsilon \in \mathbb{R} | \delta\varepsilon = 1/4; \varepsilon > 0\}$.

In case $\delta, \varepsilon \notin \Omega_1 \cup \Omega_2$ degenerate cases occur, i.e. (2) imposes either no restriction on the input-output behavior, or there is no system satisfying (2). The class of IF-OFP systems is quite general and includes even some input-output unstable systems, however, not with finite escape time.

An interesting feature of the IF-OFP property is that it can guarantee input-output stability in feedback interconnections. Considering two systems h_p, h_c satisfying (2) for some $\delta_i, \varepsilon_i, i \in \{p, c\}$ in feedback interconnection the next proposition holds

Proposition 2.1: [14] The negative feedback interconnection of h_p and h_c is finite gain L_2 stable if

$$\varepsilon_c + \delta_p > 0 \qquad \text{and} \qquad \varepsilon_p + \delta_c > 0.$$

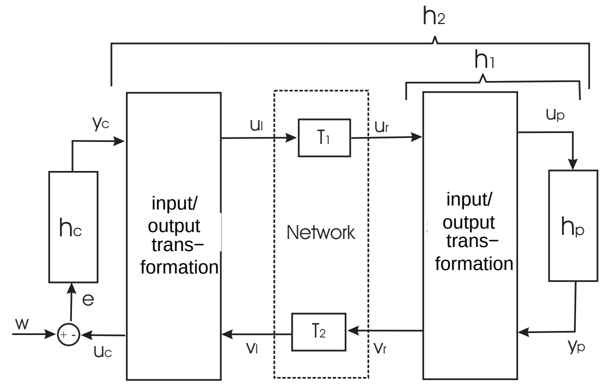

Fig. 2. NCS with input-output transformation.

Clearly, some of δ_i, ε_i can be negative if compensated by other positive values. Within the passivity formalism this can be interpreted as compensating shortage of passivity of the one system with excess of passivity of the other and vise-versa.

C. Interpretation on the Complex Plane

In case of a SISO LTI system with the transfer function $G(s)$ passivity and finite gain L_2 stability define areas on the complex plane where the Nyquist plot is confined to. The Nyquist plot of a passive system lies in the right half plane. The Nyquist plot of a finite gain L_2 stable system with gain γ lies in a circle with radius $r = \gamma$ and center the origin. Analogously, the Nyquist plot of a stable IF-OFP LTI system lies inside or outside of a specific circle on the complex plane. If $\varepsilon > 0$ the Nyquist plot $G(j\omega)$ lies in a circle with the center point $R = \frac{1}{2\varepsilon}$ and the radius $r = \frac{\sqrt{1-4\varepsilon\delta}}{2\varepsilon}$; if $\varepsilon < 0$ it lies everywhere on the complex plane except for a circle with center $R = \frac{1}{2\varepsilon}$ and radius $r = -\frac{\sqrt{1-4\varepsilon\delta}}{2\varepsilon}$. If $\varepsilon = 0$ then $\mathrm{Re}\{G(j\omega)\} \geq \delta$ holds, i.e. the Nyquist plot lies in a the plane right of δ. Passivity, $\delta = \varepsilon = 0$ can be considered a specific case of IF-OFP systems where $R \to \infty, r \to \infty$, i.e. the IF-OFP circle coincides with right half plane. Analogously finite gain L_2 stability with gain γ can be seen as a limit case of IF-OFP with $\delta = -\gamma^2\varepsilon, \varepsilon \to \infty \Rightarrow R = 0, r = \gamma$.

D. Linear Fractional Transformation

A linear fractional transformation (LFT) is a conformal mapping of the complex plane $\mathbb{C} \cup \infty$ to itself, defined by the relation

$$z \to \frac{az+b}{cz+d}. \tag{3}$$

where $a, b, c, d \in \mathbb{C}$. If $ad - bc \neq 0$ it is bijective, otherwise it is a constant, i.e. $z \to \frac{a}{c} = \frac{b}{d}$. A circle on the imaginary plane is mapped either to another circle or to a line. The same holds for a line.

III. SYSTEM DESCRIPTION

We consider a system consisting of a SISO plant and a SISO controller, i.e. $m = 1$, see Fig. 2. The plant is described by a mapping h_p from the plant input $u_p \in L_{2e}$ to the plant output $y_p \in L_{2e}$, i.e. $h_p : L_{2e} \rightarrow L_{2e}$. Analogously, the controller input-output behavior is described by a mapping $h_c \in L_{2e}$ from the control error $e = w - u_c$, $w \in L_{2e}$ being the desired value and u_c the lefthand side output of the communication channel, to the output, $y_c \in L_{2e}$, i.e. $h_c : L_{2e} \rightarrow L_{2e}$. The plant is connected to the controller by a communication network. However, instead of transmitting directly the input-output signals a transformation of them is sent. The transformation acts on the plant input/output vector $z_p^T = [u_p\, y_p]$, as well as on the lefthand side communication input/output vector $z_c^T = [y_c\, u_c]$ with the transformation matrix M accordingly

$$s_r = Mz_p \quad \text{and} \quad s_l = Mz_c \tag{4}$$

where $s_r^T = [u_r\, v_r]$, and $s_l^T = [u_l\, v_l]$ representing the values transmitted over the channel, the indices $(\cdot)_l$, $(\cdot)_r$ denote the lefthand and righthand side of the communication channel. The transformation matrix M is parameterized as a rotation matrix R and a scaling matrix B

$$M = RB \quad \text{with}$$
$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}; \quad B = \begin{bmatrix} \sqrt{b} & 0 \\ 0 & \frac{1}{\sqrt{b}} \end{bmatrix}, \tag{5}$$

with the rotation angle $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and the scaling parameter $b > 0$, both constant. The mapping by M is a bijection; it belongs to the class of *special linear transformations*, i.e. $\det M = 1$, hence is non-singular, an inverse exists. For stability the choice of the proper angle θ is important. The scaling parameter b can be freely adjusted to meet performance requirements.

Throughout the paper we assume that the closed loop system is well posed, i.e. for each input signal $w \in L_{2e}$ there exists a unique solution for the signals $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p$ that causally depends on w. Further, we assume the following system properties:

1) The time delays T_1 and T_2 are arbitrarily large but constant.
2) Plant h_p and controller h_c are IF-OFP with δ_i, ε_i with $i \in \{p, c\}$ satisfying Proposition 2.1, i.e. the feedback interconnection *without time delay* is finite gain L_2 stable.

IV. STABILITY

For the above described system the IF-OFP property of the plant with δ_p, ε_p can be exactly preserved to the subsystem h_2 independently of time delay by proper selection of the transformation angle θ, i.e. the next theorem holds:

Theorem 4.1: The subsystem h_2 is IF-OFP with δ_p, ε_p if and only if the following holds

$$\cot 2\theta = \varepsilon_p b - \frac{\delta_p}{b}, \tag{6}$$

and

$$\sin(\theta)\cos(\theta) - \frac{\delta_p}{b}\cos^2(\theta) - \varepsilon_p b \sin^2(\theta) \geq 0 \tag{7}$$

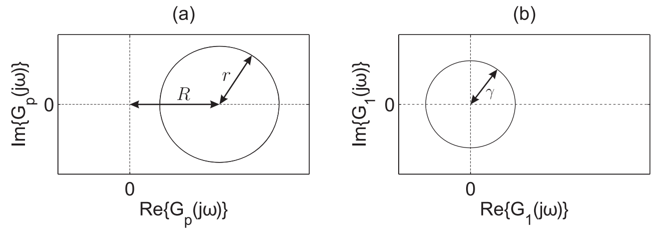

Fig. 3. (a) Nyquist plot area of the IF-OFP plant G_p. (b) Nyquist plot area of the finite gain L_2 stable subsystem $G_1(s)$.

for a proof see [7]. Equation (6) gives two solutions for $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, and (7) merely chooses between one of these two. Since the IF-OFP property is unaltered preserved to h_2 it is straightforward to conclude finite gain L_2 stability of the closed loop system.

Corollary 4.1: [7] The closed loop system with the input-output transformation (5) and the transformation parameters satisfying Theorem 4.1 is delay-independent finite gain L_2 stable.

The above result is based on the fact that the time delay operator does not change the L_2 gain of a system in series connection, as the time delay operator has a L_2 gain one. The input-output transformation transforms any IF-OFP plant to a finite gain L_2 stable system. This means that for each $\delta_p, \varepsilon_p, b$, where b is the transformation parameter, (6) (7) give always one solution for the angle θ so that the input-output behavior of the subsystem h_1 is described by

$$\|v_{r,t}\| = \|h_1(u_{r,t})_t\| \leq \gamma_{h_1}\|u_{r,t}\| \quad \forall t; \quad \gamma_{h_1} \geq 0, \tag{8}$$

The derivation of the gain γ_{h_1} can be found in [7]. For each b there is always one solution for θ, thus b is a parameter that can be chosen freely to meet performance requirements. Since the L_2 gain γ_{h_1} remains unaltered by the time delay, the left hand bijective transformation restores the original IF-OFP property to the subsystem h_2. As long as input output stability based on Proposition 2.1 is guaranteed for the initial plant and controller without the time delay, the same holds for the system with the network with arbitrary large time delay and the input-output transformation.

An intuitive graphical representation of the above can be given in the frequency domain of LTI systems on the complex plane. The above input-output transformation can be interpreted as a Linear Fractional Transformation (LFT) on the complex plane of the plant, i.e. using the transformation equations (4) it follows

$$G_1(j\omega) = \frac{\cos(\theta)G_p(j\omega) - b\sin(\theta)}{\sin(\theta)G_p(j\omega) + b\cos(\theta)} \tag{9}$$

where $G_1(j\omega)$ is the transfer function of h_1. The LFT (9) maps the initial circle that the Nyquist plot of the IF-OFP plant $G_p(j\omega)$ lies, to a finite L_2 gain circle with center the origin, see Fig. 3. The arbitrary large time delay $e^{-j\omega T}$ where $T = T_1 + T_2$ does not alter this circle as it only rotates each point around the origin. The inverse transformation, since it is a bijection, maps the finite L_2 gain circle back to the initial circle of the plant, independently of the time delay. The IF-OFP property of the plant is restored to the subsystem h_2.

Remark 1: In case of an unstable plant, the plant is locally pre-stabilized with the right hand input-output transformation. This is clear from the fact each IF-OFP plant results after the right hand transformation to a finite gain L_2 stable system described by (8).

Remark 2: In case of a passive plant the LFT (9) $\theta = 45^\circ$ recovers the well-known scattering transformation widely used for the stabilization of teleoperation systems in the presence of arbitrarily large constant time delay [8], [9]. The scattering transformation maps the right half plane to the unit circle, i.e. a passive system to a finite gain L_2 stable system with L_2 gain one.

V. Performance and Design Considerations

In this part we will consider that the stability criteria are fulfilled, i.e. the closed loop system is stable.

A. Steady State Error

The steady state of the system is not affected by the input-output transformation and the time delay. This can be seen from the fact that in the steady state $s_l = s_r$ holds, thus $z_c = M^{-1}s_l = M^{-1}s_r = M^{-1}Mz_p = z_p$. As far the steady state is concerned the controller can be designed without considering the time delay and the transformation.

B. Zero Time Delay Case

As the time delay reduces to zero, i.e. $T_1 = T_2 = 0$, $s_l = s_r$ holds and thus $z_c = z_p$, i.e. the system reduces to that without input-output transformation. Thus, for zero time delay, nominal performance is achieved.

C. Design Procedure

Based on the above a controller design procedure can be shortly described in the next steps.

1) Define the δ_p, ε_p values of the plant, e.g. from the Nyquist plot.
2) From Theorem 4.1 define the the minimum values for δ_c, ε_c of the controller so as to guarantee stability for the closed loop system.
3) Find a controller and a value for the b parameter of the transformation, so as to satisfy performance criteria under the constraints of δ_c, ε_c and the angle θ of the transformation given according to Theorem 4.1.

Step 2) is straightforward to implement. In step 1) it is desirable to determine the tightest bound for δ_p, ε_p in order to avoid conservatism. However, this is non-trivial in general. How to define the optimal controller with respect to some performance criteria in step 3) is still an open problem. A systematic controller design methodology based on the above is still subject for future research.

VI. Performance Comparison

In order to test the efficacy of the considered approach a comparison is performed with a controller designed with the classical small gain approach, and with the delay-dependent Lead-Lag controller. It should be mentioned that the latter comparison is unfair in the sense that delay-dependent methods lead usually to less conservative results.

In the lack of a systematic design methodology for the optimal controller a numerical optimization is performed. Exemplarily, the LTI plant

$$G_p(s) = \frac{36}{s^2 + 10.15s}, \tag{10}$$

is regarded, being an approximation of the experimental setup which will be used later for experimental validation. The time constant of the above system is approximately 100ms, smaller than the time delay values that will be considered in the following.

A. Controller Design

A Lead-Lag controller

$$G_c(s) = k\frac{s+a}{s+c}, \tag{11}$$

is considered where $k, a, c > 0$ are parameters to be determined in numerical optimization. The cost function, the integrated squared tracking error, given by

$$J_0 = \int_0^{t_f} e^2(\tau)\mathrm{d}\tau, \tag{12}$$

with $e(t) = y_p(t) - w(t)$ is employed for all cases in order to get a fair comparison. Here w is the reference input, y_p the output of the plant,see Fig. 2. The numerical optimization is performed for a nominal time delay of $T = 300$ms over a horizon of $t_f = 5$s using `fmincon` of the Matlab optimization toolbox. The exact design procedure for all three cases is explained in the next.

1) Input-Output Transformation: Following the design steps given in Section V-C, first the values δ_p, ε_p are determined from the Nyquist plot of (10) presented in Fig. 4. Clearly, $\mathrm{Re}\{G_p(j\omega)\} > -0.3494$, further $\mathrm{Im}\{G_p(j\omega)\} \to \infty$ as $\omega \to 0$ giving $\delta_p = -0.3494, \varepsilon_p = 0$, see Section II-B. For stability based on Theorem 4.1 $\delta_c > 0$ and $\varepsilon_c > 0.35$ must hold for the controller. The Nyquist plot of the Lead-Lag controller is a circle with center on the real axis in position R and radius r given by

$$R = \frac{k(a+c)}{2c}; \qquad r = \frac{k|a-c|}{2c}. \tag{13}$$

From these values using the equations from Section II-B follows

$$\delta_c = \frac{R^2 - r^2}{2R} > 0, \tag{14}$$

is always satisfied as long as $k, a, c > 0$. Furthermore, (13) and the requirement $\varepsilon_c > 0.35$ leads to

$$\frac{k(a+c)}{2c} < 1.4286, \tag{15}$$

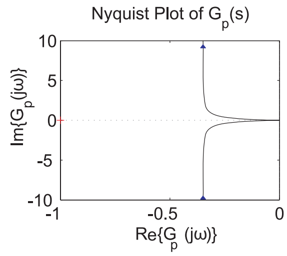

Fig. 4. Nyquist plot of the plant transfer function $G_p(j\omega)$

imposing a stability constraint to the Lead-Lag controller parameter values. The resulting constrained optimization problem can be formulated as $\min_{k,a,c,b,\theta} J_0$ subject to the equality constraint (6) and the inequality constraints $k,a,c,b > 0$, (7), and (15). The numerical optimization gives the controller for the input-output transformation

$$G_{tr}(s) = \frac{1.6160(s+2.5833)}{s+3.3628}, \tag{16}$$

with all constraints satisfied and $b = 0.1816,\ \theta = 13.73^o$.

2) Small Gain Controller: In order to achieve delay-independent stability without the transformation the small gain theorem has to be satisfied requiring $|G_0(j\omega)|_\infty = |G_p(j\omega)G_{sg}(j\omega)|_\infty < 1\ \forall\omega$ with G_{sg} the small gain controller transfer function and $|G_0(j\omega)|_\infty$ the H_∞ norm of the open loop transfer function G_0. The optimization problem is formulated similar to the previous case

$$\min_{k,a,c} J_0, \tag{17}$$

now subject to the equality constraint $k = 1/|G_0(j\omega)|_\infty$ to satisfy the small gain theorem and the inequality constraint $k,a,c > 0$. With the integrator in the plant (10) the small gain theorem cannot be satisfied with a Lead-Lag controller, thus a high pass component is added. The numerical optimization gives the controller

$$G_{sg} = \frac{0.1855(s+5.3574)}{(s+3.5258)}\frac{s}{0.001s+1}.$$

3) Delay-Dependent Lead-Lag Controller: For the Lead-Lag controller the optimization problem is represented by (17), where J_0 is given by (12), under the constraint $k,a,c > 0$ resulting in the controller

$$G_{ll} = \frac{2.6136(s+2.9028)}{(s+14.3436)}. \tag{18}$$

B. Comparison in Terms of Steady State Error

With the input-output transformation the steady state is not affected by the transformation itself as stated in Section V-A. The steady state error for the given plant (10) and the controller (16) is zero as an integrator is contained in the open loop transfer function. The same holds for the delay-dependent Lead-Lag controller (18). For small gain controllers in general, however, the steady state error does not tend to zero; for a reference step input it is $|e(t)| > \frac{1}{2}|w(t)|$ for $t \to \infty$ as straightforward computable from the requirement $|G_0(j\omega)|_\infty < 1\ \forall\omega$ and the error transfer function

$$\frac{e(s)}{w(s)} = \frac{1}{1+G_0(s)},$$

hence in terms of the steady state error the input-output transformation approach outperforms the small gain approach.

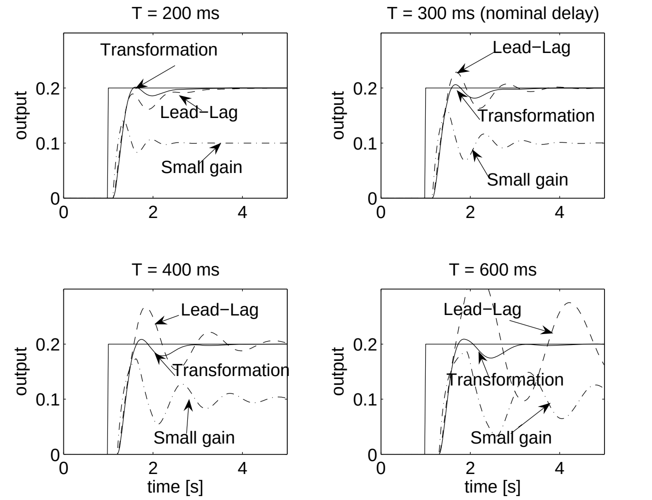

Fig. 5. Simulation step response of input-output transformation approach, small gain based controller and Lead-Lag controller for various time delay values

C. Simulations

Main focus of the comparison is the performance not only for the nominal time delay, but for different values as well, since the time delay value in NCS is not exactly known. A step input of magnitude 0.2rad is considered. The simulation results are given in Fig. 5. The proposed approach outperforms the other approaches, not only for the initially presumed time delay, but also for the other time delay values. The overshoot is small and slightly affected by the time delay value; the settling time is much smaller than with the other two approaches and does not change significantly for different time delay as well. The controller designed based on the small gain theorem gives a steady state error of approximately 50% in all cases, i.e. the theoretical

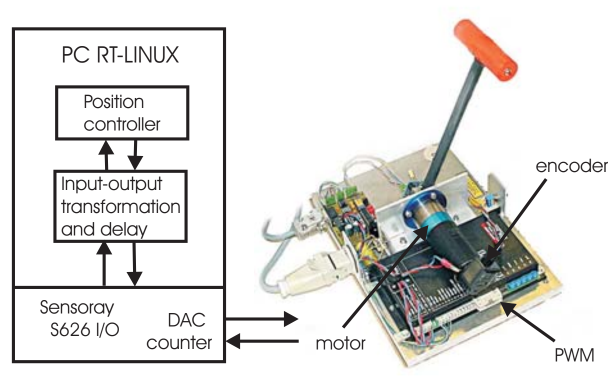

Fig. 6. Experimental testbed

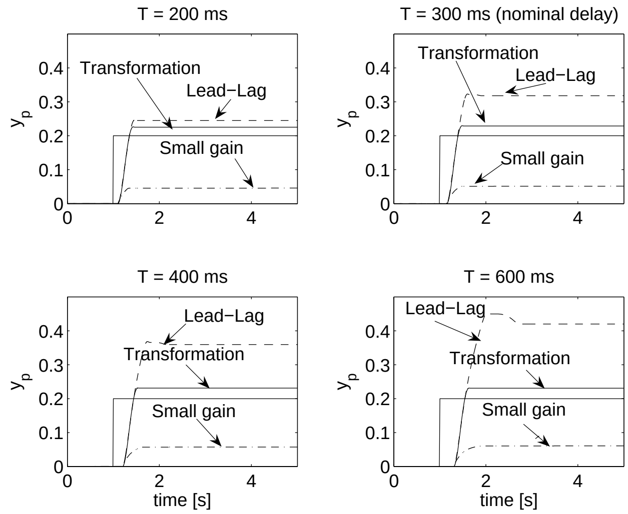

Fig. 7. Experimental step response of input-output transformation approach, small gain based controller and Lead-Lag controller for various time delay values

minimum. With increasing time delay the settling time grows significantly. The Lead-Lag controller without the input-output transformation performs well at the nominal time delay but is, as expected, sensitive with respect to time delay: still stable for the maximum time delay value tested it shows significant overshoot and settling time when the time delay increases.

D. Experiments

The experimental testbed consists of the 1DOF pendulum shown in Fig. 6 connected to a PC running under RT Linux. The original design of the pendulum can be found in [15]. The DC-motor current, resulting in a torque, is provided by the PWM amplifier operated under current control. The reference signal is given by a voltage from the D/A converter output of the I/O board. The position of the lever is measured by an optic pulse incremental encoder and processed by a quadrature encoder on the I/O board. The control loop including the controller, the input output transformation and the forward and backward time delay are implemented in MATLAB/SIMULINK blocksets. Standalone realtime code is generated directly from the SIMULINK model. All the experiments are performed with a sampling time interval of $T_A = 1$ms.

1) Experimental Results: The experimental results are presented in Fig. 7. The proposed approach outperforms even the delay-dependent lead-lag controller in the nominal time delay. Not only the smallest steady state error appears, the performance is furthermore not sensitive to the time delay value. The small gain controller seems to be inappropriate due to the very high steady state error. The steady state error with the delay-dependent lead-lag controller is high as well and increasing with increasing time delay. The observed steady state error in all cases, is due to the fact that only a first order approximation of the 1DOF actuated pendulum is used, as well as the fact that there are unmodelled nonlinearities like friction and backlash.

VII. Conclusions

A input-output transformation approach is considered in this paper for delay-independent stability in NCS with unknown constant time delay. Instead of the initial input and output the transformed signals are sent through the network. In contrast to the small gain theorem an integrator in the open loop transfer function does not prohibit the fulfillment of the stability criteria. Thus, delay-independent stability and steady state error are simultaneously guaranteed. In a comparison in simulation and experiment, the input-output transformation approach outperforms the small gain approach and a Lead-Lag controller as far as performance and sensitivity with respect to time delay is concerned. Clearly the input-output transformation approach holds great promise for NCS.

Acknowledgments

The authors are indebted to Martin Buss for his helpful advice and support.

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