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Event-based Scheduling of Multi-loop Stochastic Systems over Shared Communication Channels*

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Abstract— **This paper introduces a novel dynamic event-based scheduling mechanism for networked control systems (NCSs) composed of multiple linear heterogeneous stochastic plants whose feedback loops are closed over a shared constrained communication channel. Each subsystem competes for the channel access in order to update its own controller with true local state values. Employing an emulation-based control policy, a probabilistic scheduler allocates the communication resource according to a prioritized error-based (PEB) measure. Based on this policy, a higher chance of transmission is assigned to the subsystems with higher errors, while the other requests are blocked when the channel capacity is reached. Under some mild assumptions, the probabilistic nature of PEB scheduling scheme facilitates an approximative decentralized implementation. We evaluate the stochastic stability of the overall NCS scheduled by PEB policy in terms of networked-induced error ergodicity, by applying the drift criterion over a multi time-step horizon. Moreover, we derive uniform performance bounds for the networked-induced error variance, which demonstrates a significant reduction in comparison with static and random access scheduling schemes such as TDMA and CSMA.**

I. INTRODUCTION

The design of event-based policies to efficiently utilize the available resources such as communication and energy, is a momentous and still widely open theme in the context of networked control systems (NCSs). The scheduling policies decide how to allocate the resource in efficient fashion, but also guarantee stability and preserve control performance in networked control systems. The design of scheduling policies has seen several paradigm shifts to meet the real-time control objectives, see e.g. [1]–[12]. Event-triggered scheduling schemes are shown to outperform the time-triggered ones within the prioritizing protocols in terms of the overall performance improvement, especially when large-scale systems are of interest [7]–[12]. Try-Once-Discard (TOD), introduced in [1], is one of the basic event-based protocols which uses the current measurement with the largest discrepancy between its true and estimated values for transmission and discards the blocked data. The Maximal Allowable Transfer Interval (MATI) considers the stability of NCS with a deterministic scheduling scheme by deriving an upper bound for the time between two successive transmissions [1]–[3]. While deterministic architectures are often criticized for lack of flexibility and scalability, stochastic scheduling policies offer a more flexible and easily implementable options in particular for large-scale systems. However, MATI in its original formulation is not suited to show the stability of NCSs with stochastic scheduling schemes in general state spaces, as the intervals between transmissions are not uniformly bounded with probability one. The stability of such systems is considered in [4]–[7]. While stability of single-loop NCSs is addressed in [4], [5], stability of event-based multi-loop NCSs is yet to be comprehensively discussed.

In this paper we investigate a state-dependent probabilistic scheduling protocol for NCSs comprised of multiple heterogeneous control loops communicating over a shared channel. In our proposed architecture, the medium access is granted to the transmission requests by assigning a probability of utilizing the resource to each subsystem according to an error-dependent priority measure. As the errors are driven by the Gaussian noise process, transmissions occur randomly in event-based fashion. Unlike purely deterministic policies which require centralized knowledge about all entities, the probabilistic nature of our scheduler facilitates an approximative decentralized implementation under some mild assumptions. Each control loop is modeled as a linear time-invariant discrete-time system with the control law designed by an emulation-based approach, i.e. the controller is stabilizing in the absence of the communication network. Exploiting a drift criterion over a multi time-step interval, we show that the proposed scheduler yields stochastic stability of multi-loop NCSs in terms of Markov chain ergodicity. To investigate the performance efficiency of our approach and compare it with time-triggered and non-state-dependent randomized policies, a quadratic cost function is introduced. Subsequently, analytic upper-bounds, independent of initial values, are computed for an average cost function. Simulations show an increased performance in the mean squared networked-induced estimation error.

The remainder of the paper is organized as follows: The problem statement is described in Section II and is followed by preliminaries of stochastic stability. Section III proceeds with the stability analysis. Performance bounds are discussed in section IV, and follows by numerical results in Section V. **Notation.** Euclidean norm, and conditional expectation are denoted by $\|\cdot\|_2$, and $\mathsf{E}[\cdot|\cdot]$, respectively. $\mathcal{N}(\mu, X)$ denotes a Gaussian distribution with mean μ and covariance matrix X. If not otherwise stated, a state variable with superscript i indicates that it belongs to subsystem i. For matrices though, subscript i indicates the belonging subsystem and superscript n denotes the matrix power.

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II. PROBLEM STATEMENT AND PRELIMINARIES

We consider an NCS composed of N heterogeneous linear control loops which are coupled through a shared communication network, as schematically depicted in Fig. 1. Each individual loop consists of a linear stochastic plant $\mathcal{P}_i$, an emulation-based controller $\mathcal{C}_i$, and a sensor $\mathcal{S}_i$. An event-based scheduler decides when a state vector $x_k^i \in \mathbb{R}^{n_i}$ is an event to be scheduled for channel utilization. The plant $\mathcal{P}_i$ is modeled by the following stochastic difference equation

$$x_{k+1}^i = A_i x_k^i + B_i u_k^i + w_k^i, \quad (1)$$

where $w_k^i \in \mathbb{R}^{n_i}$ is i.i.d. with $w_k^i \sim \mathcal{N}(0, W_i)$ at each time-step k, and $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m_i}$ describe system and input matrices, respectively. Moreover, $u_k^i \in \mathbb{R}^{m_i}$ is the local control input. The initial states x_0^i are allowed to have an arbitrary distribution with bounded moments. At each time-step, the scheduler decision is presented by the binary variable $\delta_k^i \in \{0,1\}$ for every subsystem as

$$\delta_k^i = \begin{cases} 1 & \text{transmission request accepted} \\ 0 & \text{transmission request dismissed} \end{cases}$$

This implies the received signal z_k at the controller by

$$z_k^i = \begin{cases} x_k^i & \delta_k^i = 1 \\ \varnothing & \delta_k^i = 0 \end{cases}.$$

Each local system is controlled by a state feedback controller which is updated at every time step k either by the true state values x_k^i (in case $\delta_k^i = 1$) or by the estimated states $\hat{x}_k^i$ (in case $\delta_k^i = 0$). The control law γ^i is described by causal mappings of the past observations for each time step k, i.e.

$$u_k^i = \gamma_k^i(Z_k^i) = -L_i \, \mathsf{E}\left[x_k^i | Z_k^i\right] \quad (2)$$

where $Z_k^i = \{z_0^i, \ldots, z_k^i\}$ is the observation history, and L_i is the feedback gain which is assumed to be stabilizing for the ideal case when no communication channel is present. In case a transmission request is blocked the controllers are updated by the least-square estimate of the states, i.e.

$$\mathsf{E}\left[x_k^i | Z_k^i\right] = (A_i - B_i L_i) \mathsf{E}\left[x_{k-1}^i | Z_{k-1}^i\right], \quad (3)$$

with the initial distribution satisfying $\mathsf{E}\left[x_0^i | Z_0^i\right] = 0$. The estimate in (3) is well-behaved since a stabilizing L_i ensures that the closed-loop matrix $(A_i - B_i L_i)$ is Hurwitz. The network-induced error state $e_k^i \in \mathbb{R}^{n_i}$ is defined as the estimation error $e_k^i = x_k^i - \mathsf{E}\left[x_k^i | Z_{k-1}^i\right]$ and evolves as

$$e_{k+1}^i = \left(1 - \delta_k^i\right) A_i e_k^i + w_k^i. \quad (4)$$

The stability of e_k implies the overall system's stability, since the augmented state $\left[x_k^i, e_k^i\right]$ has a triangular dynamics, according to (1)-(4), implying the evolution of error state e_k^i is independent of the system state x_k^i.

The prioritized error-based scheduling policy proposed in this paper defines the probability of channel access for each subsystem at each time-step k according to the following error-dependent probabilistic measure:

$$\mathsf{P}\left[\delta_k^i = 1 \middle| e_k^j, j \in \{1, \ldots, N\}\right] = \frac{\|e_k^i\|_2^p}{\sum_{j=1}^N \|e_k^j\|_2^p} \quad (5)$$

where, $p \geq 2$ is an integer, and the probability distribution has semi-infinite support $[0, \infty)$. According to (5), the subsystems with higher error have higher access probabilities. The channel capacity constraint however, implies that only a fraction of subsystems can transmit. Therefore, the channel will be allocated probabilistically by a biased random process until the capacity of the channel is reached. Consequently, the remaining transmission requests which have not granted the channel access are blocked. It is worth noting that the random process is biased according to the assigned error-dependent probabilities. Without loss of generality and for the sake of simplicity, we consider the following constraint with probability 1 for every $k \geq 0$

$$\sum_{i=1}^N \delta_k^i = 1. \quad (6)$$

The capacity constraint can be easily generalized such that $\sum_{i=1}^N \delta_k^i = c < N$, where $c > 1$.

In order to measure the performance of our proposed scheduler, we define a quadratic cost function at each time step k for all the subsystems $j \in \{1, \ldots, N\}$ as

$$J_{e_k} = \sum_{j=1}^N \|e_k^j\|_2^2. \quad (7)$$

The PEB policy can be approximately implemented in decentralized fashion within the framework of the idealized CSMA protocol [17], under some mild assumptions. It is envisioned that the carriers are sensed instantaneously, and there are no hidden nodes. These two imply that no collision occurs. Moreover, every subsystem randomly determines its priority according to a probability distribution depending on its own error. Assuming that the backoff time is negligible with respect to the sampling interval and additionally data packets are discarded after one re-transmission trial, at the beginning of every sampling instance, each subsystem that requests for a channel access waits according to its chosen backoff interval. The duration is chosen randomly according to a distribution related to the current error norm of the individual subsystem. The subsystem with the smallest interval is awarded the channel access, while all the other subsystems are blocked. Furthermore, the mean backoff interval decreases accordingly with increasing error norm. This yields to prioritization of the control loops, as subsystems with larger error norms are more likely to transmit.

The aggregate error state $e_k \in \mathbb{R}^n$ which is defined by augmenting the individual state vectors

$$e_k = [e_k^{1\mathsf{T}}, \ldots, e_k^{N\mathsf{T}}]^\mathsf{T}, \quad (8)$$

is a homogeneous Markov chain, since the scheduling policy (5) is a randomized Markov policy depending on the most recent values of e_k. Moreover, we assume Gaussian additive noise w_k^i with $W_i > 0$, which implies that the transition kernel $P(e, \cdot)$ at any state e of the Markov chain e_k has a positive density function. Then, the error Markov chain (8) is ψ-irreducible and aperiodic.

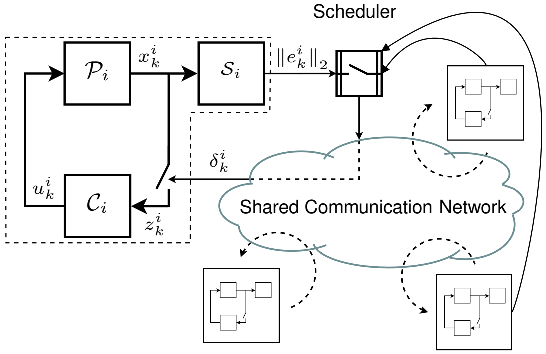

Fig. 1. An NCS with a shared communication channel.

A. Preliminaries

The stability concept used in this paper is given by f-ergodicity which is comprehensively discussed in [18], Chapter 14. We need to show that the Markov chain defined in (8) is f-ergodic rather than ergodic, since it evolves in an uncountable state space $\mathbb{R}^n$, with arbitrary initial condition. Therefore, the existence of the total variation norm cannot be ensured for infinite transition steps, e.g. in case of having coercive chains. However, by showing a Markov chain is f-ergodic, we can show the $\lim_{k\to\infty}\|P^k(e_0,\cdot)-\pi\|_f$ is independent of initial condition, where π is an invariant probability measure, $f\geq 1$, and the f-norm is defined as $\|\nu\|_f=\sup_{|g|\leq f}|\nu(g)|$. There exist several equivalent ways of showing f-ergodicity; we employ the following:

Definition 1: Let $f\geq 1$ be a real-valued function in $\mathbb{R}^n$. A Markov chain e_k is said to be *f-ergodic*, if $\mathsf{E}\left[\pi(f)\right]$ is finite, where $\pi(f)=\int f(e)\pi(de)$.
The next definition provides a notion for the Markov chain gradient with respect to a real-valued function of states.

Definition 2 (Drift for Markov chains): Let V be a real-valued function in $\mathbb{R}^n$. The drift operator Δ is defined for any non-negative measurable function V as

$$\Delta V(e_k)=\mathsf{E}\left[V(e_{k+1})\middle|e_k\right]-V(e_k),\qquad e_k\in\mathbb{R}^n. \quad (9)$$

The *f-Norm Ergodic Theorem* summarizes the f-ergodicity of Markov chains in general state spaces.

Theorem 1 ([18], Ch. 14): Suppose that the Markov chain e_k is ψ-irreducible and aperiodic and let $f(e)\geq 1$ be a real-valued function in $\mathbb{R}^n$. If a small set $\mathcal{D}$ and a non-negative real-valued function V exist such that $\Delta V(e)\leq -f(e)$ for all $e\in\mathbb{R}^n\backslash\mathcal{D}$ and $\Delta V<\infty$ for all $e\in\mathcal{D}$, then the Markov chain e_k is f-ergodic.

Remark 1: The f-norm ergodic theorem considers the ergodicity of Markov chains by exploiting drift criteria over one time-step. However, for complex practical scenarios in general state spaces it is often difficult to find the appropriate Lyapunov function to show that the one-step drift tends towards a small set, see e.g. [19]. As ergodicity is an asymptotic property of Markov chains, we instead employ a N-step drift criteria to show the ergodicity of the Markov chain over an interval with length N, [20]. The N-step drift is defined as

$$\Delta^N V(e_k)=\mathsf{E}\left[V(e_{k+N})\middle|e_k\right]-V(e_k). \quad (10)$$

III. STABILITY ANALYSIS

In this section we show the stochastic stability of NCSs with the PEB scheduling protocol in terms of f-ergodicity of the networked-induced error over an N-step interval. To employ the drift criterion, we introduce the Lyapunov candidate

$$V(e_k)=\sum_{i=1}^{N}\|e_k^i\|_2^p.$$

The N^{th}-order drift operator introduced in (10), requires the calculation of the error expectation in the last time-step $k+N$. To take into account the prior transmissions of a system, e_{k+N}^i is expressed as function of a former error value at a certain time-step $k+r'$, and the scheduling variables, as follows

$$e_{k+N}^i=\prod_{j=r'}^{N-1}\left(1-\delta_{k+j}^i\right)A_i^{N-r'}e_{k+r'}^i + \sum_{r=r'}^{N-1}\left[\prod_{j=r+1}^{N-1}\left(1-\delta_{k+j}^i\right)A_i^{N-r-1}w_{k+r}^i\right] \quad (11)$$

where, $r'\in[0,N-1]$, and we define $\prod_N^{N-1}(1-\delta_{k+j}^i)=1$ for $r'=N-1$. Exploiting the *Multinomial Theorem* for a subsystem i which has never transmitted within the past $N-1$ time-steps, we have

$$\begin{aligned}
&\mathsf{E}\left[V\left(e_{k+N}^i\right)\middle|e_k\right]=\mathsf{E}\left[\|e_{k+N}^i\|_2^p\middle|e_k\right]\\
&\leq\mathsf{E}\left[\|A_i^{N-r'}e_{k+r'}^i+\sum_{r=r'}^{N-1}A_i^{N-r-1}w_{k+r}^i\|_2^p\right]\\
&\leq\mathsf{E}\left[\|A_i^{N-r'}e_{k+r'}^i\|_2^p\right]+\mathsf{E}\left[\|\sum_{r=r'}^{N-1}A_i^{N-r-1}w_{k+r}^i\|_2^p\right] \quad (12)\\
&+\sum_{k_1,k_2<p}^{k_1+k_2=p}\frac{p!}{k_1!k_2!}\mathsf{E}\left[\|A_i^{N-r'}e_{k+r'}^i\|_2^{k_1}\|\sum_{r=r'}^{N-1}A_i^{N-r-1}w_{k+r}^i\|_2^{k_2}\right].
\end{aligned}$$

Statistical independence of $e_{k+r'}^i$ and w_{k+r}^i for $r\in[r',N-1$ yields

$$\begin{aligned}
&\mathsf{E}\left[\|A_i^{N-r'}e_{k+r'}^i\|_2^{k_1}\|\sum_{r=r'}^{N-1}A_i^{N-r-1}w_{k+r}^i\|_2^{k_2}\middle|e_k\right]\leq\\
&\|A_i^{N-r'}\|_2^{k_1}\,\mathsf{E}\left[\|e_{k+r'}^i\|_2^{k_1}\middle|e_k\right]\mathsf{E}\left[\|\sum_{r=r'}^{N-1}A_i^{N-r-1}w_{k+r}^i\|_2^{k_2}\right]
\end{aligned}$$

where, all terms in right side of the inequality are bounded except $\mathsf{E}\left[\|e_{k+r'}^i\|_2^{k_1}\middle|e_k\right]$. However, this term will remain automatically bounded, as far as $\mathsf{E}\left[\|A_i^{N-r'}e_{k+r'}^i\|_2^p\right]$ remains bounded, since $k_1<p$. Hence, we can rewrite (12) as

$$\begin{aligned}
&\mathsf{E}\left[V(e_{k+N})\middle|e_k\right] \quad (13)\\
&\leq\ \sum_{i=1}^{N}\mathsf{E}\left[\|\prod_{j=r'}^{N-1}\left(1-\delta_{k+j}^i\right)A_i^{N-r'}e_{k+r'}^i\|_2^p\middle|e_k\right]+c_i^+
\end{aligned}$$

where, c_i^+ stands for the last two bounded terms in (12). For further derivations, we classify the subsystems into two complementary sets: $\mathcal{S}_1$ contains $\bar{m}$ subsystems which have transmitted at least once in the $N-1$ time-step window, i.e. $\delta_{k+j}^{i\in\mathcal{S}_1}=1$ at least for one $j\in\{0,1,\dots,N-1\}$, and $\mathcal{S}_2$ contains $m=N-\bar{m}$ subsystems which have not transmitted at all, i.e. $\delta_{k+j}^{i\in\mathcal{S}_2}=0$ for all $j\in\{0,1,\dots,N-1\}$. Having these preliminaries, we state the stochastic stability of a multi-loop NCS triggered by the PEB scheduling policy through the following theorem.

Theorem 2: Let a multi-loop NCS consists of N heterogeneous stochastic control loops, with the stochastic plants modeled as (1), sharing a transmission channel subject to the constraint in (6), with the stabilizing control γ^i given as (2), and the channel access being scheduled by the PEB policy (5). Then, the Markov chain (8) is f-ergodic.

Proof: We investigate the drift term within three complementary and mutually exclusive cases to cover all the possible allocations of the shared communication channel, according to the PEB scheduling policy. We introduce the local parameters $M_i > 0$ as the error thresholds for each subsystem $i\in\{1,\dots,N\}$. It is worth noting that the choice of M_i's does not influence the scheduling process. As we will discuss later in section IV, they maintain a trade-off between conservativeness of the performance bounds for different cases. For now we consider the M_i's being chosen arbitrary. The cases are defined as

- l_1 Subsystem i has transmitted at least once during the most recent $N-1$ steps, i.e. $i\in\mathcal{S}_1$.
- l_2 Subsystem i has never transmitted during the most recent $N-1$ steps, i.e. $i\in\mathcal{S}_2$, and satisfies $\|e_{k+N-1}^i\|_2^p\le M_i$.
- l_3 Subsystem i has never transmitted during the most recent $N-1$ steps, i.e. $i\in\mathcal{S}_2$, and satisfies $\|e_{k+N-1}^i\|_2^p > M_i$.

Each of the aforementioned cases occurs with a probability $\mathsf{P}_{l_c}\in[0,1]$, and $\sum_{c=1}^{3}\mathsf{P}_{l_c}=1$. To satisfy (10), the following inequality must hold

$$\sum_{c=1}^{3}\mathsf{P}_{l_c}\,\mathsf{E}\left[V\left(e_{k+N}\right)\big|e_k,l_c\right]-V\left(e_k\right)\le -f(e). \tag{14}$$

From now on, we conveniently use scripts i and j for the subsystems in $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. For the first case, it is straightforward to show that $V(e_{k+N})$ is bounded if $i\in\mathcal{S}_1$, according to (13). Assuming that $\delta_{k+r'}^i=1$ is the latest transmission at time $k+r'\le k+N-1$, we have from (13)

$$\mathsf{E}\left[V\left(e_{k+N}\right)\big|e_k,l_1\right]\le\sum_{i\in\mathcal{S}_1}c_i^+, \tag{15}$$

which is dependent on only the noise values.

As for the second case l_2, we rewrite the drift in (13) by setting $r'=N-1$ as

$$\begin{aligned}
&\mathsf{E}\left[V\left(e_{k+N}\right)\big|e_k,l_2\right] \qquad\qquad (16)\\
&=\ \mathsf{E}\left[V\left(e_{k+N}\right)\big|\|e_{k+N-1}^j\|_2^p\le M_j, j\in\mathcal{S}_2\right]\\
&\le\ \sum_{j\in\mathcal{S}_2}\left[\mathsf{E}\,\|\left[1-\delta_{k+N-1}^j\right]A_je_{k+N-1}^j\|_2^p\big|e_k\right]+c_j^+.
\end{aligned}$$

To avoid the probability in (5) being undefined due to simultaneous zero error values, we introduce the following cases:

$$d_j=\begin{cases}1 & \|e_{k+N-1}^j\|_2^p\le\varepsilon_j<M_j\\ 2 & \|e_{k+N-1}^j\|_2^p>\varepsilon_j\end{cases}, \tag{17}$$

with the probability of occurrence $\mathsf{P}\left(d_1\right)=\alpha_j$ and $\mathsf{P}\left(d_2\right)=1-\alpha_j$. The law of *iterated expectation* incurs

$$\begin{aligned}
&\mathsf{E}\left[\|\left[1-\delta_{k+N-1}^j\right]A_je_{k+N-1}^j\|_2^p\big|e_k\right]\\
&=\mathsf{E}\left[\mathsf{E}\left[\|\left[1-\delta_{k+N-1}^j\right]A_je_{k+N-1}^j\|_2^p\big|e_k,d\right]\big|e_k\right]\\
&=\mathsf{P}\left(d_1\right).\,\mathsf{E}\left[\|\left[1-\delta_{k+N-1}^j\right]A_je_{k+N-1}^j\|_2^p\big|e_k,d_1\right]\\
&+\mathsf{P}\left(d_2\right).\,\mathsf{E}\left[\|\left[1-\delta_{k+N-1}^j\right]A_je_{k+N-1}^j\|_2^p\big|e_k,d_2\right]\\
&\le\varepsilon_j\alpha_j\|A_j\|_2^p+\left(1-\alpha_j\right)\|A_j\|_2^p\,\mathsf{E}\left[\|e_{k+N-1}^j\|_2^p\big|e_k\right]\\
&\le\varepsilon_j\alpha_j\|A_j\|_2^p+\left(1-\alpha_j\right)\|A_j\|_2^pM_j
\end{aligned}$$

Since M_j's are finite, we have the initial-value-independent upper bound for the drift in (16) as

$$\begin{aligned}
\mathsf{E}\big[V(e_{k+N})\big|e_k,l_2\big]&\le\sum_{j\in\mathcal{S}_2}\|A_j\|_2^p\left[\varepsilon_j\alpha_j+(1-\alpha_j)M_j\right]+c_j^+\\
&\le\sum_{j\in\mathcal{S}_2}\|A_j\|_2^pM_j+c_j^+. \qquad (18)
\end{aligned}$$

The third case considers subsystems in the set $\mathcal{S}_2$ with potentially unbounded errors at the time-step $k+N-1$. Knowing $\delta_{k'}^j=0$ for all $k'\in[k,\dots,k+N-1]$, the drift in (13) can be rewritten setting $r'=0$ as

$$\begin{aligned}
&\mathsf{E}\left[V\left(e_{k+N}\right)\big|e_k,l_3\right]\le\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\left[1-\delta_{k+N-1}^j\right]\|A_j^Ne_k^j\|_2^p\big|e_k\right]\\
&+\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\left[1-\delta_{k+N-1}^j\right]\|\sum_{h=0}^{N-1}A_j^{N-h-1}w_{k+h}^j\|_2^p\big|e_k\right]+c_j^+\\
&\le\sum_{j\in\mathcal{S}_2}\|A_j^Ne_k^j\|_2^p+\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|\sum_{h=0}^{N-1}A_j^{N-h-1}w_{k+h}^j\|_2^p\right]+c_j^+\\
&\le\sum_{j\in\mathcal{S}_2}\|A_j^N\|_2^p\|e_k^j\|_2^p+\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|\sum_{h=0}^{N-1}A_j^{N-h-1}w_{k+h}^j\|_2^p\right]+c_j^+\\
&\le\sum_{j\in\mathcal{S}_2}\|A_j^N\|_2^pV\left(e_k\right)+\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|\sum_{h=0}^{N-1}A_j^{N-h-1}w_{k+h}^j\|_2^p\right]+c_j^+
\end{aligned}$$

where, the last two terms of the latter inequality are positive and bounded for finite N. The last inequality however includes the potentially unbounded term $V\left(e_k\right)$, which would violate the f-ergodicity conditions of *Theorem 1*. To avoid this, the probability of the third case happening can be made arbitrarily close to zero by choosing appropriate error thresholds M_j, as follows If one system, say j, does not transmit during the entire interval, then there exists another subsystem, say i, which transmits more than once. Let $k+\bar{r}$ denote the most recent step in which system i transmitted. Now, the probability that system i transmits at the last step of the interval, i.e., $\delta^i_{k+N}=1$, is computed according to

$$\begin{aligned}
&\mathsf{P}\left[\delta^i_{k+N}=1\middle|\delta^i_{k+\bar{r}}=1,\|e^{j\in l_3}_{\bar{k}}\|^p_2>M_j\right]\\
&=\mathsf{E}\left[\mathsf{P}\left[\delta^i_{k+N}=1\middle|e^j_k\right]\middle|\delta^i_{k+\bar{r}}=1,\|e^{j\in l_3}_{\bar{k}}\|^p_2>M_j\right]\\
&\le\mathsf{E}\left[\frac{\|e^i_{k+N-1}\|^p_2}{\sum_{j\in\mathcal{S}_2}\|e^j_{k+N-1}\|^p_2}\middle|\delta^i_{k+\bar{r}}=1,\|e^{j\in l_3}_{\bar{k}}\|^p_2>M_j\right]\\
&\le\mathsf{E}\left[\frac{\|\sum_{r=\bar{r}}^{N-2}A_i^{N-r-1}w^i_{k+r}\|^p_2}{\sum_{j\in l_2}\|e^j_{k+N-1}\|^p_2+\sum_{j\in l_3}\|e^j_{k+N-1}\|^p_2}\middle|z_{i,j}\right]\\
&\le\mathsf{E}\left[\frac{\|\sum_{r=\bar{r}}^{N-2}A_i^{N-r-1}w^i_{k+r}\|^p_2}{\sum_{j\in l_3}\|e^j_{k+N-1}\|^p_2}\middle|z_{i,j}\right]\\
&\le\frac{\sum_{r=\bar{r}}^{N-2}\mathsf{E}\left[\|A_i^{N-r-1}w^i_{k+r}\|^p_2\right]}{\sum_{j\in l_3}M_j}=\mathrm{P}_{l_3},
\end{aligned}\tag{19}$$

where $z_{i,j}$ abbreviates the conditions of the expectation. From (19) one infers that the probability of a subsequent transmission for a certain subsystem, in the presence of large errors in subsystems and without prior transmissions, can be arbitrarily close to zero by selecting the appropriate M_j's.

The Nth order drift operator from (14) is then reduced to

$$\begin{aligned}
\Delta^N V(e_k)&=\sum_{c=1}^{3}\mathrm{P}_{l_c}\,\mathsf{E}\left[V(e_{k+N})\middle|e_k,l_c\right]-V(e_k)\\
&\le\sum_{i\in\mathcal{S}_1}c^+_i+\sum_{j\in\mathcal{S}_2}\|A_j\|^p_2\left[\varepsilon_j\alpha_j+(1-\alpha_j)M_j\right]+c^+_j\\
&+\mathrm{P}_{l_3}\left[\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|\sum_{h=0}^{N-1}A_j^{N-h-1}w^j_{k+h}\|^p_2\middle|e_k\right]+c^+_j\right]\\
&+\left[\mathrm{P}_{l_3}\sum_{j\in\mathcal{S}_2}\|A_j^N\|^p_2-1\right]V(e_k).
\end{aligned}\tag{20}$$

As it can be seen, all the terms in right hand side of the inequality (20) are bounded except the last one, which is error dependent. Therefore, the first two bounded positive terms do not endanger the drift to be negative. For the last term though, we substitute P_{l_3} according to (19) to the above inequality which yields

$$\Delta^N V(e_k)\le c^+_{l_c}+\left[\frac{\sum_{r=\bar{r}}^{N-2}\|A_i^{N-r-1}\|^p_2\,\mathsf{E}\left[\|w^i_{k+r}\|^p_2\right]}{\sum_{j\in\mathcal{S}_2}M_j}\sum_{j\in\mathcal{S}_2}\|A_j^N\|^p_2-1\right]V(e_k)$$

where, $c^+_{l_c}$ represents the bounded terms in (20). Define $f(e)=\bar{\varepsilon}_f V(e_k)-c^+_{l_c}$, $\bar{\varepsilon}_f>0$. Then, choosing M_j and $\bar{\varepsilon}_f$ such that $\left[\mathrm{P}_{l_3}\sum_{j\in\mathcal{S}_2}\|A_j^N\|^p_2-1\right]\le-\bar{\varepsilon}_f$ implies $\Delta^N V(e_k)\le-f(e)$. We can find an appropriate $\bar{\varepsilon}_f$ and a compact set $\mathcal{D}$ such that $f\ge1$, and f-ergodicity of e_k is then followed according to *Theorem 1*. p moment boundedness of the Markov chain e_k follows from the finite stationary distribution $\pi(f)$, see *Def. 1*. Clearly, if $e\in\mathcal{D}$, then $\Delta^N V(e_k)<\infty$, since $c^+_{l_c}$ and N are finite. Thus, (14) is fulfilled, and the proof is then complete according to the f-Norm Ergodic Theorem. ■

Remark 2: The probability of happening the first and second cases can also be computed considering the PEB scheduling policy (5). However, as the derived upper bounds (15) and (18) are independent of initial values, we dropped the derivations of P_{l_1} and P_{l_2} and instead considered them as unity. This just leads to more conservative upper-bounds.

IV. PERFORMANCE BOUNDS

For different network scheduling schemes, the performance efficiency is analyzed often in terms of minimizing some desired cost functions. The notion of periodic event-triggered control (PETC) is employed in [13] to obtain suboptimal bounds on a quadratic cost function, within a controller-scheduler co-design structure. In [14], [15], the performance bound is obtained for linear stochastic systems with quadratic value functions. Geometric bounds are derived for the stochastic infinite but countable Markov chains in [16]. This section presents performance bounds on the average cost, under an emulation-based control policy. Employing the quadratic cost function (7), we find uniform upper bound for the associated average cost function, assuming that the scheduler operates according to the PEB policy (5), and the stabilizing controllers are given as (2). First we state the following lemma as the main tool to obtain the upper bound.

Lemma 1 ([21]): Suppose that e_k represents a Markov chain with general state space $\mathcal{E}$. Introduce $J_{e_k}:\mathcal{E}\to\mathbb{R}$, and $h:\mathcal{E}\to\mathbb{R}$. Define the average per-period cost as

$$J_{\text{ave}}=\lim_{t\to\infty}\sup\frac{1}{t}\sum_{k=0}^{t-1}\mathsf{E}\left[J_{e_k}\right]$$

If $h(e_k)\ge0$ for all $e_k\in\mathcal{E}$, then

$$J_{\text{ave}}\le\sup_{e_k\in\mathcal{E}}\left\{J_{e_k}+\mathsf{E}\left[h(e_{k+1})\middle|e_k\right]-h(e_k)\right\}.$$

Remark 3: Lemma 1, provides the performance bound in one step transition of the Markov chain, while we evaluate the error variance over the N-step transitive interval. However, one can still use of the aforementioned lemma, if the Markov chain is ψ-irreducible and evolves in an uncountable state space. Then, we can construct a new Markov chain which contains the states of the original chain at $\{0,N,2N,...\}$ time-steps. Therefore, ψ-irreducibility and uncountability of the Markov chain is preserved for finite N. Discussions in *Remark* 3 leads us to re-express the upper bound in *Lemma 1* as follows

$$J^{\text{PEB}}_{\text{ave}}\le\sup_{e_k\in\mathcal{E}}\left\{J_{e_k}+\mathsf{E}\left[h(e_{k+N})\middle|e_k\right]-h(e_k)\right\}.\tag{21}$$

We introduce the quadratic function $h(e_k) = \sum_{j=1}^{N} e_k^{j^\mathsf{T}} e_k^j = \sum_{j=1}^{N} \|e_k^j\|_2^2$. Therefore, the bound in (21) is reduced to

$$J_{\text{ave}}^{\text{PEB}} \leq \sup_{e_k \in \mathcal{E}} \mathsf{E}\left[h(e_{k+N}) \,\middle|\, e_k\right]. \tag{22}$$

We evaluate $\mathsf{E}\left[h(e_{k+N}) \,\middle|\, e_k\right]$ by dividing the operational space of scheduler's policy into three cases, as already categorized in the proof of *Theorem 2*. As we will see in the following, it is straightforward to compute the uniform bounds for the first two cases. For the third case though, at which the errors are unbounded, the closed-form bounds are difficult to obtain. We will further obtain the uniform bound for the worst case possible of error variance. As the worst case situation, we consider the error to be increasing from one step to the next, i.e. $\|e_{k'+1}^j\|_2^2 \geq \|e_{k'}^j\|_2^2$ for $k' \in [k, \ldots, k+N-1]$.

For the first case, at which the subsystems are contained in the set $\mathcal{S}_1$, transmission will occur at least once with probability one. Adjust $r' = 0$ in (11), and knowing that $\delta_{k+r'}^i = 1$ we have

$$\begin{aligned} &\mathsf{E}\left[h(e_{k+N}) \,\middle|\, e_k, i \in \mathcal{S}_1\right] \\ &\leq \sum_{i \in \mathcal{S}_1} \sum_{r=0}^{N-1} \mathsf{E}\left[\|A_i^{N-r-1} w_{k+r}^i\|_2^2\right] \\ &\leq \sum_{i \in \mathcal{S}_1} \operatorname{tr}(C_i) \sum_{r=0}^{N-1} \|A_i^{N-r-1}\|_2^2 \end{aligned} \tag{23}$$

where, C_i is the covariance matrix of the noise signal w^i. For the case l_2 at which the subsystems are in $\mathcal{S}_2$, we use the upper bound obtained from the stability analysis as

$$J_{\text{ave}}^{\text{PEB}} \leq \sum_{j \in \mathcal{S}_2} \|A_j\|_2^2 M_j + \sum_{j \in \mathcal{S}_2} \operatorname{tr}(C_j). \tag{24}$$

We propose the performance bound for the third case l_3 within the following theorem, for the worst case possible scenario of the error variance:

Theorem 3: Let a multi-loop NCS consists of N heterogeneous stochastic control loops sharing a transmission channel subject to the constraint (6), with the stabilizing control γ^i given as (2), and the channel access being scheduled by the PEB policy (5). Suppose that the total imposed cost per time-step is defined by (7). Assuming the third case, there exists some subsystems with no transmission over $[k, k+N-1]$ steps, satisfying $\|e_{k+N-1}^j\|_2^2 > M_j$. Then the average cost function (22) is uniformly upper bounded.

Proof: See Appendix. ■

Remark 4: It can be seen that the introduced error thresholds M_i's maintain a trade-off between the conservativeness of the performance bounds (24) and (25). It is concluded that increasing the error thresholds leads to more conservative upper bound for the case l_2 while it has a reverse effect on the performance bound for the third case l_3. On the other hand, decreasing M_i's leads to a less conservative bound for case l_2, while increases the conservativeness of the upper bound in (25). The least conservative performance bounds for the average error variance will be obtained by choosing appropriate thresholds such that a trade-off is maintained between the upper bounds.

V. Numerical Results

We derive the upper bounds for the error variance in a networked system comprised of two classes of subsystems - a stable class and an unstable class of process - with system parameters $A_1 = 1.25$, $B_1 = 1$ and $A_2 = 0.75$, $B_2 = 1$, respectively. The either heterogeneous classes includes finite number of homogeneous subsystems. In both classes, the state initiates with $x_0^1 = x_0^2 = 0$ and the noise is modeled by $w_k^i \sim \mathcal{N}(0, 1)$. We assume a stabilizing deadbeat control law with $L_i = A_i$ for $i \in \{1, 2\}$ and a model-based observer in case of no data transmission.

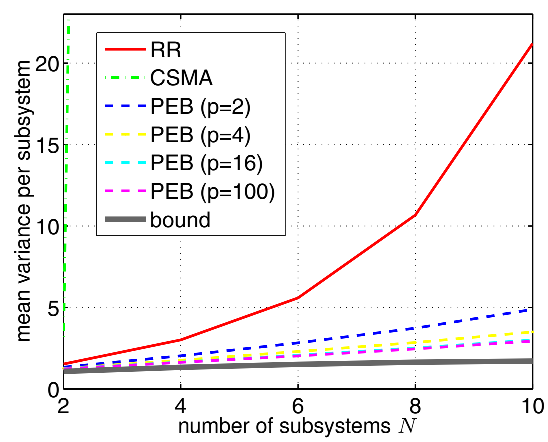

Fig. 2. Mean steady-state variance of e_k^i- Comparison for various schemes and different number of subsystems.

Fig. 2 compares the performance of the proposed PEB protocol for different p powers with other scheduling protocols for $N \in \{2, 4, 6, 8, 10\}$ in terms of the estimation error e_k^i induced by the network. The means are calculated by their empirical means through Monte Carlo simulations over a horizon of $100\,000$ samples. The lower bound is determined by relaxing the initial problem to have no resource constraint. The round robin protocol is a periodic access scheme with a sampling period of N. The CSMA protocol operates in the same fashion as the PEB protocol without prioritizing subsystems, i.e. the probability of updating the controller is $\frac{1}{N}$ for each subsystem at each time. With an increasing number of subsystems sharing the resource, the performance gap between the PEB scheduler and the other protocols becomes more evident. At the same time, the PEB scheduler deviates moderately from the lower bound, which grows slowly with increasing N. This suggests that the PEB protocol is more profitable than the round robin protocol when the resource is scarce. Moreover, by increasing the power of p, the performance of the scheduler improves, as the subsystems with higher errors get more chance to utilize the channel. In case $p \to \infty$, the scheduler behaves as the deterministic scheduler. As the simulations show, the PEB scheduler is scalable with respect to the increasing number of subsystems, compared to the other policies.

The uniform bounds (24)-(25) of the section IV are calculated for the similar networked system as one the Monte Carlo simulations are derived for, and the results are shown in the Fig. 3. It can be seen that M_j affects the performance bounds (24) and (25) in opposite direction, such that bigger M_j's result in more conservative upper bound for (24) and less conservative for (25). Thus, the error thresholds are chosen in a way to maintain a trade-off between the cases. As it can be seen in the Fig. 3, the error thresholds are chosen where the error bounds for the cases $l_1 + l_2$ and l_3 coincides. It is clear that for bigger M_j's the error variance increases due to the increase in the upper bound (24), while choosing smaller thresholds leads to increase the error variance because the upper bound (25) increases. The subplot in the down right of the Fig. 3 shows the bounds for a networked system composed of 2,4, and 6 subsystems with 1,2, and 3 unstable subsystems, respectively. Comparing with the results extracted from the Monte Carlo simulation in Fig. 2, the uniform bound seems to be conservative for higher number of subsystems. However, this result is expected, because the performance bounds in Fig. 2, which are obtained by the Monte Carlo simulations, are not necessarily obtained for the worst case possible situation, and therefore illustrate better performance. Fig. 3 also shows that the described networked control system in this paper remains bounded when it is scheduled by the PEB policy (5), since the worst case possible is bounded independent of the initial values.

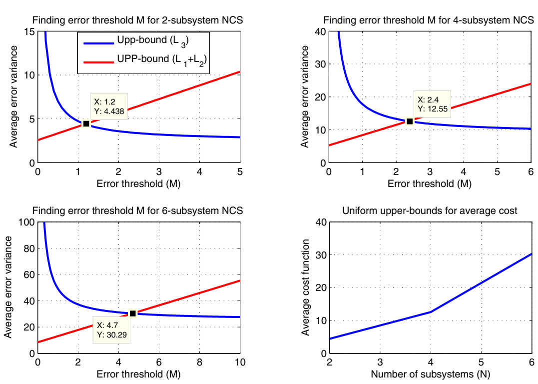

Fig. 3. Choosing threshold M for the cases $l_1 + l_2$- l_3, and the corresponding upper bounds for the mean error variance.

VI. Conclusion

This paper investigates the stability and performance of resource-constrained networked control systems by introducing Prioritized Error-Based (PEB) scheduling scheme. This policy assigns priorities dynamically for NCS which contain a finite number of stochastic linear subsystems coupled through a shared communication resource. Provided with stabilizing feedback controllers, we show the stability of the overall networked system under the employment of the PEB policy using the drift criteria for general power of 2-norm functions. Stochastic stability is shown in terms of Markov chain ergodicity and p-moment boundedness for p-power f-ergodic functions. A comprehensive performance analysis is done to find the upper bounds on the variance of the error, under employment of PEB resource allocation strategy. We derive the performance bound for the worst case possible error variance and show its boundedness employing PEB scheduling policy. Numerical simulations show the stability alongside a major performance improvement in comparison with the other protocols, e.g. Round Robin, and CSMA, especially when the number of subsystems increases.

VII. Appendix

Proof: [Theorem 3]: We find the upper bound for the error variance in N time-step ahead, for the worst case possible, when the third case (l_3) happens. We have $\delta^j_{k'} = 0$ for all $j \in \mathcal{S}_2$ and for all time-steps $[k, \ldots, k+N-1]$, then

$$\begin{aligned}
&\mathsf{E}\left[h\left(e_{k+N}\right)|e_k, l_3\right] = \mathsf{E}\left[\sum_{j\in l_3} \|e^j_{k+N}\|_2^2 \Big| e_k\right] \\
&\leq \sum_{j\in\mathcal{S}_2} \mathsf{E}\left[\|A_j^N e^j_k + A_j^{N-1} w^j_k + \ldots + w^j_{k+N-1}\|_2^2 | e_k\right] \\
&\leq \sum_{j\in\mathcal{S}_2} \|A_j^N\|_2^2 \|e^j_k\|_2^2 + \mathsf{E}\left[\|w^j_{k+N-1}\|_2^2\right] \\
&+ \sum_{j\in\mathcal{S}_2} \mathsf{E}\left[\|A_j^{N-1} w^j_k + \ldots + A_j w^j_{k+N-2}\|_2^2\right] \\
&\leq \sum_{j\in\mathcal{S}_2} \|A_j^N\|_2^2 \sum_{j\in\mathcal{S}_2} \|e^j_k\|_2^2 + \mathsf{E}\left[\|w^j_{k+N-1}\|_2^2\right] \\
&+ \sum_{j\in\mathcal{S}_2} \|A_j\|_2^2 \, \mathsf{E}\left[\|A_j^{N-2} w^j_k + \ldots + w^j_{k+N-2}\|_2^2\right]
\end{aligned}$$

The worst case possible for the subsystems $j \in \mathcal{S}_2$ happens when the subsystems are unstable, i.e. the system matrix A has at least one eigenvalue outside the unit circle, and $\|e^j_{k'+1}\|_2^2 \geq \|e^j_{k'}\|_2^2$ for $k' \in [k, \ldots, k+N-2]$. Therefore, we consider from now on that all the unstable subsystems belong to set $\mathcal{S}_2$. Now we calculate the probability of happening the third case in the worst case situation. We modify the upper bound as

$$\begin{aligned}
&\mathsf{P}\left[\delta^i_{k+N-1} = 1 \big| \|e^j_{k+N-1}\|_2^2 > \|e^j_k\|_2^2, i \in \mathcal{S}_1\right] \\
&= \mathsf{E}\left[\frac{\|e^i_{k+N-1}\|_2^2}{\sum_{j=1}^N \|e^j_{k+N-1}\|_2^2} \Big| \|e^j_{k+N-1}\|_2^2 > \|e^j_k\|_2^2, i \in \mathcal{S}_1\right] \\
&\leq \mathsf{E}\left[\frac{\|e^i_{k+N-1}\|_2^2}{\sum_{j\in\mathcal{S}_2} \|e^j_k\|_2^2} \Big| e_k, i \in \mathcal{S}_1\right] \\
&\leq \frac{\sum_{r=0}^{N-2} \|A_i^{N-r-2}\|_2^2 \, \mathsf{E}\left[\|w^i_{k+r}\|_2^2\right]}{\sum_{j\in\mathcal{S}_2} \|e^j_k\|_2^2}
\end{aligned}$$

where, the first inequality is ensured considering that $\|e^j_{k+N-1}\|_2^2 \geq \|e^j_k\|_2^2$. Applying the latter upper bound on

$\mathsf{E}\left[h\left(e_{k+N}\right)\middle|e_k, l_3\right]$ yields

$$\begin{aligned}
&\mathsf{P}_{c_{l_3}}\,\mathsf{E}\left[h\left(e_{k+N}\right)\middle|e_k, l_3\right] \\
&\leq \frac{\sum_{r=0}^{N-2}\|A_i^{N-r-2}\|_2^2\,\mathsf{E}\left[\|w_{k+r}^i\|_2^2\right]}{\sum_{j\in\mathcal{S}_2}\|e_k^j\|_2^2}\sum_{j\in\mathcal{S}_2}\|A_j^N\|_2^2\sum_{j\in\mathcal{S}_2}\|e_k^j\|_2^2 \\
&+ \frac{\sum_{r=0}^{N-2}\|A_i^{N-r-2}\|_2^2\,\mathsf{E}\left[\|w_{k+r}^i\|_2^2\right]}{\sum_{j\in\mathcal{S}_2}M_j}\xi_{j\in\mathcal{S}_2}^+ \\
&= \sum_{r=0}^{N-2}\|A_i^{N-r-2}\|_2^2\,\mathsf{E}\left[\|w_{k+r}^i\|_2^2\right]\sum_{j\in\mathcal{S}_2}\|A_j^N\|_2^2 \\
&+ \frac{\sum_{r=0}^{N-2}\|A_i^{N-r-2}\|_2^2\,\mathsf{E}\left[\|w_{k+r}^i\|_2^2\right]}{\sum_{j\in\mathcal{S}_2}M_j}\xi_{j\in\mathcal{S}_2}^+
\end{aligned} \tag{25}$$

where the term $\xi_{j\in\mathcal{S}_2}^+$ stands for $\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|w_{k+N-1}^j\|_2^2\right] + \sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|A_j^{N-1}w_k^j + \ldots + A_j w_{k+N-2}^j\|_2^2\right]$. The bound is then independent of e_k^j and therefore the worst case error variance of the third case, is uniformly upper bounded. The last step is to calculate the noise-dependent term $\xi_{j\in\mathcal{S}_2}^+$. It should be noted that unlike the other noise-dependent distributions, the distribution of the vector $W\left(A_j, w^j\right) = A_j^{N-1}w_k^j + \ldots + w_{k+N-1}^j$ is conditioned on the segment of the distribution which enlarges the noise-dependent term $A_j^{N-1}e_k^j$, either in positive or negative directions, to satisfy $\|e_{k'+1}^j\|_2^2 \geq \|e_{k'}^j\|_2^2$. In other words, depending on the sign of the elements of the vector-valued term $A_j^{N-1}e_k^j$, either positive or negative parts of the distribution $W\left(A_j, w^j\right)$ should be considered. Since the norm of the error will be measured as the average cost, the worst case would occur if the noise-dependent and error-dependent terms have similar signs, element wise, either positive or negative. Due to the symmetry of the noise-dependent distribution $W\left(A_j, w^j\right)$, both positive and negative parts of the distribution turn out to have similar values. Assume that they are positive, then we need to calculate the positive part of the distribution of $W\left(A_j, w^j\right)$. Since, the noise variables are independent, then $W\left(A_j, w^j\right)$ has a zero-mean multi-dimensional Gaussian distribution with covariance matrix $\Sigma = \left(A_j^{N-2} + \ldots + A_j + I\right)C_j$. The corresponding probability density function (pdf) is defined as

$$f\left(w\right) = \frac{1}{\sqrt{(2\pi)^{n_j}|\Sigma|}}\exp\left(\frac{-w^{\mathsf{T}}\Sigma^{-1}w}{2}\right) \tag{26}$$

where w is the n_j-dimensional noise-dependent random vector, and $|\Sigma|$ is the determinant of the covariance matrix. To calculate $\xi_{j\in s_2}^+$, we essentially need to calculate $\mathsf{E}\left[\|w\left(A_j, w^j\right)\|_2^2\middle|W \geq 0\right]$. According to the law of *unconscious statistician*

$$\begin{aligned}
&\mathsf{E}\left[\|W\left(A_j, w^j\right)\|_2^2\middle|W \geq 0\right] \\
&= \frac{1}{\sqrt{(2\pi)^{n_j}|\Sigma|}}\int_0^\infty\ldots\int_0^\infty\|w\|_2^2\exp\left(\frac{-w^T\Sigma^{-1}w}{2}\right)dw
\end{aligned}$$

which the integral can be effectively calculated. It worth to mention that the other noise-dependent term $\sum_{j\in\mathcal{S}_2}\mathsf{E}\left[\|w_{k+N-1}^j\|_2^2\right]$ is not conditionally distributed, so it can be trivially calculated. ∎

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