Minimizing the Energy per Bit for Pilot-Assisted Data Transmission over Quantized Channels

Qing Bai, Ulrich Mittmann, Amine Mezghani and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München, Munich, Germany
Email: bai@tum.de

***Abstract*—To communicate over a priori unknown channels, pilot sequences can be exploited to assist the receiver in obtaining the channel state information. The analog-to-digital converter (ADC) at the front end of the receiver samples and quantizes the input signal, including both pilot and data symbols. This results in a reduction of the receive signal-to-noise ratio (SNR) as well as deteriorated quality of channel estimation, which depends quantitatively on the bit resolution used by the ADC. In this work, we consider the point-to-point, training based communication between a single-antenna transmitter and a multi-antenna receiver over a Rayleigh block fading channel, and take into account the impact of the ADC for a joint optimization of the training length, the average receive SNR, the number of receive antennas, and the bit resolution of the ADC. Goal of the optimization is to minimize the energy per bit metric, where we include both transmit power and power dissipation of the ADC into the energy consumption model, and employ a capacity lower bound which depends on all aforementioned design parameters. Results from numerical simulations are demonstrated and analyzed, leading to a number of insightful observations and conclusions which are important for the energy efficient operation of the system.**

I. Introduction

For wireless communications, the degree of channel state information available at the transmitter and/or the receiver is a key assumption in the study of channel capacity. The ideal case of having perfect channel knowledge has been thoroughly studied, *e.g.* by [1] for flat-fading channels. In practice, the system dedicates a certain amount of time and energy for the transmission of pilot symbols which are known by the receiver. Channel estimation as well as other signal processing tasks can then be performed which provides the receiver with an imperfect channel estimate [2]. How much resources in terms of time and power should be allocated for training has been investigated in a number of works. In [3], the authors consider the training and channel estimation for a multiple-input multiple-output (MIMO) system, and maximize a lower bound on the capacity with respect to the training parameters. In [4], an energy efficiency perspective is taken and training schemes which minimize the energy cost per transmitted information bit are proposed. Due to the difficulty in obtaining exact capacity formulas, both papers treat the product of the channel estimation error and the data symbols as Gaussian noise, which gives a lower bound on capacity since Gaussian noise minimizes the mutual information.

In digital communication systems, the received analog signal is sampled and quantized into discrete-time, discrete-valued signals via the A/D conversion. The precision with which the receiver is able to access the received signal has a direct impact on the channel capacity as well as the power dissipation of the receiver. It has been reported [5] that the ADC consumes a significant amount of power when operating at high sampling rate and resolution, hence becoming a bottleneck in system performance. As a result, there have been intensive research activities in recent years on communications with low-precision A/D conversion at the receiver *e.g.* [6][7], which establish the relation between capacity degradation and the applied ADC resolution. In our previous work [8], we propose to maximize the energy efficiency of a multi-antenna receiver with respect to the vector of ADC resolutions, assuming perfect channel state information. In this paper, we further investigate the energy efficiency, but of a training based system and address the impact of the ADC on channel estimation. To this end, we evaluate the energy per bit metric of the system by deriving a capacity lower bound of the quantized channel and by setting up an energy consumption model which accounts both the transmitter and the receiver. The minimization of energy per bit with respect to the training length, the average receive SNR, the ADC resolution, and the number of receive antennas is of both theoretical and practical importance, which remains yet unexplored to our best knowledge in the literature.

The rest of the paper is organized as follows. In Section II, we set up the system model for the training based transmission scheme and discuss in particular the modeling of the quantization operation. Then we derive a capacity lower bound of the quantized channel. The minimization problem of the energy per bit metric is formulated in Section III. Numerical results are demonstrated and analyzed in Section IV, while a summary and conclusions are given in Section V. Throughout the paper, we use boldfaced letters to represent vectors and matrices. The operators $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ stand for the transpose and Hermitian of a matrix, respectively. The symbol $\mathbf{1}_M$ denotes the identity matrix of dimension $M\times M$, and $\mathrm{diag}(\boldsymbol{A})$ denotes the diagonal matrix with the same diagonal elements as matrix $\boldsymbol{A}$.

II. System Model

We start with considering a single-input single-output (SISO) system in a block fading channel, *i.e.*, the channel coefficient stays constant within one block and changes independently to a new value for the next block. The length of each block is assumed to be small enough compared to the channel coherence time. During each block, the transmitter sends first a number of pilot symbols, and then the information carrying data symbols. At the receiver side, channel estimation is performed upon the reception of all pilot symbols based on the a priori knowledge of the pilots, and the estimated channel coefficient is then used in the detection of the subsequent data symbols. Note that in real digital systems, estimation error is unavoidable not only because of the additive noise introduced at the receiver RF front end, but also because of the limited precision we have in obtaining the received pilots.

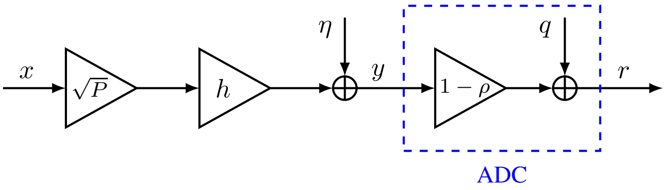


Figure 1. SISO system model with quantization at the receiver

A. Transmit signal and the wireless channel

A block diagram of the SISO system is plotted in Figure 1. Let m be the total number of symbols in one block, and l be the number of pilot symbols. For the training based transmission scheme, we restrict $m \geq 2$ and $1 \leq l \leq m-1$. The transmitted symbols within one block are given by $\sqrt{P}\boldsymbol{x}$, where P is the transmit power which is assumed constant, *i.e.*, pilot symbols and data symbols are radiated with the same power. The vector $\boldsymbol{x} \in \mathbb{C}^m$ contains unit variance symbols and can be partitioned into a training vector $\boldsymbol{x}_\mathrm{t} \in \mathbb{C}^l$ and a data vector $\boldsymbol{x}_\mathrm{d} \in \mathbb{C}^{m-l}$

$$\boldsymbol{x} = [x_1 \; x_2 \; \cdots \; x_l \; x_{l+1} \; \cdots \; x_m]^\mathrm{T} = \begin{bmatrix} \boldsymbol{x}_\mathrm{t} \\ \boldsymbol{x}_\mathrm{d} \end{bmatrix},$$

$$|x_i|^2 = 1, \; \mathrm{E}\left[|x_j|^2\right] = 1, \quad i = 1, \ldots, l, \; j = l+1, \ldots, m.$$

The corresponding unquantized received symbols are denoted with a vector $\boldsymbol{y} \in \mathbb{C}^m$ and expressed as $\boldsymbol{y} = \sqrt{P} \cdot h\boldsymbol{x} + \boldsymbol{\eta}$, where $h \sim \mathcal{CN}(0, \alpha)$ is the random channel coefficient and $\boldsymbol{\eta} \sim \mathcal{CN}(\boldsymbol{0}, \beta\boldsymbol{1}_m)$ is the noise vector. The average receive signal-to-noise ratio (SNR) before quantization is given by

$$\gamma = \frac{\alpha P}{\beta}. \tag{1}$$

We have used γ instead of P later as an optimization variable for convenience and generality, and the optimal transmit power P^* is immediately determined according to this linear mapping once the optimal average receive SNR γ^* is obtained.

B. Modeling the quantization operation

The A/D converter at the receiver samples and quantizes the received signal with a finite number of bits. We let $b \in \{1, \ldots, b_\mathrm{max}\}$ be the bit resolution of the ADC, where b_max is the maximal number of bits that the ADC can use to represent one sample. In general, the quantization operation is nonlinear, and the introduced quantization error is correlated with the input signal. By using the Bussgang theorem [9][10], we can linearize the operation as shown in Figure 1 to produce

$$\boldsymbol{r} = (1-\rho)\boldsymbol{y} + \boldsymbol{q} = \sqrt{P}(1-\rho)h\boldsymbol{x} + (1-\rho)\boldsymbol{\eta} + \boldsymbol{q},$$

where ρ is the distortion factor which depends on the ADC resolution b, and $\boldsymbol{q}$ is a noise vector which is uncorrelated with $\boldsymbol{y}$. The covariance matrix of $\boldsymbol{q}$, which depends on b via the distortion factor ρ, is given approximately by [8]

$$\boldsymbol{R}_{\boldsymbol{qq}} = \rho(1-\rho)\,\mathrm{diag}(\boldsymbol{R}_{\boldsymbol{yy}}). \tag{2}$$

With Gaussian input and optimal (in the sense of minimum distortion which requires the step size of the quantizer to scale linearly with the square root of the input power) scalar non-uniform quantizer, the distortion factor ρ attains the values given in Table I [11], where for $b > 5$ the asymptotic approximation $\rho = \frac{\pi\sqrt{3}}{2} \cdot 2^{-2b}$ can be used [12].

Table I. DISTORTION FACTOR ρ FOR DIFFERENT ADC RESOLUTIONS b

b	1	2	3	4
ρ	0.3634	0.1175	0.03454	0.009497
b	5	6	7	8
ρ	0.002499	0.0006642	0.0001660	0.00004151

C. Channel estimation

The receiver employs the linear minimum mean square error (MMSE) estimator $\hat{h} = \boldsymbol{g}_\mathrm{MMSE}^\mathrm{H}\boldsymbol{r}_\mathrm{t}$ for channel estimation, where the quantized received pilot symbols $\boldsymbol{r}_\mathrm{t}$ are given by

$$\boldsymbol{r}_\mathrm{t} = \sqrt{P}(1-\rho)h\boldsymbol{x}_\mathrm{t} + (1-\rho)\boldsymbol{\eta}_\mathrm{t} + \boldsymbol{q}_\mathrm{t},$$

and the vector of filter coefficients $\boldsymbol{g}_\mathrm{MMSE}$ is computed as

$$\boldsymbol{g}_\mathrm{MMSE} = \underset{\boldsymbol{g}}{\mathrm{argmin}}\,\mathrm{E}\left[|h - \hat{h}|^2\right] = \boldsymbol{R}_{\boldsymbol{r}_\mathrm{t}\boldsymbol{r}_\mathrm{t}}^{-1}\boldsymbol{p}_{\boldsymbol{r}_\mathrm{t}h},$$

$$\begin{aligned} \text{where } \boldsymbol{R}_{\boldsymbol{r}_\mathrm{t}\boldsymbol{r}_\mathrm{t}} &= \mathrm{E}\left[\boldsymbol{r}_\mathrm{t}\boldsymbol{r}_\mathrm{t}^\mathrm{H}\right] \\ &= \beta(1-\rho)\left(\gamma(1-\rho)\boldsymbol{x}_\mathrm{t}\boldsymbol{x}_\mathrm{t}^\mathrm{H} + (\rho\gamma+1)\boldsymbol{1}_l\right), \\ \boldsymbol{p}_{\boldsymbol{r}_\mathrm{t}h} &= \mathrm{E}\left[\boldsymbol{r}_\mathrm{t}h^*\right] = \alpha\sqrt{P}(1-\rho)\boldsymbol{x}_\mathrm{t}. \end{aligned}$$

Note that in the calculation of $\boldsymbol{R}_{\boldsymbol{r}_\mathrm{t}\boldsymbol{r}_\mathrm{t}}$, the relation (2) and the assumption that the training symbols are all of unit norm are needed. Plugging the results back and applying the matrix inversion lemma, we proceed to have

$$\boldsymbol{g}_\mathrm{MMSE} = \frac{\alpha\sqrt{P}}{\beta\left(1 + \rho\gamma + l\gamma(1-\rho)\right)} \cdot \boldsymbol{x}_\mathrm{t}.$$

The variance of the channel estimate $\hat{h}$ is computed as

$$\sigma_{\hat{h}}^2 = \mathrm{E}\left[|\hat{h}|^2\right] = \boldsymbol{p}_{\boldsymbol{r}_\mathrm{t}h}^\mathrm{H}\boldsymbol{g}_\mathrm{MMSE} = \frac{l\gamma(1-\rho)\alpha}{1 + \rho\gamma + l\gamma(1-\rho)}, \tag{3}$$

and the variance of the estimation error $e = h - \hat{h}$ follows according to the orthogonality principle as

$$\sigma_e^2 = \alpha - \sigma_{\hat{h}}^2 = \frac{\alpha(1+\rho\gamma)}{1 + \rho\gamma + l\gamma(1-\rho)}. \tag{4}$$

D. Capacity lower bound of the quantized channel

The received data symbols before and after quantization can be written respectively as

$$\begin{aligned} \boldsymbol{y}_\mathrm{d} &= \sqrt{P}(\hat{h} + e)\boldsymbol{x}_\mathrm{d} + \boldsymbol{\eta}_\mathrm{d}, \\ \boldsymbol{r}_\mathrm{d} &= \sqrt{P}(1-\rho)\hat{h}\boldsymbol{x}_\mathrm{d} + \sqrt{P}(1-\rho)e\boldsymbol{x}_\mathrm{d} + (1-\rho)\boldsymbol{\eta}_\mathrm{d} + \boldsymbol{q}_\mathrm{d}. \end{aligned}$$

In the quantized received data vector $\boldsymbol{r}_\mathrm{d}$, we consider $\sqrt{P}(1-\rho)\hat{h}\boldsymbol{x}_\mathrm{d}$ as the signal part, and the remaining summation $\boldsymbol{\eta\prime} = \sqrt{P}(1-\rho)e\boldsymbol{x}_\mathrm{d} + (1-\rho)\boldsymbol{\eta}_\mathrm{d} + \boldsymbol{q}_\mathrm{d}$ as additive Gaussian noise. To this end, the *effective instantaneous SNR* is computed as

$$\begin{aligned} \mathit{\Gamma}_\mathrm{SISO} &= \frac{\mathrm{E}\left[|\sqrt{P}(1-\rho)\hat{h}x_{\mathrm{d},i}|^2|\hat{h}\right]}{\mathrm{E}\left[|\sqrt{P}(1-\rho)ex_{\mathrm{d},i}|^2\right] + \mathrm{E}\left[|(1-\rho)\eta_{\mathrm{d},i}|^2\right] + \mathrm{E}\left[|q_{\mathrm{d},i}|^2\,|\hat{h}\right]} \\ &= \frac{P(1-\rho)|\hat{h}|^2}{P(1-\rho)\sigma_e^2 + \beta(1-\rho) + \rho(P|\hat{h}|^2 + P\sigma_e^2 + \beta)} = \frac{(1-\rho)|\hat{h}|^2}{\sigma_e^2 + \frac{\alpha}{\gamma} + \rho|\hat{h}|^2}. \end{aligned}$$

Although the true distribution of $\hat{h}$ can not be determined due to $\boldsymbol{q}_\mathrm{t}$, we approximate it as Gaussian which is asymptotically valid for large l. By using the results of (3)(4) and replacing $\hat{h}$ with an auxiliary random variable w according to

$$\hat{h} = \sigma_{\hat{h}} \cdot w, \quad w \sim \mathcal{CN}(0,1), \tag{5}$$

we further write the effective instantaneous SNR as

$$\mathit{\Gamma}_{\mathrm{SISO}} = \frac{l\gamma^2(1-\rho)^2|w|^2}{(1+\gamma)(1+\rho\gamma)+l\gamma(1-\rho)+l\gamma^2\rho(1-\rho)|w|^2}. \tag{6}$$

Let B be the transmission bandwidth. The ergodic channel capacity in bit/sec achieved during the data transmission phase is lower bounded by

$$C_{\mathrm{L,SISO}} = \mathrm{E}\left[B\log_2\left(1+\mathit{\Gamma}_{\mathrm{SISO}}\right)\right], \tag{7}$$

which is a function of the training length l, the average receive SNR γ (the transmit power P), and the distortion factor ρ (the ADC resolution b), for given m and α. The reason that the formula given in (7) is a capacity lower bound is as follows:

- the channel estimation error e is treated as random noise, while it stays constant during each block and changes independently only from block to block;
- the noise summation term $\boldsymbol{\eta}\prime$ is taken as Gaussian which is not necessarily true, especially since that the additive noise $\boldsymbol{q}$ introduced by quantization is not necessarily Gaussian [10].

Consequently, what we have considered is the worst case scenario which leads to a lower bound on the channel capacity.

E. Extension to SIMO system

Now suppose the receiver is equipped with $M > 1$ antennas and each antenna is connected to an ADC employing the same bit resolution. Assuming that the M spatial subchannels are independent, we obtain the effective instantaneous SNR as

$$\mathit{\Gamma}_{\mathrm{SIMO}} = \sum_{i=1}^{M} \frac{(1-\rho)|\hat{h}_i|^2}{\sigma_e^2 + \frac{\alpha}{\gamma} + \rho|\hat{h}_i|^2} \tag{8}$$

$$= \sum_{i=1}^{M} \frac{l\gamma^2(1-\rho)^2|w_i|^2}{(1+\gamma)(1+\rho\gamma)+l\gamma(1-\rho)+l\gamma^2\rho(1-\rho)|w_i|^2},$$

with $\hat{h}_i$ being the estimate of the channel coefficient h_i for the i-th antenna, and w_i the associated auxiliary random variable which is standard Gaussian distributed. The coefficients for maximal-ratio combining which lead to (8) are given by

$$p_i = \frac{\hat{h}_i}{\sigma_e^2 + \frac{\alpha}{\gamma} + \rho|\hat{h}_i|^2}, \quad i = 1, \ldots, M.$$

The capacity lower bound for the SIMO system follows as

$$C_{\mathrm{L,SIMO}} = \mathrm{E}\left[B\log_2\left(1+\mathit{\Gamma}_{\mathrm{SIMO}}\right)\right].$$

On the other hand, the SIMO receiver consumes more power than a SISO receiver using the same bit resolution in its ADC. We will treat this aspect in the next section where we formulate the minimization problem of the energy per bit metric.

III. Minimization of the Energy per Bit Metric

The energy per bit metric, as given by the energy consumption divided by the number of transmitted/received bits, is a common and important performance measure of communication systems. For theoretical analysis, its determination depends on how the energy consumption is computed, *i.e.*, which components of the transmitter and/or the receiver are considered as the dominating power consuming parts, and how their power dissipations depend on the system parameters. In this work we take into account the transmit power of the transmitter and the power dissipation of the ADC at the receiver for our energy consumption model. Given resolution b, the power consumption of the ADC is given by [13]

$$P_{\mathrm{ADC}} = 2c \cdot N_0 B \cdot 2^b,$$

where c is a constant design parameter, and N_0 is the noise power spectral density. The energy consumption for the transmission and reception of one block is computed as

$$E = mT_{\mathrm{s}}(P + P_{\mathrm{ADC}}) = mT_{\mathrm{s}}\left(\frac{\beta\gamma}{\alpha} + 2c\cdot BN_0 \cdot 2^b\right),$$

with T_{s} denoting the symbol duration and $\beta = BN_0$. For the SISO system, as the number of received information bits is lower bounded by $I = (m-l)T_{\mathrm{s}}C_{\mathrm{L,SISO}}$, an upper bound on the energy per bit E_{b} normalized with N_0 is computed as

$$\frac{E_{\mathrm{b,U,SISO}}}{N_0} = \frac{E}{I\cdot N_0} = \frac{m}{m-l}\cdot\frac{\gamma + 2c\alpha\cdot 2^b}{\alpha\,\mathrm{E}\left[\log_2\left(1+\mathit{\Gamma}_{\mathrm{SISO}}\right)\right]}. \tag{9}$$

For the SIMO system, noting that there are M ADC using the same resolution, we give the upper bound on E_{b} by

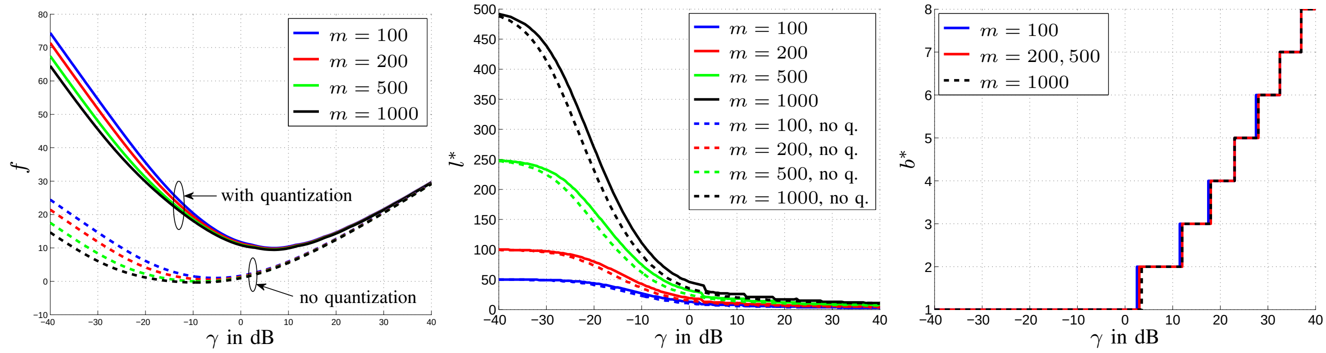
$$\frac{E_{\mathrm{b,U,SIMO}}}{N_0} = \frac{m}{m-l}\cdot\frac{\gamma + 2c\alpha\cdot 2^b M}{\alpha\,\mathrm{E}\left[\log_2\left(1+\mathit{\Gamma}_{\mathrm{SIMO}}\right)\right]}. \tag{10}$$

An optimization w.r.t. l, γ, b, and M can be formulated which minimizes this upper bound scaled with the path gain α:

$$\min_{l,\gamma,b,M} \quad f(l,\gamma,b,M) \triangleq \frac{E_{\mathrm{b,U,SIMO}}}{N_0}\cdot\alpha. \tag{11}$$

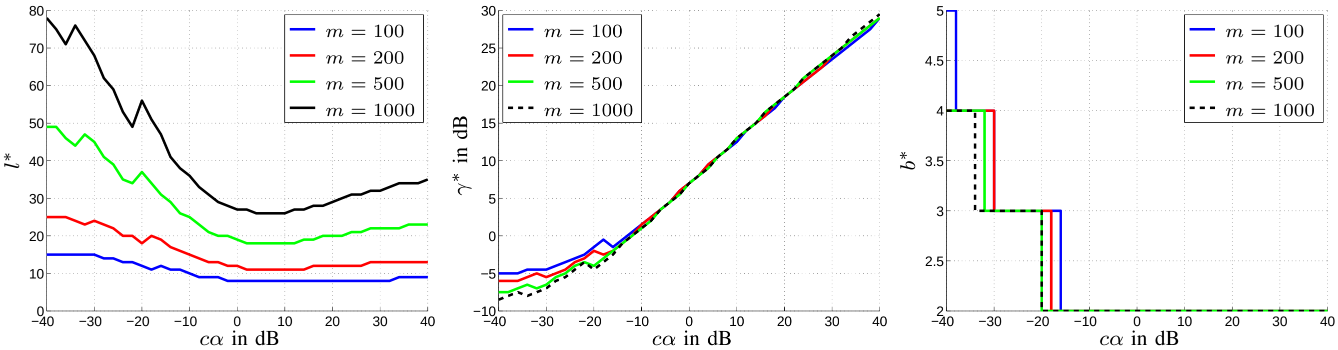
For SISO system we simply have $M = 1$. In the formulation of (10) and (11), notice that we have used the average receive SNR γ instead of the transmit power P, which allows us to cancel out the bandwidth B in our expression. This means, the signal bandwidth does not play any role in the optimization and can be used as a degree of freedom in system design to fulfill certain QoS constraints. For instance, the optimal solution of (11), denoted with $(l^*, \gamma^*, b^*, M^*)$, represents a desirable operation mode from an energy efficiency point of view. By adjusting the bandwidth in an allowable range, the parameters $(l^*, \gamma^*, b^*, M^*)$ can be employed while at the same time, different data rates can be achieved to support heterogeneous applications. Also notice that via the scaling of the upper bound on energy per bit with α in (11), it becomes clear that the optimal solution depends only on the product $c\alpha$. This means, for the numerical simulations we do not need to specify the constant c which renders our results more general with respect to different design and performance of the ADC.

Due to the fact that the optimization variables l and M are integer valued and the difficulty in the evaluation of the derivatives of f, we propose to use exhaustive search to solve (11). The expectation in (10) is computed by averaging the expression over 2×10^4 independent channel realizations.



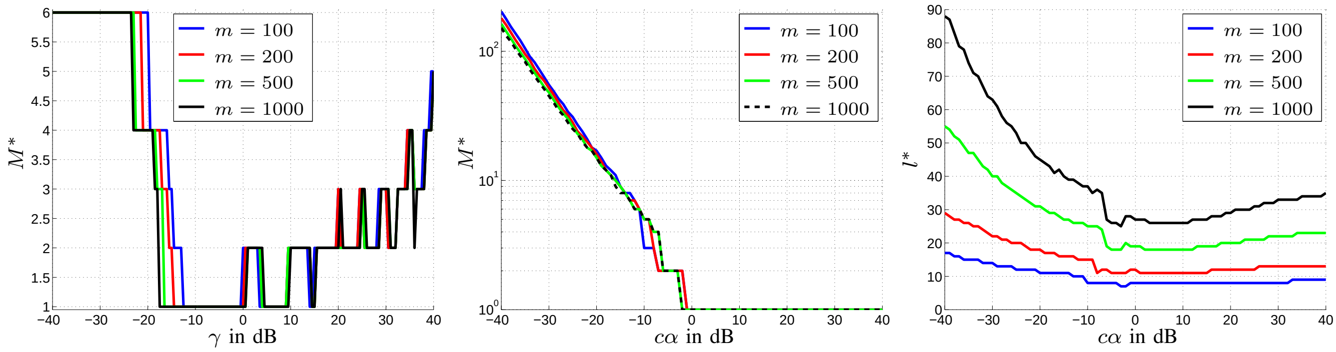
(a) Upper bound on normalized energy per bit (b) Optimal training length (c) Optimal bit resolution of the ADC

Figure 2. Minimizing the energy per bit for SISO systems, $c\alpha = 0$ dB

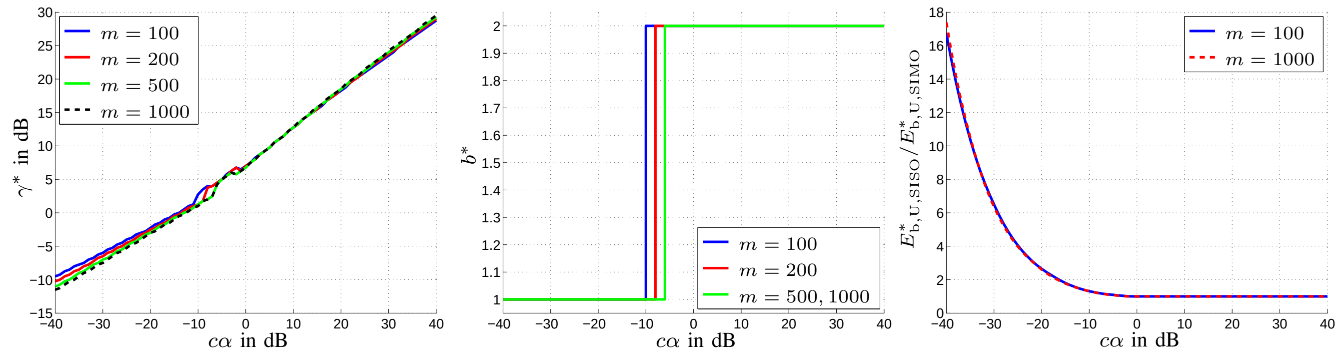


(a) Optimal training length (b) Optimal mean receive SNR (c) Optimal bit resolution of the ADC

Figure 3. Minimizing the energy per bit for SISO systems with varying $c\alpha$ values



(a) Optimal number of antennas, $c\alpha = 0$ dB (b) Optimal number of antennas (c) Optimal training length



(d) Optimal mean receive SNR (e) Optimal bit resolution of the ADC (f) Gain of SIMO system over SISO system

Figure 4. Minimizing the energy per bit for SIMO systems

IV. Numerical Results and Analysis

We first study the performance of SISO systems. In Figure 2, we show the scaled energy per bit, the optimal training length, and the optimal bit resolution as dependent on γ for a fixed $c\alpha$ value of 0 dB. For the curves illustrated in Figure 2(a), the training length and bit resolution are optimized for the quantized case. For the unquantized case, we set $c = 0$ and $\rho = 0$, and optimize only the training length. The existence of a finite γ^* can be observed, while f approaches infinity for both $\gamma \to 0$ and $\gamma \to +\infty$. Quantization leads to an increased f and a rightward shift of γ^*, meaning that more transmit power is required. On the other hand, the change in the optimal training length due to the impact of quantization is not significant, as shown in Figure 2(b), where only a small increment in l^* can be observed. Higher bit resolution can be employed with better γ, while the lowest resolution is favorable for low to medium γ, as illustrated in Figure 2(c). It can also be noticed that the optimal resolution is almost independent of the block length.

In Figure 3 the variations in l^*, γ^*, and b^* with respect to $c\alpha$ are demonstrated. Due to the switch of the optimal bit resolution, l^* and γ^* are not monotonic in $c\alpha$. For a given system with fixed c, the increment of $c\alpha$ corresponds to increasing α, *i.e.*, having better channel conditions. The optimal average receive SNR grows almost linearly in $c\alpha$ with a rather slow rate, leading in fact to a decreasing transmit power function in $c\alpha$. The tendency of using lower bit resolution under better channel conditions as illustrated in Figure 3(c) can be explained by observing the numerator of (9). When $c\alpha$ is extremely low, γ is the dominating part in the summation, and increasing b in this case improves the channel capacity without raising the power too much. On the other hand, with $c\alpha$ being large, $2c\alpha \cdot 2^b$ becomes dominant which prohibits the employment of a high resolution. This is to say, the joint optimization of γ and b literally results in a balancing between the power consumption of the transmitter and the receiver.

Numerical results for the SIMO system are depicted in Figure 4. We first show the variation of the optimal number of antennas M^* with respect to γ with fixed $c\alpha$ in Figure 4(a), and then illustrate the dependency of M^*, l^*, γ^* and b^* on varying $c\alpha$ in Figure 4(b)-4(e). For medium to high $c\alpha$, using one single antenna with 2 bit resolution of the ADC, and a rather short training sequence ($3\% - 10\%$ of the block length) gives the minimal energy per bit value. With low $c\alpha$, having more antennas helps improving the energy efficiency of the system, and the associated optimal ADC resolution is 1 bit. The increment of the optimal number of antennas is steadily fast with a close-to-linear relation with $c\alpha$. Training length in this case is increased compared to the single antenna receiver, and the percentage in block becomes lower when m is large. The optimal average receive SNR is a few dB lower in the extreme low $c\alpha$ regime compared to the SISO system, suggesting some reduction in transmit power. Due to the impact of the factor M in (10), a reversed situation in the optimal bit resolution over $c\alpha$ can be observed in Figure 4(e) for the SIMO case. In Figure 4(f) we compare the performance of SIMO and SISO systems, by showing the ratio of the minimum energy per bit between them. For medium to high $c\alpha$, there is no gain since $M^* = 1$, whereas for low $c\alpha$, the SIMO system achieves the energy per bit value which is only a small fraction of that of the SISO system, by using a rather large antenna array.

V. Conclusion

We consider the pilot-assisted data transmission between a transmitter and a receiver over a block-fading channel, and investigate the impact of quantization via A/D conversion at the receiver. We further formulate a joint optimization problem on the training length, the average receive SNR, the bit resolution of the ADC, and the number of receive antennas with the objective to minimize the energy per bit metric. Numerical results lead us to the following observations and conclusions: i. quantization increases the minimal energy required to transmit one bit and also the optimal average receive SNR, but results in a very small increment in the optimal training length; ii. for a wide range of the path gain, low ADC resolution (1 or 2 bits) yields the optimal performance; iii. while the optimal training length depends on the block length strongly, the optimal average receive SNR and the bit resolution are less influenced by different block lengths; iv. for medium to high path gain, using a single receive antenna is optimal, while for very low path gain the SISO system consumes more than 10 times of energy for one bit compared to the optimized SIMO system, which employs a large number of antennas. The optimization results we obtained indicate the set of design parameters leading to the best energy efficiency of the system, and are quite general in that they are independent of the choice of bandwidth and the A/D converter the receiver employs.

References

[1] A. J. Goldsmith and P. P. Varaiya, *Capacity of fading channels with channel side information*, IEEE Trans. on Information Theory, vol. 43, no. 6, pp. 1986-1992, November 1997.

[2] L. Tong, B. M. Sadler and M. Dong, *Pilot-assisted wireless transmissions: general model, design criteria, and signal processing*, IEEE Signal Processing Magazine, vol. 21, no. 6, pp. 12-25, November 2004.

[3] B. Hassibi and B. M. Hochwald, *How much training is needed in multiple-antenna wireless links?*, IEEE Trans. on Information Theory, vol. 49, no. 4, pp. 951-963, April 2003.

[4] M. C. Gursoy, *On the capacity and energy efficiency of training-based transmissions over fading channels*, IEEE Trans. on Information Theory, vol. 55, no. 10, pp. 4543-4567, October 2005.

[5] B. Murmann, *A/D converter trends: power dissipation, scaling and digitally assisted architectures*, in Proc. IEEE Custom Integrated Circuits Conference, pp. 105-112, September 2008.

[6] A. Mezghani and J. A. Nossek, *Analysis of Rayleigh-fading channels with 1-bit quantized output*, in Proc. IEEE International Symposium on Information Theory, pp. 260-264, July 2008.

[7] J. Singh, O. Dabeer and U. Madhow, *On the limits of communication with low-precision analog-to-digital conversion at the receiver*, IEEE Trans. on Communications, vol. 57, no. 12, pp. 3629-3639, December 2009.

[8] Q. Bai, A. Mezghani and J. A. Nossek, *On the optimization of ADC resolution in multi-antenna systems*, in Proc. 10th International Symposium on Wireless Communication Systems, August 2013.

[9] J. J. Bussgang, *Crosscorrelation functions of amplitude-distorted Gaussian signals*, Technical report no. 216, MIT, Cambridge, MA, March 1952.

[10] A. Mezghani and J. A. Nossek, *Capacity lower bound of MIMO channels with output quantization and correlated noise*, in Proc. IEEE International Symposium on Information Theory, Cambridge, MA, USA, July 2012.

[11] J. Max, *Quantizing for minimum distortion*, IEEE Trans. on Information Theory, vol. 6, no. 1, pp. 7-12, March 1960.

[12] A. Gersho and R. M. Gary, *Vector quantization and signal compression*, Springer, 1992.

[13] H. S. Lee and C. G. Sodini, *Analog-to-digital converters: digitizing the analog world*, Proceedings of the IEEE, vol. 96, no. 2, pp. 323-334, February 2008.