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Modeling Start-Up Times in Unit Commitment by Limiting Temperature Increase and Heating

Matthias Huber and Matthias Silbernagl
Technische Universität München
Munich, Germany
{matthias.huber, silbernagl}@tum.de

***Abstract*—The integration of variable renewable energy sources leads to an increased cycling of conventional power plants, necessitating a detailed model of the start-up process. Based on the recently developed temperature formulation for start-up costs in Unit Commitment, we model the off-time-dependent start-up times of thermal units by limiting temperature increase and heating. Numerical results indicate that limiting heating speed is more efficient and leads only to a moderate increase in computational time.**

***Index Terms*—Unit Commitment, Start-up Times, Power Plant Temperatures, Integration of Renewables**

NOMENCLATURE

Indices and Sets

$t \in \mathcal{T}$	Time periods, $\mathcal{T} = [1 \mathinner{..} T]$
$i \in \mathcal{I}$	Generating units
$l \in \mathbb{N}$	Cooling time

Parameters

L^t	Electricity demand [MW]
$P_i^{\max}$	Maximum power output [MW]
$P_i^{\min}$	Minimum power output [MW]
B_i	Variable production cost [cost/MWperiod]
A_i	Fixed cost while online [cost/period]
λ_i	Heat-loss coefficient, $\lambda_i \in (0,1)$ [1/period]
V_i	Variable start-up cost [cost]
$H_i^{\max}$	Maximum heating
$\Delta\text{temp}_i^{\max}$	Maximum temperature increase
F_i	Fixed start-up cost [cost]
PD_i	Offline periods prior to first period [period]

Variables

v_i^t	State of power plant, $v_i^t \in \{0,1\}$
p_i^t	Power output [MW]
temp_i^t	Temperature, normalized to $[0,1]$
h_i^t	Heating, normalized to $[0,1]$
z_i^t	Start-up status, $z_i^t \in \{0,1\}$
cp_i^t	Production costs [cost]

I. INTRODUCTION

A. Motivation

The intermittent nature of electricity production from renewable energy sources, mainly wind and solar, is leading to a higher number of start-ups of conventional thermal power plants [1], [2]. As the impact of the start-ups on the production planning rises, an accurate modeling of the start-up process becomes increasingly important for efficient system operation.

An important aspect of the start-up process is the required start-up time, which depends on the preceding offline time. Especially power plants based on steam cycles require a significant time of up to 12 hours for a cold start [3]. Moreover, when planning under uncertainty, operators may choose to keep units at operational temperature during off-time [4], or may be required to abort or vary the heating during the start-up process. Both facts emphasize the importance of modeling power plant temperatures and heating.

In [5], power plant temperatures were included as an additional variable to model start-up costs more efficiently. Based on this approach, we employ the temperature variable to model start-up times depending on the preceding offline time.

B. Literature Review

In a wide variety of Unit Commitment problems (see e.g. [6]), the start-up time is subsumed in the minimal downtime. In [7], an explicit model of an off-time-independent start-up process was introduced. This was extended by [8] to multiple start-up types for different offline times, including synchronization time, soak phase, and power trajectories. In [9], the formulation was further tightened, resulting in better computational performance. Finally, [10] utilizes the shut-down and start-up times to model an off-time dependent start-up time.

Instead of classifying the start-ups into types, [5] models the start-up costs based on the temperature of a unit. In line with this approach, we present a model of the start-up time based on limiting the heating of a unit.

C. Contribution and Paper Organization

After introducing the temperature formulation in Section II, our contributions are introduced as follows.

Section III presents two methods for modeling start-up times: by limiting heating (Section III-A) and by limiting the temperature increase (Section III-B). In both cases, the necessary constraints and the resulting start-up time and start-up costs functions are derived, which are compared in Section III-D.

Section IV considers the computational performance of the introduced models, as well as the effect of the start-up time on system costs.

Finally, we draw conclusions and give an outlook to future research in Section V.

II. The Temperature Formulation

We base our contribution on a Unit Commitment formulation which models the start-up costs by introducing power plant temperatures as in [5], without including the residual temperature inequalities.

A. Base Model

Generally, the problem is to minimize costs while the sum of the productions p_i^t has to meet the demand L^t. The costs are divided into production costs cp_i^t and start-up costs cu_i^t, leading to

$$\min \sum_{i\in\mathcal{I},t\in\mathcal{T}} cp_i^t + cu_i^t, \quad \text{s.t.} \tag{1}$$

$$\sum_{i\in\mathcal{I}} p_i^t = L^t \qquad \forall\, t \in \mathcal{T}. \tag{2}$$

The production costs are modeled as a linear function of the binary operational state v_i^t and the production p_i^t as in [11], with coefficients A_i and B_i. From [6], we further adopt the constraints of thermal power plants regarding the minimal production P_i^{min}, the maximum production P_i^{max}, maximum up and down ramping speeds RU_i and RD_i, as well as maximum ramping at start-up SU_i and shutdown SD_i.

B. The Temperature Model

If started up after an offline time l, a thermal unit incurs a start-up cost of K_i^l, which according to [12], [13] is defined as

$$K_i^l = \underbrace{V_i\,(1-\mathrm{e}^{-\lambda_i l})}_{\text{variable cost}} + \underbrace{F_i}_{\text{fixed cost}} \qquad \forall\, i\in\mathcal{I}, l\in\mathbb{N}. \tag{3}$$

Here, the fixed start-up costs F_i include labor costs as well as time-independent wear and tear costs. The variable costs V_i originate from the reheating process at start-up, where fuel needs to be burned and where the unit experiences thermal stress. Parameters are chosen such that the costs for a complete cold start equal $V_i + F_i$.

The term $(1-\mathrm{e}^{-\lambda_i l})$ in (3) is proportional to the heat loss caused by the exponential decay of the power plant temperature while offline. The temperature formulation in [5] models this loss by explicitly capturing the temperature of a unit as the new state variable temp_i^t and the amount of heating as the new variable h_i^t,

$$\text{temp}_i^t, h_i^{t-1} \in \mathbb{R}_{\geq 0} \qquad \forall\, i\in\mathcal{I}, t\in\mathcal{T}. \tag{4}$$

The operational temperature is expressed as

$$v_i^t \leq \text{temp}_i^t \qquad \forall\, i\in\mathcal{I}, t\in\mathcal{T}, \tag{5}$$

enforcing a temperature of 1 during operation. The development of the temperature is modeled as

$$\text{temp}_i^1 = \mathrm{e}^{-\lambda_i PD_i} + h_i^0 \qquad \forall\, i\in\mathcal{I}, \tag{6}$$

$$\text{temp}_i^t = \mathrm{e}^{-\lambda_i}\text{temp}_i^{t-1} + (1-\mathrm{e}^{-\lambda_i})v_i^{t-1} + h_i^{t-1} \qquad \forall\, i\in\mathcal{I}, t\in[2\,..\,T]. \tag{7}$$

In combination with the start-up status z_i^t (see [14]), the start-up costs in (3) can be modeled as

$$cu_i^t := V_i h_i^{t-1} + F_i z_i^t \qquad \forall\, i\in\mathcal{I}, t\in\mathcal{T}, \tag{8}$$

and substituted in the objective function in (1).

III. Modeling Start-up Times

This chapter presents two approaches to integrate start-up times in the temperature formulation. As noted in Section II, the start-up costs of a unit stem from its need to re-heat at start-up. Moreover, the heating speed is bounded, either due to limited heating capacity or due to the need for preventing strong material tensions induced by temperature gradients [4]. For typical thermal units, this restriction is the main cause of the start-up time.

After considering the temperature development during heating, we present the additional constraints necessary to model

- units with limited heating speed in Section III-A, and
- units with limited temperature increase in Section III-B.

For both types of units, we derive the resulting start-up time and costs, which we compare in Section III-D. The start-up time, which is defined as the number of periods during which the unit heats up, is denoted by $SUT_i(l)$. The sum of the variables h_i^t during start-up is denoted by $TH_i(l)$, such that $V_i \cdot TH_i(l)$ equals the variable start-up costs in (8).

We start by modeling the development of the temperature of a unit in a continuous model. The offline time can be split in two phases: first, the unit cools down and, subsequently, it is reheated before the start-up takes place (c.f. Fig. 1).

During the entire offline time, the unit continuously loses heat at a rate of $\lambda_i\text{temp}_i^t$. As shown in [5], the modeled temperature of unit i equals $\mathrm{e}^{-\lambda_i l}$ after l offline periods. During heating, the temperature is increased by supplying heat at a rate of $\overline{h}_i(t)$. Assuming the heating phase starts at $t=0$, the continuous temperature $\overline{\text{temp}}_i(t)$ while heating with speed $\overline{h}_i(t)$ may be modeled as

$$\begin{aligned} \overline{\text{temp}}_i(0) &= \mathrm{e}^{-\lambda_i l}, \\ \frac{\mathrm{d}\overline{\text{temp}}_i(t)}{\mathrm{d}t} &= -\lambda_i\overline{\text{temp}}_i(t) + \overline{h}_i(t). \end{aligned} \tag{9}$$

While heating, the unit continues to lose further heat. Therefore, units heat as fast as possible in a cost-minimal solution. In [5], unbounded heating is assumed which models the typical start-up costs (3). As noted, the following two sections consider the effect of limiting the heating speed and the temperature increase.

A. Limited Heating

The start-up time of a unit may stem from its limited ability to heat (c.f. Fig. 1). Assuming a heating speed of at most H_i^{max}, the continuous model for cost-minimal heating in (9) can be simplified by substituting $\overline{h}_i(t)$ with H_i^{max}. Considering the initial temperature of $\overline{\text{temp}}_i(0) = \mathrm{e}^{-\lambda_i l}$, its solution is

$$\overline{\text{temp}}_i(t) = \mathrm{e}^{-\lambda_i(l+t)} + \frac{H_i^{\text{max}}}{\lambda_i}(1-\mathrm{e}^{-\lambda_i t}). \tag{10}$$

This function fulfills the recursion

$$\overline{\text{temp}}_i(t+1) = \mathrm{e}^{-\lambda_i}\overline{\text{temp}}_i(t) + \frac{1-\mathrm{e}^{-\lambda_i}}{\lambda_i}H_i^{\text{max}}. \tag{11}$$

Since the temperature development in (7) is modeled as

$$\text{temp}_i^t = \mathrm{e}^{-\lambda_i}\text{temp}_i^{t-1} + (1-\mathrm{e}^{-\lambda_i})v_i^{t-1} + h_i^{t-1},$$

the limit on the heating speed may be expressed as

$$h_i^t \leq \frac{1 - \mathrm{e}^{-\lambda_i}}{\lambda_i} H_i^{\max} \qquad \forall\, i \in \mathcal{I}, t \in [0 \,..\, T{-}1] \tag{12}$$

in our Unit Commitment formulation. During heating, the variables temp_i^t thus discretize the continuous temperature $\overline{\mathrm{temp}}_i(t)$.

In the continuous model, the start-up heating finishes at time t^* with $\overline{\mathrm{temp}}_i(t^*) = 1$. From (10) we can derive

$$t^* = \frac{1}{\lambda_i} \ln \left(1 + \frac{1 - \mathrm{e}^{-\lambda_i l}}{H_i^{\max}/\lambda_i - 1}\right),$$

which results in $\lceil t^* \rceil$ periods of heating in the Unit Commitment formulation, and hence in a start-up time of

$$SUT_i(l) = \left\lceil \frac{1}{\lambda_i} \ln \left(1 + \frac{1 - \mathrm{e}^{-\lambda_i l}}{H_i^{\max}/\lambda_i - 1}\right) \right\rceil. \tag{13}$$

If t^* is integral, and therefore $SUT_i(l) = t^*$, the unit heats at maximal speed in the Unit Commitment problem, and the total heating equates to

$$TH_i(l) = \frac{1 - \mathrm{e}^{-\lambda_i}}{\lambda_i} H_i^{\max}\, SUT_i(l). \tag{14}$$

If t^* is not integral, and therefore $SUT_i(l) > t^*$, the unit needs to heat at sub-maximal speed in the first heat-up period to reach a final temperature of exactly 1. As this is equivalent to keeping the unit warm for the initial part of that period, it results in a slightly higher total heating than $TH_i(l)$. However, the difference is generally small (c.f. Fig. 4).

B. Maximum Temperature Increase

Alternatively, a unit may have a maximally allowed temperature change $\Delta\mathrm{temp}_i^{\max}$ (c.f. Fig. 2),

$$\begin{aligned} \overline{\mathrm{temp}}_i(0) &= \mathrm{e}^{-\lambda_i l} \\ \frac{\mathrm{d}\overline{\mathrm{temp}}_i(t)}{\mathrm{d}t} &= \Delta\mathrm{temp}_i^{\max}, \end{aligned} \tag{15}$$

which is solved by

$$\overline{\mathrm{temp}}_i(t) = \mathrm{e}^{-\lambda_i l} + t\Delta\mathrm{temp}_i^{\max}. \tag{16}$$

This function fulfills the recursion

$$\overline{\mathrm{temp}}_i(t+1) = \overline{\mathrm{temp}}_i(t) + \Delta\mathrm{temp}_i^{\max}, \tag{17}$$

and may be modeled as

$$\mathrm{temp}_i^t \leq \mathrm{temp}_i^{t-1} + \Delta\mathrm{temp}_i^{\max} \qquad \forall\, i \in \mathcal{I}, t \in [2 \,..\, T] \tag{18}$$

in our Unit Commitment formulation.

As the temperature after l offline periods equals $\mathrm{e}^{-\lambda_i l}$, the required start-up time can be derived as

$$SUT_i(l) = \left\lceil \frac{1 - \mathrm{e}^{-\lambda_i l}}{\Delta\mathrm{temp}_i^{\max}} \right\rceil. \tag{19}$$

Assume now that $\Delta\mathrm{temp}_i^{\max}$ divides $1 - \mathrm{e}^{-\lambda_i l}$ evenly, i.e. that the unit heats at maximal speed for the entire start-up time. Then, the required heating in each period equals

$$h_i^t = \mathrm{temp}_i^{t+1} - \mathrm{e}^{-\lambda_i}\mathrm{temp}_i^t = (1 - \mathrm{e}^{-\lambda_i})\mathrm{temp}_i^t + \Delta\mathrm{temp}_i^{\max}.$$

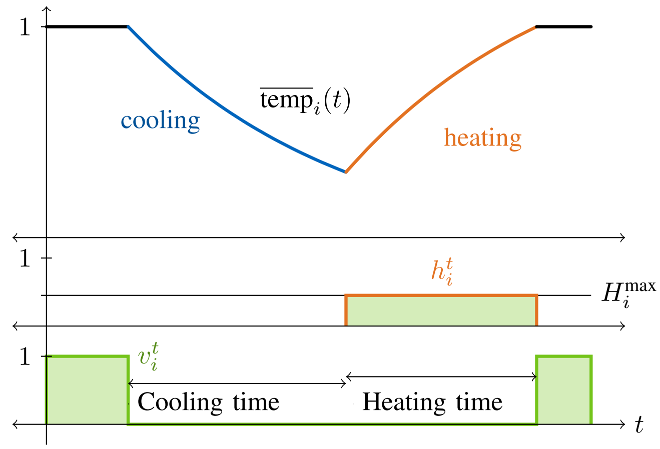

Fig. 1. Cooling and heating during the offline time of a unit with limited heating speed.

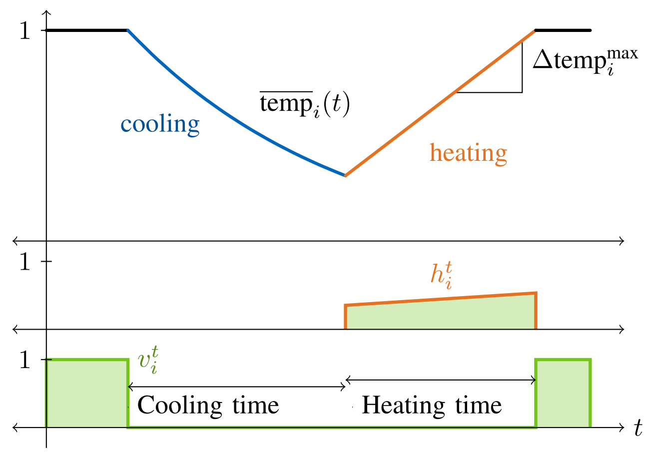

Fig. 2. Cooling and heating during the offline time of a unit with limited temperature increase per period.

Noting that the temperature in the j-th period of heating equals $\mathrm{e}^{-\lambda_i l} + (j-1)\Delta\mathrm{temp}_i^{\max}$, the sum of the heating variables can be derived as

$$TH_i(l) = (1 - \mathrm{e}^{-\lambda_i l})\left(1 + \frac{1 - \mathrm{e}^{-\lambda_i}}{2}\left(\frac{1 + \mathrm{e}^{-\lambda_i l}}{\Delta\mathrm{temp}_i^{\max}} - 1\right)\right). \tag{20}$$

If $\Delta\mathrm{temp}_i^{\max}$ does not divide $1 - \mathrm{e}^{-\lambda_i l}$ evenly, then the effective total heating slightly surpasses $TH_i(l)$; yet $TH_i(l)$ remains an excellent approximation (c.f. Fig. 4).

C. Objective Function

In (11) and (17) the continuous model of the heating process in (9) was discretized using the period length 1. Choosing a different period length $f \in \mathbb{R}_{>0}$ yields the same model, albeit with scaled parameters $\tilde{\lambda}_i = f\lambda_i$, $\tilde{H}_i^{\max} = fH_i^{\max}$ and $\Delta\tilde{\mathrm{temp}}_i^{\max} = f\Delta\mathrm{temp}_i^{\max}$.

Apart from scaling and rounding, the resulting start-up time $\tilde{SUT}_i(l)$ matches the original start-up time $SUT_i(l)$, i.e. $f\tilde{SUT}_i(l/f) \approx SUT_i(l)$. The same however does not hold for the total heating, which varies significantly depending on f.

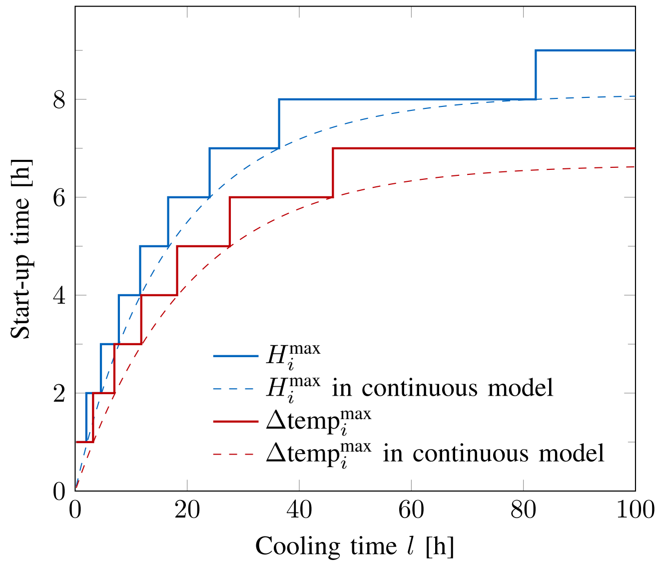

Fig. 3. Start-up times when limiting heating/temperature increase for a unit with parameters $\lambda_i = 0.05$ and $H_i^{\max} = \Delta\text{temp}_i^{\max} = 3\lambda_i$, compared to start-up times in respective continuous model.

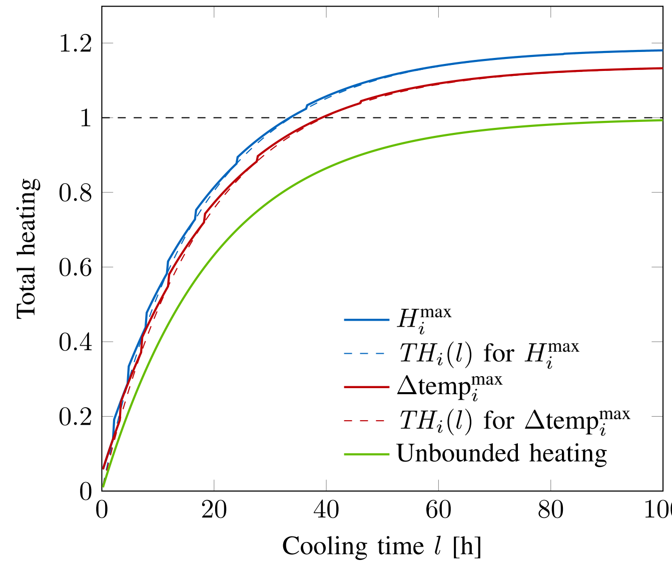

Fig. 4. Total heating when limiting heating/temperature increase for a unit with parameters $\lambda_i = 0.05$ and $H_i^{\max} = \Delta\text{temp}_i^{\max} = 3\lambda_i$, compared to approximations $TH_i(i)$ and the model with unbounded heating.

This discrepancy arises from the modeling of the variable h_i^t, which represents the temperature increase in period t due to heating; In the context of start-up times however, one should instead consider the heating speed $\overline{h}_i(t)$ in (9).

A heat increase h_i^t is equivalent to heating at speed

$$\overline{h}_i(t) = \frac{\lambda_i}{1 - \mathrm{e}^{-\lambda_i}} h_i^t$$

during period t. Introducing this factor in (8) as

$$cu_i^t := V_i \frac{\lambda_i}{1 - \mathrm{e}^{-\lambda_i}} h_i^{t-1} + F_i z_i^t \qquad \forall\, i \in \mathcal{I}, t \in \mathcal{T}, \qquad (21)$$

results in start-up costs which remain approximately equal for equivalent operational schedules, regardless of the period length f.

D. Comparison of Approaches

Due to differing power plant technology, the choice of the appropriate limitation depends on the individual unit, and some units may even require both limitations. This section compares the start-up time $SUT_i(l)$ and the total heating $TH_i(l)$ for an offline time l in both approaches.

Fig. 3 depicts the start-up time $SUT_i(l)$ for an exemplary unit with parameters $\lambda_i = 0.05$ and $H_i^{\max} = \Delta\text{temp}_i^{\max} = 3\lambda_i$, showing that limiting the heating leads to higher start-up times. As defined, $SUT_i(l)$ equals the start-up time in the continuous model, rounded up to the next integer.

Using the same example, Fig. 4 demonstrates that the approximations $TH_i(i)$ of the total heating given in (14) and (20) closely match the actual values. Furthermore, the figure shows that the required heating is highest when limiting the heating speed, and both limitations result in higher variable costs than the model with unbounded heating.

IV. Numerical Examples

This section describes the computational performance of the proposed approaches as well as the effect of the restrictions on the system costs. We compare both models to the model without start-up time, i.e. with unbounded heating.

A. Scenario

We use a scenario representing the German power system, and a forecast of demand and renewable generation in the year 2025. The dataset is described in detail in [5], and is available for download at [15]. To include different seasonal and daily patterns, each experiment considers 14 time ranges spread uniformly over the year 2025.

Depending on the examined approach, the maximum heating speed $H_i^{\max}$ and the maximum temperature increase $\Delta\text{temp}_i^{\max}$ of each unit i are set to $H_i^{\max} = \Delta\text{temp}_i^{\max} = M\lambda_i$, where the factor M is varied to highlight the impact of the strictness of the limits $H_i^{\max}$ and $\Delta\text{temp}_i^{\max}$.

B. Computational Efficiency

The introduction of either (12) or (18) leads to additional constraints, whose effect on the computational time is investigated in the following. Fig. 5 shows the change of computational time for time ranges with $T = 72$ and $T = 144$ periods, and depending on the factor M. The figure indicates that the change in computational time is not significant for small models, but may increase significantly for larger models and strict limits (small M). The model with limited heating is more efficient and only increases computational times moderately up to 200% compared to the model with unbounded heating. In contrast, the model with limited temperature increase may increase computational times by up to 600% ($M = 40$).

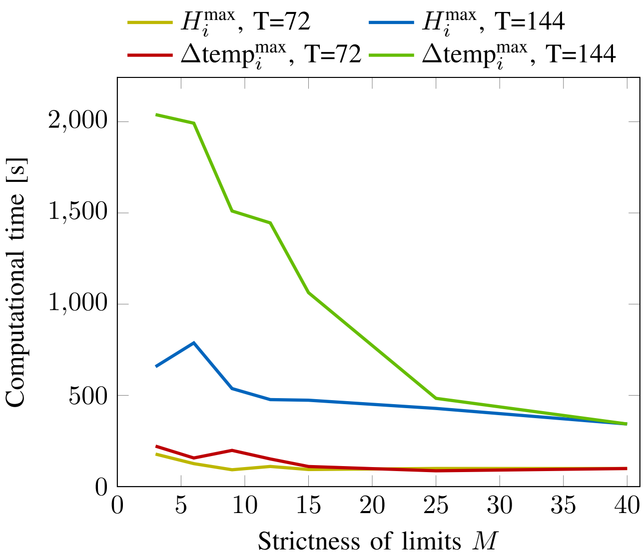

Fig. 5. Computational time with limited heating/temperature increase for test cases of two sizes and parameters $H_i^{max} = M\lambda_i$ or $\Delta temp_i^{max} = M\lambda_i$. While restricting the heating speed has only a moderate impact on computational times, limiting the temperature increase leads to an increase of up to 600%.

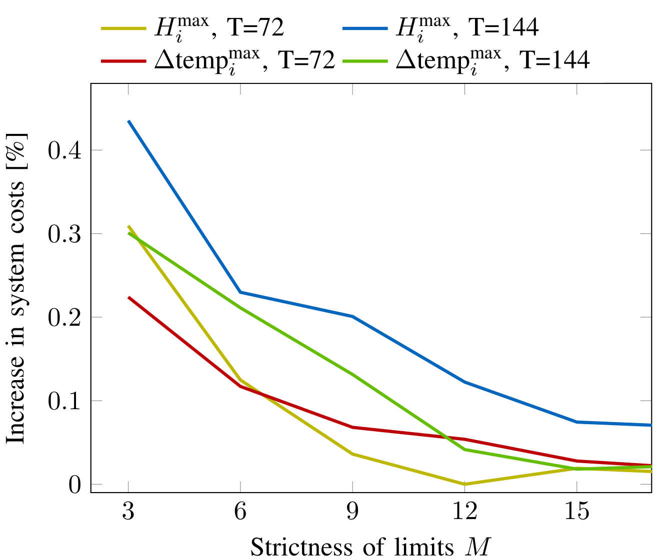

Fig. 6. Increase in system costs due to limited heating/temperature increase for test cases of two sizes and parameters $H_i^{max} = M\lambda_i$ or $\Delta temp_i^{max} = M\lambda_i$. The additional system costs amount to less than 0.5%.

C. Effects on System Costs

As highlighted in Fig. 4, the required heating for a start-up increases when modeling the start-up time. Fig. 6 analyzes the resulting increase in system costs, depending on H_i^{max} and $\Delta temp_i^{max}$. The increase amounts to less than 0.5% even for very strict limitations.

This observation applies only to a deterministic model; as noted in Section I-A, in a stochastic model the start-up time may force a unit to stay at operational temperature during offline time, possibly increasing the system costs considerably.

V. Conclusion

This paper introduced start-up times to the temperature formulation for the Unit Commitment problem. Two options were proposed: limiting the heating speed and limiting the temperature increase. Both approaches lead to different start-up times and costs and are hence useful for different applications. The approach with limited heating proved to be computationally more efficient than the approach with limited temperature increase, and raised computational times only moderately.

Apart from its computational efficiency, modeling start-up times based on the temperature has a further advantage: it allows units to remain at operational temperature during offline time by heating continuously. This behaviour can be observed in real power systems in situations of uncertain load, and may be captured in a stochastic model.

A further aspect of the start-up process is power production. Extending the temperature formulation to model start-up power trajectories is part of the current research of the authors.

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