Lehrstuhl für Regelungstechnik

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# p(t)MOR and Applications for Moving Loads

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Dynamic systems with time-varying parameters arise in numerous industrial applications, e.g. in structural dynamics or systems with moving loads. A spatial discretization of such systems often leads to high-dimensional linear parameter-varying models, which need to be reduced in order to enable a fast simulation. In the following we present time-varying parametric model order reduction (p(t)MOR) based on matrix interpolation and apply this novel framework to a system with moving load.

## Parametric Model Order Reduction

**High-dimensional parametric system:**

$$\mathbf{E}(\mathbf{p})\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p})\mathbf{x}(t) + \mathbf{B}(\mathbf{p})\mathbf{u}(t), \qquad \mathbf{p} \in \mathcal{D} \subset \mathbb{R}^d$$

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p})\mathbf{x}(t) \qquad \mathbf{x}(t) \in \mathbb{R}^N$$

**Projective pMOR:**

Choose appropriate projection matrices $\mathbf{V}(\mathbf{p}), \mathbf{W}(\mathbf{p}) \in \mathbb{R}^{N\times n}$ to approximate the state-vector by $\mathbf{x}(t) \approx \mathbf{V}(\mathbf{p})\mathbf{x}_r(t)$.

**Reduced Order Model:**

$$\overbrace{\mathbf{W}(\mathbf{p})^T\mathbf{E}(\mathbf{p})\mathbf{V}(\mathbf{p})}^{\mathbf{E}_r(\mathbf{p})}\,\dot{\mathbf{x}}_r(t) = \overbrace{\mathbf{W}(\mathbf{p})^T\mathbf{A}(\mathbf{p})\mathbf{V}(\mathbf{p})}^{\mathbf{A}_r(\mathbf{p})}\,\mathbf{x}_r(t) + \overbrace{\mathbf{W}(\mathbf{p})^T\mathbf{B}(\mathbf{p})}^{\mathbf{B}_r(\mathbf{p})}\,\mathbf{u}(t),$$

$$\mathbf{y}_r(t) = \underbrace{\mathbf{C}(\mathbf{p})\mathbf{V}(\mathbf{p})}_{\mathbf{C}_r(\mathbf{p})}\,\mathbf{x}_r(t)$$

## pMOR by Matrix Interpolation

**Individual reduction of local systems:**

$$\mathbf{E}_{r,i}\dot{\mathbf{x}}_{r,i}(t) = \mathbf{A}_{r,i}\mathbf{x}_{r,i}(t) + \mathbf{B}_{r,i}\mathbf{u}(t),$$

$$\mathbf{y}_{r,i}(t) = \mathbf{C}_{r,i}\mathbf{x}_{r,i}(t)$$

$\mathbf{V}_1, \mathbf{W}_1$ $\quad$ $\mathbf{V}_2, \mathbf{W}_2$

$p_1$ $\quad$ $p$ $\quad$ $p_2$

$\mathbf{G}_{r,1}(s)$ $\quad$ ? $\quad$ $\mathbf{G}_{r,2}(s)$

**Transformation to same coordinates:**

$$\overbrace{\mathbf{M}_i^T\mathbf{E}_{r,i}\mathbf{T}_i}^{\hat{\mathbf{E}}_{r,i}}\,\hat{\dot{\mathbf{x}}}_{r,i}(t) = \overbrace{\mathbf{M}_i^T\mathbf{A}_{r,i}\mathbf{T}_i}^{\hat{\mathbf{A}}_{r,i}}\,\hat{\mathbf{x}}_{r,i}(t) + \overbrace{\mathbf{M}_i^T\mathbf{B}_{r,i}}^{\hat{\mathbf{B}}_{r,i}}\,\mathbf{u}(t),$$

$$\mathbf{y}_{r,i}(t) = \underbrace{\mathbf{C}_{r,i}\mathbf{T}_i}_{\hat{\mathbf{C}}_{r,i}}\,\hat{\mathbf{x}}_{r,i}(t)$$

**Interpolation:**

$$\hat{\mathbf{E}}_r(\mathbf{p}) = \sum_{i=1}^k \omega_i(\mathbf{p})\hat{\mathbf{E}}_{r,i}, \quad \hat{\mathbf{A}}_r(\mathbf{p}) = \sum_{i=1}^k \omega_i(\mathbf{p})\hat{\mathbf{A}}_{r,i},$$

$$\hat{\mathbf{B}}_r(\mathbf{p}) = \sum_{i=1}^k \omega_i(\mathbf{p})\hat{\mathbf{B}}_{r,i}, \quad \hat{\mathbf{C}}_r(\mathbf{p}) = \sum_{i=1}^k \omega_i(\mathbf{p})\hat{\mathbf{C}}_{r,i}.$$

## Time-Dependent Parametric Model Order Reduction

**High-dimensional linear parameter-varying system (LPV):**

$$\mathbf{E}(\mathbf{p}(t))\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t), \qquad \mathbf{p}(t) \in \mathcal{D} \subset \mathbb{R}^d$$

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) \qquad \mathbf{x}(t) \in \mathbb{R}^N$$

**Projective p(t)MOR:**

Analogously, we aim to approximate the state-vector by

$$\mathbf{x} \approx \mathbf{V}(\mathbf{p}(t))\mathbf{x}_r,$$

$$\dot{\mathbf{x}} \approx \dot{\mathbf{V}}(\mathbf{p}(t))\mathbf{x}_r + \mathbf{V}(\mathbf{p}(t))\dot{\mathbf{x}}_r = \frac{\partial \mathbf{V}}{\partial \mathbf{p}}\,\dot{\mathbf{p}}\,\mathbf{x}_r + \mathbf{V}(\mathbf{p}(t))\dot{\mathbf{x}}_r$$

**Reduced Order Model:**

$$\mathbf{E}_r(\mathbf{p}(t))\dot{\mathbf{x}}_r = \left(\mathbf{A}_r(\mathbf{p}(t)) - \mathbf{W}(\mathbf{p}(t))^T\mathbf{E}(\mathbf{p}(t))\frac{\partial \mathbf{V}}{\partial \mathbf{p}}\,\dot{\mathbf{p}}\right)\mathbf{x}_r + \mathbf{B}_r(\mathbf{p}(t))\mathbf{u},$$

$$\mathbf{y}_r = \mathbf{C}_r(\mathbf{p}(t))\mathbf{x}_r$$

## p(t)MOR by Matrix Interpolation

**Individual reduction of local systems:**

$$\mathbf{E}_{r,i}\dot{\mathbf{x}}_{r,i} = \left(\mathbf{A}_{r,i} - \mathbf{W}_i^T\mathbf{E}_i\frac{\partial \mathbf{V}}{\partial \mathbf{p}}\,\dot{\mathbf{p}}\right)\mathbf{x}_{r,i} + \mathbf{B}_{r,i}\mathbf{u},$$

$$\mathbf{y}_{r,i} = \mathbf{C}_{r,i}\mathbf{x}_{r,i}$$

**Transformation to same coordinates:** $\mathbf{x}_{r,i} = \mathbf{T}_i\hat{\mathbf{x}}_{r,i}, \; \dot{\mathbf{x}}_{r,i} = \dot{\mathbf{T}}_i\hat{\mathbf{x}}_{r,i} + \mathbf{T}_i\dot{\hat{\mathbf{x}}}_{r,i},$

$$\overbrace{\mathbf{M}_i^T\mathbf{E}_{r,i}\mathbf{T}_i}^{\hat{\mathbf{E}}_{r,i}}\,\dot{\hat{\mathbf{x}}}_{r,i} = \overbrace{\left(\mathbf{M}_i^T\mathbf{A}_{r,i}\mathbf{T}_i - \mathbf{M}_i^T\mathbf{W}_i^T\mathbf{E}_i\frac{\partial \mathbf{V}}{\partial \mathbf{p}}\,\dot{\mathbf{p}}\,\mathbf{T}_i - \mathbf{M}_i^T\mathbf{E}_{r,i}\dot{\mathbf{T}}_i\right)}^{\hat{\mathbf{A}}_{\mathrm{new}\,r,i}}\hat{\mathbf{x}}_{r,i} + \overbrace{\mathbf{M}_i^T\mathbf{B}_{r,i}}^{\hat{\mathbf{B}}_{r,i}}\,\mathbf{u}$$

$$\mathbf{y}_{r,i} = \underbrace{\mathbf{C}_{r,i}\mathbf{T}_i}_{\hat{\mathbf{C}}_{r,i}}\,\hat{\mathbf{x}}_{r,i}$$

**Interpolation:**

$$\hat{\mathbf{E}}_r(\mathbf{p}(t)) = \sum_{i=1}^k \omega_i(\mathbf{p}(t))\hat{\mathbf{E}}_{r,i}, \quad \hat{\mathbf{A}}_{\mathrm{new}\,r}(\mathbf{p}(t)) = \sum_{i=1}^k \omega_i(\mathbf{p}(t))\hat{\mathbf{A}}_{\mathrm{new}\,r,i},$$

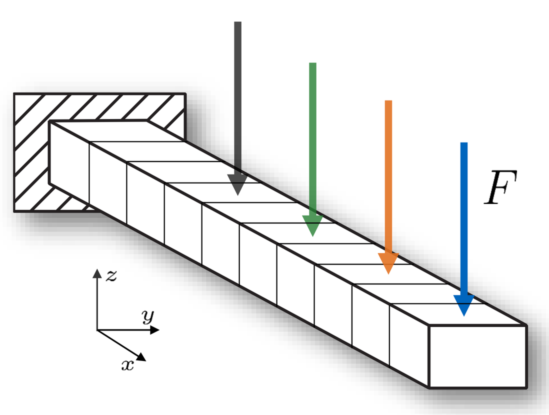
$$\hat{\mathbf{B}}_r(\mathbf{p}(t)) = \sum_{i=1}^k \omega_i(\mathbf{p}(t))\hat{\mathbf{B}}_{r,i}, \quad \hat{\mathbf{C}}_r(\mathbf{p}(t)) = \sum_{i=1}^k \omega_i(\mathbf{p}(t))\hat{\mathbf{C}}_{r,i}.$$

## Application for Systems with Moving Loads

**Systems with Moving Loads:**

- position of the acting load varies with time
- varying load position can be regarded as a time-dependent parameter of the system

→ Linear parameter-varying system

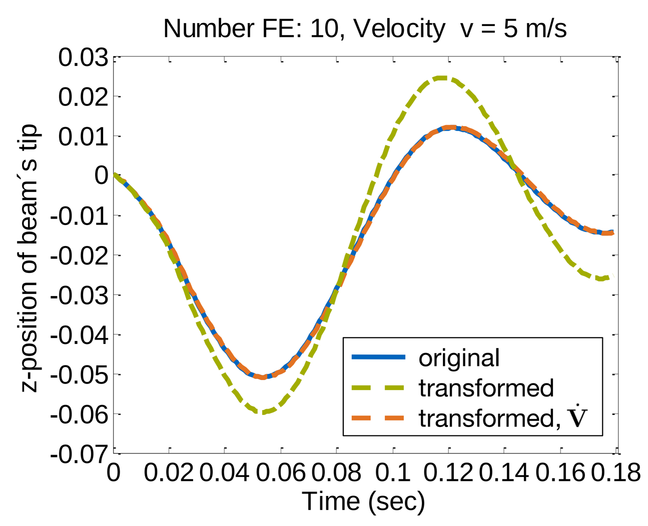



**Considered example:**

- Timoshenko beam with moving load
- Finite Element Discretization
- LPV system with parameter-dependent input matrix $\mathbf{B}(\mathbf{p}(t))$

**Relevance of $\dot{\mathbf{V}}(\mathbf{p}(t))$:**

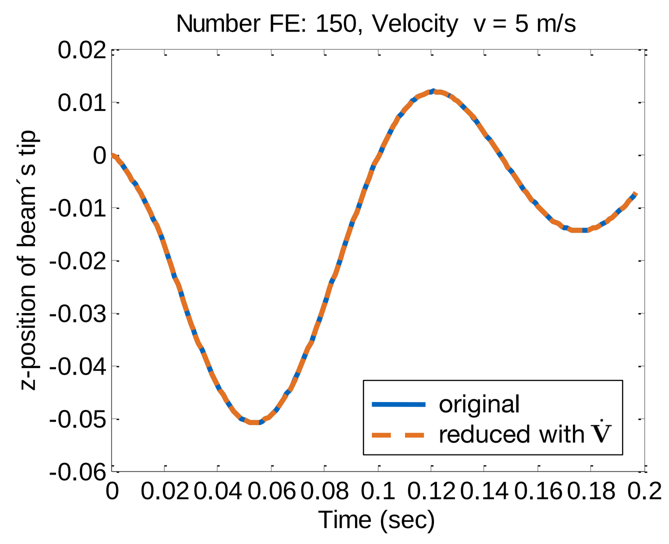
- Parameter-varying state transformation of the original model shows the importance of the consideration of $\dot{\mathbf{V}}(\mathbf{p}(t))$




## Current Work and Outlook

**Model Reduction:**

- p(t)MOR by Matrix Interpolation applied
- Order of locally reduced systems: $n = 20$




**Further study:**

- Interpretation of the new matrix $\hat{\mathbf{A}}_{\mathrm{new}\,r,i}$
- Remedial actions against unstable interpolated systems
- Application of p(t)MOR to generalized linear parameter-varying systems

## References

[1] M. Geuss, H. Panzer and B. Lohmann: „On parametric model order reduction by matrix interpolation", Proceedings of the ECC, 3433-3438, 2013.

[2] H. Panzer, J. Hubele, et al.: „Generating a Parametric Finite Element Model of a 3D Cantilever Timoshenko Beam Using Matlab", Vol. TRAC-4, 2009.

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