Least Squares Optimization of Zero Crossing Technique for Frequency Estimation of Power System Grid Distorted Sinusoidal Signals.

Thiago R. F. Mendonça, Milena F. Pinto, Carlos A. Duque
Electrical Engineering Department
Federal University of Juiz de Fora, UFJF
Juiz de Fora, Brazil.
thiago.mendonca@engenharia.ufjf.br, carlos.duque@ufjf.edu.br

***Abstract*—Reliable frequency estimation is considered crucial for several industrial applications, such as power control, power system protection and monitoring. In certain applications is required an accurate and fast frequency estimations of the sinusoidal signals. Several algorithms are found in literature able to solve this task. However, their responses under low Signal to Noise Ratio (SNR) are not reliable. In this paper is proposed an optimization by least squares of the well-known zero-crossing technique in order to enhance performance under such condition. To validate the algorithm, it was performed some comparisons with other methods using different values of frequency. The presented technique has shown better results regarding the estimation error.**

Keywords - Zero crossing, frequency estimation, least square, low SNR.

I. INTRODUCTION

Frequency estimation of a sinusoidal signal in presence of broadband noise is considered a common problem among a variety of signal processing applications [1]. For instance, in power systems, this parameter is extremely important to guarantee a safe operation, optimized power flow, efficient setting of protective relays for load shedding and many others. Additionally, for others fields of applications, it can be cited sonar, radar and nuclear magnetic resonance [2].

According to [3], due to the increasing number of electrical energy consumers and consequently higher demands for reliability of power supply, several companies are concerned about the need of a real-time wide area measurement system, which relies on the correct frequency estimation, providing useful information about the grid status. As result, accurate and fast methods are necessary.

By means of instrumentation and signal processing techniques, the acquisition is accomplished. When the noise exists and its frequency domain representation is known, it is possible to design a specific filter in order to remove it from the signal. However, in most of the power systems applications, the noise has a normal probability density function and its spectrum spreads all over the frequency domain. Thereby, it is not an easy task to completely remove it [4]. Several algorithms used for frequency estimation, assume the noise as random white noise with normal distribution and are able to estimate accurately the frequency when the SNR (Signal to Noise Ration) is higher, but their performance are degraded for low SNR. Additionally, the pdf (probability distribution function) cannot be modeled as a normal distribution for some case and under this assumption the estimation could be poor. For example, the noise may vary in form due to others factors, such as opening and closing switches at the power grid, insertion or removal of big loads and a less discussed effect, the corona noise [5].

A well-established method for estimating frequency of certain signal consists of measuring the period by counting the time interval for two consecutive zero-crossings, which gives half the period. Then, by taking two times its reciprocal, the frequency is computed [6]. Figure 1 and 2 show the zero-crossing performance for free-noise and with AWGN in the signal, respectively. It is noticed that this method is accurate for noise-free signal. However, when the noise level goes high, spurious counting could occur, generating a false frequency estimation.

Focusing on this issue, this work presents a least square optimization of the well-known zero-crossing technique when the signal is under low SNR. To demonstrate performance characteristics of this method, it is compared with an enhanced zero-crossing proposed by [7], which utilizes the histogram of the time intervals between consecutive zero-crossings and with a conventional zero-crossing algorithm applied in a filtered signal.

978-1-4799-5551-0/14/$31.00 ©2014 IEEE

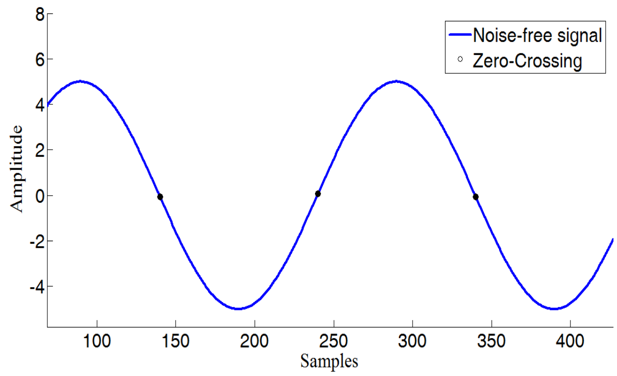

Figure 1. Zero-crossing performance for noise-free signal.

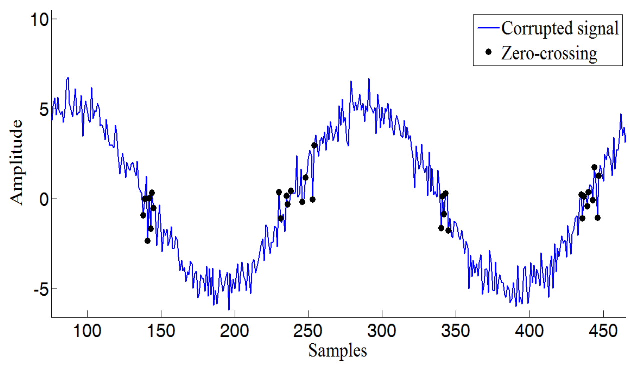

Figure 2. Zero-crossing performance for signal in presence of AWGN.

II. PROPOSED METHOD

The proposed method consists in determining an optimized line that intercepts the zero value as close as possible to the real instant. This procedure is done by the first order least squares approximation, which finds the line equation parameters, whose overall solution minimizes the sum of the squares of the errors made from the points of the straight line equation and the values in the observation data.

Figure 3 shows the straight line resulted from a first order least squares optimization applied in the observed data.

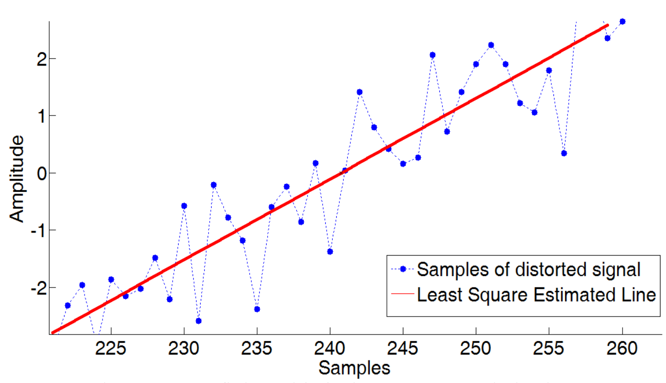

Figure 3. Data fitting with the least squares optimization.

The main principle for finding the line coefficients is presented in (1), where n is the length of the observed data vector, θ_1 and θ_2 are the slope and the intercept coefficients, respectively. Finally, the vector Y is the observed data vector.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ \vdots & \vdots \\ n & 1 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{1}$$

The data vector is chosen to be the signal samples near the zero, considering that for a few samples, the sinusoid can be approximated as linear. It is important to consider some prior information about the frequency of the signal in order to define the length of the Y vector, since that for high frequency, the portion near to zero that can be considered linear, is smaller than for lower frequencies. In other words, it is required a first approximation of the signal's frequency in other to obtain a better estimation. This first approximation is achieved by the ratio between the length of the entire signal and the number of semi-cycles sampled. To illustrate how the length of the observed data affects the optimization, the linear approximation by least squares of the signal is represented in Figure 4. It can be noticed that if the portion analyzed was bigger, the first order least square algorithm utilized for optimization, would approximate a clearly non-linear signal as linear, leading to errors.

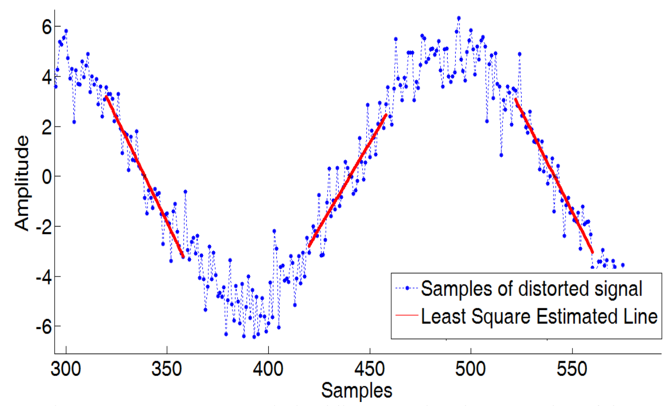

Figure 4. Least squares optimized lines crossing the zero value of the corrupted signal.

To count the number of semi-cycles, note that when in presence of noise, the spurious counting occurs at smaller intervals. As discussed in [7], by measuring these intervals, it is possible to divide in two clusters, one of small periods, indicating passage through zero due to noise distortion, and another with an approximation of the semi-cycle interval. The clusters can be seen in the histogram presented in Figure 5. Through this figure, it is possible to conclude that the analyzed signal has almost 65 falsely counted zero-crossings and near 12 semi-cycles. With this data and the length of the analyzed signal, the range of the signal close to zero that can be approximated as linear, is obtained.

At this point, the information for the Y vector of equation (1) is known and the matrix that multiplies the coefficients vector is constant. In order to solve the matrix equation, the pseudo-inverse must be computed as shown by (2), where A is the constant valued matrix of equation (1).

$$\theta = (A.A^T)^{-1}.A^T.Y \tag{2}$$

It is noteworthy that once defined the length of the observed data vector, the matrix A remains unchangeable. And the same occurs with its pseudo-inverse. Therefore, it can be calculated outside a loop, avoiding unnecessary computational effort.

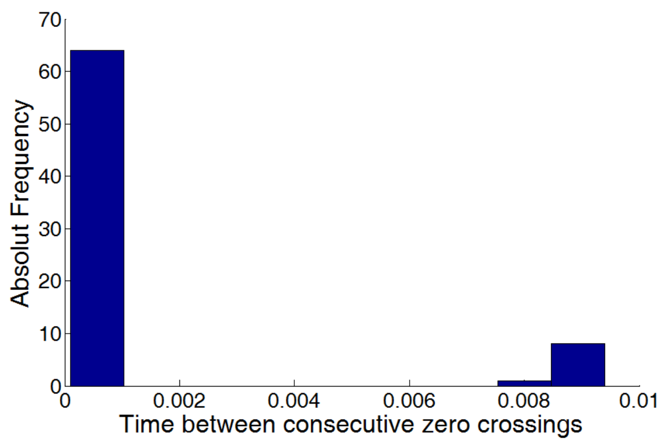

Figure 5. Observable cluster from two zero crossings intervals.

After the coefficients from equation (2) are obtained, it is necessary to do an interpolation with the aim to estimate the point in the line that intercepts the zero value. Posteriorly, the frequency is estimated by computing the reciprocal of the time interval from two consecutive passages through zero value. Note that when the measurement time is longer, more semi-cycles are available for estimating the frequency. Therefore, accuracy could be improved, since a mean value of all estimation is considered, and for longer data, the error tends to decrease. Although, it requires more time to perform the algorithm, turning it unfeasible for some applications.

The algorithm of the proposed method is presented in Figure 6. As can be seen, initially, for each semi-cycle, is necessary to count the numbers of the spurious zero-crossing in order to define the observed data vector length of (2). This information is also useful to create the constant matrix A. It can be noticed that this matrix does not vary within each iteration, allowing computation of its pseudo-inverse only once, i.e. outside of the loop. After this process, the algorithm calculates the coefficients of the least squares estimated line and through interpolation, it is possible to find the interception, which is an approximated zero-crossing value. This procedure repeats for all semi-cycles contained in the sampled signal. In case of the maximum number of iterations is reached, the method will have all necessary data for computing the time intervals between the estimated zero-crossings, allowing frequency estimation by its reciprocal.

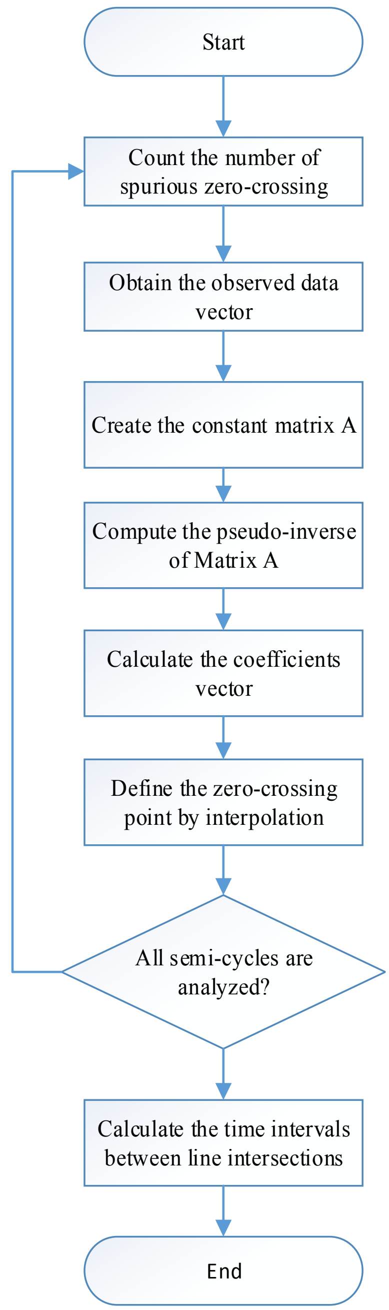

Figure 6. Proposed method algorithm.

III. RESULTS

As a form of validating the method's efficiency for frequency estimation under low SNR, a distorted sinusoidal signal was generated using a Gaussian noise with standard deviation of 0.5. Additionally, only three semi-cycles were considered in order to provide fast analysis. Besides, two methods known to work under low SNR condition were selected in order to compare with the proposed algorithm discussed in this paper. These methods are as follows.

- Optimized zero crossing by means of filtering the corrupted signal;
- Optimized zero crossing by means of statistical analysis as discussed in [7].

All three methods were executed for a hundred iterations, while the frequency of the signal was varied from 50 to 250 Hz, with the purpose to measure the mean value for each frequency. It was used a sampling rate for each fundamental frequency of the signal, ensuring that the Nyquist rate, to avoid aliasing effect, was satisfied. Sampling rate was defined as 500 times the fundamental frequency for this experiment. The results are shown in Figure 7.

It is observed that the proposed method presented lower error values compared to the others, besides to having a stable behavior in a large range of frequency. On the other hand, the method of enhanced zero crossing by means of statistics analysis has a higher error variance with large oscillations.

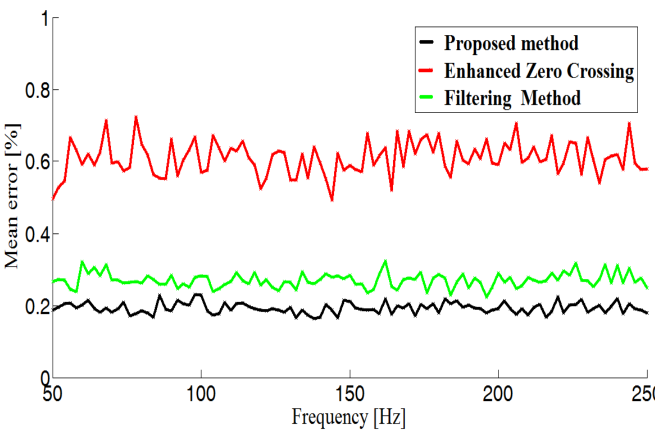

Figure 7. Comparison between the three methods.

Instead of varying the frequency, another test was implemented, changing the values of the SNR. It was also performed a hundred iterations and the mean error was considered for all three methods. The results are presented in Table I. The standard deviation used to produce the normal distribution noise for each value in Db of SNR is calculated according to (3).

$$\sigma = \sqrt{\frac{10^{-\frac{SNR}{10}}}{2}} \tag{3}$$

As can be seen from Figure 7 and Table I, the least square optimization proposed outperformed the others techniques regarding the frequency error.

The filter designed to optimize the traditional zero-crossing method, avoiding improper counting, was a fifth order Butterworth filter with a normalized cutoff frequency of 0.015. The cutoff frequency was defined considering that the sampling rate utilized for this experiment was 500 times the fundamental, so maximum representable frequency should be half of this value (i.e. 250 times the fundamental), which for the normalized scale is one. Note that the normalized fundamental frequency is 1/250, which results in 0.004. Therefore, a good value should be chosen around four times 0.004. The filtered signal can be seen in Figure 8.

TABLE I. MEAN ERROR FOR EACH METHOD CONSIDERING DIFFERENT SNR CONDITIONS.

SNR [Db]	Error [%]		
	Enhanced by least squares	Enhanced by filter	Enhanced by statistics
0.01	0.2724	0.3924	0.8461
0.1	0.2679	0.3783	0.8415
1	0.2422	0.3380	0.7598
5	0.1582	0.2153	0.5033
10	0.0913	0.1197	0.2991
15	0.0549	0.0681	0.1852
20	0.0345	0.0387	0.1165
30	0.0176	0.0138	0.0408
40	0.0149	0.0082	0.0149

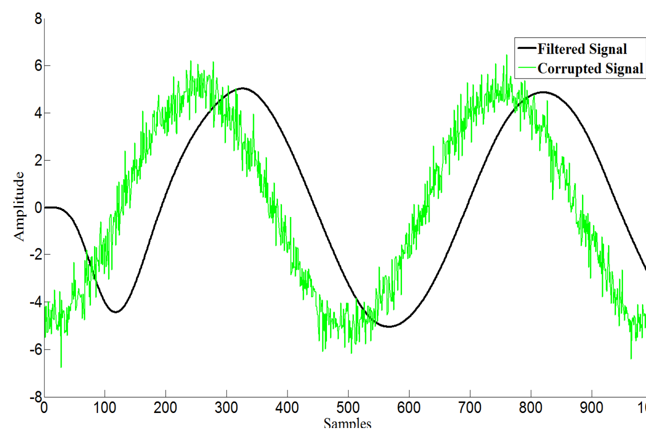

Figure 8. Noise removal by a fifth order Butterworth filter with normalized cutoff frequency of 0.015, considering sampling rate of 500 times the fundamental frequency.

Note that even after the filter process, the signal remains with a small distortion due to others components present in the white Gaussian noise spectrum. It could result in small errors when simple zero crossing algorithm is applied. This form of frequency estimation is commonly used with an analog filter, due to its computational simplicity.

Considering that typical sampling rates used in protection relays are around 32 samples per cycle, it is important to validate the algorithm for such condition. Table II shows the average frequency error obtained for different sample rates. It was analyzed ten cycles instead of only three in order to increase accuracy. It is noteworthy that as the number of samples analyzed increases, error tends to decrease. For fewer samples, it is necessary to acquire more numbers of cycles, aiming to approximate the noise distribution function.

TABLE II. AVERAGE ERROR CONSIDERING DIFFERENT NUMBERS OF SAMPLES PER CYCLE.

Samples per Cycle	Error [%]		
	Enhanced by least squares	Enhanced by filter	Enhanced by statistics
16	0.1471	0.2275	0.1982
32	0.1002	0.1154	0.1956
64	0.0710	0.0889	0.1851
128	0.0508	0.0675	0.1696

In Figure 9 is showed a visual form of validating the proposed method's efficiency. As can be seen, the line coefficients calculated from the least squares algorithm intercepts the zero value as close as the fundamental, providing accurate values of frequency estimation even under high distortion. It is observed that the corrupted signal is represented by the blue line and the fundamental and estimated line by green and red, respectively. Note that the frequency can be measured even in presence of large noises, in other words, the estimation is accomplished without the need of filter.

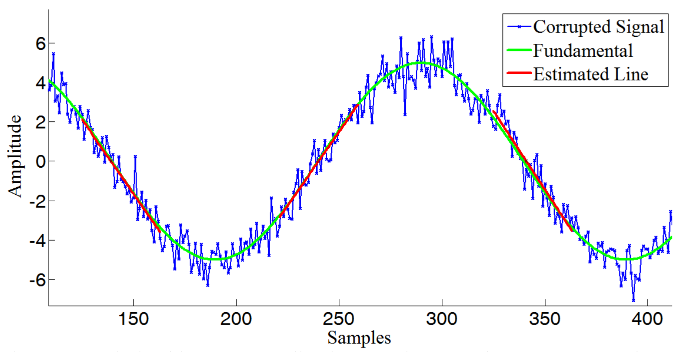

Figure 9. Optimized least squares line intercepting near the true zero crossing point.

A drawback of the proposed method is when in presence of harmonics, since the least square algorithm is efficient for normal distributed data. As shown in Figure 10, the line estimation does not cross near the fundamental, leading to unreliable values. The fundamental frequency of 60 Hz with amplitude of 5 and harmonics of 3º, 5º and 6º with amplitudes of 2, 1 and 0.5 respectively, was used to generate this figure. The results of frequency estimation are not reliable in this condition. Table III shows the simulated results of this method for a 60 Hz sinusoidal signal corrupted by harmonics of different order, with 20% of energy. It was performed only one iteration, since there is no random noise applied.

As expected, measured frequency under such conditions is not reliable. As seen in Table III, perhaps for a certain combination of harmonics (e.g. 5º and 7º) the error could be low. In this way the harmonics need to be filtered before the frequency estimation. It is worth to highlight that several frequency estimation method has similar problem and a pre-filtering must be used before estimation.

TABLE III. FUNDAMENTAL FREQUENCY ERROR OBTAINED FOR EACH COMBINATION OF HARMONICS.

Order of the harmonic	Error [%]
2º and 3º	1.199
3º and 4º	0.289
2º and 5º	0.790
5º and 7	0.005
2º and 10º	0.983

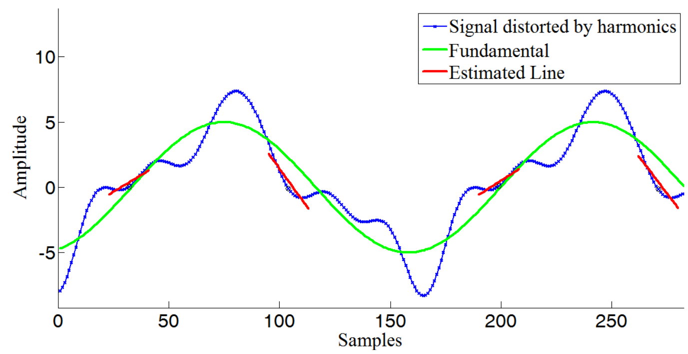

Figure 10. Least square optimization for signal in presence of harmonics.

Furthermore, filtering harmonics of a signal is not a difficult task, since they have frequencies separated from the fundamental. Problems arise in situations where inter-harmonics exist, hampering the signal filtering. For this reason, the proposed method in this paper is not recommended for this situation.

In order to prove the efficiency of the technique presented in this paper, others noise distributions were used. Table IV presents a numerical comparison between three methods using four noise distributions, which are Gaussian, Rayleigh, Uniform and Poisson. It is noticed that the enhanced by least squares technique has also exhibited better results, regardless of what noise distribution is applied. It is noteworthy that was used a normalized noise varying from -2 to 2 for all conditions.

TABLE IV. MEAN ERROR FOR EACH METHOD CONSIDERING DIFFERENT NOISE DISTRIBUTIONS.

Noise Distribution	Error [%]		
	Enhanced by least squares	Enhanced by filter	Enhanced by statistics
Gaussian	0.2424	0.3379	0.7485
Rayleigh	0.1413	0.2606	0.5974
Uniform	0.3028	0.4800	0.7729
Poisson	0.1784	0.2852	0.6409

Moreover, the method has the advantage to be used in any quantity of semi-cycles. In this way, it was performed simulations using different values of signal length, as shown in Table V. Note that, as expected, when the amount of semi-cycles increase, the error tends to lower values. However, it demands a higher time for frequency estimation, giving flexibility of usage for specific applications.

TABLE V. MEAN ERROR FOR EACH METHOD CONSIDERING DIFFERENT NUMBER OF SEMI-CYCLES.

Number of Semi-Cycles	Mean Error [%]		
	Enhanced by least squares	Enhanced by filter	Enhanced by statistics
5	0.0742	0.1548	0.3763
9	0.0386	0.0857	0.2121
13	0.0255	0.0576	0.1492
19	0.0171	0.0390	0.1018

IV. CONCLUSION

The problem of frequency estimation of sinusoidal signals that are corrupted by noise is addressed in this paper as well as the importance of frequency estimation for power system applications. Additionally, it was discussed the situations where signal to noise ratio is low and unfeasible for measuring.

Considering a small portion of the signal, located near the zero-crossing, it is possible to optimize a straight line that intercepts at a fairly close point as the fundamental, resulting in accurate frequency estimations without requiring filtering. Even though least squares is meant for normal distributed observed data, for this small portion analyzed, others noise distributions may be approximated as Gaussian, resulting in good estimation.

The results have shown the efficiency of the proposed method considering frequency error. However, it’s not useful for such applications where is required a fast estimation, once the accuracy is proportional to the number of cycles analyzed.

There is no intention of the proposed method to substitute others estimators like the Phase Locked Loop structure, commonly used to follow frequency variation. However, it is recommended to work together in such a way that whenever other structure fails, it relies on the output of this technique, avoiding oscillations of measurement.

V. ACKNOWLEDGMENT

The authors would like to thank the Federal University of Juiz de Fora, CAPES, FAPEMIG, INERGE, CNPQ and Brazilian Government for technical and financial support on this work.

REFERENCES

[1] R. Elasmi-Ksibi, H. Besbes, R. López-Valcarce and S. Cherif, “Frequency estimation of real-valued single-tone in colored noise using multiple autocorrelation lags,” Signal Processing, vol. 90, no. 7, pp. 2303–2307, Jul. 2010

[2] C. Huang, C. Lee, S. Member, K. Shih and Y. Wang, “A Robust Technique for Frequency Estimation of Distorted Signals in Power Systems,” IEEE Trans. Instrum. Meas, August 2010, vol. 59, no. 8, pp. 2026–2036, 2010.

[3] S. Chakrabarti, E. Kyriakides, T. Bi, D. Cai and V. Terzija, “Measurements get together,” *IEE power & energy magazine*, february, pp. 41–49, 2009.

[4] B. Sklar, Digital Communications - Fudamentals and Applications, Second Edition, Prentice Hall. 2001.

[5] A. Mujčić, N. Suljanović, M. Zajc and J. F. Tasič, “Corona noise on the 400 kV overhead power line - measurements and computer modeling,” Springer Electr. Eng. Journal, Vol. 86, Issue 2, January 2004, pp. 1–16.

[6] R. Aghazadeh, H. Lesani and B. Ganji, “New technique for frequency and amplitude estimation of power system signals,” Gener. Transm. Distrib. IEE Proceedings- Volume152 , Issue 3, pp. 435–440, 2005.

[7] D. Grillo, N. Pasquino, L. Angrisani and R. Schiano Lo Moriello, “An efficient extension of the zero-crossing technique to measure frequency of noisy signals,” 2012 IEEE Int. Instrum. Meas. Technol. Conf. Proc., pp. 2706–2709, May 2012.