THE IMPACT OF PARAMETER VARIATION, EXPERIMENTAL DATA AND UNCERTAINTY QUANTIFICATION FOR COMPLEX BIOMECHANICAL PROBLEMS EXAMPLIFIED FOR AAA

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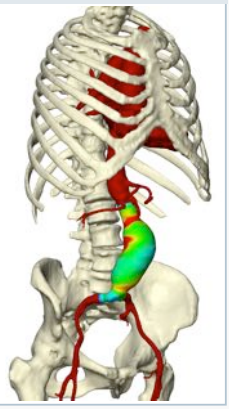
Introduction

AAA rupture risk prediction using FEM

- Computational rupture risk indicators, e.g., peak wall stress are superior to the diameter criterion [1].
- Most "patient-specific" models are based on population averaged model parameters.



Existing uncertainties

- Computational geometries, e.g., wall thickness
- Boundary conditions, e.g., intra luminal pressure
- Physical parameters, e.g., constitutive parameters

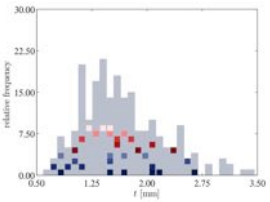
Towards more reliable rupture risk prediction

- In absence of truly patient-specific parameters: include uncertainties in the FEM analysis
- Here uncertain wall thickness is considered and its impact on peak wall stress is studied. For uncertain constitutive parameters, see [2].

Probabilistic models of AAA wall

Experimental research

- Experimental measurements reveal significant inter- and intra-patient variations for wall thickness
- Probability distribution for thickness t can be estimated from histogram
- 29 supplementary patient- and specimen-specific parameters $\boldsymbol{v}$ were gathered in addition to thickness measurements and tensile test data.



Histogram of measured thickness values. Squares of same color indicate measurements from same patient

Harnessing non-invasive information

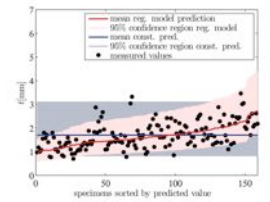
- The supplementary data can be harnessed by Bayesian regression models to obtain non-invasive predictions of patient-specific wall thickness and other quantities [3].
- Using Gaussian process (GP) regression GP prior:

$$\log t(\boldsymbol{v})|\boldsymbol{\theta}_m, \boldsymbol{\theta}_k \sim \mathcal{GP}\left(m_t(\boldsymbol{v};\boldsymbol{\theta}_m), k_t(\boldsymbol{v},\boldsymbol{v}';\boldsymbol{\theta}_k)\right)$$

- GP is conditioned on available data $\mathcal{D}$.
- The hyper-parameters $\boldsymbol{\theta}_m, \boldsymbol{\theta}_k$ are obtained by maximization of the marginal likelihood [5].
- Gaussian predictive distribution for $\log t$ depends on non-invasively accessible parameters $\boldsymbol{v}^*$ of particular patient:

$$p(\log t^*|\boldsymbol{v}^*, \mathcal{D}) = \mathcal{N}\left(m_t^*, \sigma_t^{*2}\right)$$

- Regression model allows more accurate prediction of wall thickness with reduced variance, i.e., uncertainty about t than direct estimation.



Regression model predictions and measured values for wall thickness of excised specimens. Explanatory variables used in regression model: sex, age, ruptured, subrenal diameter, CKD, DM, creatinin, erythrocytes,thrombocytes, sodium, urea, MCH, MCHC, distance to bifurcation, thrombus thickness, and NORD.

Considered probabilistic wall thickness models

- Spatial, intra-patient variations can only be captured with random field model.
- Positivity constraint: (marginal) Gaussian distribution for $\log t$
- Three different probabilistic wall models are considered:
  - Model A: Random field model, parameters estimated from available thickness measurements
  - Model B: Random field model, parameters obtained from GP regression model which predicts wall thickness distribution
  - Model C: Random variable model, distribution parameters estimated from available thickness measurements

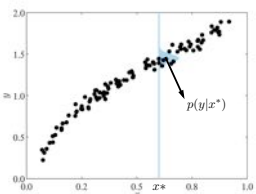
Propagation of uncertainties

Bayesian multi-fidelity Monte Carlo

- Incorporation of low-fidelity models [2][4]
- Probabilistic link between low-fidelity model x and high-fidelity y model obtained with Bayesian regression.

$$\pi_y(y) = \int p(y|x)\pi_x(x)dx$$

- Accurate and efficient for problems with high stochastic dimension
- Here: low-fidelity model simply assumes a linear relationship between local wall stress and local wall thickness.



Schematic representation of correlation between low-fidelity and high-fidelity model output.

Gaussian process Bayesian surrogate model

- If wall thickness is modeled as random variable, Bayesian surrogate models are more suitable.

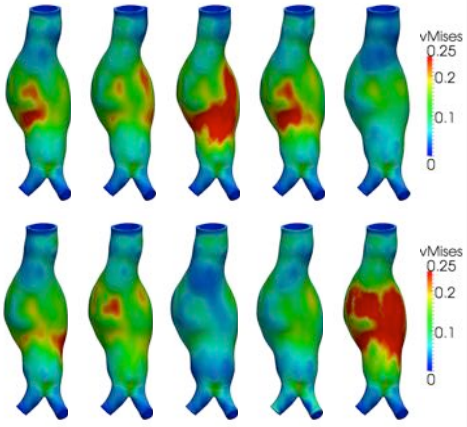
Results for exemplary AAA geometry

Patient-specific AAA model

- Full nonlinear patient-specific model with ca. 380.000 degrees of freedom
- Nonlinear constitutive models of AAA wall and ILT based on [6] and [7]
- Realizations of random wall thickness are created at runtime using pseudo-structure approach.
- Quantity of interest: von Mises peak wall stress

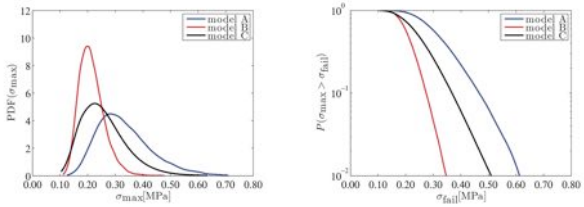
Impact on wall stress

- High variability for wall stress and peak wall stress
- Large variability in thickness can result in drastically different stress patterns for both of the random field models A and B.
- If model C is used, stress pattern is largely determined by geo-metric features and the location of peak stress stays the same.



Resulting von Mises stress σ_{vM} [MPa] for 10 realizations of model A

Probability distribution of peak wall stress



Probability distribution for peak wall stress. Left: PDF. Right: failure probability for threshold σ_{fail}

	$\mathbb{E}[\sigma_{\max}]$[MPa]	$\mathrm{var}[\sigma_{\max}]$	95% q. of $\sigma_{\max}$[MPa]
model A	0.333	0.0094	0.516
model B	0.215	0.0021	0.298
model C	0.259	0.0069	0.412

- Model A exhibits significantly larger variance than model B.
- Uncertainty about peak wall stress can be reduced by using regression models to predict the wall thickness distribution.
- Distributions for model A and model C are different, i.e., the choice of probabilistic wall model affects probability distribution of peak wall stress.
- Efficient solution of UQ problem through Bayesian multi-fidelity Monte Carlo or Gaussian process surrogate models reduces costs to acceptable level.

Conclusion and outlook

- We quantified uncertainty about peak wall stress in AAA due to uncertain wall thickness, inclusion of additional uncertain parameters is possible.
- Random variable models for the thickness can be overly simplistic
- Regional spatial variations in wall thickness are important and should not be neglected
- Study with larger patient cohort is needed to confirm preliminary results.
- 95% quantile could serve as worst case estimate for peak stress, however, definition a of probabilistic failure measure is necessary.

References:

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