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# LNA Noise Parameter Measurement

Bernhard Lehmeyer, Michel T. Ivrlač, Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München, 80290 Munich, Germany
e-mail: `{berhard.lehmeyer|ivrlac|josef.a.nossek}@tum.de`

***Abstract*—Joint measurement of stochastic parameters of a low noise amplifier (LNA), based on output power measurement for input termination impedances is considered. As an advantage of this method the noise parameters can be determined in a quick way for narrow and broad band applications without special equipment. Knowing the noise parameters helps the user designing both matching networks for given low noise amplifiers and for optimizing amplifiers. Four real parameters characterize the amplifier with the noise figure as function of the matching network. About ten different values for input termination impedances also provide an estimation of sensitivity and accuracy of this new method.**

## I. INTRODUCTION

To achieve the lowest possible noise figure by noise matching in a receiver its LNA has to be characterized regarding its stochastic parameters. Because the established methods, e.g. slide screw tuner method [1] are narrow band, time consuming and expensive, there is a need for a quick automatable method. This concept is based on the output power measurement for a set of known *input termination impedances*. As a consequence of this generator-less noise-only approach, the voltage levels at the input of the amplifier are very small and do not exceed the dynamic range of the LNA. At first theory and calculation will be presented and how to obtain noise matching in a quick way. At last a demonstration measurement of a real two stage amplifier is shown.

## II. THEORETICAL BACKGROUND

### A. Matching Strategies

For *noise matching*, the stochastic-, for *power matching* the deterministic parameters of an amplifier are needed [2] [3] [4]. When doing *sensitivity matching* both, the stochastic and the deterministic parameters are needed [5]. For *power matching* the signal source is matched to $Z_{in}$, for *noise matching* $i_N$ and $v_N$ determine the source impedance [6].

### B. Noise Figure Calculation

The noise figure of an amplifier is calculated to

$$\mathrm{NF} = 1 + \frac{E[|i_N|^2]}{4kTB\Re\{Z'_G\}}\left(|Z'_G|^2 + R_N^2 - 2\Re\{\rho Z'^*_G\}\right), \quad (1)$$

where k is the Boltzmann constant, T the absolute temperature, B the equivalent noise bandwidth, $Z'_G$ the generator´s impedance, $E[|i_N|^2]$ and $E[|v_N|^2]$ as follows the variances of noise current- and voltage source, the noise resistance $R_N$ and $\rho$ the complex correlation coefficient

$$R_N = \sqrt{\frac{E[|v_N|^2]}{E[|i_N|^2]}}, \quad (2)$$

$$\rho = \frac{E[v_N i_N^*]}{\sqrt{E[|v_N|^2]E[|i_N|^2]}}. \quad (3)$$

The noisy amplifier Fig. 1 is modeled as a noiseless amplifier [6] [7] [8] that is connected to a voltage- $v_N$ and a current noise source $i_N$ at its input [9]. The signal source, e.g. an antenna, is modeled as voltage source $v'_0$ with the transformed impedance $Z'_G$.

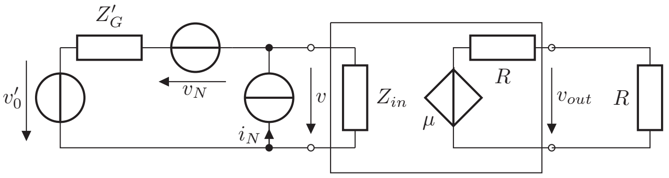

Figure 1: LNA model driven by a signal source. The loading effect has been taken into account by the gain $\mu$ of the VCVS.

### C. Extracting Parameters

With the input shorted, only $v_N$ determines the voltage $v$ across $Z_{in}$, which multiplied by the voltage gain factor $\mu$ leads to the output voltage

$$v_{out} = \frac{\mu}{2} v_N. \quad (4)$$

Open input terminals lead to $i_N$ controlling the voltage over $Z_{in}$ and therefore, the output voltage is

$$v_{out} = \frac{\mu}{2} Z_{in} i_N. \quad (5)$$

An ideal RF open is very hard to realize, because its electrical length must be identical to the one of the other terminations, while shielding and high Q materials are necessary, otherwise thermal noise would falsify the results. The input impedance must not exceed some hundred ohms at system center frequency [10]. To get $E[|i_N|^2]$, $E[|v_N|^2]$ two input terminations are needed, they can either be resistive or reactive devices. To get the imaginary part of the correlation coefficient $\Im\{\rho\}$ a reactive one and a resistive one to determine the real part $\Re\{\rho\}$ is needed. A good choice for example would be

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$0\ \Omega, 50\ \Omega, +j50\ \Omega, -j50\ \Omega$. If ever possible, *many more than 4*, for example $Z_m = R_m + jX_m$, $m = 1, ..., M$ e.g. $M = 10$, termination impedances should be used in order to improve accuracy, especially for the real part of the complex correlation coefficient. For the same reason it is advisable that several real valued impedances in the range from zero (short circuit) to some hundred Ohms are used in conjunction with purely reactive impedances (both capacitive and inductive). For this approach the amplifier´s forward transmittance $S_{21}$ has to be determined either by resistive terminations at different temperatures [11] or with another known signal fed to the input. Now two cases have to be distinguished: Reactive terminations which are noiseless, Fig. 2 and resistive terminations. The latter procedure thermal noise [12] that is modeled by voltage source $v_m$, Fig. 3, which is characterized by

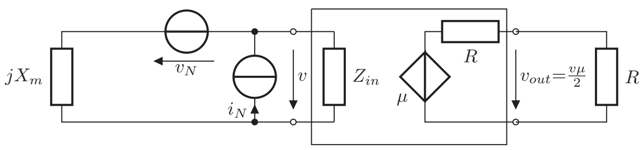

Figure 2: Amplifier with reactive termination

$$E[|v_m|^2] = 4\mathrm{k}TB\Re\{Z\}, E[v_m v_N*] = 0, E[v_m i_N*] = 0. \tag{6}$$

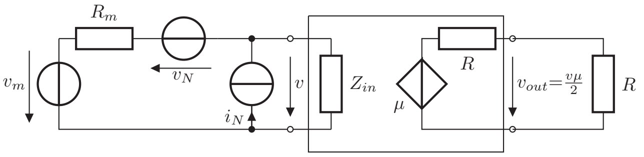

Figure 3: Amplifier with resistive termination

Usually an RF LNA has a considerable output impedance, therefore S- parameters [13] offer another possibility of describing the system. Note that the relationship between the open-circuit voltage gain and the forward transmittance $S_{21}$ is given by (7)

$$\mu = \left(1 + \frac{R}{Z_{in}}\right) S_{21}, \tag{7}$$

where $R$ is used as the port-reference resistance for specifying the S-parameters, so $S_{12} = 0$ and $S_{22} = 0$. The power delivered to the output termination is

$$\begin{aligned} P_m &= \frac{E[|v_{out}|^2]}{R} = \frac{|\mu|^2}{4R} E[|v|^2] = \\ &= \frac{|\mu|^2}{4R} E[|(v_m + v_N + i_N Z_m) \frac{Z_{in}}{Z_m + Z_{in}}|^2] = \\ &= \frac{|\mu|^2 |Z_{in}|^2}{4R|Z_m + Z_{in}|^2} (4kTBR_m + \\ &+ E[|v_N|^2] + E[|i_N|^2]|Z_m|^2) - \\ &- 2(R_m \Re\{E[v_N i_N*]\} + X_m \Im\{E[v_N i_N*]). \end{aligned} \tag{8}$$

Measure $P_m$ for different values of $Z_m$, say $Z_1, Z_2, Z_3$ up to $Z_M$ and define the vectors

$$\boldsymbol{p} = [P_1, P_2, P_3, ...P_M]^T, \tag{9}$$

$$\boldsymbol{g} = [\Re\{Z_1\},\ \Re\{Z_2\},\ \Re\{Z_3\}, ...\ \Re\{Z_M\}]^T, \tag{10}$$

$$\boldsymbol{g}_m = \left[|Z_m|^2,\ 1,\ -2\Re\{Z_m\},\ -2\Im\{Z_m\}\right]^T, \tag{11}$$

as well as the matrices:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g_1^T} \\ \boldsymbol{g_2^T} \\ \cdots \\ \boldsymbol{g_M^T} \end{bmatrix} \tag{12}$$

$$\boldsymbol{\Phi} = \mathrm{diag}_{m=1}^{M} \left(\frac{1}{|Z_m + Z_{in}|^2}\right). \tag{13}$$

With the help of these considerations this vector can be written $\boldsymbol{p}$ compactly as

$$\boldsymbol{p} = \frac{|Z_{in}|^2|\mu|^2}{4R} \boldsymbol{\Phi} \left(\boldsymbol{G\theta} + 4\mathrm{k}TB\boldsymbol{g}\right) \tag{14}$$

where the noise parameters are put into the vector

$$\boldsymbol{\theta} = \left[E[|i_N|^2], E[|v_N|^2], \Re\{E[v_N i_N^*]\}, \Im\{E[v_N i_N^*]\}\right]. \tag{15}$$

Provided, that *det* $(\boldsymbol{G^T G}) \neq 0$, one can solve this system of linear equations uniquely for $\boldsymbol{\theta}$ in the least square sense:

$$\boldsymbol{\theta}_{LS} = \boldsymbol{G}^{+} \left(\frac{4R}{|Z_{in}|^2|\mu|^2} \boldsymbol{\Phi}^{-1} \boldsymbol{p} - 4\mathrm{k}TB\boldsymbol{g}\right). \tag{16}$$

## III. MEASUREMENT

For the measurement a low noise spectrum analyzer was used. At frequencies around 100 MHz, lumped elements with a quality factor of 100 and even more are available. The terminations are connected to the amplifier by SMA connectors. Each termination is mounted on its own SMA connector. Inductors were implemented as coils of 3 mm diameter made of 0.3 $\mathrm{mm}^2$ silver wire, as capacitances RF ceramic capacitors have been used.

### A. Amplifier design

The amplifier is a two stage common emitter circuit. It is a broadband amplifier, optimized for linearity, designed to work from 20 MHz to 1.5 GHz, which uses the low noise transistor BFT66 in its input stage. Fig. 5 shows the schematic of its first stage. Here, a 15 $\Omega$ emitter resistor and between base and collector 680 $\Omega$ and 1 nF are used for feedback. This causes lower gain, a higher noise figure but improves linearity and bandwidth very much. For power supply a 15 V source is used, a 475 $\Omega$ resistor limits the collector current to 13 mA and a 100 nF ceramic capacitor blocks the RF.

If the collector current is increased to achieve better linearity the noise figure will rise, too [12]. If the base emitter diode current increases, $R_N$ will move to lower values and more shot noise will be produced. Usually input stages of measurement equipment like spectrum analyzers are optimized in a similar way. This kind of amplifier fits best to show the trade off. To keep its IP3 high the 2.7 W RF power transistor BFQ34 is used,

Figure 4: Terminations

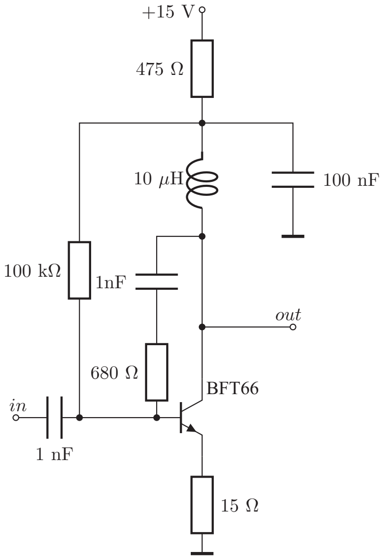

Figure 5: Amplifier 1st stage

collector to base and emitter feedback is also implemented. In the 2nd stage, due to the higher signal power a much stronger feedback is implemented. By its biasing network, the collector current is adjusted to 50 mA.

The RF encapsulated amplifier is put in an EMI shielded box. Shielding of cables and boxes have been tested with an RF power transmitter to be sure there will be no electromagnetic interference influencing the setup.

First the system parameters seen in Table I have to be determined. $Z_{\text{in}}$ and $|S_{21}|$ are known from a previous measurement with a network analyzer. In the next step the output power for each termination is measured and compared to the calculated value from plugging a least squares fit into (14).

The accuracy of the spectrum analyzer is limited to $0.1$ dB according to its data sheet. How many measurement points are needed to reach this limit? This analysis is done for the

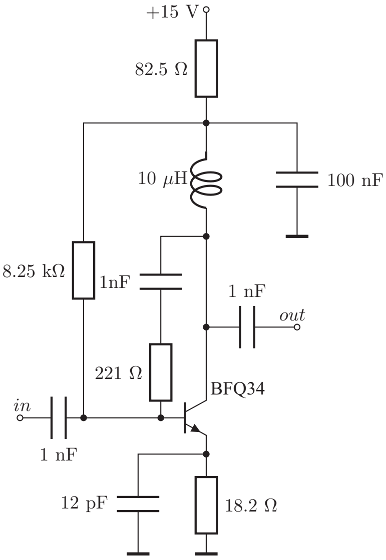

Figure 6: Amplifier 2nd stage

Figure 7: Amplifier in box

| Parameter | Value |
|---|---|
| $T$ | 290 K |
| $k$ | $1.38 \cdot 10^{-23}\ \frac{\text{VAs}}{\text{K}}$ |
| $B$ | 0.74 MHz |
| $R$ | 50 Ω |
| $Z_{in}$ | $(186 - j31.6)\ \Omega$ |
| $\lvert S_{2,1}\rvert$ | 9.55 |

Table I: System parameters

| Parameter | Value |
|---|---|
| $E[\lvert v_N\rvert^2]$ | $2.968 \cdot 10^{-13}$ V$^2$ |
| $E[\lvert i_N\rvert^2]$ | $2.844 \cdot 10^{-17}$ A$^2$ |
| $\Re\{\rho\}$ | 0.2730 |
| $\Im\{\rho\}$ | 0.1793 |

Table II: Stochastic amplifier parameter

| Parameter | Measurement | Calculation | Difference |
|---|---|---|---|
| 100 Ω | 14.0 dB$\mu$V | 13.9969 dB$\mu$V | −0.0031 dB |
| 50 Ω | 13.0 dB$\mu$V | 13.0286 dB$\mu$V | +0.0286 dB |
| 22 Ω | 12.0 dB$\mu$V | 11.9473 dB$\mu$V | −0.0527 dB |
| 0 Ω | 10.3 dB$\mu$ V | 10.3236 dB$\mu$V | +0.0236 dB |
| $-j29$ Ω | 10.7 dB$\mu$V | 10.7258 dB$\mu$V | +0.0258 dB |
| $-j43$ Ω | 10.9 dB$\mu$V | 11.0177 dB$\mu$V | +0.1177 dB |
| $-j83$ Ω | 12.0 dB$\mu$V | 11.9358 dB$\mu$V | −0.0642 dB |
| $+j12$ Ω | 10.3 dB$\mu$V | 10.2800 dB$\mu$V | −0.0200 dB |
| $+j26$ Ω | 10.4 dB$\mu$V | 10.3389 dB$\mu$V | −0.0611 dB |
| $+j50$ Ω | 10.7 dB$\mu$V | 10.6965 dB$\mu$V | −0.0035 dB |
| $+j120$ Ω | 12.5 dB$\mu$V | 12.5135 dB$\mu$V | +0.0135 dB |

Table III: Measured and recalculated powers for 11 terminations

four stochastic parameters and an overdetermined system of the equations (16), tuples of 5 to 15 points of measurement out of 23 points and the calculation of the mean error $|\Delta P|$, Fig. 8. In other words: There are subsets of measurements with only 5, 6, 7,...15 terminations and the error compared to using all 23 measurements shows, that from 13 measurements in a tuple no further improvement of accuracy is achieved.

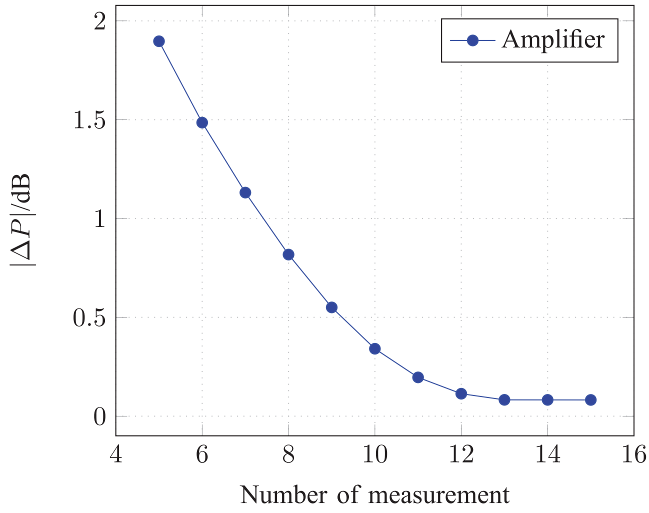

Figure 8: Mean error

With this method low noise amplifiers can be characterized in a quick way. This powerful method helps improving both matching networks and amplifiers. There are the noise figure for noise matching NF$_{NM}$, the required generator´s impedance for noise matching Z$_{\mathrm{NM}}$ and the noise figure for power matching NF$_{PM}$ calculated from these parameters, Z$_{\mathrm{PM}}$ is the impedance required for power matching are shown in Table IV

| Parameter | Value |
|---|---|
| NF$_{NM}$ | 1.3 dB |
| Z$_{\mathrm{NM}}$ | $100 - j18$ Ω |
| NF$_{PM}$ | 1.7 dB |
| Z$_{\mathrm{PM}}$ | $186 + j31.6$ Ω |

Table IV: Noise figures and source impedances

## IV. CONCLUSION

In this paper, a novel single port measurement technique for stochastic parameters of a low noise amplifier has been presented. No special input matching two port is necessary as in former established concepts. As proof of concept the measurement results for one amplifier are shown. First the deterministic parameters, reflection $S_{11}$ and forward gain $|S_{21}|$ have been measured with a network analyzer, the stochastic parameters $E[|v_N|^2], E[|i_N|^2], \Re\{\rho\}, \Im\{\rho\}$ were calculated from output power measurements with different input terminations by least squares method. Finally the recalculated output power has been compared to the measurements and it had been shown how accuracy is influenced by the number of measurements. Future investigations will extend the method to determine Z$_{\mathrm{in}}$ and $\mu$ or $S_{11}$ and $|S_{21}|$ also from power measurements only not needing any network analyzer. This will avoid possible nonlinearity problems especially in determining $S_{11}$.

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