Universität
Bielefeld

Forschungsberichte der
Technischen Fakultät
Abteilung Informationstechnik

Searching Correspondences in Colour Stereo Images - Recent Results Using the Fuzzy Integral

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Report 94-05

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Impressum: Herausgeber:
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1 Introduction

We introduce an approach for obtaining dense contour maps of objects, which is based on the combination of the images of two or more colour cameras [1]. As with all stereo techniques corresponding points in the image planes of the cameras, i.e. pixels illuminated by a certain point on the object, must be identified before the distance between the cameras and the object point is computed by triangulation. The search process should find as many correspondences as possible to enable the generation of dense range maps and the process must be accurate because errors in the pixel assignment directly affect the precision of the computed distance. With the correspondence search presented in Section 3 the function that determines the similarity between the image segments under consideration is a special application of the fuzzy integral and hence nonlinear. Through fuzzy measures, knowledge about the colour channels, e.g. their transfer function or their credibility, may be incorporated into the search process. Our first findings and some of our recent results, which are discussed in Section 4, indicate that the method is considerably more effective than the standard approach of minimizing the squared difference between line blocks. We conclude the report with a discussion of the potential of the method and directions of possible future research in Section 5.

[1]A previous version of this research report was submitted to and accepted for publication in Proc. of the Int. J. Conf. of the 4th IEEE Int. Conf. on Fuzzy Systems and the 2nd Int. Fuzzy Engineering Symp., Yokohama, Japan, 3/1995

2 Experimental Set-Up

To study the problem of finding correspondences in stereo images, an experimental set-up as shown in Figure 1 was used.

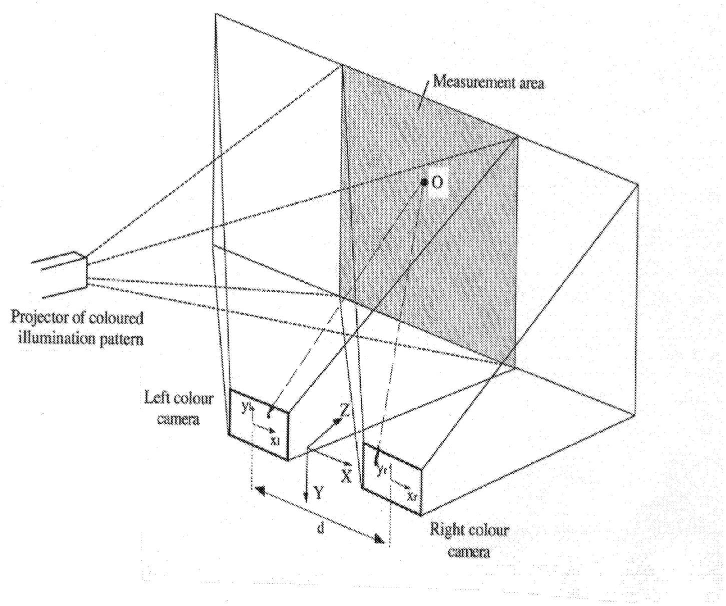


Figure 1: The basic experimental set-up for implementing the method

It resembles the usual stereo experiments (see [BF82, BY89, DA89] for good introductory papers to the field or [Kos93] for a recent survey); however, to alleviate the problem of the correspondence search, a continuous colour pattern is projected on the objects in the scene. The pattern consists of a multitude of colour hues in the horizontal direction; in the vertical direction the individual colour does not change. In other words: the colour values change only in one dimension, i.e. they vary continuously as we move over the projection plane from left to right (X-direction in Figure 1) but remain constant when we move vertically (in the Y-direction). Thus, from the projector's point of view, every vertical line (ideally an infinitesimally small stripe) of the object is marked with a unique colour.

Although the method for the correspondence search described below may be applied to passive colour stereo images without any changes, the active illumination of the scene offers several advantages for use in robotics including the potential for a very high speed of operation and hence the range image generation of fast moving objects in highly dynamic environments. The continuous colour pattern provides higher accuracy than other active triangulation techniques that employ discrete coloured light stripes or sequential binary-encoded monochrome patterns.

Since two cameras in a stereoscopic configuration are used the criterion to be met by corresponding points is that they have the same colour in both images. This colour is not necessarily equivalent to the colour that was emitted from the light source. This is the reason, therefore, that the method is perfectly well suited to non-structured scenes of uniform intensity (a search for edges or prominent object features is never necessary) but sets it apart from other colour-based techniques that employ a sequence of colour stripes whose absolute colour must be identified in an image recorded with a single camera [BK87, PJ91]. Unlike these, the proposed method is insensitive to colour changes caused by the object (provided they evenly affect both cameras) and does not depend on special cameras with high response stability over a large dynamic range (such as the method described in [Taj87]). Preliminary findings indicate that this is true for most materials with different reflectance functions, which are used for technical products (metals, all kinds of plastics, foams).

Once the correspondence search is accomplished, the distance of the image points can be calculated by simple triangulation. This implies that the accuracy of the computed distance is only limited by the ability of the cameras to differentiate between colours and their spatial resolution. There is no principal lower limit to accuracy inherent to all methods relying on discrete patterns.

3 Search for Correspondences

Traditionally, the search algorithms for intensity-based colour stereo are a straightforward extension of those used for grey-scale black and white images. Commonly, a similarity measure Ξ of the form

$$\Xi(H^{\mathrm{l}}, H^{\mathrm{r}}) = \sum_{j=-p}^{+p} |I_j^{\mathrm{l}} - I_j^{\mathrm{r}}| \tag{1}$$

or, more generally, a member of the family of L_k-norms

$$\Xi_{L_k}(H^{\mathrm{l}}, H^{\mathrm{r}}) = \left(\sum_{j=-p}^{p} |I_j^{\mathrm{l}} - I_j^{\mathrm{r}}|^k \right)^{\frac{1}{k}} , \ k > 0 \tag{2}$$

is minimized by trying several candidates for the correspondence in a local neighbourhood. Here, H^{l} and H^{r} denote blocks of adjacent pixels along a line in the left and along a line in the right image. For reasons of simplicity the disparity (i.e. the difference of absolute coordinates of the block centres) was omitted. I_j^{l} and I_j^{r} stand for the pixel intensities at the relative coordinate j and $2p+1$ is the block size.

The algorithm that is followed for finding the corresponding right pixel of a fixed pixel in the left image is conceptually very simple: First, a certain number of correspondence candidates in the right image are selected. This may be done using several criteria, the simplest (and most time-consuming) of which is to make no assumptions at all and define all points on the right image line to be candidates. Then, for each of the candidates at their points x_r (see Figure 1), the measure Ξ is computed by adding up the squared differences of the left and right pixel intensities for all the pixels in the intervall $[x_r - p, x_r + p]$, i.e. in a neighbourhood of $\pm p$ pixels. In a third step, the candidate for which the similarity measure is the lowest of all is chosen as the corresponding point.

3.1 Correspondence Search with Colour Images

In the case of colour images a pragmatic approach for finding correspondences is the intensity-based comparison of colour value relations over the two epipolar lines of the left and right image [KOS90]. The search depth can be reduced by introducing appropriate local and global disparity limits. A particular difficulty in the case of colour images is introduced by the fact that due to different observation angles the values of the colours in the two images may be different. This gives rise to the need for an intensity compensation. In a first version of our system a simple procedure was used to compensate for this: The basic idea (see [Sas93]) is that the colour reflected off a certain pixel is determined mainly by the largest of the three colour components. We assume that the colour vectors at an arbitrary pixel in the left and right image are linearly dependent, i.e. they are multiples of each other:

$$\begin{pmatrix} R^{\mathrm{l}} \\ G^{\mathrm{l}} \\ B^{\mathrm{l}} \end{pmatrix} = s \cdot \begin{pmatrix} R^{\mathrm{r}} \\ G^{\mathrm{r}} \\ B^{\mathrm{r}} \end{pmatrix} \tag{3}$$

where R, G and B are the intensities of the colour channels (with l, r indicating the left and right image). In order to scale the two colour vectors before applying the similarity measure, the factor s is determined as follows:

$$\begin{array}{llll} m := \max\left(R^{\mathrm{l}}, G^{\mathrm{l}}, B^{\mathrm{l}}\right) & & & \\ \text{IF} & m = R^{\mathrm{l}} & \text{THEN} & s := R^{\mathrm{l}}/R^{\mathrm{r}} \\ \text{ELSE IF} & m = G^{\mathrm{l}} & \text{THEN} & s := G^{\mathrm{l}}/G^{\mathrm{r}} \\ \text{ELSE} & & & s := B^{\mathrm{l}}/B^{\mathrm{r}} \end{array}$$

where for every pixel-pair of the neighbourhood within the block size $2p+1$ an individual value s is computed (two-dimensional windows are also possible, but at the expense of higher algorithmic complexity). Only after this normalization the similarity between two blocks H_F^{l} and H_F^{r} of adjacent colour pixels centered around the left and right image points $(R_0^{\mathrm{l}}, G_0^{\mathrm{l}}, B_0^{\mathrm{l}})$ and $(R_0^{\mathrm{r}}, G_0^{\mathrm{r}}, B_0^{\mathrm{r}})$ to be compared is computed according to:

$$\begin{aligned} \Xi^*(H_F^{\mathrm{l}}, H_F^{\mathrm{r}}) \quad = \quad & \sum_{j=-p}^{+p} \left[\left(R_j^{\mathrm{l}} - s_j R_j^{\mathrm{r}}\right)^2 + \left(G_j^{\mathrm{l}} - s_j G_j^{\mathrm{r}}\right)^2 \right. \\ & \left. + \left(B_j^{\mathrm{l}} - s_j B_j^{\mathrm{r}}\right)^2 + m_j^2 \left(1 - \frac{1}{s_j}\right)^2 \right] \end{aligned} \tag{4}$$

The last term of (4) was added to consider especially the difference between the intensities for which the left colour pixel has maximum intensity. The candidate for which (4) reaches a global minimum within the considered image line is chosen as the corresponding point. The scaling of the colour vectors acquired by the cameras but also the incorporation of knowledge about both different sensitivities and cross-correlations between the colour channels along with knowledge about the credibility of the data provided by the channels is possible in a natural way using fuzzy measures and the fuzzy integral. The latter has already been applied to problems of the fields of image segmentation and object recognition (see [KK94, TK92]).

3.2 Fuzzy Measures for Expressing the Relative Importance of the Colour Channels

The concept of a fuzzy measure, i.e. a set function possessing the property of monotonicity, but not necessarily that of additivity, which is used as a subjective scale for fuzziness was originated by Sugeno (see [Sug74, Sug77]). The motivation of the following presentation is the interpretation of a fuzzy measure value $g(\cdot)$ as a measure for the importance, relevance or credibility of an individual colour channel or a selection of colour channels for their contribution to the process of finding correspondences within a certain block. In principle, these may also depend on the scene, i.e. the picture contents, but this will not be considered here.

The behaviour of the fuzzy measure intuitively mirrors the real conditions, where the basic set consists of the colour channels of the cameras, $F = \{R, G, B\}$:

- If none of the colour channels is evaluated ($A = \{\} \in \mathcal{P}(F)$), the contribution of this to further insights is nil:
$$g(\{\}) = 0$$
- If all colour channels are evaluated ($A = F$), then the importance is a maximum:
$$g(\{R, G, B\}) = 1$$
- The importance of an observation based on a selection of channels $A_1 \in \mathcal{P}(F)$ is smaller or equal to the importance of an observation based on $A_2 \in \mathcal{P}(F)$ if A_2 contains additionally more channels than those already contained in A_1:
$$A_1 \subset A_2 \;\Rightarrow\; g(A_1) \leq g(A_2)$$

In the sequel we use the well-known g_λ-fuzzy measure as introduced by Sugeno. To each of the colour channels $f \in F$ an importance for its contribution to the identification of correspondences is assigned through $g_\lambda(\{f\}) \in [0, 1]$. Then, using the λ-combination rule, one obtains for all $A_1, A_2 \subset F$ and $A_1 \cap A_2 = \{\}$:

$$g_\lambda(A_1 \cup A_2) = g_\lambda(A_1) + g_\lambda(A_2) + \lambda \cdot g_\lambda(A_1) \cdot g_\lambda(A_2) \tag{5}$$

$$-1 < \lambda < \infty$$

To each element of the power set $\mathcal{P}(F)$ an unambiguous degree of importance is assigned (for further details, e.g. the determination of λ or cross-references to other types of fuzzy measures, see [SW90, WK92]).

Using fuzzy measures thus enables the incorporation of knowledge about the sensitivity of the individual colour channels into the search process by first determining the relative performance of the channels and subsequently assigning these importance values to the different measures. One of the possibilities to aggregate these values is described below.

3.3 Correspondence Search Using the Fuzzy Integral

Formally, let

$$f \in F = \{R, G, B\},$$
$$f' \in F' = \{(H^{\mathrm{l}}_R, H^{\mathrm{r}}_R), (H^{\mathrm{l}}_G, H^{\mathrm{r}}_G), (H^{\mathrm{l}}_B, H^{\mathrm{r}}_B)\}$$

and g_λ a fuzzy measure defined on F'. Then, a method Ξ^* for comparing the similarity between the two colour blocks H^{l}_F and H^{r}_F can be obtained by fuzzy integration of the method Ξ over F' with respect to the fuzzy measure g_λ:

$$\begin{aligned} \Xi^*(H^{\mathrm{l}}_F, H^{\mathrm{r}}_F) &= ⨍_{F'} \Xi(f') \circ g_\lambda(\cdot) \\ &= \sup_{\alpha \in [0,1]} \min(\alpha, g_\lambda(\{f' \,|\, \Xi(f') \geq \alpha\})) \\ &= \sup_{A \in \mathcal{P}(F')} \min[\min_{f' \in A}(\Xi(f')), g_\lambda(A)] \end{aligned} \tag{6}$$

In principle, it is irrelevant what kind of image or object features the method Ξ acts on. In any case, according to the principle of minimum specification in the possibility theory [DP88] $\min_{f' \in A} \Xi(f')$ is interpreted as the decision of maximum safety for evaluating the similarity based on a certain set A of colour channels. The fuzzy integral then maximizes the degree of the match generated by the minimum operator between this decision of maximum safety on the correspondence hypothesis and the expectation of the importance of this decision with respect to the hypothesis.

It should be noted that within this framework even crisp standard methods, which are defined only for grey scale images and thus use the information contained in the colour channels separately to compute the similarity Ξ can be combined into a method Ξ^*, which makes use of the complete colour information and the correlations between the channels. Such standard methods are the aforementioned L_k-norms or the computation of the correlation coefficient. In [WK94] it is shown how an arbitrary number of similarity measures can be aggregated into a combined measure. It is shown further how it is possible to add new similarity measures.

We shall now turn to our practical experiments, in which we first made a simulation study to explore the behaviour of a prototypical camera transfer function. We then show some results obtained from real image data and compare these with results from standard correspondence search methods.

4 Experimental Results

To simulate the search for correspondences along an epipolar line the continuous colour pattern as used by Sasse [Sas93] was generated twice and subjected to nonlinear transformations. These transformations model the transfer functions of the individual colour channels f of

f	red	green	blue
$k=1$ (left)			
a	0.05	0.02	−0.06
b	50	−14	30
$k=2$ (right)			
a	0.04	0.03	−0.06
b	45	−12	20

σ_{red}	σ_{green}	σ_{blue}
$k=1$ (left)		
0.05	0.03	0.15
$k=2$ (right)		
0.01	0.08	0.12

Table 1: The transformation parameters used in our simulations

camera k; i.e. they establish relations between the true intensity values $I_{f,k}$ impinging on the lenses of the cameras and the output values of the cameras $I^*_{f,k}$. Furthermore, Gaussian observation noise as introduced by the cameras was added.

In concrete terms, we first applied a transformation T with

$$I^*_{f,k} = T(I_{f,k}) = I_{f,k}\left(1 + a_{f,k} + \frac{\ln(I_{f,k}+1)}{b_{f,k}}\right) \tag{7}$$

where $a_{f,k}$ and $b_{f,k}$ denote suitably chosen parameters.

After this transformation we added normally distributed white noise to the colour patterns. The simulation parameters of Table 1 were used in the experiments and were chosen so as to model closely the actual behaviour of our cameras.

Finally, the noisy patterns were subjected to a colour correction according to [Sas93] to warrant comparability. The resulting left and right colour patterns were used to investigate the quality of different methods for the correspondence search.

The block-based methods Ξ that were used in our simulations for evaluating the similarity of two pixels of the left and right colour pattern for each colour channel separately (see also [DP82, WK94, ZCB87]) were:

- **MSE**: The commonly used mean squared error applied to the pixel intensities within the block size,
- **POSS**: A method based on possibility distributions; before the comparison of two pixels, their intensity values in each channel were expressed as trapezoidal possibility distributions directly dependent on the chosen simulation parameters. They were then compared applying the well known consistency measure introduced by Dubois and Prade (see e.g. [DP88]):

$$\Xi(H^{\text{l}}, H^{\text{r}}) = \sum_{j=-p}^{+p}\left[\sup_{u\in U}\min\left(\pi_{I^{\text{l}}_j}(u), \pi_{I^{\text{r}}_j}(u)\right)\right] \tag{8}$$

  where $U = \{0, 1, \ldots, u_{\max}\}$ represents the set of available intensities, $\pi_{I^{\text{l}}_j}(u)$ and $\pi_{I^{\text{r}}_j}(u)$ the trapezoidal possibility distributions,

Ξ	Δ^2
MSE	6.00
POSS	5.73
INCL	4.89

Table 2: Average squared deviations Δ^2 of the found positions from the true positions (in pixels) when the methods *MSE*, *POSS* and *INCL* with summation according to (11) and subsequ. normalization were used

- **INCL**: A method based on an evaluation of the mutual inclusion of two intensity blocks according to [Ped89]:

$$\Xi(H^{\mathrm{l}}, H^{\mathrm{r}}) = \sum_{j=-p}^{+p} \top \left[\phi(I_j^{\mathrm{l}}, I_j^{\mathrm{r}}), \phi(I_j^{\mathrm{r}}, I_j^{\mathrm{l}}) \right] \tag{9}$$

with t-norm $\top$ and grade of inclusion of I^{l} in I^{r} (prov. $I^{\mathrm{l}}, I^{\mathrm{r}} \in [0,1]$) expressed as

$$\phi(I^{\mathrm{l}}, I^{\mathrm{r}}) = sup\{c \in [0,1] \mid \top(I^{\mathrm{l}}, c) \leq I^{\mathrm{r}}\} \tag{10}$$

In our experiments we used the bounded product.

To aggregate the methods Ξ acting on the individual channels into a measure Ξ^* for the similarity of two colour pixels, the common simple summation

$$\Xi^*(H_F^{\mathrm{l}}, H_F^{\mathrm{r}}) = \sum_{f \in F} \Xi(H_f^{\mathrm{l}}, H_f^{\mathrm{r}}) \tag{11}$$

with subsequent normalization was used (we write $\sum MSE$, $\sum POSS$, and $\sum INCL$, for short).

To this end for each of all the possible windows of block size 17 in the left colour pattern (which may be considered as one line in the left image) corresponding blocks in the right colour pattern (one line in the right image) were searched. Then, the average squared deviations Δ^2 between the true positions and the positions found by the considered evaluation methods were computed (see Table 2).

As shown in Figure 2 there are obvious differences in the behaviour of the methods. The graphs show the results of the evaluation of the pattern at the fixed position 109 in the left colour pattern and the resulting correspondence evaluations Ξ^* (scaled to lie in the unit interval) after searching in the right colour pattern with the above mentioned methods.

In the ideal case the graphs should show a peak at position 109 with a fast decay to the left and right. The methods $\sum MSE$ and $\sum INCL$ output 107 as the best match whereas the clearest decision is made by $\sum POSS$ for point 108.

Additionally we tested a method Ξ^* according to Equation (4), called *MMSE* for short. Although capable of compensating intensity changes better than the simple *MSE* plus summation, the method *MMSE* with an error value of 5.92 in the multiple search simulation

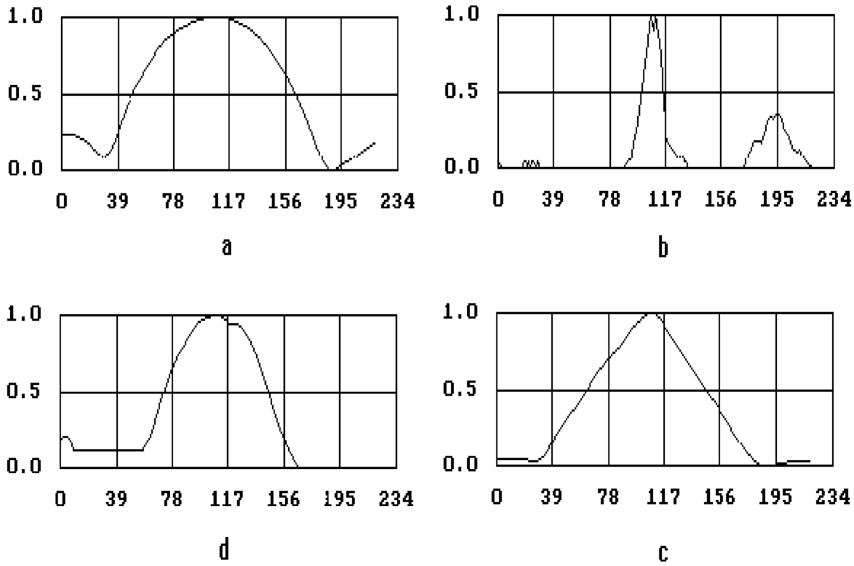


Figure 2: Resulting correspondence evaluations for searching a given left colour pattern at position 109 in the right colour pattern using the methods (a) $\sum MSE$, (b) $\sum POSS$, (c) $\sum INCL$, and (d) $\fint MSE$ with $g_\lambda(\{R\}) = 0.9$, $g_\lambda(\{G\}) = 0.2$, $g_\lambda(\{B\}) = 0.1$

and the maximum at position 107 in the last experiment performs worse than the others. This is because the derived evaluation curves were very flat; hence the matches ambiguous and errors more likely.

We conclude that the simple summation of the evaluation results produces acceptable but not really satisfactory results. Furthermore, the simple summation neglects potential differences between the colour channels and cannot integrate knowledge about these differences.

Motivated by these results, in a second step the behaviour of the fuzzy integral in the aggregation of similarity evaluations for the RGB-channels was explored. As an example the aggregation through fuzzy integration of the *MSE* evaluations for each of the three colour channels under the same simulation conditions as used before was chosen for our further experiments (using the notation $\fint MSE$, for short).

In the first experiment, the fuzzy measure g_λ was set to the coarse values $(0.9, 0.2, 0.1)$. The result of the multiple search experiment was already much better ($\Delta^2 = 5.1$) than the value of the simple method $\sum MSE$ (see also Figure 2(d)) and the one obtained with the *MMSE* evaluation. Interestingly, the result was also better than that obtained with $\sum POSS$ and approached $\sum INCL$'s quite closely.

In a second experiment, g_λ was tuned more carefully to the simulation using $(0.9, 0.7, 0.5)$. This produced the best result ($\Delta^2 = 3.97$) of all experiments and clearly showed the superiority of the fuzzy integral to simple summation.

To further verify the potential of the fuzzy integral to integrate knowledge about the recording system, a third experiment was made with the complements $(0.1, 0.3, 0.5)$ of the values, which have been experimentally validated. As expected, this results in the worst of all errors; it also shows that the values of g_λ must be carefully determined (e.g. in a series of experiments) so as to reflect the true behaviour of the system as represented by the fuzzy measure g_λ.

Based on the experience gathered in the simulation runs, a number of experiments were carried out with real data, which were acquired with a white automotive part. The colour pattern was the same as the one used in the simulation; it was projected using a standard slide projector. The images were recorded using two one-chip CCD cameras (Sony XC-711P), which have a resolution of 756×581 pixels, and were written directly into the frame grabber (ITI FG-100Q).

It should be noted that, when an identical scene is taken using the aforementioned two cameras, the two colour images are by no means identical; in particular the colour values vary drastically between the images. This was also the reason, therefore, that in [Sas93] for applying the method *MMSE* the two cameras were replaced with only one, whose location was changed from the left to the right position to record the left and right image, which is rather impractical. With the combination *MSE* and aggregation through the fuzzy integral, however, both images were recorded at the same time with the two cameras; the difference of the cameras is implicitly represented in the g_λ fuzzy measure. The test images are shown in Figure 3.

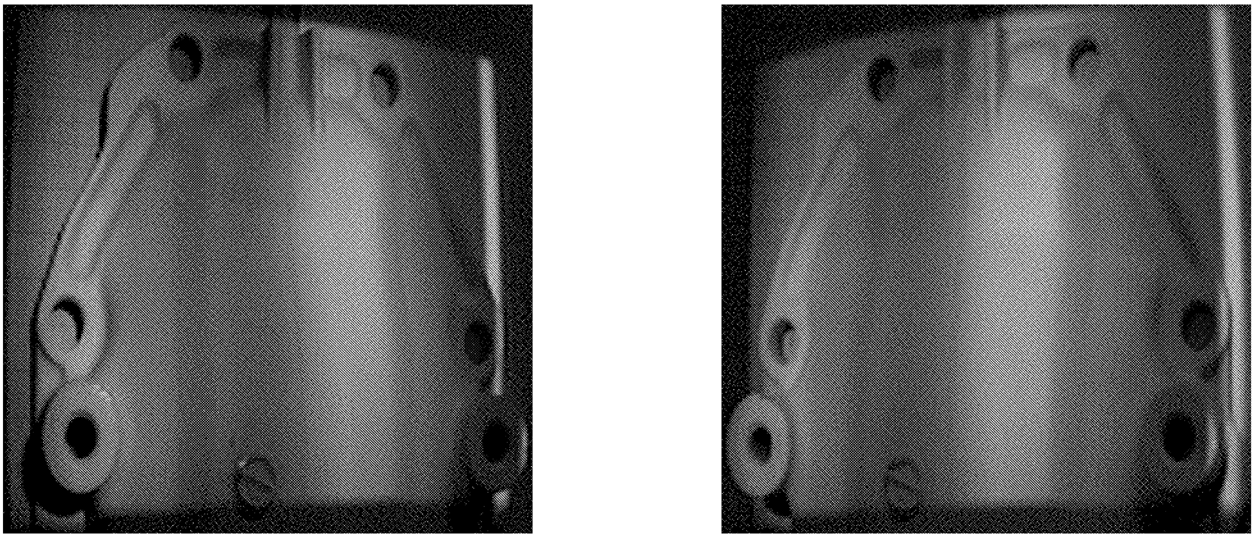


Figure 3: The test image, a white automotive part illuminated by the colour pattern from the left and right camera's points of view

As in the simulation, the correspondence search was performed using windows of block size 17. In a first series of tests to reduce the computation time, only a section of the whole image containing $42 \times 191 = 8022$ pixels was searched for correspondences.

The results of the different considered methods in the form of correspondence maps are shown in Figure 4(a)-(d). The light grey part of each figure shows the pixels that were assigned unambiguous and exact matches, the dark grey part shows pixels that were assigned correspondences but with associated uncertainty and the black part shows pixels that could not be assigned any correspondences at all. The latter were not illuminated by the slide projector and hence could not be assigned corresponding pixels in the other image. The inclination of the image section is due to the fact that sight-looking cameras were used, which is far more flexible than the usual set-ups with parallel optical axes. A match was defined to be exact if a correspondence P^r in the right image found for a fixed point P^l in the left image produced P^l as its correspondence when the left image was searched with P^r fixed.

If the match is not exact, then there is a remaining difference or uncertainty. The found correspondence was then marked as ambiguous and is shown in the dark grey part of the figures. The difference may either be due to the respective method's inability to find the exact corresponding pixels (e.g. it produces the result of 107 instead of 109) or due to unavoidable differences in the colour channels.

Note that with the simple pattern that was projected a large number of matches was reached in all those areas in which a mixed colour was projected, whereas in those sections in which there was only one channel active a much smaller number of correspondences was found. This situation can easily be improved if a more complicated pattern is projected; for a discussion of this issue, see [Sas93].

Figure 4(a) shows the results of the *MSE* evaluation plus the simple RGB-aggregation through summation ($\sum MSE$); of the 8022 pixels under consideration 2821 exact matches were found, while the other 5004 could not be assigned an exact match.

Figure 4(b) shows the results of the proposed combination, *MSE* evaluations for each of the colour channels plus aggregation through the fuzzy integral according to (6) ($\fint MSE$). Here, the density values of the fuzzy measure were adjusted coarsely according to the intuitive guesses of different experienced users. The values were $(0.9, 0.4, 0.1)$. This means that the red channel is over-weighted against the blue channel. Of the 8022 pixels 3252 were correctly assigned matches while for 4573 no unambiguous partners were found. This shows that even with weighting factors chosen only roughly, the nonlinear behaviour of the fuzzy integral is better suited to model the influences the cameras have on the true colour values than simple linear evaluation models.

If, as in Figure 4(c), the fuzzy measure is selected more carefully based on a series of tests of the channel performances of the different object regions, the results are even better. Here, the fuzzy measure was tuned to our best knowledge of the behaviour of the channels, i.e. their ability to find correspondences individually. With these values $(0.9, 0.7, 0.5)$ the results were 3367 correct matches and 4458 uncertain matches where the uncertainty in most cases was also much reduced. This was better than what was achievable with all the other methods.

Figure 4(d) shows a comparison with the method *MMSE*. As expected, this method is better than pure $\sum MSE$, but it cannot reach the number of matches obtained by $\fint MSE$: 3061 pixels were found and 4764 were not assigned unambiguously.

To complete this section we show some of our recent results concerning the whole test images (see Figure 3) that were produced using a C-Code-implementation on a Sun-SPARC-Workstation.

Ξ^*	unambiguous matches		ambiguous matches	
$\sum MSE$	32785	($\approx$ 43.9%)	38318	($\approx$ 51.3%)
$\sum WMSE$	34568	($\approx$ 46.2%)	36658	($\approx$ 49.0%)
$⨍ MSE$	37720	($\approx$ 50.5%)	33534	($\approx$ 44.9%)
$MMSE$	34941	($\approx$ 46.7%)	36287	($\approx$ 48.5%)

Table 3: Resulting correspondence evaluations using the methods $\sum MSE$, $\sum WMSE$ with $w_r = 0.6, w_g = 0.3$, and $w_3 = 0.1$, $⨍ MSE$ with $g_\lambda(\{R\}) = 0.9$, $g_\lambda(\{G\}) = 0.7$, $g_\lambda(\{B\}) = 0.5$, and $MMSE$ for the test images in Figure 3

Figure 5 shows the correspondence maps generated with the methods $\sum MSE$ and $⨍ MSE$. Here, a total number of 74760 pixels was searched for correspondences using windows of block size 17. The corresponding range maps (brighter grey levels represent shorter distances between the object and the stereo camera system) are shown in Figure 6.

Using the *MSE*-evaluations combined with the simple RGB-aggregation through summation results in 32785 unambiguous and 38318 ambiguous matches; the application of a fuzzy integral $(0.9, 0.7, 0.5)$ for fusing the single channel *MSE*-evaluations leads to a significant improvement in the system's performance, i.e. 37720 exact and 33534 ambiguous matches.

In further experiments we tested some weighted linear combinations of the *MSE*-evaluations ($\sum WMSE$, for short), i.e.

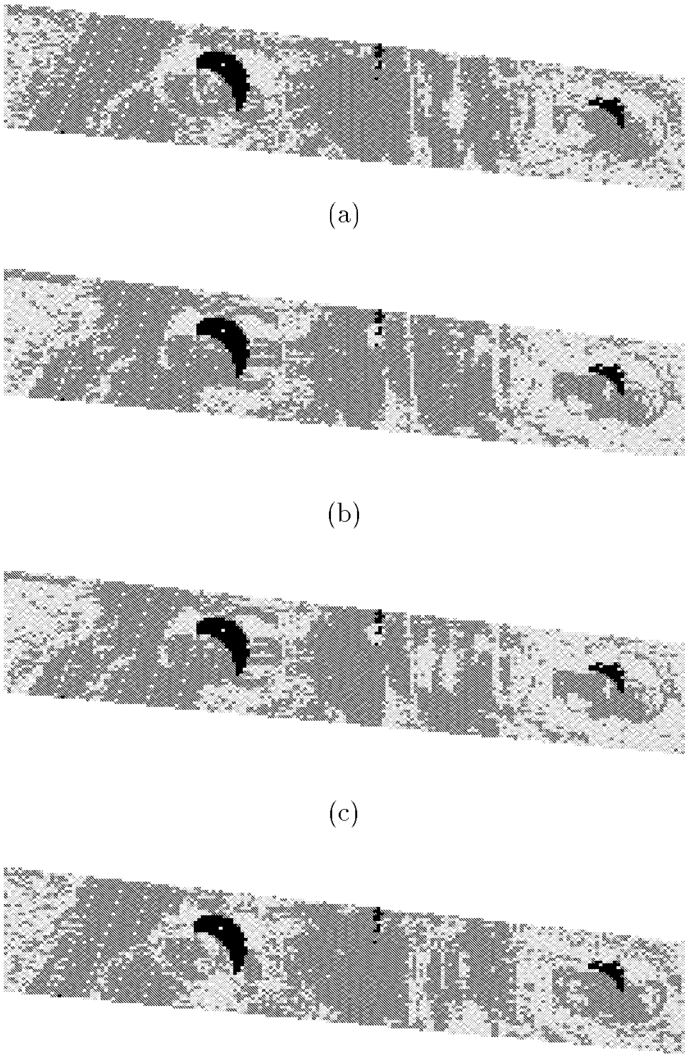
$$\Xi^*(H_F^{\mathrm{l}}, H_F^{\mathrm{r}}) = \sum_{j=-p}^{+p} \left[w_r \cdot \left(R_j^{\mathrm{l}} - R_j^{\mathrm{r}}\right)^2 + w_g \cdot \left(G_j^{\mathrm{l}} - G_j^{\mathrm{r}}\right)^2 + w_b \cdot \left(B_j^{\mathrm{l}} - B_j^{\mathrm{r}}\right)^2 \right] \qquad (12)$$

As expected, these performed worse (e.g. 34568 correctly marked matches using weights $w_r = 0.6, w_g = 0.3$, and $w_3 = 0.1$) than the nonlinear method using the fuzzy integral.

The method *MMSE* generated 34941 unambiguous and 36287 ambiguous matches. Interestingly, even using the sophisticated and computationally expensive colour correction method proposed in [Sas93] the results (35343 exact and 28250 ambiguous matches) cannot match the results produced using the fuzzy integral.

The aforementioned results are summarized in Table 3.

The question to be answered in the future is, of course, whether methods similar to *MMSE* or *POSS* plus the fuzzy integral outperform the simple *MSE* method. Though very likely, this remains to be proven.



(a) (b) (c) (d)

Figure 4: Correspondence maps showing exact (light grey), ambiguous (dark grey), and no matches (black) with the used methods (a) *MSE* plus summation, (b) *MSE* plus fuzzy integration based on coarse tuned fuzzy measure, (c) *MSE* plus fuzzy integration based on fine tuned fuzzy measure, and (d) *MMSE*

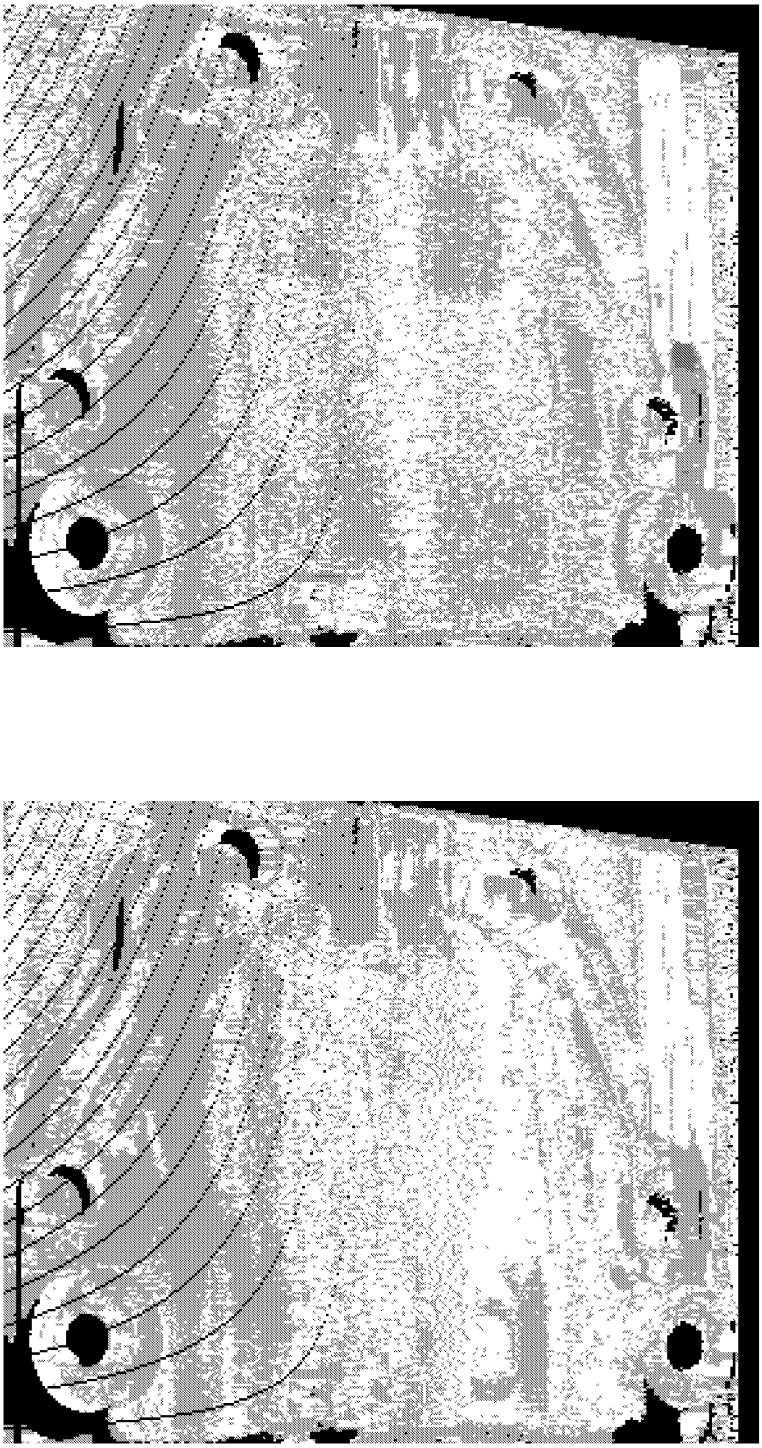
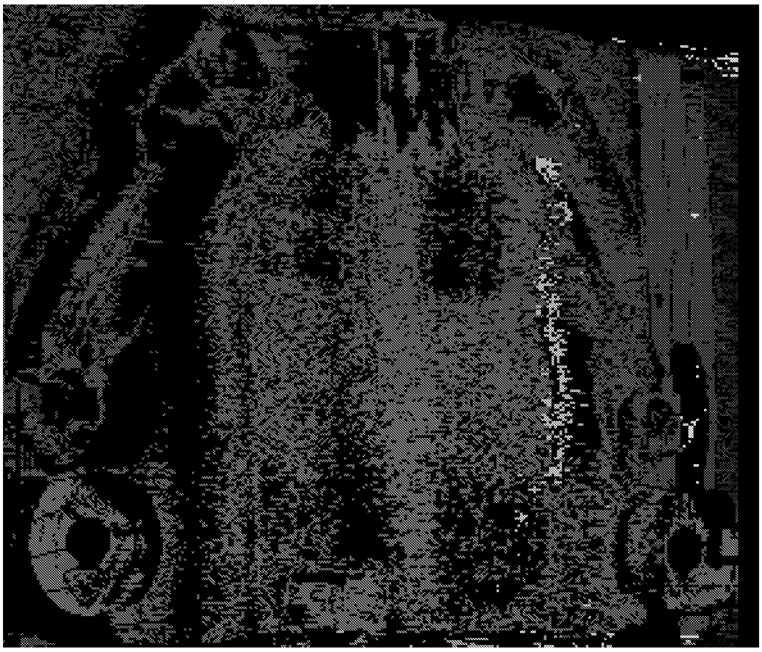


Figure 5: Correspondence maps showing exact (light grey), ambiguous (dark grey), and no matches (black) with the methods *MSE* plus summation (top) and *MSE* plus fuzzy integration (bottom)



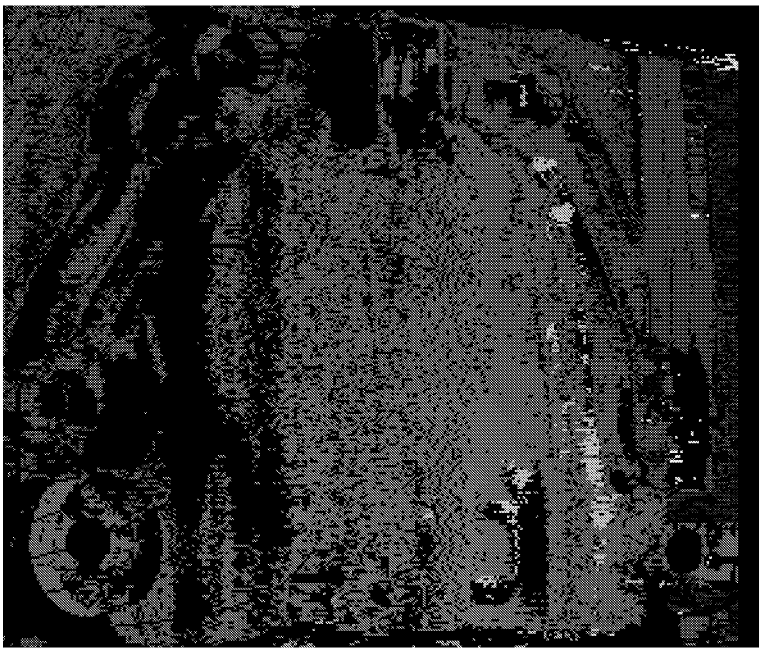


Figure 6: Distance maps computed with the methods *MSE* plus summation (top) and *MSE* plus fuzzy integration (bottom)

5 Conclusions

It was shown that the fuzzy integral can be applied successfully to the problem of correspondence search in colour stereo images. Further tests will reveal what classes of scenes may profit from this approach and how great the gain is over simpler methods.

We shall concentrate mainly on two lines of research: the first is along the adequate modelling of the nonlinearities in the colour channels, the correlation between them (in particular with one-chip CCDs) and the implications of this choice of the parameters. The second line is the improvement of the stereo matching using only one-dimensional intensity information, i.e. the evaluation of the single channels before the aggregation. Finally, instead of simple or slightly modified *MSE* methods, we shall explore the utilization of adaptive membership functions to model the behaviour of the channels.

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