Downlink Precoder and Equalizer Designs for Multi-User MIMO FBMC/OQAM

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***Abstract*—In this contribution we propose two new iterative precoder and equalizer designs for the *Downlink* (DL) scenario of *Multi-User* (MU)-MIMO systems which employ *Filter Bank based Multi-Carrier* (FBMC) with *Offset-Quadrature Amplitude Modulation* (O-QAM). In a MU-MIMO DL scenario, we must design our per-subcarrier filters to compensate the *inter-symbol* and *inter-carrier interference* (ISI and ICI) present in an FBMC/OQAM system in addition to *multi-user interference* (MUI). The first method presented takes advantage of the *Mean Squared Error* (MSE)-duality to design *Minimum MSE* (MMSE)-based precoders and equalizers. The second method looks at maximizing the *Signal-to-Leakage Ratio* (SLR) in the transmitter and the *Signal-to-Interference plus Noise Ratio* (SINR) in the receiver. Through numerical simulations we will evaluate the performance of these methods and compare them to recent approaches found in the literature.**

I. Introduction

In recent years, FBMC systems have received attention as a promising alternative to *Orthogonal Frequency Division Multiplexing* with *Cyclic-Prefix* (CP-OFDM) for the physical layer of the new 5-th generation mobile communication systems (5G). CP-OFDM is already a widely employed multi-carrier solution due to the simple equalization given the CP and an efficient implementation using the *Fast Fourier Transform* (FFT). However, this comes at the price of a loss in spectral efficiency due to the CP, which is extremely long in the presence of highly frequently selective channels. Furthermore, CP-OFDM comes with difficult synchronization requirements in the *Base Station* (BS) and the *User Equipment* (UE).

Due to the spectrally well designed *Synthesis* and *Analysis Filter Bank*s, (SFB) and (AFB), at the transmitters and the receivers [1], FBMC systems have a much lower out-of-band radiation compared with CP-OFDM [2]. By introducing the O-QAM, FBMC/OQAM systems do not require a CP and thus have an improved spectral efficiency. Using an appropriate design of the pulse shaping filters limits the ICI whilst contributing to more ISI within each sub-carrier. Furthermore, FBMC/OQAM systems are more efficient in the presence of highly frequency selective channels compared with CP-OFDM. This comes at the price of slightly higher computational complexity [3], [4].

To take advantage of the MU-MIMO DL scenario with *Space Division Multiple Access* (SDMA), we must introduce a multi-tap, fractionally spaced, complex valued, finite impulse response filters in the transmitters and/or receivers. These should be designed to mitigate the ISI, ICI and MUI.

In [5], a non-linear *spatial Tomlinson Harashima precoder* (STHP) design was introduced which showed promising results compared with CP-OFDM. However, this design was limited to a MU-MISO with a flat channel frequency response. The authors additionally looked at a block diagonalization design in [6] to mitigate the MUI and use a zero forcing based design to remove the remaining interference. In [7], the non-linear STHP design from [5] was generalized and a further iterative precoder and equalizer design was introduced to accommodate a multi-stream MU-MIMO scenario. However, this design was again limited to a flat channel frequency response. Furthermore, in [8] the authors look into splitting the computational complexity between the transmitter and receiver. They used two linear designs based on a maximization of the SLNR and SINR in the transmitter and receiver, respectively.

In [9], an iterative design for a *quasi* MMSE-based precoder filters and MMSE-based equalizer filters was introduced for the MU-MISO DL scenario. This design was extended in [10] to the MU-MIMO DL scenario and compared with an SLR-based precoder design. However, in both designs only a single tap, real valued equalizer with a *Maximal-Ratio Combining* (MRC) design was used at the receivers.

In this contribution we propose two new iterative designs for the single stream MU-MIMO DL scenario. The first takes advantage of the MSE-duality, [11], [12], between UL and DL scenarios, such that we only need to design MMSE-based MIMO equalizers and transform them into precoders. In the second method an iterative design will extend the SLR design in [10] to accommodate complex valued multi-tap equalizers at the receiver that maximize the SINR.

This paper is organized as follows; in Section II we briefly describe the MU-MIMO FBMC/OQAM model we investigated. In Section III and Section IV the two proposed precoders and equalizers designs will be discussed. Finally, in Section V and Section VI we will discuss the simulation results and draw our conclusions.

II. FBMC System Model

In a MIMO FBMC/OQAM system, the SFB in each transmitter antenna combines the M, complex valued QAM input signals $d_k^s[m]$ generated at a rate of $1/T_s$, into a single, complex valued signal generated at a higher rate of M/T_s. The signal is transmitted across a highly frequency selective additive white Gaussian noise channel to the receiver. In our system, M corresponds to the total number of sub-channels and M_u to the number of sub-carriers we transmit across. k corresponds to the sub-carrier index and s to the user index. The AFB separates the received signal back into its M_u components at a low rate per sub-carrier.

The first operation in the SFB is the O-QAM staggering $\mathcal{O}_k$ of the input symbols $d_k^s[m]$. The input symbol $d_k^s[m]$ is split into its real and imaginary parts, up-sampled by a factor of 2, then depending on which sub-carrier we observe, either the $\Re\{d_k^s[m]\}$ or $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed by $T_s/2$ and finally these components are added together. When the sub-carrier index k is even, the $\Re\{d_k^s[m]\}$ symbol is delayed and when the sub-carrier index is odd, the $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed. Therefore, the symbol $x_k^s[n]$ at the output of our $\mathcal{O}_k$ operation has an O-QAM structure, i.e., each symbol is either purely real or purely imaginary at a symbol rate $[n]$, which is double the symbol rate of the input signals $d_k^s[m]$. Due to this characteristic of the O-QAM symbols, there is a phase change of $\pi/2$ between immediately adjacent sub-carriers, ensuring orthogonality between sub-carriers. At the receiver, the AFB applies O-QAM de-staggering to reconstruct the complex QAM $\hat{d}_k^s[m]$ symbols at the original symbol rate from the equalized $\hat{x}_k^s[n]$ symbols.

After the O-QAM staggering the signals $x_k^s[n]$ are filtered by the multi-tap precoders, upsampled by $M/2$ and pulse-shaped by narrowband filters that allow a good spectral containment of each sub-carrier. At the AFB, similar filters are applied, a downsampling by $M/2$ and filtering by the equalizers are performed to the signals before the O-QAM de-staggering.

Efficient realization of the FBMC system can be achieved by taking advantage of exponentially modulated filters on both SFB and AFB given by

$$h_k[r] = h_\mathrm{p}[r] \exp\left(\mathrm{j}\,\frac{2\pi}{M}\,k\,\left(r - \frac{L_\mathrm{p}-1}{2}\right)\right),\ r = 0,\ldots,L_\mathrm{p}-1,$$

where $h_\mathrm{p}[r]$ is a lowpass narrowband prototype filter, here a *Root Raised Cosine* (RRC), with length $L_\mathrm{p} = KM+1$, with K representing the overlapping factor of the symbols in the time domain. K should be kept as small as possible not only to limit the complexity, but also to reduce the time-domain spreading of the symbols and the transmission latency. Furthermore, by taking advantage of the polyphase decomposition of $h_\mathrm{p}[r]$ all the filtering can be performed at a rate of only $2/T_s$. The complex modulation is perfomed by a FFT.

To minimize the complexity in the calculations of the equalizer and precoder filters, we set $K = 4$ and the roll-off factor of our RRC filter equal to one. Thus, the frequency response of the filter $\mathbf{h}_k$ only significantly overlaps with the two adjacent filters.

In our MU-MIMO FBMC/OQAM DL system, we have assumed the BS to have a total of N_t transmitter antennas, each with an SFB and each UE to have a total of $N_{\mathrm{r_s}}$ receiver antennas. In the MU-MIMO UL system we assume that the BS has $N_\mathrm{r} = N_\mathrm{t}$ receiver antennas and each UE has $N_{\mathrm{t_s}} = N_{\mathrm{r_s}}$ transmitter antennas. The total number of users is U.

To simplify the system model we define the following notation, $h_{l,r,j}^s[n] = \left(h_l * h_{\mathrm{ch},r,j}^s * h_k\right)[t]\ |_{t=nM/2}$. This represents the interference from the BS antenna j in sub-carrier l into the UE antenna r of user s in sub-carrier k. Where $l \in \{k-1, k, k+1\}$, $k \in \{1,\ldots,M_\mathrm{u}\}$, $s \in \{1,\ldots,U\}$, $j \in \{1,\ldots,N_\mathrm{t}\}$ and $r \in \{1,\ldots,N_{\mathrm{r_s}}\}$. To simplify notation we do not include the sub-script index of the receiver sub-carrier since the interference is always relative to k. Furthermore, in the following derivations we will stack or sum the vectors of equivalent channels over the antennas to further simplify the notation. The resulting filter has the length

$$Q = \left\lceil \frac{2(L_\mathrm{p}-1) + L_\mathrm{ch}}{M/2} \right\rceil,$$

with the prototype filter length and channel impulse response length, L_p and L_ch, respectively.

After the O-QAM staggering operation, the sequences of input symbols, $\mathbf{x}_k^s[n]$, have the structure

$$\mathbf{x}_k^s[n] = \begin{cases} \begin{bmatrix} \alpha_k^s[m] & \mathrm{j}\beta_k^s[m] & \alpha_k^s[m-1] & \cdots \end{bmatrix}^T, & k \text{ is odd},\\ \begin{bmatrix} \mathrm{j}\beta_k^s[m] & \alpha_k^s[m] & \mathrm{j}\beta_k^s[m-1] & \cdots \end{bmatrix}^T, & k \text{ is even}, \end{cases}$$

where $\alpha_k^s[m]$ and $\beta_k^s[m]$ represent the real and imaginary part of complex modulated QAM input symbol. In the following sections we work with a purely real notation and therefore define a purely real input sequence as $\mathbf{x}_k^s[n] = \mathbf{J}_k \tilde{\mathbf{x}}_k^s[n]$, i.e., $\tilde{\mathbf{x}}_k^s[n] \in \mathbb{R}^{B+Q-1}$ with

$$\mathbf{J}_k = \begin{cases} \operatorname{diag}\begin{bmatrix} 1 & \mathrm{j} & 1 & \mathrm{j} & \cdots \end{bmatrix}, & k \text{ is odd},\\ \operatorname{diag}\begin{bmatrix} \mathrm{j} & 1 & \mathrm{j} & 1 & \cdots \end{bmatrix}, & k \text{ is even}. \end{cases}$$

The matrix $\mathbf{J}_k$ extracts the imaginary j 's from the input signal. In the following derivations, we will multiply the transposed convolution matrices of the equivalent channels with $\mathbf{J}_k$ and work with purely real notation. It can be shown, [9] and [13], that calculating the precoder or equalizer filters with either the real or imaginary part of the input symbol both result in the same filters.

III. MSE-Duality based Precoder and Equalizer Design

In this section we discuss an iterative algorithm to design joint MMSE-based precoder and equalizer filter for the MU-MIMO DL scenario. However, it should be noted that we only design receiver filters from both the BS and UE perspectives. In the following sections we will use the notation $(\check{\bullet})$ and $(\hat{\bullet})$ to indicate the DL scenario and the UL scenario, respectively. In Algorithm 1, each step only depends on variables from the same iteration, thus to simplify notation, we will exclude the iteration index in the derivations that follow.

Our algorithm starts by initializing the equalizer filters in the DL scenario as a simple delay, i.e., the unit vector $\mathbf{e}$ with a 1 at the position $\lceil B/2 \rceil$. The initial DL to UL (DL/UL) MSE-duality transformation in Step 3 sets the scaling factor $\check{\gamma}$ in the (0)th iteration equal to 1 which means the precoder filter in the UL scenario is also a delay.

As already mentioned we only need to design MMSE-based equalizers for the UL and DL scenarios as seen in Step 6 and Step 9. In Step 7, we use the UL/DL MSE-duality transformation to calculate the DL precoder filter and in Step 10, we have to transform the DL equalizer filter into the UL precoder filter for the next iteration. Finally, our algorithm ends after a predefined number of iterations, n, have been executed. Algorithm 1 can be applied to all of the MSE-duality transformation, in step 7 and 10 we observe that our designs keeps the UL/DL and DL/UL MSE-duality transformations the same for all iterations.

Algorithm 1 Joint MMSE-based Precoder and Equalizer Design using the MSE-duality Transformations

1: **Initialization:**
2: $\check{\mathbf{w}}^v_{k,(0)} = \mathbf{e}_{\lceil \mathrm{B}/2 \rceil} \quad \forall v, k$
3: $\check{\gamma}_{(0)} = 1 \Rightarrow \hat{\mathbf{b}}^v_{k,(1)} = \mathbf{e}_{\lceil \mathrm{B}/2 \rceil} \quad \forall v, k$
4: $i = 1$
5: **repeat**
6: $\quad \hat{\mathbf{w}}^v_{k,(i)} = \arg\min \mathrm{E}\left[\left|\hat{\alpha}^v_{k,(i)}[n] - \alpha^v_{k,(i)}[n-\nu]\right|_2^2\right]$
7: $\quad \hat{\gamma}_{(i)}$ ▷ UL/DL MSE-duality transformation
8: $\quad \check{\mathbf{b}}^v_{k,(i)} = \hat{\gamma}_{(i)} \hat{\mathbf{w}}^v_{k,(i)}$
9: $\quad \check{\mathbf{w}}^v_{k,(i)} = \arg\min \mathrm{E}\left[\left|\check{\alpha}^v_{k,(i)}[n] - \alpha^v_{k,(i)}[n-\nu]\right|_2^2\right]$
10: $\quad \check{\gamma}^s_{(i)}$ ▷ DL/UL MSE-duality transformation
11: $\quad \hat{\mathbf{b}}^v_{k,(i+1)} = \check{\gamma}^v_{(i)} \check{\mathbf{w}}^v_{k,(i)}$
12: $\quad i = i + 1$
13: **until** $i = n$

A. Base station Perspective

In this sub-section we investigate the MU-MIMO UL scenario. We define a multi-tap, fractionally spaced equalizer $\hat{\mathbf{w}} \in \mathbb{C}^{L_{\mathrm{eq}}}$ per user, sub-carrier and BS receiver antenna and a multi-tap, fractionally spaced precoder $\hat{\mathbf{b}} \in \mathbb{C}^{B}$ per user, sub-carrier and UE transmitter antenna. In our system we have U decentralized users, each with N_{r_s} transmitter antennas. Each user transmits sequences $\mathbf{x}^1_k, \cdots, \mathbf{x}^U_k$ of *independent and identically distributed* (i.i.d.) and Gaussian distributed input signals in the sub-carriers $k \in \{1, \ldots, M_{\mathrm{u}}\}$ to the N_{r} centralized BS receiver antennas. Furthermore, we assume the O-QAM input symbols to be have half the variance of the QAM input symbols σ^2_d, i.e., $\mathrm{E}\left[\tilde{\mathbf{x}}^s_k[n]\tilde{\mathbf{x}}^{s,T}_k[n]\right] = (\sigma^2_{\mathrm{d}}/(2U))\mathbf{I} = \sigma^2_{\mathrm{M}}\mathbf{I}$. In the UL scenario, the real part of our received signal for user v in sub-carrier k is defined as

$$\hat{\alpha}^v_k[n] = \hat{\mathbf{w}}^{v,\mathrm{T}}_k \left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \hat{\mathbf{P}}^s_l \tilde{\mathbf{x}}^s_l[n] + \hat{\mathbf{\Xi}}_k \right), \tag{1}$$

where $\hat{\mathbf{P}}^s_l$ is the transposed convolution matrix of the equivalent UL channel. We have added the UL precoder filter into the total transmission channel, i.e., $\hat{p}^s_{l,r,j} = \hat{b}^s_{l,r} * h^s_{l,r,j}$ where s, l, j, r represent the user index, sub-carrier index, BS receiver antenna index and UE transmitter antenna index, respectively. Furthermore, $\hat{\mathbf{\Xi}}_k \in \mathbb{R}^{2BN_{\mathrm{t}} \times 1}$ contains the stacked real and imaginary parts of $\mathbf{\Gamma}_k \eta_j$ with $\mathbf{\Gamma}_k$ as an $M/2$ downsampled, transposed convolution matrix of $\mathbf{h}_k$ which filters the noise $\boldsymbol{\eta}_j$. We assume the additive noise is Gaussian distributed with $\boldsymbol{\eta}_j[n] \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, \sigma^2_\eta \mathbf{I}\right)$.

The optimization problem we wish to minimize is expressed with respect to the UL MSE $\hat{\epsilon}^v_k$ as

$$\begin{aligned} \hat{\mathbf{w}}^v_k &= \underset{\hat{\mathbf{w}}^v_k}{\arg\min}\, \mathrm{E}\left[\left|\hat{\alpha}^v_k[n] - \alpha^v_k[n-\nu]\right|^2\right], \\ &= \underset{\hat{\mathbf{w}}^v_k}{\arg\min}\, \hat{\epsilon}^v_k, \end{aligned} \tag{2}$$

where we define ν as the transmission latency in our system. This optimization represents Step 6 of Algorithm 1. We solve the optimization problem in (2) similar to [13], arriving at an MMSE-based equalizer filter for all receiver antennas,

$$\hat{\mathbf{w}}^v_k = \left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \sigma^2_{\mathrm{M}} \hat{\mathbf{P}}^s_l \hat{\mathbf{P}}^{s,\mathrm{T}}_l + \hat{\mathbf{R}}_\eta \right)^{-1} \sigma^2_{\mathrm{M}} \hat{\mathbf{P}}^v_k \mathbf{e}_\nu. \tag{3}$$

Given the MMSE-based equalizer we are left with a simplified, closed form expression for the UL MSE per user and per sub-carrier defined as

$$\hat{\epsilon}^v_k(\hat{\mathbf{w}}^v_k) = \sigma^2_{\mathrm{M}} \left(1 - \mathbf{e}^{\mathrm{T}}_\nu \hat{\mathbf{P}}^{v,\mathrm{T}}_k \hat{\mathbf{w}}^v_k \right), \tag{4}$$

where we define the matrices, stacked over the BS receiver antennas, as

$$\hat{\mathbf{w}}^{v,\mathrm{T}}_k = \left[\begin{array}{ccc} \bar{\mathbf{w}}^{v,\mathrm{T}}_{k,1} & \cdots & \bar{\mathbf{w}}^{v,\mathrm{T}}_{k,N_{\mathrm{r}}} \end{array}\right] \in \mathbb{R}^{1 \times 2BN_{\mathrm{r}}}, \tag{5}$$

$$\tilde{\mathbf{P}}^s_{l,r,j} = \mathbf{J}_l \mathbf{P}^s_{l,r,j} \in \mathbb{C}^{B \times (B + L_{\mathrm{eq}} + Q - 2)}, \tag{6}$$

$$\bar{\mathbf{P}}^s_{l,r,j} = \left[\begin{array}{cc} \Re\left\{\tilde{\mathbf{P}}^s_{l,r,j}\right\} & \Im\left\{\tilde{\mathbf{P}}^s_{l,r,j}\right\} \end{array}\right]^{\mathrm{T}} \tag{7}$$

$$\hat{\mathbf{P}}^s_l = \left[\begin{array}{ccc} \sum_{r=1}^{N_{\mathrm{t_s}}} \bar{\mathbf{P}}^s_{l,r,1} & \cdots & \sum_{r=1}^{N_{\mathrm{t_s}}} \bar{\mathbf{P}}^s_{l,r,N_{\mathrm{r}}} \end{array}\right]^{\mathrm{T}}, \tag{8}$$

$$\hat{\mathbf{R}}_\eta = \mathrm{blockdiag}\left[\begin{array}{ccc} \bar{\mathbf{R}}_{\eta_1} & \cdots & \bar{\mathbf{R}}_{\eta_{N_{\mathrm{r}}}} \end{array}\right], \tag{9}$$

$$\bar{\mathbf{R}}_{\eta,k} = \begin{bmatrix} \mathbf{R}_{\eta,k,1} & \mathbf{R}_{\eta,k,2} \\ -\mathbf{R}_{\eta,k,2} & \mathbf{R}_{\eta,k,1} \end{bmatrix} \in \mathbb{R}^{2B \times 2B}, \tag{10}$$

$$\text{with } \mathbf{R}_{\eta,k,1} = \frac{\sigma^2_\eta}{2} \left(\mathbf{\Gamma}^{(R)}_k \mathbf{\Gamma}^{(R),T}_k + \mathbf{\Gamma}^{(I)}_k \mathbf{\Gamma}^{(I),T}_k \right) \in \mathbb{R}^{B \times B}, \tag{11}$$

$$\mathbf{R}_{\eta,k,2} = \frac{\sigma^2_\eta}{2} \left(\mathbf{\Gamma}^{(R)}_k \mathbf{\Gamma}^{(I),T}_k - \mathbf{\Gamma}^{(I)}_k \mathbf{\Gamma}^{(R),T}_k \right) \in \mathbb{R}^{B \times B} \tag{12}$$

We use the notation $(\bar{\bullet})$ to indicate taking the real and imaginary part of a vector and stacking them on top of each other, i.e., $\bar{\mathbf{x}} = \left[\Re\{\mathbf{x}\}, \quad \Im\{\mathbf{x}\}\right]^{\mathrm{T}}$.

Now we move onto the MU-MIMO DL scenario where, from the BS perspective, we can define the real part of our receive signal for user v in sub-carrier k as

$$\check{\alpha}^v_k[n] = \sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \check{\mathbf{b}}^{s,\mathrm{T}}_l \check{\mathbf{Q}}^v_{kl} \tilde{\mathbf{x}}^s_l[n] + \Re\left\{ \check{\mathbf{w}}^{v,\mathrm{T}}_k \mathbf{\Gamma}_k \boldsymbol{\eta}^v[n] \right\}, \tag{13}$$

where $\check{\mathbf{b}}^s_l$ is defined as the dual stacking vector to $\hat{\mathbf{w}}^s_l$ with $N_{\mathrm{t}} = N_{\mathrm{r}}$. Furthermore, $\check{\mathbf{Q}}^v_{kl} = \left[\sum_{r=1}^{N_{\mathrm{r_s}}} \bar{\mathbf{Q}}^v_{kl,r,1} \cdots \sum_{r=1}^{N_{\mathrm{r_s}}} \bar{\mathbf{Q}}^v_{kl,r,N_{\mathrm{t}}}\right]^{\mathrm{T}}$ is the equivalent DL channel to move the DL equalizer into the total transmission chain, i.e., $\check{q}^{v}_{kl,r,j} = \check{w}^{v}_{k,r} * h^{v}_{l,r,j}$. Again, we assume that the input signals $\tilde{\mathbf{x}}^{s}_{k}[n]$ and the noise are i.i.d. and Gaussian distributed with an equivalent distribution to that defined in Section III-A.

The optimization problem in the DL scenario we would require to minimize is defined as

$$\begin{aligned}\check{\mathbf{b}}^{v}_{k} &= \arg\min_{\check{\mathbf{b}}^{v}_{k}} \mathrm{E}\left[\left|\check{\alpha}^{v}_{k}[n] - \alpha^{v}_{k}[n-\nu]\right|^{2}\right],\\ &= \arg\min_{\check{\mathbf{b}}^{v}_{k}} \check{\epsilon}^{v}_{k} \quad \text{s. t.} \sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}} \left\|\check{\mathbf{b}}^{v}_{k}\right\|_{2}^{2} \leq M_{\mathrm{u}}U.\end{aligned}$$

By plugging (13) into the argument of our optimization problem, we arrive at a close formed expression for the DL MSE from the BS perspective as

$$\begin{aligned}\check{\epsilon}^{v}_{k} = \sigma^{2}_{\mathrm{M}}\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1} \check{\mathbf{b}}^{s,\mathrm{T}}_{l}\check{\mathbf{Q}}^{v}_{kl}\check{\mathbf{Q}}^{v,\mathrm{T}}_{kl}\check{\mathbf{b}}^{s}_{l} - 2\mathbf{e}^{\mathrm{T}}_{\nu}\check{\mathbf{Q}}^{v,\mathrm{T}}_{k}\check{\mathbf{b}}^{v}_{k} + 1\right)\\ +\check{\mathbf{w}}^{v,\mathrm{T}}_{k}\check{\mathbf{R}}^{v}_{\eta}\check{\mathbf{w}}^{v}_{k}.\end{aligned} \tag{14}$$

B. BS-Side MSE-duality Transformations

In this sub-section we investigate the four different methods, from the BS perspective, of transforming our UL MIMO system into an equivalent DL MIMO system using the MSE-duality principle as introduced in [12] and [11]. In our iterative Algorithm 1, we are now at the UL to DL MSE-duality transformation in Step 7. In all the MSE-duality transformations, the total power is preserved [11], [12], i.e., $\sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\|\hat{\mathbf{b}}^{v}_{k}\|_{2}^{2} \leq M_{\mathrm{u}}U$. A more detailed description of the MSE-duality transformations for a MU-MISO FBMC/OQAM system can be found in [14].

1) UL/DL System-Wide Sum-MSE: First, we define a relation between the DL and UL filters with a single scaling factor for all users and sub-carriers such that

$$\check{\mathbf{b}}^{s}_{l} = \hat{\gamma}\hat{\mathbf{w}}^{s}_{l} \quad \text{and} \quad \check{\mathbf{w}}^{v}_{k} = \hat{\gamma}^{-1}\hat{\mathbf{b}}^{v}_{k}, \quad \hat{\gamma} \in \mathbb{R}_{+}. \tag{15}$$

In the next step we set the system-wide sum-MSE equal between the UL and the DL scenarios , i.e., $\sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\hat{\epsilon}^{v}_{k} \overset{!}{=} \sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\check{\epsilon}^{v}_{k}$, where the relation $\overset{!}{=}$ implies both sides of the equation must be equal. By solving this equation we can calculate a single scaling factor

$$\hat{\gamma}^{2} = \frac{\sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\hat{\mathbf{b}}^{v,\mathrm{T}}_{k}\check{\mathbf{R}}^{v}_{\eta}\hat{\mathbf{b}}^{v}_{k}}{\sum_{v=1}^{U}\sum_{k=1}^{M_{\mathrm{u}}}\sigma^{2}_{\mathrm{M}}\left(\mathbf{e}^{\mathrm{T}}_{\nu}\hat{\mathbf{P}}^{v,\mathrm{T}}_{k}\hat{\mathbf{w}}^{v}_{k} - \sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{w}}^{s,\mathrm{T}}_{l}\hat{\mathbf{P}}^{v}_{kl}\hat{\mathbf{P}}^{v,\mathrm{T}}_{kl}\hat{\mathbf{w}}^{s}_{l}\right)}. \tag{16}$$

2) UL/DL User-Wise Sum-MSE: Next, we define a relation between the DL and UL filters with a scaling factor per user such that

$$\check{\mathbf{b}}^{s}_{l} = \hat{\gamma}^{s}\hat{\mathbf{w}}^{s}_{l} \quad \text{and} \quad \check{\mathbf{w}}^{v}_{k} = \left(\hat{\gamma}^{v}\right)^{-1}\hat{\mathbf{b}}^{v}_{k}, \quad \hat{\gamma}^{s} \in \mathbb{R}_{+}. \tag{17}$$

Following this, we set the user-wise sum-MSE equal between the UL and the DL system , i.e., $\sum_{k=1}^{M_{\mathrm{u}}}\hat{\epsilon}^{v}_{k} \overset{!}{=} \sum_{k=1}^{M_{\mathrm{u}}}\check{\epsilon}^{v}_{k}, \quad \forall v \in \{1,\ldots,U\}$. We end up with the following system of linear equations,

$$\mathbf{A}^{s}\begin{bmatrix}\left(\hat{\gamma}^{1}\right)^{2}\\ \vdots\\ \left(\hat{\gamma}^{U}\right)^{2}\end{bmatrix} = \underbrace{\begin{bmatrix}\sum_{k=1}^{M_{\mathrm{u}}}\hat{\mathbf{b}}^{1,\mathrm{T}}_{k}\check{\mathbf{R}}^{1}_{\eta}\hat{\mathbf{b}}^{1}_{k}\\ \vdots\\ \sum_{k=1}^{M_{\mathrm{u}}}\hat{\mathbf{b}}^{U,\mathrm{T}}_{k}\check{\mathbf{R}}^{U}_{\eta}\hat{\mathbf{b}}^{U}_{k}\end{bmatrix}}_{\mathbf{y}^{s}}, \tag{18}$$

where the matrix $\mathbf{A}^{s} \in \mathbb{R}^{U\times U}$ has strictly positive main diagonal elements, defined as

$$[\mathbf{A}^{s}]_{v,y} = \begin{cases}\sum_{k=1}^{M_{\mathrm{u}}}\sum_{l=k-1}^{k+1}\sigma^{2}_{\mathrm{M}}\left(\mathbf{e}^{\mathrm{T}}_{\nu}\hat{\mathbf{P}}^{v,\mathrm{T}}_{k}\hat{\mathbf{w}}^{v}_{k}\right.\\ \qquad\left.-\hat{\mathbf{w}}^{v,\mathrm{T}}_{l}\hat{\mathbf{P}}^{v}_{kl}\hat{\mathbf{P}}^{v,\mathrm{T}}_{kl}\hat{\mathbf{w}}^{v}_{l}\right), & \text{if } v = y,\\ -\sum_{k=1}^{M_{\mathrm{u}}}\sum_{l=k-1}^{k+1}\sigma^{2}_{\mathrm{M}}\left(\hat{\mathbf{w}}^{y,\mathrm{T}}_{l}\hat{\mathbf{P}}^{v}_{kl}\hat{\mathbf{P}}^{v,\mathrm{T}}_{kl}\hat{\mathbf{w}}^{y}_{l}\right), & \text{if } v \neq y.\end{cases} \tag{19}$$

3) UL/DL Sub-Carrier-Wise Sum-MSE: Next, we define a relation between the DL and UL filters with a scaling factor per sub-carrier such that

$$\check{\mathbf{b}}^{s}_{l} = \hat{\gamma}_{l}\hat{\mathbf{w}}^{s}_{l} \quad \text{and} \quad \check{\mathbf{w}}^{v}_{k} = \hat{\gamma}^{-1}_{k}\hat{\mathbf{b}}^{v}_{k}, \quad \hat{\gamma}_{k} \in \mathbb{R}_{+}. \tag{20}$$

Next we set the sub-carrier-wise sum-MSE equal , i.e., $\sum_{v=1}^{U}\hat{\epsilon}^{v}_{k} \overset{!}{=} \sum_{v=1}^{U}\check{\epsilon}^{v}_{k}, \quad \forall k \in \{1,2,\ldots,M_{\mathrm{u}}\}$. We end up with the following system of linear equations

$$\mathbf{A}^{k}\begin{bmatrix}\hat{\gamma}^{2}_{1}\\ \vdots\\ \hat{\gamma}^{2}_{M_{\mathrm{u}}}\end{bmatrix} = \begin{bmatrix}\sum_{v=1}^{U}\hat{\mathbf{b}}^{v,\mathrm{T}}_{1}\check{\mathbf{R}}^{v}_{\eta}\hat{\mathbf{b}}^{v}_{1}\\ \vdots\\ \sum_{v=1}^{U}\hat{\mathbf{b}}^{v,\mathrm{T}}_{M_{\mathrm{u}}}\check{\mathbf{R}}^{v}_{\eta}\hat{\mathbf{b}}^{v}_{M_{\mathrm{u}}}\end{bmatrix}, \tag{21}$$

where the tri-diagonal matrix $\mathbf{A}^{k} \in \mathbb{R}^{M_{\mathrm{u}}\times M_{\mathrm{u}}}$ has strictly positive elements on the main diagonal and strictly negative off-diagonal elements, defined as

$$\left[\mathbf{A}^{k}\right]_{k,m} = \begin{cases}\sum_{v,s=1}^{U}\sigma^{2}_{\mathrm{M}}\left(\mathbf{e}^{\mathrm{T}}_{\nu}\hat{\mathbf{P}}^{v,\mathrm{T}}_{k}\hat{\mathbf{w}}^{v}_{k}\right.\\ \qquad\left.-\hat{\mathbf{w}}^{s,\mathrm{T}}_{k}\hat{\mathbf{P}}^{v}_{k}\hat{\mathbf{P}}^{v,\mathrm{T}}_{k}\hat{\mathbf{w}}^{s}_{k}\right), & \text{if } k = m,\\ -\sum_{v,s=1}^{U}\sigma^{2}_{\mathrm{M}}\left(\hat{\mathbf{w}}^{s,\mathrm{T}}_{m}\hat{\mathbf{P}}^{v}_{km}\hat{\mathbf{P}}^{v,\mathrm{T}}_{km}\hat{\mathbf{w}}^{s}_{m}\right), & \text{if } |k-m|=1,\\ 0 & \text{else.}\end{cases} \tag{22}$$

4) UL/DL User and Sub-Carrier-Wise MSE: Finally, we define a relation between the DL and UL filters with a scaling factor per user and per sub-carrier such that

$$\check{\mathbf{b}}^{s}_{l} = \hat{\gamma}^{s}_{l}\hat{\mathbf{w}}^{s}_{l} \quad \text{and} \quad \check{\mathbf{w}}^{v}_{k} = \left(\hat{\gamma}^{v}_{k}\right)^{-1}\hat{\mathbf{b}}^{v}_{k}, \quad \hat{\gamma}^{s}_{k} \in \mathbb{R}_{+} \tag{23}$$

We then set the user and sub-carrier-wise MSE equal between the UL and the DL system, i.e., $\check{\epsilon}^{v}_{k} \overset{!}{=} \hat{\epsilon}^{v}_{k}, \quad \forall v \in \{1,2,\ldots,U\}$ and $\forall k \in \{1,2,\ldots,M_{\mathrm{u}}\}$. We end up with the following system of linear equations

$$\begin{bmatrix}\mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \mathbf{0}_{U} & \cdots & \mathbf{0}_{U}\\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \mathbf{A}_{2,3} & \ddots & \vdots\\ \mathbf{0}_{U} & \ddots & \ddots & \ddots & \mathbf{0}_{U}\\ \vdots & \ddots & \ddots & \ddots & \mathbf{A}_{M_{\mathrm{u}}-1,M_{\mathrm{u}}}\\ \mathbf{0}_{U} & \cdots & \mathbf{0}_{U} & \mathbf{A}_{M_{\mathrm{u}},M_{\mathrm{u}}-1} & \mathbf{A}_{M_{\mathrm{u}},M_{\mathrm{u}}}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\gamma}}_{1}\\ \hat{\boldsymbol{\gamma}}_{2}\\ \hat{\boldsymbol{\gamma}}_{3}\\ \vdots\\ \vdots\\ \hat{\boldsymbol{\gamma}}_{M_{\mathrm{u}}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{\kappa}_{1}\\ \boldsymbol{\kappa}_{2}\\ \boldsymbol{\kappa}_{3}\\ \vdots\\ \vdots\\ \boldsymbol{\kappa}_{M_{\mathrm{u}}}\end{bmatrix}, \tag{24}$$

where $\mathbf{A}_{k,m} \in \mathbb{R}^{U\times U}$ and $\check{\boldsymbol{\gamma}}_k \in \mathbb{R}_+^U,\ k \in \{1,\ldots,M_{\mathrm{u}}\}$. The elements on the *Right-Hand-Side* (RHS) are defined as

$$\boldsymbol{\kappa}_k = \left[\hat{\mathbf{b}}_k^{1,\mathrm{T}}\check{\mathbf{R}}_\eta^1\hat{\mathbf{b}}_k^1, \quad \cdots, \quad \hat{\mathbf{b}}_k^{U,\mathrm{T}}\check{\mathbf{R}}_\eta^U\hat{\mathbf{b}}_k^U\right]^T. \tag{25}$$

We define the matrices $\mathbf{A}_{k,k}$ and $\mathbf{A}_{k,m}$ for $m \neq k$ as follows

$$[\mathbf{A}_{k,k}]_{v,s} = \begin{cases} \sigma_{\mathrm{M}}^2\left(\mathbf{e}_\nu^{\mathrm{T}}\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^v \right. \\ \qquad \left. -\hat{\mathbf{w}}_k^{v,\mathrm{T}}\hat{\mathbf{P}}_k^v\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^v\right), & \text{if } v = s, \\ -\sigma_{\mathrm{M}}^2\hat{\mathbf{w}}_k^{s,\mathrm{T}}\hat{\mathbf{P}}_k^v\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^s, & \text{if } v \neq s, \end{cases} \tag{26}$$

$$[\mathbf{A}_{k,m}]_{v,s} = \left\{-\sigma_{\mathrm{M}}^2\hat{\mathbf{w}}_m^{s,\mathrm{T}}\hat{\mathbf{P}}_m^v\hat{\mathbf{P}}_m^{v,\mathrm{T}}\hat{\mathbf{w}}_m^s, \quad \text{if } |k-m| = 1. \right. \tag{27}$$

C. User Equipment Perspective

In this sub-section we move on to look at the design of the DL MIMO MMSE-based equalizer filter from the UE's perspective. The following derivations represent Steps 9 and 10 in our iterative Algorithm 1.

From the UE perspective, the real part of the received signal for user v in sub-carrier k can be defined as

$$\check{\alpha}_k^v[n] = \check{\mathbf{w}}_k^{v,\mathrm{T}}\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\check{\mathbf{P}}_l^{sv}\tilde{\mathbf{x}}_l^s[n] + \Re\left\{\boldsymbol{\Gamma}_k\boldsymbol{\eta}^v\right\}\right), \tag{28}$$

where we define $\check{\mathbf{w}}_k^v$ as the multi-tap, fractionally spaced DL equalizer. Again, we assume i.i.d. input symbols $\mathbf{x}_k^s[n]$ and AWGN noise as defined in Section III-A. The transposed convolution matrix $\check{\mathbf{P}}_l^{sv}$ is the equivalent DL channel to move the DL precoder into the total transmission chain, i.e., $\check{p}_{l,r,j}^{sv} = \check{b}_{l,j}^s * h_{l,r,j}^v$.

The optimization problem we wish to minimize on the UE-side, is expressed with respect to the DL MSE $\check{\epsilon}_k^v$ as

$$\begin{aligned}\check{\mathbf{w}}_k^v &= \underset{\check{\mathbf{w}}_k^v}{\arg\min}\,\mathrm{E}\left[\left|\check{\alpha}_k^v[n] - \alpha_k^v[n-\nu]\right|^2\right], \\ &= \underset{\check{\mathbf{w}}_k^v}{\arg\min}\,\check{\epsilon}_k^v,\end{aligned} \tag{29}$$

To this end, the MMSE-based equalizer filter per user, and sub-carrier in the DL system is calculated as

$$\check{\mathbf{w}}_k^v = \left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\sigma_{\mathrm{M}}^2\check{\mathbf{P}}_l^{sv}\check{\mathbf{P}}_l^{sv,\mathrm{T}} + \check{\mathbf{R}}_\eta^v\right)^{-1}\sigma_{\mathrm{M}}^2\check{\mathbf{P}}_k^v\mathbf{e}_\nu. \tag{30}$$

Given the MMSE-based DL equalizer we are left with a simplified, closed form expression for the UE-side DL MSE ,

$$\check{\epsilon}_k^v\left(\check{\mathbf{w}}_k^v\right) = \sigma_{\mathrm{M}}^2\left(1 - \mathbf{e}_\nu^{\mathrm{T}}\check{\mathbf{P}}_k^{v,\mathrm{T}}\check{\mathbf{w}}_k^v\right). \tag{31}$$

Now we move on to the MU-MIMO UL scenario from the UE perspective. The real part of our received signal for user v in sub-carrier k is defined as

$$\hat{\alpha}_k^v[n] = \sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,\mathrm{T}}\hat{\mathbf{Q}}_{kl}^{vs}\tilde{\mathbf{x}}_l^s[n] + \hat{\mathbf{w}}_k^{v,\mathrm{T}}\Re\left\{\hat{\boldsymbol{\Gamma}}_k\hat{\boldsymbol{\eta}}\right\}, \tag{32}$$

where we define $\hat{\mathbf{b}}_k^v$ as a multi-tap, fractionally spaced precoder and $\hat{\mathbf{w}}_k^v$ is the UL MMSE-based equalizer designed in (3). The transposed convolution matrix $\hat{\mathbf{Q}}_l^{sv}$ is the equivalent UL channel to move the UL equalizer into the total transmission chain, i.e., $\hat{q}_{kl,r,j}^{vs} = \hat{w}_{k,r}^v * h_{l,r,j}^s$. Using (32) we can calculate the UL MSE as

$$\begin{aligned}\hat{\epsilon}_k^v =& \sigma_{\mathrm{M}}^2\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,\mathrm{T}}\hat{\mathbf{Q}}_{kl}^{vs}\hat{\mathbf{Q}}_{kl}^{vs,\mathrm{T}}\hat{\mathbf{b}}_l^s - 2\hat{\mathbf{b}}_k^{v,\mathrm{T}}\hat{\mathbf{Q}}_k^v\mathbf{e}_\nu + 1\right) \\ &+ \hat{\mathbf{w}}_k^{v,\mathrm{T}}\hat{\mathbf{R}}_\eta\hat{\mathbf{w}}_k^v\end{aligned} \tag{33}$$

D. User Equipment-Side MSE-duality Transformations

In this sub-section we investigate the two possible methods, from the UE perspective, of transforming our DL equalizer filters into equivalent UL precoder filters. These are similar to the transformations introduced in Sub-Section III-B, however, since we have decentralized users, we concluded that spreading the transmit power over the users was not meaningful. Therefore, we end up with only two forms of DL/UL MSE-duality transformations, i.e., the *DL/UL User-Wise Sum-MSE* and the *DL/UL User and Sub-carrier-Wise MSE* transformation. For consistency we matched the DL/UL MSE-transformation to the MSE-transformation used in the UL/DL scenario, i.e., where we summed over the sub-carriers we used the *DL/UL User-Wise Sum-MSE* transformation and where we summed over users or set the individual MSEs equal we used the *DL/UL User and Sub-carrier-Wise MSE* transformation.

1) DL/UL User-Wise Sum-MSE: Again, we define a relation between the UL and DL filters with a scaling factor per user such that

$$\hat{\mathbf{b}}_l^s = \check{\gamma}_{(i)}^s\check{\mathbf{w}}_l^s \quad \text{and} \quad \hat{\mathbf{w}}_k^v = \left(\check{\gamma}_{(i)}^v\right)^{-1}\check{\mathbf{b}}_k^v, \quad \check{\gamma}^s \in \mathbb{R}_+. \tag{34}$$

By summing over all sub-carriers as in Section III-B2 and setting this up for all users, we end up with a system of linear equations in the same form as (18). Now the RHS of the equation is defined as

$$\mathbf{y}^s = \left[\sum_{k=1}^{M_{\mathrm{u}}}\check{\mathbf{b}}_k^{1,\mathrm{T}}\hat{\mathbf{R}}_\eta^1\check{\mathbf{b}}_k^1 \quad \cdots \quad \sum_{k=1}^{M_{\mathrm{u}}}\check{\mathbf{b}}_k^{U,\mathrm{T}}\hat{\mathbf{R}}_\eta^U\check{\mathbf{b}}_k^U\right]^{\mathrm{T}}, \tag{35}$$

and the matrix $\mathbf{A}^s$ is defined as

$$[\mathbf{A}^s]_{v,y} = \begin{cases} \displaystyle\sum_{k=1}^{M_{\mathrm{u}}}\sum_{l=k-1}^{k+1}\sigma_{\mathrm{M}}^2\left(\mathbf{e}_\nu^{\mathrm{T}}\check{\mathbf{P}}_k^{v,\mathrm{T}}\check{\mathbf{w}}_k^v \right. \\ \qquad \left. -\check{\mathbf{w}}_l^{v,\mathrm{T}}\check{\mathbf{P}}_{kl}^{vs}\check{\mathbf{P}}_{kl}^{vs,\mathrm{T}}\check{\mathbf{w}}_l^v\right), & \text{if } v = y, \\ \displaystyle-\sum_{k=1}^{M_{\mathrm{u}}}\sum_{l=k-1}^{k+1}\sigma_{\mathrm{M}}^2\left(\check{\mathbf{w}}_l^{y,\mathrm{T}}\check{\mathbf{P}}_{kl}^{vy}\check{\mathbf{P}}_{kl}^{vy,\mathrm{T}}\check{\mathbf{w}}_l^y\right), & \text{if } v \neq y. \end{cases} \tag{36}$$

2) DL/UL User and Sub-Carrier-Wise MSE: Finally, we define a relation between the UL and DL filters with a scaling factor per user and per sub-carrier such that

$$\hat{\mathbf{b}}_l^s = \check{\gamma}_l^s\check{\mathbf{w}}_l^s \quad \text{and} \quad \hat{\mathbf{w}}_k^v = \left(\check{\gamma}_k^v\right)^{-1}\check{\mathbf{b}}_k^v, \quad \check{\gamma}_k^s \in \mathbb{R}_+. \tag{37}$$

We then set the UL and DL MSE equal for each user and sub-carrier similar to Section III-B4. Again, we end up with a system of linear equations similar to 24. The elements on the RHS of the system of equations are defined as

$$\boldsymbol{\kappa}_k = \left[\check{\mathbf{b}}_k^{1,\mathrm{T}}\hat{\mathbf{R}}_\eta^1\check{\mathbf{b}}_k^1, \quad \ldots, \quad \check{\mathbf{b}}_k^{U,\mathrm{T}}\hat{\mathbf{R}}_\eta^U\check{\mathbf{b}}_k^U\right]^T. \tag{38}$$

We define the tri-diagonal matrices $\mathbf{A}_{k,k}$ and $\mathbf{A}_{k,m}$ for $m \neq k$ as follows

$$[\mathbf{A}_{k,k}]_{v,s} = \begin{cases} \sigma_{\mathrm{M}}^2 \Big(\mathbf{e}_\nu^{\mathrm{T}} \check{\mathbf{P}}_k^{v,\mathrm{T}} \check{\mathbf{w}}_k^v \\ \qquad - \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{P}}_k^{v} \check{\mathbf{P}}_k^{v,\mathrm{T}} \check{\mathbf{w}}_k^v \Big), & \text{if } v = s, \\ -\sigma_{\mathrm{M}}^2 \check{\mathbf{w}}_k^{s,\mathrm{T}} \check{\mathbf{P}}_k^{vs} \check{\mathbf{P}}_k^{vs,\mathrm{T}} \check{\mathbf{w}}_k^s, & \text{if } v \neq s, \end{cases} \tag{39}$$

$$[\mathbf{A}_{k,m}]_{v,s} = \left\{ -\sigma_{\mathrm{M}}^2 \check{\mathbf{w}}_m^{s,\mathrm{T}} \check{\mathbf{P}}_{km}^{vs} \check{\mathbf{P}}_{km}^{vs,\mathrm{T}} \check{\mathbf{w}}_m^s, \quad \text{if } |k-m| = 1 \right. \tag{40}$$

IV. SLR-based Precoder and SINR-based Equalizer Design

The second design method we consider is a precoder designed to maximize the SLR and the equalizer designed to maximize the SINR. The SLR precoder design is similar to [10], however, in this contribution we employ complex valued, multi-tap equalizers.

The iterative alternating SLR/SINR design Algorithm 2 is similar to Algorithm 1 used to design the MMSE-based precoder and equalizer filters. Again, we initialize the SINR equalizer filter as a simple delay vector with a one at position $\lceil \mathrm{L}_{\mathrm{eq}}/2 \rceil$. The SLR-maximizing precoder is designed taking the equalizer filter from the previous iteration into account and the SINR-maximizing equalizer is designed taking the precoder filter from the current iteration into account. Analogously to the MSE-Duality derivations, we have omitted the iteration index (i) in the following derivations to simplify notation.

Algorithm 2 Joint SLR-based Precoder and SINR-based Equalizer Design

1: **Initialization:**
2: $\check{\mathbf{w}}_{k,(0)}^v = \mathbf{e}_{\lceil \mathrm{L}_{\mathrm{eq}}/2 \rceil} \quad \forall v, k$
3: $i = 1$
4: **repeat**
5: $\quad \check{\mathbf{b}}_{k,(i)}^v = \underset{\check{\mathbf{b}}_{k,(i)}^v}{\arg\max}\ \mathrm{SLR}_{k,(i)}^v(\check{\mathbf{w}}_{k,(i-1)}^v)$
6: $\quad \check{\mathbf{w}}_{k,(i)}^v = \underset{\check{\mathbf{w}}_{k,(i)}^v}{\arg\max}\ \mathrm{SINR}_{k,(i)}^v(\check{\mathbf{b}}_{k,(i)}^v)$
7: $\quad i = i + 1$
8: **until** $i = n$

A. SLR-Maximizing Precoder

In this section we design a complex valued, multi-tap precoder filter based on a maximization of the SLR, i.e., Step 5 of Algorithm 2. To derive a closed form expression for the SLR we must define the ICI, ISI, MUI and the noiseless received symbol for user v in sub-carrier k. Again, we assume i.i.d. input symbols $\mathbf{x}_k^s$ and AWGN noise as defined in Section III-A.

The sum ICI leaked into the imaginary part of the two adjacent sub-carriers is defined as

$$c_k^v = \sum_{l=k-1, l\neq k}^{k+1} \mathrm{E}\left[\left| \check{\mathbf{b}}_k^{v,\mathrm{T}} \check{\mathbf{\Phi}}_l^v \tilde{\mathbf{x}}_k^v[n] \right|^2 \right], \tag{41}$$

where $\check{\mathbf{\Phi}}_l^v \in \mathbb{R}^{2B\mathrm{N_t} \times (Q+B+L_{\mathrm{eq}}-2)}$ is a stacking matrix of the transposed convolution matrix of the equivalent channel channel, $q_{l,r,j}^v = \check{w}_{l,r}^v * h_{l,r,j}^v$ and we define

$$\mathbf{\Phi}_{l,r,j}^v = \left[\Im\left\{ \tilde{\mathbf{Q}}_{l,r,j}^v \right\} \quad -\Re\left\{ \tilde{\mathbf{Q}}_{l,r,j}^v \right\} \right]^{\mathrm{T}} \in \mathbb{R}^{2B \times (Q+B+L_{\mathrm{eq}}-2)}.$$

The ISI within sub-carrier k of user v is defined as

$$s_k^v = \mathrm{E}\left[\left| \check{\mathbf{b}}_k^{v,\mathrm{T}} \check{\mathbf{Q}}_k^{v,(\mathrm{ISI})} \tilde{\mathbf{x}}_k^v[n] \right|^2 \right] \tag{42}$$

with the matrix $\check{\mathbf{Q}}_k^{v,(\mathrm{ISI})} = \check{\mathbf{Q}}_k^v(\mathbf{I} - \mathbf{e}_\nu \mathbf{e}_\nu^{\mathrm{T}})$. Furthermore, the MUI leaked into the other users and their adjacent sub-carriers is defined as

$$u_k^v = \sum_{\substack{s=1 \\ s\neq v}}^{U} \sum_{\substack{l=k-1 \\ l\neq k}}^{k+1} \mathrm{E}\left[\left| \check{\mathbf{b}}_k^{v,\mathrm{T}} \check{\mathbf{\Phi}}_l^s \tilde{\mathbf{x}}_k^v[n] \right|^2 + \left| \check{\mathbf{b}}_k^{v,\mathrm{T}} \check{\mathbf{Q}}_k^s \tilde{\mathbf{x}}_k^v[n] \right|^2 \right]. \tag{43}$$

Additionally, we require the effective channel of user v in sub-carrier k from BS transmitter antenna j to all UE receiver antennas N_{r} for the noiseless received signal, defined as

$$\check{\mathbf{q}}_k^{v,(\mathrm{eff})} = \sum_{r=1}^{N_{\mathrm{r}}} \mathbf{q}_{k,r,j}^{v,(\mathrm{eff})} = \sum_{r=1}^{N_{\mathrm{r}}} \mathbf{Q}_{k,r,j}^v \mathbf{e}_\nu. \tag{44}$$

Finally, using (41), (43), (42) and (44), we end up with a closed form expression for the SLR of user v in sub-carrier k, expressed in (45). The precoder for user v in sub-carrier k is the maximizer of $\check{\mathbf{b}}_k^v = \underset{\check{\mathbf{b}}_k^v}{\arg\max}\ \mathrm{SLR}_k^v$, where the solution is calculated as the principal eigenvector corresponding to the maximum eigenvalue of the matrix $\{\mathbf{C}^{-1}\mathbf{A}\}$. Since the eigenvectors are already normalized to one, no further normalization of the precoder filters is required.

B. SINR-Maximizing Equalizer

In this section we design a complex valued, multi-tap equalizer vector based on a maximization of the SINR, i.e., Step 6 of Algorithm 2. To derive a closed form expression for the SINR we must again define the ICI, MUI, ISI, filtered noise and the noiseless received symbol for user v in sub-carrier k. Furthermore, we assume i.i.d. input symbols $\mathbf{x}_k^s[n]$ and AWGN noise as defined in Section III-A. The ICI received from the adjacent sub-carriers is defined as

$$\tilde{c}_k^v = \sum_{\substack{l=k-1 \\ l\neq k}}^{k+1} \mathrm{E}\left[\left| \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{P}}_l^{vv} \tilde{\mathbf{x}}_l^v[n] \right|^2 \right], \tag{46}$$

where $\check{\mathbf{P}}_l^{vv} \in \mathbb{R}^{2B\mathrm{N_t} \times (Q+B+L_{\mathrm{eq}}-2)}$ is a stacking matrix of the transposed convolution matrix of the equivalent channel channel, $p_{l,r,j}^{sv} = \check{b}_{l,j}^s * h_{l,r,j}^v$, equivalent to the matrix defined in Section III. The MUI received from the other users and from the adjacent sub-carriers is defined as

$$\tilde{u}_k^v = \sum_{\substack{s=1 \\ s\neq v}}^{U} \sum_{l=k-1}^{k+1} \mathrm{E}\left[\left| \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{P}}_l^{sv} \tilde{\mathbf{x}}_l^s[n] \right|^2 \right]. \tag{47}$$

The ISI within sub-carrier k of user v is defined as

$$\tilde{s}_k^v = \mathrm{E}\left[\left| \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{P}}_k^{v,(\mathrm{ISI})} \tilde{\mathbf{x}}_k^v[n] \right|^2 \right] \tag{48}$$

with the the ISI matrix equivalent to that found in Section IV-A, i.e., $\check{\mathbf{P}}_k^{v,(\mathrm{ISI})} = \check{\mathbf{P}}_k^{vv}(\mathbf{I}_Q - \mathbf{e}_\nu \mathbf{e}_\nu^{\mathrm{T}})$.

We require the effective channel of user v in sub-carrier k from BS transmitter antenna j to all UE receiver antennas N_{r} for the noiseless received signal, defined as

$$\mathrm{SLR}_k^v = \frac{\check{\mathbf{b}}_k^{v,\mathrm{T}} \overbrace{\left(\check{\mathbf{q}}_k^{v,,(\mathrm{eff})} \check{\mathbf{q}}_k^{v,,(\mathrm{eff}),\mathrm{T}}\right)}^{\mathbf{A}} \check{\mathbf{b}}_k^v}{\check{\mathbf{b}}_k^{v,\mathrm{T}} \underbrace{\left(\sum_{s=1}^{U} \sum_{\substack{l=k-1 \\ l \neq k}}^{k+1} \check{\mathbf{\Phi}}_l^s \check{\mathbf{\Phi}}_l^{s,\mathrm{T}} + \sum_{\substack{s=1 \\ s \neq v}}^{U} \check{\mathbf{Q}}_k^s \check{\mathbf{Q}}_k^{s,\mathrm{T}} + \check{\mathbf{Q}}_k^{v,(\mathrm{ISI})} \check{\mathbf{Q}}_k^{v,(\mathrm{ISI}),\mathrm{T}} \right)}_{\mathbf{C}} \check{\mathbf{b}}_k^v} \tag{45}$$

$$\check{\mathbf{p}}_k^{v,(\mathrm{eff})} = \sum_{j=1}^{N_\mathrm{t}} \mathbf{p}_{k,r,j}^{v,(\mathrm{eff})} = \sum_{j=1}^{N_\mathrm{t}} \mathbf{P}_{k,r,j}^v \mathbf{e}_\nu. \tag{49}$$

Additionally, we define the filtered noise of user v in sub-carrier k as $n_k^v = \mathrm{E}\left[\left|\check{\mathbf{w}}_k^{v,\mathrm{T}} \mathbf{\Gamma}_k \eta[n]\right|^2\right] = \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{R}}_\eta^v \check{\mathbf{w}}_k^v$.

Finally, a closed form expression for the SINR of user v in sub-carrier k, which the equalizer filter should maximize, is expressed in (50). The equalizer for user v in sub-carrier k is the maximizer of $\check{\mathbf{w}}_k^v = \underset{\check{\mathbf{w}}_k^v}{\arg\max} \mathrm{SINR}_k^v$, whereby the solution is calculated as the principle eigenvector corresponding to the maximum eigenvalue of the matrix $\{\tilde{\mathbf{C}}^{-1}\tilde{\mathbf{A}}\}$. Since the equalizers are calculated as eigenvectors normalized to one, we must scale the equalizer filters with the factor $\alpha = \sqrt{\check{\mathbf{w}}_k^{v,\mathrm{T}} \tilde{\mathbf{A}} \check{\mathbf{w}}_k^v}$ such that the symbols can be correctly decoded in the UEs.

V. Simulation Results

In this section we discuss the simulation results of the MSE-duality based design and the SLR/SINR based design for the DL MU-MIMO scenario. The channel realizations are from the *Wireless World Initiative New Radio* (WINNER II) project [15]. We transmit data across $M_\mathrm{u} = 210$ of the available $M = 256$ sub-carriers per user and per transmitter antenna with a sampling rate of $f_s = 15.36$ MHz, giving a sub-carrier spacing of 60 kHz. We randomly generate 16-QAM symbols and take a block length of 1000 symbols per sub-carrier. The channel impulse response is $L_\mathrm{ch} = 169$ taps. With these system configurations, especially due to $L_\mathrm{ch} = 169$ and the highly frequency selective channel, a CP-OFDM system would have required a CP with a minimum length of 168 taps [3], [4]. This limits the data-throughput of the CP-OFDM to more than 50%, therefore we do not include a direct comparison in the simulation results. We take the quantity of E_b/N_0 to be a pseudo-*Signal-to-Noise Ratio* (SNR) per user for the MU-MIMO simulations. We take the uncoded *Bit Error Rate* (BER) and MSE as an average over all users, and we average over 400 randomly generated channel realizations.

We have a precoder length of $B = 5$ taps and an equalizer length of $L_\mathrm{eq} = 3$ taps. Throughout our simulations we have a system with $N_\mathrm{t} = 4$ BS transmitter antennas and $U = 2$ users, each with $N_{\mathrm{r}_1} = N_{\mathrm{r}_2} = 2$ receiver antennas. We stopped both of our iterative algorithms after $n = 4$ iterations.

In Fig. 1 we see the uncoded BER versus SNR for the two iterative precoder and equalizer design algorithms introduced in this paper. We have compared our two iterative designs with an SLR-based precoder design with a real value, single tap equalizer from [10]. We observe that the MSE-duality based designs show a better performance over the whole SNR regime. Furthermore, in the high SNR regime, the MSE-duality transformations with the *System-wide Sum-MSE* and *User-wise Sum-MSE* UL/DL transformation show performance gains of more than 5dB compared with the other designs. This is attributed to the fact that these methods allow the total transmit power to be spread across all sub-carriers depending on the channel conditions, somewhat like an inverse waterfilling power allocation scheme.

Fig. 2 shows the MSE versus SNR of the two different iterative designs. We observe that all four MSE-duality based designs outperform the SLR/SINR based design in the low SNR regime due to the fact, that the SLR-precoder design does not take the noise variance into account, leading to worse MSE values.

In Fig. 3 we see the convergence, in dB, of the two different iterative designs. The solid curves show the MSE convergence of the MSE-duality based designs, and the dashed curve shows the SINR convergence of the SLR/SINR design. After 4 iterations, the values does not significantly improve anymore, which is why we stopped our algorithms after $n = 4$.

VI. Conclusions

We have presented two schemes for precoder and equalizer design for MU-MIMO DL FBMC/OQAM systems. Both are iterative and the first one is an MMSE design based on the MSE-duality and the second one maximizes the SLR and SINR in an alternating fashion. We can see that both methods present a similar uncoded BER performance where the MMSE based designs outperform the SLR/SINR based design and another current precoder design over the whole SNR regime. Moreover, we can see that all the iterative algorithms converge.

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$$\mathrm{SINR}_k^v = \frac{\breve{\mathbf{w}}_k^{v,\mathrm{T}} \overbrace{\left(\breve{\mathbf{p}}_k^{v,(\mathrm{eff})} \breve{\mathbf{p}}_k^{v,(\mathrm{eff}),\mathrm{T}}\right)}^{\tilde{\mathbf{A}}} \breve{\mathbf{w}}_k^v}{\breve{\mathbf{w}}_k^{v,\mathrm{T}} \underbrace{\left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \breve{\mathbf{P}}_l^{sv} \breve{\mathbf{P}}_l^{sv,\mathrm{T}} - \breve{\mathbf{P}}_k^{vv} \breve{\mathbf{P}}_k^{vv,\mathrm{T}} + \breve{\mathbf{P}}_k^{v,(\mathrm{ISI})} \breve{\mathbf{P}}_k^{v,(\mathrm{ISI}),\mathrm{T}} + \sigma_{\mathrm{M}}^{-2} \breve{\mathbf{R}}_\eta^v\right)}_{\tilde{\mathbf{C}}} \breve{\mathbf{w}}_k^v} \tag{50}$$

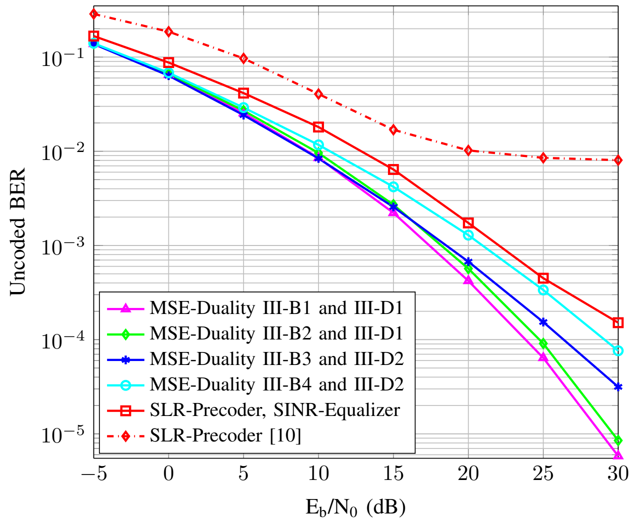

Figure 1. BER of the two iterative designs for $N_\mathrm{t} = 4$, $N_\mathrm{r} = 2$ and $U = 2$.

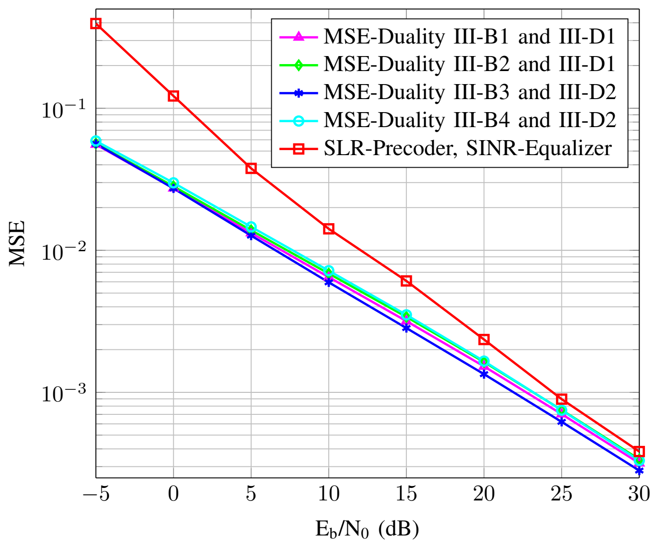

Figure 2. MSE of the two iterative designs for $N_\mathrm{t} = 4$, $N_\mathrm{r} = 2$ and $U = 2$.

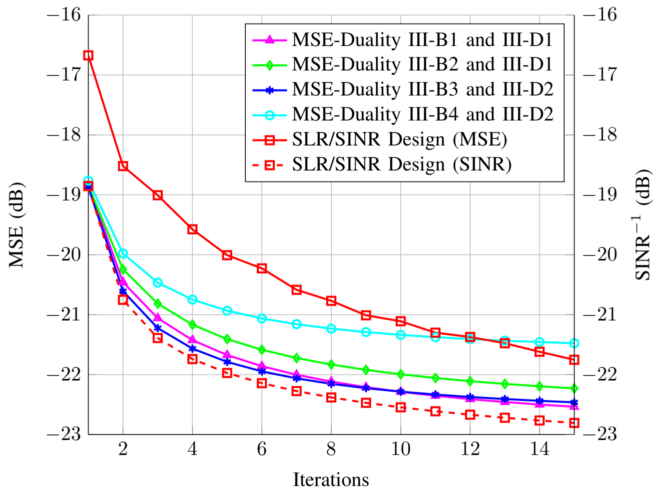

Figure 3. Algorithm convergence at $\mathrm{E_b/N_0} = 5$dB of the iterative designs for $N_\mathrm{t} = 4$, $N_\mathrm{r} = 2$ and $U = 2$.

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