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Magnetic electron-drift vortex modes in an inhomogeneous quantum plasma

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Abstract. Dispersion relation for the low-frequency magnetic electron-drift vortex (MEDV) mode in an inhomogeneous dense quantum plasma has been derived. New class of purely growing instabilities are found to exist on a time scale of the order of ion plasma period. We found that the MEDV mode become unstable when the propagation direction is perpendicular to the equilibrium density gradients. We believe that the present investigation would be useful to understand the wave phenomena and in the study of magnetic field generation in laser-produced plasmas as well as in dense space plasmas, where quantum effects are expected to play a dominant role.

1. Introduction

The field of quantum plasmas has generated a lot of interest in the plasma physics community owing to its wide range of applicability. Numerous investigations have been carried out in dense astrophysical environments [1, 2] in dusty plasmas [3, 4] (such as white dwarfs and neutron stars), in microelectronic devices [5], in intense laser-beam-produced plasmas [6], in nonlinear optics [7, 8], etc. to understand the quantum effects on the behavior of linear and nonlinear wave propagation in these systems. The quantum plasmas are characterized by high densities and low temperatures in sharp contrast to the low density and high temperatures that constitute the classical plasmas. When the plasma is cooled to extremely low temperatures, the de-Broglie wavelength of the charge carriers becomes comparable to the dimension of the system under consideration. In such a situation, the plasma behaves like a Fermi gas and quantum mechanical effects are expected to play a significant role in the behavior of charged particles [9–13].

The approaches that are frequently employed for quantum plasmas are the Schrödinger-Poisson, the Wigner-Poisson, and the Dirac–Maxwell which describe the statistical and hydrodynamic behavior of the plasma particles at quantum scales. These models are the quantum analogues of fluid and kinetic models of the classical plasma physics. Manfredi [14] wrote a review article on the Schrodinger–Poisson and the Wigner–Poisson models in a collisionless quantum plasma and introduced the rudimentary ideas in this regard. The quantum hydrodynamic model (QHD) is an extension of the classical fluid model in a plasma. The basic set of QHD equations describe the momentum and energy transport of the charged species. The departure from the classical model lies in the fact that an additional term, the so called 'Bohm potential', is introduced in the equation of motion of the charged particles. In the limit that the quantum effects go to zero, the classical fluid equation of motion is retrieved in accordance with the correspondence principle.

The QHD model has also been employed to study the propagation of linear and nonlinear waves in inhomogeneous quantum plasmas. Taibany and Wadati [11] studied the dynamics of nonlinear quantum dust acoustic wave in a non-uniform quantum dusty plasma and found that the formation of solitons showed a dependence on a critical value of plasma parameters unlike a homogeneous plasma. Shukla and Stenflo [15] found new drift modes in non-uniform quantum magnetoplasmas and observed that the electron-drift wave frequency got significantly modified by the electron Bohm potential term. Haque and Saleem [16] proposed that monopolar and dipolar quantum vortices could appear in uniform dense plasmas. Recently, Masood *et al.* [17] investigated the linear and nonlinear properties of ion-acoustic waves in an inhomogeneous and dissipative quantum magnetoplasma with sheared ion flows parallel to the ambient magnetic field. It was shown that the shear flow parallel to the magnetic field could drive the quantum ion-acoustic wave unstable. Stationary solutions of the nonlinear equations that govern the quantum ion-acoustic waves were also obtained. It was found that electrostatic monopolar, dipolar, and vortex street type solutions can appear in such a plasma. It was observed that the inclusion of the quantum statistical and Bohm potential terms significantly modified the scalelengths of these structures. Furthermore, it was shown that vortices form on a very short scalelength i.e. of the order of ion Larmor radius ρ_i in quantum magnetoplasmas. Quite recently, Shukla and Eliasson [18] discussed in great detail nonlinear electrostatic electron-ion plasma waves, novel aspects of three-dimensional electron fluid turbulence and nonlinear coupled intense electromagnetic waves and localized plasma wave structures in dense quantum plasma system.

It is well known that a non-uniform unmagnetized electron plasma with stationary ion background supports the magnetic electron-drift vortex (MEDV) mode [19–23]. The mode is flute-like, purely magnetic, and having frequency (proportional to v_{te}/L_{ne}, where v_{te} is the electron thermal velocity and $L_{ne} = n_{e0}/|\nabla n_{e0}|$ is the electron density scalelength) [24]. Physically, the generation of magnetic filed can be correlated with the electric force which is balanced with the electron pressure gradient force. In a non-uniform plasma, the curl of electric field is related with time varying magnetic filed whose strength can be of the order of megagauss, when the equilibrium density and temperature gradients are non-parallel. In a dense quantum plasma, the kinetic pressure is effectively replaced with Fermi pressure alongwith small correction term known as Bohm potential term, and as a result, the physical mechanism for the generation of MEDV mode in quantum plasma remains the same (i.e. the first order baroclinic type effect). The only difference is that the electron temperature perturbations is now replaced with a Fermi temperature perturbation. Since Fermi temperature is a function of density of plasma and consequently, in a quantum plasma, the baroclinic term appears in the form of $\nabla n_{e0} \times \nabla n_{e1}$. This mode is closely related to several well-known dissipative instabilities in collisional plasmas. The MEDV mode is shown to have a frequency identical to that of the magnetic electron-drift waves first studied by Chamberlain [26] with regard to the ionospheric plasmas. MEDV modes are, however, different in the sense that the perturbation magnetic field is polarized along the external field, thus causing magnetic compression [27]. In this paper, we derive the nonlinear equations governing the space–time evolution of MEDV mode in an inhomogeneous quantum plasma. New dispersion relation has been derived for the low-frequency MEDV mode in a non-uniform quantum plasma. Several interesting limiting cases are also discussed. It is found that the modes become unstable when the propagation direction is perpendicular to the equilibrium density gradients. The results of our investigation should be useful to understand wave phenomena in dense space and laboratory plasmas, where quantum effects are expected to play an important role.

2. Mathematical formulation

We consider a non-uniform dense quantum plasma whose constituents are the electrons and ions in the presence of electromagnetic disturbances. We consider a model in which electric field is along x- and y-axis, the equilibrium density of ion and electron density gradients are along the x-axis and the perturbations are along y-axis. The magnetic field $\mathbf{B}$ is of the form $\hat{z}B_1$, where $\hat{z}$ is the unit vector along the z-axis.

The dynamics of the model is governed by the continuity equation

$$\partial_t n_j + \nabla \cdot (n_j \mathbf{v}_j) = 0. \tag{2.1}$$

The equation of motion is given by

$$m_j n_{j0} \left(\partial_t + \mathbf{v}_j \cdot \nabla\right) \mathbf{v}_j = q_j n_{j0} \mathbf{E} - \nabla \left(2T_{Fj} n_{j1}\right) + n_{j0} \mathbf{F}_{Qj}, \tag{2.2}$$

where n_j, $\mathbf{v}_j$, and m_j are the number density, the fluid velocity, and the mass of jth species (j equals e for the electrons and i for the ions), $q_e = -e$, $q_i = e$, where e is the magnitude of the electron charge, T_{Fj} is the Fermi temperature where

$$T_{Fj} = \frac{\hbar^2}{2m_j}(3\pi^2)^{2/3} \left(n_{j0}\right)^{2/3}.$$

Here we assume that the plasma particles for a three-dimensional zero-temperature Fermi gas obey the following pressure law:

$$p_j = \frac{1}{3} \frac{m v_{Fj}^2}{n_{j0}^2} n_{j1}^3,$$

where $v_{Fj} = \sqrt{2T_{Fj}/m_j}$ is the Fermi speed, and T_{Fj} is the Fermi temperature expressed in the energy units. Here $\mathbf{F}_{Qj}$ is the quantum force

$$\mathbf{F}_{Qj} = \frac{\hbar^2}{4m_j n_{j0}} \nabla \nabla^2 n_{j1}.$$

The electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are governed by the Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B} \tag{2.3}$$

and the Ampere's law is

$$\nabla \times \mathbf{B} = \frac{4\pi e}{c}\left(n_{i0}\mathbf{v}_i - n_{e0}\mathbf{v}_e\right), \tag{2.4}$$

where the displacement current is ignored because we are dealing with electromagnetic waves, the phase velocity of which is much smaller than the speed of light c. The governing equation is closed with the help of Poisson's law,

$$\nabla \cdot \mathbf{E} = 4\pi e\left(n_i - n_e\right). \tag{2.5}$$

The equilibrium electron number density is denoted by $n_0(n_0 = n_{i0} = n_{e0})$, where the equilibrium drifts are taken zero for both ions and electrons ($\mathbf{v}_{i0} = \mathbf{v}_{e0} = 0$). Since we consider a plasma in which there are no background fields, i.e. $\mathbf{B}_0$ and $\mathbf{E}_0$ are taken to be zero, and there are no equilibrium drifts. Furthermore, in equilibrium, electron equation of motion (2.2) dictates that quantum Bohm potential term would balance the Fermi pressure for dense quantum plasmas. The perturbation magnetic field is assumed to have a form $\widehat{z}B_1$, where $\widehat{z}$ is the unit vector along the z-axis. Let us suppose that B_1 is proportional to $\exp(iky - i\omega t)$, where k is the wave number pointing in the y direction and ω is the wave frequency.

Suppose that the phase velocity of the mode is much larger than the ion Fermi velocity or the ions are consider to be cold as compare to the electrons and ignoring the quantum force in (2.2). The ion equation of motion can be written as

$$\partial_t \mathbf{v}_{i1} = \frac{e\mathbf{E}_1}{m_i}. \tag{2.6}$$

Similarly, from the electron equation of motion, we get

$$\partial_t \mathbf{v}_{e1} = \frac{e\mathbf{E}_1}{m_e} + \frac{\nabla\left(2T_{Fe}n_{e1}\right)}{n_0 m_e} + \frac{\mathbf{F}_{Qe}}{m_e}. \tag{2.7}$$

Subtracting (2.6) and (2.7), we get

$$\partial_t\left(\mathbf{v}_{i1} - \mathbf{v}_{e1}\right) = e\mathbf{E}_1\left(\frac{1}{m_e} + \frac{1}{m_i}\right) + \frac{\nabla\left(2T_{Fe}n_{e1}\right)}{n_0 m_e} + \frac{\mathbf{F}_{Qe}}{m_e}.$$

Using the Ampere's law,

$$\mathbf{v}_{i1} - \mathbf{v}_{e1} = \frac{c}{4\pi e n_0}\nabla \times \mathbf{B}_1$$

Substituting $(\mathbf{v}_{i1} - \mathbf{v}_{e1})$ in (2.7) and taking curl, we get

$$\frac{c}{4\pi e}\partial_t \nabla \times \left(\frac{\nabla \times \mathbf{B}_1}{n_0}\right) = \frac{e}{m_e}\nabla \times \mathbf{E}_1 - 2T_{Fe}\frac{\nabla n_0 \times \nabla n_{e1}}{n_0^2 m_e} + \frac{\nabla \times \mathbf{F}_{Qe}}{m_e}. \tag{2.8}$$

Here we have used the limit $m_i \gg m_e$:

$$-\frac{\partial}{\partial t}\left(\frac{\nabla n_0}{n_0^2} \times \left(\nabla \times \mathbf{B}_1\right)\right) - \frac{1}{n_0}\frac{\partial}{\partial t}\nabla_\perp^2 \mathbf{B}_1 = -\frac{4\pi e^2}{c^2 m_e}\left(\frac{\partial \mathbf{B}_1}{\partial t}\right)$$

$$-\frac{(8\pi e T_{Fe})}{c}\frac{\nabla n_0 \times \nabla n_{e1}}{n_0^2 m_e} - \frac{\pi e \hbar^2 \nabla n_0 \times \left(\nabla \nabla^2 n_{e1}\right)}{c m_e^2 n_0^2},$$

$$\partial_t\left(\nabla_\perp^2 - \frac{\omega_{pe}^2}{c^2}\right)\mathbf{B}_1 = \frac{(8\pi e T_{Fe})}{c}\frac{\nabla n_0 \times \nabla n_{e1}}{n_0 m_e} + \frac{\pi e \hbar^2 \nabla n_0 \times \left(\nabla \nabla^2 n_{e1}\right)}{c m_e^2 n_0},$$

where $\omega_{pe}^2 = 4\pi e^2 n_0/m_e$ is the electron plasma frequency

$$(\lambda_e^2\nabla_\perp^2 - 1)\partial_t \mathbf{B}_1 = 2cT_{Fe}\frac{\nabla n_0 \times \nabla n_{e1}}{en_0^2} + \frac{c\hbar^2\nabla n_0 \times (\nabla\nabla^2 n_{e1})}{4em_e n_0^2}, \tag{2.9}$$

where $\lambda_e = c/\omega_{pe}$ is the electron skin depth, and defines $1/L_{ne} \equiv |\nabla n_0|/n_0$. Applying Fourier transform to (2.9), we get

$$\omega\left(1 + k^2\lambda_e^2\right) B_1 = \frac{2kc}{en_0 L_{ne}}\left(T_{Fe} - \frac{k^2\hbar^2}{8m_e}\right) n_{e1}. \tag{2.10}$$

In order to find n_{e1}, we use continuity equation

$$\partial_t n_{e1} + \nabla\cdot(n_0\mathbf{v}_{e1}) = 0. \tag{2.11}$$

From the Ampere's law, we may write $\mathbf{v}_{e1} = \mathbf{v}_{i1} - \left(c/4\pi e n_0\right)\nabla\times\mathbf{B}$, therefore (2.11) can be written as

$$\partial_t n_{e1} + \nabla\cdot\left[n_0\mathbf{v}_i - \frac{c}{4\pi e}\nabla\times\mathbf{B}\right] = 0,$$

and its Fourier transform gives

$$-i\omega n_{e1} + n_0\nabla\cdot\mathbf{v}_{i1} + \mathbf{v}_{i1}\cdot\nabla n_0 = 0. \tag{2.12}$$

Inserting the value $\mathbf{v}_{i1}$ from (2.6) after its Fourier transform in (2.12), we get

$$i\omega n_{e1} + \frac{n_0 e}{i\omega m_i}\left(\nabla\cdot\mathbf{E}_1 + \mathbf{E}_1\cdot\frac{\nabla n_0}{n_0}\right) = 0. \tag{2.13}$$

In order to find $\nabla\cdot\mathbf{E}_1$, we use Poisson's equation. In Poisson's equation to find n_{e1} and n_{i1}we use continuity and equation of motion of electron and ion.

$$m_e n_0 \partial_t \mathbf{v}_{e1} = -en_0\mathbf{E}_1 - \nabla(2T_{Fe}n_{e1}) + n_0\mathbf{F}_{Qe1}$$

Taking divergence of above equation, we get

$$\nabla\cdot(\mathbf{v}_{e1}n_0) = -ie\frac{\nabla\cdot(n_0\mathbf{E}_1)}{m_e\omega} - \frac{i\nabla\cdot\nabla(2T_{Fe}n_{e1})}{m_e\omega} + \frac{i\nabla\cdot\left(n_0\mathbf{F}_{Qe1}\right)}{m_e\omega}. \tag{2.14}$$

Inserting (2.14) in the continuity equation of electron

$$\partial_t n_{e1} + \nabla\cdot(n_0\mathbf{v}_{e1}) = 0,$$

and after Fourier transform, we get

$$n_{e1} = \frac{-en_0}{m_e\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)}\left(\nabla\cdot\mathbf{E}_1 + \mathbf{E}_1\cdot\frac{\nabla n_0}{n_0}\right), \tag{2.15}$$

where $\mathbf{v}_F \equiv (2T_{Fe}/m_e)^{1/2}$ is the Fermi electron thermal velocity. Similarly, n_{i1} can be calculated from the continuity and momentum equation,

$$n_{i1} = \frac{en_0}{m_i\omega^2}\left(\nabla\cdot\mathbf{E}_1 + \mathbf{E}_1\cdot\frac{\nabla n_0}{n_0}\right). \tag{2.16}$$

Inserting the values of (2.15) and (2.16) in (2.5), we get

$$\nabla\cdot\mathbf{E}_1 = \left[\frac{4\pi n_0 e^2}{m_i\omega^2} + \frac{4\pi n_0 e^2}{m_e(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2)}\right]\left(\nabla\cdot\mathbf{E}_1 + \mathbf{E}_1\cdot\frac{\nabla n_0}{n_0}\right),$$

$$\nabla \cdot \mathbf{E}_1 = \left[\frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2)}\right]\left(\nabla \cdot \mathbf{E}_1 + \mathbf{E}_1 \cdot \frac{\nabla n_0}{n_0}\right),$$

where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ is the electron plasma frequency and $\omega_{pi} = \sqrt{4\pi n_0 e^2/m_i}$ is the ion plasma frequency:

$$\nabla \cdot \mathbf{E}_1 = \frac{\omega_{pi}^2 \left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right) + \omega_{pe}^2 \omega^2}{\left[\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)\left(\omega^2 - \omega_{pi}^2\right) - \omega^2 \omega_{pe}^2\right]} \mathbf{E}_1 \cdot \frac{\nabla n_0}{n_0}. \tag{2.17}$$

Inserting the (2.17) in (2.12), we get

$$n_{e1} = \frac{e n_0 \left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)}{m_i \left\{\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)\left(\omega^2 - \omega_{pi}^2\right) - \omega^2 \omega_{pe}^2\right\}} \mathbf{E}_1 \cdot \frac{\nabla n_0}{n_0}. \tag{2.18}$$

Inserting the value of n_{e1} from (2.17) in (2.10) yields the following result:

$$\begin{aligned} &\omega B_1 \left(1 + k^2 \lambda_e^2\right)\left\{\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)\left(\omega^2 - \omega_{pi}^2\right) - \omega^2 \omega_{pe}^2\right\} \\ &= \left(\frac{2kcT_{Fe}}{L_{ne}} - \frac{k^3 c\hbar^2}{4L_{ne} m_e}\right)\left(\frac{\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)}{m_i}\right) E_{1x} \frac{\partial_x n_0}{n_0}. \end{aligned} \tag{2.19}$$

Taking the Fourier transform of Faraday's law, we have

$$E_{1x} = (-\omega/ck) B_1. \tag{2.20}$$

Inserting (2.20) in (2.19), we get

$$\begin{aligned} &\left(1 + k^2 \lambda_e^2\right)\left\{\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)\left(\omega^2 - \omega_{pi}^2\right) - \omega^2 \omega_{pe}^2\right\} \\ &= L_{ne}^{-2}\left\{\left(\omega^2 - \mathbf{v}_F^2 k^2 - \hbar^2 k^4/4m_e^2\right)\left(\frac{k^2 \hbar^2}{4m_i m_e} - \frac{2T_{Fe}}{m_i}\right)\right\}, \end{aligned} \tag{2.21}$$

where $L_{ne} = |\partial_x n_0| / n_0$. Defining, $\lambda_{Fe} = \mathbf{v}_F/\omega_{pe}$, $\mathbf{v}_F = \left(2T_{Fe}/m_e\right)^{1/2}$, $c_{FS} = \sqrt{2T_{Fe}/m_i}$, the Fermi electron wavelength, Fermi velocity and the quantum ion-acoustic speed, respectively, in (2.21), we get

$$\begin{aligned} &\left(1 + k^2 \lambda_e^2\right)\left\{\left[\omega^2 - \mathbf{v}_F^2 k^2 \left(1 + k^2 \lambda_q^2\right)\right]\left(\omega^2 - \omega_{pi}^2\right) - \omega^2 \omega_{pe}^2\right\} \\ &= -c_{FS}^2 L_{ne}^{-2}\left(1 - k^2 \lambda_q^2\right)\left[\omega^2 - \mathbf{v}_F^2 k^2 \left(1 + k^2 \lambda_q^2\right)\right], \end{aligned} \tag{2.22}$$

where $\lambda_q = \hbar/2\, m_e \mathbf{v}_F$. It is difficult to find analytical solution of the above dispersion relation. We shall, therefore, present here several interesting limiting cases.

3. Limiting cases

Case 1:

Let us consider a wave whose wave frequency is much larger than the ion plasma frequency $\omega \gg \omega_{pi}$ and the wave phase velocity is much smaller than the Fermi electron thermal speed $\mathbf{v}_F$ so for this case $\omega/k \ll \mathbf{v}_F$. In this limit, the dispersion relation (2.22) takes the following form:

$$\left(1 + k^2 \lambda_e^2\right)\left\{\left[\mathbf{v}_F^2 k^2 \left(1 + k^2 \lambda_q^2\right)\right]\left(\omega^2 - \omega_{pi}^2\right) + \omega^2 \omega_{pe}^2\right\} = -c_{FS}^2 L_{ne}^{-2} \mathbf{v}_F^2 k^2 \left(1 - k^4 \lambda_q^4\right).$$

If we assume that $\omega \gg \omega_{pi}$, we get

$$\omega^2\left(1+k^2\lambda_e^2\right)\left\{\mathbf{v}_F^2k^2\left(1+k^2\lambda_q^2\right)+\omega_{pe}^2\right\}=-c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right)$$

or

$$\omega^2=\frac{-c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right)}{\left(1+k^2\lambda_e^2\right)\left[\mathbf{v}_F^2k^2\left(1+k^2\lambda_q^2\right)+\omega_{pe}^2\right]}, \tag{3.1}$$

which is dispersion relation of the coupled electron-drift mode with quantum ion-acoustic and electron Fermi mode for dense quantum plasma. For $k^4\lambda_q^4<1$ the quantum MEDV mode becomes unstable for highly dense non-uniform plasma. Which shows that electron quantum effects seem to play very important role for long-wavelength perturbations (i.e. $\lambda>\lambda_q$, where λ is the perturbation wavelength) on the growth of these modes. This mode disappears for uniform density case. As discussed in the introduction section that in a classical electron-ion plasma with fixed ions, MEDV mode has a frequency proportional to proportional to v_{te}/L_{ne}, but in a dense quantum plasma, the electron thermal velocity is replaced with $\mathbf{v}_F$. As a result, we obtain a new mode which is essentially coupled qunatum-acoustic and electron-drift dispersive type mode. The dispersion comes from the quantum wavelength correction term.

Case 2:

Consider the case in which the wave frequency is very close to the ion plasma frequency $\omega\simeq\omega_{pi}$ and the wave phase velocity is much smaller than the Fermi electron thermal speed $\mathbf{v}_F$ so for this case $\left(\omega/k\ll\mathbf{v}_F\right)$. In this limit, dispersion relation (2.22) becomes

$$\left(1+k^2\lambda_e^2\right)\omega^2\omega_{pe}^2=-c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right),$$

$$\omega^2=-\frac{c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right)}{\omega_{pe}^2\left(1+k^2\lambda_e^2\right)},$$

$$\omega^2=-\frac{c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right)}{\omega_{pe}^2\left(1+k^2\lambda_e^2\right)}. \tag{3.2}$$

For $k^4\lambda_q^4<1$, the growth rate is given by the relation

$$\gamma\simeq\frac{c_{FS}}{L_{ne}}\left\{\frac{k^2\lambda_{Fe}^2\left(1-k^4\lambda_q^4\right)}{\left(1+k^2\lambda_e^2\right)}\right\}^{1/2}. \tag{3.3}$$

The normalized growth rate γ/ω_{pi} has been plotted by varying the number density from $n_{e0}=10^{26}$ cm^{-3} to 10^{25} cm^{-3} as a function of kL_{ne} and is shown in Fig. 1 by choosing some typical parameters [28] of laser-based plasma compression scheme for which $n_{e0}=10^{26}$ cm^{-3}, $\omega_{pe}=5.64\times10^{17}$ s^{-1}, $\omega_{pi}=1.32\times10^{16}$s^{-1}, $\lambda_{Fe}=2.95\times10^{-9}$ cm, $\mathbf{v}_F=1.66\times10^9$ cm s^{-1}, $c_{FS}=3.88\times10^7$ cm s^{-1}, $L_{ne}=10^{-7}$ cm. It is evident from the graph that for high-density quantum plasma case, the growth rate is higher as compared to the low-density case.

Case 3:

Consider the case in which $\omega\ll\omega_{pi}$ and $\omega/k\ll\mathbf{v}_F$. In this limit, the general dispersion relation takes the following form:

$$\left(1+k^2\lambda_e^2\right)\left\{\mathbf{v}_F^2k^2\left(1+k^2\lambda_q^2\right)\left(\omega^2-\omega_{pi}^2\right)+\omega^2\omega_{pe}^2\right\}=-c_{FS}^2L_{ne}^{-2}\mathbf{v}_F^2k^2\left(1-k^4\lambda_q^4\right) \tag{3.4}$$

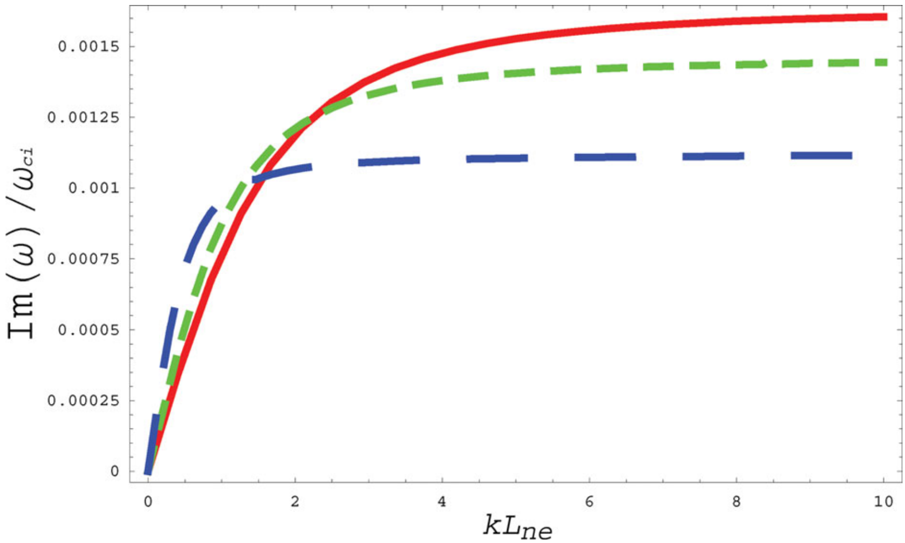

Figure 1. (Color online) Plot of normalized growth rate γ/ω_{pi} as the function of kL_n by varying the number density $n_{e0} = 10^{26}$ cm^{-3} (red curve), 0.5× 10^{26} cm^{-3} (green curve), and 10^{25} cm^{-3} (blue curve) is shown in the figure by choosing some typical parameters of laser-based plasma compression scheme.

or

$$\omega^2 = \mathbf{v}_F^2 k^2 \left(1 + k^2\lambda_q^2\right) \left[\frac{m_e}{m_i} - \frac{c_{FS}^2 \left(1 - k^2\lambda_q^2\right)}{L_{ne}^2 \omega_{pe}^2 \left(1 + k^2\lambda_e^2\right)}\right],$$

since $m_i \gg m_e$, the above equation can be rewritten as

$$\omega^2 = -\frac{\lambda_{Fe}^2 k^2 L_{ne}^{-2} c_{FS}^2 \left(1 - k^4\lambda_q^4\right)}{\left(1 + k^2\lambda_e^2\right)}, \tag{3.5}$$

which predicts an instability for $k^4\lambda_q^4 < 1$.

In summary, we derive a dispersion relation for the MEDV mode in an inhomogeneous quantum plasma. New dispersion relation has been derived for the low-frequency MEDV mode in a non-uniform quantum plasma. New class of purely growing modes that occurs on the time scale of ion plasma period for dense quantum plasma are identified. It is found that the modes become unstable when the propagation direction is perpendicular to the equilibrium density gradients and for $k^4\lambda_q^4 < 1$. We expect that the present investigation would play an important role in the study of magnetic field generation in dense plasma systems such as prevalent in laser-produced plasma (ICF), where high-power laser interacts with solid pellet material and produces inhomgeneous dense plasma [28]. Therefore, the results of our present investigation should be useful to understand wave phenomena in dense space and laboratory plasmas, where quantum effects are expected to play an important role.

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