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Complexity Analysis of Large CoMP Areas with Sparse Massive MIMO Channel Matrices

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Abstract

Joint transmission coordinated multipoint (JT CoMP) has been identified as a potential differentiator for future 5G radio systems due to its superior interference mitigation capabilities. Especially the combination with massive MIMO seems to be promising as it results in sparse overall channel matrices with a relatively low number of relevant channel components, which for example reduces the feedback overhead for reporting of channel state information (CSI). JT CoMP faces several other challenges from synchronization to CSI outdating, but here the focus will be on the complexity for the precoding over large massive MIMO cooperation areas. It will be derived how the typical sparse number of relevant channel components per user equipment can be exploited to reduce the number of floating point operations (FLOPs) by a factor of ten compared to state of the art solutions for the calculation of the Moore Penrose pseudo inverse of the channel matrix.

Research Area

- Future 5G below 6 GHz spectral capacity and coverage to be maximized with tight cooperation
- JT CoMP over adjacent sites in combination with Massive MIMO to combat interference and support large multi user MIMO gains
- Many JT CoMP aspects ranging from user grouping and clustering up to channel prediction have been evaluated

Coordination is essential between the entire network

- Cooperation area (CA) forms cell clusters to support JT CoMP along with Massive MIMO
- Cooperation over the cells equipped with Massive MIMO antenna array made by means of Grid of Beam (GoB) concept (See Figure 1) is considered as:

8 beams in elevation × 2 beams in azimuth × 2 polarizations = 32 beams per cell

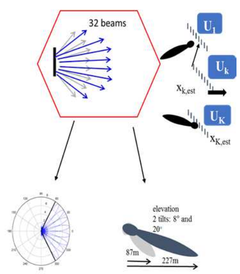

Fig. 1: Grid of Beam Concept

Problem Statement

Current understanding is that massive MIMO gains for below 6GHZ will rely mainly on MU MIMO like spatial multiplexing of ten or more users.

Most JT CoMP algorithms like for example zero forcing require for accurate precoding the calculation of the Moore Penrose pseudo inverse, where processing complexity in number of FLOPs (Floating Point) can be expressed as $2np^2 + 2n^3$ for a matrix size of $n{\times}p$ applying the common Singular Value Decomposition (SVD) method.

Utilizing the the state of the art algorithms, in case the transmission time interval (TTI) has 1ms length and the frequency band sub divided into 100 physical resource blocks (PRB) per TTI for a 90 × 288 size of matrix, we will have $10^2 \times 10^3 \times 10^6 = 10^{11}$ FLOPs only for precoding process.

Such processing power would bring a significant burden for a future evolved node B (eNB).Furthermore, precoding complexity described above neglects any special matrix structure.

Considering typical JT CoMP channel matrix, containing up to %80 or more of the channel components are closed to a threshold, a novel algorithm that essentially calculates the Moore-Penrose inverse of sparse channel components matrix used in precoding generation process at the base station can be proposed to reduce the complexity.

Theoretical Performance Analysis

Typical JT CoMP channel matrix contains approximately to %80 or more of channel components are very closed to zero (See Figure 2).

Proposed Approach: set the channel components below a certain power threshold to zero having minor impact to the precoding performance:

- Different power threshold brings different level of sparsity
- Employ minimum power levels with respect to the strongest channel component per UE
- Set all those matrix elements to zero for which the Rx power is below the power of the strongest channel component minus the threshold value

Efficient sparce Moore-Penrose Inverse Algorithm:

Our below analysis extends the known results for so-called *geninv* method of Moore-Penrose Inverse computation. More specifically, we target to investigate the number of FLOPs if using such method with respect to sparsity of the matrix.

If we denote *H* as complex-valued channel components matrix of size m ×n where m < n, we could consider the symmetric positive matrix *HH'* of size m × m and rank of r ≤ n where *H'* corresponds to adjoins of the channel matrix H. By using the usual Cholesky factorization of matrix *HH'*, the matrix *L* of size n × r and consequently its transpose *L'* of size r × n is obtained where: $HH' = L'L$. (1)

Using the general relation concerning the product of two matrices A and B, we have: $(AB)^{+}=A'B' (A'ABB')^{+}$, (2)

Where $(AB)^{+}$ represents the Moore-Penrose inversion of the product of two matrices A and B. If B=A' and A is n×r matrix of rank r, then from (2) we have: $(AA')^{+}=A'A (AA')^{-1} (AA')^{-1}$. (3)

Preposition 1. The Pseudo Inverse of matrix H defined above can be obtained by the following expression :

$$H^{+}=H'L (L'L)^{-1}(L'L)^{-1} L' . \qquad (4)$$

Considering the Eq.(4), we may divide this process into three following steps:

$$P1=L \times (L'L)^{-1}, \quad P2=P1 \times P1', \quad P3=P2 \times H'. \qquad (5)$$

The *Spgeninv* algorithm applies as follows to each equation given in (5):

- Store the location of non-zero elements in the matrix before reordering,
- Reorder the sparse matrix applying Reserve Cuthill-Mckee (RCM) algorithm
- Store the location of non-zero elements after reordering,
- Multiply two new reordered matrices together with avoiding unnecessary multiplication and additions.

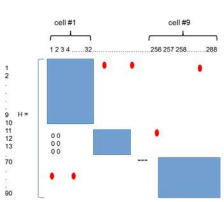

Fig. 2: Typical JT CoMP Channel Matrix

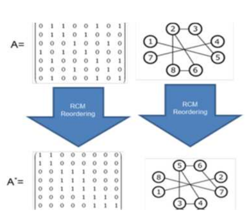

Fig. 3: Reverse Cuthill-Mckee Algorithm

System Model

Proposed integrated system is assumed to support massive MIMO along with JT CoMP to increase the spectral efficiency. Forming cell clusters in the network denoted as cooperation areas (CA), the served users are expected to gain through cooperation. Here we limit our analysis to a single cooperation area comprising three adjacent sites with three cells per site, i.e. overall 9 cells (See Figure 4a). Each cell is equipped with a massive MIMO antenna array, which might consist of 1000 or more antenna elements. This high number of antenna elements is reduced to a limited set of effective beams or antenna ports by using the so called fixed grid of beam (GoB) concept. For example a precoding matrix V generating eight beams in elevation and two in azimuth direction would result in combination with two polarizations per beam in overall 32 beams per cell (See Figure 1b).

In case of FDD, UEs report relevant channel components which are above a certain power threshold relative to the strongest channel component to the eNB, which calculates in case of single cell MU MIMO the precoding matrix W. The MIMO channel precoder matrix W is obtained through the pseudo inversion of the sparse channel components matrix H. As such, the form of the matrix W is dependent on the certain power threshold applied. The higher power threshold relative to the strongest channel component, the higher sparse W there is.

For JT CoMP up to 9 cells will use a single precoder spanning all cells with an accordingly high matrix dimension of e.g. K=90 UEs times N=288 antenna ports, which is significantly larger compared to todays 3GPP LTE matrix dimensions of e.g. 4×4 or maximal 8 ×8 .

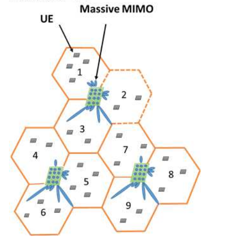

Fig. 4a: Cooperation Area Architecture

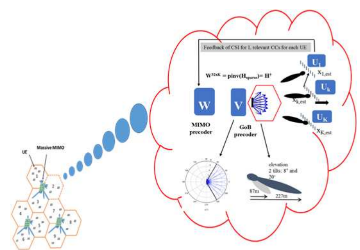

Fig. 4b: JT CoMP Precoding Process

System Parameters

For evaluation, two different channel models have been applied. First scenario belongs to a quadratic CoMP channel filled with 40 users and 288 beams over 3 site cooperation areas (See Figure 4a and Table 1). The same model has been applied to second scenario with a large 3D CoMP channel where only 3 cooperation sites out of existing 7 sites covered by 90 UEs have been focused (Figure 5). As such, three selected cooperation sites in second scenario will have the same parameters shown in Table 1. LTE downlink physical layer parameters used in both scenarios obtaining performance metric are provided in Table 2.

Channel parameters	value
Number of sites	3
Number of beams per cell	32
Number of cells per site	3
Number of BSc per site	3
Channel taps	24
Number of UEs per site	30
Number of total beams	288

Table 1: JT CoMP Channel Parameters

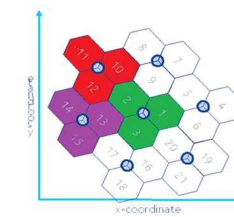

Fig. 5: JT CoMP Architecture (Scenario 2)

Downlink PHY Layer parameter	value
Channel Bandwidth (MHz)	20
FFT size	2048
Occupied sub-carriers	1200
Sub-carrier spacing (KHz)	15
Sampling Frequency (MHz)	30.72
Noise level (dBm)	-124.6
Number of Resource Blocks	100

Table 2: JT CoMP Channel Parameters

Method	Inter-Site Cooperation	Receiving Power Threshold (dB)
SVD	55344466	10,15,25
Geninv	4986003	10,15,25
SP-Geninv	278013	10
SP-Geninv	512925	15
SP-Geninv	617588	25

Table 3: Average Number of FLOPs

Numerical Results

Table 3 compares the average number of FLOPs for calculation of 100 precoders for 100 physical resource blocks for different scenarios applying three methods.

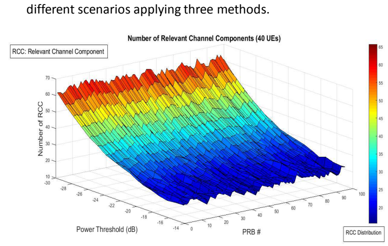

Fig. 6: Relevant Channel Components Distribution (Scenario 1)

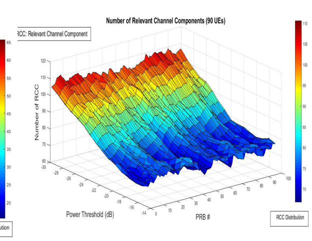

Fig. 7: Relevant Channel Components Distribution (Scenario 2)

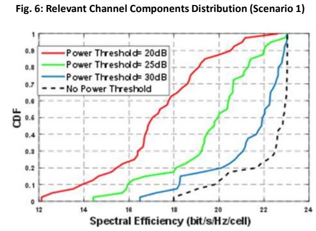

Fig. 8: Spectral Efficiency (Scenario 1)

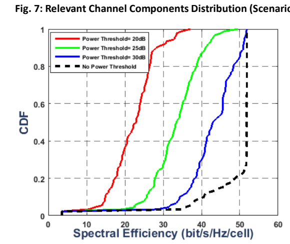

Fig. 9: Spectral Efficiency (Scenario 2)

Conclusions

- Novel algorithm to exploit the sparsity level of massive MIMO channel matrices reducing the complexity
- Reducing interference between neighboring cells through introducing power threshold mechanism with moderate degradation of precoding performance