Energy Coupling in Optical WDM Systems with Frequency-Dependent Attenuation Profile

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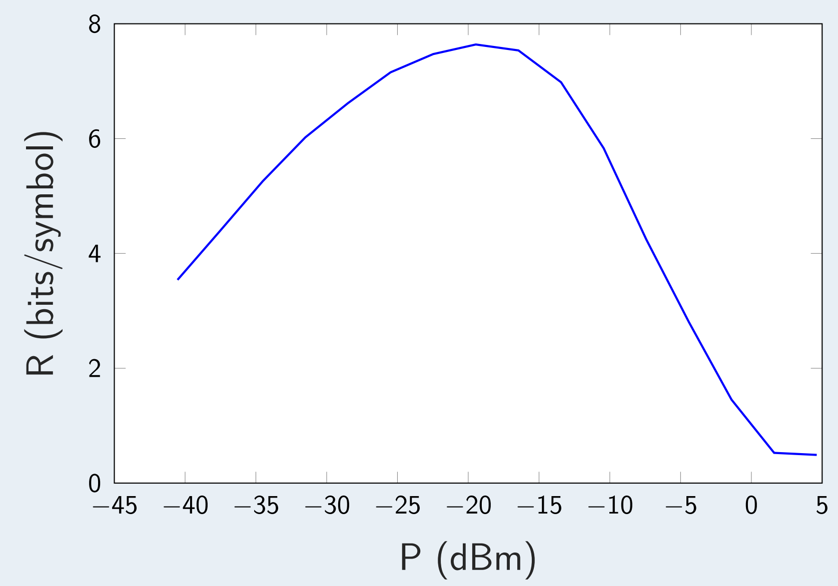
Introduction

- Capacity of linear fiber optic systems **has reached a peak** due to the nonlinearity of the channel.
- Multi-channel (WDM) systems also suffer from **channel coupling** due to spectral broadening.

Achievable rate of an WDM system with 5 channels



$B_{\text{channel}} = 20\,\text{GHz}, B_{\text{guard}} = 5\,\text{GHz}, \text{distance} = 2000\,\text{km}.$

Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial q(z,t)}{\partial z} = -\frac{\alpha}{2}q(z,t) - j\frac{\beta_2}{2}\frac{\partial^2 q(z,t)}{\partial t^2} + j\gamma\,|q(z,t)|^2\,q(z,t)$$

Attenuation

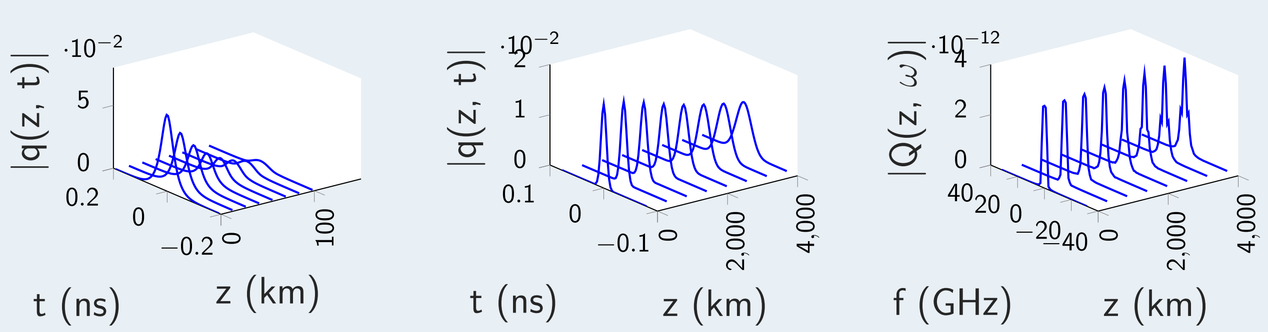
- Exponential decay in power.

Dispersion

- Linear term.
- Causes pulse broadening in time.

Nonlinearity

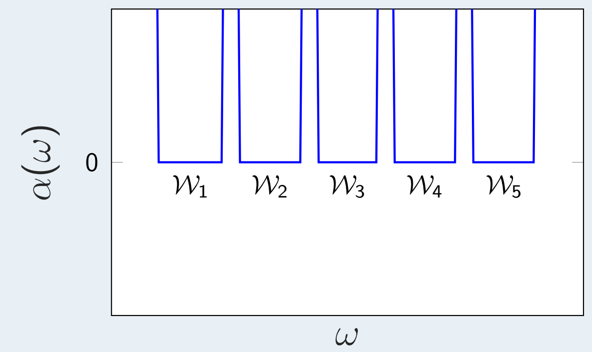
- Causes frequency mixing (spectral broadening, SPM, XPM, FWM).



The nonlinear term causes channel coupling in WDM systems!

Idea: Frequency-dependent attenuation profile

$$\alpha(\omega) = \begin{cases} 0, & \omega \in \mathcal{W} \\ \infty, & \text{otherwise.} \end{cases}$$



Such a system:

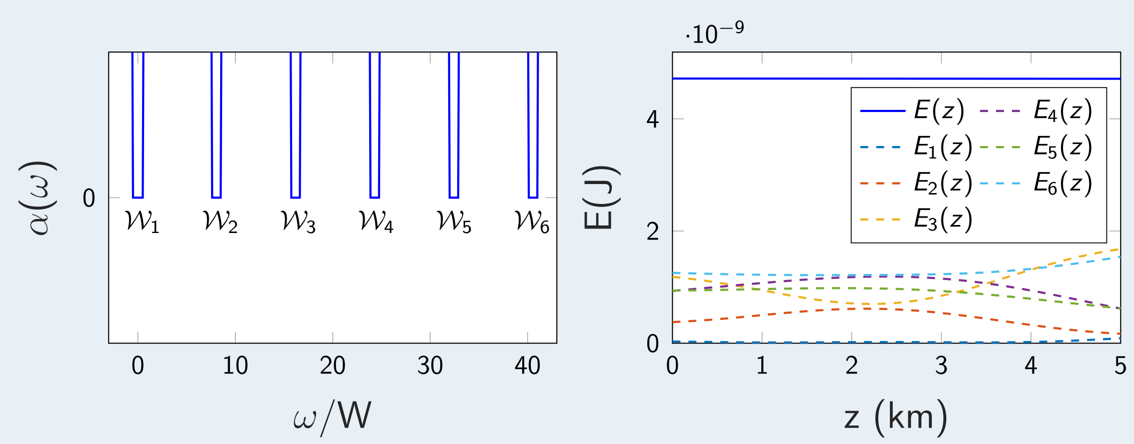
- does not allow spectral broadening,
- and turns out to still **preserve the total energy** of the signal:

$$E(z) = \frac{1}{2\pi}\int_{\mathcal{W}} |Q(z,\omega)|^2\,\mathrm{d}\omega = E(0) \quad \forall z.$$

Energy coupling between channels

However, the **energy in each individual channel**, $E_n(z)$ is not preserved. Four-Wave Mixing (FWM) still causes coupling:

$$\frac{\mathrm{d}}{\mathrm{d}z}E_n(z) = -\frac{\gamma}{4\pi^3}\Im\left\{\int_{-\infty}^{\infty}[Q(z,\omega)*Q(z,\omega)]\cdot[Q_n(z,\omega)*Q(z,\omega)]^*\,\mathrm{d}\omega\right\},$$



Condition for absence of energy coupling

We derived the following condition that ensures the absence of **energy** coupling between channels ($\mathrm{d}E_n(z)/\mathrm{d}z = 0$):

$$(\mathcal{W}_{n_1} + \mathcal{W}_{n_2}) \cap (\mathcal{W}_n + \mathcal{W}_{n_3}) = \emptyset,\ \forall \{n_1, n_2\} \neq \{n, n_3\}, \qquad (1)$$

where $+$ denotes the *sum of intervals*:

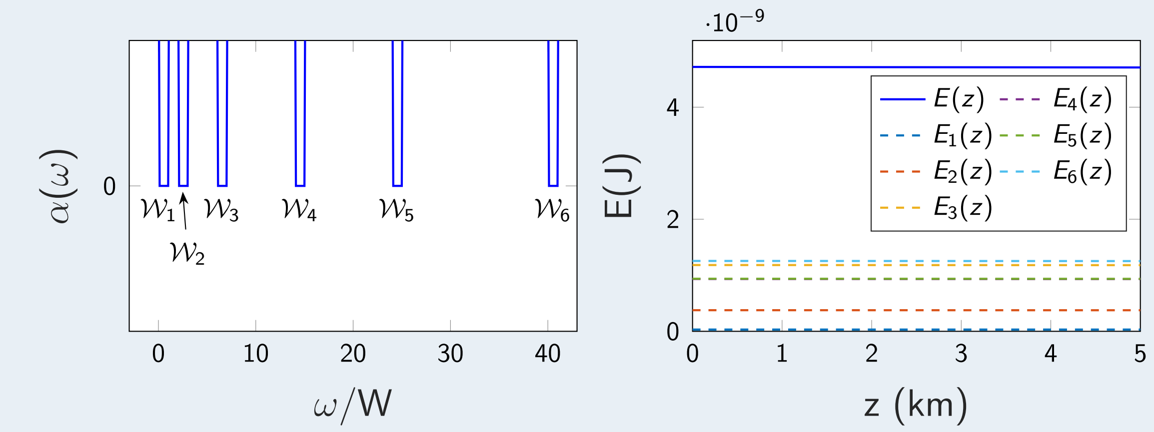
$$[\omega_{11}, \omega_{12}] + [\omega_{21}, \omega_{22}] = [\omega_{11} + \omega_{21}, \omega_{12} + \omega_{22}].$$

Design of an energy-decoupled system

- For channels with equal width W, condition (1) forces the use of a *Sidon sequence* [1] to place the channel centers ω_n:

$$\omega_n = 2m_nW, \quad m_n \text{ is a Sidon sequence}$$

$$m_{n_1} + m_{n_2} \neq m_n + m_{n_3},\ \forall \{n_1, n_2\} \neq \{n, n_3\},$$



$$\beta_2 = -21.67\,\text{ps}^2/\text{km}, \gamma = 1.26\,\text{W/km}, W = 2\pi\cdot 1\,\text{GHz}$$

- The maximum spectral efficiency of Sidon sequences for an N-channel system is:

$$\eta(N) = \frac{NW}{\omega_N - \omega_1 + W} \in \mathcal{O}(1/N).$$

i. e. an energy-decoupled N-channel system can asymptotically fill **at most** a fraction $1/N$ of the spectrum.

Conclusions

- The frequency-dependent attenuation profile **prevents spectral broadening** and **conserves the total energy** of the system.
- There is still **energy transfer between channels**, which can be avoided by using a Sidon sequence.
- The **maximum spectral efficiency** of an energy-decoupled N-channel system is $\mathcal{O}(1/N)$, which is very inefficient. To design a more efficient system, energy coupling needs to be allowed.

References

[1] A. M. Mian and S. D. Chowla, "On the B2 sequences of Sidon," *Proc. National Academy of Sciences of India, Sect. A*, vol. 14, no. 3-4, 1944.

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