Stochastic structural dynamic analysis with random damping parameters

K. Sepahvand[1], F. Saati Khosroshahi, C. A. Geweth and S. Marburg
Chair of Vibroacoustics of Vehicles and Machines
Department of Mechanical Engineering
Technical University of Munich, 85748 Garching, Germany

ABSTRACT
Assigning deterministic values to damping parameters in many numerical simulations of structural dynamics is a very difficult task owing to the fact that such parameters possess significant uncertainty. In this paper, the modal damping parameters are considered as random variables. The generalized polynomial chaos (gPC) expansion is employed to capture the uncertainty in the parameters and dynamic responses. Finite element model of damped vibration analysis of a composite plate is served as deterministic black-box solver to realize responses. The range of uncertainties and the probability distributions of the parameters are identified from experimental modal tests. A set of random collocation points are generated for which the constructed gPC expansions are used to generate samples of damping parameters as deterministic inputs to the FEM model. This yields realizations of the responses which are then employed to estimate the unknown coefficients. The results show while the responses are influenced from the damping uncertainties at the mid and high frequency ranges, the uncertainty impact at lower modes can be safely ignored. Furthermore, the method indicates also a very good agreement compared to the sampling-based Monte Carlo FEM simulations with large number of realizations. This leads to a very efficient simulation in terms of computational time.
Keywords: stochastic structural dynamics, random damping , polynomial chaos, I-INCE Classification of Subjects Number(s): 47.3

1. INTRODUCTION

Transient analysis (known also as time-history analysis) plays a very important role in investigation of dynamic behavior of structures. The technique used to determine the dynamic response under various time-dependent loads to calculate the time-varying kinematic responses as deformations and consequential strain-stress results in the structure. The time-dependency of this type of analysis requires considering the effect of structural damping, especially, where the structure is under step or impulse loading conditions, i.e. suddenly load variation in a fraction of time. On the other hand, unlike dynamical properties such as mass and stiffness, the structural damping prediction is often the most difficult process due to the complicated damping mechanism and related inherent uncertainties [1]. An efficient and reliable estimation of the responses are achieved by considering uncertainties, particularly, when one deals with engineering reliability analysis.

The uncertain structural damping parameters may be represented as random variables having an expected mean value and a variation range denoted by the standard deviation [2]. This is, however, true if one makes sure that damping parameters can be estimated as Gaussian random variables. For non-Gaussian uncertain parameters, the whole range of uncertainty cannot be captured only by means of the mean value and the standard deviation [3, 4]. Stochastic finite element method (SFEM) based on sampling methods, e.g. Monte Carlo (MC) simulations, then is the simplest way and straightforward to quantify the structural responses under uncertainty in

[1]Email: k.sepahvand@tum.de

modeling parameters. However, the accuracy of such a method depends extremely on the number of samples and, consequently, very expensive in terms of computation time[1]. For that reason, in this paper, the spectral based method [5] for uncertainty quantification is employed in which the generalized polynomial chaos (gPC) expansion [6–12] plays the major role.

This work uses the gPC expansion for representation of the modal damping parameters of the structures in transient dynamic analysis. The information on the uncertainty range and the probability distributions of the damping parameters are obtained from experimental modal tests on 100 samples of plates having identical nominal topology. The transient responses are then represented in an explicit form as a random process approximating by means of the gPC expansion with unknown time-dependent deterministic functions. A deterministic FEM model of the plate operates as a black-box to realize samples of the responses, on a set of collocation points generated in the random space. The modal superposition method is employed to estimate transient responses. In the modal superposition method, the transient structural responses under the loading condition is estimated from the linear combination of the eigenvectors obtained in a modal analysis. This yields in an efficient sparse gPC expansion with a few unknown functions representing the responses. The realizations then are served to estimate the unknown functions. This provides the major advantage of using limited number of realizations to estimate the system responses which normally require large number of samples when using the MC method. Furthermore, any commercial or developed in-house FEM code can be used to accomplish desirable structural responses considering parameter uncertainty [12].

This paper is prepared as follows: the stochastic spectral representation of random transient analysis with random modal damping is given in the next section. Numerical and experimental results are given in the section 3, and the final section discusses the conclusions.

2. MODAL-SUPERPOSITION BASED STOCHASTIC TRANSIENT DYNAMIC ANALYSIS OF STRUCTURES

It is assumed that a set of experimental data on the modal damping parameters are available, so that the uncertain modal damping parameters can be represented as random variables having predefined probability density function (PDF) as discussed in [12]. The numerical finite element solution of the structural transient analysis can be performed in full, reduced and modal-superposition methods. The first method uses the full system matrices to calculate the transient responses while the second one condenses the problem size by using master degrees of freedom and reduced matrices. Both methods are more practically powerful because they allow all types of nonlinearities to be included in the solution. The modal-superposition method uses the normal modes of the structure modal analysis to estimate the nodal displacement responses in the transient analysis. It is faster and less expensive than the first method for many problems while accepting modal damping. The coupled equations of motion then are transformed into a set of independent uncoupled equations from which the dynamic responses of the original system are estimated from the superimposing responses of the uncoupled equations.
For the FEM model of a structural system having n degrees of freedom, let the modal damping ratios $\zeta_r,\ r = 1, 2, \ldots, m$ for $m \leq n$ eigenvectors be represented as uncertain parameters with specific PDF. The stochastic uncoupled modal equations can be then given as

$$\ddot{\boldsymbol{q}}(t, \boldsymbol{\xi}) + \text{diag}\left[2\zeta_r(\boldsymbol{\xi})\omega_r\right] \dot{\boldsymbol{q}}(t, \boldsymbol{\xi}) + \text{diag}\left[\omega_r^2\right] \boldsymbol{q}(t, \boldsymbol{\xi}) = \boldsymbol{\Phi}\boldsymbol{F}(t) \quad (1)$$

in which $\boldsymbol{q} = \{q_1, q_2, \ldots, q_m\}^T$ is a vector of size $m \times 1$ representing the modal amplitudes, $\boldsymbol{\Phi} = [\phi_1|\phi_2|\ldots|\phi_m]$ is an $n \times m$ modal matrix with ϕ_i as the ith-eigenvector, ω_r denotes the natural circular frequency of mode r and $\boldsymbol{F}$ represents the vector of external nodal loads. To develop the spectral stochastic form of Eq. (1), the modal damping parameter and the modal amplitudes are discretized employing the gPC expansions. That is, for the model damping parameter ζ_r

$$\zeta_r = \sum_{i=0}^{N_\zeta} a_{r_i} \Psi_i(\boldsymbol{\xi}) = \boldsymbol{a}_r^T \boldsymbol{\Psi}(\boldsymbol{\xi}) \quad (2)$$

[1]for n pseudo-random samples and a variance of σ^2, the convergence rate of MC is $\frac{\sigma}{\sqrt{n}}$.

in which $\boldsymbol{a}_r^T = \{a_{r_0}, a_{r_2}, \ldots, a_{r_{N_\zeta}}\}$ denotes the vector of deterministic coefficients and $\boldsymbol{\Psi}(\boldsymbol{\xi}) = \{\Psi_0, \Psi_1, \ldots, \Psi_{N_\zeta}\}^T$ is the vector of orthogonal random basis function. Accordingly, using the same random basis for the modal amplitudes, one can write

$$\boldsymbol{q}(t,\boldsymbol{\xi}) = \sum_{j=0}^{N_q} \{Q(t)\}_j \, \Psi_j(\boldsymbol{\xi}) = \boldsymbol{Q}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{3}$$

where $\boldsymbol{Q}(t)$ is a matrix of unknown deterministic gPC coefficient vector $\{Q(t)\}_j$ in its row $j+1$. Substituting the expansions in Eq. (1) yields to

$$\ddot{\boldsymbol{Q}}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) + \text{diag}\left[2\boldsymbol{a}_r^T\boldsymbol{\Psi}(\boldsymbol{\xi})\omega_r\right]\dot{\boldsymbol{Q}}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) + \text{diag}\left[\omega_r^2\right]\boldsymbol{Q}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) = \boldsymbol{\Phi}\boldsymbol{F}(t) \tag{4}$$

This equation denotes modal-superposition based spectral stochastic FEM (sSFEM) of transient dynamic analysis of structures with random modal damping parameters. Knowing the gPC coefficients of the damping parameters, the goal is usually the calculation of unknown functions $\{Q(t)\}_j$. To this end, the corresponding random error $\epsilon(t,\boldsymbol{\xi})$ due to the discretization of the random parameters and response has to be minimized, i.e.

$$\epsilon(t,\boldsymbol{\xi}) = \ddot{\boldsymbol{Q}}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) + \text{diag}\left[2\boldsymbol{a}_r^T\boldsymbol{\Psi}(\boldsymbol{\xi})\omega_r\right]\dot{\boldsymbol{Q}}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) + \text{diag}\left[\omega_r^2\right]\boldsymbol{Q}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) - \boldsymbol{\Phi}\boldsymbol{F}(t) \tag{5}$$

There are two broad classes of methods that can be used to minimize this error, namely, Galerkin projection and collocation-based methods. While the former requires access to the data structures of the FEM model or analytical governing equations to derive a system of deterministic equations for the stochastic modes of the solution, the error in the latter is set equal to zero at sample collocation points in the random space. Herein, the FEM model or the governing equations are employed as solver/black box. The realized uncertain parameters on collocation points are then served as deterministic inputs to the FEM solver to evaluate samples of responses. The major advantage is using any deterministic numerical and analytical procedure for extracting the structural vibration responses, cf. [12] for more details. Once the coefficient functions $\{Q(t)\}_j$ are calculated, the vector of stochastic dynamic responses of the original system, $\boldsymbol{u}$, is then estimated as

$$\boldsymbol{u}(t,\boldsymbol{\xi}) \approx \sum_{i=1}^{m} \{\phi_i\}\, q_i(t,\boldsymbol{\xi}) = \boldsymbol{\Phi}\boldsymbol{q}(t,\boldsymbol{\xi}) = \boldsymbol{\Phi}\boldsymbol{Q}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) = \tilde{\boldsymbol{Q}}(t)\boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{6}$$

where $\tilde{\boldsymbol{Q}}(t) = \boldsymbol{\Phi}\boldsymbol{Q}(t)$. The first coefficient function $\tilde{Q}_0(t)$ denotes the expected function of the dynamic response. Given the vector $\boldsymbol{\xi}$ as a set of Gaussian variables, the orthogonal Hermite polynomials can be employed to represent the stochastic responses. For instance, considering the first 6 random modal damping parameters as $\zeta_r(\xi_r),\ r = 1,\ldots,6$, the 2-order gPC expansion of $\boldsymbol{u}$ is represented as

$$\boldsymbol{u}(t,\boldsymbol{\xi}) = \sum_{i=0}^{2} \tilde{Q}_i(t)H_i(\boldsymbol{\xi}) = \sum_{i=0}^{2} \tilde{Q}_i(t)H_i(\xi_1,\ldots,\xi_6) \tag{7}$$

The expansion includes a complete basis of 6-dimensional Hermite polynomials up to a fixed total 2nd-order specification. The total number of terms N in the expansion is given by [7]

$$N = \frac{(6+2)!}{2!6!} = 28 \tag{8}$$

Therefore, the expansion in Eq. (7) is stated as

$$\begin{aligned}
\boldsymbol{u}(t,\boldsymbol{\xi}) = {} & \tilde{Q}_0(t) + \tilde{Q}_1(t)\xi_1 + \tilde{Q}_2(t)\xi_2 + \tilde{Q}_3(t)\xi_3 + \tilde{Q}_4(t)\xi_4 + \tilde{Q}_5(t)\xi_5 + \tilde{Q}_6(t)\xi_6 \\
& + \tilde{Q}_7(t)(\xi_1^2 - 1) + \tilde{Q}_8(t)\xi_1\xi_2 + \tilde{Q}_9(t)\xi_1\xi_3 + \tilde{Q}_{10}(t)\xi_1\xi_4 + \tilde{Q}_{11}(t)\xi_1\,\xi_5 + \tilde{Q}_{12}(t)\xi_1\,\xi_6 \\
& + \tilde{Q}_{13}(t)(\xi_2^2 - 1) + \tilde{Q}_{14}(t)\xi_2\xi_3 + \tilde{Q}_{15}(t)\xi_2\xi_4 + \tilde{Q}_{16}(t)\xi_2\,\xi_5 + \tilde{Q}_{17}(t)\xi_2\,\xi_6 \\
& + \tilde{Q}_{18}(t)(\xi_3^2 - 1) + \tilde{Q}_{19}(t)\xi_3\xi_4 + \tilde{Q}_{20}(t)\xi_3\xi_5 + \tilde{Q}_{21}(t)\xi_3\,\xi_6 + \tilde{Q}_{22}(t)(\xi_4^2 - 1) \\
& + \tilde{Q}_{23}(t)\xi_4\xi_5 + \tilde{Q}_{24}(t)\xi_4\xi_6 + \tilde{Q}_{25}(t)(\xi_5^2 - 1) + \tilde{Q}_{26}(t)\xi_5\xi_6 + \tilde{Q}_{27}(t)(\xi_6^2 - 1)
\end{aligned} \tag{9}$$

The first coefficient function $\tilde{Q}_0$ denotes the deterministic (mean) value of the random response. Due to the independent modal damping parameters in modal superposition analysis, the multiplicative terms in the above expansion, i.e. $\xi_i\xi_j,\ 0 < i,j \leq 6$, are vanished. This yields the reduced-order expansion with 13 unknown terms.

3. NUMERICAL AND EXPERIMENTAL STUDY

The application of the above mentioned theory is examined to determine the transient response of fiber-reinforced composite (FRC) plate made of symmetric 12 layers with 60% glass fibers oriented in $[0/90]_s$ in epoxy matrix under an impulsive excitation having amplitude of 100 N in the FEM model. The impulse is applied on the upper right corner of the plate over one integration time step of 0.0003 [s]. The plate exhibits dimensions of $250 \times 125 \times 2$ mm^3 and the black-box FEM model is constructed having nominal elastic parameters of $E_{11} = 47.45,\ E_{22} = 9.73,\ G_{12} = 4.00$ (all in Gpa) with the major Poisson's ratio of $\nu_{12} = 0.24$. The experimental modal analysis has been conducted on 100 sample plates having identical nominal topology to realize the damping parameters and their uncertainty ranges. The responses are measured by continuously scanning of 35 points on plates using Laser Doppler Vibrometer (LDV). The randomness in the damping parameters is well represented by the lognormal distributions compared to results obtained from the maximum likelihood estimation (MLE). The identified PDF are shown in Fig. 1 for the first 6 modes. Once the PDF types of the parameter are identified, the procedure introduced in [7] is

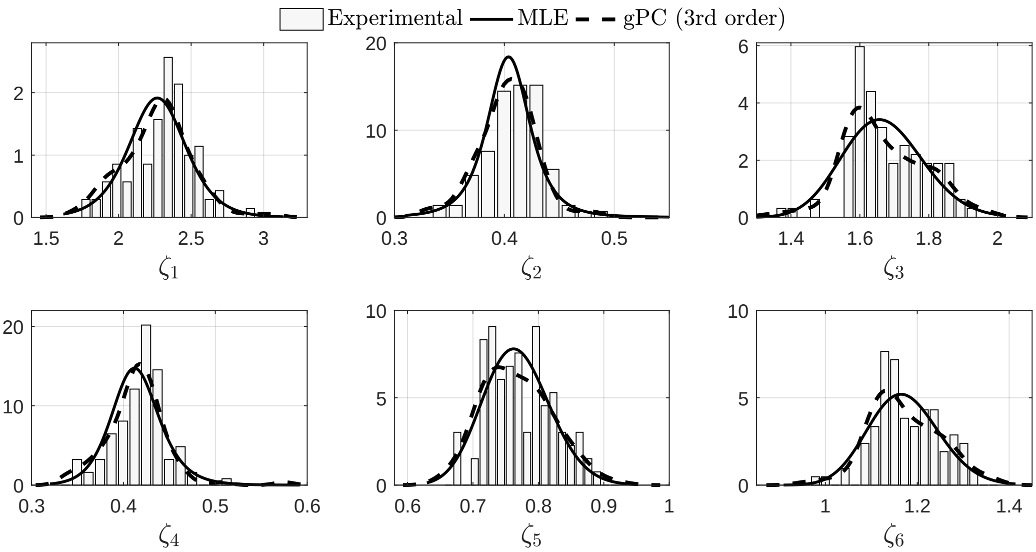

Figure 1 – The identified PDF of the first 6 experimental modal damping parameters [12]

adopted to calculate the unknown coefficients of the gPC expansions. The numerical algorithm presented in [12] is employed to realize samples of the transient responses in time domain of $[0, 0.3]$ seconds. For that, the numerical Matlab® code given in [12] is adopted to generate 60 collocation points from the 4th roots of multi-dimensional Hermite polynomials. Accordingly, the FEM code developed in [12] is used as black-box solver of the transient dynamic analysis of the plate. As preliminary to transient dynamic analysis using modal-superposition, the modal analysis of the plate for calculation of the first 20 modes are performed. The stochastic time-dependent response at the node in which load acts, cf. Eq. (6), is approximated by 2nd-order gPC expansion having multi-dimensional orthogonal Hermite polynomials of 6 random variables, $H(\xi_1, \xi_2, \ldots, \xi_6)$, each of which represents randomness in 6 modal damping parameters. The response realizations are served to calculated the unknown deterministic functions $\tilde{Q}_i(t)$. The results are shown in Fig. 2 for the zeroth-order function $\tilde{Q}_0(t)$ and the $\tilde{Q}_i(t),\ i = 1, \ldots, 4$. As expected, $\tilde{Q}_0(t)$, the expected deterministic response, diminishes as the load is removed in a short period of time due to the damping. The convergence of the higher order terms as time increases can be detected from the right plot in Fig. 2. It is also demonstrated that the higher order terms converge to zero faster than the lower order terms.

To ensure the accuracy of the method, the mean and the standard deviation functions derived from the gPC expansion of the stochastic dynamic response compared to the results obtained from 1000 MC realizations are shown in Fig. 3. As clearly observed, while no difference between the expected functions is detected, the variance functions show some disagreements when time increases.

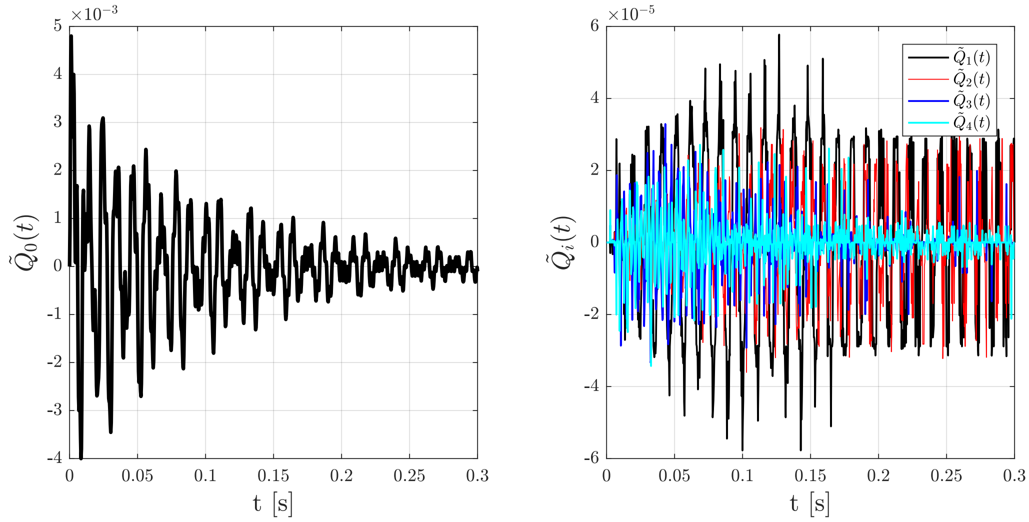

Figure 2 – The first gPC coefficient function $\tilde{Q}_0(t)$ (left) represents the mean value of the responses. The higher terms show the range of uncertainty in the response due to the randomness in damping parameters. The right plot demonstrates the coefficients $\tilde{Q}_i(t),\ i = 1, 2, 3, 4$.

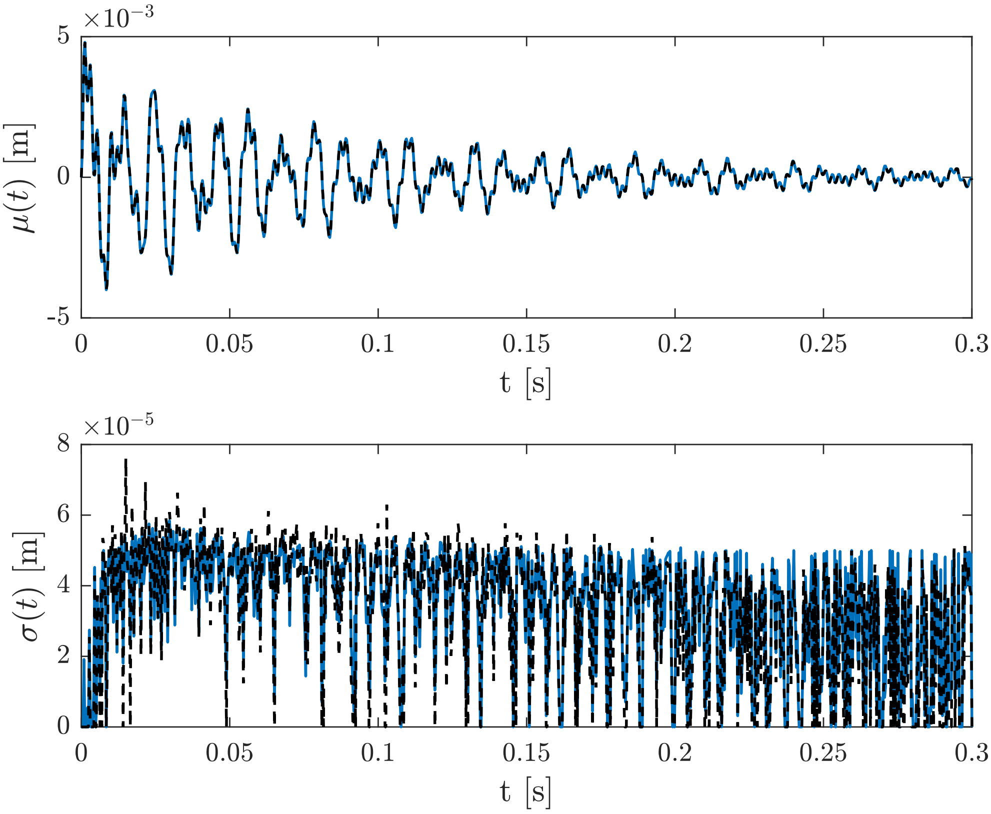

Figure 3 – The mean value $\mu(t)$ and the standard deviation $\sigma(t)$ of the dynamic response constructed from the 2nd-order gPC expansion (dashed line) using 60 collocation points compared to the results obtained from 1000 MC realizations (bold lines)

4. CONCLUSIONS

The stochastic FEM is used to investigate the impact of damping parameters on the dynamic response of structures. The major contribution of the study is using experimental data for constructing the gPC expansions of the damping parameters. The numerical FEM solver utilizes the modal-superposition method which effectively decreases the simulation cost in terms of computational time. A few realizations of the system responses were required compared to sampling based methods such as MC simulations. Thanks to collocation based stochastic simulation, the procedure can be implemented to complex structural systems with uncertain parameters for which the FEM model serves only as deterministic solver/black-box.

REFERENCES

[1] Kareem A, Sun WJ. Dynamic response of structures with uncertain damping. Engineering Structures. 1990;12(1):2–8.

[2] Canor T, Blaise N, Denoël V. Efficient uncoupled stochastic analysis with non-proportional damping. Journal of Sound and Vibration. 2012;331(24):5283–5291.

[3] Sepahvand K, Marburg S. On Construction of Uncertain Material Parameter using Generalized Polynomial Chaos Expansion from Experimental Data. Procedia IUTAM. 2013;6:4–17.

[4] Sepahvand K, Marburg S. Identification of composite uncertain material parameters from experimental modal data. Probabilistic Engineering Mechanics. 2014;37:148–153.

[5] Maitre OPL, Knio OM. Spectral Methods for Uncertainty Quantification. Netherlands: Springer; 2010.

[6] Sepahvand K. Uncertainty quantification in stochastic forward and inverse vibration problems using generalized polynomial chaos expansion. TU Dresden; 2009.

[7] Sepahvand K, Marburg S, Hardtke HJ. Uncertainty quantification in stochastic systems using polynomial Chaos expansion. International Journal of Applied Mechanics (IJAM). 2010;2:305–353.

[8] Sepahvand K, Marburg S, Hardtke HJ. Stochastic free vibration of orthotropic plates using generalized polynomial chaos expansion. Journal of Sound and Vibration. 2012;331(1):167–179.

[9] Sepahvand K, Marburg S. Random and Stochastic Structural Acoustic Analysis. In: S A Hambric SHS, Nefske DJ, editors. Engineering Vibroacoustic Analysis: Methods and Applications. New York: John Wiley & Sons; 2016. p. 305–338.

[10] Sepahvand K, Marburg S. Non-sampling inverse stochastic numerical-experimental identification of random elastic material parameters in composite plates. Mechanical Systems and Signal Processing. 2015;54-55:172–181.

[11] Sepahvand K. Spectral stochastic finite element vibration analysis of fiber-reinforced composites with random fiber orientation. Composite Structures. 2016;145:119–128.

[12] Sepahvand K. Stochastic finite element method for random harmonic analysis of composite plates with uncertain modal damping parameters. Journal of Sound and Vibration. 2017;400:1–12.