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Variation of Reference Strategy - A novel approach for generating optimized cutting patterns of membrane structures

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Abstract

Finding the plane shape of the double curved surfaces is a well-known challenge for every design engineer dealing with either fiber reinforced plastics lightweight designs or textile architectural membranes. A novel approach for generating optimized cutting patterns including nonlinear isotropic and anisotropic material behavior is presented. The so-called Variation of Reference Strategy can be seen as an inverse approach, defining the nodal positions in the material configuration as design variables holding the spatial configuration fixed. Thereby, the stress-free state of the cutting pattern which is an important characteristic of the manufacturing process is preserved. In order to demonstrate the abilities and robustness of the Variation of Reference Strategy several numerical examples considering different kind of materials are presented.

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Keywords:

1. Introduction

In various fields of engineering a significant rise of lightweight design concepts can be observed, e.g. membrane antennas, stringers and ribs of fabric reinforced materials (aeronautic/aerospace engineering), frame components and car body panels (automotive engineering), membrane rooftops and pneumatic structures (architectural membranes). The basic idea of lightweight design is an increasing load carrying behavior due to a combination of curvature, i.e.

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anticlastic or synclastic surfaces, and high-tensile materials, like high-performance alloys, carbon fiber reinforced plastics (CFRPs) or coated textiles. Since the wrought material is mostly a plane textile, the shape of the blank, the so-called cutting pattern, is very important in the sense of manufacturing. Cutting patterns are determined by an inverse process starting from the intended final shape of the erected structure. The manufacturing process bringing the plane wrought material into its final shape is known as drape process or short draping. The optimal cutting pattern is found if – considering draping – the shape deviation of the real erected structure from the intended one is minimal. As a matter of fact the standard situation of continuum mechanics is inverted as the shape of the deformed structure (erected structure) is known but that of the undeformed one (the reference configuration or cutting pattern) is unknown. Consequently, theory and numerical simulation methods have to deal with the variation of the reference geometry to find the optimal pattern which explains the name of the method presented here. It will be shown that the thoroughly discussion of all aspects needs considerable modifications of standard theory and methods but pays off in most robust and precise techniques of unknown quality so far.

Considering the non-developable characteristics of a double curved surface, it is not possible to find an unique corresponding blank. Hence, getting a solution of the mentioned problem means achieving a compromise w.r.t. some predefined criteria, i.e. an optimization problem has to be solved. Gründig et al. [1] came up with the idea of using an affine map between the plane configuration (2D) and the spatial configuration (3D) defining the element edges as design variables. Other kinematics based approaches are summarized in Topping and Iványi [2]. The major drawback of these approaches is the disregard of the material behavior. In order to avoid this, Maurin and Motro [3] developed the so-called Stress Compensation Method. In a first step, an orthogonal projection of the spatial configuration(3D) into a plane leads to a non-stress-free intermediate configuration(2D). Followed by a second step, a least squares approach minimizes the difference between these residual stresses and a predefined stress state by varying the contour of the intermediate configuration. The approach published by Kim and Lee [4] is based on the stress compensation method. But in contrast to Maurin and Motro [3], not only the displacements of the contour of the intermediate configuration but also the internal displacements are defined as design variables. The methodology for optimized cutting patterns published by Haug et al. [5], [6] is based on solving a standard structural mechanics problem. In a first step, the desired material characteristics are assigned to a not necessarily pre-stressed structure by means of the so-called metric retrieval method. In a second step, the structure is divided into different subdomains defined by geodesic boundaries, also called geodesic gores. Ina next step, each gore is separately forced into an appropriate plane. Finally, the optimized cutting patterns are found by finding the equilibrium state of the statically determinant supportedflat gores allowing only in-plane motions of the nodes. Linhard et al. [7], [8] introduced two also mechanically motivatedapproaches1 for solving the optimization problem. According to Maurin and Motro, Linhard et al. also minimize the difference between a predefined stress state and the residual stresses. But instead of using an intermediate configuration, Linhard et al. directly used the spatial configuration for computing the residual and predefined stresses. Additionally, Bletzinger et al. [9] and Bletzinger and Linhard [10] incorporated their approaches into an overall design strategy for architectural membranes. Further enhancements on the design of membrane structures considering cutting patterns has been published by Dieringer et al. [11].

A more general approach finding the stress-free (unloaded) configuration for a given set of boundary conditions (displacements and tractions) was published by Govindee and Mihalic[12] and Govindee and Mihalic [13]. According to the theory of finite deformations, the mapping of a point in the material configuration to the corresponding point in the spatial configuration is defined by a so-called motion. Govindee and Mihalic came up with the idea defining the inverse motion of a point as primary unknown.

Similar questions are arising in biomechanical applications. Gee et al. [14] addressed the problem of finding the unloaded configuration when patient-specific geometries are directly reconstructed out of in vivo imaging data.

In this contribution a novel approach for generating optimized cutting pattern will be presented. The key idea of the presented method, so-called Variation of Reference Strategy (VaReS), is to consider the elastic potential of the membrane arising due to the residual stresses. The paper is organized as follows. Section 2 shortly presents the important equations of nonlinear continuum mechanics. Section 3 will introduce and explain the fore-mentioned Variation of Reference Strategy in the very detail. Section 4 will draw the link between the Variation of Reference Strategy and a nonlinear kinematic description of plane structures within the context of finite elements. A general interface to hyperelastic material models will be introduced in section 5. The convergence behavior and the robustness of the presented approach is investigated in Section 6. Section 7 will show the good convergence of the presented method followed by other examples. Finally, a summary of the achieved results can be found in section 8.

2. Nonlinear continuum mechanics for thin membrane structures

Thin structures are characterized by the in-plane dimensions (l1 and l2) being significantly larger than their dimension in normal direction l3, i.e. l1; l2 >> l3. Assuming a constant distribution of the normal stresses over the thickness, the kinematic of the membrane can be reduced to its mid-surface. Since dry or coated fabrics exhibit almost no resistance against bending, only the in-plane actions, so-called membrane actions, are responsible for load carrying. Plane stress conditions are applied. For the cutting pattern and draping simulation large deformations have to be considered. The position vectors for a surface point P(θ 1; θ 2) are denoted as X(θ 1; θ 2) and x(θ 1, θ 2) in the undeformed and deformed configuration, respectively. As well, Gi and gi denote the tangential and normal vectors in either configuration, Fig. 1. Generally, capital letters are used for material quantities and small letters are used for spatial quantities. The basic quantities to determine strain are the deformation gradient F, the right Cauchy-Green tensor C, and the Green-Lagrange strain tensor E. The second Piola-Kirchhoff stress tensor S is used as stress measure. Refer to e.g. Holzapfel [15] for further details.

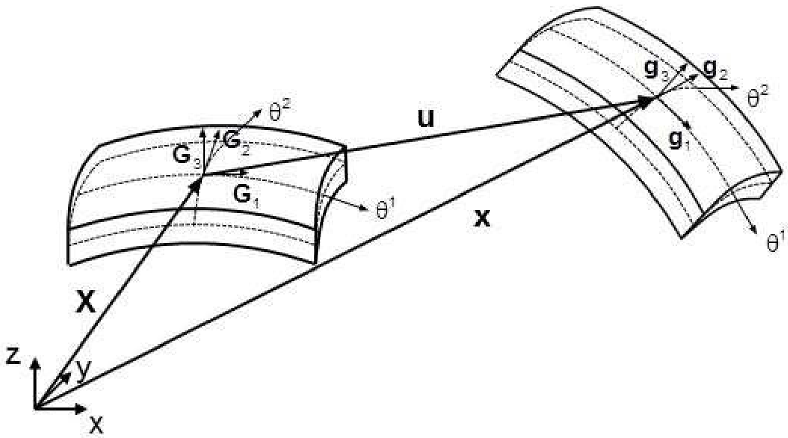

Fig. 1. Nonlinear kinematics of thin membrane structures.

2.1. Anisotropic hyperelasticity for fabrics

Hyperelastic, anisotropic material behaviour with a certain elastic potential Ψ is assumed. It are defined in terms of invariants I1, I2, and I3 of the right Cauchy Green tensor C and further invariants I4+2(i-1) which represent the anisotropic fiber orientations Φi of anisotropic fabric material where Mi being the corresponding structural tensors Mi = Φi ⊗ Φi:

$$I_{4+2(i-1)}(\mathbf{C},\mathbf{M}) = \mathbf{\Phi}^i \cdot \mathbf{C}\,\mathbf{\Phi}^i = tr\,\mathbf{C}\mathbf{M}^i$$
$$I_{5+2(i-1)}(\mathbf{C},\mathbf{M}) = \mathbf{\Phi}^i \cdot \mathbf{C}^2\,\mathbf{\Phi}^i = tr\,\mathbf{C}^2\mathbf{M}^i \tag{1}$$

Various anisotropic material laws have been published which are suitable for fabrics, see Vidal-Sallé et al. [16] and Y. Aimène et al. [17], Balzani et al. [18], Schröder and Neff [19], Bonet and Burton [20], Reese et al. [21], etc.

3. Variation of reference strategy (vares)

Consider a given non-developable surface S which defines the intended shape of the structure. Every point P $\in$ S is described by its spatial position vector x. To every 3D spatial position x an unique 2D material position X is assigned representing the point on the 2D cutting pattern plane to be determined.

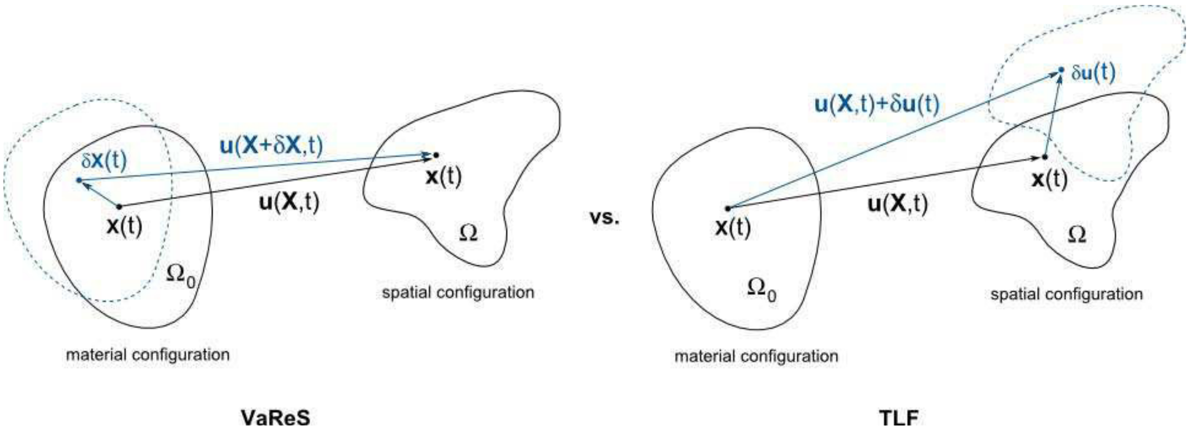

Fig. 2. Comparison: VaReS (left) vs. Total Lagrange Formulation (TLF) (right)

3.1. The shape-optimization problem to find the optimal cutting pattern

The aim of the following optimization problem is to find that material composition X of the cutting pattern such that the deformation X $\rightarrow$ x generates the least potential energy Π2D$\rightarrow$3D(X) for a given spatial composition x. This can be seen as a reinterpretation of the Stress Compensation Method published by Maurin and Motro [3]. But instead of minimizing the difference of residual stresses and predefined stresses, the elastic potential Π due to these stresses is now considered. Opposite to standard continuum mechanics, now, the coordinates X of reference shape are the unknowns instead of the spatial coordinates x as usually. Thus, the optimization problem is formulated in terms of X and will be called Variation of Reference Strategy or short VaReS. Additionally, a prestress field may act on the intended structure (as it is the case e.g. for tent like structures or fabric roofs):

(2)

$$\min_{\mathbf{X}\in\Omega_0} \rightarrow \Pi_{total}(\mathbf{X}) = \Pi_{2D\rightarrow 3D}(\mathbf{X}) - \Pi_{prestress}(\mathbf{X})$$

3.2. Stationary point: variational formulation

The optimal cutting pattern X is defined by the first variation of equation (2) which has to vanish:

(3)

$$\delta\Pi_{total}(\mathbf{X}) = D_{\delta\mathbf{X}}\Pi_{total}(\mathbf{X}) = 0$$

Assuming hyperelastic material behavior in terms of the energy density function Ψ, the Green-Lagrange strains E and 2nd Piola Kirchhoff stresses S and applying the chain rule gives as governing equation equations:

(4)

$$\delta\Pi_{total}(\mathbf{X}) = \int_{\Omega^{2D}} \left(\mathbf{S}_{2S\to 3D} - \mathbf{S}_{prestress}\right) : D_{\delta\mathbf{X}}\mathbf{E}(\mathbf{X})\, d\Omega^{2D} + \int_{\Omega^{2D}} \left(\Psi_{2S\to 3D} - \Psi_{prestress}\right) : D_{\delta\mathbf{X}}\, d\Omega^{2D} = 0$$

Note, that the variation of the integration domain has to be considered as well. In contrast to other methods e.g. by Linhard VaReS ensures the objectivity of the material by considering the “forward” erection procedure to determine deformation whilst the “inverse” cutting pattern problem is solved**.**

4. Finite element formulation

Standard finite element discretization techniques can be applied for the solution of the nonlinear equations (4). However, additional terms for residual force vector and stiffness matrix appear because of using the discrete material coordinates X_a defining the shape of the cutting pattern as unknowns and by the variation of the integral domain.

Straight forward the components R_a of the residual force vector R are:

$$R_a = l_3 \int_0^1\int_0^1 \det J \left(\mathbf{S}_{2D\to 3D} - \mathbf{S}_{prestress}\right) : \frac{\partial \mathbf{E}}{\partial X_a}\, d\xi d\eta + l_3 \int_0^1\int_0^1 \frac{\partial \det J}{\partial X_a}\left(\Psi_{2D\to 3D} - \Psi_{prestress}\right) d\xi d\eta = 0 \quad (5)$$

with l_3 being the thickness and det J defining the mapping of the material domain into the parametric space ξ, η. Further linearization of the residual force vector R gives the nonlinear tangential stiffness matrix K. Finally, a highly nonlinear system of equations has to be solved. As a consequence, standard pseudo-time stepping techniques (path following methods, numerical continuation), as known from other nonlinear structural problems, can successfully be applied together with Newton-Raphson schemes for the iterative solution within each time step.

5. Examples

The following 3 examples will show the capabilities of the presented Variation of Reference Strategy. A section of a cylinder (see section 5.1) will serve as a benchmark problem comparing both damping and pseudo-time stepping. Additionally, different material models (isotropic and anisotropic) were applied in order to outline the generality of the presented Variation of Reference Strategy, see section 5.2.

5.1. Section of a cylinder

As reference to an analytical solution of the cutting pattern problem, a 160° cylindrical segment is used as a benchmark. The applied material model is based on a Neo- Hooke strain-energy function. An orthogonal projection onto the x-y-plane serves as an initial guess, see figure 3 (left). Due to the projection method, the elements on both ends of the strip are heavily distorted. Hence, at least 3 pseudo-time steps were necessary. As expected, the final shape of the optimized cutting pattern is a rectangle of area equal to the 160° segment, see figure 3 (right). VaReS can be proved to be stable and robust. As a consequence of the consistent linearization the convergence properties are proved as intended by the chosen discretization and trial functions.

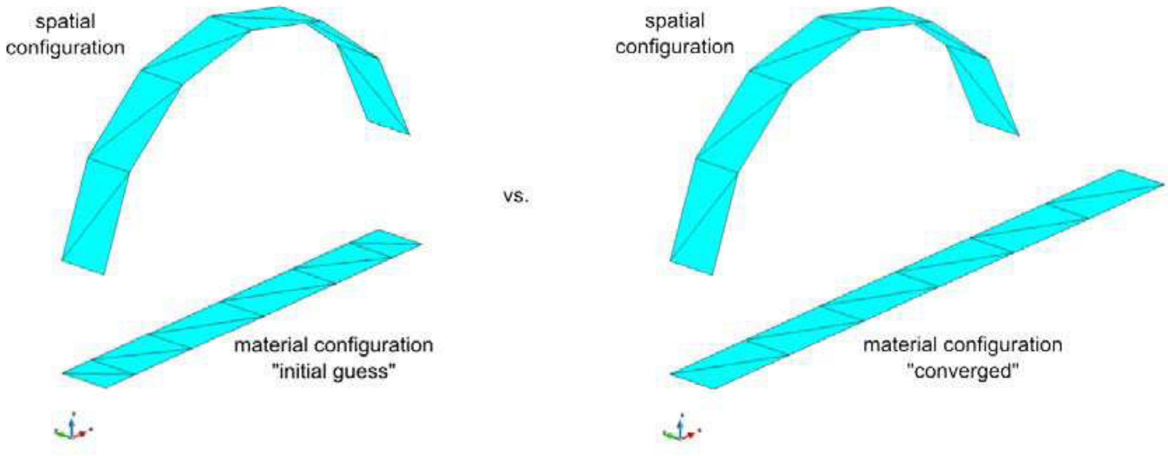

Fig. 3. 160° segment: initial guess (left) and converged solution (right)

5.2. 30deg rib

The following examples deal with the cutting pattern generation of a generic rib segment (30° arc) of fiber composite structure. Figure 4 shows the plane initial guess and the corresponding determinant of the deformation gradients (J = det(F)) as a measure for the resulting displacement field.
Two different material models were used, the Neo-Hooke model (see section 5.2.1) and the NCF model introduced by Vidal-Sallé et al. [16] and Y. Aimène et al. [17] (see section 5.2.2).

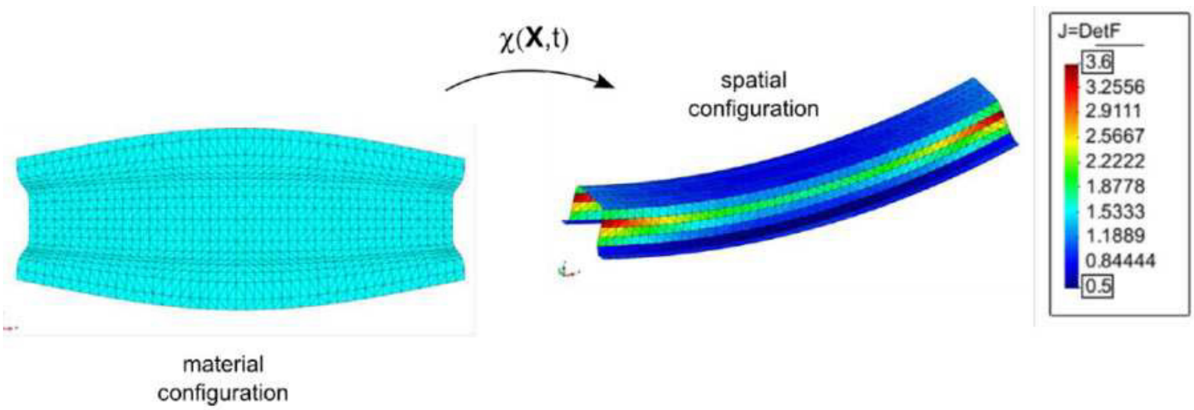

Fig. 4. 30° rib: plane initial guess (left) and corresponding deformation gradients (right)

Neo-Hooke

The challenge is to controlling the large element distortions. Consequently, 5 pseudo-time steps have been taken. Figure 5 shows the determinant of the deformation gradients corresponding to each initial guess. The initial guess of the first pseudo-time step and the converged solution of the last pseudo-time step are shown in figure 6. Again, the method converges optimally.

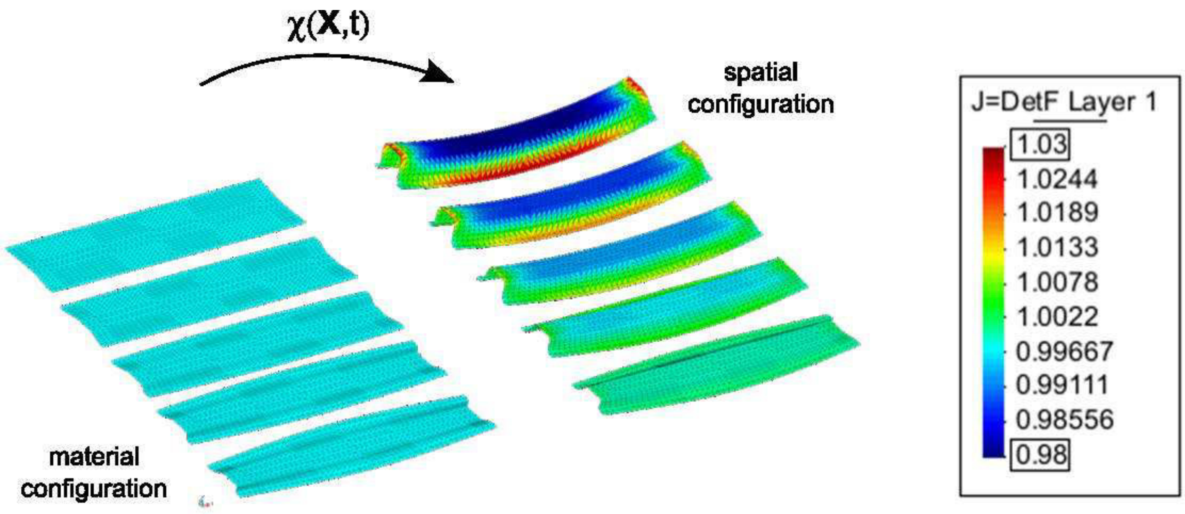

Fig. 5. 30° rib: det(F) at the beginning of each pseudo-time step

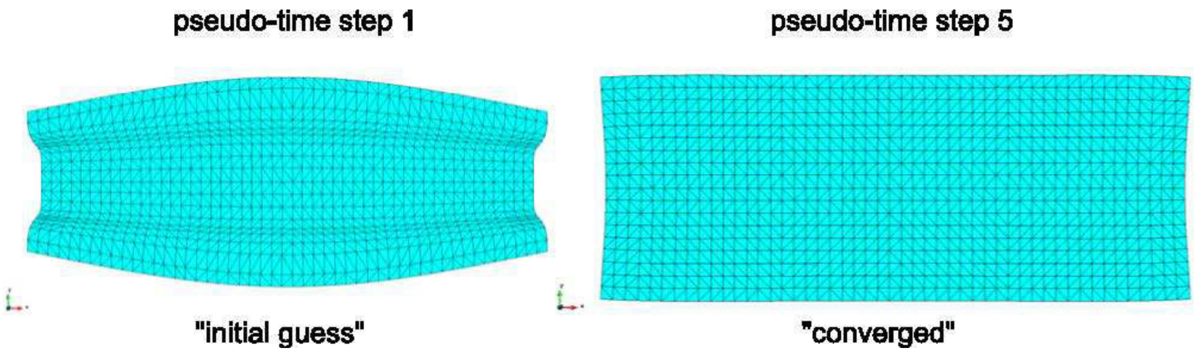

Fig. 6. 30° rib: Initial guess (left) vs. optimized cutting pattern (right

Strain-energy function for Non-Crimped Fabrics

Considering an anisotropic material behavior, the shape of the optimal cutting pattern strongly depends on the material orientation. In order to investigate these effects, a strain-energy function for non-crimped fabrics (see Y. Aimène et al. [17]) is used

$$\Psi(\mathbf{C}) = \sum_{i=0}^{r} \frac{1}{i+1} A_i \left(I_4^{i+1} - 1\right) + \sum_{j=0}^{s} \frac{1}{j+1} B_j \left(I_6^{j+1} - 1\right) + \sum_{k=1}^{t} \frac{1}{k} C_k \left(\frac{1}{I_4 I_6} tr\left(\mathbf{CMCM}^2\right)\right)^k \tag{6}$$

The coefficients of the polynomials for a glass plane weave are listed in table 1 and table 2 published in Y. Aimène et al. [17]). Both, 0°/90° and 45°/-45° fiber orientations have been under investigation. Figures 7 and 8 show the corresponding optimized cutting patterns and the resulting fiber distortions. Comparing both cutting patterns clearly shows the influence of the fiber direction. Furthermore, an analysis of the stresses in warp and weft direction provides a deeper understanding of potential areas where wrinkles are more likely to be expected. Figures 9 and 10 show the corresponding fiber stresses for the achieved cutting patterns.

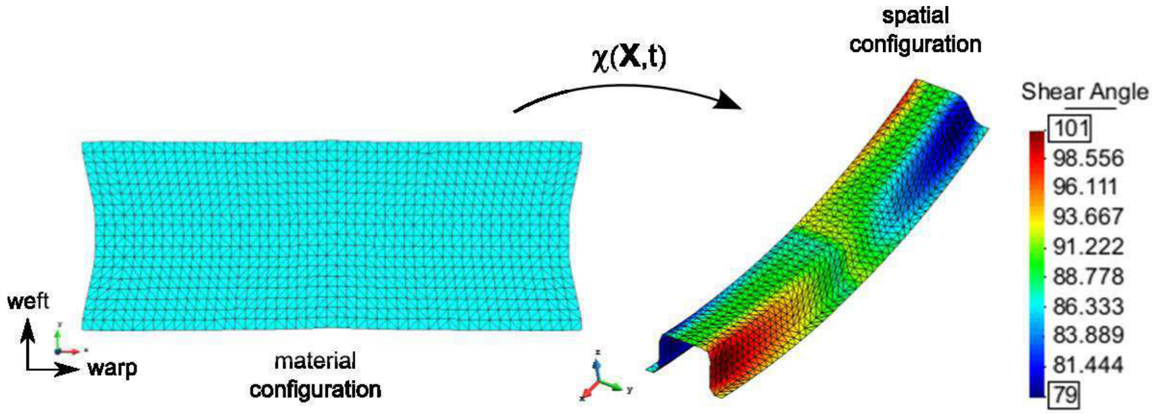

Fig. 7 30° rib: NCF 0°/90° - optimized cutting pattern (left) and fiber distortions (right)

Finally, figure 11 shows the dilemma every design engineer has to deal with, if a so-called stack of different plies has to be designed. For example, the application of the shape of the 45°/-45° configuration for a 0°/90° ply leads to areas with unprofitable compressive stresses in warp direction at the upper part of the rib, Fig. 11.

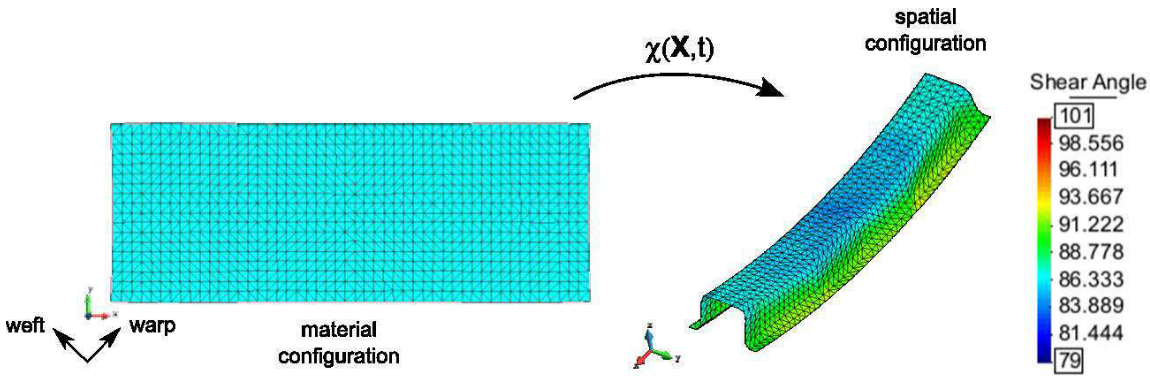

Fig. 8 30° rib: NCF 45°/-45° - optimized cutting pattern (left) and fiber distortions (right)

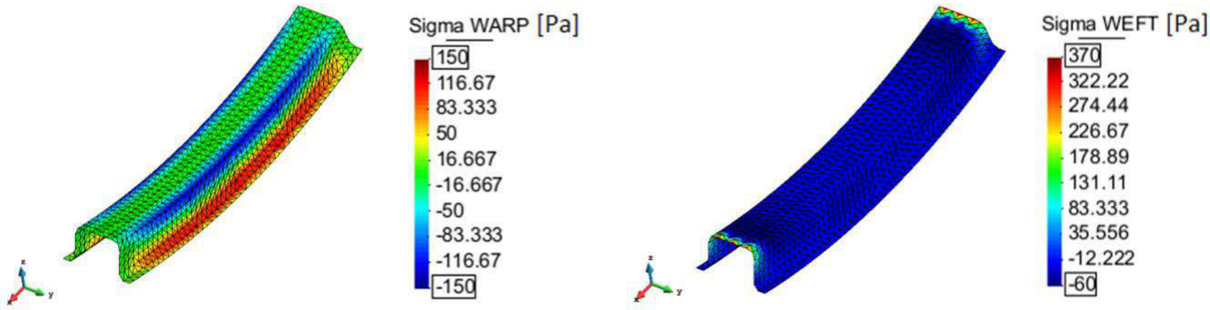

Fig. 9 30° rib: 0°/90° ply - warp-stress (left) and weft-stress(right)

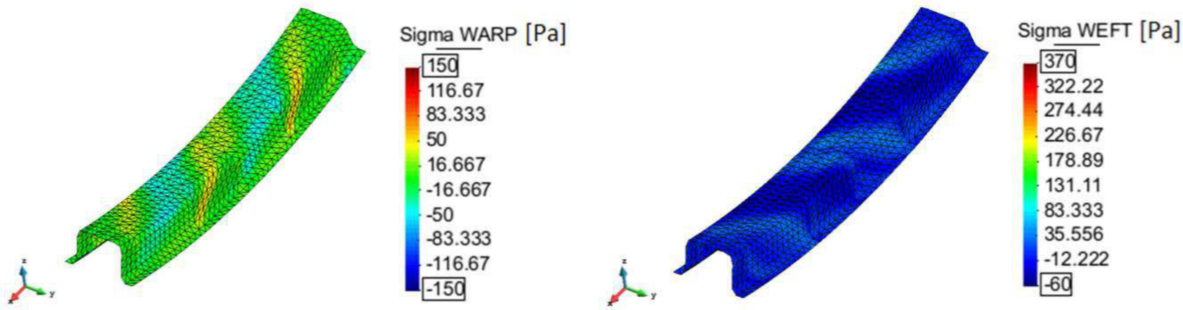

Fig. 10 Figure 24: 30° rib: 45°/-45° ply - warp-stress (left) and weft-stress(right)

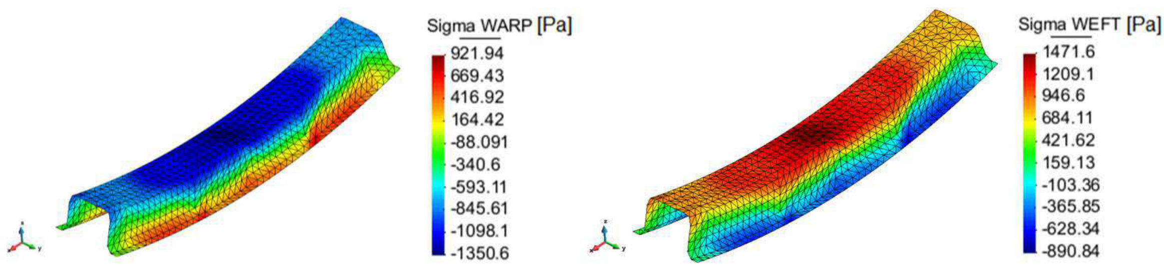

Fig. 11 Figure 25: 30° rib: Unprofitable cutting pattern - warp-stresses (left) and weft stresses (right)

6. Conclusions

The presented Variation of Reference Strategy deals with the challenge of finding the optimized plane shape for a given double-curved structure. The novel numerical approach is based on an unconstrained optimization problem, namely minimizing the stationary potential energy of the structure. At a first glance, the problem looks very similar to a Total Lagrange Formulation used for standard structural analysis. A direct comparison between both Variation of Reference Strategy and Total Lagrange Formulation clearly shows their similarities and differences. Both approaches are using the same conjugated work pair, namely Green-Lagrange strains in combination with 2nd Piola-Kirchhoff stresses. I.e. all integrals are established and evaluated with respect to the material configuration. Within a Total Lagrange Formulation the material configuration of a structure is known where the spatial configuration of the structure under loading conditions is of interest. The primary variables are the displacements of each node. However, when dealing with the Variation of Reference Strategy the spatial configuration of a double-curved structure is fixed in space (and therefore known) and the material configuration of the structure (cutting pattern) needs to be found. Hence, the primary variables are the nodal positions in the material configuration.

A major property of the presented method is the high nonlinearity of the governing equations and therir sensitivity with respect to the initialization of the Newton-Raphson scheme. The pseudo-time stepping turns out to be a very powerful remedy coming along with small number of iterations within each pseudo time step. Finally, VaReS can be easily applied to all kind of isotropic and anisotropic hyperelastic material models allowing for the cutting pattern and draping simulation of fabric structures of one or more layers.

The problem of intra-ply shear locking arising due to highly anisotropic materials such as CFRPs is addressed to further ongoing research by the chair of structural analysis.

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