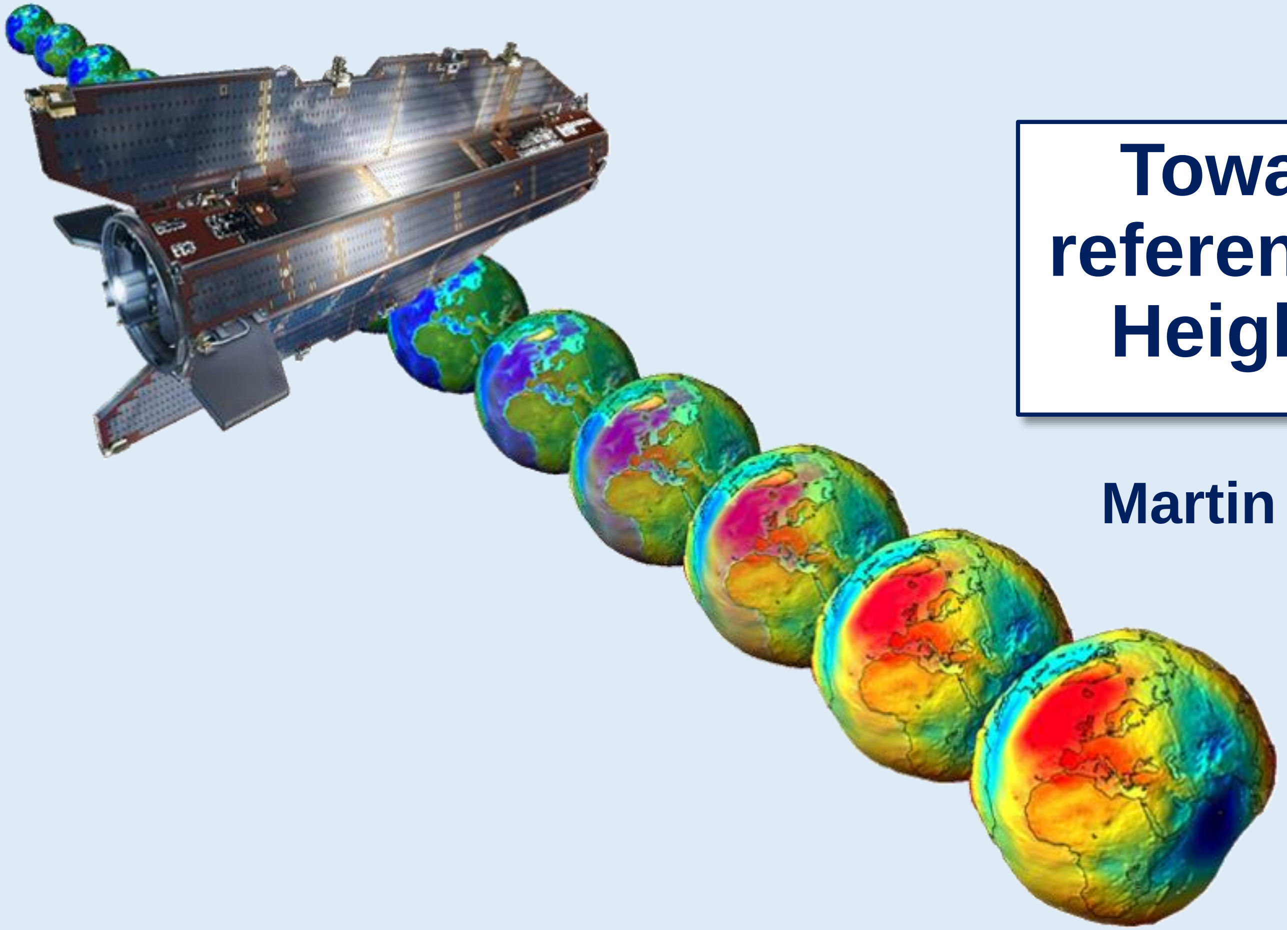
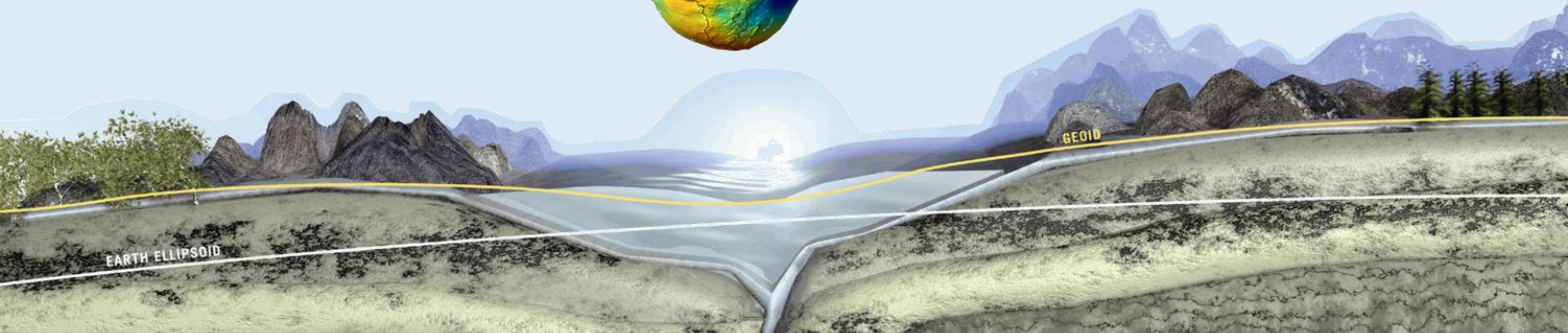


Towards a 1 mm geoid at height reference points of the International Height Reference System (IHRS)

Martin Willberg, Philipp Zingerle & Roland Pail

Chair of Astronomical and Physical Geodesy

Technical University of Munich (TUM)



Global Geodetic Reference Frame

$$\boxed{P = P\left(\mathbf{X}, W, \mathbf{g}(\mathbf{X}) = -\partial W / \partial H\right)}$$

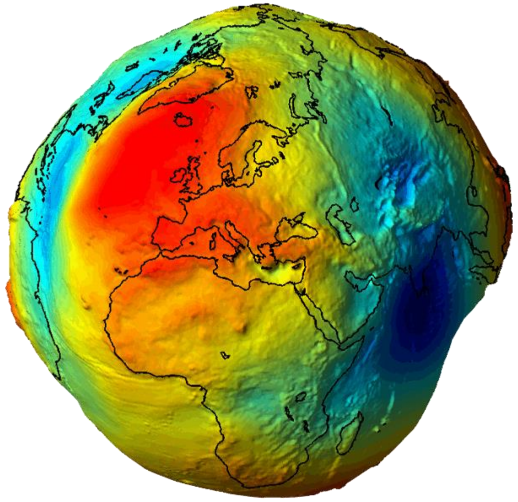
Geometric Reference Frame

- International Terrestrial Reference System (ITRS)
- Position $\mathbf{X}$ and rotation

[Angermann, 2018]

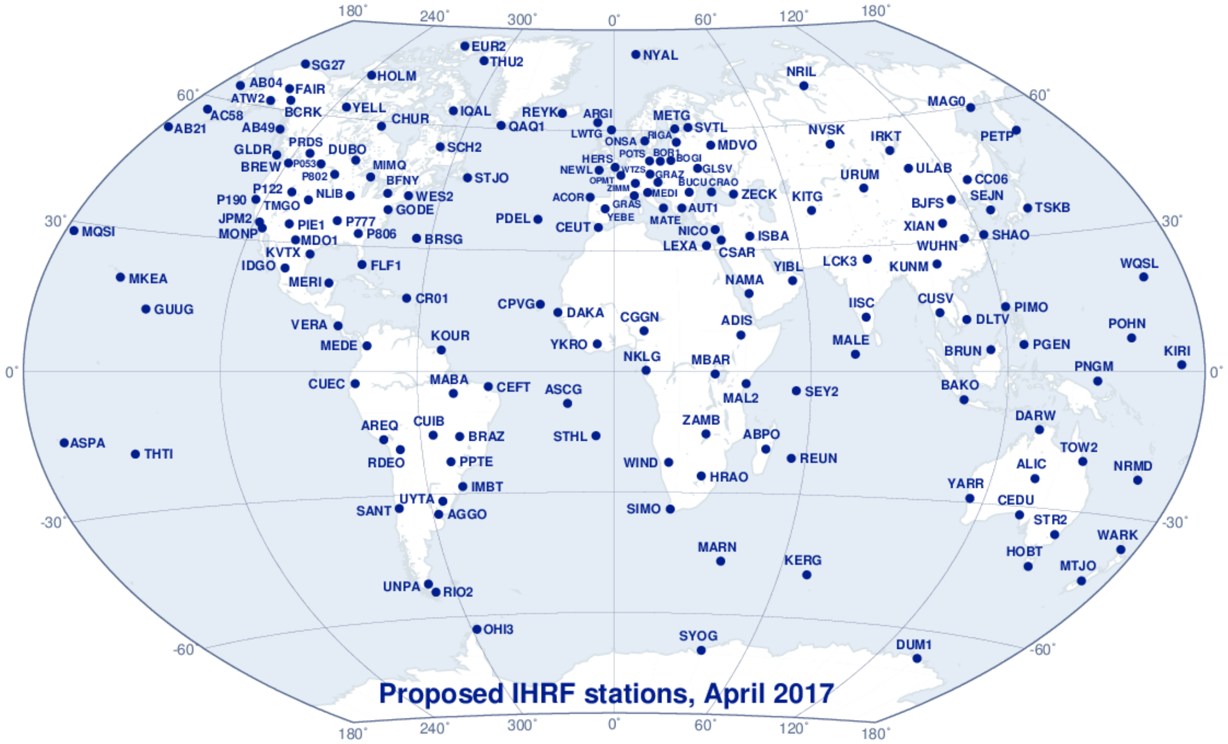
Physical Reference Frame

- Gravity field
  - Height over equipotential surface H [m]
  - Potential values W [m²/s²]
  - (Gravity vector $\mathbf{g}(\mathbf{X})$ [m/s²])
- International Height Reference System (IHRS)
- A main objective of the IAG



International Height Reference System (IHRS)

- Worldwide unified reference system
- System and frame declaration analogue to ITRS/ITRF
  - Standards and conventions
  - Global distribution of stations
- Goal: sea level variations, mass displacements
- IAG Resolution 2015:
  “an integrated global geodetic reference frame with millimeter accuracy must be implemented.”
- Realized by high precision gravity potential values at reference stations
- Poster Sánchez & Madzak (X3.40): GGOS activities related to the implementation of the IHRS



[Sanchez & Madzak, 2018]

Research topics:

1. How can we quantify the **errors**?

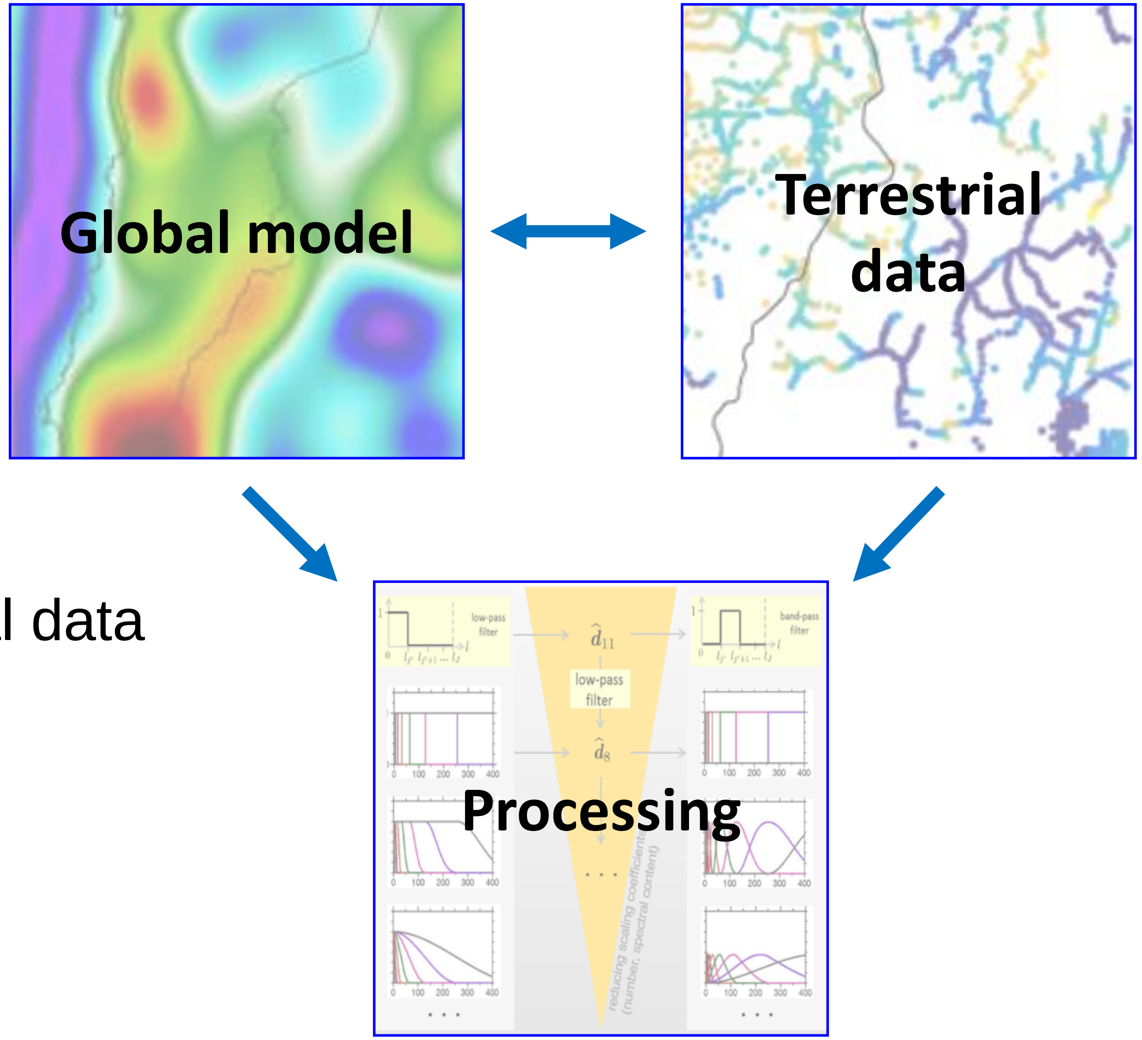
2. Which quality have **global models** and which role do they play?

3. What influence has the distribution of **terrestrial data**?

4. How can we improve the **processing**?

Requirements & quality criteria:

- 1. IHRS definitions
- 2. Global model
- 3. Terrestrial data
- 4. Processing



Challenges:

- Combine global model and terrestrial data
- Different processing methods
- Stochastic modelling

Requirements & quality criteria:

- 1. IHRS definitions
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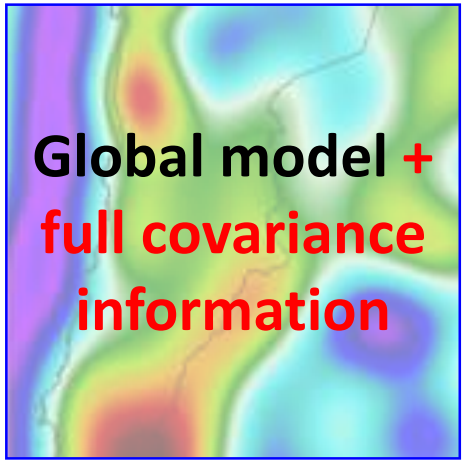
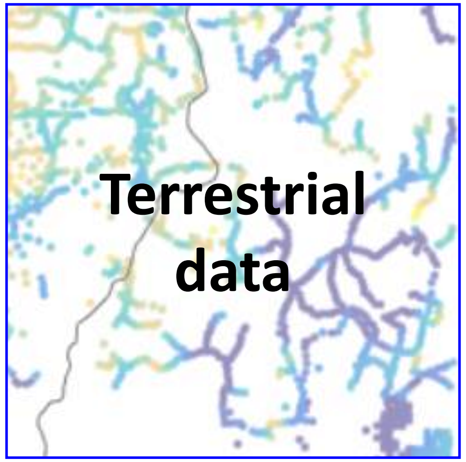
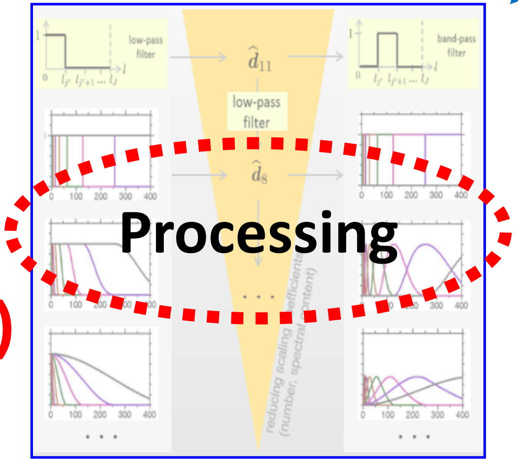




Challenges:

- Combine global model and terrestrial data
- Different processing methods
- Stochastic modelling

Least Squares Collocation (LSC)



Include full covariance information to system

Standard LSC methods:

- Assume global models as error-free
- Include error degree variances to collocation
- Approximate global model accuracy with commission error

→

Covariances:

- From signal or Tscherning-Rapp degree variances
- Assumes
  - coefficients as uncorrelated
  - no dependency of order m

Add covariances from global model

- Correlations between input points (also between input and output)
- Consistent stochastic information of global model
- All stochastic effects are included

→

Covariances:

- From propagation of a global model (from normal equations)
- Considers:
  - correlation between coefficients
  - order-dependency of covariance

Include full covariance information to system

TUM IAPG

- Basic Least Squares Collocation (LSC)

$$s = C_{sl}(C_{ll}^{noise} + C_{ll})^{-1} l$$

- LSC with Remove-Compute-Restore (RCR)
  (Assumption: global model is error free)

$$s = [C_{sl}^{topo}\left(C_{ll}^{noise} + C_{ll}^{topo}\right)^{-1}\underbrace{(l - l^{global} - l^{topo})}_{l^{remove}}] + \underbrace{s^{global} + s^{topo}}_{s^{restore}}$$

- l = input (gravity anomaly Δg)
- s = output (geoid height N)
- C_{ll} = auto covariance
- C_{sl} = cross covariance (transformation Input – Output)

- LSC with full global model covariance

$$s = [(\boldsymbol{C}_{\boldsymbol{sl}}^{\boldsymbol{global}} + C_{sl}^{topo})\left(C_{ll}^{noise} + \boldsymbol{C}_{\boldsymbol{ll}}^{\boldsymbol{global}} + C_{ll}^{topo}\right)^{-1}\underbrace{(l - l^{global} - l^{topo})}_{l^{remove}}] + \underbrace{s^{global} + s^{topo}}_{s^{restore}}$$

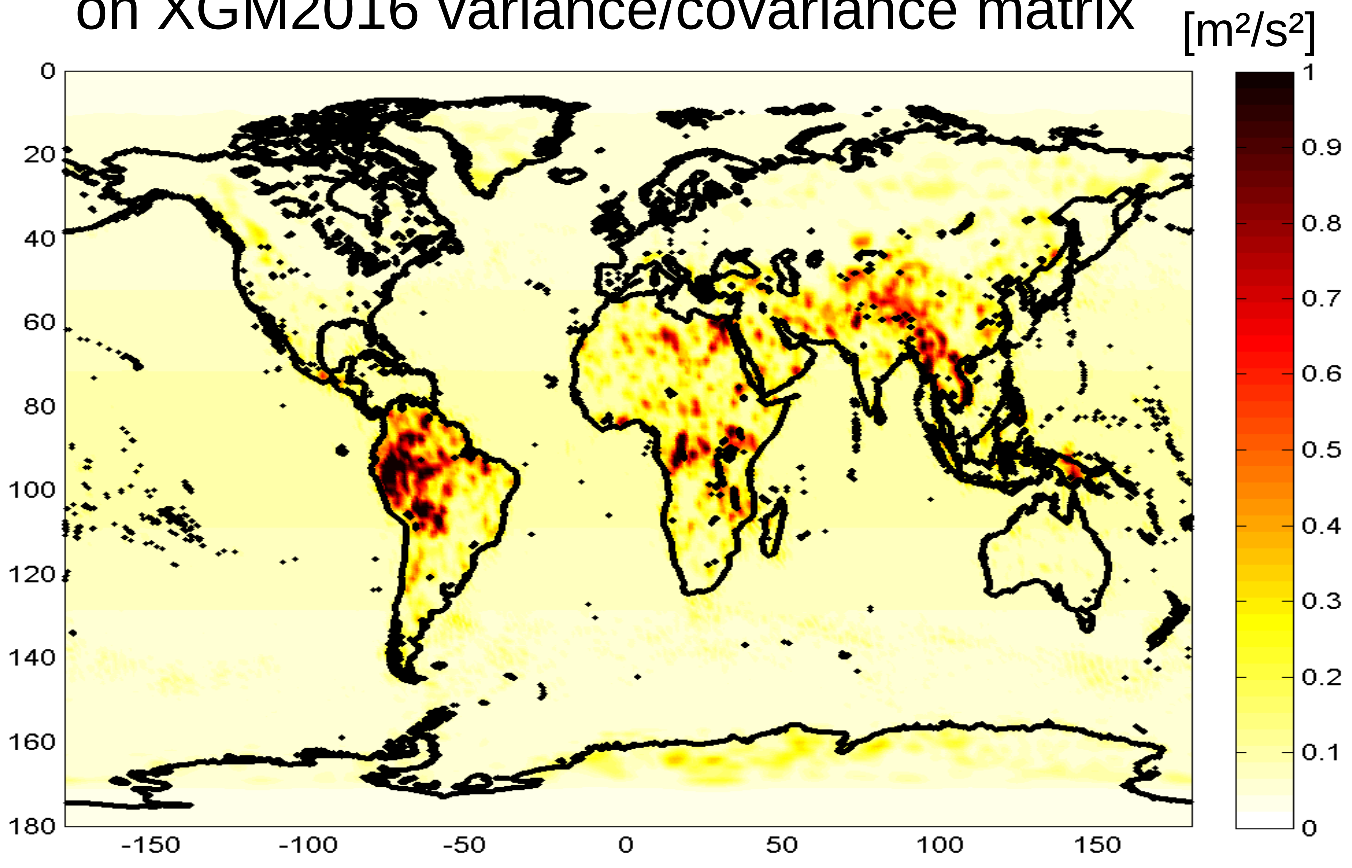
NEW COVARIANCES

Side note: XGM2016

TUM IAPG

- Experimental Geopotential Model (XGM) 2016
- Maximum degree 719 (equals to 15 min of spatial resolution)
- Calculated from
  - Satellite model GOCO05s
  - 15’ x 15’ global equi-angular grid of gravity anomalies (by NGA)
- Regional weighting depending on terrestrial data quality and homogeneity
- Potential [$0.1 m^2/s^2$] ≙ Geoid height [0.01m]
- In the following: accuracy decreased by factor 3

Full error propagation to gravity potential based on XGM2016 variance/covariance matrix



[Pail et al., 2017]

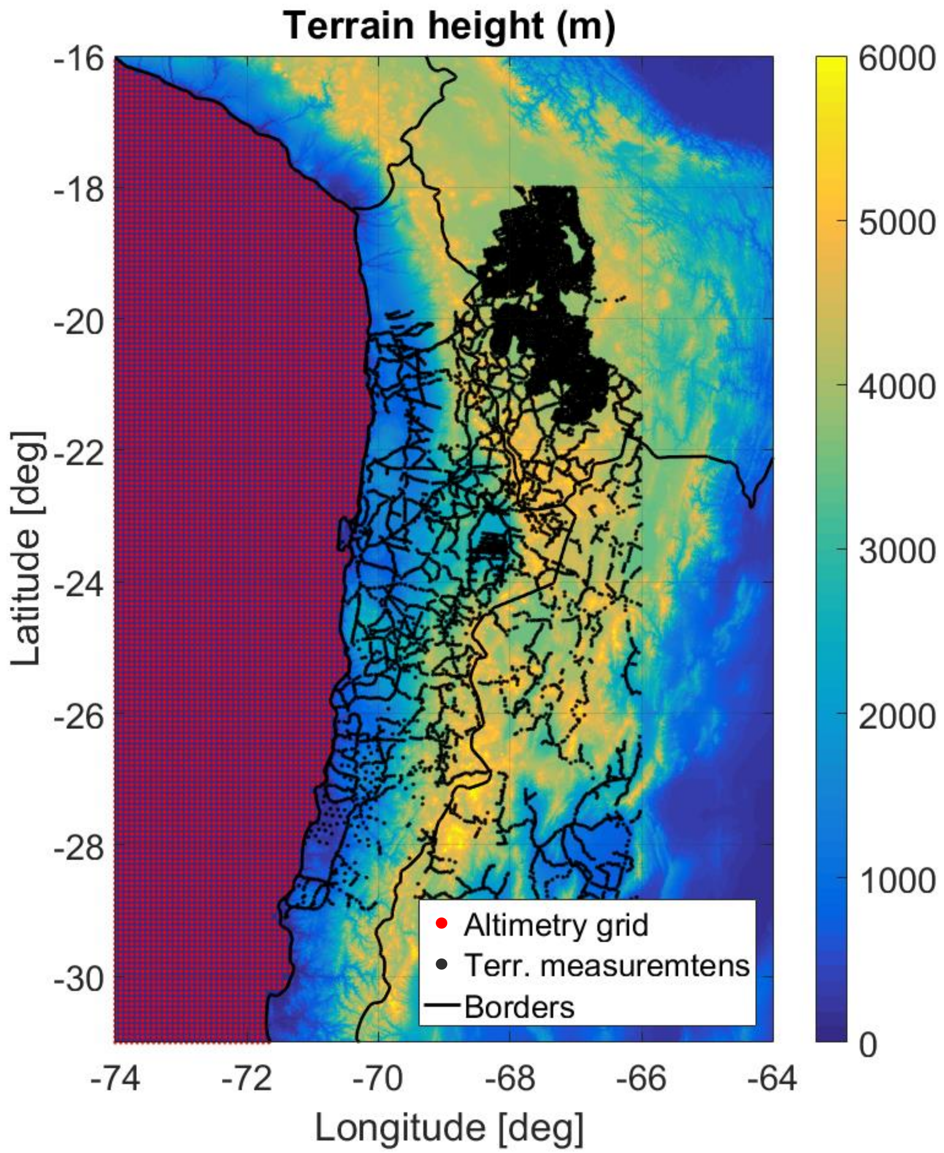
Collocation experiment

Test case in the South American Andes

- Terrestrial measurements and altimetry
- Synthetic data
- Heterogeneous data distribution

Collocation parameters

- Input: gravity anomalies Δg (14613 points)
- Output: geoid heights N (regular grid)
- Truth: spherical harmonic synthesis to 2190 (OGMOC model) (Gruber & Fecher, X.3.41)
- Global model for reduction: XGM2016
- Topography for reduction: dV_ELL_Earth2014



Collocation experiment

TUM IAPG

Noise of model

- Noise directly to SH-coefficients
- Simulates commission error
- Calculated from global model accuracy

Noise of observations

- White noise 1 mGal
- Not related to degree

Degree

2

$\boxed{200}$

2190

- RCR model l^{global}
- Covariances $C_{sl/ll}^{global}$

$\boxed{\text{Max. model degree}}$

- RCR topography l^{topo}
- Covariances $C_{sl/ll}^{topo}$

$$s = \underbrace{[C_{sl}^{global} + C_{sl}^{topo}]}_{\text{Cov. Input-output}} \underbrace{\left(C_{ll}^{noise} + C_{ll}^{global} + C_{ll}^{topo}\right)^{-1}}_{\text{Cov. of input}} (\underbrace{l}_{\text{Input}} \underbrace{- l^{global} - l^{topo}}_{\text{Remove}})] + \underbrace{s^{global} + s^{topo}}_{\text{Restore}}$$

Collocation experiment

TUM IAPG

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$\boxed{719}$ $\boxed{\text{Max. model degree}}$

- RCR topography l^{topo}
- Covariances $C_{sl/ll}^{topo}$

2190

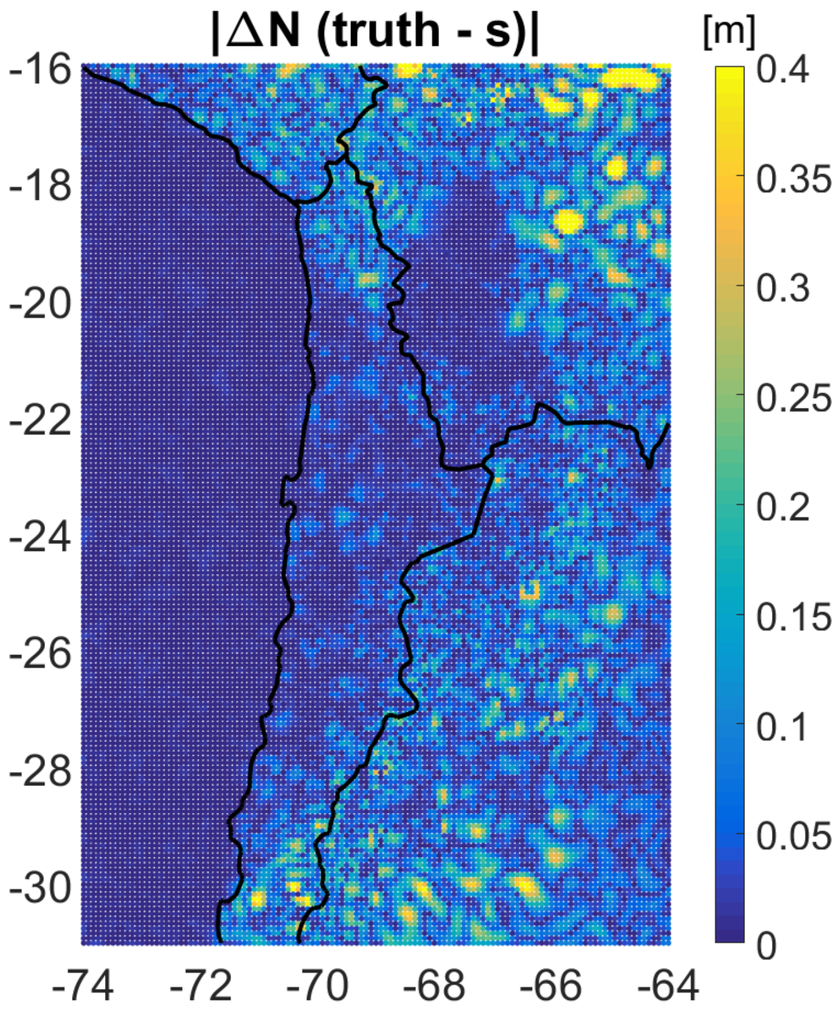
$$s = \underbrace{[C_{sl}^{global} + C_{sl}^{topo}]}_{\text{Cov. Input-output}}\underbrace{\left(C_{ll}^{noise} + C_{ll}^{global} + C_{ll}^{topo}\right)^{-1}}_{\text{Cov. of input}}(\underbrace{l}_{\text{Input}} \underbrace{- l^{global} - l^{topo}}_{\text{Remove}})] + \underbrace{s^{global} + s^{topo}}_{\text{Restore}}$$

Collocation experiment

TUM IAPG

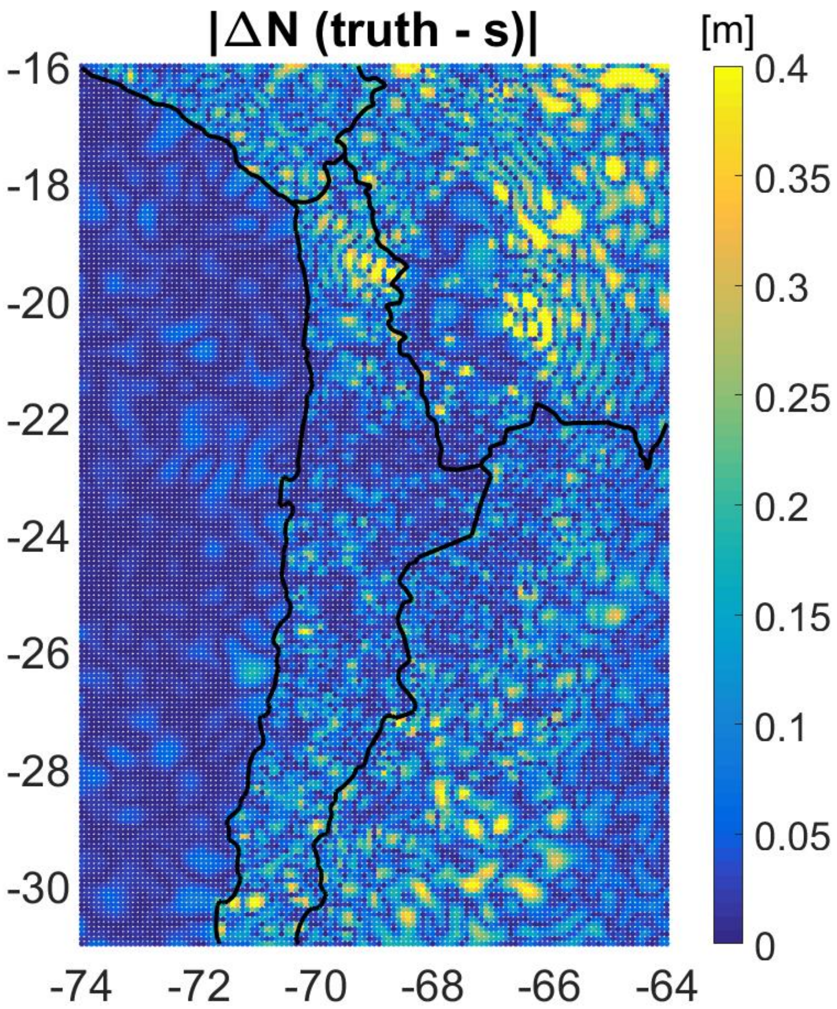
Collocation with full model covariance

Max. model degree: 719



Classical collocation with RCR

Max. model degree: 719



Collocation experiment

Collocation with full model covariance

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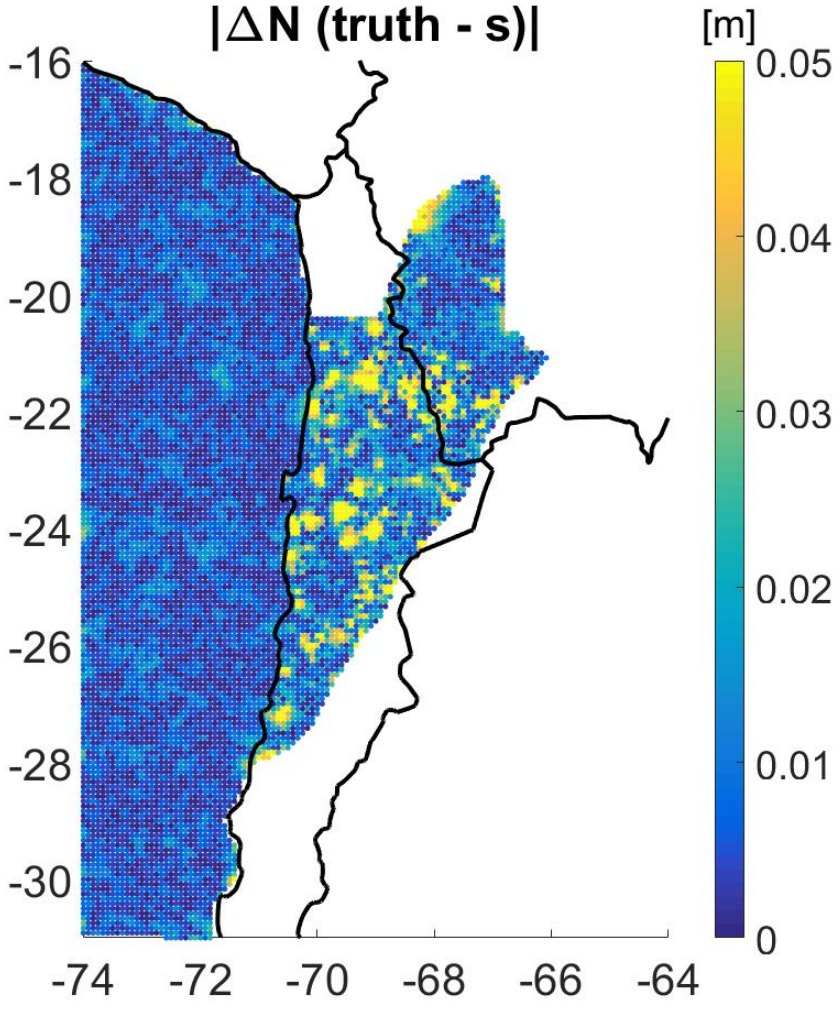

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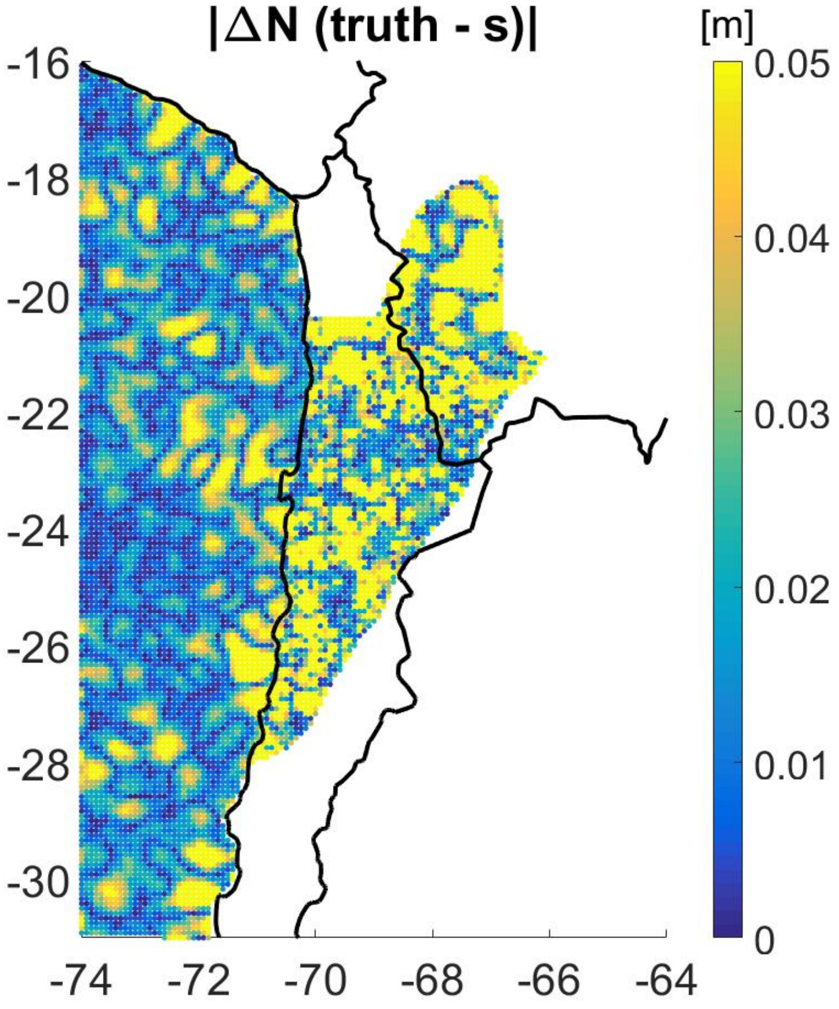

Collocation experiment

Collocation with full model covariance

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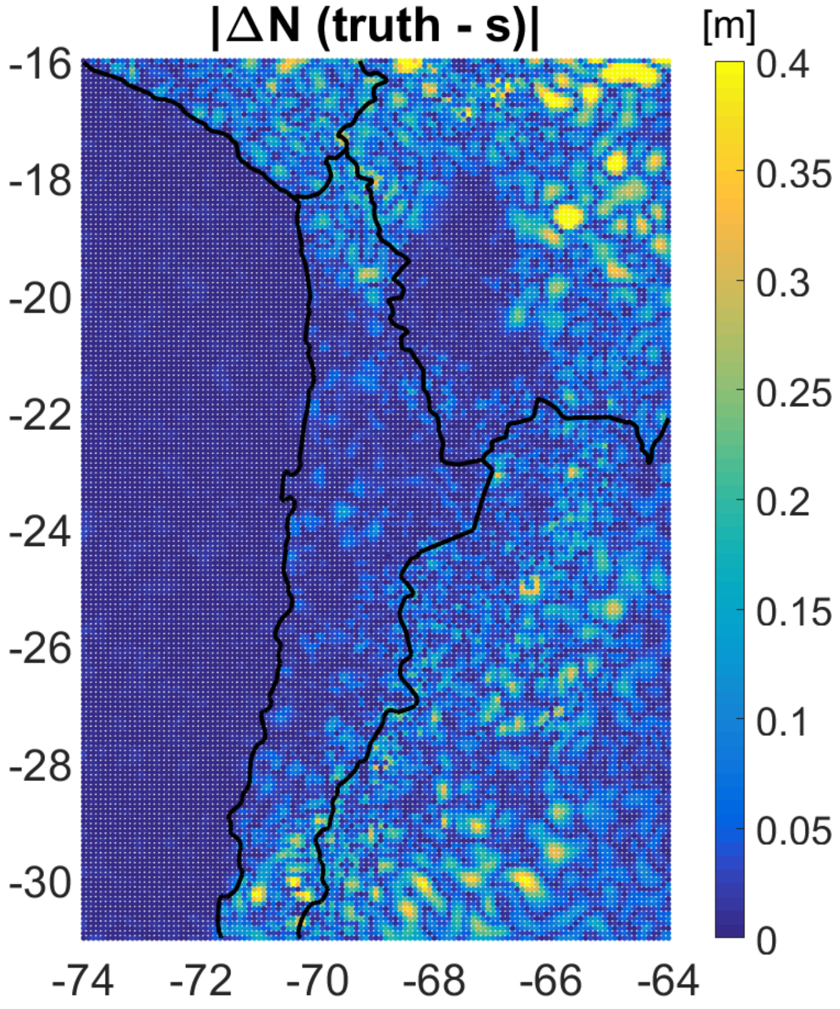

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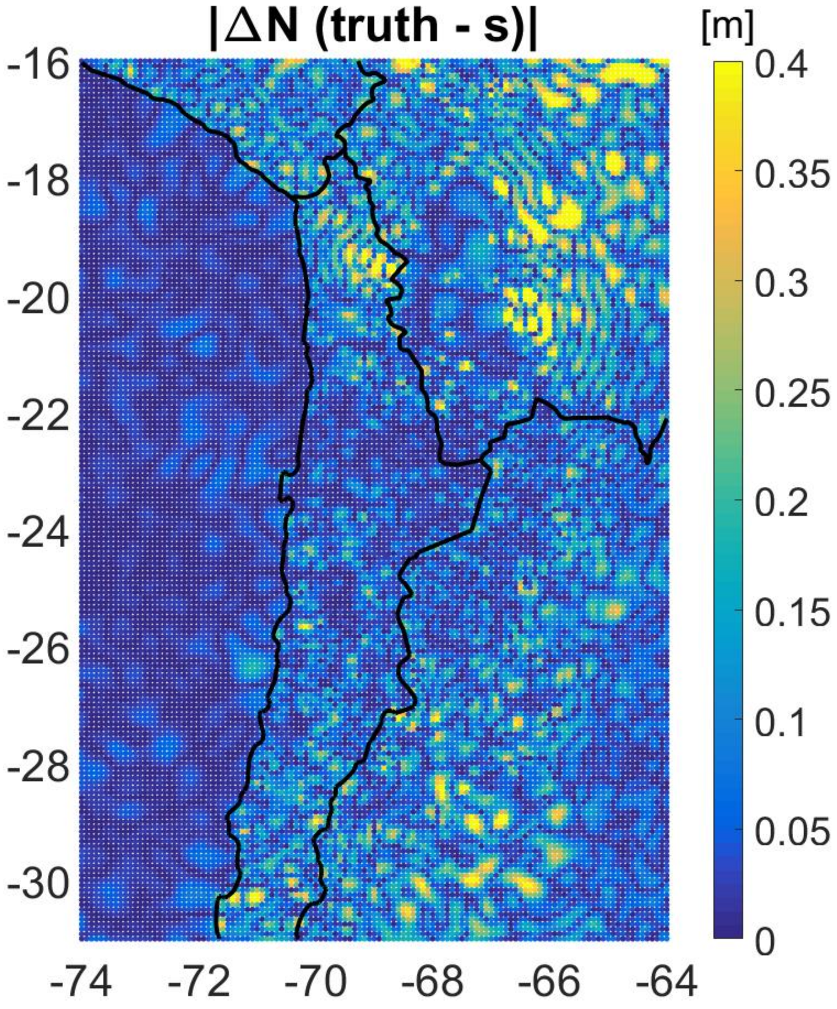
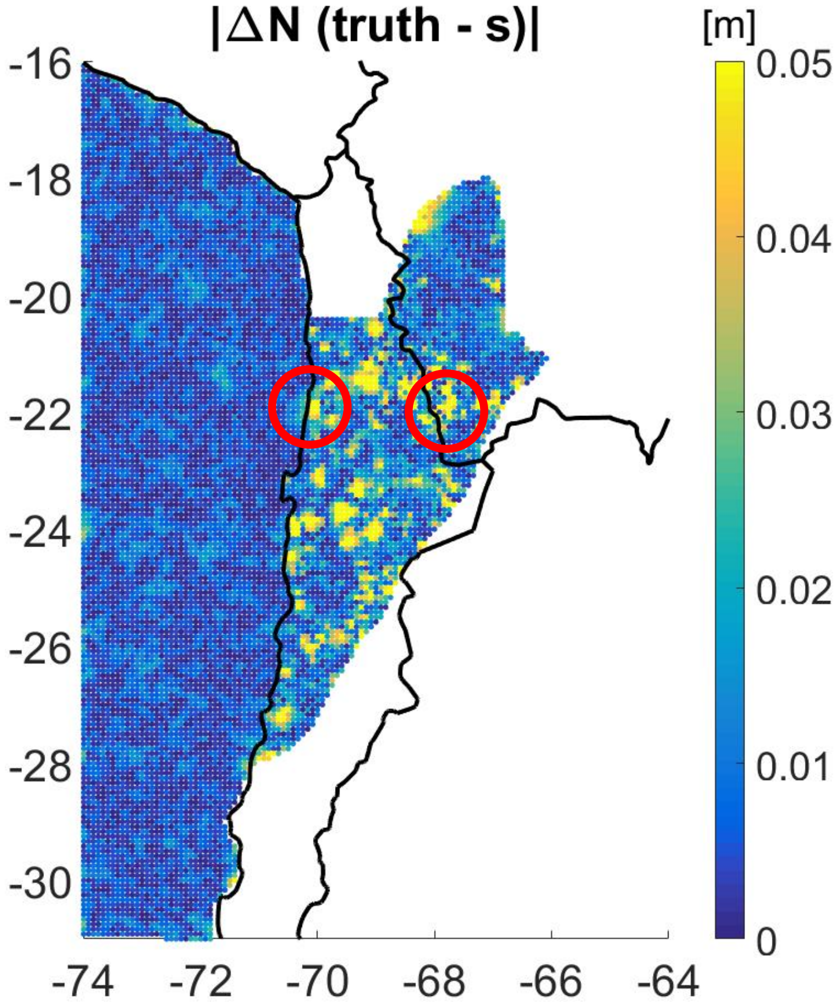


Collocation experiment

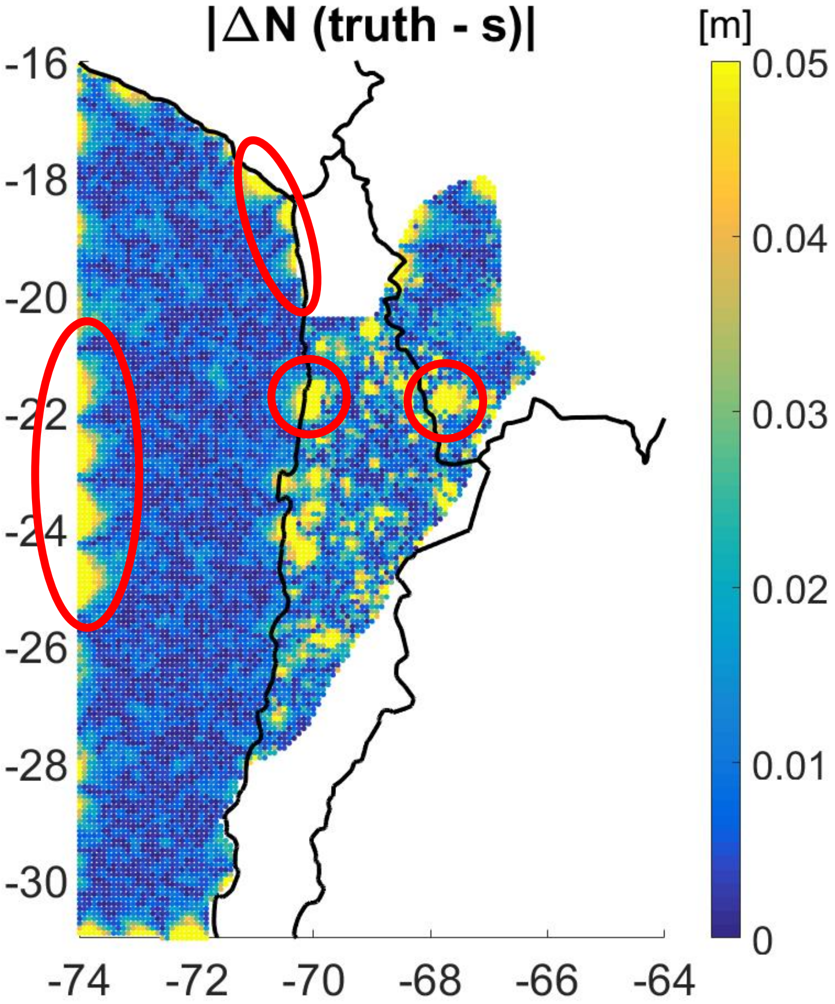
Collocation with full model covariance

Max. model degree: 719



Classical collocation with RCR

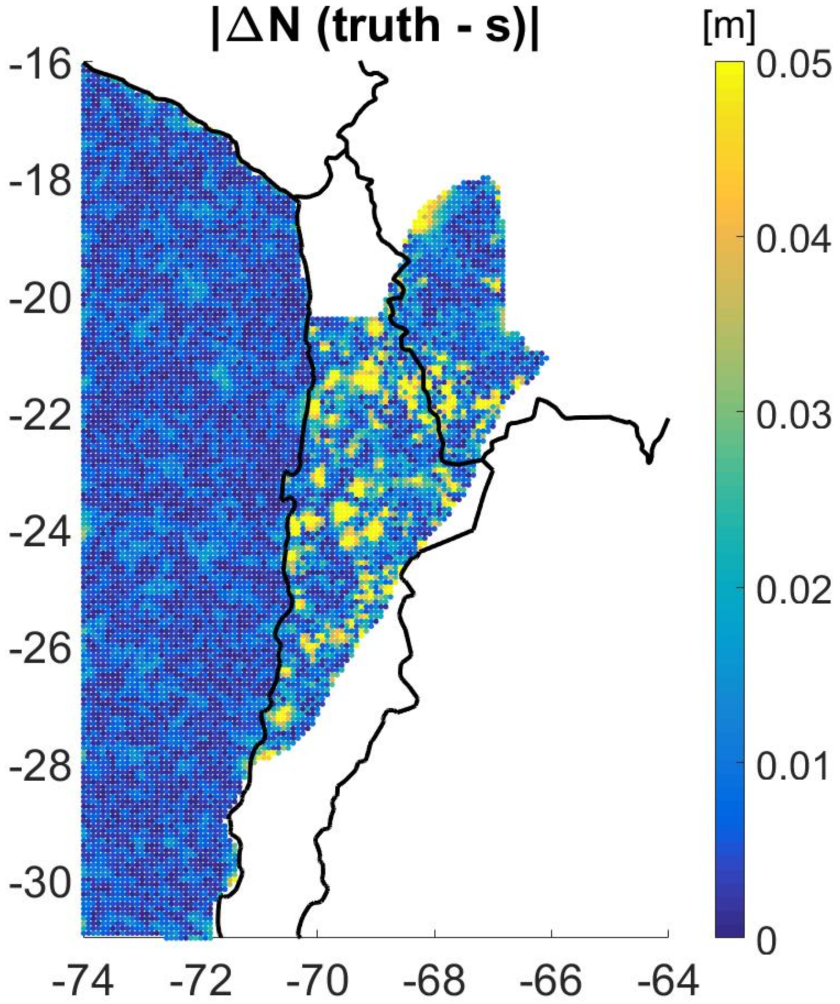
Max. model degree: 200



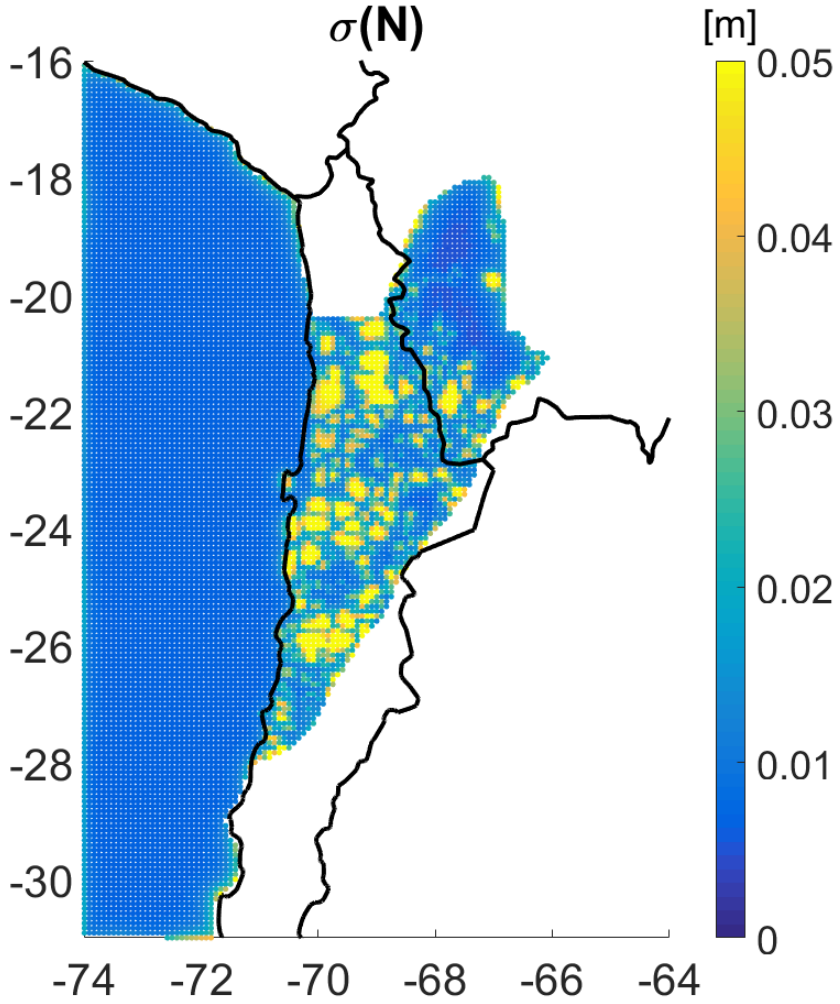
Collocation experiment

Collocation with full model covariance

Max. model degree: 719



Accuracy of collocated geoid height N



Summary & outlook

1. How can we quantify the **errors**?
   - Consistent stochastic modelling
   - Error estimation for potential values with all included effects

2. Which quality have **global models** and which role do they play?
   - Use quality of high-resolution global models as basis

3. What influence has the distribution of **terrestrial data**?
   - Used for densification
   - Decrease amount of terrestrial data

4. How can we improve the **processing**?
   - Include full covariance matrix of a global model to LSC