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# RANDOM ACOUSTIC METAMATERIALS

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Acoustic metamaterials (AMM) having periodic resonators are well known in the field of acoustic absorber. The major issue, however, is that such AMM are able to band acoustic wave in specific frequency band. In this paper, the spectral-based stochastic method is employed to develop random AMM to overcome the above mentioned issue. Here, the stochastic finite element method (FEM) is applied to random distributed AMM to get maximum band gap. To this end, a FEM model with random distributed resonators is considered to investigate the maximum band gap in the frequency range. The spectral-based stochastic simulation is used to minimize the cost in terms of computational time.
Keywords: Acoustic Metamaterials, Stochastic FEM, Vibro-acoustic, Polynomial Chaos

## 1. Introduction

Metamaterials (MM) are man-made materials having extraordinary properties like negative effective mass and stiffness. Acoustic MM (AMM), as a type of MM, are used to reduce noise and vibrations [1, 2]. The most common type of such materials is made using very thin membrane having adjustable concentrated masses, known as resonators [3, 4]. They exhibit a negative effective mass at specific frequency ranges. Here, a strongly reduced acoustic transmission loss at low frequency is achieved by negative effective density of the structures when the acoustic wave passing through it. During the past decade, many research have been conducted to investigate negative mass density [5–7], negative bulk module [8] or on both negative mass and bulk module [9, 10].

The major issue relating AMM is, however, adjusting parameters to tune the frequency bands and the location of the highest transmission loss. In fact, once the parameters are estimated and the structure made of AMM is produced, the re-adjusting the parameters is not more possible. An effective method to overcome such drawback is to use stochastic determination of the parameters in the design stage, as introduced in this paper. Here, the spectral stochastic simulation method is applied to adjust parameters. Such simulation yields very effective and a wide domain for selection of parameters of the resonators. The results shows that due to considering random input parameters, the amplitude of the vibration is considerable reduced.

This paper is organized as follows: the next section describes the spectral method for representation of the random input parameters and output responses. Section 3 presents the numerical results for a typical AMM having small resonators with random parameters. The conclusions are given in the last section.

## 2. Spectral representation of random parameters

The spectral characterization methods are the key advantage for the efficient stochastic reduced basis representation of uncertain parameters in modeling. This is because these methods provide a similar application of the deterministic Galerkin projection and collocation methods to reduce the order of complex systems. In this way, it is common to employ a truncated expansion to discretize the input random quantities of the structure and system responses. The unknown coefficients of the expansions then can be calculated based on the model outputs. Let us consider the uncertain parameter $P(\xi_i)$ where $\xi \in \Omega$ is the random variable characterizing the uncertainty in the parameter and $\Omega$ denotes the random space. Under the limited variance of the parameter, i.e. $\sigma_p^2 < \infty$, the parameter can be approximated as [11]

$$P(\xi) = \sum_{i=0}^{N} p_i \Psi_i(\xi) \tag{1}$$

This is the truncated generalized Polynomial chaos (gPC) expansion of the parameter up to oder of $N$. The deterministic coefficients $p_i$ are calculated as

$$p_i = \frac{1}{h_i^2} \int_\Omega P(\xi)\Psi_i(\xi) f(\xi)\, \mathrm{d}\xi \qquad\qquad i = 1, \ldots, N \tag{2}$$

in which $f$ is the joint probability density function (PDF) of random variable $\xi$ and $h_i$ denotes the norm of polynomials defined as

$$h_i^2 = \int_\Omega \Psi_i^2(\boldsymbol{\xi}) f(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} \tag{3}$$

Considering uniform distributed uncertain parameters, the orthogonal Legendre polynomials are used as random basis, i.e. $\Psi_i(\xi) = L_i(\boldsymbol{\xi})$. Accordingly, the PDF $f(\xi) = 1$ for orthogonality domain of $[-1, 1]$. The following MATLAB® code then is used to estimate the coefficients $p_i$.

MATLAB® code 1: Claculation of coefficients in Eq. (2)

```
%  random parameter with unifrom PDF: U(a,b)
clc,clear all
a=0.5;b=1.5;
syms xi
N=2;
L=[legendreP(0:N-1, xi)]; % Legendre Poly. up order n-1
rho_xi=1; % density of \xi
P=0.5*(a+b)+0.5*(b-a)*xi;
Hi=int(P.*L(1:N)*rho_xi,xi,-1,1);
hi2=int(L(1:N).^2*rho_xi,xi,-1,1);
pi=eval(Hi./hi2)
```

For AMM having random multi-resonators, each parameter has to be approximated by the gPC with individual random variable and orthogonal basis. Having $n$ uncertain parameters leads to a $n$-dimensional random vector of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_n\}$.

## 2.1 Spectral representation of random responses

Analogy to input uncertain parameters, stochastic responses of AMM structures can be approximated using the gPC expansion with unknown deterministic functions. In this paper, the frequency response function (FRF) of the AMM considered as output random. It is approximated by multi-dimensional gPC expansion of random vector $\boldsymbol{\xi}$ as

$$\mathrm{FRF}(\omega, \xi_1, \ldots, \xi_n) = \sum_{j=0}^{s} b_j(\omega)\Phi_j(\xi_1, \ldots, \xi_n) \tag{4}$$

in which $b_j$ are unknown deterministic depending on the frequency $\omega$ and $\Phi_j$ are multi-dimensional orthogonal basis. The basis are constructed from the tensor product of the individual random basis used to approximate the input uncertain parameters, see for details [11]. As the response of the system is not known a priori, the unknown functions can not be estimated. For that, the non-intrusive stochastic method is used in which realizations of system response are generated at a set of collocation points. The roots of higher order polynomial $\Psi_{i+1}(\boldsymbol{\xi})$ can be simply used as collocation points. Depending on the number of unknown functions $s+1$, a set of $2(s+1)$ realizations of the system response are recommended for a very good estimation of the unknown functions. More details on the method can be found in [12–17].

## 3. Numerical case study

To demonstrate the impact of random resonators, a multi-DOF vibration model is considered, as shown in Fig. 1. The details of deterministic solution of such typical model is given in [18]. The spring and damping constants of the resonators are considered as uncertain parameters given by

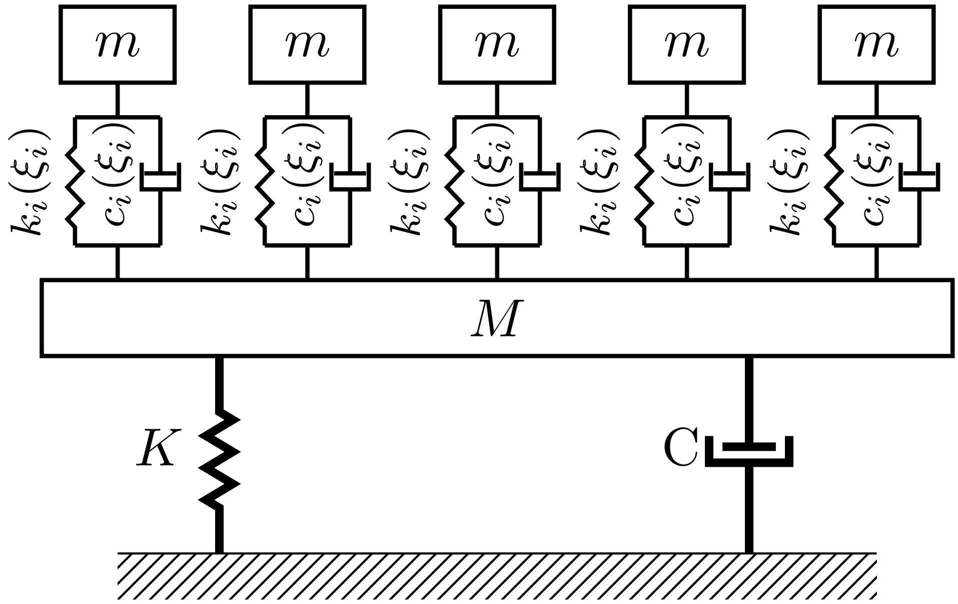


Figure 1: Locally resonant structure with multiple periodic random arrays of attached spring-mass resonators; $M = 100$ kg, $K = 400\pi^2 M$ N/m, $C = 0.10\sqrt{MK}$ Ns/m and $m = 2.0$ kg.

$k_i(\xi_i) = 400\pi^2 m\beta(\xi_i)$ and $c_i(\xi_i) = 0.10\sqrt{mk_i(\xi_i)}$ where $\beta$ is a uniform distributed random variable having minimum and maximum values of $(\beta_{min}, \beta_{max}) = (0.5, 1.5)$. The first order gPC expansion of $\beta$ is constructed using orthogonal Legendre polynomials $L_i(\xi_i)$ as

$$\beta(\xi_i) = p_0 L_0(\xi_i) + p_1 L_1(\xi_i) + \dots \qquad i = 1, 2, \dots, 5 \tag{5}$$

in which employing MATLAB® code given in the section 2 the gPC coefficients are calculated as $p_0 = \frac{1}{2}(\beta_{min} + \beta_{max}) = 1.0$ and $p_1 = \frac{1}{2}(\beta_{max} - \beta_{min}) = 0.5$. Note that the higher order coefficients are zero, i.e. $p_i(i \geq 2) = 0$. The constructed PDF of $\beta$ from the first order gPC expansion compared with theoretical function is given in Fig. 2. As shown the first order gPC expansion has very high accuracy to represent the uncertain parameters.

The deterministic finite element model of the system is performed using ANSYS and the APDL code is employed as black-box model for the stochastic simulation. The harmonic analysis of the system is performed with applied force of amplitude $F = 200$ N on the main mass $M$. The frequency response function (FRF) of the system at position of $M$ is considered as random output approximated using second order gPC expansion as given in Eq. (4). The expansion includes 21 unknown functions

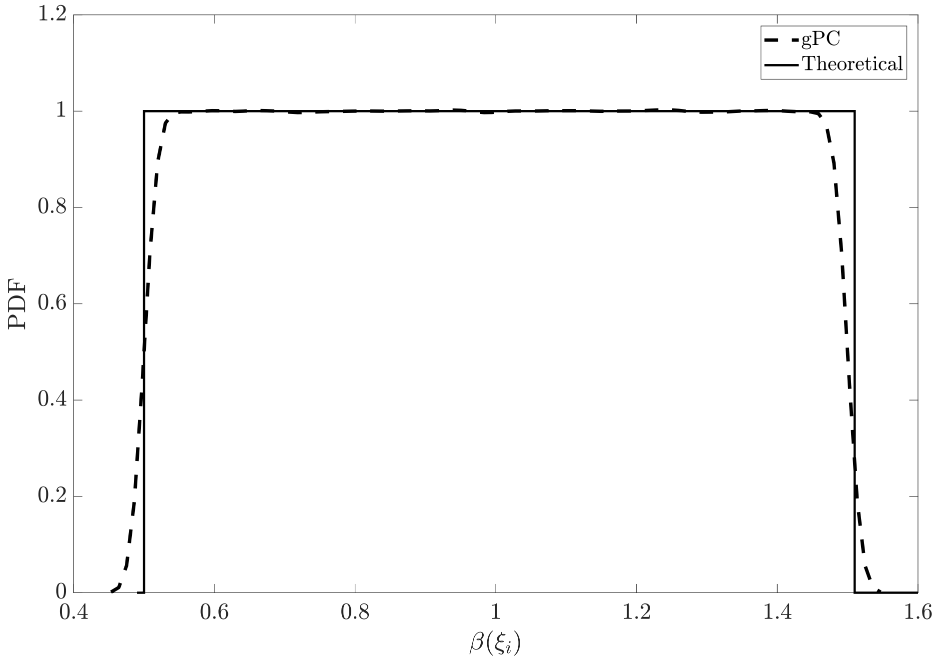


Figure 2: The constructed distribution from the first order gPC compared with theoretical PDF

which are estimated from realizations of the system at collocation points. A set of 50 collocation points are generated from the roots of 3rd-order Legendre polynomial. The system model is executed for realizations of the uncertain parameters at each collocation points. Once the realizations are known, a least-square minimization procedure is used to calculate the unknown deterministic functions $b_i(\omega)$. This leads to a closed from for the random FRF depending to uncertainty in the random input parameters. The impact of random resonators on the amplitude of the main structure is shown in Fig. 3. The dotted line shows the results for deterministic system with identical resonators. Trivially, the whole system acts as 2-DOF model in this case with two distinct resonances. The impact of resonators are appeared as increasing the stiffness, damping and mass of the system with light reduction of the amplitude peaks compared to the response of the main structure without resonators, shown as bold dotted line. This shows that using identical resonators has not much effective impact for vibration reduction. In contrast, the shaded area shows the whole domain of uncertainty associating the response. Clearly, adjusting proper values for parameters of the resonators in this area yields effective amplitude reduction.

## 4. Conclusions

In the past decade, the field of acoustic metamaterials has grown. Basic design concepts and experimental demonstrations show a wide range of extreme effective acoustic properties. It is desired to design acoustic metamaterials with the wide range of frequency band. In this paper, a stochastic procedure has been proposed to achieve optimal parameter adjustment. The non-intrusive stochastic simulation provide us to use developed deterministic models of metamaterials ss black-box which is very attractive for effective simulation of large systems. As demonstrated, the spectral stochastic simulation yields very wide range of parameters selection for the metamaterials compared to deterministic results with a considerable reduction of vibration amplitude.

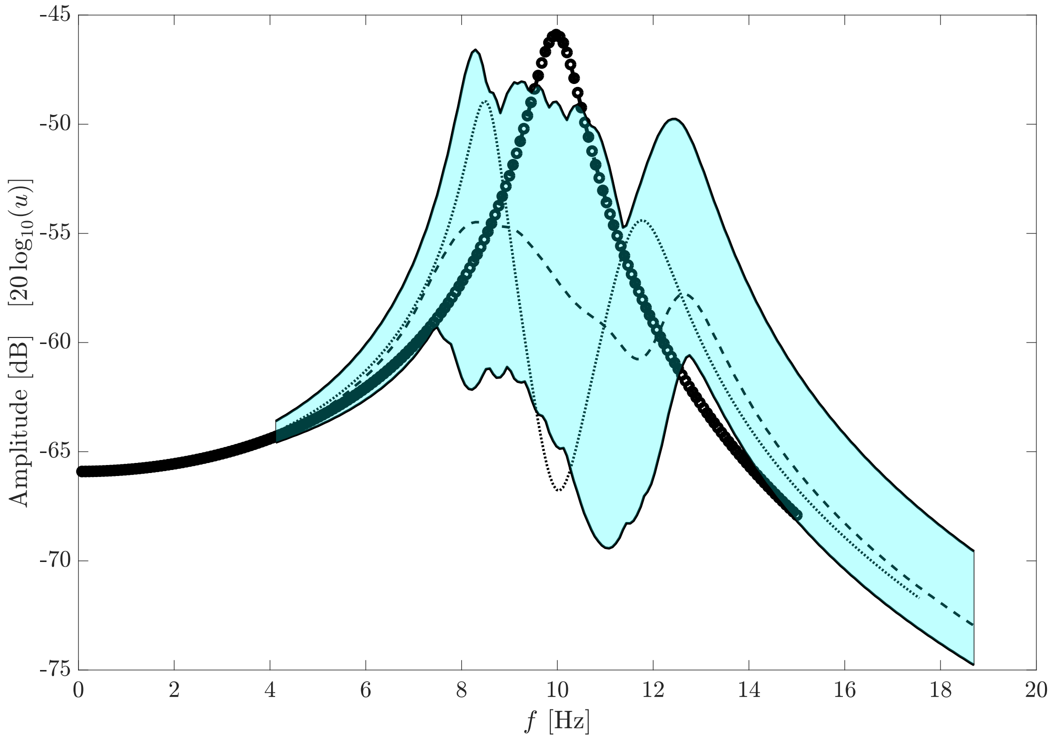


Figure 3: Shaded area: the random FRF; dashed line: the mean FRF; dotted line: the system FRF with similar deterministic resonators; and bold dotted line shows the response of main structure without resonators, $\omega = 2\pi f$.

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