Chair of Scientific Computing
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Partitioned multirate coupling schemes for the heat equation in preCICE

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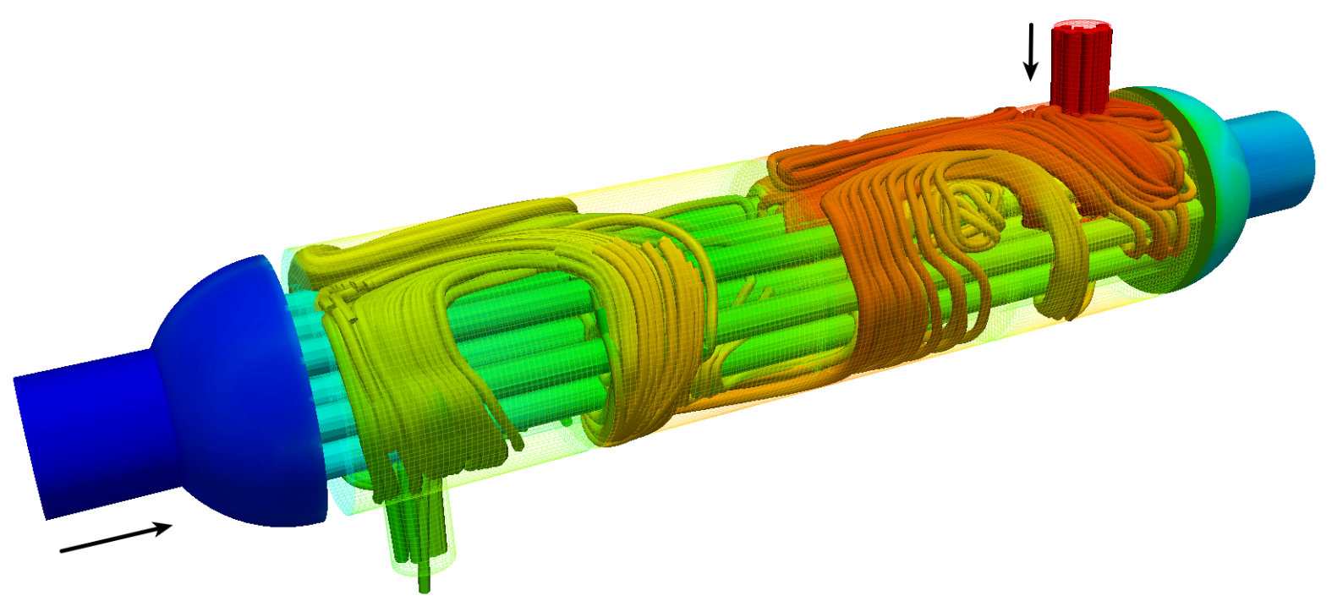
Multirate partitioned multi-physics

Motivation
The efficient simulation of multi-physics phenomena is an important task in research and industry. Currently, there is a high demand for flexible time stepping methods that allow to account for multirate characteristics (i.e. different resolution in time) of the different physical domains [1].



Shell and tube heat exchanger [2].

Our goal
We look for an algorithm that supports high order multirate time stepping. On this poster two different multirate coupling schemes are presented and applied to a model problem [3,4].

Partitioned heat equation

The partitioned heat equation is used as a model problem:

$$(\rho c_p)_m \frac{\partial u_m}{\partial t} - \lambda_m \Delta u_m = 0,\ x \in \Omega_m$$
$$u_m = 0,\ x \in \partial\Omega.$$

The material properties ($\lambda_m, (\rho c_p)_m$) may differ, if different materials are used on the subdomains. We add coupling conditions at the interface $\Gamma = \Omega_1 \cup \Omega_2$:

$$u_1 = u_2,\ x \in \Gamma$$
$$\lambda_1 \frac{\partial u_1}{\partial n_1} = -\lambda_2 \frac{\partial u_2}{\partial n_2},\ x \in \Gamma$$

The first coupling condition guarantees consistency of temperature u, the second consistency of heat flux q on Γ.

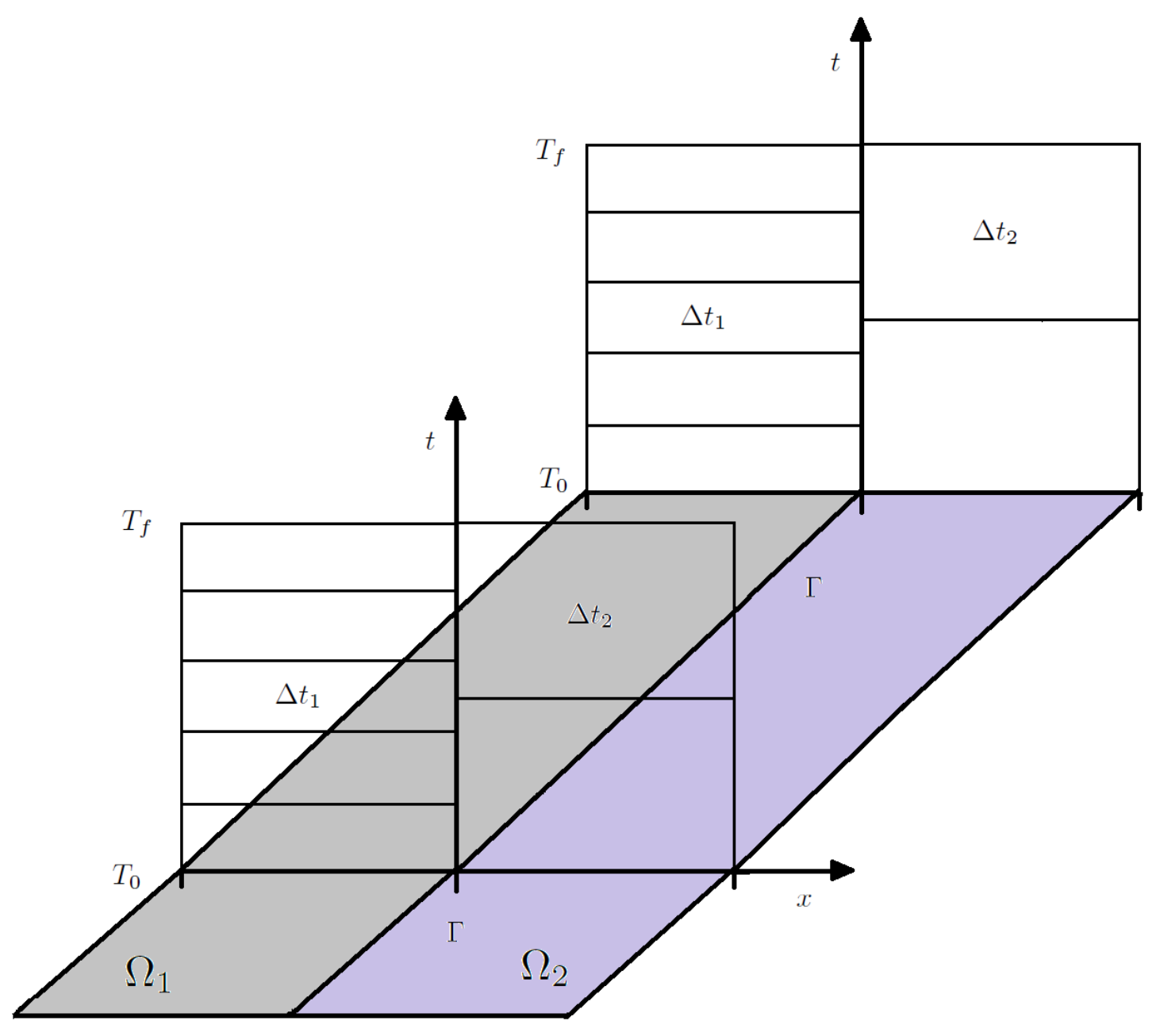


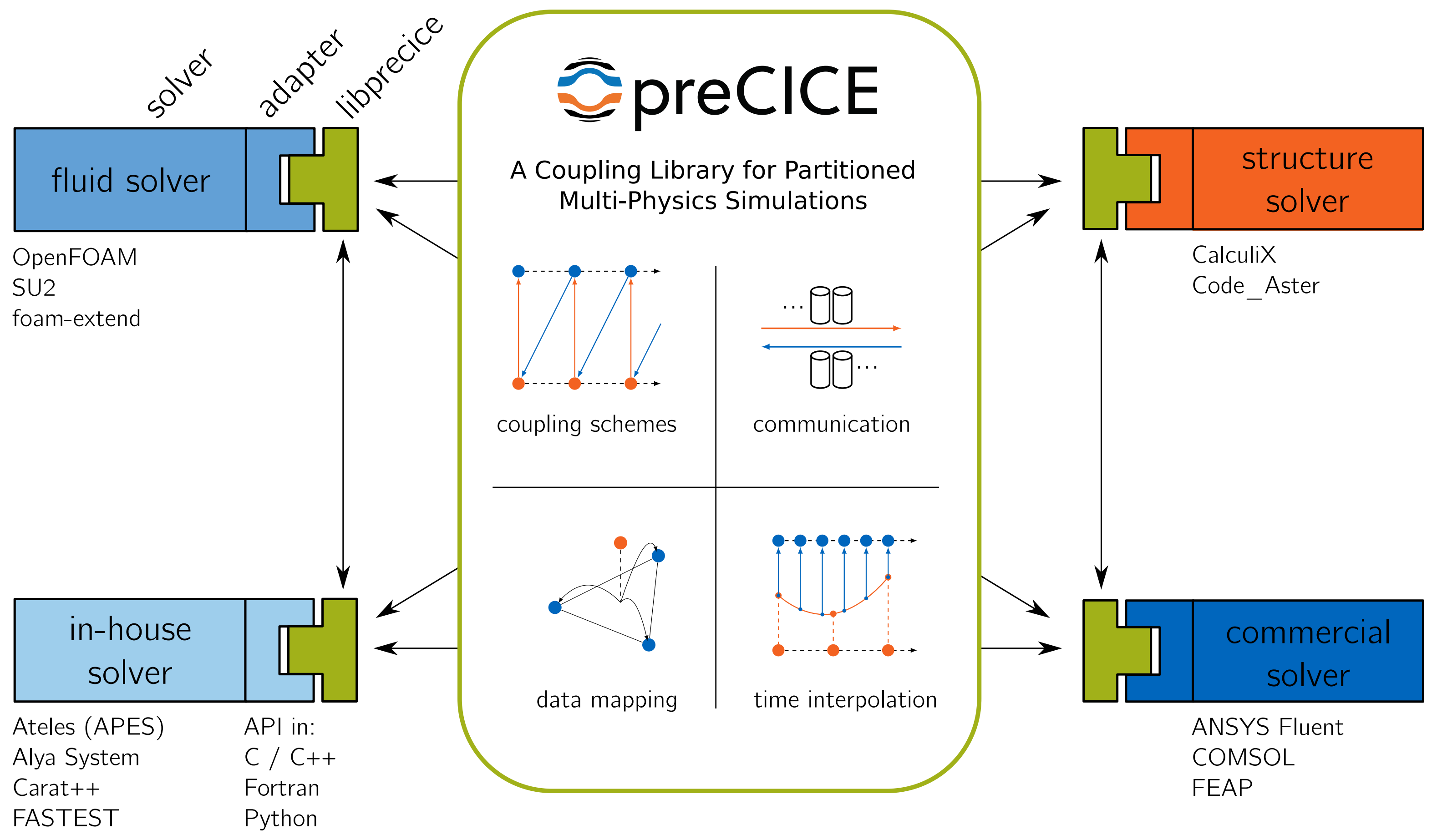
Figure from [2].

Discretization
Spatial discretization is realized through the application of FEM. We only consider uniform meshed that are matching at the coupling interface.

Time stepping takes place inside a common time window $[T_0, T_f]$. Implicit Euler with constant timestep size Δt_m is used. Differing timestep sizes $\Delta t_1 \neq \Delta t_2$ allow us to implement multirate time stepping for the two subdomains $\Omega_{1,2}$.

preCICE

The coupling library preCICE [5] is used for realization of the partitioned approach. preCICE follows a library approach that allows minimally invasive coupling, where the solvers are treated as black-boxes [6]. The solvers are extended by a simple adapter interfacing with the preCICE API, while implementation details of the solvers remain hidden [7]. preCICE is written in C++ and offers API bindings for different languages (Python, C, Fortran).



solver
adapter
libprecice

fluid solver
OpenFOAM
SU2
foam-extend

in-house solver
Ateles (APES)
Alya System
Carat++
FASTEST

API in:
C / C++
Fortran
Python

preCICE
A Coupling Library for Partitioned Multi-Physics Simulations
coupling schemes
communication
data mapping
time interpolation

structure solver
CalculiX
Code_Aster

commercial solver
ANSYS Fluent
COMSOL
FEAP

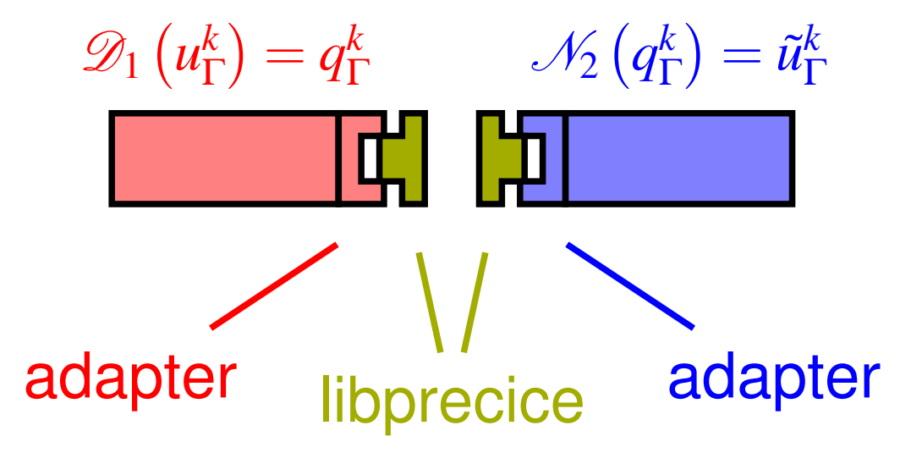
Black-box coupling with preCICE

We use the following black-box solvers:

- $\mathscr{D}_m$ accepts the temperature u_Γ as a Dirichlet boundary condition, solves the heat equation on Ω_m and returns the flux q_Γ corresponding to the solution u_m.
- $\mathscr{N}_m$ accepts the flux q_Γ as a Neumann boundary condition, solves the heat equation on Ω_m and returns the temperature u_Γ corresponding to the solution u_m.

For details refer to [8]. On basis of the solvers we implement the following coupling schemes using preCICE:

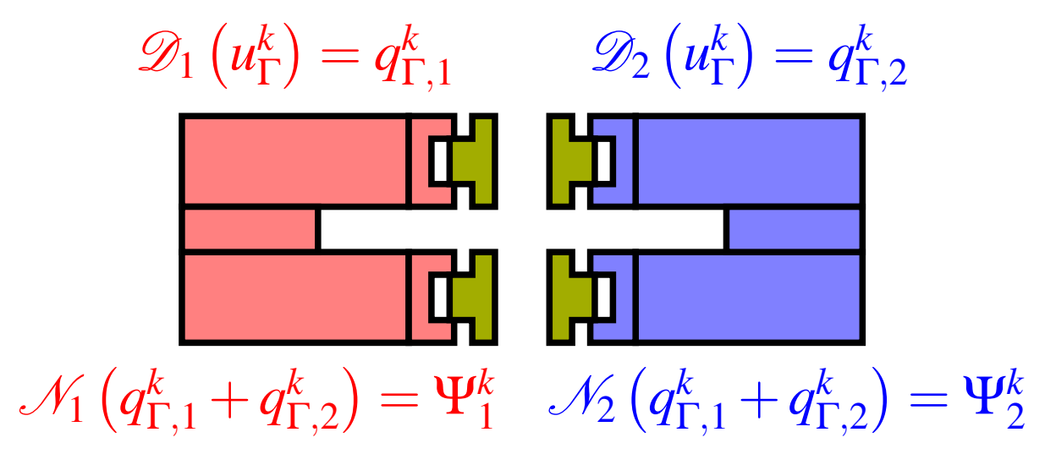
Dirichlet-Neumann (DN)



We use an underrelaxation scheme provided by preCICE to speed up convergence:

$$u_\Gamma^{k+1} = \theta \tilde{u}_\Gamma^k + (1-\theta)\, u_\Gamma^k$$

Neumann-Neumann (NN)



We implemented the following acceleration scheme in our adapter:

$$u_\Gamma^{k+1} = u_\Gamma^k - \theta\left(\Psi_1^k + \Psi_2^k\right)$$

For both coupling schemes we use an **optimal underrelaxation parameter** θ_{opt} to speed up convergence [3, 9]. *Remark:* The analysis to determine θ_{opt} only applies to the 1D, non-multirate case. However, we also use it for 2D multirate scenarios as an estimator (see [3]).

Numerical Experiments

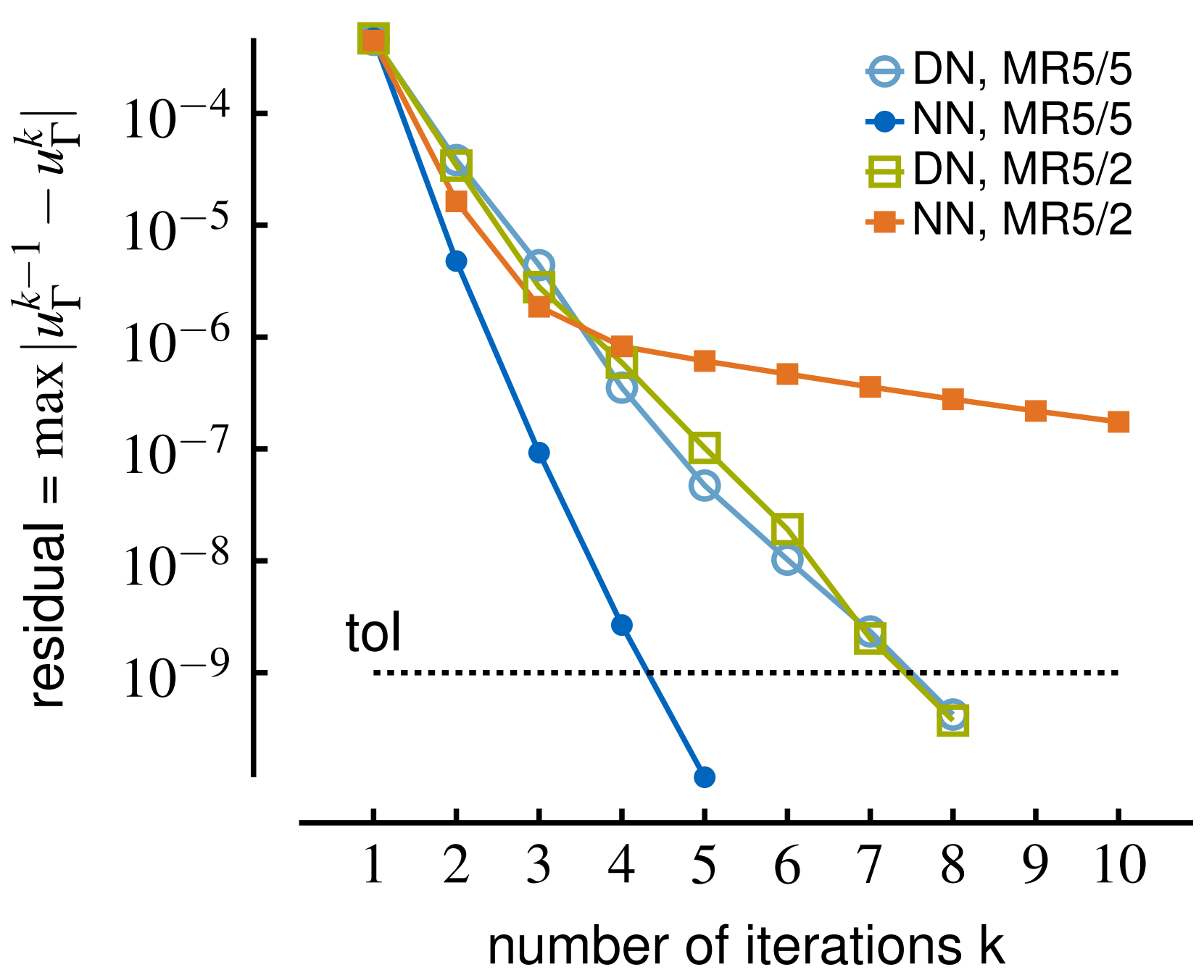
We evaluate the performance of the two coupling schemes through the number of coupling iterations (k) needed to reduce the residual of u_Γ below a certain threshold (tol).

1D without multirate
We use this setup to validate our implementation of DN and NN coupling in preCICE:

- 1D heat equations on $\Omega_{1,2}$
- different material combinations on $\Omega_{1,2}$ (Air-Steel, Air-Water, Water-Steel).
- use θ_{opt} from [3] for NN and from [9] for DN.
- use non-multirate setup: MR1/1 ($\Delta t_1 = \Delta t_2 = T_f$)

Obervations: Convergence after a single iteration for all tested material combinations and coupling schemes. Good agreement of preCICE implementation and reference implementation in pure Python.

2D with multirate



- 2D heat equations on $\Omega_{1,2}$
- materials combination Water-Steel
- use θ_{opt}
- use multirate setups:
  - MR5/5 ($\Delta t_{1,2} = T_f/5$)
  - MR5/2 ($\Delta t_1 = T_f/5$, $\Delta t_2 = T_f/2$)

Obervations: Good convergence for identical timestep size, convergence degrades for NN if $\Delta t_{1,2}$ differ. Good agreement of preCICE implementation and reference implementation.

Conclusions & Outlook

- **DN and NN multirate coupling schemes** can be implemented in preCICE by extending the adapter correspondingly.
- For **DN coupling** the acceleration schemes of preCICE can be used, for **NN coupling** the relaxation scheme had to be implemented in the adapter.
- The proposed coupling schemes allow for **multirate time stepping in preCICE**.
- The use of **Quasi-Newton** acceleration schemes with multirate time stepping requires further research.
- A **high order waveform relaxation** approach as in [2,5] should be evaluated next.

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www.precice.org
github.com/precice