Technische Universität München
Munich School of Engineering

Pumped Hydropower Storage Optimization and Trading Considering Short-Term Electricity Markets

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Vollständiger Abdruck der von der Munich School of Engineering der Technischen Universität München zur Erlangung des akademischen Grades eines

Doktoringenieurs (Dr.-Ing.)

genehmigten Dissertation.

Vorsitzender: Prof. Dr. Gunther Friedl

Prüfer der Dissertation:

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2. Prof. Dr. David Wozabal

Die Dissertation wurde am 08.01.2019 an der Technischen Universität München eingereicht
und durch die Munich School of Engineering am 11.12.2020 angenommen.

Preface

This thesis was written during my time as Asset Portfolio Manager at EnBW and as Ph.D. student at the Chair of Renewable and Sustainable Energy Systems at TU München between August 2014 and January 2018.

During the first two years of my thesis, EnBW enabled me to work on my Ph.D. project fulltime and to learn a lot about trading and asset portfolio management. Here I was given the opportunity to use high-performance computing clusters and to access all relevant data only available in a big utility. The last one and a half years, I worked as Asset Portfolio Manager and finished my thesis at night and during many weekends and holidays.

A special thanks goes to Prof. Thomas Hamacher, who encouraged this cooperation between industry and academia in order to combine the best from both sides. He offered advice when needed and allowed for freedom in the choice of research topics - both were highly appreciated. Philipp Kuhn and Christiane Hamacher fantastically helped in terms of organizational questions.

A tremendous amount of support came from my EnBW supervisor Rainer Hoffmann, who invested a lot of his personal time. I also want to thank Philipp Seydel for introducing me to many important people in the organization and my former manager Clemens Krauß for numerous critical discussions about market behavior.

Furthermore, I want to thank the paper co-authors and Ph.D. fellows Fabian Ocker, Christian Will, Michael Burkhardt and especially Christoph Brunner, who spent countless hours giving me feedback on various topics. Thanks also to Laura Pfeifer, Judith Vesper and Isabell Mau for writing their final theses under my supervision, all provided significant input for my work.

Finally, I would like to thank my family, friends and especially my wife Stephanie for her unwavering support and incredible understanding. A very big 'thank you' to all of you!

Sebastian Braun
Karlsruhe, 11th November 2018

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List of Symbols

Variables

α	approximation of the expected return (cutting planes)
$\alpha(i,l)$	sort function of balancing power prices
$\beta(i,j)$	sort function of balancing work prices
ϵ	slack variable
λ	water value/ dual variable/ marginal costs [€/1000m³]
p	pump position [MW]
q	quantity supplied, trading volume [MW]
σ	empirical variance of the expected return
s	spillage [1000m³]
τ	limits of new time intervals
u	turbine position [MW]
v	reservoir filling level [1000m³]
x	decision variable (reservoir filling, turbine, pump, spillage, slack)
y	second stage decision variable
$\underline{z}$	lower bound of expected return
$\overline{z}$	upper bound of expected return
$z(\cdot)$	binary decision variable, selects own bids, depends on i,j,k,l,m

Indices

h	hourly time stages
i,j	price cluster
i,j,k	position of balancing work bids
i,l,m	position of balancing power bids
m	machines
r	reservoirs
l	inflow scenario
k	scenarios
n	sample size
qh	quarter-hourly time stages
$s,\tilde{s}$	price scenarios
t	time stages

Parameter

$AA(i)$	activation probability of a bid
$AP(i)$	work price [€/MWh]
A	regression matrix
$a(q)$	ask side merit order [€/MWh, MW]
B	technology matrix
$b(q)$	bid side merit order [€/MWh, MW]
b	target value of constraints
c	prices [€/MWh]
C	price forward curve

Δ	additional quantity in the market [MW]
$\mathcal{E}(q)$	costs to produced quantity q [€/MWh]
η	efficiency of turbine/ specific discharge turbine [% or 1000m³/MWh]
γ	fixed discount factor
$\mathcal{G}$	grid charges
I, J	set of clusters/ number of trial solutions backward step [€/MWh]
K	number of trial solutions (SDDP and MCSDDP)
ℓ	price sensitivity factor or price response [€/MW]
L	number of inflow scenarios per time step
$LP(i)$	balancing power price [€/MW]
M	number of price cluster per time step
$\mathcal{M}$	allowed set in deterministic two-stage model
m∈ $\underline{rm}$	machine below reservoir
$m \in \underline{mr}$	machine above reservoir
N	samples size for prices and inflows (SDDP)
p^{min}, p^{max}	limits for pump power with $t = 1, \dots, T$ and $m = 1, \dots, M$
ρ	efficiency of pump/ specific charge with pump [% or 1000m³/MWh]
R	iteration index
s^{min}, s^{max}	limits for spillage
Θ	sorted price curves
T	time horizon
u^{min}, u^{max}	limits for turbine power [MW]
v^{in}	inflows [1000m³]
v^{start}	start reservoir filling level [MW]
v^{end}	end reservoir filling level [1000m³]
v^{min}, v^{max}	limits for reservoir filling level [1000m³]
X_1	x_1-share of M (permitted quantity of deterministic two-stage model)
$X_2(x_1)$	x_2-share of M (permitted quantity of deterministic two-stage model) depend. on x_1
$defX_2$	domain of definition of X_2

Stochastic sets

$\mathcal{A}$	σ-Algebra
$\omega \in \Omega$	elementary event
Ω	set of events
$(\Omega, \mathcal{A}, \mathcal{P})$	probability space
$\mathcal{P}: \mathcal{A} \rightarrow [0,1]$	probability measure
$\tilde{\mathcal{P}}$	empirical distribution
$\phi_{i,j}$	transition probability from price cluster i to j
ψ_l	probability of inflow scenario l
S	number of scenarios
$\mathbb{S}_{t,s}$	set with t and s corresponding scenarios
ξ	random vector
$\tilde{\xi}$	sample of random vector ξ
Ξ	discrete stochastic process
Ξ^{price}	set of all discrete price scenarios
Ξ^{inflow}	set of all discrete inflow scenarios
ζ	discretized price process (stage dependent)

Functions

$\mathcal{C}(\cdot,\cdot)$	return function under decision x and event ω or inflow δ and price ζ
$F_{\xi}(x)$	distribution function
$G(q)/G$	profit of quantity supplied/ profit margin
$\mathcal{Q}(\cdot)$	expected return function/ "profit-to-go" function under the decision x and if necessary price ζ
$\tilde{\mathcal{Q}}(\cdot)$	empirical expected return function under decision x
$\mathcal{S}(m)$	shadow price of turbine or pump [€/MWh]

1. Introduction

Beginning in the 1980s the electricity sector began to change, pushed by strong endeavors of countries around the globe, towards liberalized power marks. The European Commission directives of 1996, 2003 and 2009 (Directive 72/EC, 2009; Steve, 2004) were the main drivers for the liberalization in Europe. The idea was an interconnected and common market for all participants (Directive 72/EC, 2009) taking into account the overarching goal of an environmentally friendly, secure and cost-efficient energy supply. Many other countries pursued similar liberalization goals such as the United States, Norway or Chile.

The doctrine was clear, the energy sector has two new cornerstones: customers and markets. Utilities had to overcome the regulated past, starting to concentrate on actively managed revenues, react on customers and to accept the competition. The focus changed from a region associated oligopolistic dispatch towards a profit based use of the company's production capacity. This came along with novelties such as price forecasting, price based optimization and smart grids.

The latest development shows a way towards more and more short-term trading (EPEX Spot, 2017b). In Germany about 103.6 GW of RES are installed by now producing a third of 2016 gross electricity demand (Umweltbundesamt, 2017). The major share of this production depends on the fluctuating primary energy sources of sun and wind. Since the sun does not always shine and the wind does not always blow when the demand is high the produced renewable energy need to be stored until the demand is high which can be done with pumped hydropower storages. Since pumped hydropower plants were former mainly used to store energy at night to be shifted to the peak demand times over the course of the day, this dispatch process changes significantly. Operators of pumped hydropower plants need to react on an increasing number of short-term electricity markets and prices that are strongly influenced by RES. These challenges tear at the very fabric of profitability so that without new optimization methods, management and steering the absolutely required storages for the Energiewende cannot be operated economically.

1.1. Research Question and Contributions

This work focuses on modeling scheduling problems for pumped hydropower storages and to determine bidding strategies which maximize revenues from selling and buying power on the various short-term electricity markets.

With the liberalization of the electricity sector in the beginning of the 20th century the bidding problem started to attract much attention in industry and academia. More and more short-term markets were introduced lining up along the timeline. As one of the first, Klaboe and Fosso (2013) recognized in their review on optimal bidding on various markets that bidding on more than one market results in a link between the markets in terms of price, capacity, energy and after all opportunities. Klaboe and Fosso define two kind of bidding strategies: separate bidding and process coordinated bidding. For the first, both markets are considered independently of each other. The second strategy suggests that all subsequent markets should be considered when bidding on the first market. From a theoretical point of view, this is because bidding on the first market reduces flexibility for the following markets and therefore comes with a cost. These costs can be quantified when explicitly modelled.

Nevertheless, a general preference of one strategy over the other has not been analyzed or given in literature so far. Whereas two markets are compared in some papers an overall bidding concept considering multiple or all short-term electricity markets, such as day-ahead, intraday as well as balancing markets, has not been designed yet. Given this overall research question this work combines and contributes to several fields from market analysis, mathematical optimization and partly also political framework evaluations. It finally proposes an overall optimization and bidding concept.

Germany introduced several new electricity markets and flexible trading products to enable higher shares of variable RES production and can be seen as a role model for other countries in transition. This multifaceted market structure results in a complex hydro power scheduling problem which is analyzed in this thesis. Until now, literature constantly lags behind and fails to keep up with market development as well as to provide suitable optimization methods and bidding strategies. This work provides a solution to this problem as well as explains when separated and when process coordinated bidding should be applied. This probably becomes relevant in other countries as well, when additional electricity markets need to be introduced to enable higher shares of variable RES.

The key to identifying optimal bids for various markets is the knowledge about the opportunities within markets. As most complex problems, this work suggests dividing the multi-market bidding problem into smaller problems to be composed into an overall solution again. Therefore, the specific contributions are presented in four separate listings hereafter that are already oriented towards the structure of the content chapters. The respective research questions are always stated before the listing.

By means of the new quarter-hourly day-ahead market in Germany, introduced in December 2014, a multi-day-ahead-market optimization is modeled to answer the research questions, if and how the new market can contribute to higher revenues for operators of flexibility providing power plants and how the respective bids should look like.

- This work states that the consideration of the more fluctuating quarter-hourly day-ahead market increases the profit of flexible pumped hydropower storages significantly.
- Furthermore, the thesis proposes to analyze every market that is considered in a multi-market optimization in detail. Through this, it is demonstrated that the price on the quarter-hourly day-ahead auction is strongly influenced by the variable RES of the sun, whereas no significant influence of wind power could be quantified. This comes along with an observed zig-zag effect and limited liquidity in the quarter-hourly day-ahead auction. (Braun & Brunner, 2018)
- This work formulates and solves the extended quadratic problem formulation considering the observed limited price sensitivity on the quarter-hourly day-ahead market. Two formulations are presented to consider either new or consistent auction participants. By means of extensive experiments the optionalities within the two markets are determined and optimal bids generated for each market (Braun, 2016b).

In literature, the consideration of stochastic inflows or prices in hydropower storage optimization is an important factor. Therefore, more literature exists on modeling stochastic optimization than on all other hydropower storage optimization topics together. The reviews by Wallace and Fleten (2003) and Labadie

(2004) give a good overview whereas very limited literature can be found on the stochastic optimization of pumped hydropower storages. Furthermore, existing stochastic optimization models focus on long term scheduling rather than considering short-term electricity markets. Given this research gap, this work performs a study including stochastic prices and inflows in a short-term pumped hydropower storage optimization considering even quarter-hourly products.

- One insight of the study is that the stochastic solution dispatches the storages more conservatively to react on possible high or low prices. Accordingly, this is relevant for weekly pumped hydropower storages that are highly influenced by the occurrence and force of periods with high wind feed-in and respectively low electricity prices during these times.
- In order to consider dependent prices between two quarter-hours this work suggests a multi-cut stochastic dual dynamic (MCSDDP) approach which leads to an even more conservative storage dispatch but lower revenues.
- Furthermore, this thesis demonstrates that the positive effect of stochastic optimization on revenue and reservoir filling level adherence is significantly lower for pumped hydropower storage systems in comparison to hydropower storages without pumps.
- Therefore, the work states that using linear models for the short-term dispatch of pumped hydropower storages is a reasonable simplification of the complex decision problem. This facilitates the consideration of other effects that were up to now too computationally expensive to model when using stochastic optimization.

The next following market along the timeline is the intraday market. This market is getting more and more attention and the trading volume quintupled since 2011 (EPEX Spot, 2017b). Faria and Fleten (2011) provided research on this field of combining day-ahead and intraday markets. Nevertheless, they concluded that including intraday when bidding day-ahead does not increase profit or influence bids of storages significantly. The highly fluctuating intraday prices in Germany point to another direction and open up the research question, if separate bidding would lead to better results as coordinated bidding for pumped hydropower storages.

- This work presents that the reason for the findings of Faria and Fleten (2011) is the specific market design with a very liquid hourly day-ahead market and an intraday market that is used to balance deviations that occurred after the day-ahead auctions. By means of this, the day-ahead market bidding should not be influenced by intraday market bidding. Vice versa, the day-ahead market results are the basis for the independent intraday optimization.
- An intraday optimization approach is proposed in this thesis, based on the algorithm of Lu, Chow, and Desrochers (2004). The algorithm is extended towards grid charges, accounting for unavailabilities as well as non-monotonous prices (Braun & Hoffmann, 2016).
- The advantage of the extended algorithm is a very short runtime. Based on extensive research and tests, this thesis addresses the continuous intraday markets decision problem with a continuously operating algorithm. The constantly occurring changes are therefore immediately considered and converted into bids.

Pumped hydropower storages are often equipped with highly flexible pump and turbine machines. Therefore, it is important to consider balancing markets in the pumped hydropower storage scheduling decision as well. Comparing just the price fluctuations and the possible resulting profits, it could be suggested to trade all electricity in the intraday or the balancing market rather than day-ahead. This is no practical solution because limited liquidity, power plant restrictions and the market structure are not considered. Therefore, the key research question is how much and for what price to bid the respective hydro storage power and capacity on the balancing markets.

- A broad overview on the European balancing markets is given to get an understanding of the complexity in market design, pricing and product quality (Ocker, Braun, & Will, 2016).
- This work shows the strong relationship between balancing and energy only markets. This means, before the balancing auction, it needs to be estimated what profit can be generated either in the balancing market or in a combination of all energy only markets, namely day-ahead and intraday markets, together. This is because the balancing auction results cannot be reversed, and capacities are not available any more.
- This work formulates an integrated non-linear optimization approach that considers the intraday market and the frequency restoration reserve (also called secondary reserve); representatively for energy only and balancing markets respectively. This opens up the possibilities to exploiting optionalities across the markets in a most optimal way.
- It is stressed that the resulting problem is non-linear and not solvable for real-world problems. A linearization and an extensive case study are presented (Braun & Burkhardt, 2015).

1.2. Outline

The overall research question of this work is how to optimize and trade pumped hydropower storages on short-term energy markets. To provide answers to this question, this thesis is structured into part A, B and C (see Figure 1). Part A is the introduction into the most important fields and topics that are relevant for this thesis. Part B presents methods, models and results and part C provides conclusion, future research questions as well as an outlook into flexibility demand in the future.

In part A, three important presets are introduced: energy markets, hydropower storages and optimization methods. The first preset, energy markets, discussed in chapter 2, is required to take the decision on the relevant electricity markets and how to generate the income stream. This could be for example the hourly day-ahead auction but also a mix of intraday and balancing energy markets. As highlighted in orange in Figure 1, this work focuses on short-term energy markets rather than the futures market or other revenues such as black start capacity provision. The second preset deals with everything related to the physical hydropower storage itself and is presented in chapter 3. The wide range of hydropower storage types with miscellaneous characteristics and different technical possibilities is introduced. Furthermore, the various input parameters such as prices and inflows are defined. Beyond, special focus is on water values based bidding. The third preset includes the optimization methods needed for pumped hydropower optimizations. Depending on the application, separate mathematical problem formulations and optimization techniques are required and therefore presented in chapter 4, such as linear, non-linear, stochastic and dynamic programming.

This triangle of presets is the foundation for the methodologies and results provided in part B. In an optimal, ideal world all energy markets and all storage types are solved with a superior optimization technique providing an optimal solution, with arbitrary time resolution and within no time. This is not possible due to computational obstacles. Nevertheless, this work tries to get one step closer to this utopia. This means that chapter 5, 6, 7 and 8 cover the most important short-term income streams for pumped hydropower storages. Each chapter solves one or a combination of several energy markets and storage types with a selected optimization method. Therewith, chapter 5 provides a multi-market optimization approach considering day-ahead auctions. That means several auction-based markets with different time resolutions and price sensitivity characteristics can be considered and a case study for the German market is presented. Chapter 6 determines the value of stochastic optimization considering even a quarter-hourly time resolution. Special care has been taken considering stochastic price and inflow forecasts as well as analyzing systems with different reservoir sizes both in combination with and without pumps. Chapter 7 demonstrates how the optimal day-ahead market production schedule can still be adopted during intraday trading, exploiting the very short-term flexibility. For the continuous intraday market also a continuously optimizing algorithm is developed. Chapter 8 deals with the complex multi-market problem of bidding into balancing as well as energy only markets. Due to the allocation within the markets and the balancing work and power price merit order sorting a non-linear optimization is formulated. The joint optimization fosters the decision on how much and for what price to bid on the different markets.

Part C provides conclusion, future research and outlook of this thesis. Therefore, chapter 9 is the pivotal point, bringing together the solutions given in part B and drafts a guidance to approach the hydropower scheduling problem considering short-term electricity markets. A final overview on the optionalities and future research for the pumped hydropower storage scheduling problem considering various short-term markets is given. Further, chapter 10 gives an outlook, defining and estimating future flexibility demand, a possible pricing outside the thermal merit order and deriving consequences for pumped hydropower storages and the presented optimization tools.

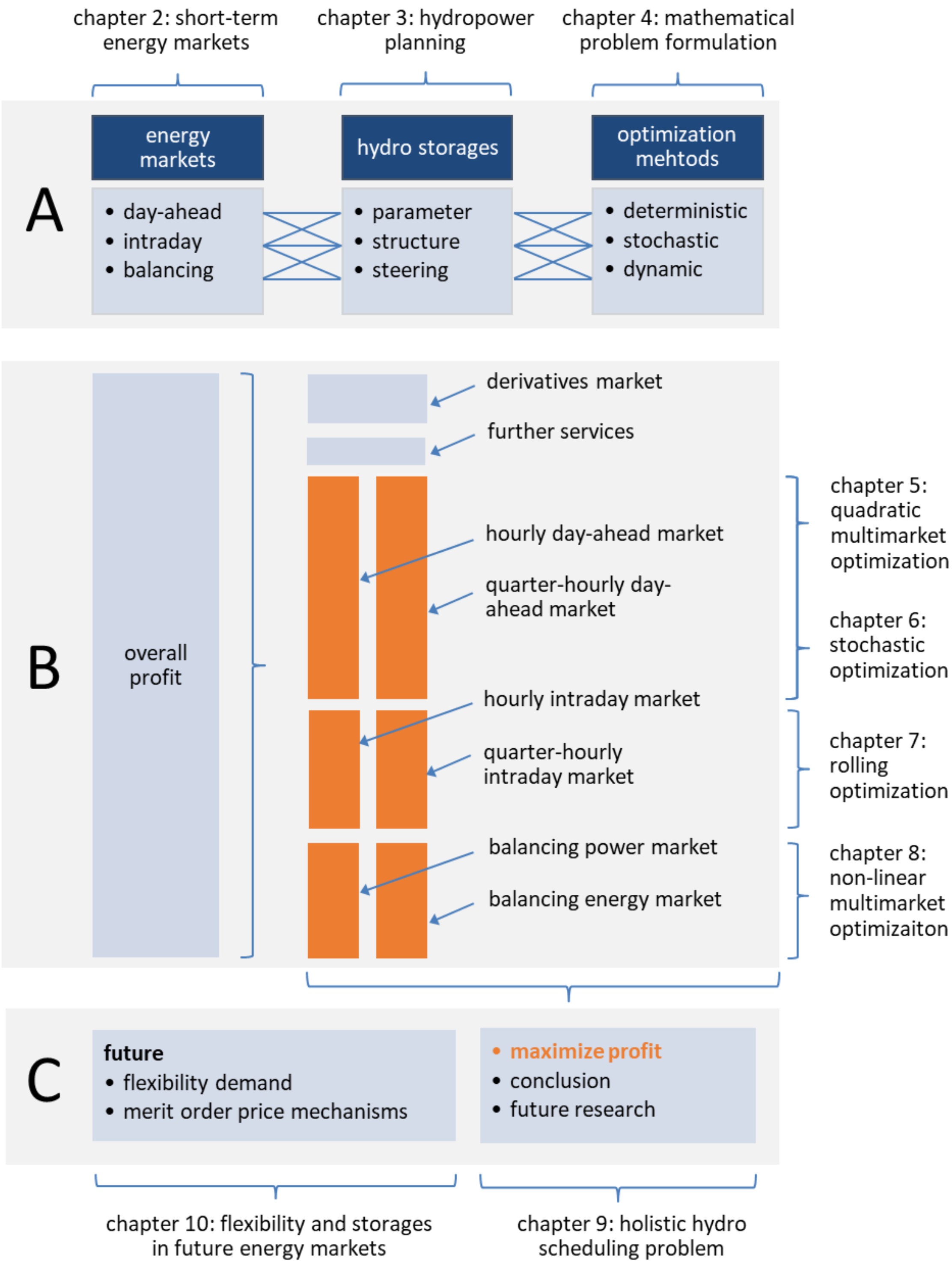


Figure 1 Structure of the thesis

A. Energy Economics and Mathematical Foundations

A widely used source of flexibility are pumped hydropower storages. The significant quantity of energy that can be stored, the good conversion efficiency and the flexibility of provision reason pumped hydropower as one of the most attractive option for large scale electricity storage and flexibility provision. Worldwide, as of 2017, it is reported that hydroelectric pumped storage accounts for over 96% of all active tracked storage installations worldwide, resulting in a capacity of 168 GW (DOE, 2017).

Whereas pumped storages exist since more than 100 years the operation objective is changing due to new challenges of the liberalization and the RES penetration. Especially the increasing share of volatile RES from wind and sun are affecting the characteristics of market prices. To align with the new production mix new short-term energy markets with higher time resolutions and trading possibilities until shortly before delivery were introduced. This unveils great possibilities for flexible pumped hydropower storages but demands a rethinking of operation, optimization, forecasting and the dispatch itself.

To approach these challenges part A lays the foundation introducing energy markets in chapter 2, hydropower plants in chapter 3 and mathematical optimization methods in chapter 4.

2. Short-Term Energy Markets

With the liberalization in the late nineties and the beginning of the millennium countries around the globe as the United States, Argentina or the European Union introduced electricity markets to foster the transformation from a regulated and monopoly based supply of electricity into a system of competition. The benefits were seen in a more cost efficient, since increased competitive, as well as more transparent system with price signals and diversification of supply (OECD & IEA, 2005). With this development, a whole new field for business opportunities was born. Utilities, banks and other players forged trading units to participate in upcoming opportunities. That, furthermore, generated the demand for new research fields including market analyses, price forecasting, risk management and optimization models.

Chapter 2.1 provides a summary of trading possibilities, the characteristics of electricity prices and the policy framework in Germany. Furthermore, chapter 2.2 introduces the day-ahead, 2.3 the intraday and 2.4 the balancing power markets. For each market the key characteristics are given, an overview on the existing European market designs is presented and the corresponding German market including an overview on historic prices is explained in detail.

2.1. Introduction

2.1.1. Characteristics of Electricity Prices

Whereas commodities such as copper, oil or gold can be easily shipped and stored and therefore internationally traded electricity is difficult to store and predominantly transportable via power grids (Burger, Klar, Müller, & Schindlmayr, 2004). This results in an increased complexity for electricity markets, prices as well as transmission and production planning. To understand these additional complexities, it is crucial to choose the right methods and models on all stages of the production chain.

Therefore, prices on power markets follow some general rules which are explained below, based on the work of Jameson et. al. (1999) and Johnson and Barz (1999) with some extensions to bring it up to date:

- Electricity prices have a tendency to oscillate around a long-term average, also called the mean reversion. This fluctuation is caused by the price inelastic, cyclical, mean-reverting nature of demand, limited grids and storages, the encyclical feed-in of wind-mills and PV and the cost structure of generation.
- Time depending effects can be seen over the course of hours, days, weeks and even years. Each cyclical fluctuation has a defining frequency and magnitude that depends on the patterns of weather, economic activity, the regions generation structure, market coupling and transmission lines.
- The volatility of the electricity price often increases with the price level. Generation is moved to the margin when their price level is reached. Higher price levels bring generation on their upper

limits and the next generation units are switched on. Due to massive RES feed-in, similar effects can be seen for very low-price levels when generation units are switched off.

- Occasional price peaks occur in unusual load conditions or when important generation or transmission units suffer outages. In this case, inelastic demand meets a steep supply curve and prices can skyrocket for a short time.

As mentioned, the general principle of electricity markets is that storage is limited but possible in for example pumped hydropower or battery storages. Strong volatility enables high returns for possible storages and thus storage is basically a question of costs. If unlimited capacity of storages with no cost would exist, the price for energy would be smoothed (Johnson & Barz, 1999, p. 11). Since storage is economically strained, the real-time prices must reflect the limited storage capacity. Furthermore, location and extension of transmission lines between grid territories influences the system costs.

Effective models to forecast energy prices are essential for all subsequent processes. These include, just to mention the most important, structuring, pricing, trading, financial and physical risk management, contract valuation, operating policies for generation and transmission, expansion plans etc. Mistakes or inaccuracies influence all downstream processes and have severe effects for all market participants.

Four types of price models can be accentuated from literature that model basic diffusion processes with and without jumps such as sudden, short-term and discontinuous price changes.

- Brownian Motion $dP_t = \mu_t d_t + \sigma dB_t$
- Orstein-Uhlenbeck mean reversion $dP_t = \kappa(\alpha_t - P_t)d_t + \sigma dB_t$
- Geometric Brownian motion $dP_t = \mu_t P_t d_t + \sigma P_t dB_t$
- Geometric mean reversion $dP_t = \kappa\left(\alpha_t + \frac{\sigma^2}{2} - InP_t\right)P_t d_t + \sigma P_t dB_t$

As mentioned above all characteristics, mean reversion, time dependencies, volatility and price peaks are very important to model prices. Table 1 shows that just the geometric mean reversion is able to consider all features. Not considering one of these key aspects leads to unrealistic results. For example, calculating the value of a peak load hydropower storage plant without price peaks is a substantial undervaluation. Additionally, the geometric mean reversion with jump process shows the best log-likelihood values in the short-term (1h) but is outperformed by the geometric mean reversion without jump process in the long-term (1 day to 1 week) (Johnson & Barz, 1999). Other popular approaches for price scenario generation are based on econometric time series models, such as GARCH (Faria & Fleten, 2011) and ARIMA (Plazas, Conejo, & Prieto, 2005) or use machine learning methodologies for example neuronal networks.

Table 1 Comparison of models for energy price modulation (Johnson & Barz, 1999)

models	Brownian motion	Orstein-Uhlenbeck mean reversion	Geometric Brownian motion	Geometric mean reversion
mean reversion		x		x
time depending effects	x	x	x	x
volatility			x	x
occasional price spikes		x		x

2.1.2. Trading on Electricity Markets

Market Pools and Bilateral Contracts

Generally, two ways of trading electricity are available: market pools and bilateral contracts, see Figure 2. Market pools, the predominant way of trading, are centralized marketplaces where market participants merchandize. The markets are managed by a private enterprise or an independent system operator (ISO) that clears the market, maintains reliability and controls the interconnector capacities. All market participants submit quantity-price pairwise bids they are willing to sell or buy for. The operator ranks and matches the buying and selling offers for every bidding period so that the lowest price of the selling offer and the buying offer with the highest price are matched (Ott, 2003). The market clearing price (MCP) is either based on uniform pricing or pay-as-bid. In case of a uniform pricing auction every buyer receives, and seller delivers the energy at the same MCP. In case of pay-as-bid, the individual bidding price is paid to the bidder when allocated (Li, Shi, & Qu, 2011).

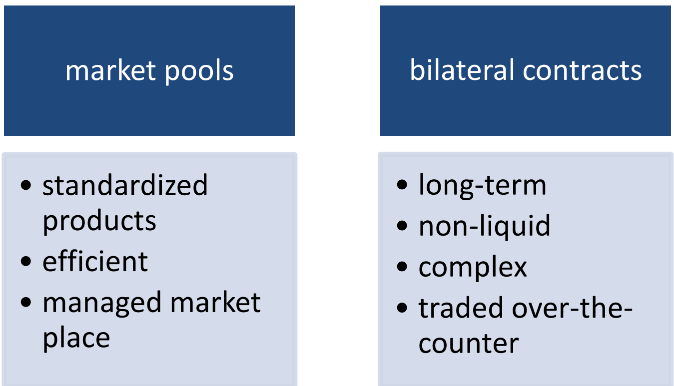


Figure 2 Market pools and bilateral contracts

Whereas market pools are based on trading standardized products as efficient as possible; bilateral contracts are traded over-the-counter and are flexible in terms of participants, clearing mechanism and products. In this case, the traded contracts can be off-standard, long-term, non-liquid or peculiarly complex. The valuation and trading of such products is normally more time consuming and expensive. This work focuses on market pools referred as energy markets. If the term trading is used during this work, this always denotes the consideration of an underlying market. In chapter 2.2 to 2.4 the relevant energy markets for the short-term are introduced.

Merit Order Effect

Spot markets are the corner stone of liberalized energy markets. Various different forms of market designs, market clearing mechanisms and products exist. All spot markets have the characteristic price formation in common that can be explained with the merit order effect. In the merit order the available power plants line-up with their different variable costs, power plants with low variable costs in the beginning the ones with higher variable costs at the end of the merit order. Power plant operators have the incentive to bid their marginal costs as long as more power plant capacity is available as needed to meet the demand. According to several studies, the production capacity in Germany as well as Europe is sufficient to meet the electricity demand now and in the near future, even during the annual peak demand (BMWi, 2016a; ENTSO-E, 2017c). Furthermore, the grid capacity in Germany, including a cross border capacity of about 20 GW to the neighbor states, is sufficient, so that the produced energy can be transported to the consumers. This means that the MCP is highly correlated with the marginal costs of the last power plant that is in the money (Ockenfels, Grimm, & Zoettl, 2008, p. 70). The interception of the inflexible demand with the merit order defines the MCP.

Figure 3 presents a classic merit order constellation with a major share of thermal power plants. All power plants with marginal costs below the MCP are needed to meet the demand. Whereas the price sensitivity of the demand is very inelastic the demand itself varies strongly and the respective point of intersection is shifted right or left. Consequently, the MCP shows a significant volatility. The difference between marginal costs and MCP is the producer surplus which is used to pay-off the fix costs, such as investment, labor and maintenance costs (Nicolosi & Fürsch, 2009; Ockenfels et al., 2008).

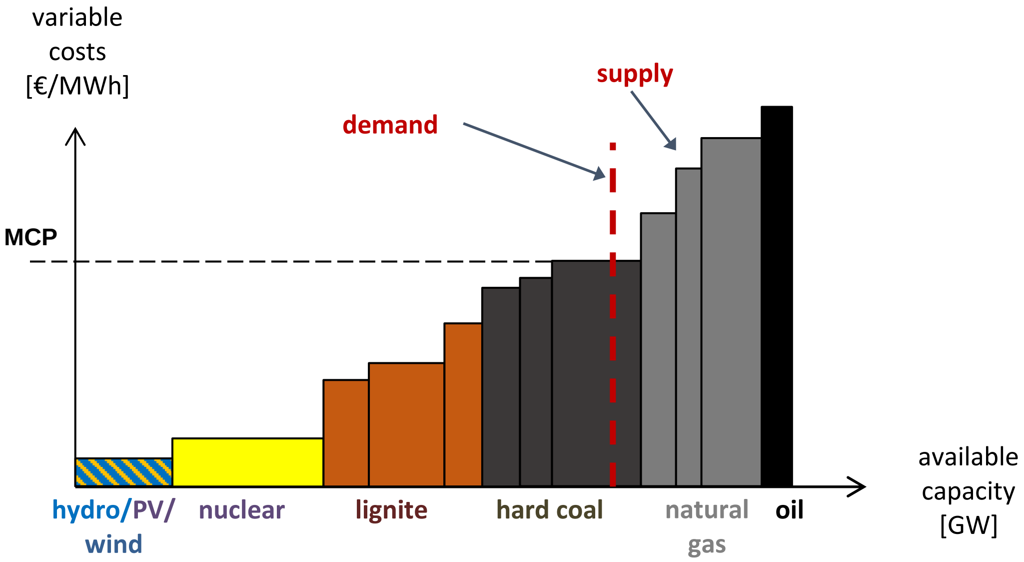


Figure 3 Exemplary merit order with limited RES

In the recent years, the electricity market in Germany changed due to a now 29% RES share in gross electricity production in 2016 (AGEB, 2017, p.28) which is constantly increasing. RES replaces the former thermal production. Depending on the EEG-remuneration regime this is due to an unlimited priority feed-in laid down in § 11 of the RES-Act, or, if the direct marketing regime holds, due to the variable costs close to zero (Gestz zur grundlegenden Reform des Erneuerbare-Energien-Gesetzs und zur Änderung weiterer Bestimmungen des Energiewirtschaftsrechts, 2014). That means the high investment and maintenance costs are neglected in the short-term dispatch decision and RES enqueue right at the beginning of the merit order. With an unaltered demand, the MCP of the new merit order is below the one without RES, see Figure 4. This results in significantly reduced full load hours of all conventional power plants that are at least partly pushed out of the money as well as a significantly lower producer surplus for the power plants in the money (Nicolosi & Fürsch, 2009). Hitherto and even more in the future, the RES are shaping the spot price structure.

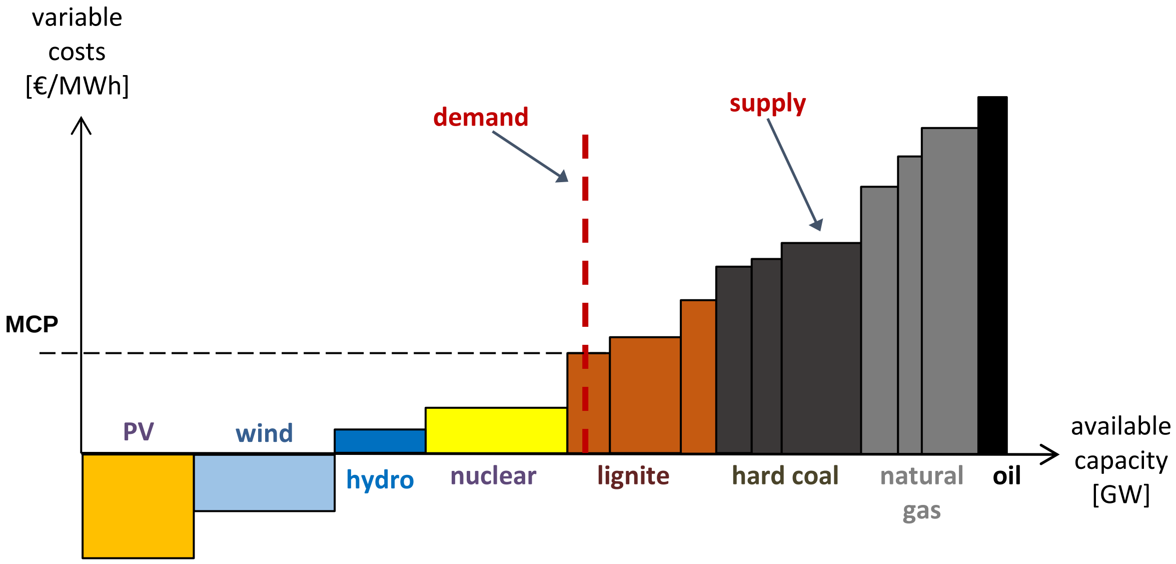


Figure 4 Merit order with significant RES

With the increasing installation of solar and wind power capacity the number of hours during the year increases in which the residual load is low enough that even base load, such as nuclear or run-of-river power plants, need to reduce their production. The residual load is defined as the load minus the weather-dependent production of wind and solar power. Reasonably, the high start-up costs and the limited load change rate are determining the price structure of the respective power plant bids for the hours during as well as before and after such an event. Often a negative contribution margin is accepted in a few hours to minimize the overall costs. This could result in short-term negative price bids which are allowed on EPEX Spot since 2008 (EPEX Spot, 2017b). Even in these situations the merit order based pricing works and ensures an optimal allocation of resources (Brunner, 2014a; Götz, Henkel, Lenck, & Lenz, 2014; Nicolosi & Fürsch, 2009). Deep negative prices are furthermore an effect of the RES support regime in Germany, since the point of switch-off is not at zero but just when the absolute of the negative price is higher as the EEG-remuneration of the respective RES. The remuneration of RES varies strongly; 40-200 €/MWh for wind power, 80-500 €/MWh for PV and 130-170 €/MWh for biomass (Gesetz für den Vorrang Erneuerbarer Energien (Erneuerbare-Energien-Gesetz-EEG), 2000; Gesetz für den Vorrang Erneuerbarer Energien (Erneuerbare-Energien-Gesetz-EEG), 2017).

Structure of Short-Term Electricity Markets

The electricity markets introduced during the liberalization are: derivatives, day-ahead, intraday and balancing markets, Figure 5. On the derivatives or futures market, yearly, quarterly and monthly products are offered to be traded already years before delivery. Also, weeks, weekends and days can be traded on the derivatives market whereas not all products can be traded liquidly at all times. This market is followed in time by balancing and day-ahead markets. The auctioning of balancing power and energy does not follow a uniform market design. The day-ahead market contains hourly, rarer half- and even quarter-hourly products that are allocated in a unified pricing auction taking place one day-ahead of delivery. A few years after the introduction of derivative and day-ahead markets also intraday markets were established with hourly and in some countries half- and quarter-hourly trading products. One main difference to the day-ahead market is the widespread continuous pay-as-bid trading until a certain time before physical delivery. The day-ahead and the intraday market are designated as spot markets.

Figure 5 Overview over German power markets for standard trading products

Electricity markets are the link between producer, retailer and consumer. Market participants are utilities, sizable end-consumers, trading houses and local municipal utilities. Nearly all European countries have implemented spot markets consisting of a day-ahead and an intraday market by now. The task of politics is to improve the existing market designs permanently to enable even higher shares of generation from weather driven RES as cost efficient as possible and without risking security of supply.

2.2. Day-Ahead Markets

Day-ahead markets are the corner stone of the European market liberalization. Nearly all European countries have implemented day-ahead markets using an auction-based market design in which producers and sellers trade electricity.

Therefore, day-ahead markets have by far the highest turnover in terms of electricity traded of all spot markets (EPEX Spot, 2017b). It is important to note that day-ahead markets are sometimes misleadingly referred as sport markets. Nevertheless, spot markets are an umbrella term for day-ahead and intraday markets. The latter is discussed in chapter 2.3.

Today energy markets are expounded to the influence of variable RES. This includes especially the trading strategy for unbalanced day-ahead schedules of the RES solar and wind as well as ramps of inflexible thermal power plants. In the first part of this chapter the general auction design and key characteristics of day-ahead markets are presented including an overview over the existing European day-ahead market landscape. Thereafter, the German day-ahead markets are discussed including a historic recap of the prices in the last years. To sketch the specialties of and differences between short-term power markets the price level and the dissimilar trading volumes of the hourly and quarter-hourly day-ahead markets are analyzed.

2.2.1. Market Design

All European day-ahead markets are based on a similar market framework in which electricity is allocated within multiple buyers and sellers in a uniform pricing auction (Weron, 2006a). That means, the same price is paid for a fixed allocated number of identical units of a homogenous commodity, here electricity. Uniform pricing auctions are also known as one shot auctions or clearing price auctions. Every market participant submits a quantity in MW and a respective price per unit, in €/MWh, the buyer is willing to pay. The bids are normally submitted sealed to the auctioneer and are not available for the other market participants. The auctioneer, for example the EPEX SPOT SE, allocates beginning at the highest bidder until the supply is met. The residual MCP is the same for every bid. The auction design can be seen as an example in which the MCP mechanism is used to establish a benchmark price index for other energy markets. Technical analysis of this auction type (Krishna, 2009) as well as reviews on optimal spot market bidding are available (Baillo, Ventosa, Rivier, & Ramos, 2004; Kristoffersen & Fleten, 2010).

The key differences throughout Europe, including the shares of variable RES and the existing short-term trading possibilities, can be see for day-ahead markets in Table 2. Most exchanges offer hours and blocks of several hours to be traded. The EPEX Spot allows half-hourly trading for the UK. Germany and Austria offer the possibility to trade quarter-hours as well, whereas Germany performs two auctions and Austria clears both hourly and quarter-hourly markets at once. The Polish market TGE prepares also two day-ahead auctions and additionally provides a continuous trading possibility (TGE, 2017). Nevertheless, the influence of the increased share of RES and thus the need for more flexible market designs and products comes much more to the fore in the intraday and balancing power markets rather the day-ahead market. This explains why their market designs vary much stronger to align with the respective regulatory environment.

Table 2 Empirical analysis of European day-ahead spot markets. Data retrieved from the energy exchange operators: (EPEX Spot, 2017b), (Belpex, 2017), (Nord Pool, 2017), (Hupx, 2017), (GME, 2017), (APX, 2017), (TGE, 2017), (OMIE, 2017), (Opcom, 2017), (South Pool, 2017), (EXAA, 2017), (PXE, 2017)

<table>
<tr><th rowspan="2">country</th><th rowspan="2">solar energy sources</th><th rowspan="2">wind energy sources</th><th colspan="3">day-ahead market</th></tr>
<tr><th>design and pricing rules</th><th>trading products</th><th>main market place</th></tr>
<tr><td>Austria</td><td>1%</td><td>6%</td><td rowspan="16">MUP auction</td><td>quarters, hours, blocks</td><td>EXAA, EPEX Spot</td></tr>
<tr><td>Belgium</td><td>0%</td><td>6%</td><td rowspan="6">hours, blocks</td><td>EPEX Spot, Belpex</td></tr>
<tr><td>Czech Republic</td><td>4%</td><td>1%</td><td>PXE</td></tr>
<tr><td>Denmark</td><td>2%</td><td>43%</td><td rowspan="3">Nord Pool</td></tr>
<tr><td>Estonia</td><td>0%</td><td>9%</td></tr>
<tr><td>Finland</td><td>0%</td><td>1%</td></tr>
<tr><td>France</td><td>1%</td><td>4%</td><td>EPEX Spot</td></tr>
<tr><td>Germany</td><td>7%</td><td>11%</td><td>quarters, hours, blocks</td><td>EXAA, EPEX Spot</td></tr>
<tr><td>Hungary</td><td>0%</td><td>2%</td><td rowspan="8">hours, blocks</td><td>HUPX</td></tr>
<tr><td>Italy</td><td>8%</td><td>5%</td><td>GME</td></tr>
<tr><td>Latvia</td><td>0%</td><td>2%</td><td rowspan="2">Nord Pool</td></tr>
<tr><td>Lithuania</td><td>1%</td><td>7%</td></tr>
<tr><td>The Netherlands</td><td>0%</td><td>5%</td><td>EPEX Spot, APX</td></tr>
<tr><td>Norway</td><td>0%</td><td>2%</td><td>Nord Pool</td></tr>
<tr><td>Portugal</td><td>1%</td><td>27%</td><td>OMIE</td></tr>
<tr><td>Romania</td><td>3%</td><td>14%</td><td>OPCOM</td></tr>
</table>

country	solar energy sources	wind energy sources	day-ahead market		
			design and pricing rules	trading products	main market place
Slovenia	0%	0%			South Pool
Spain	6%	22%			OMIE
Sweden	0%	9%			Nord Pool
Poland	0%	6%	MUP auction, continuous PaB		TGE
United Kingdom	0%	9%	MUP auction	halves, hours, blocks	Nord Pool, EPEX Spot

Abbreviations: *MUP*=marginal uniform pricing

2.2.2. German Hourly Day-Ahead Auction

For the German market region bids for hours and blocks of variable length can be handed in at the EPEX Spot until 12 pm and the results are published 40 min later (EPEX Spot, 2017b). The minimum trading volume is 0.1 MW and -500 €/MWh to 3000 €/MWh define the allowed price range. The intersection between the cumulated bid and ask curve marks the MCP which is decisive for all market participants.

Trading Volume

The hourly day-ahead auction is the predominant short-term energy market in Germany that accounts for more than 85% of the short-term traded energy, see Figure 6. Each quarter-hourly market accounts for less than 5% of the overall short-term market trading. As the largest physical spot market in terms of trading volume many financial products settle on this physical electricity price (Phelix) which is the average of all traded hours and blocks for one hour (EPEX Spot, 2017b).

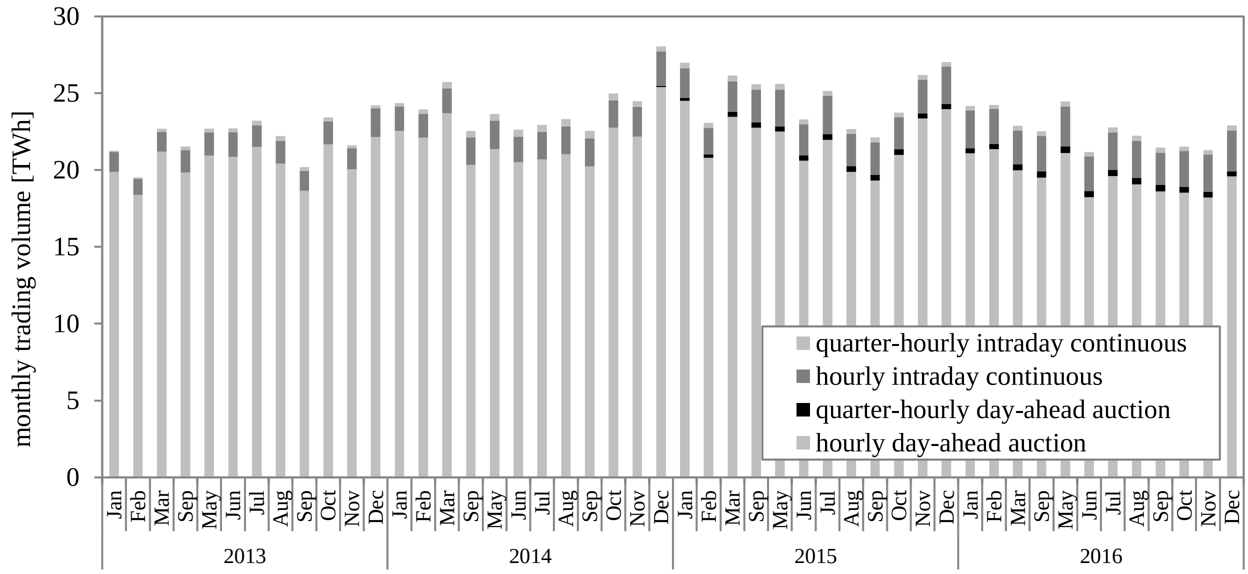


Figure 6 Market volumes of the short-term energy markets in Germany. Data derived from (EPEX Spot, 2017b)

Historic Market Prices

In Figure 7 the average historic hourly day-ahead prices since the year 2000 are depicted and reveal significant price changes over the recent years. This is especially interesting since it unveils how market prices have been influenced in history by changes of the environment conditions.

Whereas the young day-ahead market in the early stages between 2000 and 2004 (green lines) had been determined by a low-price level and flat curves. In the years 2005 until 2008 the day-ahead prices spiked especially during midday and the evening (blue lines). The high price level did not last for long. Investments in new production capacities, the financial crisis in 2009 and the reduced energy demand provoke a turnaround of the always rising energy prices. Since 2008 a reduction of the price level can be seen (blue, then red and later yellow lines). A drop of coal prices after mid 2008 forced the price reduction even further (Schernikau, 2017).

The installation of solar power in Germany form 2006 on, including especially the boom years 2010, 2011 and 2012, had a strong influence on the shapings (BNetzA, 2017). Until 2009 the daily price peak was between 11 and 12 o'clock. But with the increased solar power based production this peak diminished over the course of the years. It is not uncommon that on sunny days the prices over midday drop below the night prices. Today especially the low commodity prices (Schernikau, 2017) and the overcapacities due to the installation of promoted RES pressure the market prices.

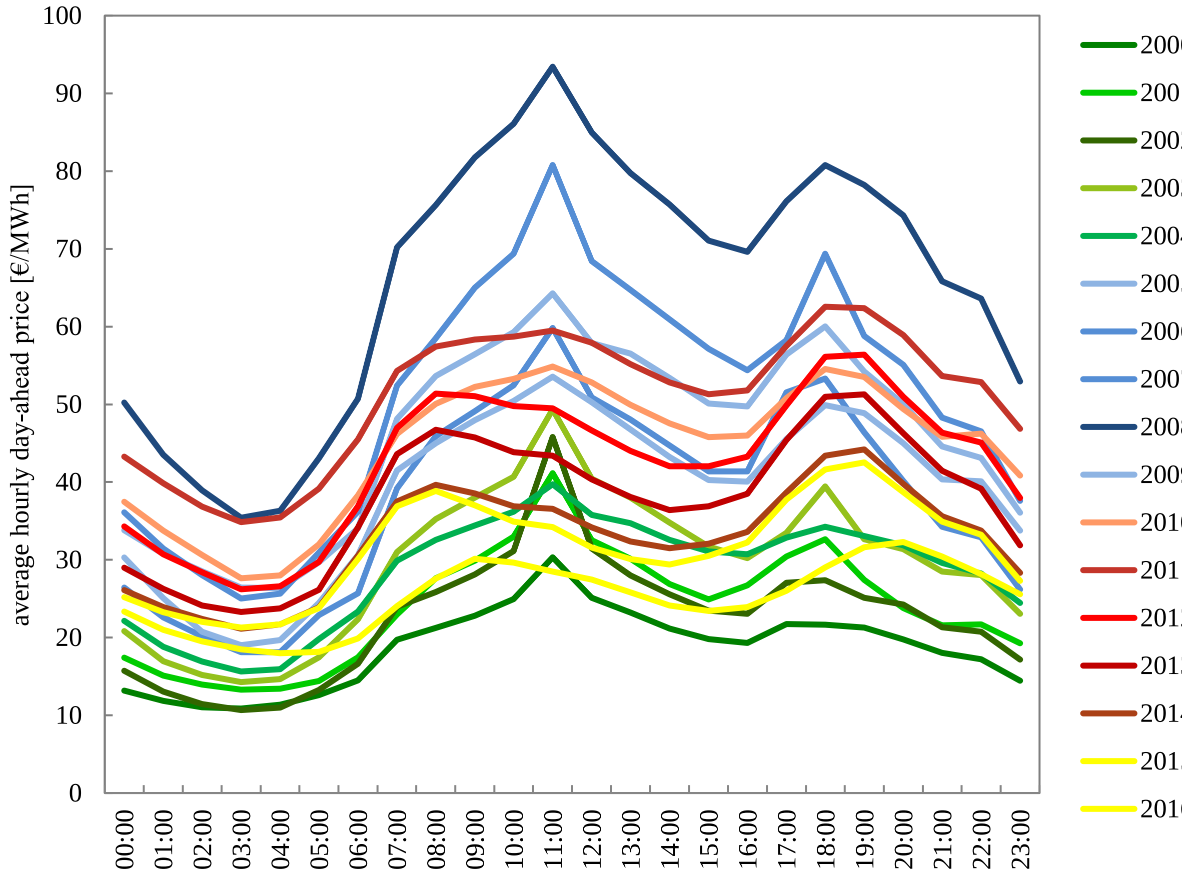


Figure 7 Average German hourly day-ahead market prices for the 16 historic years. Data derived from (EPEX Spot, 2017b)

The standard deviation of the last sixteen years of the hourly day-ahead auction is illustrated in Figure 8. Similar effects as for the absolute market price can be seen. In the beginning the standard deviation was relatively low but increased quickly and resulted in 2005 and 2009 in two peak years. Even clearer is the sudden change from 2010 on due to the installation of solar power. The highly volatile prices during midday diminished entirely. The time since 2010 is characterized by low standard deviations, despite the variable RES boom or the nuclear power plant shutdowns after the maximum credible accident in Fukushima.

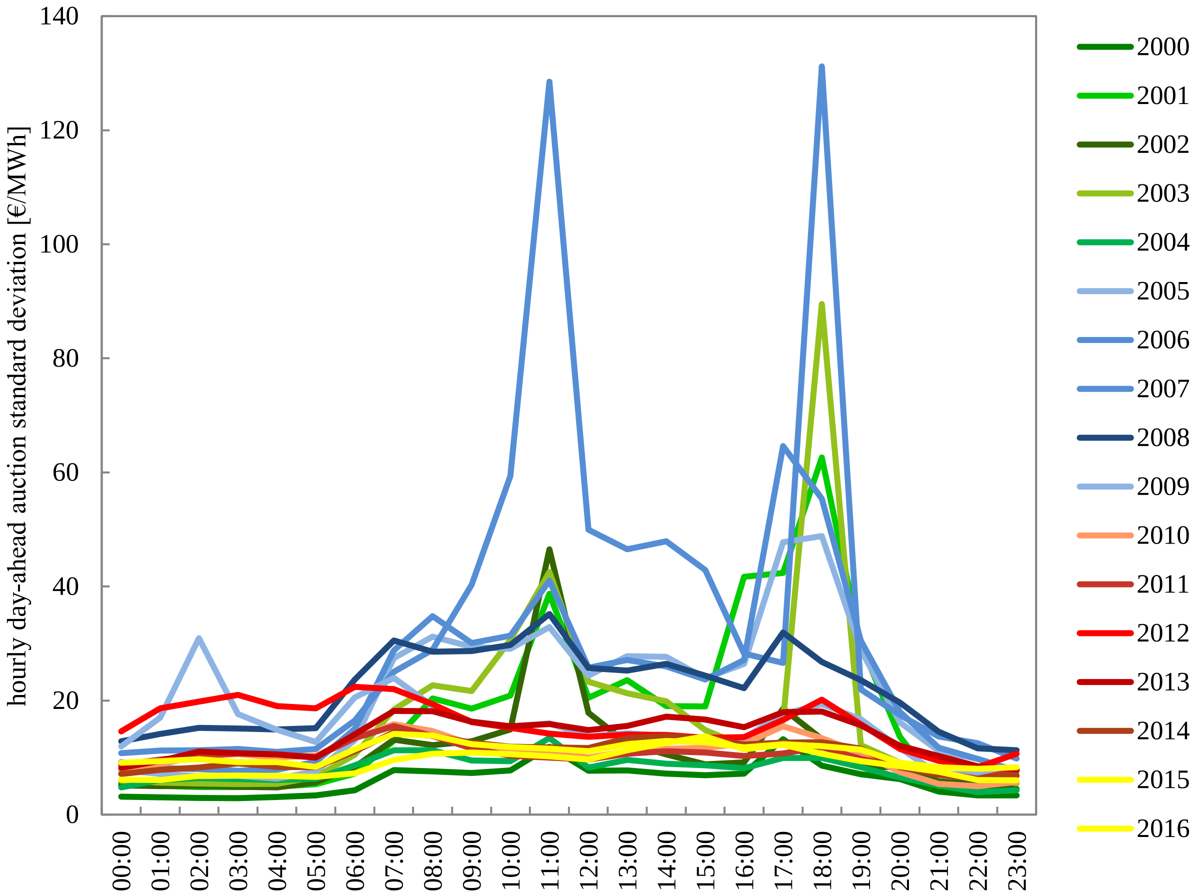


Figure 8 Average German hourly day-ahead market prices standard deviation for the 16 historic years. Data derived from (EPEX Spot, 2017b)

2.2.3. German Quarter-Hourly Day-Ahead Auction

As the only country in Europe, Germany performs two day-ahead auctions, of which the second is the new quarter-hourly day-ahead auction for the German energy market launched in December 2014. According to the power exchange EPEX Spot, this was a reaction to an increased day-ahead demand for shorter order types to reduce the magnitude of unbalanced quarter-hourly schedules already day-ahead (EPEX Spot 2015b, p.2). Similar to the hourly day-ahead auction at 12 pm, the quarter-hourly day-ahead auction is based on marginal uniform pricing, also called clearing price auction (EPEX Spot, 2017b). For a technical analysis of this auction type the interested reader is referred to (Krishna, 2009).

However, the main advantages of the quarter-hourly day-ahead auction in comparison to the continuous intraday trading are the bundling of liquidity during regular office hours (Hagemann & Weber, 2013). Therewith, the new day-ahead auction is especially beneficial for smaller market participants since it gives the possibility to trade all 96 quarters of the next day without providing around the clock shifts for continuous trading. Furthermore, the concentration of market volume at a certain point in time enables higher market liquidity compared to continuous intraday trading. Sufficient liquidity is a key for economic welfare, goal of many market reforms (Amihud, 2002) and increases the overall market efficiency.

Volume

With the introduction of the quarter-hourly day-ahead auction in December 2014 the overall day-ahead trading volume increased considerably, whereas the trading volume of the continuous quarter-hourly intraday market decreased in the first following months, see Figure 9. The trend of increasing quarterly trading volume stopped in mid-2015. The trading volume for both quarter-hourly markets during the second half year of 2015 stagnated at about 0.3 to 0.4 TWh per month. This can be converted to an average trading turnover of about 500 MW for each quarter-hourly product which seems sparsely in comparison to approximately 28,000 MW for the hourly day-ahead market.

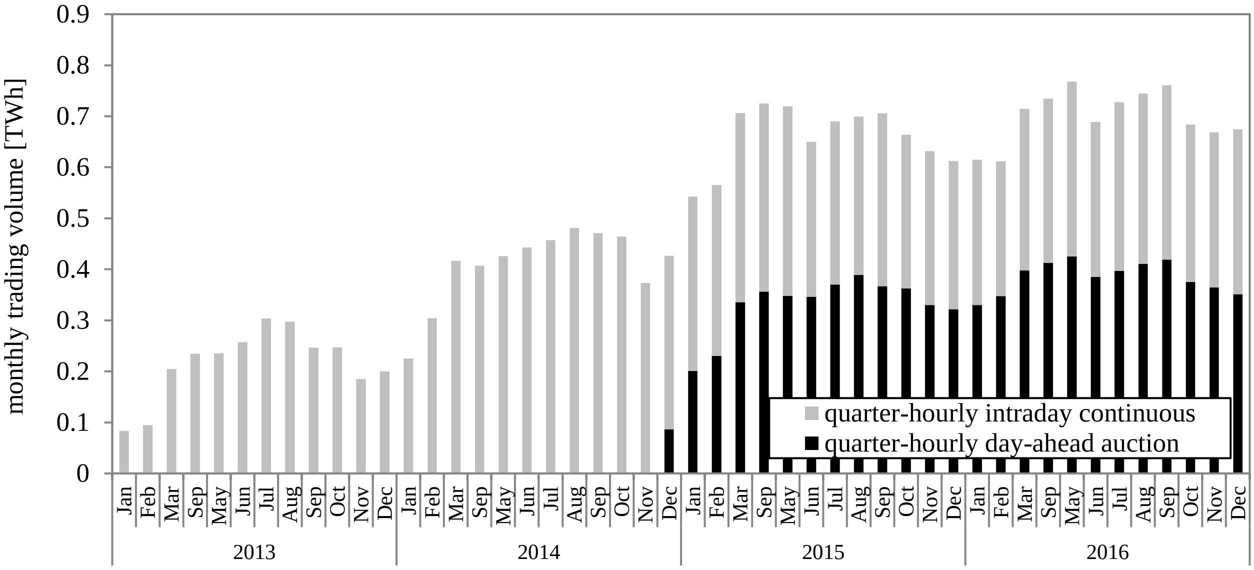


Figure 9 Market volume of the quarter-hourly day-ahead auction and the quarter-hourly intraday countinuous market. Data derived from (EPEX Spot, 2017b)

Historic Market Prices

Figure 10 presents the fluctuating prices of the quarter-hourly day-ahead market (grey line) in comparison to the hourly day-ahead market (black dotted line) for the course of the average day of the year 2015. It can be seen that in some times of the day the quarter-hourly price oscillates more as in others.

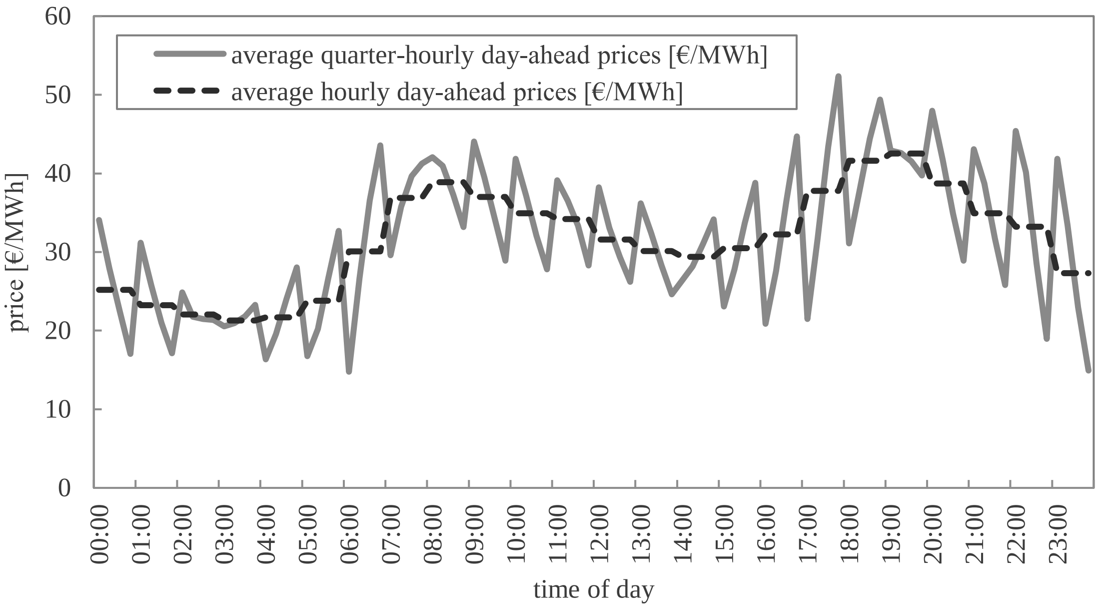


Figure 10 Average hourly and quarter-hourly day-ahead MCP over the course of the day for Germany in 2015. Date derived from (EPEX Spot, 2017b)

Furthermore, Figure 11 illustrates the size of the spreads between the lowest and the highest quarter-hourly day-ahead MCP within one hour. That means in 2015 in more than 40 % of the hours the difference between the MCP of the first and the last quarter of one and the same hour was more than 15 €/MWh. Despite these intense fluctuations of the quarter-hourly market prices the average price levels of all short-term energy markets are nearly identical (EPEX Spot, 2017b) and can be assumed to be arbitrage free (EPEX Spot, 2015b).

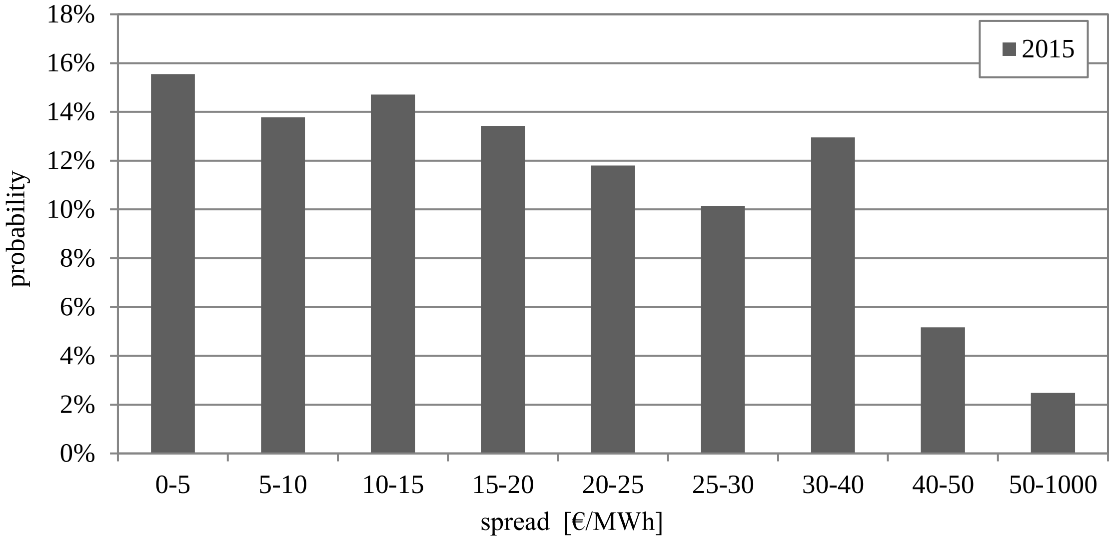


Figure 11 Quarter-hourly day-ahead auction price analyzation: spread between the highest and the lowest volume weighted average quarter-hourly intraday price within one hour (VWAPS). Data derived from (EPEX Spot, 2017b)

2.3. Intraday Markets

Intraday markets are short-term electricity markets which cover the time span between day-ahead auction and physical gate closure, the time when the system operator does not allow changes of the planned schedules anymore (Weber, 2010). Predominantly, trading and delivery take place on the same day. Historically, the optimal production was determined one day ahead and intraday deviations where balanced either within the operator’s own power plant portfolio or resulted in the activation of balancing power. With the introduction of intraday markets unbalanced plant outages, changes in load expectations and variations in RES production could be traded during the intraday. Therefore, the reliability of the schedules has been increased and the demand for balancing energy decreased; a more economically allocation in the short-term has been achieved.

In this part, the intraday market design and modelling including the overview on the EU intraday markets is presented in chapter 2.3.1. In chapter 0 the German intraday market is introduced focusing especially on the characteristics resulting from the continuous trading.

2.3.1. Market Design

Most Intraday markets in Europe are organized as continuous double auctions. Three exceptions are Spain, Portugal and Italy which perform repeated closed and centrally cleared marginal uniform pricing auctions during the intraday (GME, 2017; OMIE, 2017). Both market types are so called multi-actor markets (Klaboe & Fosso, 2013). These intraday auctions are similarly organized as the auctions in the day-ahead market, just more frequently and with a shorter time to delivery. The advantage is the pooling of liquidity in a few auctions which is at the same time the notable disadvantage, since the deviations cannot be balanced at all time. A plant outage right after the auction would mean to wait until the next auction or in case it was the last auction before delivery to stay unbalanced.

Comparing, the continuous intraday provides an around the clock trading possibility. Bids and asks are listed in an order book for each product. If supply and demand orders have the same price and size the orders are executed. Every submitted and executed order needs to be delivered. Continuous trading has the great advantage that every market participant may balance unplanned deviations whenever they occur. All other market participants have the possibility to actively manage their power plants as real options in the intraday with the possibility but not the obligation to adopt planned production, if the prices change.

Nearly all European countries have implemented intraday markets by now. Table 3 presents an overview over European intraday markets including some important market and regulatory characteristics as the last possible trading before delivery, the already mentioned design and pricing rule as well as the respective market place. Furthermore, the share of the variable RES solar and wind are prepared to show the link between market design and production mix. Politics is driven to improve the existing market designs to enable more and more weather driven RES feed-in as cost efficient as possible and without risking security of supply. It can be assumed that the more RES are installed the more flexible the market design. For example, the gate closure time and the last possible trading before delivery varies strongly from 180 min in Poland (TGE, 2017) with a major share of hard coal fired power plants and 5 min in the Netherlands and Belgium (Belpex, 2017) with higher cross-border exchanges and an increasing importance of wind power.

The continuous intraday market design with a pay as bid pricing rule is predominant in Europe; whereas Italy, Spain and Portugal, as already mentioned, implemented marginal unified pricing auctions during the intraday. Most markets are organized by EPEX Spot, Nord Pool or a corporation with one of these two energy exchanges.

Table 3 Empirical analysis of European intraday markets. Data retrieved from the energy exchange operators (APX, 2017; Belpex, 2017; EPEX Spot, 2017b; EXAA, 2017; GME, 2017; Hupx, 2017; Nord Pool, 2017; OMIE, 2017; Opcom, 2017; PXE, 2017; South Pool, 2017; TGE, 2017)

country	solar energy sources	wind energy sources	intraday market			
			last possible trading before delivery	design and pricing rules	trading products	main market place
Austria	1%	6%	30min	continuous PaB	quarters, halves, hours, blocks	EPEX Spot, EXAA
Belgium	0%	6%	5min		hours, blocks	Belpex
Czech Republic	4%	1%	n/a	n/a	n/a	n/a
Denmark	0%	43%	60min	continuous PaB	hours, blocks	Nord Pool
Estonia	0%	9%				
Finland	0%	1%				
France	1%	4%	30min			EPEX Spot
Germany	7%	11%	30min, 0min within own balancing group		quarters, halves, hours, blocks	
Hungary	0%	2%	120min			HUPX
Italy	8%	5%	250min	MUP auction	hours, blocks	GME

country	solar energy sources	wind energy sources	intraday market			
			last possible trading before delivery	design and pricing rules	trading products	main market place
Latvia	0%	2%	60min	continuous PaB		Nord Pool
Lithuania	1%	7%				
Norway	0%	2%	60min			Nord Pool
Poland	0%	6%	3hours			TGE
Portugal	1%	27%	195min	MUP auction		OMIE
Romania	3%	14%	90min	continuous PaB		OPCOM
Slovenia	0%	0%	60min		quarters, hours, blocks	South Pool
Spain	0%	6%	195min	MUP auction	hours, blocks	OMIE
Sweden	0%	9%	60min	continuous PaB		Nord Pool
The Netherlands	0%	5%	5min			APX Power NL
United Kingdom	0%	9%	75min;			APX Power UK

Abbreviations: *MUP*=marginal uniform pricing, *PaB*=pay as bid

2.3.2. German Intraday Continuous

The German intraday continuous market, enforced by the energy act of 2005, started in September 2006 at the power exchange EPEX Spot. The continuous market is based on closed order book pay-as-bid trading in which the market participants offer a quantity in MW and a respective price they either want to buy or sell for. Such a quantity price pair is called an order. Every order entered during continuous trading does immediately fill the order book. Every such order remains in the order book until the order is deleted, executed or the trading time is exhausted. Two orders are executed when a sell and a buy order have the same price and quantity; the orders are filled. As underlying periods, hours, half-hours, quarter-hours and blocks of hours can be traded. Orders can be executed partly with the smallest allowed quantity of 0.1 MW within a price range of -9,999 € and +9,999 €. The duration of continuous trading is limited to a timeframe between opening and closing of trading. Hourly trading starts at 3pm on the day before delivery, followed by the quarter-hourly trading that starts at 4pm for the following day, see Figure 12. Each product can be traded until 30 minutes before delivery begins and until 0 minutes before delivery in the own grid control area. The trading takes place at 7 days a week and 24 hours a day. There are no breaks during continuous trading.

Furthermore, EPEX Spot enacted some guidelines to limit high-frequency atomized trading, as for example with a maximum number of bids entered without being executed. Machines that trade following algorithmic based logics are present on the market since a few years. Nevertheless, it needs to be mentioned that the trading on energy markets cannot be compared with the high-frequency stock or currency trading.

Instead of activating balancing energy when deviating from the planned schedule, market participants trade missing or surplus electricity due to load deviations, power plant outages and RES forecast changes into the continuous intraday market. Along with savings for the single market participants also the balancing demand and in consequence also the balancing prices decreased significantly with the introduction of intraday markets.

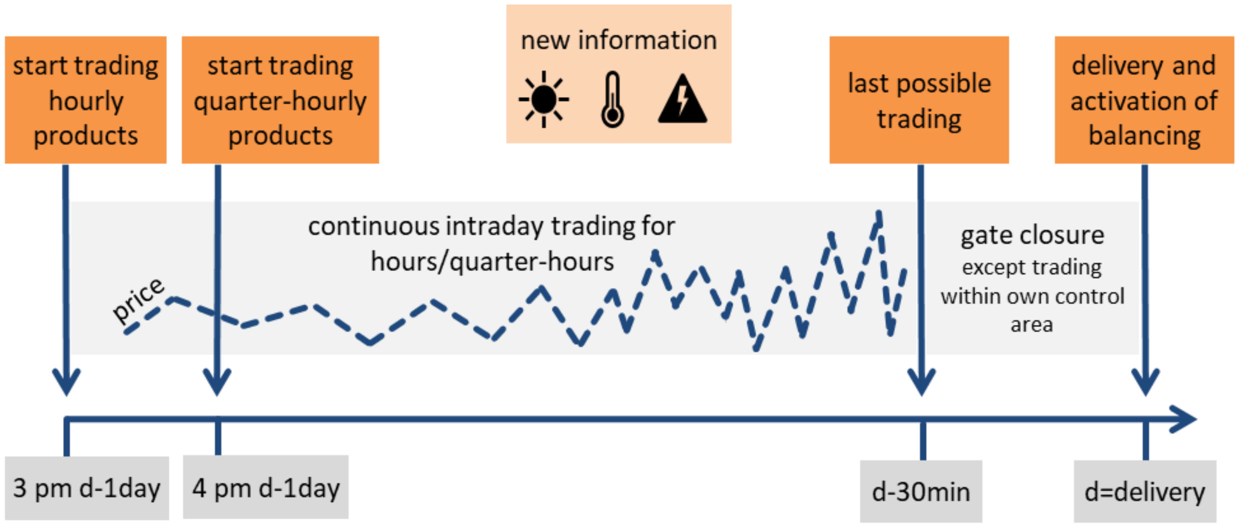


Figure 12 Illustration of the continuous intraday trading in Germany

As graphically presented in Figure 12, the trading activity increases strongly towards the end of the trading period. The clear majority of trading takes place in the last hours before gate closure. This is reasonable since market participants have to close their open positions until the end of the trading period. During the trading period the price of one and the same product can vary strongly. In Figure 13 and Figure 14 the average price spread between the highest and the lowest price paid for one hour or quarter-hour over the whole trading period is illustrated for the year 2015. In 24 % of the hours the difference between the highest and the lowest price paid for one and the same hour was more than 20 €/MWh. This was the case for more than 75 % of the quarter-hours. In 18 % of the quarter-hours the spread was higher than 40 €/MWh. The fluctuations increase the optional value of flexible assets during the intraday.

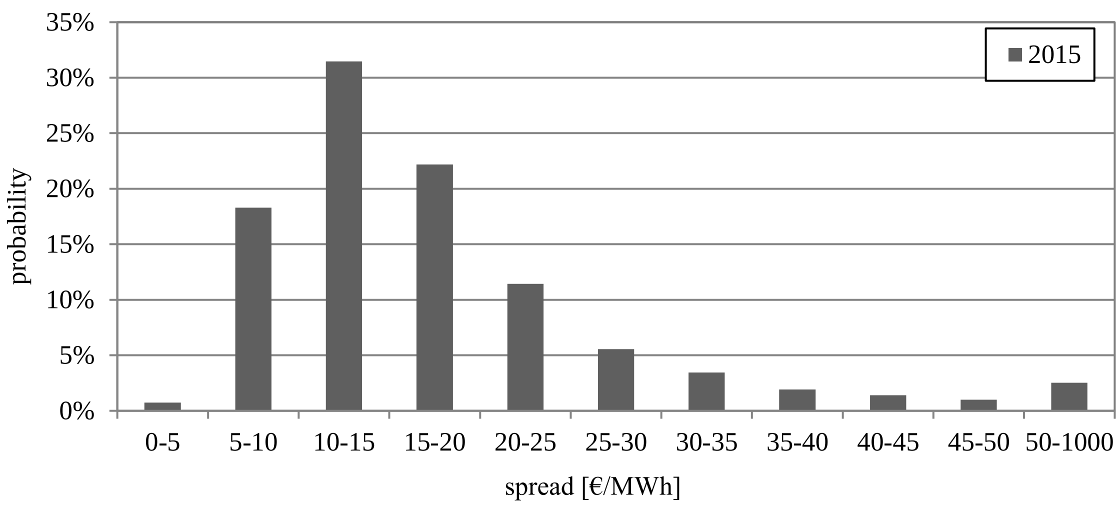


Figure 13 Hourly intraday continuous price analyzation: spread between the highest and the lowest price traded for one and the same product over the course of the whole trading period (intraday high-intraday low). Data derived from (EPEX Spot, 2017b)

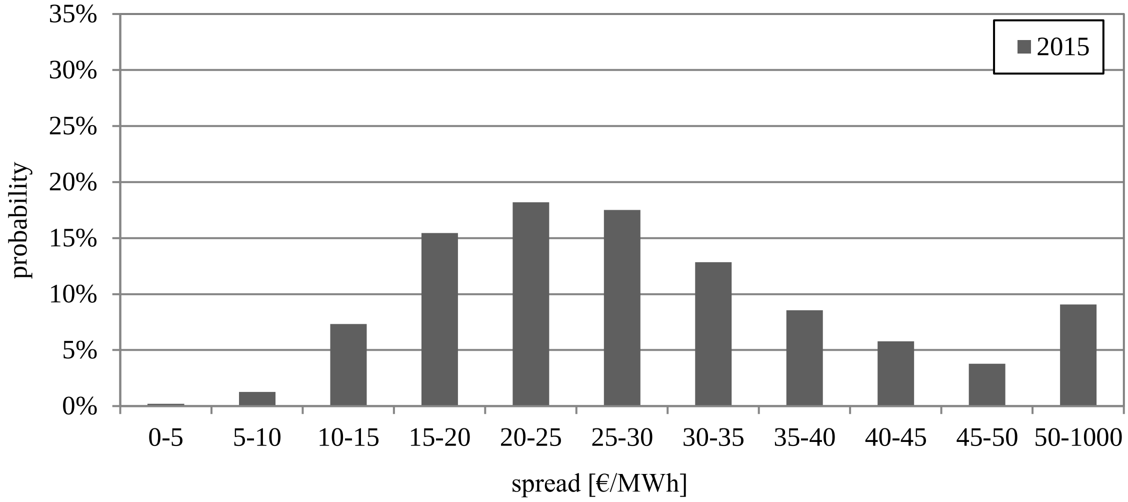


Figure 14 Quarter-hourly intraday continuous price analyzation: spread between the highest and the lowest price traded for one and the same product over the course of the whole trading period (intraday high- intraday low). Data derived from (EPEX Spot, 2017b)

A continuous source for intraday trading demand are variable RES. Solar power can be forecasted relatively good and is therefore traded for the most part already into the quarter-hourly day-ahead market. Although, for example dissolving mist, which is always a challenge for meteorologists, can result in significant solar power intraday trading. In contrast, a schedule precise wind forecast for the next day is relatively difficult. Whereas the overall amount of wind might be estimated relatively good, the exact point in time when the wind blows is much more difficult to foresee. Therefore, the intraday market is the most important market for wind millers. In times when the gradient of the production capacity at the money cannot follow the in- or decreases of the residual load a characteristic zig-zag-price effect is formed. During the intraday this is, for example, the case when a low-pressure area with strong wind is passing over Germany. The resulting price spread for one hour can be significant. Figure 15 demonstrates the probability distribution of the volume weighted average price spread of the quarter-hours within one hour. This means that in more than 20 % of the hours the price spread between the first and the last quarter of an hour was more than 30 €/MWh in 2015. Whereas the price spreads in 2016 where significantly lower they increased again in 2017.

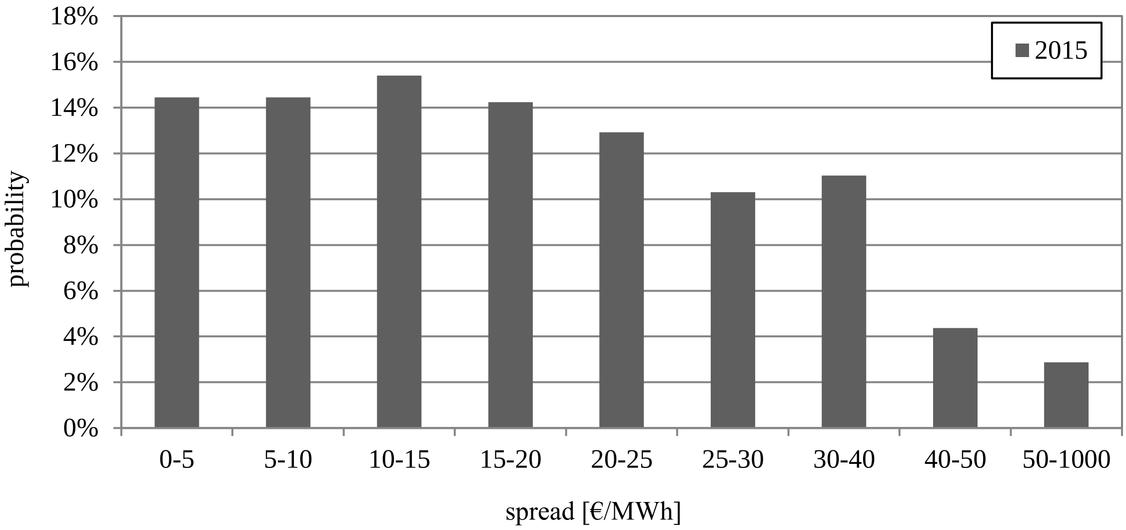


Figure 15 Quarter-hourly intraday continuous price analyzation: spread between the highest and the lowest volume weighted average quarter-hourly intraday price within one hour (VWAPS). Data derived from (EPEX Spot, 2017b)

2.4. Balancing Energy Markets

A secure operation of electric devices requires a constant net frequency in alternating current (AC). If too much (little) energy is supplied to the energy grid, the net frequency will rise (drop). Hence, energy supply and demand need to be balanced permanently. Since energy can neither be saved easily nor cheaply, there is a necessity for an ancillary service – the so-called balancing power. If the frequency drops (rises) positive (negative) balancing power is needed, e.g. by increasing (decreasing) the load level of a power plant.

2.4.1. Market Design

Before the liberalization, the grid balancing was provided by the regional utility monopolists. The system was optimized as a whole and the costs allocated to all electricity users. Since the liberalization and the unbundling of grid and production, the responsibility for grid stability has been with the TSO. Therefore, the TSO in most counties introduced balancing energy markets to buy the needed balancing power from operators with flexible capacity. The costs of procurement are attributed to the market participants that cause imbalances and to the end-user via grid tariffs.

Due to the growing use of renewable energy sources the European electricity markets face tremendous transformations. Strong action is taken to harmonize the various energy-only and the balancing markets in Europe (ENTSO-E, 2017a), since the increasing volatility on the production side will require additional flexible products (Fraunhofer IWES, 2015). A unified European electricity market is supposed to lead to a higher degree of competition and in the long-term promote efficiency. Major electricity systems, as the continental Europe UCTE and the Scandinavian NORDEL system have been united to the European

Network of Transmission System Operators for Electricity (ENTSO-E) and with the size of the synchronized area the costs for every single participant is supposed to decrease.

For the harmonization, the ENTSO-E discerns three “qualities” of balancing power (“three-quality pattern”), namely the Frequency Containment Reserve (FCR), the Frequency Restoration Reserve (FRR) and the Replacement Reserve (RR) (ENTSO-E, 2013). First, FCR is used to limit deviations from the frequency, then automatically-activated FRR is utilized to restore the frequency. This is replaced by a manually-activated FRR and as a final measure RR is activated if required. All suppliers need to prequalify to participate in the double auction based on power bids (€/MW) for the provision itself and sometimes energy bids (€/MWh) if energy is activated.

Below, an overview is given on the European balancing markets of the 24 ENTSO-E members based on Ocker, Braun and Will (2016). This includes a qualitative analysis of potential drivers for market designs. The information is qualitatively aggregated into tangible findings and illustrated through the use of specific market examples.

European Balancing Power Markets Analysis

The overview on the European balancing power markets is given in Table 4. As market designs vary considerably, minor simplifications of the real market structures were inevitable. Supplementary information is available by way of additional download material, linked to the paper Ocker, Braun and Will (2016).

The European balancing power markets are analyzed along three categories:

- energy only market characteristics
- balancing power market characteristics and
- auction characteristics

First, general power market characteristics have strong implications on the ancillary services market designs. Historically, the key driver for the power market design is the underlying electricity mix. Therefore, share of gross electricity consumption served from variable RES, namely production from wind and photovoltaics, is reported in Table 4. The variable RES-share, given as the ratio between net electricity produced from wind and solar power and the electrical energy available for consumption, is used as an indicator for this increasing volatility, which can have a significant impact on required balancing power and implications for the design of these markets (Bevrani, Ghosh, & Ledwich, 2010; Stadler, 2008). As the necessity for a flexible adjustment of production levels is sometimes known in advance, most countries implemented short-term trading options such as intraday markets. Intraday markets allow load serving entities to avoid balancing activities by trading for delivery on the same day. The latest time before physical delivery on the primary power market when an intraday market trading option is still available is reported in Table 4, in order to investigate a possible impact on the design of balancing markets (ENTSO-E, 2015). Cross border trading is not considered.

Secondly, balancing market characteristics describe the implementation of each balancing power market quality whereas the focus is on FCR as well automatically- and manually-activated FRR. First, it is examined whether these three qualities are applied or if certain market qualities are non-existent. If existent, for each market quality whether the provision of balancing power is a compulsory service or is procured with the help of an auction. In case of the latter, the bid elements (power and/or energy bid) are presented and whether positive and negative balancing power are distinguished. Furthermore, the auction frequency (yearly, monthly, weekly, daily) as well as the activation strategy (merit-order or pro-ratio/parallel) are discussed. Lastly, the number of delivery time slices, their duration (e.g. 24x1h for a daily procurement) and the minimum power offer are stated, since these are fundamental to assess a particular market's flexibility.

Finally, auction characteristics in Table 4 discuss pricing and scoring rules of the respective markets. Pricing options are uniform pricing, pay-as-bid or a combination of these. The scoring rule describes how the winners of the auction are determined. Both rules have significant impact on the bidding behavior of suppliers (Ocker, Belica, Ehrhart, & Karl-Martin, 2016).

Table 4 Empirical analysis of European balancing power markets. For supplementary information on the sources for this table please refer to the following document: (Ocker et al., 2016)

	power market characteristics		balancing power market characteristics			auction characteristics	
country	variable RES share (2014)	latest possible trading	FCR (automatic)	FRR (automatic)	FRR (manually)	pricing rule	scoring rule
Austria	7.3%	30min	PB; s; w; m.-o.; 1x168h; 1MW	PB&EB; ±; w; m.-o.; Mo-Fr 8am-8pm, rest; 5MW	PB&EB; ±; w; m.-o.; 42x4h; 5MW	PaB	lowest PBs
Belgium	9.2%	5min	TP; ±; m; n/a.; base, peak, offpeak; 1MW	PB&EB; ±; m; m.-o.; base, peak, offpeak; 5MW	PB&EB; ±; y; n/a.; base, peak, offpeak; 5MW	PaB	SP
Czech Republic	4.4%	Day-ahead	PB; s; d; n/a; 24x1h; n/a	PB; ±; d; p; 24x1h; n/a	PB; s; d; m.-o.; 24x1h; n/a	UP	lowest PBs
Denmark (DK1/DK2)	44.7%	60min	PB; ±; d; n/a; 6x4h; 0,3MW	PB; s; m; p.; 24x1h; 0,3MW	PB&EB; ±; d; n/a; 24x1h; 10MW	UP (DK1), PaB&UP (DK2)	n/a
Estonia	8.7%	60min	provided by russian TSO	TP; n/a; n/a; m.-o.; 24x1h; 5MW	TP; ±; n/a; n/a; 24x1h; 5MW	PaB	n/a
Finland	1.4%	60min	n/a; s; n/a; n/a; 24x1h; 1MW	EB; ±; n/a; p; 24x1h; 10MW	non-existent	UP	n/a
France	5.6%	30min	compulsory, regulated prices	compulsory, regulated prices	TP; ±; y; m.-o.; n/a; 10MW	PaB	n/a
Germany	18.2%	30min	PB; s; w; m.-o.; 1x168h; 1MW	PB&EB; ±; w; m.-o.; Mo-Fr 8am-8pm, rest; 5MW	PB&EB; ±; d; m.-o.; 6x4h; 5MW	PaB	lowest PBs

	power market characteristics		balancing power market characteristics			auction characteristics	
country	variable RES share (2014)	latest possible trading	FCR (automatic)	FRR (automatic)	FRR (manually)	pricing rule	scoring rule
Hungary	1.9%	120min	PB; ±; n/a; n/a; 24x1h; n/a	PB&EB; ±; n/a; m.-o.; 24x1h; n/a	PB&EB; ±; n/a; m.-o.; 24x1h; n/a	PaB	n/a
Iceland	0.0%	Day-ahead	TP; s; w; m.-o.; 24x1h; 1MW	TP; s; w; m.-o.; 24x1h; 1MW	TP; ±; w; m.-o.; 24x1h; 1MW	UP	lowest TPs
Italy	13.1%	250min	compulsory, regulated prices	EB; s; d; p; 24x1h; 1MW	EB; s; d; m.-o.; 24x1h; 1MW	PaB	n/a
Latvia	2.1%	60min	provided by russian TSO	manual: n/a; ±; n/a; m.-o.; 24x1h; n/a	non-existent	n/a	n/a
Lithuania	13.7%	60min	provided by russian TSO	manual: TP; n/a; d; m.-o.; 24x1h; 5MW	TP; n/a; d; m.-o.; 24x1h; 5MW	UP	lowest TPs
The Netherlands	6.4%	5min	PB; s; w; m.-o.; 1x168h; 1MW	PB&EB; ±; d/y; m.-o.; n/a; 4MW	PB&EB; ±; d/y; m.-o.; n/a; 20MW	PaB & UP	lowest PBs (FCR), n/a
Norway	2.0%	60min	PB; s/±; d/w; n/a; 24x1h; 1MW	PB&EB; ±; w; p; n/a; 1MW	non-existent	UP	n/a
Poland	6.0%	180min	EB; ±; n/a; n/a; 24x1h; n/a	EB; ±; n/a; n/a; 24x1h; n/a	EB; ±; n/a; m.-o.; 24x1h; n/a	UP	SP
Portugal	27.9%	195min	compulsory, no compensation	PB; ±; d; p; 24x1h; n/a	PB&EB; ±; d; m.-o.; 24x1h; n/a	UP	lowest PBs
Romania	18.4%	90min	compulsory, no compensation	TP; ±; d; m.-o.; 24x1h; n/a	TP; ±; d; m.-o.; 24x1h; n/a	UP	lowest TPs
Slovenia	2.1%	60min	compulsory, no compensation	PB&EB; n/a; y; p; 24x1h; n/a	PB&EB; n/a; y; m.-o.; 24x1h; n/a	PaB	n/a
Spain	28.3%	195min	compulsory, no compensation	PB; ±; d; p; 24x1h; n/a	PB&EB; ±; d; m.-o.; 24x1h; n/a	UP	lowest PBs
Sweden	9.2%	60min	PB&EB; s; d/w; n/a; 24x1h; n/a	PB&EB; ±; w; p; n/a; n/a	non-existent	PaB	n/a
Switzerland	1.6%	60min	PB; s; w; m.-o.; 1x168h; 1MW	PB; s; w; p.; n/a; 5MW	PB; ±; w; m.-o.; 6x4h; 1MW	PaB	lowest PBs
Serbia	0,0%	Day-ahead	non-existent	TP; ±; d; p; 24x1h; n/a	TP; ±; d; n/a; 24x1h; n/a	UP	lowest TPs

	power market characteristics		balancing power market characteristics			auction characteristics	
country	variable RES share (2014)	latest possible trading	FCR (automatic)	FRR (automatic)	FRR (manually)	pricing rule	scoring rule
United Kingdom	11.9%	75min	PB&EB; ±; m; n/a; Mo-Fr, Sa, Su; 10MW	PB&EB; ±; m; n/a; Mo-Fr, Sa, Su ; 10MW	PB&EB; s; m; n/a; Mo-Fr, Sa, Su; 50MW	PaB	n/a

Abbreviations: ***manual***=manual activation; **PB**=power bid and/or **EB**=energy bid or **TP**=total price; **s**=symmetric product (no distinction between positive and negative balancing energy) or ±=distinction between positive and negative balancing power; procurement: **d**=daily, **w**=weekly, **m**=monthly or **y**=yearly; **m.-o.**=merit-order activation of balancing energy or **p**=pro-ratio/parallel activation of balancing energy; **24x1h**=24 one-hour time slices per day; **5MW**=minimum power offer is 5MW; **PaB**=Pay-as-Bid pricing or **UP**=Uniform pricing (for EB and/or PB); **SP**=Stochastic Programming or **lowest PBs/TPs**=lowest capacity bids/total prices are considered until balancing demand is met; ***n/a***=parameter not available (e.g. not published)

A wide range of gross electricity consumption served from variable RES among the 24 evaluated countries can be found, spanning from 0 % in Serbia and Iceland to almost 45 % in Denmark. In 21 countries, there are intraday trading options for electricity which, however, does not imply equal levels of flexibility. More than half of these countries have trading options of 60 min or less before delivery, whereas especially southern European countries such as Portugal, Spain and Italy can trade only up to 195 min before delivery. 19 countries apply the three-quality pattern introduced by the ENTSO-E. While automatically-activated FRR is part of nearly every market, FCR and manually-activated FRR are not as abundantly used. Especially smaller countries often compel market players to supply FCR or even rely on larger neighboring countries for this service, such as Russia for the Baltic states. Both manual and automatic activation of balancing energy occurs.

Regarding balancing power market design, nearly every constellation of power bid and/or energy bid is applied throughout the three qualities. 23 countries generally distinguish positive from negative balancing power, especially for automatically- and manually-activated FRR. One exemption is the FCR-cooperation between Austria, Germany, the Netherlands and Switzerland which procures FCR without the distinction of positive and negative balancing power (symmetric product). Only Italy is not at all distinguishing between the products. The frequency of balancing power procurement is highly diverse, ranging from a daily to a yearly auction. The activation strategy for balancing energy power on the other hand is almost consistent throughout the European markets: Merit-order activation is used mainly, merely a few countries activate pro-ratio/parallel. The number of time slices, their duration and the minimum size of the power offer vary greatly between the countries and balancing power qualities.

With regard to the applied pricing rule, the picture is also incoherent: In ten countries uniform and in eleven countries pay-as-bid pricing is used. If uniform pricing is used for the procurement of balancing power, this price either depends on an exogenous market price or on the submitted energy bids of the suppliers. The scoring rule is either based on a total price for balancing power and energy, only on the price for balancing power or on a stochastic optimization program minimizing total costs.

Furthermore, four key drivers for the current configurations of the balancing power markets are identified:

- share of volatile RES,

- short-term flexibility,
- market coupling and
- inconsistency in auction characteristics.

These key drivers are explained in detail below, since it can be assumed that a mayor change in these key factors may lead to a modification in market design as well.

Share of Volatile Renewable Energy Sources

Climate concerns and geopolitical circumstances lead to increased interest in power production from renewable sources such as wind and photovoltaics. In order to incentivize large scale investments in variable RES and generate economies of scale, subsidy programs were implemented in many European countries. These were largely successful but now raise questions on the integration of a more volatile production into an only recently liberalized market structure. Generally, integrating variable RES into the power system can have two opposing effects on the balancing market: On the one hand, more variable RES generally induce higher production fluctuations and more balancing power is needed. As a result, the price for balancing should increase. On the other hand, variable RES with low marginal costs reduce day-ahead and intraday prices and may push the existing power plants out of the merit-order. As a consequence, displaced conventional production capacity pushes onto the balancing market and reduces prices there. However, an isolated operation on the balancing power market is not viable for conventional base-load power plants with high ramp-up costs. If regulators do not want to subsidize deficient conventional power plants, balancing power must in the long-run also be supplied by variable RES. The balancing market integration of variable RES can reduce balancing costs (Jansen, Speckmann, & Schwinn, 2012) along with further omitting carbon emissions by conventional production.

The analysis shows that countries with higher shares of variable RES predominantly have flexible auctioning procedures as apparent in a greater number of time slices with shorter maximum durations. Furthermore, auction frequencies are higher and the minimum size of power offers tends to be smaller.

Two exemplary markets, France and Denmark, are discussed to elucidate the transition from a market with a high share of conventional production towards a market with a very high share of wind power plants. While both countries have substantially reduced their CO2-emissions in recent years (EEA, 2017) they achieved this with very divergent production mixes, market structures and liberalization levels. In France less than 6% of the electricity consumed is supplied from variable RES while about 77% of the electricity consumed is produced in nuclear power plants, the highest share in the world (NEI, 2017). Consequently, these power plants are obliged to provide FCR and FRR to the grid. The French auction-based market for RR has changed very little since 2003 (RTE, 2016). The auction takes place once a year allocating blocks of positive and negative RR. Since just two big power plant operators operate on the market the surcharges are flexibly allocated to the power plants within each portfolio. Therefore, the operator is able to compensate unavailable production capacity within their portfolio.

Denmark on the other hand, driven by a very high share of wind power integration, opened the balancing market for variable RES. Wind power generation in Denmark corresponded to a share of about 42% of the Danish electricity consumption in 2014 (cf. Table 3). The wind parks are owned by various companies. Balancing power procurement according to the French model would not be suitable since the small suppliers are not able to guarantee balancing power for a whole year with their limited production capacity and volatile production. Therefore, Denmark changed their markets towards variable RES market integration in three steps: (1) Denmark installed a system to easily prequalify wind power plants for balancing provision, (2) made the auction process available for more participants by performing auctions daily and (3) reduced the traded time slices to a length of four (FCR) and one hour (FRR and RR) (Energinet, 2017). The wind energy feed-in forecasts are reliable enough to estimate wind production for the following day and to precisely assess available gas power capacity to be placed on the balancing market. The Danish system was the first to integrate variable RES into the power system and now serves as an innovation example for future, flexible market structures.

Short-Term Flexibility

Except for the Czech Republic, Iceland and Serbia, all European countries introduced intraday markets that are either based on a regular auction (Spain, Italy and Portugal) (Weber, 2010) or on continuous trading (all others) during the day of delivery. The TSO receives binding production plans of every power plant operator within its grid area. Depending on the market structure, operators are allowed to change this plan until a certain time before delivery. Changes in production have to be balanced on the intraday market for as long as possible. Only the resulting imbalances after the market closure are balanced by the TSO, who in turn procured the balancing power earlier on the balancing market.

In Germany two different short-term markets are available: The quarter-hourly day-ahead auction and the intraday continuous market (EPEX Spot, 2015a). The former was introduced at the end of 2014 and is an additional measure to balance the increasing power supply from solar power plants on the day before delivery (Braun, 2016b). The latter allows trading until 30min before delivery within the entire market area since 2011 and therefore is especially relevant for fluctuations in wind energy supply. By introducing two complementary short-term markets, the German electricity market became highly flexible and can balance volatile supply with-out an increased demand for balancing power (Ocker & Ehrhart, 2017). In fact, the procured balancing capacity fell by 20% between 2008 and the end of 2015 (Hirth & Ziegenhagen, 2015).

As the example of Germany shows, it can be argued that short-term flexibility on intraday markets could reduce the demand for balancing power in other European countries as well. Nevertheless, a more rigorous evaluation of this hypothesis is a promising topic for further research.

Market Coupling

Beyond the consideration of individual national market designs, a trend for international cooperation via coupling of national electricity markets is addressed. Market coupling describes the act of joining physically connected but systematically separated markets via implicit auctions on the respective trading platforms. In coupled markets cross-border transmission capacity is not traded in explicit auctions but rather part of the pricing procedure on national power exchanges. Infrastructure can therefore be used more efficiently, and the resulting greater market area has more participants and a higher liquidity. On the spot market coupling is already being applied and should result in increased competition and lower prices. Nevertheless, this trend is relevant for all power markets and is a stepping stone towards a single

European market (ENTSO-E, 2015; EPEX Spot, 2017a). In the case of balancing power, a positive balancing requirement in one balancing region can often be compensated with a negative one in another. By coupling balancing markets, this pooling effect can be harnessed through an economic mechanism and lead to higher supply security. The effect was e.g. observable upon the introduction of cooperation mechanisms between the four TSOs in Germany in the springs of 2009 and 2010: Immediately after introduction, dispatched balancing energy significantly dropped along with monthly volatility (BNetzA, 2015).

Further efforts aim to promote the international cooperation between TSOs and eventually drive joining of markets. Most employ the categorization into three quality levels and differentiate between positive and negative balancing power. With the help of the ENTSO-E further convergence in market design is foreseeable. Such reconciliation would lead to additional cost reductions and efficiency gains in the procurement of balancing power. In an initial step, the International Grid Control Cooperation (IGCC) of TSOs in Denmark, Germany, the Netherlands, Belgium, Switzerland, Austria, the Czech Republic and most recently France and Belgium, closely cooperate for the minimization of FRR-activation. Their respective net balancing needs are communicated and cleared, therefore compensating opposite requirements (Regelleistung.net, 2016b). Austria, Germany, the Netherlands and Switzerland also have a joint market for 793 MW of FCR. A maximum of 30% (at least 90 MW) of the national FCR-need can be exported to partnering countries, which has led to significant cost reductions after initiation (Regelleistung.net, 2016a). Especially smaller markets may profit from more participants and higher liquidity through joint markets. In fact, Denmark and Belgium currently consider participating in the existing scheme (Energinet, 2015).

If balancing markets continue to converge, a further cost reduction for balancing power can lead to a higher market efficiency and therefore to an increase in public welfare. The ENTSO-E offers a viable platform for this process.

Inconsistency in Auction Characteristics

As seen in the previous sections, some of the characteristics of balancing power markets can be traced back to the respective structure of the power market as a whole. However, the greatly varying auction characteristics, namely the scoring and pricing rules, do not directly correspond to the power market characteristics, see Table 4. This discontinuity is approached by considering the relevant literature on auction design and present a theoretical discussion beyond the characteristics of the balancing power markets (e.g. auction frequency or timing).

Auction rules have a direct impact on the bidding behavior of the suppliers and should therefore be designed according to the goal of the auction, e.g. lowest procurement costs possible. From a theoretical stance, balancing power is procured in multi-part auctions since bidders must be compensated for both reserving capacity and delivering balancing energy if called for. This type of auction is discussed for the procurement of a wide range of goods, e.g. for highways construction (Stark, 1974) or for weapon system tenders (Che, 1993). Procuring balancing power with multi-part auctions was first analyzed by Bushnell and Oren (1994). They develop conditions for scoring rules to result in efficient winner selection. Since then politics and research try to find remuneration schemes to incentivize suppliers to bid their true costs and provide a cost efficient and secure balancing.

The heterogeneity in auction design could be a consequence of the complexity of multi-part auctions for balancing power procurement. National regulators define their own design due to the lack of theoretical research on robust and thus applicable auction designs.

It can be concluded from the balancing power market overview and analysis, of the 24 countries that are members of the ENTSO-E that no predominant market design can be found in comparison to the day-ahead and intraday markets in Europe. Certain elements of this heterogeneity, e.g. auction frequency, timing and duration of time slices, seem to be influenced by the frame-work conditions of the respective power market. The three identified key drivers for these conditions are: the share of variable RES in the electricity mix, the short-term flexibility for trading and pan-European market coupling. On the other hand, the inconsistency in auction characteristics seems to be caused by the complexity of multi-part auctions for balancing power procurement. National legislation has to decide, if a further integration of European balancing power markets is desirable, i.e. concerning the target of national energy independence and adjust their market design accordingly.

2.4.2. German Balancing Energy Markets

In Germany, the three balancing power markets, sometimes referred as ancillary services, are organized as suggested by the ENTSO-E and described in chapter 2.4.1. Figure 16 illustrates the activation process after a disturbance in the grids frequency. In the first seconds the spinning FCR, in Germany also named primary reserve (Primärregelleistung) is passively activated. Shortly afterwards also FRR, the secondary reserve (Sekundärregelleistung), is actively activated and has to be provided by the operators within 30 sec. In case of a long-term disturbance also the RR, in Germany stated as minute or tertiary reserve (Minutenreserve), is called up. Beyond that, with a gate closure time of 30 min, the German intraday market gives the possibility to quickly react on unplanned deviations. Short-term deviations of the planned schedule can be traded on the intraday market to reduce the need for balancing energy activation.

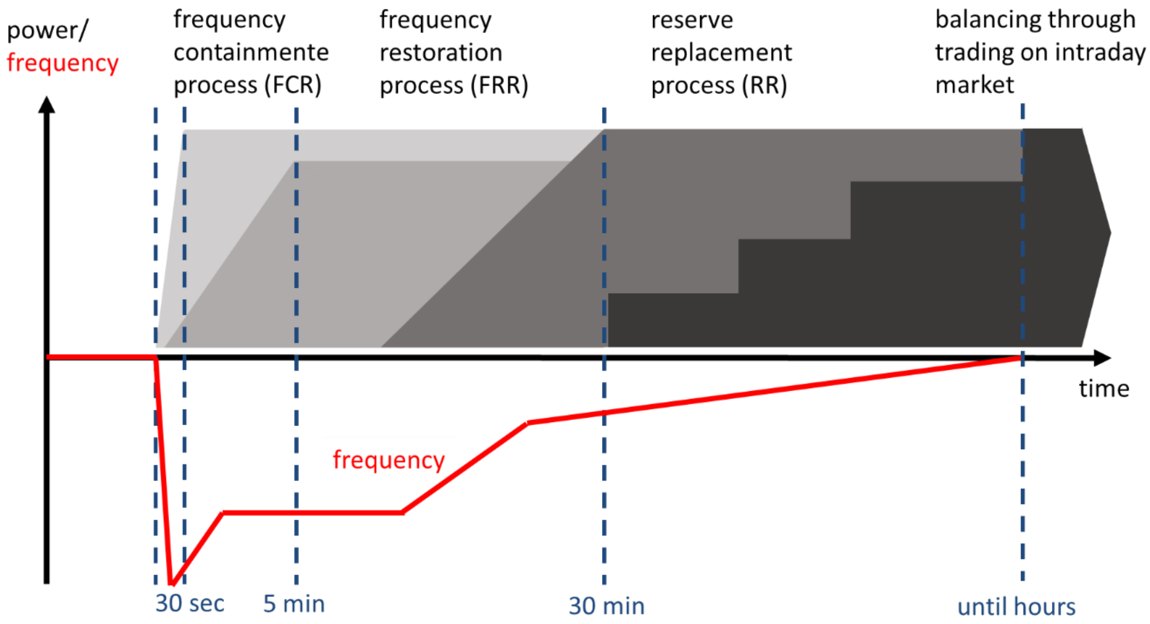


Figure 16 The chronological activation after a fault in grid frequency. Data derived from (Regelleistung.net, 2017a)

In chapter 2.4.1 four key drivers for balancing energy are identified: the share of variable RES, short-term trading possibilities and market coupling. The influence of these factors can be seen in Figure 17 presenting the demand for the various balancing energy products over the course of the last six years. Despite an increasing demand for balancing power is expected due to significant higher shares of variable RES, the overall balancing power demand is constant or slightly decreased. Better short-term trading possibilities, a more flexible market design and market coupling are counter-effects.

Below, the FCR, FRR and RR market including auctioning procedure, activation process, price development and auction volumes are presented.

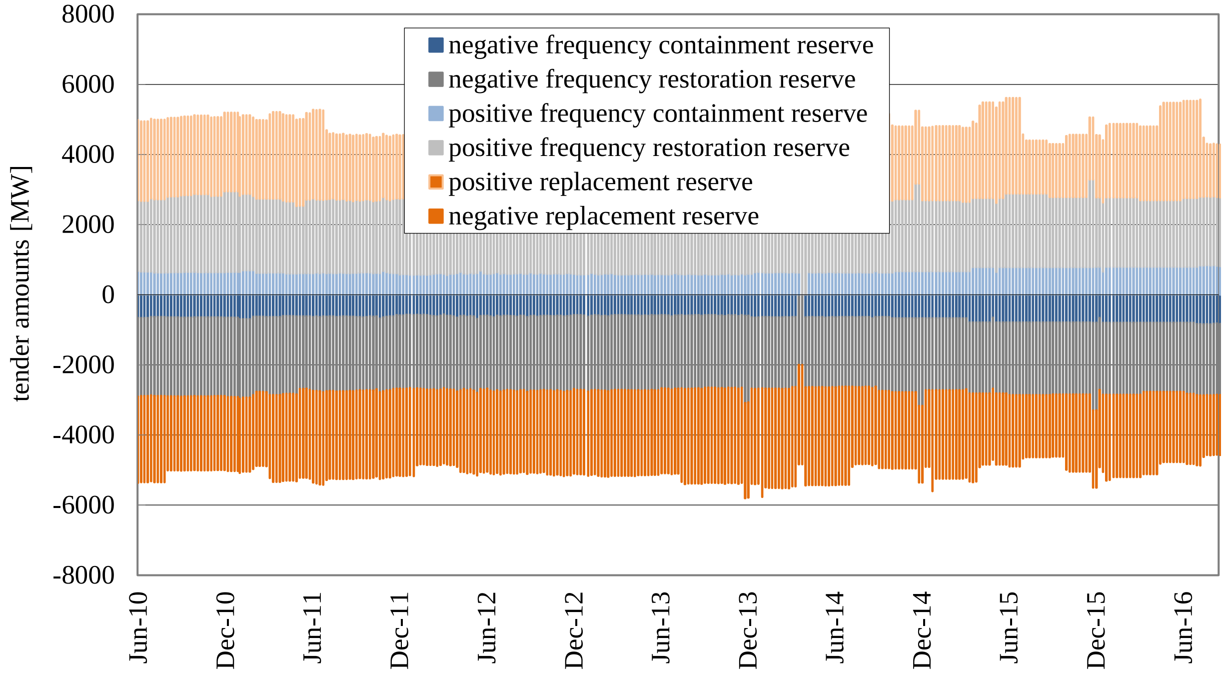


Figure 17 Balancing energy demand of the German TSOs or the group of TSOs in which the German TSOs is organized. Data derived from (Regelleistung.net, 2017a)

German Frequency Containment Reserve

The German FCR is the first balancing market that is activated after a fault in the grid. Since December 1st, 2007 the four German TSO organize a joint FCR auction to buy the frequency containment demand. Every plant that is allowed to participate in the FCR auction went through a prequalification process organized by the respective connection-TSO. The so-called connection-TSO (Anschluss-ÜNB) is the TSO responsible for the grid region in which the bidder is connected to the grid.

Before the weekly auction, the TSOs publish the demand for FCR in accordance with §6 (2) Strom NZV. The product length is one week and the auction takes place Tuesday at 3pm one week before delivery starts. Since 2011 the corner stones of the auctioning process are regulated by the German federal network agency (BNetzA) in BK6-10-097. Orders can be handed in with a bid size increment of +/- 1 MW (5 MW before 2011). No separate products exist for positive or negative provision. The activation is not compensated separately and capacity need to be always available to be activated. This means, the price for one unit need to include the positive and negative provision costs and the costs for activation.

The already mentioned internationalization of the balancing power markets has strong influence on the Germany FCR auction. Already in 2012, Swissgrid joined the German TSO auctioning platform followed by the Dutch Tennet NL in 2014, the Austria APG in 2015, the Belgium TSO Elia in 2016 and the French RTE in 2017. A cooperation with the Danish TSO Energinet.dk is planned. The participating network operators state that the common procurement increases the liquidity in the market and opens up new sales markets for operators (Regelleistung.net, 2017b). The common demand for FCR is about 1250 MW since 2017. Further the FCR exports are limited to a maximum of 30 % of a country's FCR demand or at least 90 MW. This means for Germany 173 MW. In case of auction failures or technical reasons the market can be decoupled if needed. (Regelleistung.net, 2017b)

The demand for FCR between June 2010 and June 2016 is depicted in Figure 18. With the progressive international consolidation, the demand for FCR of the whole area increased slightly but decreased significantly if the share for each country would be seen separately. Resulting, the costs for every single TSO to buy the FCR provision dropped considerably.

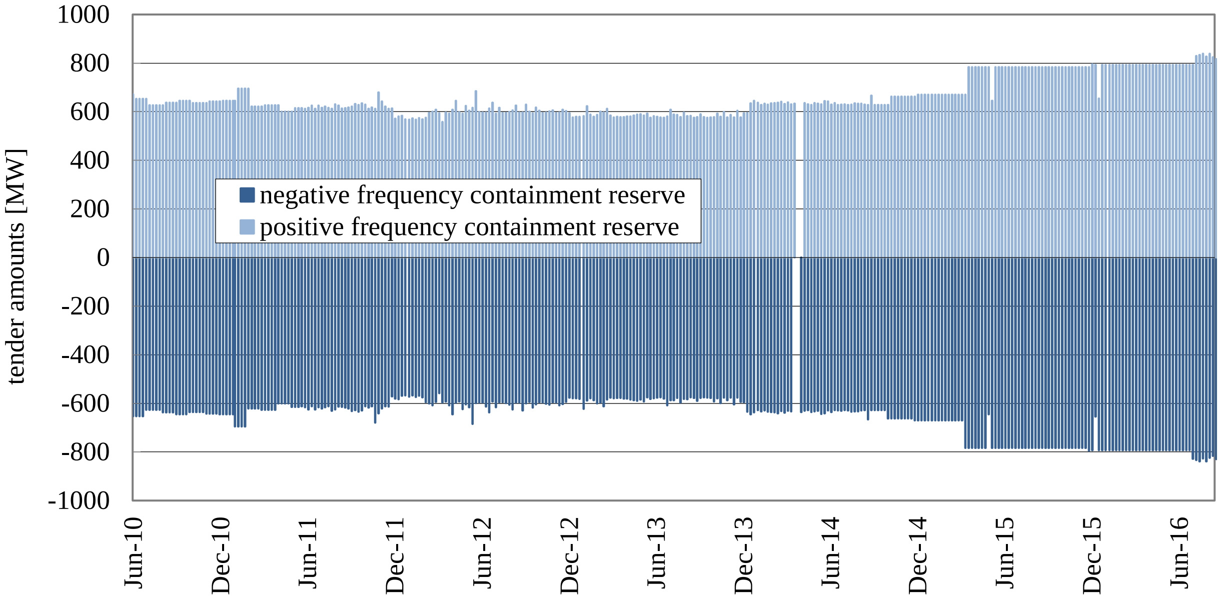


Figure 18 Positive and negative FCR demand of the German TSOs or the group of TSOs in which the German TSOs is organized. Data derived from (Regelleistung.net, 2017a)

Whereas the demand for FCR changes significantly, the price development is not that clear, see Figure 19Figure 18. The market prices (blue line) from 2012 to 2015 were bullish. In the recent years, a bearish price development can be seen. Since 2013, the Christmas and sometimes also the Easter week resulted in extreme high prices in comparison to the long-term average (blue dotted line), so that the volatility seems much higher as it actually is in the rest of the year.

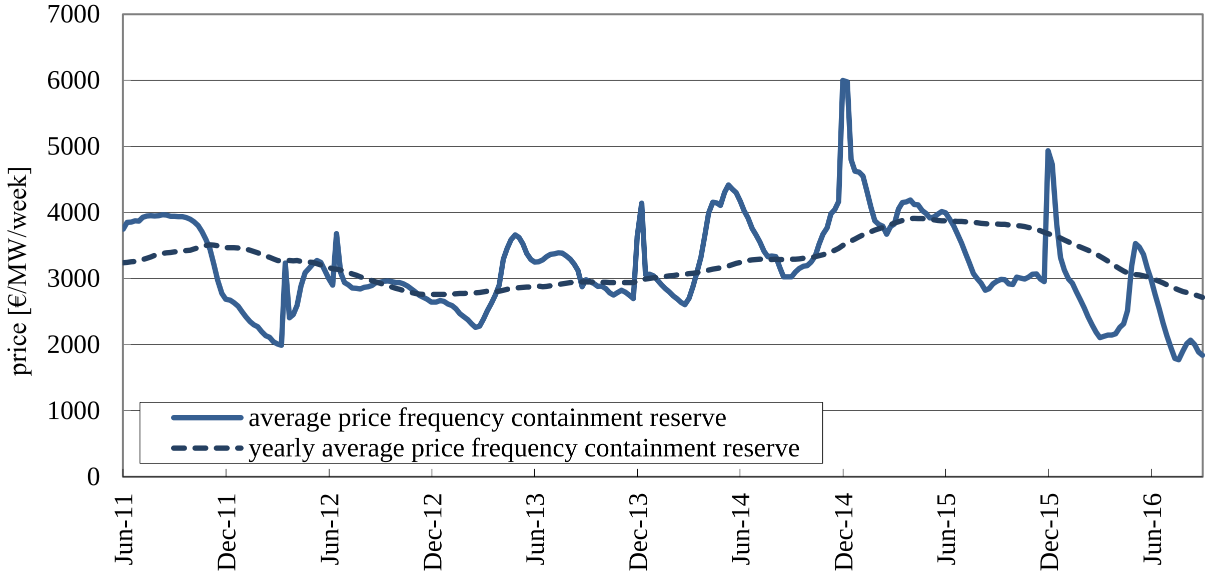


Figure 19 Prices for FCR in Germany from 2011 to 2016. Data derived from (Regelleistung.net, 2017a)

German Frequency Restoration Reserve

Since December 1st, 2007 the four German TSOs buy their FRR demand together in one auction. Therefore, just the net FRR demand of the four TSO grid regions leads to the activation of the first FRR bid. Although, the prequalification to participate in the FRR auction is still organized by the local TSOs that are responsible for the gird in which the power plant or the demand side is connected, no matter highest, high or low voltage level. In comparison to the extensive market coupling of FCR the FRR is limited to the German Austrian market region. No coupling with further countries exist, although planned as described in chapter 2.4.1. This is also evident in Figure 20 as the tendering quantities have not been changing noticeable over the recent years. Just the Christmas week is an exception in which the TSOs demand significantly more balancing provision.

The product lengths are one week (until 2007 one month) and the auction takes place Wednesday one week before delivery starts. The key points for the market organization are defined in BK6-10-098. Since the year 2011 the minimum bid size is 5 MW (before 10 MW) with an increment of 1 MW. The four available products are separated into positive and negative as well as in peak and off-peak times. The peak time is defined from 8 am to 8 pm at normal work days and off-peak as the time between 8 pm and 8 am on normal work days as well as the complete weekend and public holidays.

After an activation, the capacity needs to be delivered completely within 5 minutes to replace the FCR. Upon 30 seconds, at least 1 MW need to be delivered. All market participants are connected with their control system to the FRR activation signal of the TSO in real-time, which operates the frequency controller.

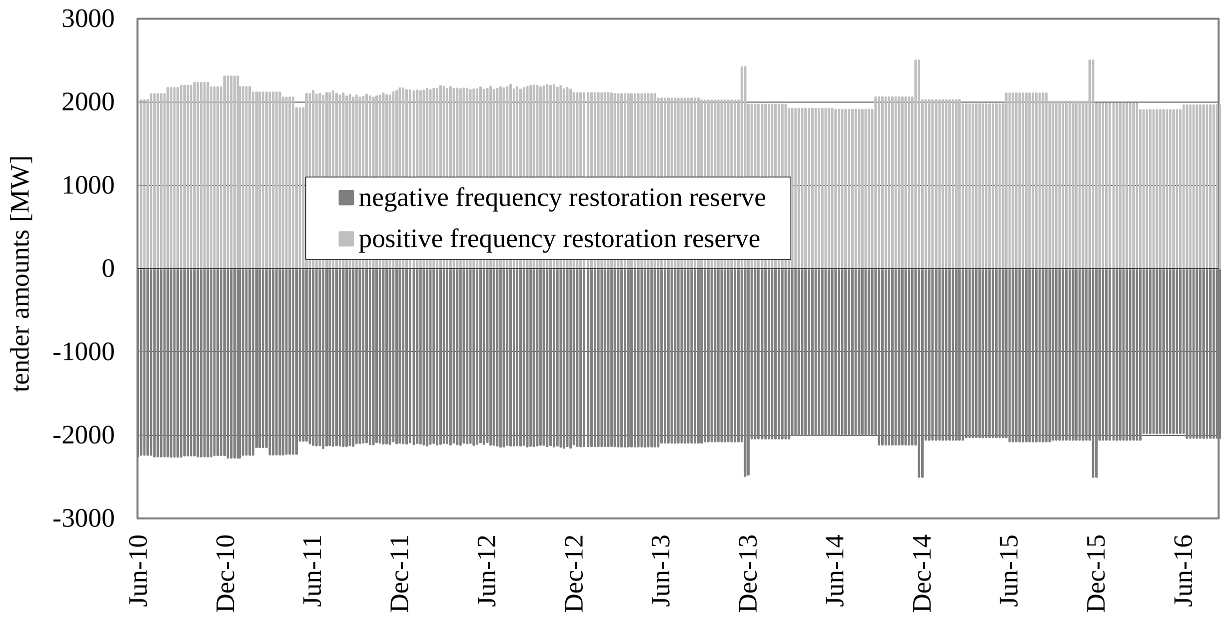


Figure 20 FRR demand of the German TSOs or the group of TSOs in which the German TSOs is organized. Data derived from (Regelleistung.net, 2017a)

The FRR is allocated within the bidders in a double auction of which the first, the power price auction is crucial for the acceptance. In this pay-as-bid pricing auction the TSO sorts all bids in a merit order and allocates the lowest power price bids until the tendering amount is filled. The last accepted bid sets the published marginal price and the volume weighted average of all accepted bids defines the average capacity price, see Figure 21. The average power prices, and the marginal price even more, fluctuate extremely for positive (orange line) as well as negative balancing (blue line). It is therefore likely that strategic bidding is present on the market. Nevertheless, a long-term shrinking price trend can be identified especially looking on the rolling yearly average prices (orange and blue dotted lines) coming from more than 1000 €/MW per week in 2010 to less than 200 €/MW per week marginal price in 2015. Today the price for the negative peak product decreased to a price of 0 €/MW for one week. This trend can be reasoned with the flexibilization of the market including smaller bid sizes of 5 MW and the increased number of market participants. A significant share of FRR has been provided by pumped hydropower storages. Today more and more market participants prequalify further capacities. Balancing with pools of RES is possible as well.

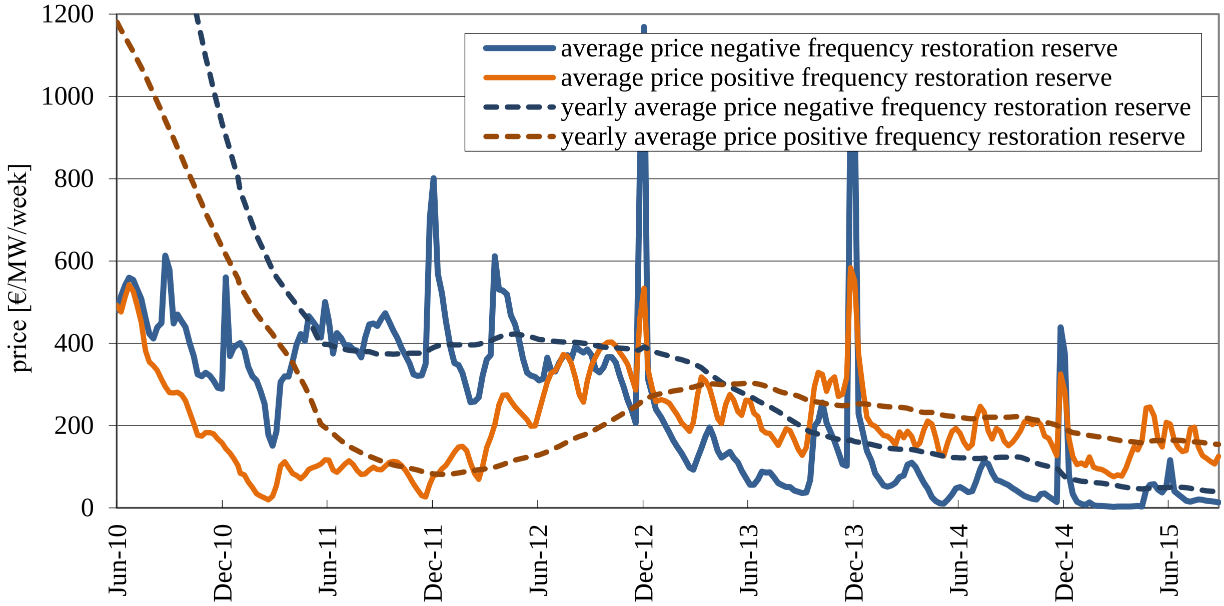


Figure 21 Prices for positive and negative FRR in Germany from 2010 to 2016. Data derived from (Regelleistung.net, 2017a)

After a bid got accepted in the power price allocation, it is also included in the work price merit order. In the work price merit order all bids are sorted again ascending by the energy price in €/MWh. In case the TSO need energy, the activation starts with the lowest price bid. That means the first bid has the highest and the last bid the lowest activation probability. The remuneration for the operator can be estimated multiplying the activation probability with the offered work price. The pay-as-bid work price tendering ensures a cost-effective allocation of the energy demand for the TSO. In the predominant case, the activation does not exceed 500 to 1000 MW which means that bids at the position of 2000 MW are seldom activated. Bidders with these positions speculate with extremely high prices for the activation of the complete merit order. This happens just a few times a year since the TSO can activate RR (minute reserve) if this is cost efficient. Therefore, it is also not reasonable to present the average offered work price in a separate figure.

Figure 22 and Figure 23 present the activation (black line) of balancing energy for the German Austrian balancing region for two exemplary weeks. The quantity of activation can differ strongly during days or weeks. For storage based bidders this needs to be considered since reservoirs or batteries are limited in the quantity of energy. To facilitate comparison, the sorted average activated energy per 15 min is visualized in the figures as well.

Since the intraday continuous market is often used to balance deviation before delivery, a correlation with the activation of FRR can be perceived; for example, after a significant positive activation over 10 to 20 min the intraday prices increase as well, because, the source of unbalance might need to buy the missing quantities on the intraday markets.

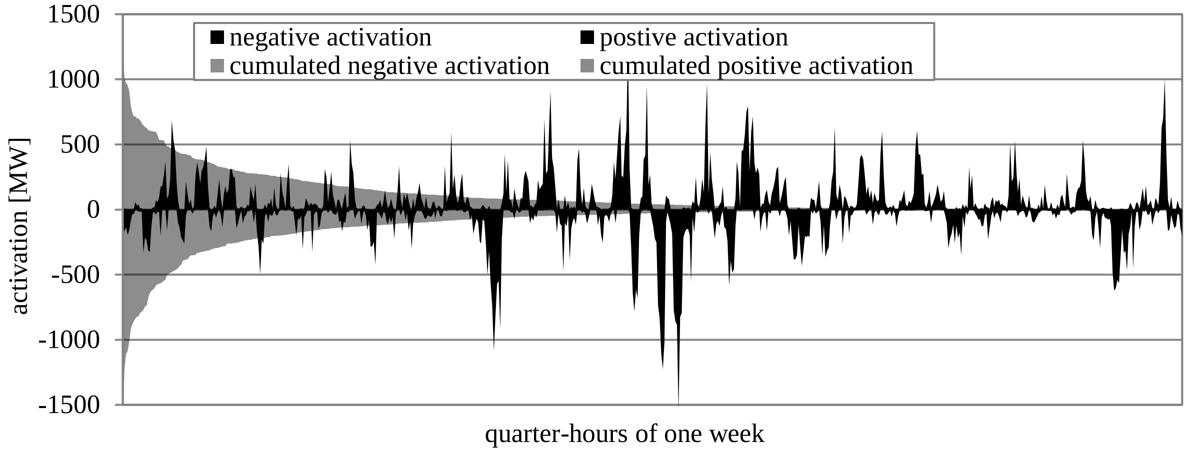


Figure 22 Positive and negative activated balancing demand for FRR for the German Austrian combined grid region in black and the sorted activation in grey. Both illustrated for the week from 2017-4-10 to 2017-4-16. Data retrieved from (Regelleistung.net, 2017a).

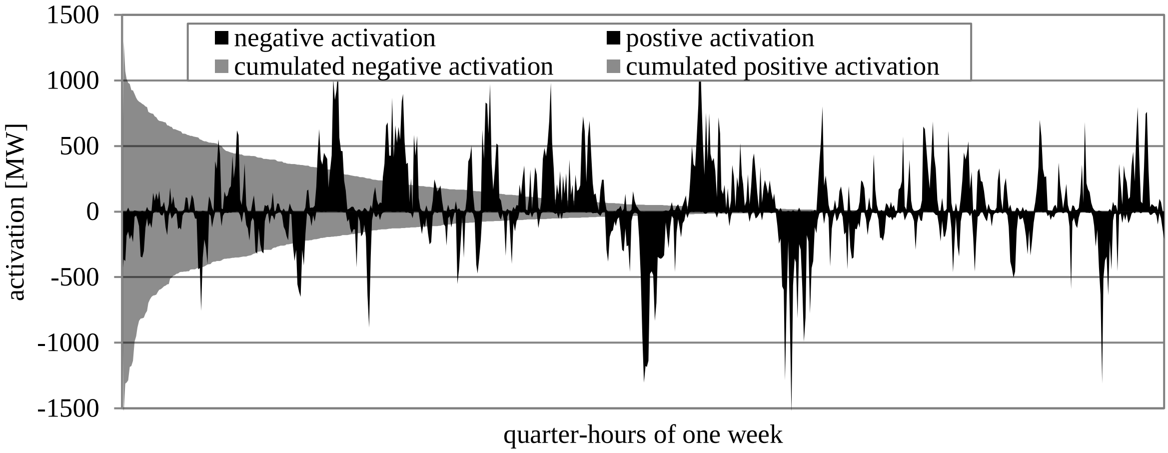


Figure 23 Positive and negative activated balancing demand for FRR for the German Austrian combined grid region in black and the sorted activation in grey. Both illustrated for the week from 2017-4-17 to 2017-4-23. Data retrieved from (Regelleistung.net, 2017a).

Beside the remuneration for delivery of balancing energy also the deviation from this planned and reported schedule is penalized by the TSO with the compensation energy price (Ausgleichsenergiepreis). The compensation energy price is based on the marginal price of the activated energy in the five-minute average. The high costs for deviations, strengthen the fulfilment of the planned schedule. The other way around; stabilizing the grid when producing more if to less energy is feed-in and producing less if too much energy is in the grid is remunerated with the compensation energy price. However, doing this actively is a legal grey area.

German Replacement Reserve

RR (Minutenreserve or Tertiärregelung) is organized in a common auction of the four TSOs in Germany since December 1st, 2006 and the prequalification is managed by the regional TSOs. After extensive consultation in 2011 the auctioning process rules were defined in BK6-10-099. In the daily auction, six positive and negative products are separated in time slots of four hours.

The minimum bid quantity is 5 MW (until 2011 10 MW) and the maximum quantity is limited to 25 MW blocks. Bids are always activated as a whole which is advantageous for operators with for example sizeable gas power plants. Apart from that, the 5 MW minimum quantity can be pooled with several prequalified smaller units. Also, the pooling of third party units within one balancing area is allowed. With the rules for more flexibilization and the pooling of capacity to reach the 5 MW level new players have been able to enter the market such as cogeneration units and emergency power generators.

The development of the tendering quantity over the last years can be seen in Figure 24. Due to the development of the intraday continuous market with quarter-hourly products and more important, the trading until 30 min before delivery across balancing energy zones and even until delivery within one balancing zone, promotes the possibility to balance demand at short notice. This has reduced the RR activation significantly (Regelleistung.net, 2017a). Therefore, in 2016 also the tendering quantity were decreased by the TSOs.

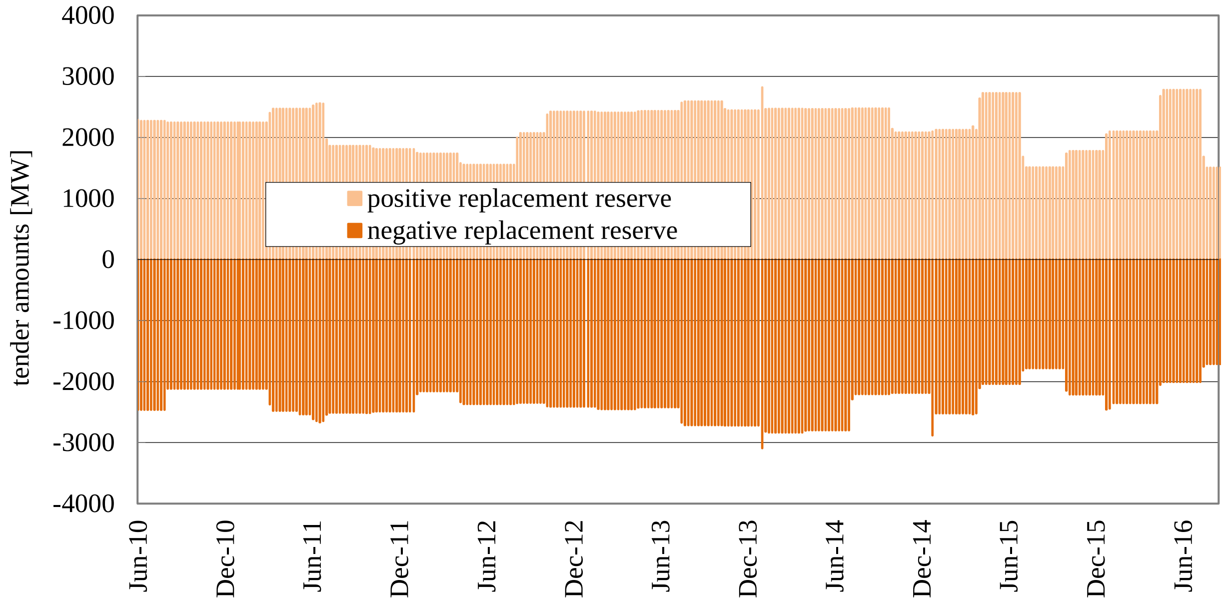


Figure 24 RR demand of the German TSO or the group of TSOs in which the German TSO is organized. Data derived from (Regelleistung.net, 2017a)

The activation of RR takes place with a lead time of 15 min. RR is not needed to stabilize the grids frequency but to balance too much or too less electricity production. Since the delivery of energy is the most important part of RR, it is, similar to FRR, also remunerated in a double auction.

In the first pay-as-bid pricing auction for the provision of power the TSOs sort all bids in a merit order and allocates the lowest power price bids until the tendering amount is filled. The last accepted bid sets the published marginal price and the volume weighted average of all accepted bids defines the average capacity price (orange and blue line), as presented in Figure 25. The rolling yearly average price for RR (orange and blue dotted line) presents the price erosion very clearly. With the flexibilization of the RR auction including shorter product lengths, lower minimum bids, production pooling and lower prequalification standards more and more bidders participated in the auction. The above described reduced demand but mainly the available thermal capacity that are pushed out of the energy only market due to the merit-order-effect reasons the price development.

After a bid is accepted due to the power price it is also included in the work price merit order. In the work price merit order all bids are sorted again ascending by the price in €/MWh. In case the TSOs need energy, the activation starts with the lowest priced bid. That means the first bid has the highest and the last bid the lowest activation probability. The final remuneration for the operator is the multiplication of the activation probability with the offered work price. The pay-as-bid work price tendering ensures a cost-effective allocation of the energy demand for the TSOs and has a positive effect on the grid charges for the end-consumer.

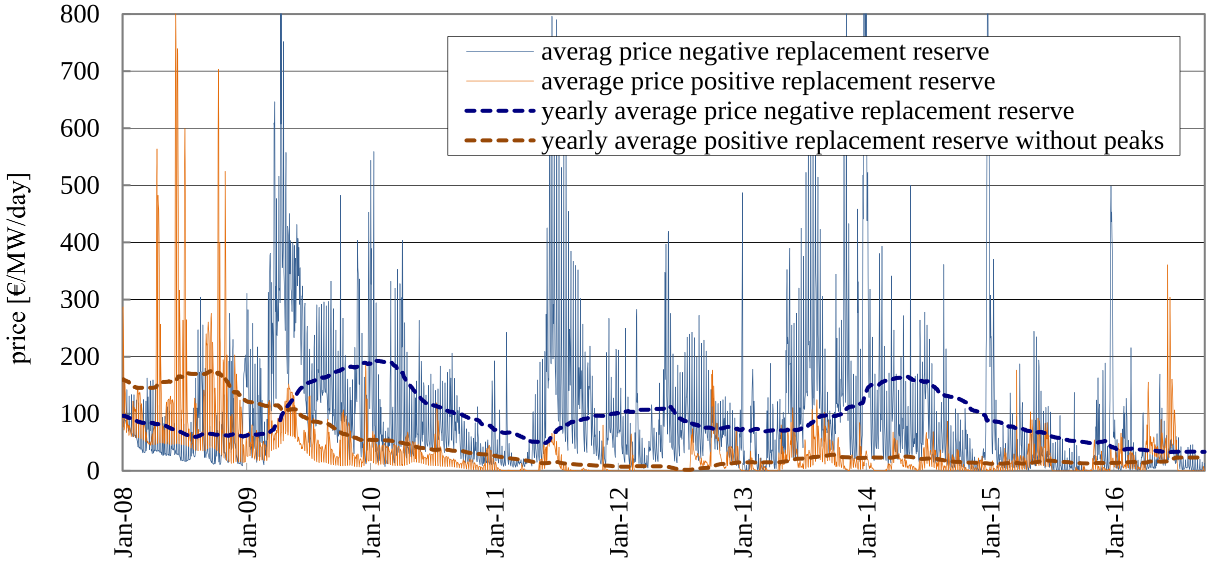


Figure 25 Prices for positive and negative RR in Germany from 2008 to 2016. Data derived from (Regelleistung.net, 2017a)

3. Hydropower Planning

After the introduction of short-term energy markets in chapter 4 this chapter provides all information needed on hydropower storages to solve the hydropower scheduling problem. Generally, hydropower storages are the most cost efficient as well as common electricity storage technology in the world that is sufficiently scalable in terms of power and work capacity, see Table 5. Whereas for example lithium-ion batteries are in the ascent due to e-mobility and its extensive use in electrical appliances, large scale energy storage over a long period of time is still difficult. In terms of optimization method and market structure, all approaches provided in this thesis to optimally utilize hydropower storages can be applied on other storage technologies as well. Nonetheless, hydropower plants, and surely other technologies as well, face characteristic technology related challenges that influence optimization and marketing. This is the reason why this chapter attends to the characteristics of the hydropower technology itself.

Table 5 Comparison of various electricity storage technologies. Source (Evans, Strezov, & Evans, 2012) and (Gaudard & Romerio, 2014)

storage technology	efficiency [%]	power [MW]	capital [$/kW]	capital [$/kWh]	self-discharge time [per day]
mechanical					
pumped hydropower	65-85	100-5000	600	100	very small
compressed air	50-89	3-400	800-2000	50-100	small
flywheel	93-95	0.25	350	5000	100 %
electrical/magnetical					
supercapacitor	90-95	0.3	300	2000	20-40 %
superconducting magnetic	95-98	0.1-10	300	10,000	10-15 %
thermal					
cryogenic energy	40-50	0.1-300	300	30	0.5-1 %
HT-TES	30-60	0-60		60	0.05-1 %
chemical					
battery	60-90	0-40	300-4000	400-2500	0.1-20 %

Chapter 3.1 starts with the different construction forms of hydropower plants. A special focus is on the distinction between reservoir systems with and without pumps. Characteristics of the plant layout, machine types, penstocks, water retaining system etc. are discussed as well. The relevant parameters such as prices, grid charges, inflows and technical restrictions are discussed in chapter 3.2. Thereafter, chapter 3.3 presents the most common model structure based on decomposition. Especially relevant for the transformation of the model results into reality are steering parameter which are subject of chapter 3.4.

3.1. Introduction

The demand for mechanic and later electric energy is prevalent since ancient times and hydropower has been a source to be used in many ways. The first significant commercial hydroelectric power plant was built at Niagara Falls in the year 1879. Rivers and lakes with natural inflows provided water to be released. Until today hydroelectric power is viewed as a substantial part for economic development without adding substantial amounts of carbon to the atmosphere (Howard, 2013). Since hydropower is manifold this chapter gives an overview on the most common construction types, especially pumped hydropower storages.

3.1.1. Construction Types

Every hydropower plant is based on a reservoir and a work water supply system as well as machines to generate electricity, see Figure 26. The water-retaining structure can be of natural origin such as lakes or rivers or constructed using a dam. Reservoirs are normally filled directly with natural inflows or indirectly via tunnels and water catchments from rivers or glacier melting water. Reservoirs connected to pump machines can be additionally filled with water from lower reservoirs. Storages need to be distinguished from run-of-river plants which normally have a very small reservoir that is mainly used to increase the water head. Nevertheless, many run-of-river plants can vary the filling level (Schwellbetrieb) for peak power production and to increase turbine efficiency; the transition to hydropower storages is then fluent.

Figure 26 Penstock and machine house of the Walchensee power plant and the lower reservoir Kochelsee in southern Germany constructed in 1924 with an installed peak power of 124 MW

The work water supply system is a penstock connecting reservoirs and machines, as for example seen in Figure 27. The distance between the upper reservoir and the beginning of the slope often need to be bridged with a tunneled water route and is locked with a surge chamber or water lock (Wasserschloß). A surge chamber absorbs sudden rises of pressure due to rapid changes in water velocity. When the load increases, additional supply of water is provided, if the load decreases, the water moves backwards and gets stored. The higher the distance between the storage reservoir and the machines in the power house the more sizeable the surge tank. The penstock is responsible for a significant share of the efficiency losses in hydropower storages due to friction.

Figure 27 Penstock under construction for the Obervermuntwerk II in alpine Austria with a planned peak power of 360 MW and a flow through of 160 m³/s

The energy that can be retrieved in the machine from the water depends on the pressure in the penstock, i. e. the water column. In the turbine, the potential energy is transformed into rotational kinetic energy and further into electric energy in the generator. Afterwards the water flows into the lower reservoir or a river system. The machines are either located above the ground in a machine house or underground in a cavern in the mountain. Machines for pumping either need to be located below the water level of the lower reservoir to ensure pre-compression or the pre-compression is provided with an additional pump. The most common turbine systems, in this order, are Francis, Pelton (see Figure 28) and Kaplan machines. Francis machines are widely used in pumped hydropower storages as well as run-of-river plants and are suitable for medium water heads and medium flow though rates. Francis turbines are for example used in the famous three gorges dam and the Niagara fall plants. Pelton machines are specified for systems with high water heads and low flow through rates. The Kaplan turbines are determined for run-of-river power plants handling low water heads and sizeable flow through rates. The efficiency of the hydropower machine and the generator provoke the predominate share of the efficiency losses in the whole system.

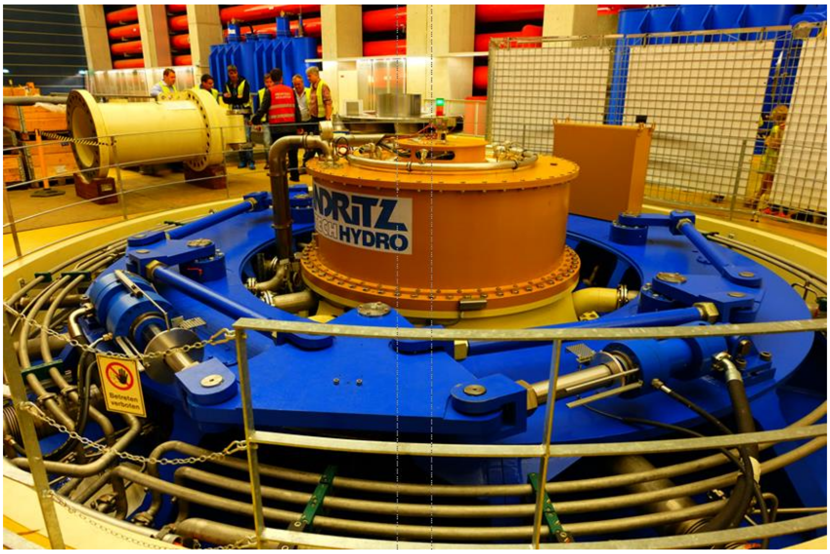


Figure 28 Six nozzles Pelton machine with a peak power of 150 MW, a flow through of 25.3 m³ and a water velocity at the injectors of 450 km/h as a part of the Kops II power plant constructed in 2008

Hydropower reservoirs can be classified by their reservoir size, machine type, water head and the difference in height between upper and lower reservoir. Furthermore, a hydropower plant is often linked to other power plants and is part of a bigger hydrological system. Below, storages without pumps, storages with pumps and various reservoir sizes will be introduced.

Table 6 presents an overview on how the different types of systems can be used on energy only and balancing markets as well as for grid support. In the first row, the plants are separated into systems with and without pumps. The second row distinguishes between run-of-river, pump storages with small reservoirs and seasonal storages with sizeable reservoirs. Pump storages are the most flexible devices and therefore deliver mostly peak and balancing power. But in comparison to run-of-river, it need to be considered that not all hydropower storages necessarily deliver renewable energy. Pumped hydropower storages are dependent on the energy mix consumed for pumping. Since construction types of hydropower storages are manifold, this classification should help the reader to get an overview and is not exhaustive.

Table 6: Classification of hydropower plants

<table>
<tr><th colspan="3" rowspan="2">machine:
reservoir size:
services:</th><th colspan="2">turbines</th><th colspan="2">turbines and pumps</th></tr>
<tr><th>run-of-river</th><th>large storage</th><th>large storage</th><th>small storage</th></tr>
<tr><td rowspan="3">energy only</td><td colspan="2">base load</td><td rowspan="2">+</td><td>+/-</td><td>+/-</td><td>-</td></tr>
<tr><td colspan="2">renewable/inflow</td><td>+</td><td colspan="2">+/-</td></tr>
<tr><td colspan="2">flexible</td><td rowspan="7">-</td><td colspan="3" rowspan="10">+</td></tr>
<tr><td rowspan="6">balancing reserves</td><td rowspan="2">frequency containment</td><td>positive</td></tr>
<tr><td>negative</td></tr>
<tr><td rowspan="2">frequency restoration</td><td>positive</td></tr>
<tr><td>negative</td></tr>
<tr><td rowspan="2">replacement</td><td>positive</td></tr>
<tr><td>negative</td></tr>
<tr><td rowspan="3">grid support</td><td colspan="2">black start</td><td rowspan="3">+</td></tr>
<tr><td colspan="2">active losses compensation</td></tr>
<tr><td colspan="2">voltage support</td></tr>
</table>

3.1.2. Pumped Hydropower Storages

With the beginning of the 20th century, technologies such as nuclear and coal fired power plants came up and delivered significant shares of the growing hunger for energy during the industrialization in Europe and North Amerika. Nevertheless, these thermal power plants were constructed to run as base load power plants around the clock. Therefore, daily pumped storages were constructed storing energy at night when demand is low and delivering in times of high demand during the day.

Daily pumped hydropower storages have small reservoirs and large machines in comparison to the reservoir size. For a daily pumped storage, the charge cycle is normally one day but on weekends this could also be a whole weekend if the prices do not allow a complete charge cycle in one day. A charge cycle can be defined as the process starting from one specific state of charge, reaching complete discharge and full charge and ending with the original state of charge. A second type of storage, so called seasonal storages, were installed as well, mainly in the mountains with large reservoirs and in some cases also with pumps. Theses power plants stored melting water, rain or pumped water over the course of months to level out fluctuating demand during the year. For example, higher demand in winter for heating in northern countries.

Generally, pumped hydropower storages are used as peak load power plants and for the provision of grid stabilization products such as balancing power. The average overall cycle efficiency of pumped hydropower storages is about 70-80%; mainly resulting from losses in the machines, the penstock and the generator (Wagner & Mathur, 2011). The combination of reservoir, water head and machine size determines the hydropower plant as daily, weekly or seasonal storages.

A sizable storage, as for example the Kops lake in Vorarlberg, see Figure 29, with a reservoir size of 42 million m³ and a water head of about 800 m, was used as seasonal storage with natural inflows. In the summer the water has been stored for peak demand during the winter. Since the construction in the year

1969 the power plant, Kopswerk I, is equipped with one machine with a peak power capacity of 247 MW. After a refurbishment in the years 2004 till 2008 further 525 MW of peak power (Kopswerk II) were installed. With about 127.45 GWh energy content the complete discharge before the refurbishment took about 512 hours. After the refurbishment, this could be done in 165 hours. The power plant turned from a seasonal storage to a weekly storage.

Figure 29 Kops lake in the Austrian Alps, 1809 meters above sea level with a maximum reservoir filling level of 42 million m³ water which equals an energy content of 127.45 GWh

3.2. Relevant Parameters

The relevant planning parameters embody input data that is required for the hydropower scheduling optimization. The results of the optimization hinges on this input. Relevant planning parameters are for example sales prices, grid charges and inflows. A significant share of input data is subject to discussion due to uncertainty, motion, market dynamics or political decisions. In this part, the different input parameters and the respective challenges are introduced, aiming to process the available data for an optimal use. Prices are discussed in chapter 3.2.1, grid charges in 3.2.2, inflows in 3.2.3, as well as technical restriction in 3.2.4.

3.2.1. Electricity Prices

The prices for electricity are one of the most important input factors of the optimization, because they substantially determine the income that can be generated with a power plant. The electricity price itself depends on numerous parameter such as demand, variable RES infeed, the power plant merit order, plant availabilities, temperature, gross border capacities etc. Furthermore, all prices are linked to a date of expiry and a product. Not all products can be traded at all time and at full liquidity.

For the European countries, the hourly day-ahead market is the lead market which is therefore most often used in pumped hydropower scheduling optimizations. Nevertheless, a consideration of additional markets can be of interest if the time structure or the remuneration system is different as for example in the quarter-hourly intraday market in Germany with a shorter time resolution and continuous trading. Whereas the average prices are similar, i.e. arbitrage free, this market provides significant higher optionalities in terms of fluctuating prices.

Every market needs to be analyzed on its additional value for the power plant. Whereas base load power plants could completely rely on hourly products a highly flexible power plant, such as pumped hydropower storages, should also consider the quarter-hourly and the balancing power markets. In this thesis, a multi-market optimization approach is suggested since an optimization model should reflect all the relevant markets on which the power plant is traded to realistically determine the trading decision. A further argument to include additional markets into the optimization, is that power plants represent real options, i.e. that even a power plant that is not in the money in the first market has an optional value on the next market due to unforeseen price changes; it has the right but not the obligation to be switched on. Vice versa, if it is in the money on the first market it has the right but not the obligation to be switched off on the second. In the long term, this is referred to hedge optimization. In the short-term it will be referred to as the value of optionality.

If the relevant markets are identified a price forecast need to be generated. Either a point prognosis, a so-called price forward curve, or a price distribution with respective probabilities is created. To make use of the latter either scenario based or stochastic optimization is needed which makes the optimization significantly more complex and the curse of dimensions gets to a problem, see chapter 6.

Since price forecasting is not subject of this thesis the interested reader is referred to Weron (2014b) who gives an up-to-date overview on relevant reviews and survey publications. Several books on electricity price forecasting exist of which the following can be suggested:

- Shahidehpour, Yamin, and Li (2002) presents in chapter 3, pp. 57-113 the basics of electricity price forecasting including the price formation, volatility and exogenous variables. He adds a neuronal network based price forecasting module and gives a performance evaluation.
- Weron (2006a) discusses in chapter 4, pp.101-155 a wide range of modeling approaches and analyses the practical application of the four statistical methods for day-ahead forecasting: ARMA-type, ARMAX, GARCH-type and regime-switching. Further, he discusses quantitative stochastic models for derivatives pricing, such as jump-diffusion models and Markov regime-switching.
- Zareipour (2008) provides an overview in chapter 3-4 on the pages 52-105 over linear time series model such as ARIMA, ARX and ARMAX and non-linear models as for example regression splines and neural networks and tests these methods forecasting the hourly electricity prices for the Ontario power market.

3.2.2. Grid Charges

For both feed-in and feed-out, grid charges have to be paid. The charge is normally paid based on a power and a work price for every kWh feed-in or -out. Strong variations exist between different voltage levels and counties.

For pumped hydropower storages it is further the question on whether grid charges have to be paid twice, first for the electricity consumed from the grid to be stored and second the electricity that is feed-in again at a later point in time. This hinges on the classification of storages as electricity end-consumer and has a significant impact on the profitability of pumped hydropower storages. Normal electricity consumer are households or companies, paying for the energy itself, taxes, grid charges and sometimes a RES support fee, as exemplary presented for a German customer in Figure 34.

A final consumer is defined by §3 Nr. 25 EnWG and §5 Nr. 24 EEG as a natural or legal person that consumes energy. Whereas in Austria and Germany pumped hydropower storages are classified as final consumers, in Switzerland they are not (Hildmann, Priker, Schaffner, Sprend, & Ulbig, 2014, p. 16). From a technical point of view the classification as consumers makes sense, whereas economically, the grid charge is a double burden for storages in general. These different legal situations can cause distortion especially in terms of investment incentives. Involved companies are very cautious dealing with long-term investments because regulatory, political and market changes have a major influence on the profitability of pumped hydropower storages; even shutdowns of existing facilities are under discussion due to a shrinking value of flexibility in the current market design (Hildmann et al., 2014, p. 18).

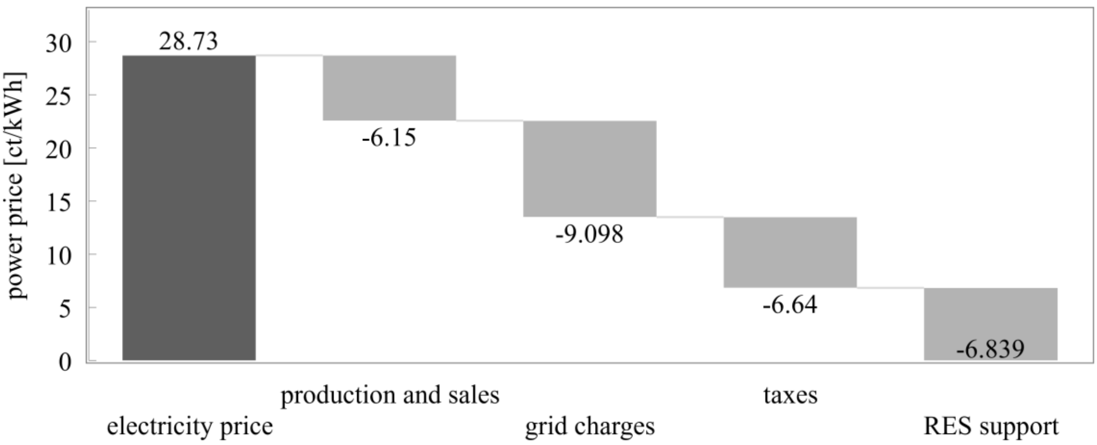


Figure 30 Electricity price for an average household in Germany with 3500 kWh consumption in May 2016. Data derived from (BDEW, 2016)

In Germany, storages have to pay grid charges and concessions. RES support must not be paid by storage operators (§ 9 Nr. 2 StromStG i.V.m. § 12 Abs. 1 Nr. 2 StromStV). Whereas the normal grid charge would make energy storage unprofitable, many operators sue for reduced grid charges as a published list of lawsuits, concerning pumped hydropower storages operators with the object to be released from grid charges shows (BMWi, 2017b). Furthermore, many exceptions exist e.g. for new build pumped hydropower storages (§ 118 Abs. 6 S. 1 und 3 EnWG) or extended storage plants which are used for grid stabilization (§ 118 Abs. 6 S. 2 und 4 EnWG). Additionally, most storage operators decide to benefit from reduced network charges for accepting an a-typical network gird usage (§ 19 Abs. 2 S. 1 StromNEV). That means that the grid operator defines some critical hours over the course of a day in which the storage operator is prohibited to use pumps to reduce the maximum grid load and in the long-term avoid grid extensions. In return, the operator receives a reduced grid charge tariff.

With the passing of the new Electricity Market Act (Strommarktgesetz) on June 23, 2016 including the adjustments of the Committee on Economic and Energy Affairs (Ausschuss für Wirtschaft und Energie) (BT-Drs. 18/8915) the grid charge raising systematic (Netzentgeldsystematik) for storages in the German energy market has been changed. It relates to the special forms of grid utilization regulated in § 19 StromNEV. Through § 19 Abs. 4 StromNEV, the network operators have to offer an individual network charge to final consumer who extract electricity from the grid exclusively for storage and subsequently feed the recovered electricity back into the grid at a later point in time. This has the effect that only the difference between the stored and generated energy, i. e the storage losses, are charged, since only these amounts are permanently consumed from the grid. The argumentation is based on the assumption that final end consumers pay grid charges for every MWh consumed. Therefore, the new regulations avoid a double taxation of stored and re-fed-in electricity quantities.

Additionally, challenges based on operating regulations and construction permits are faced by nearly every company that wants to construct and operate power plants. Such permits are implemented in terms of environmental or safety issues. Hydropower storages face regulations such as: the amount of water released to rivers at a time, spillage, reservoir minimum or maximum filling levels for tourism or environmental aspects, minimum river levels etc. Many hydropower reservoirs are also restricted in terms of flood control or irrigation. Special for many hydropower storages is a charge for the natural inflows used in the power plant to generate energy. This so called “water penny” is normally paid to the local authorities. Thermal power plants that use river cooling normally pay a charge for the water as well.

3.2.3. Water Inflows

Hydropower reservoirs can be filled with water form natural inflows and/or by pumping up water from a lower reservoir. Inflows into reservoirs depend on direct or indirect precipitation. Often the water is transported via rivers and water catchments to the reservoir itself. The sum of inflows determines the quantity of energy that can be processed. The location of the reservoir is crucial for the estimation of the natural inflows. Mainly the climatic region and the height above sea level determine the inflow characteristics.

Reservoirs in high mountain ranges as the Alps are made to catch inflows during a very short period of time in the summer, since precipitation is retained as snow and ice which melts if the temperature arises. The exemplary alpine reservoir in Figure 31 illustrates the primary inflow between May and September. This means that not just the short-term weather conditions such as precipitation and temperature are relevant to estimate inflows, but also the quantity of and the water content in the snow. Reservoirs in low mountain ranges of Germany can rely on relatively constant reservoir inflows as can be seen in Figure 31. This is mainly because the precipitation is comparatively even distributed over the year and also in winter the temperatures sometimes rise above zero so that snow can melt. Negative inflows can result from seepage or evaporation. Furthermore, in countries such as Brazil, Switzerland or Norway, hydropower has a predominant share in energy production and try and wet years need to be balanced as well. Therefore, future inflows are estimated with long-term scenario trees and probability distributions that are solved with scenario or stochastic optimizations, resulting in higher complexity.

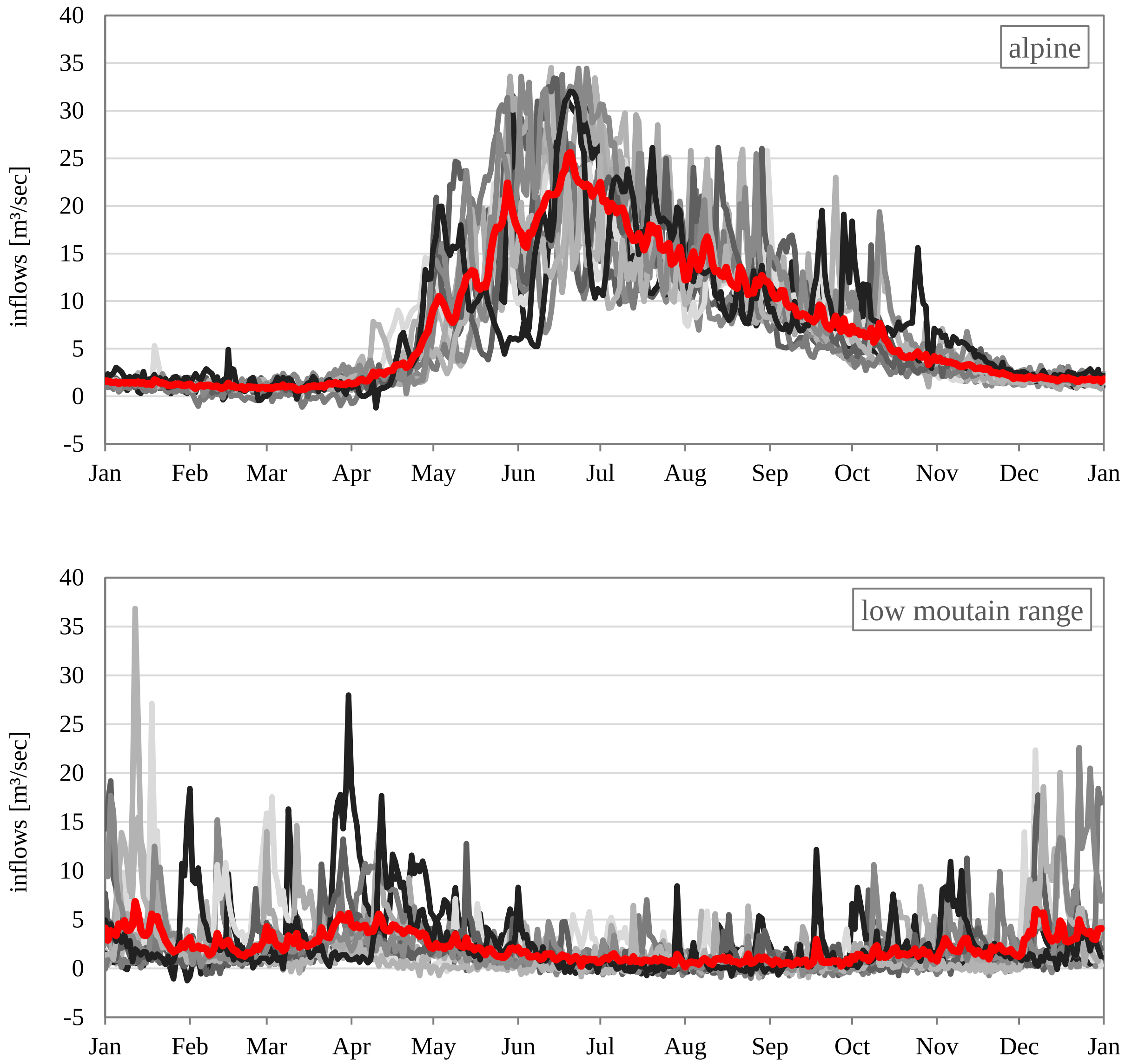


Figure 31 Constructed water inflow variations of an exemplary hydropower reservoir in alpine Austria and in low-mountain-range Germany, the red line depicts the average. Scale and fluctuations do not reflect real data. Source: Braun (2015a)

For pumped hydropower storages, in many cases, just the consideration of average inflows is sufficient and the additional complexity of stochastic inflows can be economized. With no pumps installed or the quantity and the time when the inflows come at risk a stochastic estimation is strongly recommended.

A precise inflow forecast of the near future is needed if the dispatch schedule is influenced by short-term inflows. For example, if water need to be released beforehand to catch all forecasted inflows to avoid spillage or secure flood protection. This is especially the case for small reservoirs or reservoirs at their limits. Additionally, a short-term forecast could be of interest for environmental or contractual subjects.

3.2.4. Technical Restrictions

Beside market prices, grid charges and inflows, the fourth important input for hydropower scheduling are the plants' technical restrictions. These challenges often result from complex physical structures that need to be considered in mathematical models. Models are simplified replications of the real-world and many subtleties are modeled insufficient. The challenge is further to build a model that describes the reality well enough to provide sufficient real-life decision support for the optimized system and is still mathematical tractable. Technical restrictions may add nonlinear, mixed integer, stochastic or ulterior complexity to the mathematical models. Such optimization challenges are discussed for example in Belsnes, Gjengedal, and Fosso (2005, p. 3).

- Turbine efficiencies, turbine outflow, gate opening times, spillage or flow delays between reservoirs behave nonlinear and induce nonconvexities (Diniz & Maceira, 2008). For instance, the turbine efficiency hinges on the rate between water flow and energy production. Full-load and partial-load operation can be distinguished. A Pelton machine can provide an efficiency of 80% with a flow through rate of just 20%; a Francis machine has less than 40% efficiency at this flow through rate (Wagner & Mathur, 2011). The maximum efficiency of a Pelton turbine is about 90 %, Francis or Kaplan turbines reach about 95 % at an optimal flow through of about 75 % (Wagner & Mathur, 2011). Often-uttered use cases are partial-load situations, for example due to balancing energy provision. This means, the level of modeled details may depend on the considered energy markets as well.
- Standstill and start-up costs, start-up and shut-down times, forbidden operating zones, minimal flow through rates of machines, retrenched optimization zones, etc. lead to mixed integer problems. In comparison to turbines that are highly flexible in the amount of power produced, pumps can normally just be switched on and off (Wagner & Mathur, 2011). To reach a specific negative schedule a hydraulic short circuit is used for which the pump operates at full power and a turbine at a flexible production level connected to the same penstock. The latest engineering developments introduced variable speed pumps that extend the operating zones and improved the efficiency by use of a motor-generator with frequency converter, as used for example in Hongrin Léman in Switzerland in 2011 (Voith Hydro, 2006).
- Complex topologies such as dependencies between pumps and turbines, between upper and lower reservoirs, large systems of reservoirs, rivers, water catchments and power stations increase the size of optimization problems. Depending of the optimization method the optimization problem can increase exponentially with the number of reservoirs. Furthermore, many hydropower plants are build and operated by a consortium of companies due to high

investment costs and significant risks. Such constructions with more than one shareholder and operator are opposed to further challenges.

Which of these parameters is considered should be revised for every single power plant optimized. On the one hand, some parameters might be neglected in the optimization if their influence is small and understood, on the other hand many parameters surely need to be modeled in detail to obtain the required results.

3.3. Decision Problem Structure

A general approach is to build up a model that is based on the physical layout of the power plant portfolio, estimate prices and inflows to optimize this against the energy markets. Such a model can be rather simple or highly sophisticated. This dependents on: the length of the optimization period, how many technical restrictions are considered as well as how detailed prices and inflows are regarded. The overall bidding problem is generally so complex that it is not solved at once. Therefore, the problem is decomposed and simplified. The following approaches can be catalogued:

- separation in time: long-, mid- and short-term
- separation by power plant: seasonal storage, daily storage, systems with pumps, hydro-thermal portfolios
- separation by electricity market: futures, day-ahead, intraday, balancing

The most common decomposition approach is to separate the optimization in time, so-called time-depended approaches (Fosso & Belsnes, 2004). These are explained in 3.3.1 and 3.3.2. A separation by power plant can make sense if optimization period and electricity markets differ. For example, seasonal hydropower storages are optimized already years or months ahead to be bid on futures and day-ahead markets. Daily pumped hydropower storages are optimized just days or weeks ahead with a focus on the short-term markets such as day-ahead and intraday markets. A separation by market is valuable if several different kinds of markets are considered independently. Generally, the decomposition is often a mix of various approaches.

3.3.1. Decomposition

With the decomposition in time four generally independent problems can be created, each addressing one part of the hydropower optimization time line, see Figure 32.

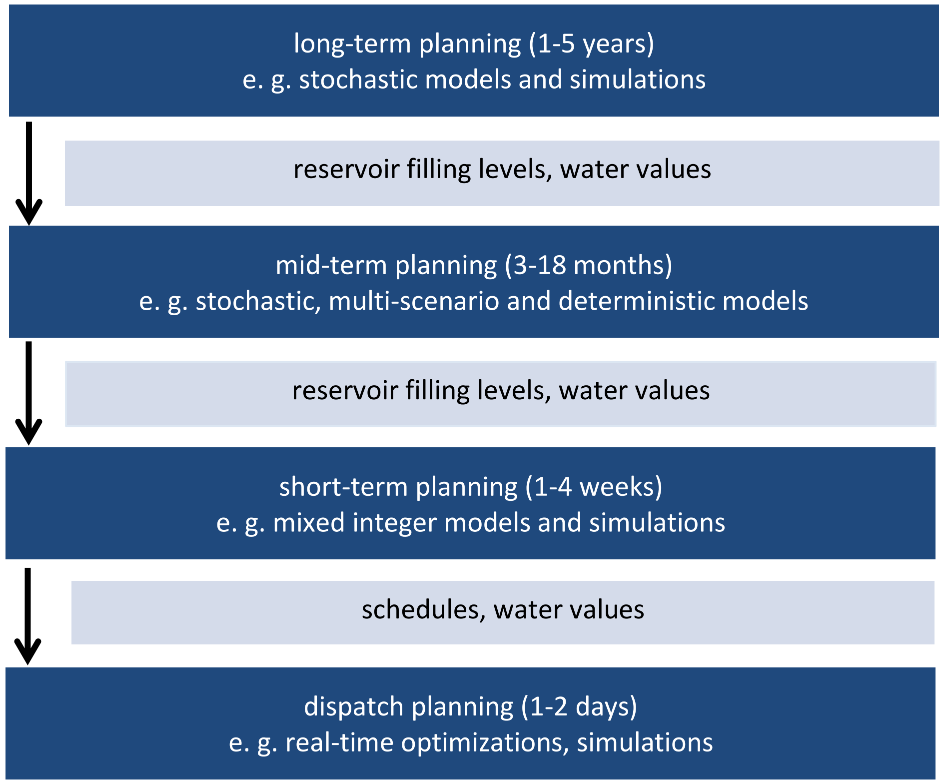


Figure 32 Decomposition in time of the hydropower scheduling problem, source: (Fosso & Belsnes, 2004)

Below these four quasi separate optimization problems are introduced and discussed along the timeline beginning with the long-term and ending with the short-term.

- The long-term optimization is built to incorporate the stochastic of the future. To ensure solvability the respective systems are aggregated with broad time steps. The goal of long-term models is resource scheduling, expansion planning, price forecasting or risk hedging. Generally long-term models provide boundary conditions to mid-term models using water values or end reservoir filling levels (Fosso & Belsnes, 2004, p. 1323). The predominant way of optimizing long-term models is based on stochastic approaches whereas the level of detail is comparatively low. Nevertheless, stochastic models do not always show the best results as the comparison of the stochastic NEWAVE and the deterministic ODIN model for the long term hydropower scheduling of the interconnected Brazilian system shows (Zambelli, Soares Filho, Toscano, Santos, & Silva Filho, 2011).
- The mid-term optimization is responsible for the transition from long- to short-term. Therefore, it represents a more detailed structure and a finer time resolution as in the long-term model to provide more precise boundaries. The mid-term model is used to generate water values or target reservoir filling levels for the short-term. Belsnes and Fosso (2004) state that the Norwegian mid-term models have the same time increment resolution as the long-term model, normally weeks, whereas the physical topology is already modelled as detailed as in the short-term model.

- The classic short-term model uses the water values or reservoir filling levels of the mid-term model to optimize the short-term hydropower scheduling problem, finding the maximum return for selling the energy on the respective short-term markets. If no markets are present, the income maximization is substituted by a cost minimization while covering the load. In liberalized systems, the goal is a decision support for day-ahead, balancing and intraday markets. Therefore, the optimal time increment should equal the time resolution of the offered products on the markets and the result of the optimization is an optimal unit commitment plan, water values and reservoir filling levels. Often a successive linear programming approach is suggested that is supposed to efficiently solve large systems without incorporating non-linear aspects such as machine efficiencies (Fosso & Belsnes, 2004). Generally, the more efficient the model the more details can be considered.
- Unit commitment planning models can be used for real-time optimization of the already existing schedules. The objective can be either to find a cost-minimal production schedule matching the already sold obligations day-ahead with production capacities assuming that no energy can be traded anymore or a maximization of profit considering the relevant intraday price changes. Due to the highest level of detail and regular optimizations during the day the common approaches in power industry and literature are mostly based on linear and mixed integer linear optimization (Billinton & Fotuhi-Firuzabad, 2000).

Generally, such an extensive decomposition is not necessary for every power plant system. How a problem is decomposed depends on the setting. For example, long-term optimization is just necessary, if input factors from three to five years ahead influence the dispatch here and now. This is the case for huge reservoir systems in Brazil or Norway that store inflows over years but not for pumped hydropower storages in Central Europe. If no intraday market exists and the load is fix, a regular intraday dispatch planning is also not necessary. If the problems are not extremely complex it is also common to skip the mid-term planning model. This work focusses on the short- and very short-term optimization since these sections underwent the strongest transformation process due to the Energiewende as outlined in the motivation.

3.3.2. Composition

While taking a decision to decompose something there should also be an idea of how to compose the problem again and how to achieve a proper coupling between models. Generally, a coupling in volume, e. g. reservoir filling levels and a coupling in costs, e. g. water values, can be distinguished. Both methods have advantages and disadvantages as listed in Table 7 and are widely used in literature and practice.

The important point for the exchange between models along the timeline is that sufficient information is provided for the interfaces and that no important information get lost. Long-term models use aggregations in topology and time resolution to limit the size of the model. An exemplary complexity reduction could mean that upper reservoirs are aggregated to one reservoir and prices are not given hourly but monthly. Inevitable, this leads to consistency problems between models with different levels of detail. Especially technical and mathematical principles hamper with these details (Fosso & Belsnes, 2004, p. 1324).

Table 7 Advantages and disadvantages of volume and cost coupling in hydropower planning, partly derived from (Fosso & Belsnes, 2004, p. 1324).

	advantages	disadvantages
volume coupling, filling level for all reservoirs at the end of a period	• flexibility in choosing mathematical mid- and short-term methods	• inconsistency in cascaded systems, because downstream reservoir levels depend on upstream water release • inflexible in terms of moving discharge within different periods • no price dependence
cost coupling, the resource water is priced with a marginal cost function, i. e. water values	• flexibility in scheduling process • expected value storing one additional unit of water in the reservoir • water release is a function of the market price (during the bidding period)	• transferability of marginals on the whole reservoir • cost for each reservoir is just an approximation • marginal cost function depends on filling levels of all other reservoirs in the system

A disadvantage of cost coupling is the complexity due to the dependence of the water value of one single reservoir on the filling levels of all other reservoirs in the system. For one reservoir, the future expected income can be plotted as a function of the reservoir filling level, see Figure 33. This is a concave function because the higher the reservoir filling level the higher the profit, whereas the marginal utility diminishes due to the increasing risk of spillage (Fosso & Belsnes, 2004, p. 1324). The slope of this function is the change of the expected profit due to a marginal change in reservoir content. For the various filling levels this derivative is denoted as the marginal water value. Solving this problem with a stochastic approximation approach (e. g. SDDP), the non-linear function is approximated and described by a set of linear functions. This example is one dimensional because just one reservoir is considered. With each additional reservoir one dimension is added.

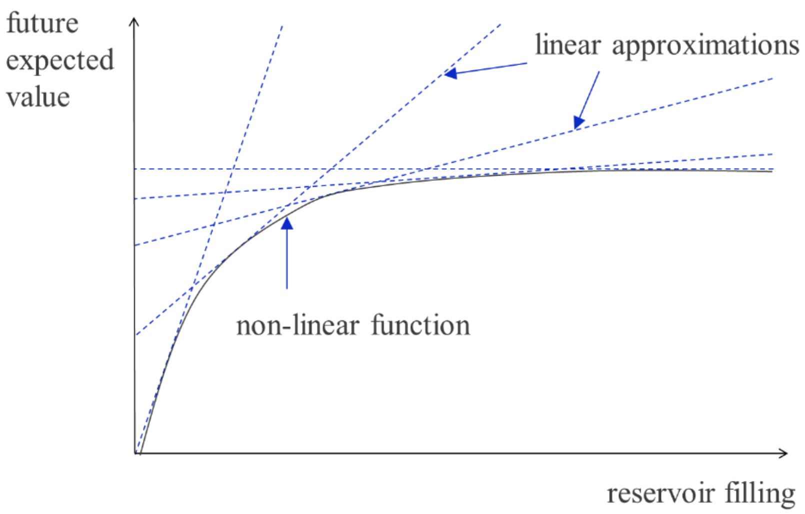


Figure 33 Exemplary calculation of water value as a function of the reservoir filling level using cutting planes approximations of the future expected value function. The SDDP method was introduced by Pereira and Pinto (1985)

3.4. Steering Parameters

The steering logic, water management or dispatch control describes the critical point of transferring the results of optimization models into a real-life dispatch. From a hydropower storage operator perspective, the objective is to achieve an optimal use of existing resources. An optimal use of a plant can mean on the one hand a cost and maintenance minimizing dispatch and on the other hand a profit maximization using the various energy markets. Whereas the focus in this work is on the latter both ways need a suitable transformation of the model outputs to a real-life dispatch. Unfortunately, the steering of the actual dispatch is often neglected in literature. Whereas case studies are normally presented, there is a big difference between calculating an optimal dispatch and realizing this dispatch in real-world due to unforeseeable changes and a higher complexity in practice. The strong share of literature is focused on to enhance the maths, tweaken computer run-time or elongate time periods rather than improving the usability of model results in real-life. This might be explained with the distance between research and practice, the jump in complexity when applying models in practice or due to unpublished case studies including confidential information. Especially the latter can be questioned since joint work often guides progress for everyone. Nevertheless, this work tries to include the steering and back testing of real-world problems.

Steering of hydropower storages is an important factor for the overall success. One crucial point is that models are abstractions of the real-world and inflows, prices, topology and technical restrictions are just mapped as good as possible. The dispatch need to be robust and resilient against unforeseeable changes in these parameters or effects such as the activation of balancing power or outages. Furthermore, most hydropower storages are not steered and traded completely automatically, which means that the human factor should not be underestimated.

Short-term optimization problems for steering the hydropower dispatch are solved repetitively with a specific time interval of for example one day or one week. For the respective interval until the next calculation the results are used as decision support whereas changes in the input parameters are ignored until the next calculation. Due to runtime infeasibilities the easiest approach, to compute the model when changes occur, is seldom possible. Therefore, the steering need to be robust enough for small and medium changes in the underlying data or includes a stochastic factor that considers the probability of such changes. Below an overview is given on the solutions provided for the different optimization methods.

The significant number of available optimization methodologies provide a wide field of steering possibilities for pumped hydropower storages. The most common steering parameters are dispatch schedules, end reservoir filling levels, historic costs for pumping and water values.

- An optimal dispatch schedule is a direct result of all deterministic calculations or can be received from stochastic optimizations for different price realizations. The application of schedule based steering is limited to fields where the calculated results equal the real-world dispatch. This could be the case for a predefined demand that is satisfied. In liberalized power markets with unexpected conditions or price changes a fixed deterministic schedule is most of the time too rigid.
- Target reservoir filling levels can be set within a day, week or even a year. In a first step the maximum number of hours the power plant can produce or generate until the target reservoir filling level is reached need to be calculated. In a second step, the most lucrative hours for the dispatch are selected. This approach has been widely used to steer daily pumped hydropower storages. The main disadvantage is the inflexibility and the reliance on inflow or price forecast changes. About 10 years ago, in Germany, the load profile was met by nuclear and coal powered base load production and induced hydropower storage pumping and storing during the night as well as generation during the demand peaks at noon and in the evening as so called peak load power. The steering of these pumped hydropower storages with a reservoir filling level of 100% in the morning and 0% in the late evening was simple and feasible that time. Nevertheless, todays German Energiewende with various switches between pumping and generating mode as well as constantly changing price profiles due to significant shares of variable RES feed-in make the target reservoir filling level steering very difficult.
- A seldom approach concentrates on historic pump costs. The water in the upper reservoir induced costs for pumping. Therefore, the value of the water in the upper reservoir can be calculated as the quantity pumped to the upper reservoir times the respective historic price. To take the decision on when to release the water again, the dispatch prices need to be higher than the value of the stored water based on the pumping prices. Nevertheless, this valuation lacks to adopt to changes in price level. For example, after the reservoir is filled the prices decrease to a lower average level at infimum. The water will never be used again, although, spreads could be realized on the new price level.
- The most common method to provide a robust power plant dispatch steering are water values which are sometimes referred as shadow prices, marginals, incremental or opportunity costs. The next chapter 3.4.1 addresses the calculation and characteristics of water values in detail.

3.4.1. Water Values

Water values are widely used, heterogeneous steering parameters that describe the value of the water in a reservoir. The water value gives a dispatcher the chance to compare the present state with the future. If the yield in the present is higher than the water value, the water should be released and energy produced. Otherwise the water is better saved for the future. Such steering parameters are similar to the hand-over parameters between long-, mid- and short-term models. The water values calculation approaches differ within the used mathematical optimization methods.

For deterministic LP and MILP the water values can be retrieved from the dual variables of the reservoir filing level equations. This approach is mathematically speaking a sensitivity calculation. The filling levels describe the resources in the reservoirs. A marginal change of the resources, the reservoir filling level constraints, corresponds to a change in the overall yield in the objective function. This change in profit is calculated for every reservoir and every time step and is described by the dual variables. Since all the previous resources are already allocated within the time stages in the objective function, the one unit of water is used at the next fortunate possibility which is either now or at a later point in time. Accordingly, this marginal value solves the decision between here and now or wait and see for better prices that exceed the dual variable. If the prices for the calculation would be exactly the same as in reality, then the water value-based steering should result in exactly the same power plant dispatch as planned. For the mathematical formulation of duality in linear programming, see chapter 4.1.1. To obtain dual variables from MILPs an intermediate step is required. First, the MILP is solved regularly and secondly, as an LP with the integer variables as constants so that the dual variables of the reservoir filling can be obtained. This is necessary to bypass a possible change of the integer variables in the primal basis.

Stochastic optimization includes uncertainty. Solving for example the deterministic equivalent of a stochastic model yields to a set of optimal points and decisions for each path and time step. This has the great advantage that for every real-life situation an equivalent set were calculated in the model providing optimal steering. This is similar to the optimal dispatch in linear optimization, if the prices were known beforehand. But since solving the whole stochastic problem is too time-consuming, approximation techniques were introduced. The well-known stochastic optimization technique SDDP solves faster but does not generate optimal points for each realization of the underlying data, e. g. prices or inflows. The problem is just solved for several scenarios that include the information of all input data and generates the in average optimal strategy. Therefore, the expected value of the water in the reservoirs can be used. SDDP water values can be retrieved form the approximations of the sub problems using the lowest valued active cutting plane, see Figure 33. For further information see chapter 6.1.

For the utilization of water values, it is important to understand origin and the mode of action. Water values, dual variable or cutting plane based, refer to a specific point in time and are based on an underlying power plant schedule. Furthermore, the interpretation of water values for seasonal storages and daily storages is different. For a seasonal storage, the water value is not very sensitive towards price changes, since normally enough water is available in the reservoir to participate in high prices; the water level changes little in comparison to the overall volume size during two optimizations. The interpretation of water values for daily pumped storages is different and difficult, since the sensitivity in respect to the prices is high. That means that the power plant steering is very fragile in terms of price changes during the operation. Hence, this method is better used for large reservoirs or, if the optimization is performed more often, also for daily pumped hydropower storages. In practice, such water values should not change significantly over the time between two optimizations. If the period between two optimizations is too long, so that the water values fluctuate, the application of water values based steering can lead to a circular reference. Nevertheless, if the time between two calculations is sufficiently small, as for example every hour, this drawback could be bypassed even for daily pumped hydropower storages.

3.4.2. Shadow Prices

For marketing pumped hydropower storages a suitable transition of the water values into steering parameters is necessary. The transition is needed because of two reasons: Water values, as described above, chapter 3.4.1, are difficult to use as direct trading support since they are mostly given in $[€/m^3]$, whereas the electricity is traded in $[€/MWh]$. Furthermore, the water value is given for every reservoir but does not indicate when to use a specific pump or a turbine. Below, an approach is introduced that solves the transition problem and defines the needed bidding strategy. A second approach from literature is presented afterwards. At last, a few obstacles are pointed out.

The marginal value of the water $\lambda_{t,r}$ for reservoir $r \in R$ in time step $t \in T$ is distinguished from the shadow price $\mathcal{S}_{t,m}$, which determines the price of releasing one unit of water through a respective turbine or pump machine $m \in M$ in time step $t \in T$. The marginal water values are obtained from the dual variables of a linear program, or retrieved form the active cutting planes of SDDP. Determining the difference between the upper and the lower reservoir water value, multiplied with the flow through rate $q(m)$ of the respective turbine or pump machine m in-between the two reservoirs and dividing with the power of the turbine $u(t)$ or pump $p(t)$ results in the turbine and pump shadow prices $\mathcal{S}_{t,m}^{u}$ and $\mathcal{S}_{t,m}^{p}$. The term $\overline{rm}$ describes all reservoirs r located above the machine m in the cascade. Vice versa, $\overline{mr}$ defines all reservoirs r that are located below the machine m. The dual variables of the reservoir filling level equations are given in the unit $[€/m^3]$, the flow through rate in $[m^3/h]$ and the power of the machines in $[MW]$. The shadow prices are now given in $[€/MWh]$ and can provide decision support for real-life trading and dispatch. With a turbine efficiency η_m and a pump efficiency ρ_m in $\left[\frac{m^3}{MWh}\right]$,

$$\eta_m = \frac{q_m}{u^{max}}, \qquad m = 1, \dots, M \qquad (1)$$

$$\rho_m = \frac{q_m}{p^{max}}, \qquad m = 1, \dots, M \quad t = 1, \dots, T \qquad (2)$$

the shadow price of a turbine $\mathcal{S}_{t,m}^{u}$ is defined as:

$$\mathcal{S}_{t,m}^{u} = \left(\sum_{\overline{rm}} \lambda_{t,r} - \sum_{\overline{mr}} \lambda_{t,r}\right)\eta_m \qquad m = 1, \dots, M \quad r = 1, \dots, M \quad t = 1, \dots, T \qquad (3)$$

and for the pumps $\mathcal{S}_{t,m}^{p}$ as:

$$S_{t,m}^{p} = \left(\sum_{\overline{rm}} \lambda_{t,r} - \sum_{\overline{mr}} \lambda_{t,r}\right)\rho_m \qquad \begin{array}{l} m = 1, \dots, M \\ r = 1, \dots, M \\ t = 1, \dots, T. \end{array} \tag{4}$$

Obviously, using the turbine efficiency and discharge rate, the reservoir filling level can also be converted to $[MWh]$ before the optimization which avoids the consideration of the unit $[m^3]$ in the optimization. Nevertheless, this has the drawback that different efficiencies at different operating states of the turbine cannot be considered. Furthermore, complex systems cannot be modeled anymore, as for example an upper reservoir that is connected to two lower reservoirs on different elevations and via two different machines.

A good way to convey the principle of water values is the price duration curve method, as illustrated in Figure 34. The prices of a price forward curve are sorted in a descending order. Furthermore, it is assumed that the reservoir performs just one load cycle during this year with an efficiency of for example $\eta_m \cdot \rho_m = 75$ %. By following the price duration curve from the highest and from the lowest price side until the plant efficiency is reached, the first price for releasing water and first price for pumping water can be found. These prices set the marginal water value. The hours for water release equal the hours for pumping times the exemplary efficiency. This approach is just a simplified illustration with two different efficiencies. If the expected prices over the course of the year fluctuate a new price duration curve and new marginal water values need to be considered.

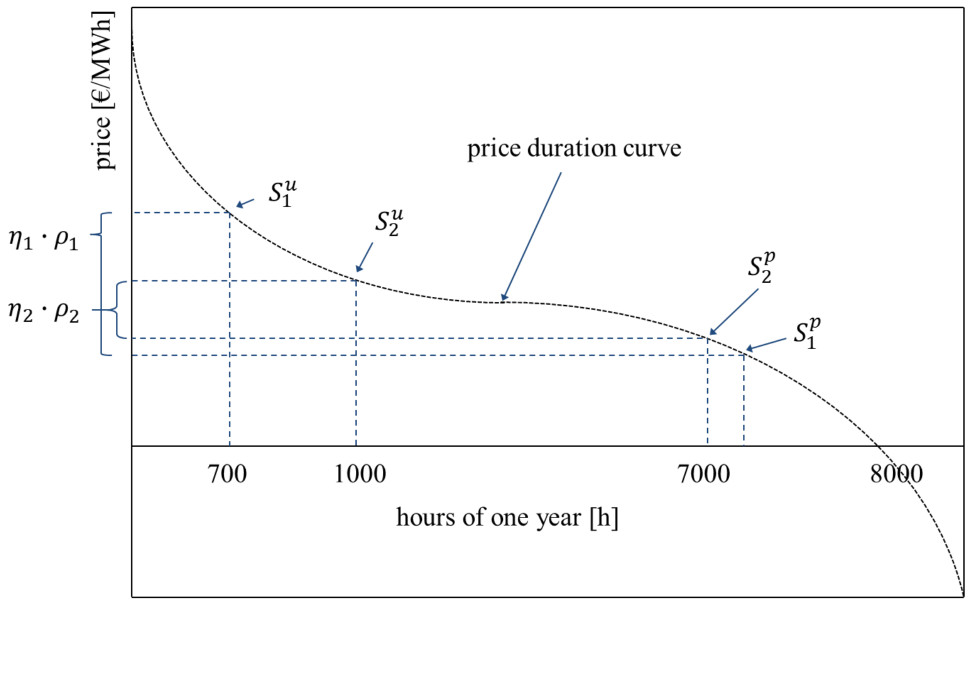


Figure 34 Exemplary price duration curve over the course of one year. Efficiency, reservoir size and inflows determine the number of pumping and generating hours and therefore the marginal prices.

3.4.3. Strategy

This part continuous with the actual trading and the application of the shadow prices on the markets. The quantity of energy that can be traded for the shadow price depends on the size of the reservoirs. Theoretically, the shadow price, as defined above, holds just for a marginal small unit of energy, e. g. 1 MWh. Nevertheless, for seasonal pumped hydropower storages the shadow price is also applicable to larger quantities of energy, because a short-term dispatch or inflows do not have a significant influence on the reservoir level itself. The smaller a reservoir the more limited is the use of the here introduced shadow price.

The calculation of shadow prices is further strongly connected to the definition of a trading strategy. The strategy suggested here is rather simple. If the shadow price of the turbine is below the observed market price the turbine is used with full power. The pump is used at full power if the shadow price for the pump is above the observed market price. Otherwise turbine and pump stand still.

One obstacle of this approach are unprecise bidding curves. This is also due to the simplifications in the optimization models. In theory, adding up the pumps with their respective water values followed by the water values of the turbines lead to the bidding curve. But even for simple systems the resulting bidding curve is neither continuous nor linear. This is because of various reasons, such as specific operating zones of turbines, the integer characteristic of pumps (which can mainly just be switched on and off) but also due to varying start-up times.

Another approach to support the bidding process, is to firstly calculate water values and secondly apply them in an operational simulation (Abgottspon, 2015b, p. 133). The Simulation can be a MILP using the prior calculated water values as opportunity costs to find the optimal hourly operation. For every possible quantity of energy produced a simulation is done to find the optimal dispatch of each pumped hydropower storage. The simulation takes into account the operating zones of the turbines and the physical layout of the power plant portfolio. The results define the marginal production cost curve of the overall pumped hydropower storage system. This cost curve can be used with the already defined trading strategy. This approach is more complicated and might be necessary for a more detailed resolution of water values.

Most approaches in literature consider just the hourly day-ahead market in the optimization and for the determination of the bidding strategy. This needs to be enhanced since the various kinds of daily as well as seasonally pumped hydropower storages are dispatched on all short-term energy markets to exploit these price spreads as well. Therefore, one challenge for steering strategies results from the consideration of more than one market in the optimization and trading, which of course increases the complexity significantly and will be discussed in part B in detail.

4. Mathematical Problem Formulation

In this chapter, the hydropower scheduling optimization problem is formulated, and the notational framework introduced. The formulations of the canonical models are based on the operations research and mathematical programming community notations. The purpose of the schedule optimizations is either a minimization of costs or a maximization of revenues while fulfilling all restrictions and obligations. The following equations are always needed for the problem formulation:

- objective function (maximize return)
- reservoir balancing equation
- maximum and minimum reservoir filling levels
- maximum and minimum turbine and pump capacity
- reservoir filling levels at the beginning and the end

The reservoir filling level at the end denotes the water value beyond the optimization period.

Energy can be generated with turbines and feed into the grid when water is released from an upper into a lower reservoir. Reservoirs are either filled by natural inflows or water that is pumped form a lower reservoir into the upper while consuming energy from the grid. When to release water and use resources depends on the expectations on the future and therefore are a trade-off between selling now or later. This generates an optimization problem that is coupled in time over the whole planning horizon.

To solve the hydropower scheduling problem, a bouquet of possibilities is available. It need to be distinguished between the problem formulation and the optimization technique or solution approach. Different solution techniques demand specific problem formulations. The most restrictive problem formulations are linear problems. Such problems can be generally solved by every solution technique. A non-linear problem formulation limits the optimization techniques to non-linear programs and heuristics. Stochastic optimization techniques require a stochastic problem formulation. Which optimization technique is applied depends on the available input parameters, the intended optimization results, the limits given by computer and software engineering as well as the advantages and disadvantages of the optimization techniques itself. The latter are concisely presented in Table 8 for linear, mixed-integer linear, non-linear, stochastic as well as stochastic dynamic programming. Furthermore, also heuristic methods are included although not applied in this thesis. Linear methods can be used to include a high level of details and still find an optimal solution in very limited time. With non-linear problems all kind of facets can be modelled but rather solved anymore. Such problems should therefore be avoided as far as possible. For stochastic problems several methods have been developed that facilitate to solve even large problems, although the course of dimensions is still the greatest challenge. Heuristics are the most flexible approaches, but the quality of the solutions is difficult to estimate.

In this chapter, the mathematical problem formulations for the here mentioned optimization techniques are formulated. Beginning with a short historic overview and the origin of the optimization techniques the respective general problem formulations are presented. Afterwards the problems are applied on a two-stage pumped hydropower scheduling problem and subsequently extended to a multistage pumped hydropower storage problem. In chapter 4.1 the deterministic, in chapter 4.2 the stochastic and in chapter 4.3 the (stochastic) dynamic problem is introduced.

Table 8 Comparison of optimization techniques. Based on (Dorfner, 2017; Kuhn, 2013)

	advantages	disadvantages
linear programming	• finds global optimal solution • scales well with large systems • standard software available	• very limited for modelling complexity e. g. stochastics, integers, non-linearities
mixed-integer linear programming	• quality of the solution can be assessed • scales quite well with large systems • standard software available • representation of integer values possible	• limited for modelling complexity e. g. stochastics non-linearities
non-linear programming	• high degree of freedom • nearly all complexities can be modelled	• scales unfavorable with large systems • quality of solution not defined (convexity issues)
stochastic programming	• quality of the solution can be assessed • representation of stochastic	• curse of dimensionality • scales unfavorable with large systems
(stochastic) dynamic programming	• quality of the solution can be assessed • scales ok with large systems • representation of stochastic	• scales unfavorable with large systems • complex modelling • no standard software available
heuristics	• high degree of freedom • nearly all complexities can be modelled • solves normally very quickly • bouquet of solution methods	• quality of solution difficult to estimate • no standard software available

4.1. Deterministic Problem

Already in the year 1940 George Danzig introduced linear programming as a method to solve mathematical problems. Linear Optimization, until today, is wildly used to solve complex problems that are formulated in a particular structure of objective function and constraints. The objective function gives the solution to the min- or maximization formulation. Whereas profit, size or distance are common maximizations, time and costs pertain to the class of minimization problems. The constraints determine the limited and often restricted resources. The optimal solution of such problems is called a policy. Although small problems with just a few constraints can be solved graphically or with Danzig's simplex tableaus, computer algorithms for deterministic programming can solve problems with millions of variables and thousands of constraints. One challenge that cannot be tackled with deterministic optimization is an unknown future. Unknown are for example the weather conditions of the next year that can be either stochastically or deterministically estimated. Deterministic means that the result of the calculus is unique for any input. The process to generate this output is called the algorithm.

4.1.1. General Formulation

Linear Program

A linear optimization problem solved with a linear program or optimization is the task to maximize or minimize a linear function satisfying linear constraints. The function to be minimized or maximized $f(x)$ is called objective function and the variables x are named decision variables. In vector notation, the problem formulates as:

$$P: \quad \min_{x} \quad f(x) = c^T x \tag{5}$$
$$s.t. \quad Ax \leq b, \qquad x \geq 0,$$

where c is a vector of costs coefficients or resources, the resource limitation is A and the limit of resource availability is described with b.

This formulation is a prominent subtype of convex optimization. It is called standard or canonical form because every linear optimization problem can be converted into this form. Nevertheless, the assumption of linearity in the objective function and the constraints limits the amount of problems that can be solved significantly.

Dual Program

Optimization problems can be viewed form two sides, the primal problem and the dual problem. The strong duality principle states that for every primal problem with an optimal solution there is a respective dual problem with the identical solution. This holds true for convex problems that fulfill the Karush-Kuhn-Tucker conditions including the constraint qualifications (Boyd & Vandenberghe, 2004). In all other cases the duality gap, the difference between the primal and the dual solution is strictly positive and weak duality holds true. In this event the solution of the dual problem provides a lower bound to the solution of the original primal minimization problem and vice versa an upper bound of the primal maximization problem. The dual problem for the linear primal problem formulates as

$$D: \quad \max_{y} \quad g(y) = b^T y \tag{6}$$
$$s.t. \quad A^T y \geq c, \qquad y \geq 0,$$

with $g(y)$ as the optimal solution value and y the dual variables, representing the marginal value of a resource.

The economic interpretation of the dual problem is as important as the solution of the problem itself. In economic theory, every limited resource has a value. The primal problem allocates a value to the limited resource. The dual problem calculates the marginal value of the resource. The marginal value is the change of the optimal solution of the objective function, if a limiting resource (here b) in the primal problem changes by one unit. This change is used to quantify the value of the resource for the optimization problem.

Non-linear Program

A non-linear problem is a generalization of a linear problem. Therefore, non-linear programming or optimization is the task to maximize or minimize a function satisfying constraints with at least one of them being non-linear. Non-linear problems are much more complicated to solve due to convexity issues. The standard form of the no-linear optimization problem formulates as:

$$\begin{aligned} P: \quad & \min_{x \in \mathcal{X}} \quad f(x) = c^T x & & \qquad (7) \\ & s.t. \quad g(x) \leq 0, & x \in \mathbb{R}^n & \\ & \qquad\;\; h(x) = 0, & x \in \mathbb{R}^n, & \end{aligned}$$

with $x \in \mathbb{R}^n$ as the space of possible solutions. The solution space of the objective function to be minimized is $f: \in \mathbb{R}^n \rightarrow \mathbb{R}^m$. Nevertheless, due to the standard form every minimization problem can be converted in a maximization problem and vice versa by negating the objective function. Furthermore, $g: \mathbb{R}^n \rightarrow \mathbb{R}^m$ represents m inequality constraints and $h: \mathbb{R}^n \rightarrow \mathbb{R}^p$ images p equality constraints. Consequently, $\mathcal{F}$ as the constraint set of feasible solutions is a subset of the solution domain $\mathcal{X}$ which satisfies all constraints.

$$\mathcal{F} = \{x \in \mathcal{X} | g(x) \leq 0 \wedge h(x) = 0\} \qquad (8)$$

The above formulated standard form can be formulated equivalently using the constrained set definition and without any explicit constraint:

$$\min_{x \in \mathcal{F}} \quad f(x). \qquad (9)$$

Nonetheless, for solving optimization problems the explicit formulation is essential.

Quadratic Program

The simplest form of non-linear problem formulations are quadratic problems, whereas it can be differentiated between quadratic formulations in the objective function, the constraints or both. The problem can be formulated similarly to the initial linear problem with the n-dimensional vector c and the $n \times n$-dimensional matrix Q. The matrix A is $m \times n$-dimensional and the vector b is m-dimensional. That means the problem has n variables and m constraints. The objective of the optimization is to find the optimal value for the n-dimensional vector x, with x^T as the transpose. Below an example with a quadratic term integrated into the objective function is presented.

$$\begin{aligned} P: \quad & \min_{x} \quad f(x) = \frac{1}{2} x^T Q x + c^T x & & \qquad (10) \\ & s.t. \quad Ax = b, & x \geq 0 & \end{aligned}$$

If $Q = 0$, the problem is linear. If the matrix Q and in case of quadratic constraints the respective matrixes are positive semidefinite, the problem is part of the class of convex optimization problems which is a strong advantage for solving. In a particular case, the quadratic problem can be reformulated as a second order cone problem (SOCP) which is also part of convex optimization.

Mixed Integer Program

A special form of linear and non-linear programs are mixed integer (non-) linear programs. The allowance of integers, often binaries with $\{0,1\}$ results in a non-continuous optimization. Such problems are stated as discrete optimizations. There is no structural difference to the linear optimization problem formulation with at least one integer variable. If all components of x are integer values the problem is assigned to the group of integer programming. For at least one x_j of x as integer the problem is referred as mixed integer programming. With the domain $x \in S$ the problem can be generally formulated with $S = \mathbb{R}^r \times \mathbb{Z}^{n-r}$ as:

$$P: \quad \min_{x \in S} \quad f(x) = c^T x \tag{11}$$
$$s.t. \quad Ax \leq b, \qquad x \geq 0.$$

Parts of the x vector are therefore real numbers and other integers.

Convexity

With the first and second derivative of a function the optimality conditions for local minima and maxima can be given. The derivative is a local information of a function and therefore also spatially limited. Nevertheless, with convexity, as a global characteristic, every local optimum is also a global optimum. This follows from the following definition and corollary.

Definition: A set $M \subseteq \mathbb{R}^n$ is convex, if $\forall x, y \in M, \lambda \in (0,1): (1-\lambda)x + \lambda y \in M$ is true, which means the connection of any two points in M is part of M. For a convex amount $M \subseteq \mathbb{R}^n$ the function $f: M \to \mathbb{R}$ is convex (on M), if $\forall x, y \in M, \lambda \in (0,1): f\big((1-\lambda)x + \lambda y\big) \leq (1-\lambda)f(x) + \lambda f(y)$. That means the graph of the function f runs below every of its secants.

The convexity of continuous differentiable functions can be shown using the first order Taylor expansion. The C^1-characterzation of convexity states the following: On a convex amount $M \subseteq \mathbb{R}^n$ is a function $f \in C^1(M, \mathbb{R})$ convex, if and only if $\forall x, y \in M, f(y) \geq \langle \nabla f(x), y - x \rangle$ holds true. The central theorem for continuous differentiable convex optimization problems is a far-reaching intensification of the Fermat's theory. For the corollary $f \in C^1(M, \mathbb{R})$ be convex, then, the critical points of f are exactly the global minima of f. Hesse matrices are important to prove convexity of twice continuously differentiable functions. The theorem of the C^2-characterzation of convexity states: A function $f \in C^2(\mathbb{R}^n, \mathbb{R})$ is convex, if and only if $\forall x \in \mathbb{R}^n: D^2 f(x) \succcurlyeq 0$ holds true.

The convexity property is very useful for every kind of iterative search algorithm. In case the solution algorithm yields a minimum, the search can be stopped since every local minimum is also a global minimum and the remaining solution space no longer needs to be searched. For the above formulated optimization problems, the domain $\mathcal{X}$ must be convex, the equality constraints h must be affine and the inequality constraints g must be a vector of convex functions.

4.1.2. Application to Hydropower Optimization

The classic deterministic linear optimization is suitable and practically used in nearly every short- and mid-term optimization and sometimes also in long-term optimization of hydropower scheduling problems. For deterministic method contemplation, prices, inflows and other stochastic variables are taken as known beforehand. Although deterministic optimization is often criticized and entitled to be inferior it is still widely used in practice. Developments, such as multi scenario or successive linear programming increased the field of application. The usability of linear optimization in terms of computation time is very high (Belsnes et al., 2005). In this part the general hydropower scheduling problem is formulated maximizing the profit and using the following constraints, parameters and variables.

State variable:

- reservoir filling level [1000m³]: $v_{t,r}$

Decision variables:

- turbine capacity [MW]: $u_{t,m}$,
- pump capacity [MW]: $p_{t,m}$,
- spillage [1000m³]: $s_{t,r}$,

Sets:

- time stages [flexible, e. g. hourly]: $t = 1, \dots, T$
- reservoirs: $r = 1{,}2$
- machines: $m = 1$
- machine below reservoir: m$\in \overline{rm}$
- machine above reservoir: $m \in \overline{mr}$

Parameters:

- prices [€/MWh]: c_t,
- inflows [1000m³]: $v_{t,r}^{in}$,
- specific discharge turbine [1000m³/MWh]: η_m,
- specific charge pump [1000m³/MWh]: ρ_m,
- limits for spillage [1000m³]: $s_{t,r}^{min}, s_{t,r}^{max}$,
- limits for filling level [1000m³]: $v_{t,r}^{min}, v_{t,r}^{max}$,
- start filling level [1000m³]: $v_{t,r}^{start}$,
- end filling level [1000m³]: $v_{t,r}^{end}$,
- limits for turbine capacity [MW]: $u_{t,m}^{min}, u_{t,m}^{max}$,
- limits for pump capacity [MW]: $p_{t,m}^{min}, p_{t,m}^{max}$,
- limits for filling level [1000m³]: $v_{t,r}^{min}, v_{t,r}^{max}$,

- limits for turbine capacity [MW]: $u_{t,m}^{min}, u_{t,m}^{max}$,

The general problem is firstly formulated for a simple two reservoir system connected with a hydropower machine consisting of a pump and a turbine to transfer the water within the reservoirs. This is illustrated in Figure 35. Further important parameters are the reservoir filling levels which also depend on inflows and spillages. The very first problem is just formulated for two time stages. Secondly, the problem is extended considering multiple time stages and, furthermore, an optional pumped hydropower storage layout to describe even complex systems.

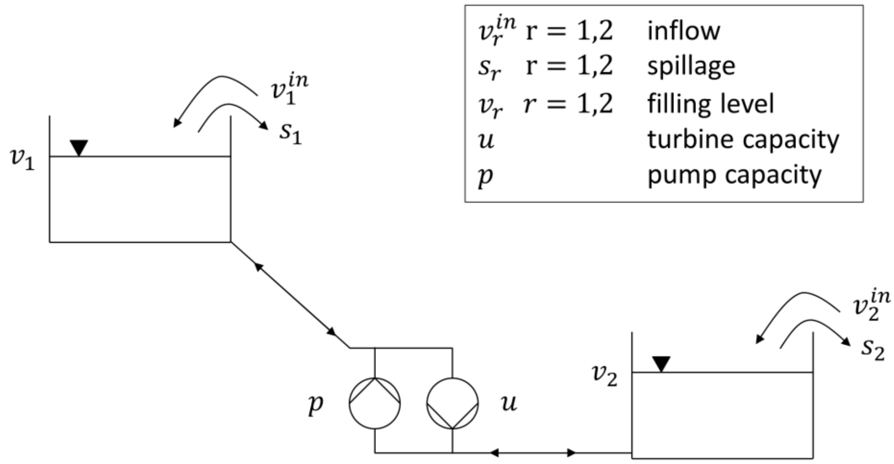


Figure 35 Diagram of a pumped hydropower storage with two reservoirs

Two Stage Problem

The two-stage problem formulates as:

$$\begin{array}{lll}
\max\limits_{u,p,s,v} & c_1(u_1 - p_1) + c_2(u_2 - p_2) & \qquad (12) \\
s.t. & v_{1,1} + s_{1,1} - \rho p_1 + \eta u_1 = v_1^{anf} + v_{1,1}^{in} & \\
 & v_{1,2} + s_{1,2} + \rho p_1 - \eta u_1 = v_2^{anf} + v_{1,2}^{in} & \\
 & v_{2,1} - v_{1,1} + s_{2,1} - \rho p_2 + \eta u_2 = v_{2,1}^{in} & \\
 & v_{2,2} - v_{1,2} + s_{2,2} + \rho p_2 - \eta u_2 = v_{2,2}^{in} & \\
 & v_r^{min} \le v_{t,r} \le v_r^{max} & t,r = 1,2,\ t = 1,2 \\
 & 0 \le p_t \le p^{max} & t = 1,2 \\
 & 0 \le u_t \le u^{max} & t = 1,2 \\
 & 0 \le s_{t,r} & t,r = 1,2
\end{array}$$

with the objective function at the first place and, in this order, the constraints reservoir filling level for $r = 1, t = 1$, reservoir filling level for $r = 2, t = 1$, reservoir filling level for $r = 1, t = 2$, reservoir filling level for $r = 2, t = 2$, reservoir limits, pump machine limits, turbine machine limits and spilling. The parameter v_r^{min} and v_r^{max} define the minimum and maximum filling levels of the reservoirs, v_r^{anf} are the start filling levels in 1000m³ of the reservoir $r = 1{,}2$ and the pump and turbine capacity limits are described by p_t and u_t for the two time stages $t = 1{,}2$.

The problem can be formulated in matrix notation by aggregating the restrictions. For simplification, below, x_t denotes for the term $(u_t - p_t)$. Furthermore, the reservoir balancing equations have been formulated to bring all parameters on the right-hand side and to provide straight forward mathematical formulation. To better understand the hydropower storage system the notation can be rearranged so that the reservoir filling level equations have just the reservoir filling of the time stage before on the left-hand side. Furthermore, inequality constraints of the given example can be reformulated to equality constraints using slack variables as it is done in the following canonical form (Pereira & Pinto, 1991).

$$
\begin{aligned}
P: \quad & \max_{x_1, x_2} \quad c_1^T x_1 + c_2^T x_2 \\
& s.t. \quad A_1 x_1 = b_1 \\
& \qquad\ \ B_2 x_1 + A_2 x_2 = b_2 \qquad\qquad x_1, x_2 \geq 0
\end{aligned} \tag{13}
$$

With the vectors $b_2 \in \mathbb{R}^{m_1}$ and $b_2 \in \mathbb{R}^{m_2}$ as well as $c_1 \in \mathbb{R}^{n_1}$ and $c_2 \in \mathbb{R}^{n_2}$ the $A_1 \in \mathbb{R}^{m_1 \times n_1}$ and $A_2 \in \mathbb{R}^{m_2 \times n_2}$ are described as regression matrices and $B_2 \in \mathbb{R}^{m_2 \times n_1}$ as technology matrix.

Here, the construction and the filling of the vectors and matrices is explained whereas in the following work this is shortened by using matrix notation. The vectors $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$ include the decision, state and slack variables.

$$
x_1 = \begin{pmatrix} v_{1,1} \\ s_{1,1} \\ p_1 \\ u_1 \\ v_{1,2} \\ s_{1,2} \\ \epsilon_{1,1} \\ \epsilon_{1,2} \\ \epsilon_{1,3} \\ \epsilon_{1,4} \\ \epsilon_{1,5} \\ \epsilon_{1,6} \end{pmatrix} \qquad x_1 = \begin{pmatrix} v_{2,1} \\ s_{2,1} \\ p_2 \\ u_2 \\ v_{2,2} \\ s_{2,2} \\ \epsilon_{2,1} \\ \epsilon_{2,2} \\ \epsilon_{2,3} \\ \epsilon_{2,4} \\ \epsilon_{2,5} \\ \epsilon_{2,6} \end{pmatrix}
$$

The entries of ϵ_r, with $r = 1, \dots, 6$ characterize the slack variables that are used to generate equality constraints. The coefficient vectors in the objective function are filled up with nulls.

$$c_1 = \begin{pmatrix} 0 \\ 0 \\ c_1 \\ -c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad c_2 = \begin{pmatrix} 0 \\ 0 \\ c_2 \\ -c_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The regression and technology matrices are defined as below:

$$A_1 = A_2 = \begin{pmatrix} 1 & 1 & -\rho & \eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & -\eta & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} v_1^{anf} + v_{1,1}^{in} \\ v_2^{anf} + v_{1,2}^{in} \\ v_1^{max} \\ v_2^{max} \\ v_1^{min} \\ v_2^{min} \\ p^{max} \\ u^{max} \end{pmatrix} \quad b_2 = \begin{pmatrix} v_1^{anf} + v_{2,1}^{in} \\ v_2^{anf} + v_{2,2}^{in} \\ v_1^{max} \\ v_2^{max} \\ v_1^{min} \\ v_2^{min} \\ p^{max} \\ u^{max} \end{pmatrix}.$$

Multistage Problem

The just described two stage problems can also be formulated as a multistage program with infinity time stages. The return of each time step is summarized in the objective function. The reservoir balancing equations are defined for every time step to secure consistent filling levels. The remaining restrictions are valid for the whole-time period. The crucial point that links the time steps together is the question on when to release the water. The extended problem formulates as follows:

$$
\begin{aligned}
P: \quad & \max_{x_1,\dots,x_T} \quad \sum_{t=1}^{T} c_t^T x_t + c_t^T x_t & & (14)\\
& s.t. \quad A_1 x_1 = b_1 & & \\
& \qquad B_t x_{t-1} + A_t x_t = b_t & t = 1,..,T & \\
& \qquad x_1, x_2 \geq 0, & t = 1,\dots,T &
\end{aligned}
$$

If more than two reservoirs need to be considered the problem can be easily extended. But it is important to define how the storages are connected.

The above introduced two-reservoir hydropower scheduling problem can be formulated as a multistage multi-reservoir problem:

$$
\begin{aligned}
\max_{u,p,s,v} \quad & \sum_{t=1}^{T} c_t (u_t - p_t) & & (15)\\
s.t. \quad & v_{t,r} = v_{t,r}^{anf} - s_{t,r} + v_{t,r}^{in} + \rho p_{t,m} - \eta u_{t,m} & \forall t, r, m & \\
& v_r^{min} \leq v_{t,r} \leq v_r^{max} & \forall t, r & \\
& 0 \leq p_{t,m} \leq p^{max} & \forall t, m & \\
& 0 \leq u_{t,m} \leq u^{max} & \forall t, m & \\
& 0 \leq s_{t,r} & \forall t, r\ . &
\end{aligned}
$$

With r as the number of reservoirs, m the number of turbine and pump machines as well as t the time stages of the optimization.

4.2. Stochastic Problem

Stochastic programming generalizes deterministic programming towards using random parameters, whereas the probability distribution for these parameters is assumed to be known or is estimated. The goal of stochastic optimization models is therefore to find a policy that is feasible for all or most expressions of the input parameters.

Dealing with uncertainties in optimization problems began in the early 50's and late 60's. One of the first stochastic programming applications was hydropower scheduling. The need for stochastic programming originated in the challenge to estimate and consider future inflows. If not one precise forecast could be given at least an inflow probability distribution were guessed. A then generated stochastic optimization based policy was feasible for various realizations of the actual inflows. Massé, as was Little, were pioneers which applied stochastic programming as one of the first to a simple hydropower storage optimization problem more than 60 year ago. (Little, 1955; Masse, 1946). And to be noted, no computers existed that time to solve such complicated problems. Whereas the basic problem still exists it has been extended as numerous books on stochastic programming show (Birge & Louveaux, 2011; Kall & Mayer, 2011; King & Wallace, 2012).

4.2.1. General Formulation

The classical two-stage linear stochastic problem is formulated similar to deterministic problems adding an expectation on the future, with at least one random vector. It is formulated as:

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x + E_\xi[Q(x,\xi)] & & \qquad (16) \\ s.t. \quad & Ax = b & x \geq 0\,, & \end{aligned}$$

with $Q(x, \xi)$ as the optimal value of the second-stage problem which is formulated as:

$$\begin{aligned} \min_{y \in \mathbb{R}^m} \quad & q(\xi)^T y & & \qquad (17) \\ s.t. \quad & T(\xi)x + W(\xi)y = h(\xi) & y \geq 0\,. & \end{aligned}$$

The first stage decision variable is $x \in \mathbb{R}^n$ and the second stage decision variable is $y \in \mathbb{R}^m$. The data for the second stage problem is contained in the random vector $\xi(q, T, W, h)$, also referred as probability vector. On the first stage, a "here-and-now" decision x must be made before the realization of the uncertain data in the random vector ξ is revealed. At the second stage, after a realization of ξ becomes available, the respective problem is solved.

At the first stage, the problem plus the expected value of the second stage is solved. The second stage problem is either seen as the optimal behavior when the uncertain data is known or is considered as a recourse action with the compensation Wy for a possible inconsistency of the system $Tx \leq h$. The value of the recourse action is defined by $q^T y$.

Crucial point is the unknown parameter that is modeled as a random vector ξ with a known probability distribution that is part of the second stage data. The distribution assumption is an important part of the problem formulation. A common approach is the discretization of the random vector ξ into scenarios.

For the multistage optimization of pumped hydropower storages either scenario trees (Jacobs et al., 1995) or Benders cuts (Shapiro, 2012) can be used, whereas the latter solves a specialized problem using a stochastic dual decomposition procedure, also referred as stochastic dual dynamic programming (SDDP) (Pereira & Pinto, 1991), see chapter 4.3.

4.2.2. Application to Hydropower Optimization

In this part, energy prices and inflows are considered as not given beforehand. A stochastic pumped hydropower storage model is introduced to deal with this uncertainty. No influence on the price, i.e. unlimited market depth, is assumed. The uncertainty is defined by random vectors. The triple $(\Omega, \mathcal{A}, \mathcal{P})$ defines the probability space with the set of events Ω, the σ-Algebra $\mathcal{A}$ over Ω and the probability measure $\mathcal{P}: \mathcal{A} \to [0,1]$. Let ξ be the random vector on $(\Omega, \mathcal{A}, \mathcal{P})$. The distribution function $F_\xi(x) =$

$\mathcal{P}(\{\omega: \xi(\omega) \leq x\})$ of ξ is known. Let ξ_t be the random vector of the set of elements Ω_t for each time step $t = 1, \dots, T$ and $\xi = (\xi_2, \dots, \xi_T)$. With $\omega \in \Omega$ is a possible set of events. The conditional set of events is given by $(\Omega_t | \omega_{t-1})$, if $\omega_{t-1} \in \Omega_{t-1}$ is realized.

Multistage Model

A multistage formulation is superior to a single-stage optimization because decisions at an early stage affect decisions at later stages. The myopic view in the rolling horizon manner solving every single-stage problem separately does not take that into account. The problem is defined as non-anticipative. That means that every decision on each time step needs to be taken without the information of the realization of the stochastic process on the following time steps, see Figure 36.

$$
\begin{aligned}
\max_{x_1} \quad & c_1^T x_1 + \mathbb{E}_\xi \left[\max_{x_2(\omega_2)} c_2(\omega_2)^T x_2(\omega_2) + \mathbb{E}_\xi \left[\dots \right. \right. \\
& \qquad\qquad \left. \left. + \mathbb{E}_\xi \left[\max_{x_T(\omega_T)} c_T(\omega_T)^T x_T(\omega_T) \right] \dots \right] \right] \\
s.t. \quad & A_1 x_1 = b_1 \\
& B_2 x_1 + A_2 x_2(\omega_2) = b_2(\omega_2), && \forall \omega_2 \in \Omega_{t-1}, \\
& B_t x_{t-1}(\omega_{t-1}) + A_t x_t(\omega_t) = b_t(\omega_t), && t = 3, \dots, T,\ \forall \omega_{t-1} \in \Omega_{t-1},\ \omega_t \in (\Omega_t | \omega_{t-1}) \\
& x_1, x_t(\omega_t) \geq 0, && t = 2, \dots, T
\end{aligned}
\tag{18}
$$

The random vector $\xi_t(\omega_t) = (c_t(\omega_t), b_t(\omega_t))$ on stage t includes an energy price c and reservoir inflows b.

Figure 36 Decision process of a multistage model

Scenario Tree Formulation

In an optimal case, all points of a continuous distribution are possible. But this would require to numerically solve multidimensional integrals which is normally not possible (Rebennack, 2016). Therefore, the discretization of the distribution of ξ is a possible solution. Hence, the constructed scenario tree consists of scenarios $s = 1, \dots, S$ with probabilities $p_s > 0$ and $\sum_{s=1}^{S} p_s = 1$, see Figure 37. By means of that scenario tree the whole stochastic program can be approximated and solved as a single linear program. This problem is the so called extensive form. If the random vector has a discrete distribution the respective linear program denotes as deterministic equivalent of the stochastic problem in scenario tree formulation. The interested reader is referred to Pflug and Pichler (2014, p. 23) for further information on scenario tree formulations.

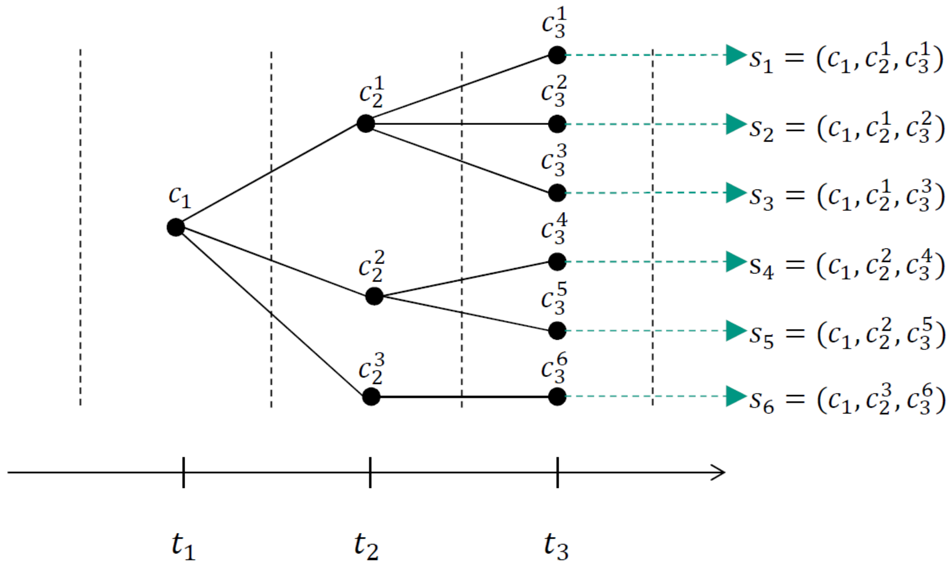


Figure 37 Scenario tree price development with $t = 3$ and $S = 6$

Deterministic Equivalent of the Multistage Model

The above introduced multistage model can be reformulated as a deterministic equivalent.

$$\max_{x_1, x_{t,s}} \quad c_1^T x_1 + \sum_{s=1}^{S} p_s \sum_{t=2}^{T} c_{t,s}^T x_{t,s} \tag{19}$$

$$\begin{aligned} s.t. \quad & A_1 x_1 = b_1 & & \\ & B_t x_{t-1,s} + A_2 x_{t,s} = b_{t,s} & s = 1, \dots, S, \quad & t = 2, .., T \\ & x_{t,s} = x_{t,\check{s}} & \forall s = 1, \dots, S \wedge \tilde{s} \in \mathbb{S}_{t,s}, \quad & t = 2, \dots, T \qquad (20) \\ & x_{t,s} \geq 0, & s = 1, \dots, S, \quad & t = 2, \dots, T \end{aligned}$$

The variables $x_{t,s}$ denote the decision at time stage t in the scenario s. The set $\mathbb{S}_{t,s}$ includes all scenarios $\tilde{s}$, which correspond at time stage t with scenario s, this applies to all s with $\tilde{s} = 1, \dots, S$ and $t = 2, \dots, T$. Equation (20) is needed for the non-anticipative modelling and secures that just the previous steps are known but not the future, see Figure 38. Comparing with Figure 37, Figure 38 ought to emphasize the importance of equation (20).

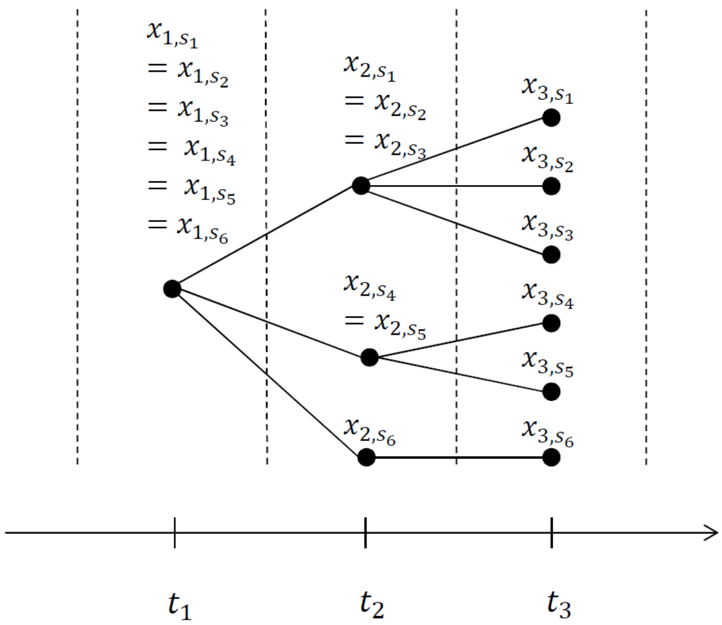


Figure 38 Scenario tree of decision variables with $t = 3$ and $S = 6$ and the respective non-anticipative conditions

To approximate stochastic processes a representative and sufficient size of discretization is crucial. This results in sizeable scenario trees with a high number of variables. Most of the time these scenario trees exceed a critical size so that the linear problem gets practically unsolvable. This effect is called the curse of dimensions (Bellman, 1954; Bellman & Dreyfus, 2010)(Bellman, 1954).

4.3. Stochastic Dynamic Problem

Dynamic programming is the separation of a problem into a sequence of sub problems which are all together easier to solve than the original problem. The principle divide-and-conquer has been developed after the second world war by three different research groups in parallel. The mathematical community named it multistage stochastic dynamic programming, computer scientists called it reinforcement learning and in the field of operations research it is stated as Markow decision process. The breakthrough came in the year 1954 when Richard Bellman introduced the term dynamic programming (Bellman, 1954) which sets the fundament for the whole research field of dynamic programming (Bellman & Dreyfus, 2010). Therefore, equation (21) is also called Bellman's equation,

$$V_t(S_t) = \min_{a \in A}(C(S_t, a) + \gamma \sum_{s' \in S} p(s' \mid S_t, a) V_{t+1}(s')) \tag{21}$$

where S_t is the state at time t, a the typically discete action in set A, $C(S_t, a)$ the cost of being in state S_t and taking action a, γ fixed discount factor, $p(s' \mid s, a)$ the probability of transition to state $S_{t+1} = s'$ if one is in state $S_t = s$ and take a as well as $V_t(s)$ the value of being in state $S_t = s$ at time t and following the optimal policy from t onward.

Today the basic concepts have been extended in all three fields of research. Good overviews from the perspective of reinforcement learning (Bertsekas, 2005; Sutton & Barto, 2010; Szepesvári, 2010) and operations research (Puterman, 2009) are given. An review of different notational systems is given as well (Powell & Meisel, 2016a, 2016b).

As stochastic hydropower scheduling problems, also stochastic dynamic hydropower scheduling problems were of interest for dynamic programming forerunners from the beginning. Nevertheless, it took 10 years from Bellman's publication until the first single reservoir problem was solved by Young (1967) due to computational problems. Between the years 1970 and 1990, solving hydropower problems was subject of active research. The main results of that time were general stochastic dynamic programming algorithms, improved inflow models, considering multi reservoirs, reliability of constraints as well as hydro thermal optimizations (Abgottspon, 2015b). The work of Yakowitz (1982) and Stedinger, Sule, and Loucks (1984) will give an overview, as it is not possible to reference all work done here.

4.3.1. General Formulation

The multistage stochastic problem can be separated into sub problems. The objective function calculates the profit from the current time step plus a function that sums up the future expected profit independently of the current time step. This corresponds to the dynamic programming approach of Bellman (1954).

Below, the dynamic formulation of the deterministic two stage program is derived and described. The feasible set is defined as followed:

$$M \coloneqq \{(x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} | A_1 x_1 = b_1, B_2 x_1 + A_2 x_2 = b_2, x_1, x_2 \geq 0\}, \tag{22}$$

with x_1 as the complicating variable. Below, the feasible set M is now separated into the x_1 and the x_2 component.

$$\begin{aligned} X_1 &\coloneqq \{x_1 \in \mathbb{R}^{n_1} | A_1 x_1 = b_1, x_1 \geq 0\} \\ X_2(x_1) &\coloneqq \begin{Bmatrix} \{x_2 \in \mathbb{R}^{n_2} | B_2 x_1 + A_2 x_2 = b_2, x_2 \geq 0\}, & if\ x_1 \in X_1 \\ \emptyset, & else \end{Bmatrix} \end{aligned} \tag{23}$$

The domain of the function X_2 is described as:

$$def X_2 = \{x_1 \in \mathbb{R}^{n_1} | X_2(x_1) \neq \emptyset\}, \tag{24}$$

so that M can be declared as

$$M = \cup_{x_1 \in def X_2} (X_2(x_1) \times \{x_1\}). \tag{25}$$

The decomposition theorem 2.5.1 (Stein, 2016) allows the following decomposition, if all sub problems are solvable, which is normally the case (Vesper, 2017).

$$\begin{aligned}\max_{(x_1,x_2)\in M} c_1^T x_1 + c_2^T x_2 &= \max_{x_1 \in defX_2} \max_{x_2 \in X_2(x_1)} c_1^T x_1 + c_2^T x_2 \\ &= \max_{x_1 \in defX_2} c_1^T x_1 \max_{x_2 \in X_2(x_1)} c_2^T x_2\end{aligned} \tag{26}$$

The inner maximization problem is defined by Q and is described as below

$$\begin{aligned}Q(x_1) \quad &:= \max_{x_2 \in X_2(x_1)} \quad c_2^T x_2 \\ &= \max_{x_2} \quad c_2^T x_2 \\ &\quad s.t. \quad A_1 x_2 = B_2 x_1,\ x_2 \geq 0.\end{aligned} \tag{27}$$

The second part of this equation is valid, because $x_1 \in X_1 \supseteq defX_2$ is by definition $X_2(x_1) = \{x_2 \in \mathbb{R}^{n_2} | B_2 x_1 + A_2 x_2 = b_2,\ x_2 \geq 0\}$. If $x_1 \in X_1 \backslash defX_2$ then $X_2(x_1) = \emptyset$ and therefore $Q(x_1) = -\infty$, and the problem is inconsistent. If one $\hat{x}_1 \in defX_2$ exists and one $\tilde{x}_1 \in X_1 \backslash defX_2$, then $Q(\hat{x}_1) > Q(\tilde{x}_1) = -\infty$ holds true. In this case, the optimality value does not even change if the originally feasible set of $defX_2$ would be extended by $X_1 \backslash defX_2$. If none $\hat{x}_1 \in defX_2$ exists, the problem is unsolvable, which has been excluded before. Concluding, the adjoining of $X_1 \backslash defX_2$ to the outer optimization problem does not change the solution. Because either the sub problem is solvable, resulting in $x_1 \in defX_2$ or the sub problem is unsolvable. The following decomposition is obtained.

$$\begin{aligned}P: \quad &\max_{x_1} \quad c_1^T x_1 + Q(x_1) \\ &s.t. \quad A_1 x_1 = b_1 \\ &\qquad\ \ x_1 \geq 0\end{aligned} \tag{28}$$

$$\begin{aligned}Q(x_1) \quad &= \max_{x_2} \quad c_2^T x_2 \\ &s.t. \quad A_2 x_2 = b_2 - B_2 x_1 \\ &\qquad\ \ x_1 \geq 0\end{aligned} \tag{29}$$

These two problems define the dynamic formulation of the deterministic program from equation (14). With two time stages $T = 2$ also the stochastic optimization problem from equation (18) can be formulated as a dynamic program.

4.3.2. Application to Hydropower Optimization

Stochastic dynamic programming is also an important field for hydropower optimization. One of the most widely known hydropower optimization methods is SDDP introduced by Pereira and Pinto (1985), (1991). Especially in practice, dynamic programming has several advantages. In comparison to standard multistage stochastic optimizations the implementation is more efficient, and it is easier to model real-life problems and decisions (Abgottspon, 2015a). Furthermore, the complexity just rises linearly with the number of time stages. Following the principle of divide and conquer in dynamic programming the original problem is split into independent sub problems that than can be easily solved in parallel. A good capability of parallelization sets the basis for high performance computing using several CPUs. If a problem can be separated into parallel tasks with little effort it is called an embarrassingly parallel or in computer science a good workload problem (Foster, 1995; Herlihy & Shavit, 2012). The opposite are inherently serial problems that cannot be parallelized at all, which are problems that are depend and need communication between the tasks.

Two Stage Dynamic Program

The deterministic as well as the stochastic two stage program can be dynamically formulated. For the dynamic formulation, $\mathcal{Q}(x_1)$ is stated the profit-to-go function form the second stage. The function $\mathcal{C}(x_1, \omega)$ describes the return of the realization ω after the decision x_1. The problem formulates as:

$$\begin{aligned} \mathcal{C}(x_1, \omega) \quad &= \max_{x_2} \quad c_2(\omega)^T x_2(\omega) \\ &s.t. \quad B_2 x_1 + A_2 x_2(\omega) = b_2(\omega) \\ &\qquad\quad x_2(\omega) \geq 0. \end{aligned} \tag{30}$$

The expected value of $\mathcal{C}(x_1, \omega)$ is the profit-to-go function:

$$\mathcal{Q}(x_1) \quad = \quad \mathbb{E}_\xi[\mathcal{C}(x_1, \omega)]. \tag{31}$$

The probability vector $\xi(\omega) = (c_2(\omega), b_2(\omega))$ includes the uncertainness of prices and inflows. The first stage problem is also called deterministic equivalent:

$$\begin{aligned} P: \quad &\max_{x_1} \quad c_1^T x_1 + \mathcal{Q}(x_1) \\ &s.t. \quad A_1 x_1 = b_1 \\ &\qquad\quad x_1 \geq 0 \end{aligned} \tag{32}$$

The problems (30), (31) and (32) form together the dynamic formulation of the stochastic maximization problem in (18) with $T = 2$.

Multistage Dynamic Programming

As for the linear and the stochastic also the dynamic two-stage program can be extended to a multistage formulation. The dynamic program formulation is applied on the stochastic multistage model in equation (18). $\mathcal{C}$ is stated as the value function for any but fixed event ω and $\mathcal{Q}$ as the expected value or profit-to-go function of $\mathcal{C}$. The value function $\mathcal{C}$ receives on the last stage the optimal point of the penultimate stage x_{T-1} as well as the event ω_T for the random vector ξ as parameters. The formula is stated as follows:

$$\mathcal{C}_T(x_{T-1}, \omega_T) \quad := \max_{x_T} \quad c_T(\omega_T)^T x_T \tag{33}$$

$$
\begin{aligned}
s.t. \quad & B_T x_{T-1} + A_T x_T = b_T(\omega_T) \\
& x_T(\omega_T) \geq 0.
\end{aligned}
$$

The expected value function or profit-to-go function is defined on time stage $(t+1)$ as

$$
\mathcal{Q}_{t+1}(x_t) = \mathbb{E}_\xi[\mathcal{C}_{t+1}(x_t, \omega_{t+1})] \quad t = 1, \dots, T-1. \tag{34}
$$

Generally, the value function at time stage t $(t = 2, \dots, T-1)$ receives the optimal point from time stage $t-1$ as a parameter as well as the event ω_T for the probability vector ξ_T. This can be formulated as follows:

$$
\begin{aligned}
\mathcal{C}_t(x_{t-1}, \omega_t) \quad := \max_{x_T} \quad & c_t(\omega_t)^T x_t(\omega_t) + \mathcal{Q}_{t+1}(x_t) \\
s.t. \quad & B_t x_{t-1} + A_t x_t(\omega_t) = b_t(\omega_t) \\
& x_t(\omega_t) \geq 0.
\end{aligned} \tag{35}
$$

The problem of the first stage is stated as deterministic equivalent. The objective function includes the expected value function of the second stage and therefore implicitly the future incomes or revenue.

$$
\begin{aligned}
P: \quad \max_{x_1} \quad & c_1^T x_1 + \mathcal{Q}_2(x_1) \\
s.t. \quad & A_1 x_1 = b_1 \\
& x_1 \geq 0
\end{aligned} \tag{36}
$$

The expected value function $\mathcal{Q}_t, t = 2, \dots, T$ is also named profit-to-go function since it determines the future revenues depending on the current decision.

Mathematical Solvability

For the solution of the hydropower scheduling problem the deterministic and the stochastic problem need to be distinguished. The solvability of both problems can be validated with the theorem of Weierstrass.

First, assuming the requirements for the deterministic two stage model that the inflows and the target reservoir filling levels are not negative and the start reservoir filling levels are not lower as the end filling level.

- As a linear function, the objective function is constant.
- A linear problem has a feasible set and therefore also a closed set.
- The feasible set is limited by the following constraints

$$
v_r^{min} \leq v_{t,r} \leq v_r^{max} \tag{37}
$$

$$
0 \leq u_{t,m} \leq u^{max} \tag{38}
$$

$$
0 \leq p_{t,m} \leq p^{max} \tag{39}
$$

$$0 \leq s_{t,r} \leq \max\left(v_r^{anf}, v_{t,r}\right) + v_{t,r}^{in} - v_{t,r} + \rho p_{t,m} + \eta u_{t,m} \\ \leq \max\left(v_r^{anf}, v_r^{max}\right) + v_{t,r}^{in} - v_r^{min} + \rho p^{max} + \eta u^{max} \tag{40}$$

which are valid for $r, m = 1{,}2$.

- The feasible set is non-empty, since the decision of not using turbines or pumps is always included. In this case, the inflows are spilled.

Therefore, the conditions for the theorem of Weierstrass are fulfilled and the problem is solvable. For multistage problems, an analogue rationale is possible. The objective function is still linear and therefore constant. The constraints are limited and therefore a feasible set. Possible reasons for insolvabilities can be empty sets or a lack of bounds. If the feasible set is empty, there is always an error since the hitherto described solution is always feasible. A lack of bounds is often reasoned by missing constraints or bounds for variables that need to be added.

Second, the stochastic optimization problem with a two-reservoir system and two time stages is scrutinized. Assuming the requirements that no negative inflows and target reservoir filling levels lower or equal to the start filling levels are required.

- The objective function is, as integral of a constant function, constant.
- Due to the linear problem description, the set is feasible and closed.
- The feasible set is limited by the following constraints, whereas the first three equations (37), (38) and (39) as well as the spillage constraints (40) are analogue to the deterministic case.

$$0 \leq s_{t,r} \leq v_r^{anf} + v_r^{in} - v_{t,ri} + \rho p_t + \eta u_t \\ \leq v_r^{anf} + v_r^{in} - v_r^{min} + \rho p^{max} + \eta u^{max} \tag{41}$$

- The equation (41) applies for all $\omega \in \Omega$ and $r, m = 1{,}2$. The feasible set has a probability of 1.
- The trivial solution is feasible. Therefore, the feasible set is non-empty.

Again, the solvability follows with the theorem of Weiserstrass. This approach is also feasible for sizeable models with multiple periods. The challenge is again to avoid empty or unbounded sets that are most of the time a result of mistakes in modeling.

Model Extensions

The here described two-reservoir model consists of one pump and one turbine connecting two reservoirs. Nevertheless, it can be easily extended towards multi-reservoirs. For every reservoir and time step one reservoir equation need to be added, which is the same for the deterministic, stochastic and dynamic problem formulation. The interconnections between the reservoirs need to be considered as further constraints. Equivalently, further pumps and turbines need to be specified and the connection to the reservoirs defined. Extensions are also possible in terms of gird charges, risk aversion or non-linear relations such as the interdependence between reservoir filling levels and turbine power due to the water head effect. A great extension represents the consideration of balancing power markets. Beside the provision especially the activation of balancing power adds further non-linearities to possible models.

B. Multi-Market Optimization and Trading

Already 70 years ago, Massé (1946) formulated the principal optimal rule: produce when the marginal utility of production is higher than the marginal utility of the expected production; else wait. To find an optimal strategy is sometimes also called dispatch control, hydropower scheduling or simple profit optimization problem, see chapter 4. This thesis pursues the target to improve analysis, optimization and trading over the whole short-term marketing process based on the new framework conditions. The whole short-term marketing process includes all relevant short-term electricity markets. The new framework conditions are characterized by the German Energiewende and the strong increase of variable RES feed-in which demand a rethinking of the usage of existing pumped hydropower storages.

To combine, all markets, all uncertainties and all technical restrictions in one optimization, is, with known methodologies, computational intractable. Therefore, it is suggested to decompose the overall hydro power scheduling problem along the timeline as suggested in literature, see chapter 3.3.1, but additionally consider the structure of the various electricity markets, see chapter 2, and the given mathematical problems, see chapter 4. Figure 39 presents an overview over solution methods and classifies by solvability and powerfulness. Whereas the linear problem solves very fast, the number of problems that can be solved with this system is the smallest. With a non-linear system many problems can be solved on the costs of computational run-time. Also, the difficulty to determine the quality and optimality of the solutions is considered. This means a significant challenge is to choose the appropriate optimization method for each problem considering an adequate level of detail and the computation time needed.

Using the given methods and models, chapter 5 describes how to find an optimal solution for the multi-market hourly and quarter-hourly day-ahead auction scheduling problem. A high level of details and challenges such as the limited liquidity on the quarter-hourly market are tackled with a quadratic optimization. The implications of not considering prices and inflows as uncertain are tackled and determined in chapter 6 using SDDP approaches applied on the quarter-hourly day-ahead auction. The next markets in the time line are the hourly and quarter-hourly intraday continuous markets which are considered in chapter 7. These markets appeared to be so volatile that also a continuous optimization is applied taking the results of the foregoing day-ahead auctions into account. To keep the optimization time within a range of seconds, the level of detail is low and no stochastic inputs could be considered. To balance deviations of production or demand after the intraday market gate closure, balancing products are activated. The auctioning for these products highly dependents on the profit of the other energy only markets. Therefore, a non-linear problem as well as a linearized simplification are introduced in chapter 8 considering both balancing and energy only markets. How the introduced optimization methods and tools intertwine and should be used by a storage operator is always laid out in each chapter and underpinned with respective case studies. To give an outlook, in part C chapter 9 the overall bidding approach and the optimization strategies are summarized.

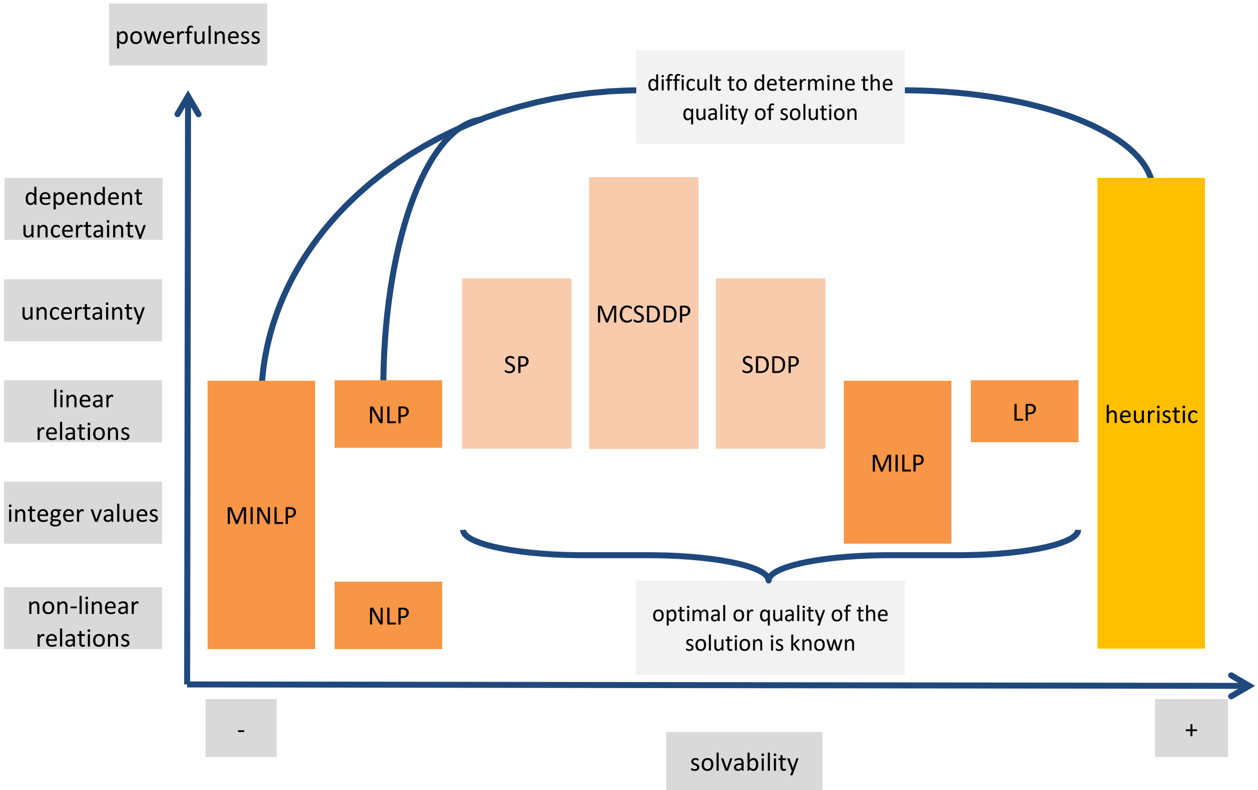


Figure 39 Overview over applied solution methods in hydropower scheduling

5. Quadratic Optimization of Quarter-Hourly and Hourly Day-Ahead Market

The short-term trading within the liberalized energy markets in Europe has gained more and more significance over recent years. The continuously increasing trading demand also led to the introduction of new spot products (EPEX Spot, 2015a, p.20). One example therefore is the launch of the quarter-hourly day-ahead auction for the German market area in December 2014. According to the power exchange EPEX Spot, this was a reaction to an increased day-ahead demand for shorter order types to reduce the magnitude of unbalanced quarter-hourly schedules already day-ahead (EPEX Spot 2015b, p.2).

In this chapter, an electricity storage optimization approach is presented that incorporates short-term energy markets with various temporal resolutions. This means that two or more markets can be considered in the objective of the optimization leaving it up for the solution algorithm to bid for the most lucrative products on each market. Therefore, chapter 5.1 gives an overview over the newly introduced German quarter-hourly day-ahead auction, which reasons the motivation for this multi-market optimization. This is followed by a literature review. In chapter 5.2, particular attention is paid to the limited price sensitivity on the new market which is quantified and compared to the results derived from the hourly day-ahead auction analysis. This market analysis is based on a paper written by Braun and Brunner (2018). In chapter 5.3, a multi-market quadratic optimization approach is presented including a negative price response based on the calculated price sensitivity. First, a two-stage optimization, based on Braun (2016b), and second, an extended multistage method is introduced. Finally, chapter 5.4 presents the numerical results of two case studies, including optimal schedules, profit, and steering parameters showing the practical applicability. Subsequently, a discussion and a conclusion are provided.

5.1. Introduction

This chapter expounds the influence of variable RES on the newly introduced Germany day-ahead market. This includes especially the trading strategy for unbalanced quarter-hourly day-ahead schedules of the RES solar and wind as well as ramps of inflexible thermal power plants. A distinct zigzag price pattern can be observed on the quarter-hourly market that might be caused by the interaction with the hourly day-ahead auction due to volume and liquidity differences. A higher price sensitivity on the quarter-hourly market than on the hourly market can be expected. For market participants, a deep understanding of the new market is crucial to grasp the interdependencies with other short-term markets and especially, to know how sensitive market prices are if additional amounts of energy are bid into the market.

The characteristics and the influence of different production types, such as solar and wind, on the quarter-hourly market will be described in 5.1.1. The resulting price changes open up new optimization and profit possibilities for pumped hydropower storages. Furthermore, chapter 5.1.2 presents the dimensions of market liquidity to derive a price sensitivity that can be applied on the German day-ahead markets. Chapter 5.1.3 provides a literature overview on valuable multi-market hydropower optimization approaches focusing on spot market bidding including any kind of price maker consideration. In the afterwards following chapters this market and price sensitivity analysis is used in the optimization.

5.1.1. Quarter-Hourly Day-ahead Auction

As a consequence of the need for shorter day-ahead order types especially because of the increasing solar power production, politics, BNetzA and EPEX Spot have worked on improving the market conditions and therefore introduced the quarter-hourly day-ahead auction in December 2014. In addition to the already existing hourly day-ahead auction and the quarter-hourly and hourly intraday continuous markets, this new market is the fourth short-term energy-only EPEX Spot market for the German market area. The bidding for this new auction takes place at 3 pm day-ahead, three hours after the hourly day-ahead auction. The EPEX Spot named the new market 15-min. Intraday Auction as it marks the opening of the continuous intraday trading for the following day that starts afterwards. Nevertheless, since this auction takes place day-ahead, the appellation quarter-hourly day-ahead auction is used throughout this paper to avoid confusion with the continuous intraday trading.

The chapter is subdivided into three parts to specifically analyze this new auction in terms of RES impact, price structure and the influence on the pumped hydropower storage dispatch. This helps to understand the special nature of this market and sets the foundation for the price sensitivity comparison of this new market with the existing hourly day-ahead market in the next chapter. For a general descriptive introduction of the market in terms of price level and trading volume see the basics chapter 2.2.3.

Impact of Renewable Energy Sources

One of the reasons for the introduction of the quarter-hourly day-ahead auction in Germany is the better integration of RES (EPEX Spot, 2015b, p.2). Therefore, below the correlation between different types of varying RES and the new quarter-hourly day-ahead auction are analyzed. Figure 40 a) depicts the 1,800 quarter-hours of the year 2015 with the highest gradient in solar power production in comparison to the trading volume for the respective quarter hour. The solar power production corresponds to all units that are sold on the market by the TSO under the RES act regime. For this solar power production, a strong Person product-moment correlation with a coefficient of $p = 0.82$ can be determined. All other quarters are less interesting since the quarter-hourly load gradients and therewith the difference between hourly and quarter-hourly RES trading is negligible. For PV, the latter is especially the case during night hours. The strong correlation hints that gradients of solar power production are significant for the quarter-hourly day-ahead auction. This is comprehensible as the solar power production forecast for the next day is already very accurate and follows minute-precise the course of the sun (ENTSO-E, 2017b).

In addition, Figure 40 b) illustrates also the 1,800 quarter-hours with the highest change in wind power production for one quarter of an hour and the corresponding trading volume of this quarter hour. The calculation is also based on the production data of wind units supported by the RES act which accounted for the predominate share of wind parks in Germany. The Person product-moment correlation coefficient for these 1,800 quarter-hours of the year 2015 amounts to $p = 0.35$. Hence, compared to PV the amount of wind power traded, and therefore the influence, on the quarterly day-ahead auction seems rather limited. This conclusion appears reasonable since already the day-ahead wind forecasts in quarter-hourly resolution are quite unprecise and likely to change again until delivery (ENTSO-E, 2017b).

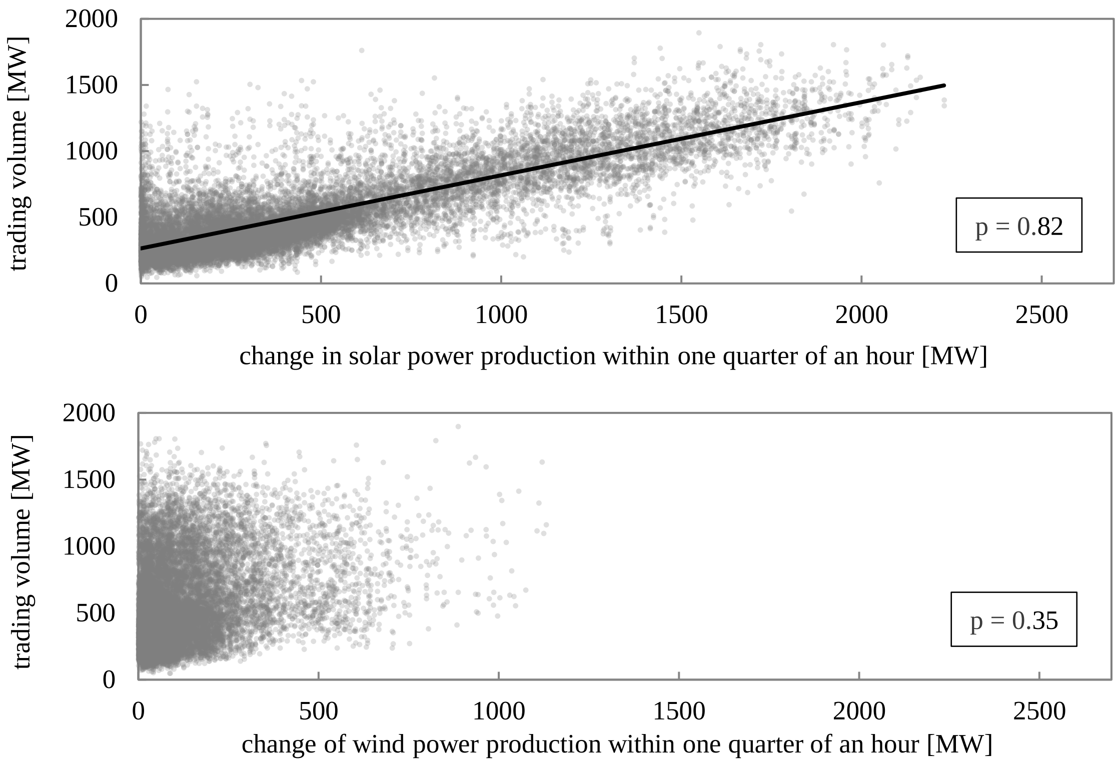


Figure 40 Correlation coefficient p for the 1800 quarter- hours with the highest gradients in solar power production (above) and wind power production (below) within one quarter of an hour and the quarter-hourly day-ahead auction trading volume. Data derived from (ENTSO-E, 2017b; EPEX Spot, 2017b)

Price Structure

In this subchapter, the characteristic price structure of the quarter-hourly day-ahead auction in comparison to the hourly day-ahead auction is explained. The price level over the course of the day generally depends on the underlying merit order and the residual load, i.e. the load subtracted by the variable solar and wind generation. This can be seen in Figure 41 comparing the daily pattern of the average historic hourly prices of 2015 (dashed black line) with the average vertical grid load of 2015 in quarter-hourly time resolution (solid black line). The vertical grid load specifies the load fed-in from the higher transmission grid level into the subordinate distribution grid which adumbrates the residual load comparatively good, as most RES units feed into the grid at medium and low voltage level while conventional thermal power stations that produce the remaining residual load are usually connected to the grid at high voltage level. In addition, the average quarter-hourly prices (solid grey line) of 2015 in Figure 41 show a strong zigzag effect around the hourly prices. To be more specific, the amplitude of the zigzag follows the gradient of the residual load. The firmer the increase or decrease of the residual load the higher is the demand for quarter-hourly trading. The slope of the residual load is a result of the usual demand ramps in the morning and the late evening as well as the solar ramps around midday, as to be seen in Figure 42. These are exactly the quarter-hours with the most distintive zigzag price pattern on an annual average.

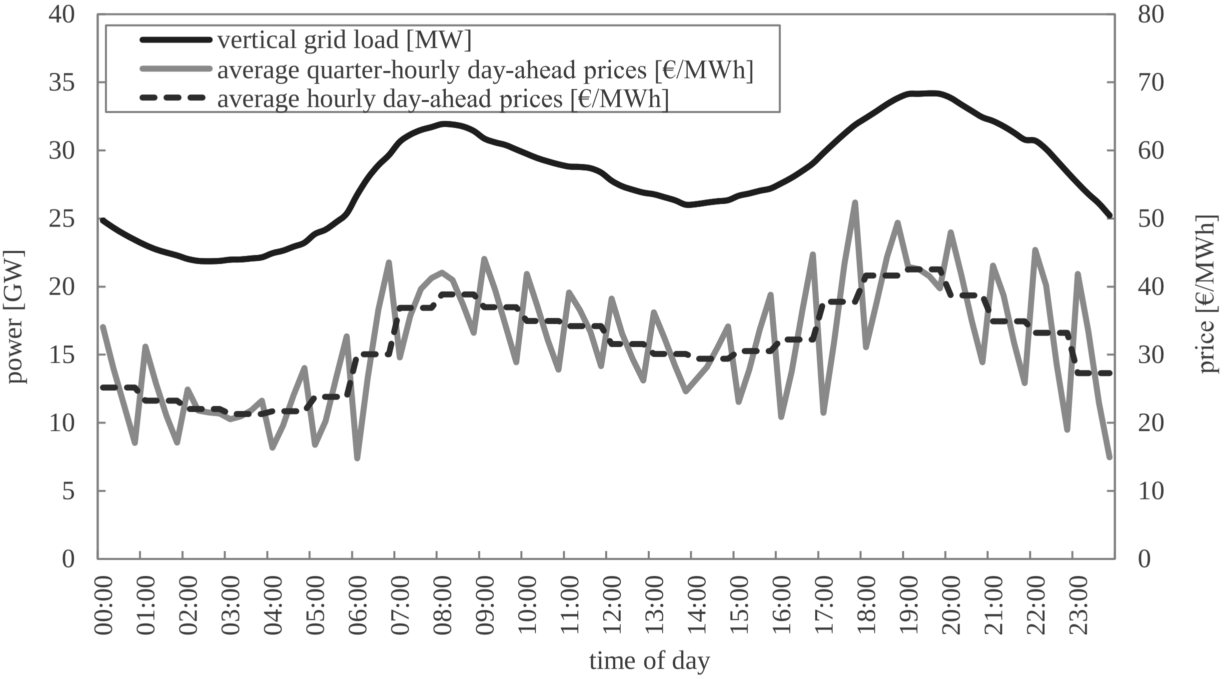


Figure 41 Average hourly and quarter-horly day-ahead MCP over the course of the day compared to the average quarter-hourly vertical grid load for Germany in 2015. Data derived from (ENTSO-E, 2017b; EPEX Spot, 2017b)

The origination of the zigzag effect is exemplarily explained for solar power below but works similar for the zigzags induced by load gradients and ramps of thermal power plants. The strong influence of solar generation is visible in the change of the quarter-hourly trading volume over the course of the year, see again Figure 9. The general way of trading solar power into the hourly and the quarter-hourly day-ahead auctions is illustrated in Figure 42. The solid black line depicts the average solar energy production in Germany. This curve can be approximated by the light grey hourly stepwise bars symbolizing the expected bidding on the hourly day-ahead auction at 12 pm. The difference between the minute-precise forecast and the hourly stepwise function is the remaining amount (solid black bars) that is traded in the quarter-hourly day-ahead auction at 3 pm. During the morning hours when the sun rises, solar power producers have a short position and therefore act as buyers in the first quarter hour and vice versa in the last quarter hour. In the afternoon, this zigzag pattern is the other way around, they hold a long position in the first quarter hour and short positions in the last quarter. Due to the significant difference of trading volume, see again Figure 6 with an average trading turnover of 500 MW for each quarter-hourly product and the difference in price sensitivity that will be analyzed in chapter 5.2.1, it seems plausible to hedge 40,000 MW installed PV capacity (BMWi, 2017a) as well as inflexible ramps of thermal power plants and load at the hourly day-ahead auction and use the quarter-hourly auction only to balance the remaining differences.

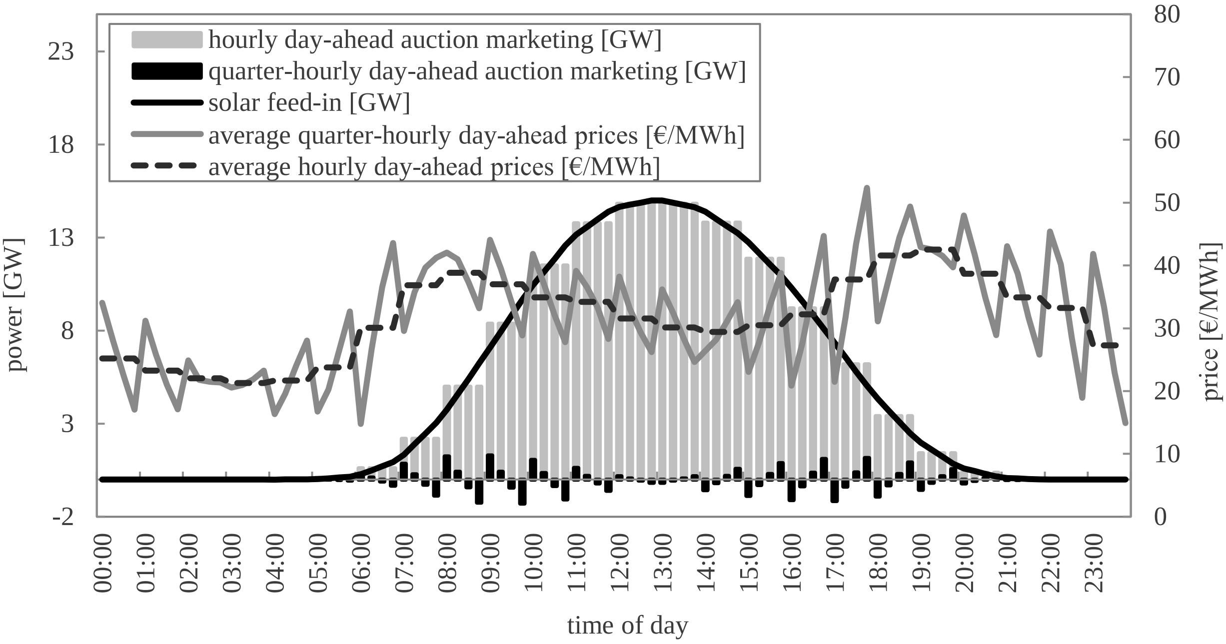


Figure 42 Exemplary solar power trading on hourly and quarter-horly day-ahead auctions in Germany. Data derived from (ENTSO-E, 2017b; EPEX Spot, 2017b)

Influence on the Pumped Hydropower Storage Dispatch

On the one hand, significant flexibility is needed due to variable production profiles of wind and solar power as well as gradual load change rates of thermal power plants. On the other hand, flexibility is provided especially by flexible production and storage capacities but possibly also from demand side management. One of the most important sources for flexibility in Europe are seasonal and daily pumped hydropower storages. As many power plants, pumped hydropower storages are under pressure in terms of profitability due to a lower price level on the spot markets but most notably due the flattened price spreads between peak and off-peak in the recent years (EPEX Spot, 2017b). The introduction of a quarter-hourly auction at 3pm day-ahead (EPEX Spot, 2015a) opens new possibilities for flexible storages.

Figure 43 and Figure 44 illustrate the exemplary dispatch of a pumped hydropower storage. Assuming a shadow price for water release of 42 €/MWh and pumping of 30 €/MWh, the figures show that electricity is generated when the price is above the shadow price und energy is consumed as long as the price is below the pump shadow price. Figure 43 presents the dispatch for a historic hourly day-ahead auction price and Figure 44 demonstrates the dispatch based on the quarter-hourly day-ahead auction price. For the same time period, the price spreads as wells as the total generation and pumping time are significantly higher in the quarter-hourly market for the same time period.

In order to profit from these changes, forecasts, control systems and real-time data transfer is needed to be improved as well as the corresponding data processing and optimization methods. Furthermore, in this example the machine is switched 17 times from pump to generation mode during the 7 days when dispatched according to the hourly day-ahead market. When traded on the quarter-hourly auction the machine is switched 129 times from pumping to generating mode. This means that the integration of the quarter-hourly market does not only has an effect on the profitability but also on the resilience of the machines and the cycle efficiency.

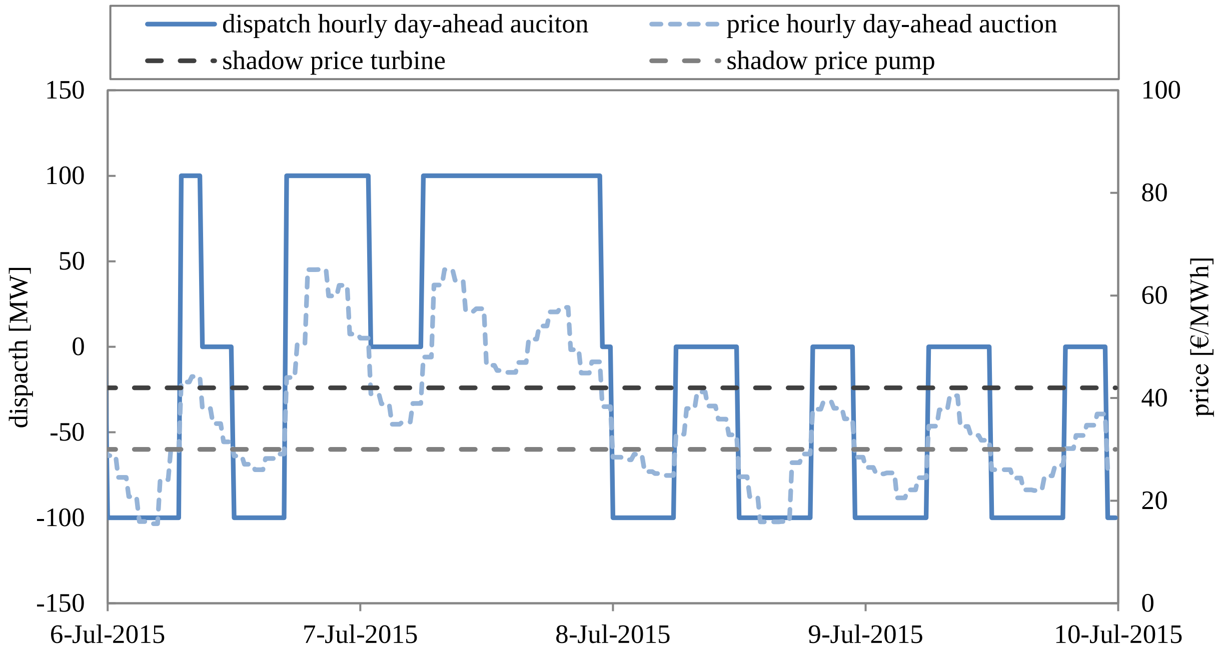


Figure 43 Exemplary dispatch of a pumped hydropower storage on the hourly day-ahead auction, data derived from (EPEX Spot, 2017b).

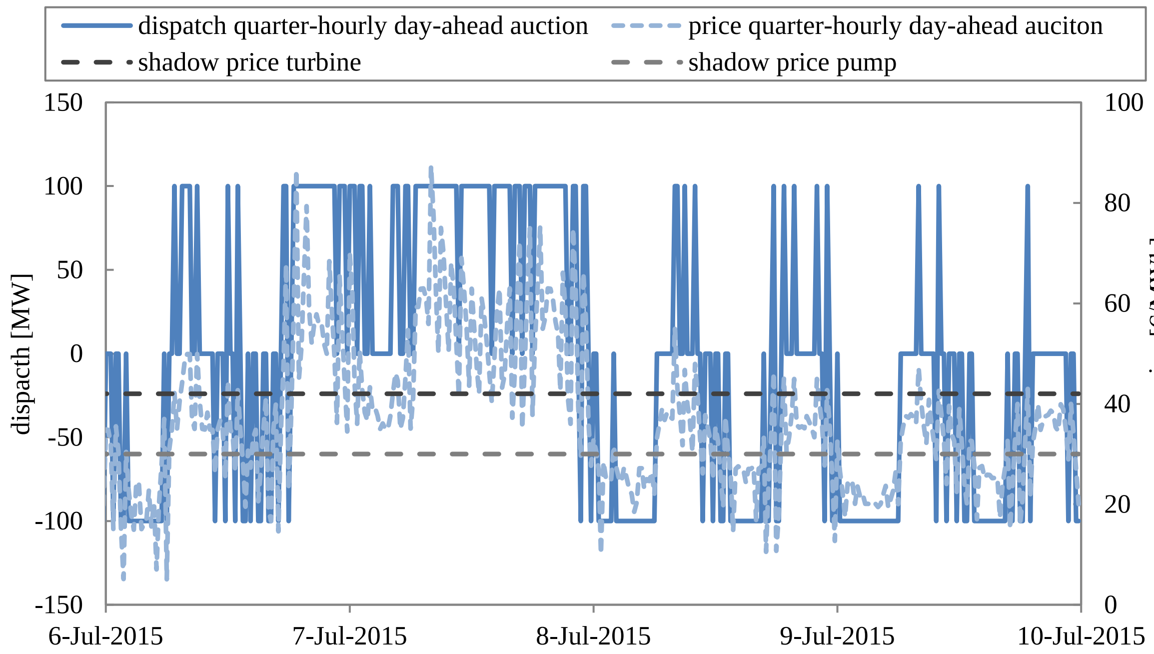


Figure 44 Exemplary dispatch of a pumped hydropower storage on the quarter-hourly day-ahead auction, data derived from (EPEX Spot, 2017b).

5.1.2. Dimensions of Market Liquidity

Market liquidity is a performance indicator for the efficiency of a market. Therefore, liquidity itself can be defined from a trader's perspective as the ability to buy or sell at any size and time without influencing the price with one's orders (Amihud, 2002; Amihud & Mendelson, 1986; Arnott & Wagner, 1990). Literature distinguishes between six dimensions to measure liquidity in a market:

- Bid-ask spread (Amihud & Mendelson, 1986; Hagemann & Weber, 2013; Paukste & Raudys, 2013) and with focus on trading costs (Liu, 2006), spreads (Mayston, Kempf, & Yadav, 2008) and tightness (Kyle, 1985)
- Resiliency (Foucault, Kadan, & Kandel, 2005; Kyle, 1985) that has been used by (Hagemann & Weber, 2013)
- Market depth (Kyle, 1985) and with focus on price impact (Amihud, 2002; Liu, 2006; Weber, 2010)
- Trading activity (Hagemann & Weber, 2013)
- Delay and search costs (Amihud & Mendelson, 1986; Liu, 2006)
- Short-run volatility (Ahn, Bae, & Chan, 2001; Engle, Fleming, Ghysels, & Nguyen, 2012; Handa & Schwartz, 1996)

However, for diverse market designs and considering availability of information different dimensions can be appropriate to measure market liquidity. For example, the European Commission (2007) states the trading volume as an important financial meter for trading activity and liquidity in electricity markets while others stress the market depth, i.e. the ability of market participants to find counterparts, as a significant factor to address market liquidity.

A further attempt is the calculation of price sensitivity as a combination of bid-ask spread, market depth and trading activity (Goyenko, Holden, & Trzcinka, 2008; Kempf, 1999). The advantage of this price sensitivity calculation is that all three approaches can be measured using the characteristics of demand and supply functions. The domain restrictions of demand and supply determine the market depth, the intersection gives indication on the trading activity and the gradient of both functions at their intersection provides the bid-ask spread. Therefore, price sensitivity seems most adequate to analyze the liquidity of auction based energy markets by combining three dimensions.

5.1.3. Literature Review

Generally, literature on hydropower optimization and basic spot markets bidding is numerous. Reviews on short-term power generation and bidding (Kristoffersen & Fleten, 2010), different optimization techniques (Labadie, 2004) and stochastic programming in hydropower scheduling (Klaboe & Fosso, 2013; Labadie, 2004; Wagner & Mathur, 2011; Wallace & Fleten, 2003) are given. Further literature exists on intraday market bidding, see chapter 7, as well as considering balancing power markets, see chapter 8. Nevertheless, this chapter focuses on unified pricing auction based bidding and almost no literature exists on the just introduced quarter-hourly auction based bidding.

Although, high short-term volatility promises significant revenues (Nogales, Contreras, Conejo, & Espinola, 2002), it need to be considered that the quarter-hourly day-ahead but also intraday and balancing markets are smaller in terms of trading volume as the hourly day-ahead market. Therefore, traders are not able to commit any desired quantity to these markets. This implicates for the bidding process that price-taker as well as price-setter need to be considered in the optimization process.

Literature on multi-market bidding lists two approaches to tackle the challenges of price-setter bidding. The first approach considers the reduced liquidity in the short-term by limiting the absolute quantity to be traded on these markets (Deng, Shen, & Sun, 2006; Faria & Fleten, 2011), which is a rather simple strategy. The second approach assumes a negative price response in for example day-ahead (Baillo, Cerisola, Fernandez-Lopez, & Bellido, 2006; Boomsma, Juul, & Fleten, 2014; Ugedo & Lobato, 2010), intraday (Löhndorf, Wozabal, & Minner, 2013) and balancing markets (Boomsma et al., 2014; Plazas et al., 2005). Table 9 provides a short overview on additional literature that can be found dealing with limited liquidity in day-ahead markets. Most authors combine the consideration of market power with stochastic optimization techniques on the costs of reducing the optimization horizon to one day. Below it will be outlined how a precise consideration of price sensitivity can be combined with a detailed power plant layout and a long optimization horizon.

Table 9 Literature on day-ahead market bidding considering price maker behavior in at least one market

authors	markets/objective								technology consideration of			method/ horizon/ country/ main findings
	day-ahead			intraday			balancing					
	price taker	price maker	oligopoly	price taker	price maker	oligopoly	price taker	price maker	hydropower	thermal	pumps	
(Baillo et al., 2006)			x			x		x	x	x		multistage SP/ Spain/ no comparison of separate and coordinated bidding
(Baillo et al., 2004)		x					x		x	x	x	MILP and Benders decomposition/ 1 day/ risk neutral spot bidding with multiple scenarios in hydro-thermal system
(Boomsma et al., 2014)	x	x					x	x	x			multistage SP/ generic example/ coordinated bidding increased profit with 2% if assuming no market power and 1% if assuming market power in the balancing market
(Ugedo et al., 2006)			x			x		x	x	x	x	MILP/ 1 day/ risk neutral, no comparison of separate and coordinated bidding
(Ugedo & Lobato, 2010)		x						x	x	x	x	MILP/ 1 day/ risk neutral optimization with multiple scenarios

5.2. Price Sensitivity of Hourly and Quarter-Hourly Day-Ahead Markets in Germany

Reason for the new quarter-hourly day-ahead market was the need to trade shorter periods than just hours day-ahead to minimize open positions in the more volatile continuous intraday trading. This includes especially the increased production capacity and the resulting trading strategy for unbalanced quarter-hourly day-ahead schedules of solar generation, the ramps of inflexible thermal power plants and the changing load.

Within the current design of short-term electricity markets the key characteristics of the new quarter-hourly day-ahead auction are explained in the first part of this chapter. The dissimilar trading volumes of the hourly and quarter-hourly day-ahead markets and the price zigzag effect of the latter are analyzed and explained. To sketch the specialties of and differences between short-term power markets a method is derived to analyze the price sensitivity of day-ahead markets. Thereafter, the findings of the hourly and quarter-hourly day-ahead auction price sensitivity calculations in Germany for the year 2015 and 2016 are presented. Afterwards, a critical discussion addresses whether the current market design is sufficient or should be further improved and concludes the major findings.

5.2.1. Calculating Price Sensitivity

In this chapter, the price sensitivity of the German quarter-hourly day-ahead auction is determined. The calculation is based on the bid and ask curves derived from (EPEX Spot, 2017b).

Market Clearing Price

Every auction participant hands in one or a set of orders each including the following information: delivery period, product, order direction (buy/sell), quantity q and price c. The energy exchange sorts all buy offers in descending order to receive the ask side merit order $a(q)$ and all sell offers in ascending order to obtain the bid side merit order $\mathrm{b}(q)$. The intersection of both stepwise demand and supply functions defines the MCP c_{MC}^{*} and quantity q_{MC}^{*}. For strictly monotonous increasing functions this point is explicitly defined. For the more general monotonous increasing stepwise functions three different types of intersections can be observed, see Figure 5. In a) and c) it is illustrated that at the intersection, either c_{MC}^{*} or q_{MC}^{*} is not explicitly defined. Normally, the respective exchanges do not publish how c_{MC}^{*} or q_{MC}^{*} are calculated in these cases. Therefore, it is suggested to determine the intermediate of the intersection between ${c_{MC}^{*}}^{+}$ and ${c_{MC}^{*}}^{-}$ or ${q_{MC}^{*}}^{+}$ and ${q_{MC}^{*}}^{-}$ and to compare this with the published MCP. If both do not correspond either block orders or other intersection techniques need to be considered as well.

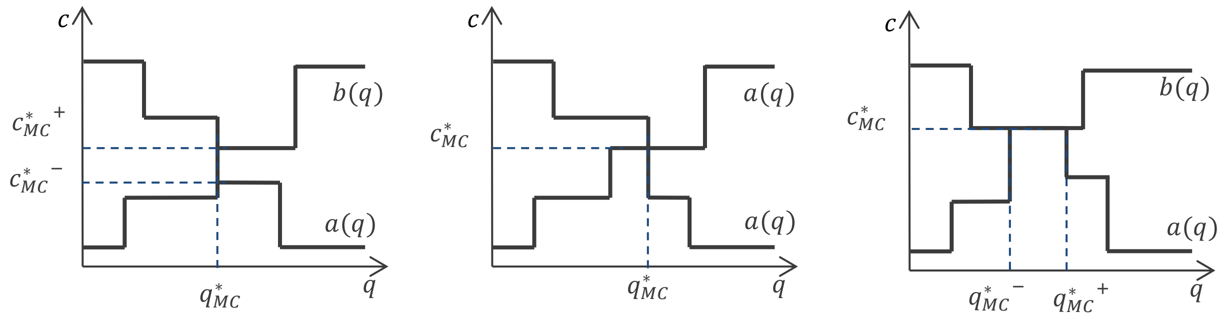


Figure 45 Intersection between ask and bid side merit order and the resulting MCP for three different intersection possibilities. Due to the stepwise character, the intersection is not always explicit.

The price sensitivity l determines the change of the MCP c^{*}_{MC} if an additional quantity Δ is added to the market. It can be distinguished between additional quantity on the bid side $b(q + \Delta)$ and additional quantity on the ask side $a(q + \Delta)$. To determine the price sensitivity, first the bid or ask side merit order is shifted, second the new MCP calculated and third, the price sensitivity computed as the difference of the new MCP minus the original MCP.

The price effect is always equal for a seller reducing (increasing) offers or a buyer lowering (increasing) demand since the two curves intersect at the same price sections just mirrored. This holds true for the intersection of a monotonously increasing (bid) and a monotonously decreasing (ask) function. However, the trading volume changes according to the particular slope of bid and ask curves.

This can be proofed graphically, see Figure 46, defining that f be a strictly monotonically increasing function and g be a strictly monotonically decreasing function. It follows that $(f, g), (f^{*}, g), (f, g^{*})$ have exactly one intersection given that $f^{*}(x) = f(x - a) \wedge g^{*}(x) = g(x + a) \quad \forall x$. Be (t, u) the intersection of f^{*} and g and (v, r) the intersection of g^{*} and f then it need to be demonstrated that $f^{*}(t) = g^{*}(v)$, $u = f^{*}(t) = g(t) = g^{*}(t - a)$ and $u = f^{*}(t) = f(t - a)$ are true. It can be followed that $g^{*}(t - a) = f(t - a) = u$ and g^{*} and f have an intersection at $(t - a, u)$. Since g^{*} and f have just one intersection which is (v, r), $t - a = v$ holds true and from $g^{*}(t - a) = g^{*}(v) \Rightarrow f^{*}(t) = g^{*}(v)$. This is illustrated on the left of Figure 46 for strict monotonous increasing (decreasing) functions and on the right extended for monotonous increasing (decreasing) functions.

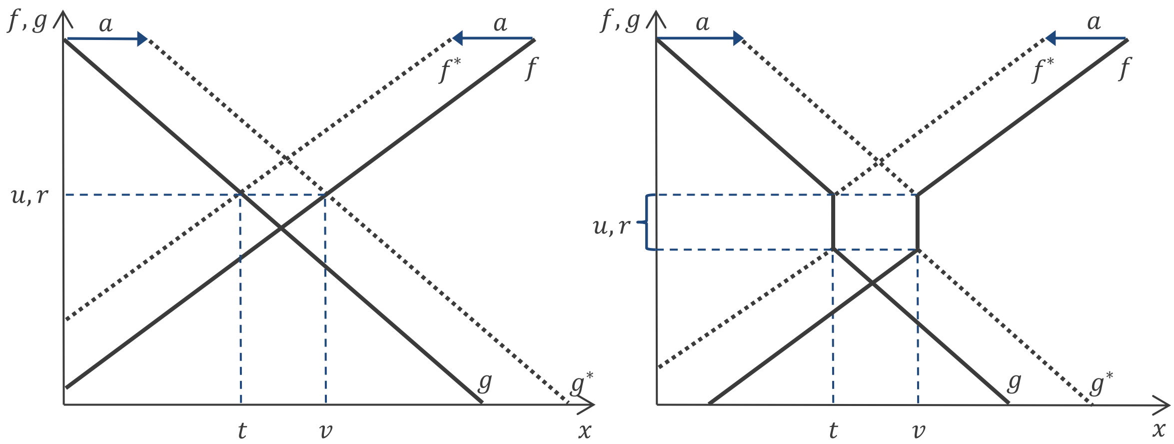


Figure 46 Graphical proof for shifting demand and supply in opposite directions by the same quantity leads to the same MCP. On the left the strict monotonous increasing (decreasing) functions and on the right monotonous increasing (decreasing) functions.

Exemplary Shift of Bid and Ask Curves

Figure 47 depicts an EPEX Spot quarter-hourly day-ahead auction example from October 10th, 2015 for the delivery period from 7:15 pm to 7:30 pm. All offers of designated sellers are cumulated in the bid and all offers of identified buyers are collected in the ask curve. The intersection of the two curves sets the MCP. For the exemplary time period that was 69.81 €/MWh with a trading volume of 350 MW.

Based on this starting point, the price sensitivity can be calculated for new market participants e.g. a pumped hydropower storage operator with a capacity of 200 MW. Therefore, in a first step, the bid curve is shifted by 200 MW to identify the potential price increase due to a market participant willing to buy 200 MW in this auction. In a second step, the ask curve is shifted by 200 MW as well, to derive the possible price decrease a market participant must be willing to accept selling these additionally quantities of energy in the auction. The example for a shift of +200 MW on the buy side is plotted in the center of Figure 47, showing that the potential MCP would shift up to 73.06 €/MWh with an increase of market volume to 392 MW. Vice versa, the expected MCP would drop to 68.37 €/MWh while the trading volume expands even further to 503 MW, if the sell curve is shifted by +200 MW, see to the right of Figure 47.

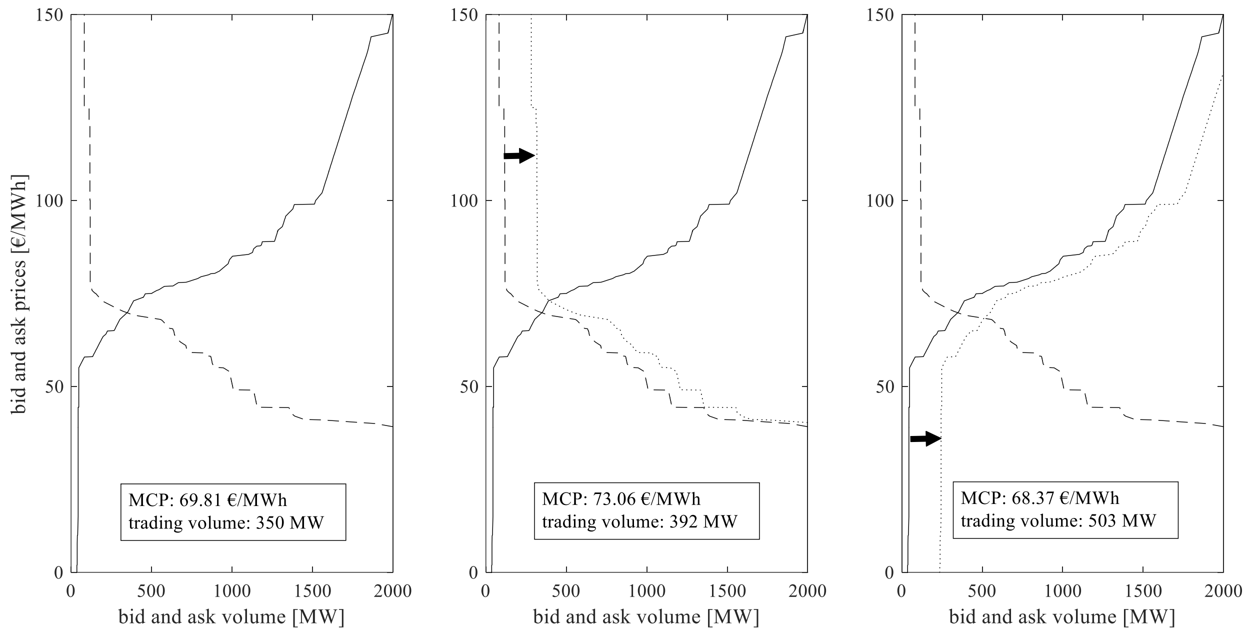


Figure 47 Exemplary bid (solid line) and ask curve (dash-dot line) of the quarter-hourly day-ahead auction for October 10th, 2015 7:15 pm to 7:30 pm showing the MCP and the trading volume (left), with additional 200 MW on the ask side (center) and the bid side leading to a new ask curve (dashed line), MCP and trading volume (right) in each respective case. Data derived from (EPEX Spot, 2017b)

The resulting spread between the original and the shifted MCP can be interpreted as price sensitivity. In this example, the price sensitivity amounts to 3.25 €/MWh per 200 MW for demand and -1.44 €/MWh per 200 MW for supply. That means, a buyer willing to purchase 200 MW for that period should expect to pay 3.25 €/MWh additionally to the observed MCP in the quarter-hourly day-ahead auction. Vice versa, a seller willing to trade 200 MW more might has to accept a markdown of -1.44 €/MWh.

As in the example the price sensitivity calculation can be determined for any time step, product and quantity bid into the market. Computing a detailed course of sensitivity of a specific product is done by running through all incremental quantities bid into the market. This can be done for any uniform market designs based on bid and ask curves, whereas the specifics of the respective electricity market need to be considered. For example, the price on the German quarter-hourly day-ahead auction is limited to -500 and $3000\ €/MWh$ with increments not smaller than $0.1\ €/MWh$.

Results for the German Hourly and Quarter-Hourly Day-Ahead Auctions

As emphasized, the trading volume in the hourly day-ahead market is about 70 times higher than in the quarter-hourly day-ahead market, see Figure 6. Therefore, it is expected that the quarter-hourly market has a higher price sensitivity than the hourly market. That this expectation holds true is shown below by comparing the results for the price sensitivity calculations of the quarter-hourly and the hourly day-ahead auction in Germany for the year 2015 and 2016. The year 2015 is the first complete year after the introduction of the German quarter-hourly day-ahead market in December 2014. The hourly and respectively the quarter-hourly results have been aggregated, clustered and depicted in box plots that present the median in the middle as well as the 25% and 75% quantiles at the bars ends. While the whiskers mark the 5% and 95% quantiles, the dots beyond the whiskers indicate outliers. The results for the two day-ahead auctions are analyzed separately, firstly, the long-established hourly and secondly, the quarter-hourly auction.

Figure 48 displays the price sensitivity boxplots of the coupled German and Austrian hourly day-ahead EPEX Spot auction of all 8760 hours of 2015 and 8784 hours of 2016. The capacity shifts of 50 MW to 6400 MW are set by doubling the additional bid and ask quantity in each step. Between 200 and 3200 MW additional ask quantity, and between 200 and 6400 MW additional bid quantity, the price sensitivity increases almost linear by 0.25 €/MWh per 100 MW bid or ask shift. For shifts up to 1600 MW most of the calculated price deviations for the hourly day-ahead auction are within the box of the boxplot, i.e. they are within the 25% and 75% quantile which indicates a low standard deviation. Generally, from 1600 MW on, a significant number of outliers are visible with noticeable prices over 100 €/MWh. Some of these outliers are likely to originate from exceptional circumstances such as the solar eclipse on March 20th, 2015. Considering the logarithmic scale of the abscises in Figure 48, the median continues to increase nearly linear with additional volume of bid and ask shifts. Furthermore, it can be noticed that for the leading hourly market bid side shifts show lower fluctuations than ask side shifts. This can be explained by a currently strong supply side of production capacity on the German electricity market.

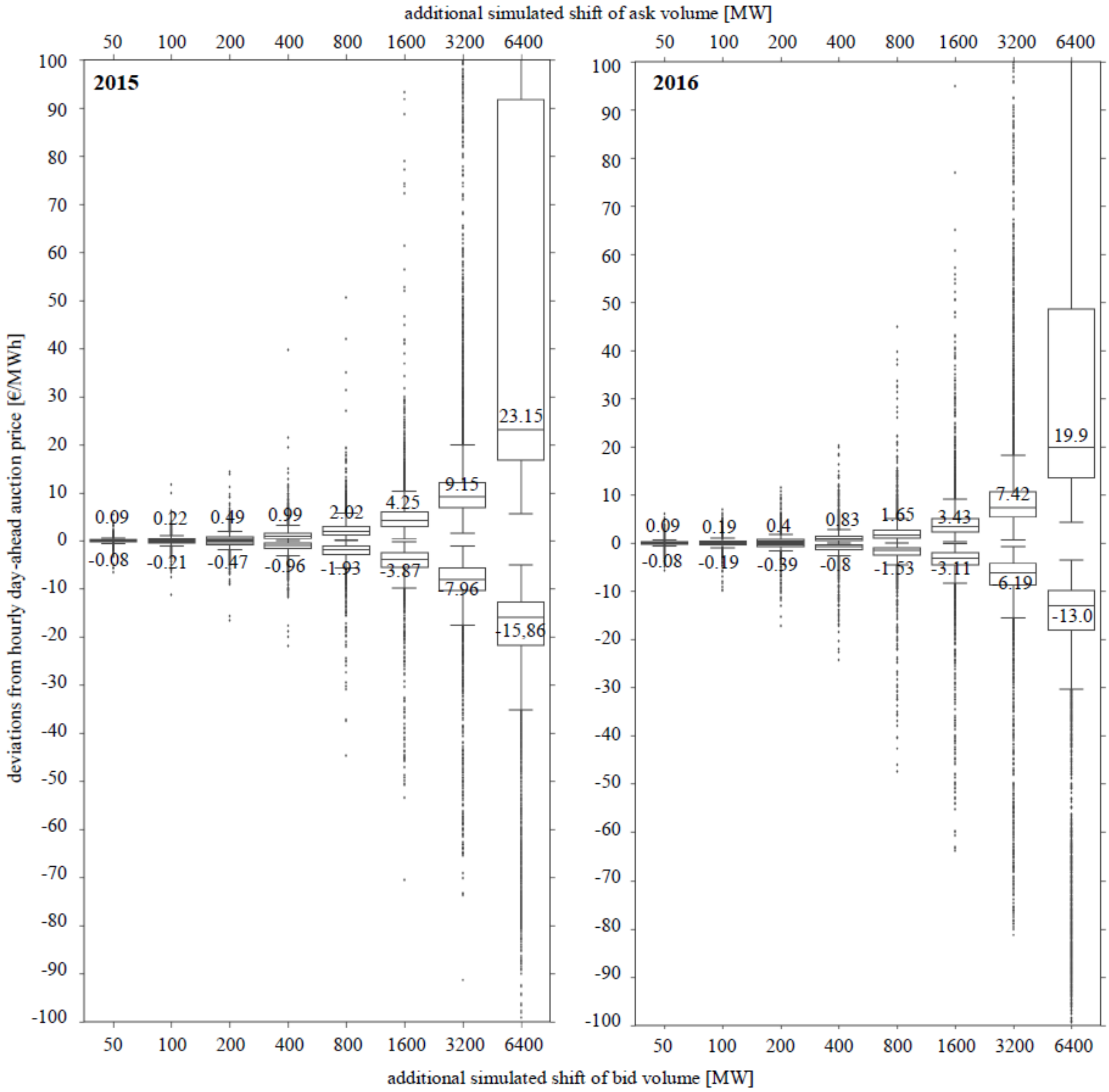


Figure 48 Price sensitivity on the German hourly day-ahead auctions in 2015 and 2016. The numbers show the median of the price sensitivity for each quantity. The bars and points on the positive side of the diagram present the additional simulated shift of ask quantity and the negative the simulated shift of bid quantity. Data derived from (EPEX Spot, 2017b)

The price sensitivity boxplots of the German quarter-hourly EPEX Spot day-ahead auction are shown in Figure 49, summarizing the sensitivity results for all 35040 quarter-hours of the year 2015 and 35126 quarter-hours of the year 2016 for capacity shifts of 50 MW to 1600 MW. Again, the additional bid and ask quantity in each step are doubled. The different limitations of the maximal quantity shift for the quarter-hourly market is based on the observation that the cumulated bid and ask curves rarely exceed 2000 MW and that the average quantity of actual executed orders per quarter-hour is less than 500 MW. The results show that up to 800 MW additional bid and ask quantity the price increase of the median is again almost linear. However, with about 1.5 €/MWh per 100 MW it is six times higher than for the hourly day-ahead auction. And again, for higher changes of bid and ask quantities the price sensitivity increases also disproportionately. But in contrast to the hourly day-ahead auction, the results indicate that the disparity of the price sensitivity on ask and on bid side is less distinct. This rather similar pricing of flexibility in both directions might be a hint that the quarter-hourly auction just levels out the deviations between the hourly products and the continuous course of the residual load.

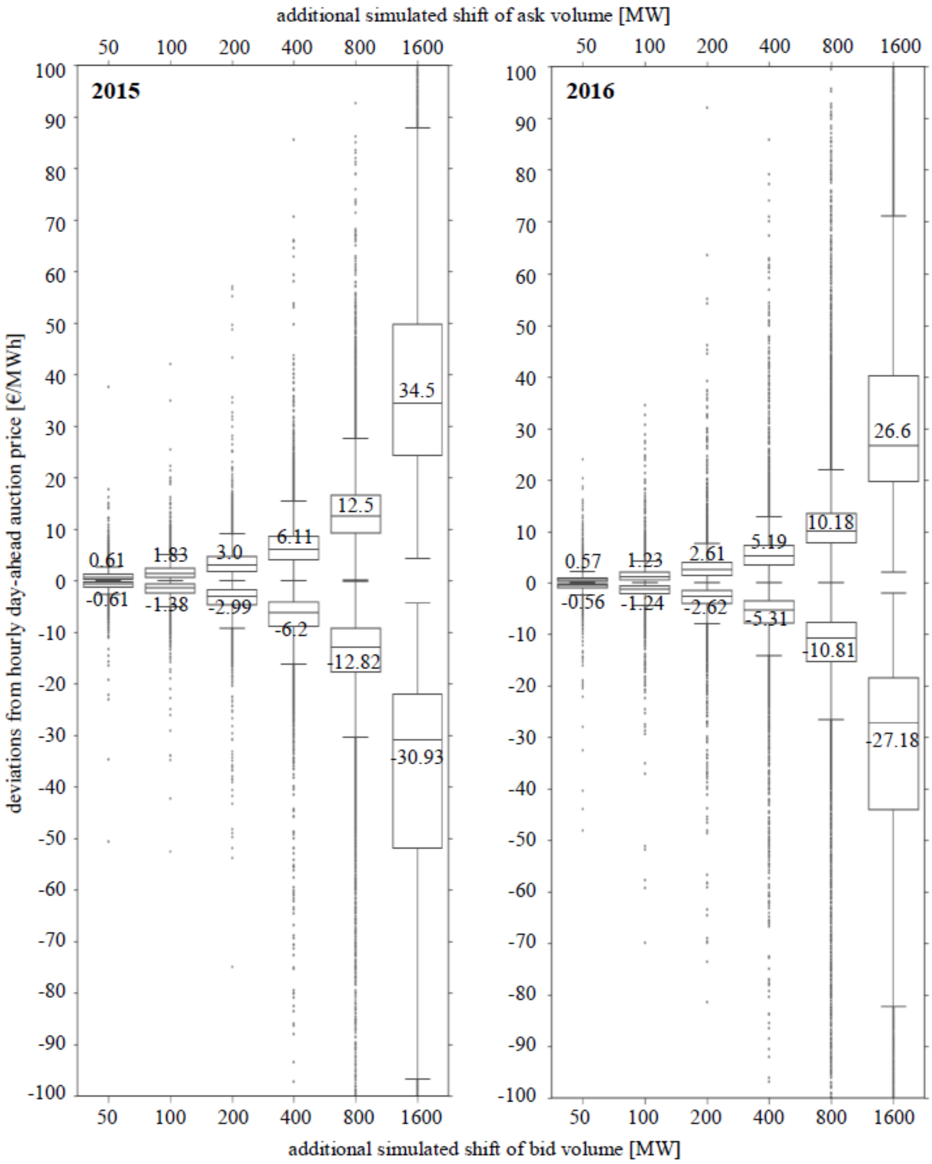


Figure 49 Price sensitivity on the German quarter-hourly day-ahead auctions in 2015 and 2016. The numbers show the median of the price sensitivity for each quantity. The bars and points on the positive side of the diagram present the additional simulated shift of ask quantity and the negative the simulated shift of bid quantity. Data derived from (EPEX Spot, 2017b)

Furthermore, just available and highly flexible technologies bid into the quarter-hourly merit order. This means the quarter-hourly "flexibility merit order" is always steeper and shorter as the hourly merit order. Also, the spread between the lower and upper end of the boxplot opens much faster than for the quarter-hourly market, indicating not just a higher sensitivity but also a larger variance of prices.

5.2.2. Critical Discussion

In this part, the quantitative price sensitivity calculation from the previous chapter is broadened and a qualitative discussion is added. In the first part, motivation and key drivers for the quarter-hourly day-ahead auction are revised. It is continued with the implication of divergent price sensitivities on the hourly and quarter-hourly day-ahead auctions for market participants and the challenge of providing an efficient market regime that supports RES as well as flexibility. Finally, the results are discussed in the context of further developments of the short-term power market design.

Impetus for the new quarter-hourly day-ahead market was the need to trade shorter time periods than just hours in order to minimize risky open positions in the quarter-hourly continuous intraday trading already day ahead. As pointed out in chapter 5.1.1, one main driver that boosted this requirement for new short-term power products was the increased production capacity of solar power (see Table 2 and on the left of Figure 40). Since the day-ahead forecasts for solar feed-in is more precise in comparison to onshore and offshore wind, the quarter-hourly day-ahead market is solar dominated. This can be substantiated by comparing forecast and actual generation data for Germany published for example by ENTSO-E Transparency Platform (ENTSO-E, 2017b). Hence, as the day-ahead forecast of wind is less reliable it can be assumed that the trading of wind power forecast deviations is rather limited to the intraday continuous market. However, to verify the latter argumentation further research is needed.

In the quarter-hourly day ahead market a distinctive zigzag price formation is apparent, see Figure 41. Three influencing factors can be enumerated: the trading of solar power ramps around midday as well as the gradients of consumption and thermal power plant ramps throughout the course of the day. Due to the characteristic two stage market design with highest liquidity on the hourly and lower liquidity on the quarter-hourly auction, hedging solar generation, inflexible thermal power plants and demand at the hourly day-ahead auction and using the quarter-hourly auction only to balance the remaining differences appears reasonable from an economic perspective. Generally, both day-ahead markets are nearly arbitrage free comparing the EPEX Spot day-ahead auction results, i. e. over a certain period of time the average price of the respective four quarter-hours equals the hourly price.

Regardless of flexibility support and zigzag price curve explanations, from a fundamental perspective, it seems questionable that the last quarter of the previous and the first quarter of the next hour deviate disproportionately compared to changes of residual load as it is for instance the case in the morning and in the afternoon or evening (see Figure 41). Ceteris paribus, if there are just changes of solar production and variations of the overall demand that is covered by thermal production, the prices should continuously follow the height and the gradient of the solar residual, i.e. overall electricity demand subtracted by solar generation. On the left-hand side of Figure 50 the solar power trading regime with the actual hourly and quarter-hourly market prices is presented and on the right a possible resulting single quarter-hourly price curve as dotted black line if both markets would be cleared together is sketched. It could be that a joint hourly and quarter-hourly day-ahead auction would ensure a more efficient market clearing due to the bundling of market liquidity. Furthermore, from a fundamental perspective, the zigzag prices would smoothen since there is no reason for an extreme price difference between the last and the first quarter of two neighboring hours. However, the example of the Energy Exchange Austria (EXAA), that clears both quarter-hourly and hourly day-ahead auction together since 2014, is still characterized by a zigzag course (EXAA, 2014, p.4, 2017). If that is due to the case that market participants with need for quarter-hourly day-ahead products stick to their hedge oriented bidding strategies on the more liquid subsequent EPEX auction for the same market areas where both day-ahead auctions with their significantly different trading volumes are timely separated or if there are other reasons that prevent a smoother quarter-hourly price structure should be subject to further research. Generally, it can be assumed that in parts with a strong change of the residual load the quarter-hourly prices result from a specific flexibility merit order which corresponds to a merit order without inflexible power plants. A flexibility merit order is therefore significantly steeper resulting in pronounced price changes.

The present two stage market design with its respective price sensitivities has consequences for most market participants. A solar producer, for instance, that would bid the whole generation only in the more price sensitive quarter-hourly day-ahead market is likely to cause higher self-induced price changes than in the less price sensitive hourly day-ahead market. With the shift towards auction-based support tariffs (BMWi, 2015, p.81), more solar power plants will be traded by the owners or a third-party under the so-called direct marketing regime. Hence, without the guarantee of fixed feed-in tariffs, price sensitivity and price differences of the various short-term markets already need to be included during the investment decision of new solar power plants. The same accounts for operators with flexible capacity. To maximize revenues, they have to take into account the price sensitivities at all short-term markets. The business case of e. g. storage units is to deliver energy in high price periods and store energy during times with low prices. The growth in solar power generation is likely to increase these temporal price spreads especially in the quarter-hourly day-ahead market. The idea of the further development of the German electricity market is that such price signals will incentivize investment in additional flexibility, especially if scarcity prices are permitted (BMWi, 2015, p.48). However, in the long-term these higher spreads might also lead to cyclical fluctuations of available capacities for storage and other flexibilities similar to the economic effect known as pork cycle. This means, that on the one hand, an increased price volatility induced by more solar power will enhance the profitability of e.g. storage units and therewith attract more investment in such flexible capacities. But on the other hand, the more flexibility is provided to the market the smaller are the spreads between the quarter-hours. Hence, the required mix of different measures to meet the future level of system flexibility will only be reached at lowest cost if an appropriate market design is in place that enables a technology neutral competition between all flexibility alternatives (Brunner, 2014b, p.8). This perception is also shared by the Federal Ministry of Economic Affairs and Energy, which published the white paper “An electricity market for Germany’s energy transition” in 2015 showing first approaches to develop a market design that fosters additional flexibilities in a competitive environment (BMWi, 2015).

It can be expected that if the future energy system becomes more flexible it is also likely to affect the trading volumes and therewith the price structure and sensitivities of today’s spot markets for power. Hence, the introduction of a quarter-hourly day-ahead auction is a first step towards an adequate market design for power systems with a high share of variable RES. And it may serve as a possible role model for other countries with liberalized energy markets, particularly when seeking to increase their share of solar power generation. But there are also other market mechanisms, such as additional intraday clearing auctions that are already in place in Spain, Portugal or Italy, see Table 2 or Table 3. Again, further research is needed to evaluate which set of market instruments and new spot products in future electricity markets are needed to integrate higher shares of variable RES most sufficient and thus improves the economic welfare.

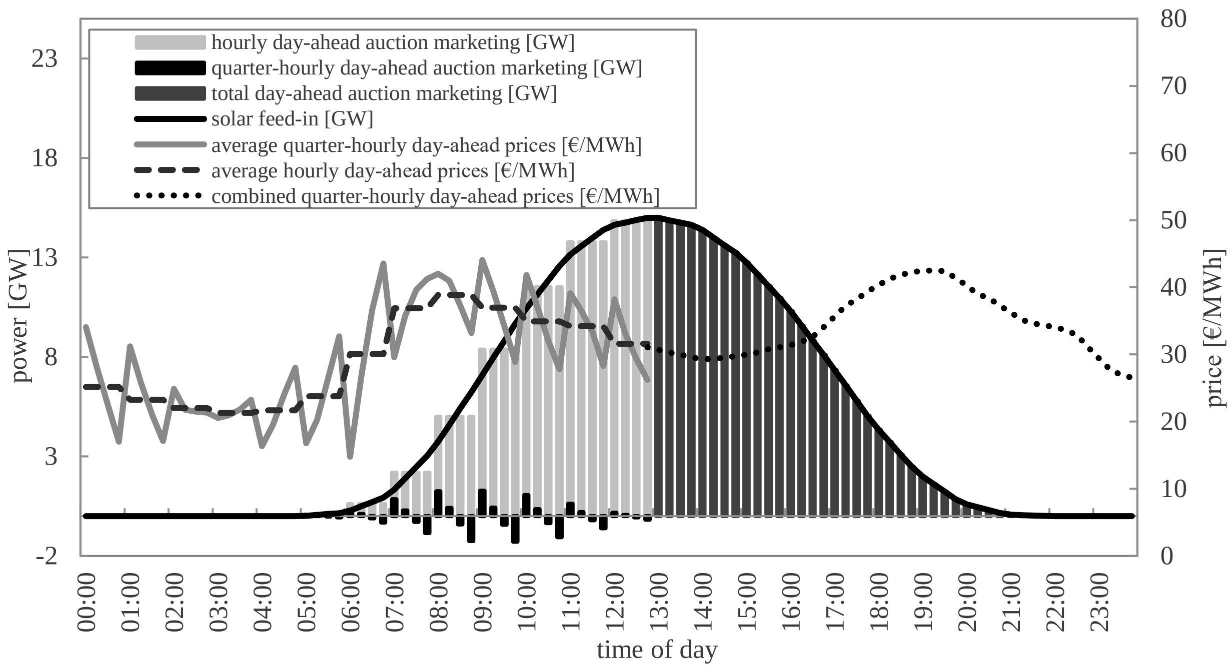


Figure 50 Exemplary solar power trading on hourly and quarter-hourly day-ahead auctions; on the left based on the existing two-stage market design and on the right the estimated price and quantity for an integrated market, Data derived from (EPEX Spot, 2017b)

5.3. Quadratic Multi-Market Optimization

Especially for flexible pumped hydropower storages the new German quarter-hourly day-ahead auction has been a possibility to realize higher spreads than in the proven hourly day-ahead auction. With electricity prices as the most important value driver, it seems logical to integrate also quarter-hourly prices into the pumped hydropower optimization (Braun, 2015a).

As a result of the previous market analysis, the limited price sensitivity on the quarter-hourly market need to be considered to receive realistic model results. Hence chapter 5.3.1 generally explains how price sensitivity should be considered in hydropower scheduling optimizations. Afterwards, the model set up for the two following multi-market optimizations is defined in chapter 5.3.2. In chapter 5.3.3, a two-stage solution approach based on Braun (2016b) is presented. Here, first an hourly day-ahead market optimization is performed followed by a quarter-hourly post-optimization based on the already calculated schedules. This approach is further developed and presented as an integrated model in chapter 5.3.4.

5.3.1. General Remarks Considering Price Sensitivity

Two different possibilities of including the price sensitivity ℓ in the profit calculation can be distinguished and need to be considered. The main question is, whether the bidder has been active on the market before, meaning the bidders historic offers are already included in the bidding curves that were used to determine the price sensitivity, or not. The difference between both possibilities is leveraged with the quantity the new bidder is offering additionally to the market so that the overall trading volume increases. Both effects are consecutively discussed. Generally, the maximization of the profit $G(q)$ formulates as price $c(q)$ times quantity q minus costs $\mathcal{E}(q)$:

$$G(q) = c(q) \cdot q - \mathcal{E}(q). \tag{42}$$

The price sensitivity ℓ is the induced price change for additional energy in the market. For $\ell = 0$ the price is not influenced by the quantity supplied or demanded and therefore complete inelastic. The greater ℓ the stronger is the influence of quantity changes on the price. To determine the effect of the price sensitivity on the price the following approach can be used.

$$\begin{aligned} c(q) &= c_0 - \ell q \\ \ell &= -c'(q) \end{aligned} \tag{43}$$

This corresponds to a new bidder in the market that wants to find out what profit one can expect entering the market. The respective profit can be determined using

$$G(q) = c_0 q - l q^2 - \mathcal{E}(q). \tag{44}$$

In case a market participant has already a share in the market and wants to include price sensitivity into the optimization, this is more difficult to estimate. There are two possible ways to consider price sensitivity in this event. Firstly, the price sensitivity calculation itself is adjusted by the market participant's historic trading volume or secondly, the optimization distinguishes between the energy that has been typically traded q_0 for the typical price c_0 on the market and quantities that are traded in the market additionally. The result is the overall quantity q that is planned to be traded into the market. Then, just the additionally traded quantity $(q - q_0)$ is strained with the price sensitivity ℓ:

$$c(q) = c_0 - \ell(q - q_0). \tag{45}$$

$$G(q) = (c_0 + \ell q_0)q - \ell q^2 - \mathcal{E}(q). \tag{46}$$

If the quantity supplied q is equal to the typical quantity q_0, the market price is, ceteris paribus, the same.

A crucial point is to determine the typical quantity supplied q_0. This can be done using the optimality condition of the objective function and insert q for q_0. I. e. when the profit is maximized then q is the profit optimizing quantity of the bidder. For the latter, the derivative of the profit function is needed.

$$G'(q) = c_0 + \ell q_0 - 2\ell q - \mathcal{E}'(q) \tag{47}$$

Further, set $q_0 = q$ to receive

$$\tilde{G}'(q) = c_0 - \ell q - \mathcal{E}'(q), \tag{48}$$

and respectively

$$\tilde{G}(q) = c_0 q - \frac{1}{2}\ell q^2 - \mathcal{E}(q). \tag{49}$$

Concluding, a new bidder should use equation (44) and a bidder that bids regularly the quantity q_0 into the market should rely on (47). In the following problem formulation as well as in the case study an already active bidder is considered since most pumped hydropower storages are existing installations. This approach is also applicable for the flexible demand side, e.g. for the investment decision in a new PV plant the operator should consider equation (44) to determine the future power plant's income.

5.3.2. Model Set Up

In this part, the model set up, for the pumped hydropower storage optimization approach, is presented in the next two chapters 5.3.3 and 5.3.4, incorporating short-term energy markets with various temporal resolutions. This means that two or more markets can be considered in the objective function of the optimization leaving it up to the solution algorithm to bid on the most lucrative markets.

The multi-market model considers grid charges, efficiencies, multiple time steps and prices (e. g. quarter-hourly and hourly), inflows, hydraulic short circuit ability, spillage as well as the possibility to optimize all different kind of pumped hydropower storages (e. g. daily, weekly and seasonally). The equations of the optimization problem are defined based on the hydropower scheduling problem defined in chapter 4.1.2. The following symbols are used:

State variable:

- filling level [1000m³]: $v_{t,r}$, $v_{t(t),r}$

Decision variables:

- turbine power [MW]: $u_{t,m}$, $u_{t(t),m}$
- pump power [MW]: $p_{t,m}$, $p_{t(t),m}$
- spillage [1000m³]: $s_{t,r}$, $s_{t(t),r}$

Additional decision variables for the two-stage optimization:

- turbine power quarter-hourly [MW]: $u^{qh}_{t(t),m}$,
- pump power quarter-hourly [MW]: $p^{qh}_{t(t),m}$,
- turbine power hourly [MW]: $u^{h}_{t,m}$,
- pump power hourly [MW]: $p^{h}_{t,m}$,

Indices:

- time stages [hourly]: $t = 1, \dots, T$
- time stages [quarter-hourly]: $\mathcal{t}(t) = 1, \dots, \mathcal{T}$
- reservoirs: $r \in R$
- machines: $m \in M$
- machine below reservoir: m$\in \underline{rm}$
- machine above reservoir: $m \in \underline{mr}$

Parameters:

- hourly prices [€/MWh]: c_t^h,
- quarter-hourly prices [€/MWh]: $c_{\mathcal{t}(t)}^{qh}$
- inflows [1000m³]: $v_{t,r}^{in}$, $v_{\mathcal{t},r}^{in}$
- specific discharge turbine [1000m³/MWh]: η_m,
- specific charge pump [1000m³/MWh]: ρ_m,
- grid charges [€/MWh]: $n_{t,m}$, $n_{\mathcal{t}(t),m}$
- limits for spillage [1000m³]: s_r^{min}, s_r^{max}
- limits for filling level [1000m³]: v_r^{min}, v_r^{max}
- start filling level [1000m³]: $v_{t,r}^{start}, v_{\mathcal{t}(t),r}^{start}$
- end filling level [1000m³]: $v_{T,r}^{end}, v_{\mathcal{T},r}^{end}$
- limits for turbine capacity [MW]: u_m^{min}, u_m^{max},
- limits for pump capacity [MW]: p_m^{min}, p_m^{max}
- limits for filling level [1000m³]: v_r^{min}, v_r^{max},
- limits for turbine capacity [MW]: u_m^{min}, u_m^{max},
- price sensitivity factors [€/100MW] $\ell_{\mathcal{t}(t)}^{qh,sell}, \ell_{\mathcal{t}(t)}^{qh,buy}$

5.3.3. Two-Stage Optimization

This part introduces the multistage mid- to short-term model based on Braun (2016b). On the first stage, the general optimization is introduced, aiming to find the optimal production schedule and water values for the hourly day-ahead market. On the second stage, the model is extended and performs a post-optimization using the quarter-hourly day-ahead auction prices and the hourly schedules of the foregoing optimization.

First Stage Optimization

The first stage optimization has an hourly time resolution $t = 1, .., T$. For the profit maximization problem, the price spread and the absolute height of the price c_t are important. The profit is summed up over the time t and machines m consisting of pumps and turbines with a respective capacity of $p_{t,m}^{max}$ and $u_{t,m}^{max}$. Further, r denotes the reservoirs. The location of machines in the reservoir cascade is modeled using $\underline{rm}$ which indicates machines located below a reservoir and $\underline{mr}$ which marks machines located above a reservoir. The efficiencies describe the proportion of the water flow rate in $1000m^3$ per produced or captured energy in MWh. The objective function determines the profit restricted by the constraints:

$$
\begin{array}{llll}
P: & \max\limits_{u,p,s} = & \sum_{t,m} c_t^h u_{t,m}^h - \left(c_t^h + n_{t,m}\right) p_{t,m}^h & \\
& s.t. & v_{t,r} = v_{t-1,r} + v_{t,r}^{in} - s_{s,t} - \sum_{m \in \underline{rm}} (u_{t,m}^h \eta_m) + & m \in M, \\
& & \qquad \sum_{m \in \underline{mr}} (p_{t,m}^h \rho_m) & t = 2, \dots, T \\
& & v_{t,r} = v_r^{start} & r \in R, t = 1 \\
& & v_r^{min} \leq v_{t,r} \leq v_r^{max} & r \in R, \\
& & & m \in M, t \in T \\
& & v_{t,r} = v_r^{end} & r \in R, t = T \\
& & 0 \leq s_{t,r} \leq s_r^{max} & r \in R, t \in T \\
& & 0 \leq p_{t,m}^h \leq p_m^{max} & m \in M, t \in T \\
& & 0 \leq u_{t,m}^h \leq u_m^{max} & m \in M, t \in T.
\end{array}
\tag{50}
$$

The most important constraints of the optimization are the reservoir filling level equations for every reservoir in the cascade. The reservoir filling level $v_{t,r}$ in time t and reservoir r is a summation of the filling level $v_{t-1,r}$ in the time stage $t-1$, plus the water that is released from an above located reservoir $u_{t,m}\eta_m$ in time stage t, plus the water that is pumped from a below located reservoir $p_{t,m}\rho_m$ in time stage t, subtracting the water that is pumped into the upper reservoir $p_{t,m}\rho_m$ in time stage t and subtracting the water that is released into the below located reservoir $u_{t,m}\eta_m$ in time stage t. If inflows $v_{t,r}^{in}$ are considered they are added as well.

The reservoir filling levels $v_{t,r}$ have to be within the minimum and maximum reservoir filling levels v_r^{min}, v_r^{max}. The restrictions of reservoir and machines are defined independently in time, assuming to hold over the whole optimization period. Nevertheless, they can also be modeled for every time stage if necessary. The end or target reservoir filling level $v_{T,r}^{end}$ should be an experienced value or the intermediate result determined in a preoptimization with a longer optimization period. The spillage $s_{t,r}$ is an option to release water in case the reservoir is filled, and the inflows exceed the flow through rates of the turbines. This variable is important to avoid insolvabilities, whereas it should be taken care of that the model does not shift water within reservoirs without being considered in the objective function. Whether grid charges $n_{t,m}$ are considered depends on the location of the system and the respective regulatory framework. At least in some countries grid charges are an authoritative parameter.

Second Stage Quarter-Hourly Optimization

The quarter-hourly day-ahead market can additionally be exploited using a post-optimization based on the results of the hourly day-ahead market optimization.

On the second-stage post-optimization a quarter-hourly time resolution is used, e. g. over the course of one year, $\mathcal{t}(t) = 1, \dots, \mathcal{T}$. As introduced in chapter 5.2.1, the hourly day-ahead market can be assumed as a sufficient liquid market. However, considering other energy only markets, either quarter-hourly day-ahead or intraday market, it is crucial to take price sensitivity into account. This means, it is not possible to trade any desired quantity at the given price. A price sensitivity factor $\ell_{\mathcal{t}(t)}^{qh}$ for the German quarter-hourly day-ahead auction is described and determined in chapter 5.2.1. The suggested price sensitivity depends on the shape of bid and ask curves and determines the quantity related price effect. Therefore, the optimization problem transforms into a quadratic problem:

$$\max_{u,p,s} = \sum_{t(t),m}\left[\left(c_{t(t)}^{qh} - u_{t(t),m}^{qh} \cdot \frac{1}{2} \ell_{t(t)}^{qh,sell}\right) \cdot u_{t(t),m}^{qh} - \left(c_{t(t)}^{qh} + n_{t(t),m} + p_{t(t),m}^{buy} \cdot \ell_{t(t)}^{qh,buy}\right) \cdot p_{t(t),m}^{qh}\right] \tag{51}$$

The optimal production schedule of the first stage optimization $\left(u_{t,m}^{h}{}^{*} p_{t,m}^{h}{}^{*},\right)$ for $t = 1, \dots, T$ and $m = 1, \dots, M$ is used as input parameter for the second stage model. With $u_{t,m}^{h}{}^{*}$ considered as an already done sell-trade and $p_{t,m}^{h}{}^{*}$ as an already done buy-trade on the hourly day-ahead auction. In the trading equation (52) this information is used. Therefore, the final production $\left(u_{t(t),m} - p_{t(t),m}\right)$ depends on the already traded and the quarter-hourly day-ahead auction quantity.

$$u_{t(t),m} - p_{t(t),m} = u_{t,m}^{h} - p_{t,m}^{h} + u_{t(t),m}^{qh} - p_{t(t),m}^{qh} \tag{52}$$

All further constraints correspond to the hourly optimization but with a quarter-hourly adjusted time resolution.

This two-stage model provides important information for the multi-market bidding, such as optimal power plant schedules with respective shadow prices for hourly and quarter-hourly day-ahead market and considering the observed price sensitivity. The two-stage model is a straight forward approach since it resembles the temporal chronology of the auctions. But this approach has one significant disadvantage. Shadow prices are generated for both markets separately, which also means that for the same time stage two different shadow prices are theoretically possible. This can be explained by the way how the shadow prices are calculated, see 3.4. Although the models solve for optimal solutions, in times without dispatch or when the marginal price is far away from the market price, the water values are not defined uniquely. Such effects can lead to incomprehensible results and are difficult to communicate in practice.

Therefore, in the next chapter 5.3.4 both problems are solved in one integrated optimization. This means just one reservoir filling level equation and one shadow price applying for both markets, which makes the trading more traceable. Nevertheless, both optimizations find optimal solutions and lead to the same profit and production schedules.

5.3.4. Combined Optimization

For the integrated model, beside $t = 1, .., T$ a further time set for the higher resolution time stages is introduced as well, with $t(t) = 1, \dots, \mathcal{T}$. The ratio of $\mathcal{T}/T$ determines the number of intermediate steps of the finer time resolution. For a quarter-hourly time resolution this is 4, if 5 min products are considered this is 12. This means the formulation is flexible in terms of possible further markets and time resolutions that may be introduced in the future. Nevertheless, for readability, the different parameters and variables are assigned with the super script letters h and qh for the hourly and quarte-hourly day-ahead markets. The new problem writes as below:

$$
\begin{array}{lll}
P: \quad \max\limits_{u,p,s} = & \sum_{t,m}\left[p^h_{t,m} + c^h_t u^h_{t,m} - \left(c^h_t + n_{t,m}\right)p^h_{t,m}\right] + \sum_{\text{ŧ}(t)}\left[\left(c^{qh}_{\text{ŧ}(t)} - u^{qh}_{\text{ŧ}(t),m} \cdot \right.\right. & (53) \\
 & \left.\left.\frac{1}{2}\ell^{qh,sell}_{\text{ŧ}(t)}\right) \cdot u^{qh}_{\text{ŧ}(t),m} - \left(c^{qh}_{\text{ŧ}(t)} + n_{\text{ŧ}(t),m} + p^{buy}_{\text{ŧ}(t),m} \cdot \ell^{qh,buy}_{\text{ŧ}(t)}\right)\right] & \\
s.t. & v_{\text{ŧ}(t),r} = v_{\text{ŧ}(t)-1,r} + v^{in}_{\text{ŧ}(t),r} - s_{\text{ŧ}(t),s} \qquad m \in M, & (54) \\
 & -\sum_{m\in \underline{rm}}(u^h_{t,m}\eta_m) + \sum_{m\in \underline{mr}}\left(p^h_{t,m}\rho_m\right) \qquad r \in \mathrm{R}, & \\
 & \qquad t \in T, & \\
 & +\sum_{\text{ŧ}(t)}\left(-\sum_{m\in \underline{rm}}(u^{qh}_{\text{ŧ}(t),m}\eta_m) + \sum_{m\in \underline{mr}}(p^{qh}_{\text{ŧ}(t),m}\rho_m)\right) \qquad \text{ŧ}(t) = 2, \dots, \mathcal{T} & \\
 & v_{\text{ŧ}(t),r} = v^{start}_{\text{ŧ}(t),r} \qquad r \in R, t = 1, & \\
 & \qquad \text{ŧ}(t) = 1 & \\
 & v^{min}_{\text{ŧ}(t),r} \le v_{\text{ŧ}(t),r} \le v^{max}_{\text{ŧ}(t),r} \qquad r \in R, m \in M, & \\
 & \qquad t \in T, \text{ŧ}(t) \in \mathcal{T} & \\
 & v_{\text{ŧ}(t),r} = v^{end}_{\text{ŧ}(t),r} \qquad r \in R, t = T, & \\
 & \qquad \text{ŧ}(t) = \mathcal{T} & \\
 & 0 \le s_{\text{ŧ}(t),r} \le s^{max}_r \qquad r \in R, t \in T, & \\
 & \qquad \text{ŧ}(t) \in \mathcal{T}. & \\
 & 0 \le p^h_{t,m} \le p^{max}_m \qquad m \in M, t \in T & \\
 & 0 \le u^h_{t,m} \le u^{max}_m \qquad m \in M, t \in T & \\
 & 0 \le p^{qh}_{\text{ŧ}(t),m} \le p^{max}_m \qquad m \in M, t \in T, & \\
 & \qquad \text{ŧ}(t) \in \mathcal{T} & \\
 & 0 \le u^{qh}_{\text{ŧ}(t),m} \le u^{max}_m \qquad m \in M, t \in T, & \\
 & \qquad \text{ŧ}(t) \in \mathcal{T} & \\
 & 0 \le p^h_{t,m} + p^{qh}_{\text{ŧ}(t),m} \le p^{max}_m \qquad m \in M, t \in T, & \\
 & \qquad \text{ŧ}(t) \in \mathcal{T} & \\
 & 0 \le u^h_{t,m} + u^{qh}_{\text{ŧ}(t),m} \le u^{max}_m \qquad m \in M, t \in T, & \\
 & \qquad \text{ŧ}(t) \in \mathcal{T} &
\end{array}
$$

In the objective function (53), the remuneration possibilities of both markets are considered. To secure the important arbitrage free characteristic within markets, it is suggested to calculate c^h_t as the average of the four quarters $c^{qh}_{\text{ŧ}(t)}$. This is not necessary when the operator has a dedicate price prognosis for each market. Then the model will automatically exploit the arbitrage within the markets as good as possible considering the given price sensitivity. In the reservoir filling level equation (49) the usage of the machines in both time steps is regarded. Therefore, all reservoir related variables and parameters are now defined based on the finer temporal time resolution $\text{ŧ}(t)$. To comply with the machine capacity limits neither $p^h_{t,m}$, $p^{qh}_{\text{ŧ}(t),m}$ nor a combination of both is allowed to exceed p^{max}_m. And the same applies for u^{max}_m with $u^h_{t,m}$, $u^{qh}_{\text{ŧ}(t),m}$ or a combination of both.

As a result, optimal power plant schedules for each market and one shadow price for both markets are determined. This simplifies the shadow price based power plant steering.

5.4. Numerical Results

In this chapter, the introduced quadratic optimization approaches are applied on hydropower scheduling problems and solved using the commercial solver CPLEX (IBM ILOG CPLEX Optimization Studio is a Simplex based system to solve optimization problems) and programmed in General Algebraic Modeling System (GAMS). Below two different kinds of case studies are performed.

The first case study presents the results of the already published multistage approach in chapter 5.4.1 including a real-world large-scale pumped hydropower storage portfolio case study with more than 20 reservoirs in the Alps and the Black Forrest, based on Braun (2016b). As electricity markets, the German hourly and quarter-hourly day-ahead markets are considered. For the latter, a price sensitivity of 0.1 €/MWh/100MW is considered.

The second case study is based on the enhanced multi-market optimization approach described in 5.3. A combination of exemplary pumped hydropower storages is defined, modeled and optimized to illustrate relevant effects of the multi-market optimization. The pumped hydropower storages are optimized inter alia considering hourly and quarter-hourly day-ahead markets to determine the optionalities within these markets.

5.4.1. Two-Stage Multi-Market Optimization

The aim of this EnBW pumped hydropower storage portfolio based case study is to show the huge impact of quarter-hourly markets on the optimization schedule as well as the applicability of the model to large scale systems.

Model Setup

For this example, the problem has been implemented as a multistage quadratic optimization in the optimization software GAMS. To illustrate the results of the presented optimization a calculation from January 1st, 2015 until April 30th, 2015 has been performed using the historic hourly and quarter-hourly day-ahead auction prices. An exemplary price profile for a week and a day in January can be seen in Figure 51. The price fluctuations and therefore the higher spreads for pumped hydropower storages motivates the consideration of quarter-hourly prices in the optimization

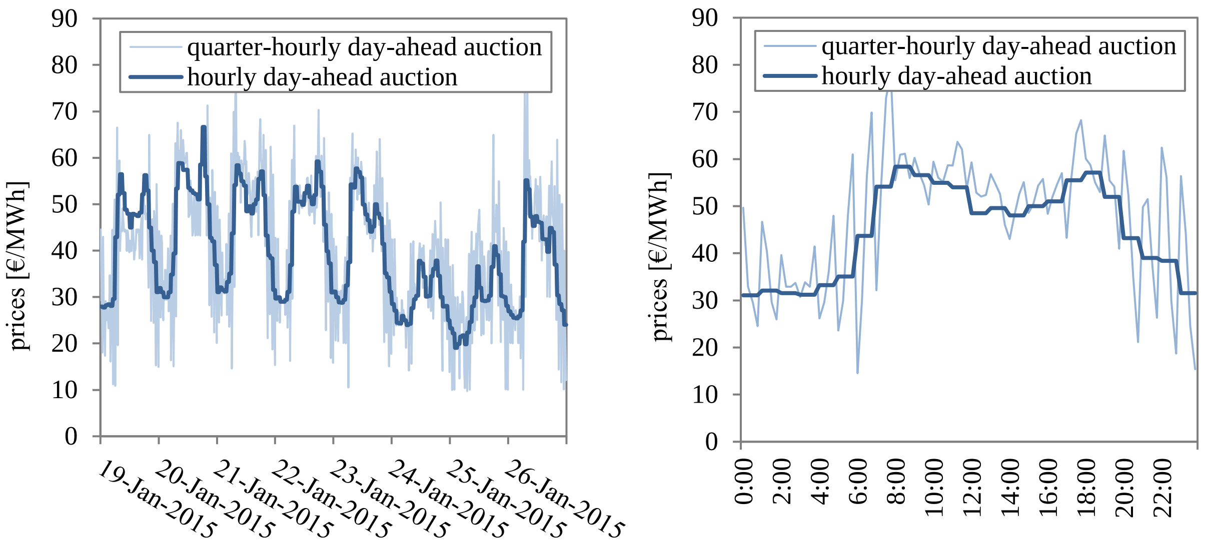


Figure 51 Exemplary hourly and quarter-hourly day-ahead auction prices over the course of one week on the left and for January 21st, 2015 on the right. Data derived from (EPEX Spot, 2017b)

The optimization has been applied on a real-world large-scale pumped hydropower storage portfolio with about 2 GW pumps and 3 GW turbine power. Every machine is defined by an efficiency rate, gird charges, flow through rates, hydraulic short circuit ability, as well as minimum and maximum capacity. Furthermore, the portfolio is a mix of daily, weekly and seasonally pumped hydropower storages including inflows. To find out more about the impact of quarter-hourly prices on profitability and shadow prices in different storage systems; reservoir with daily, weekly and seasonally pumping cycles are calculated and analyzed separately in chapter 5.4.2.

Model Results

Following the steps of the introduced two-stage optimization in chapter 5.3.3, the first stage of the optimization generates an optimal production schedule which should be bid into the hourly day-ahead market. This production schedule is presented in Figure 52, denoted by grey area. It is subsequently input for the second stage optimization and considered as an already taken sell or a buy position that has to be yielded. The second stage optimization is a post-optimization that adjusts the already calculated, and traded, hourly production schedule utilizing the quarter-hourly day-market prices. The difference between both schedules is traded into the quarter-hourly market as buy and sell within the physical limits of the power plants. This includes for example that at no time the total capacity of pumps or turbines are exceeded. The blue line in Figure 52 presents the resulting production schedule combining the sell and buy positions of both markets.

The extensive changes of the production plan after the second stage optimization are a consequence of the high price fluctuations on the quarter-hourly day-ahead auction as can be seen in Figure 51 for the same period. In some quarter-hours, the complete sell position of the hourly schedule is repurchased on the quarter-hourly market. Beyond, more energy is bought on the quarter-hourly market increasing the utilization factor, as for example in hour 7am. Vice versa for example 3 GW are sold on the quarter-hourly market in hour 11pm based on a previous buy position in the hourly market.

This optimization just makes sense for pumped hydropower storage machines that are highly flexible and can be switched off and on for single quarter-hours, otherwise the induced balancing power not complying the planned schedule could be higher as the additional profits.

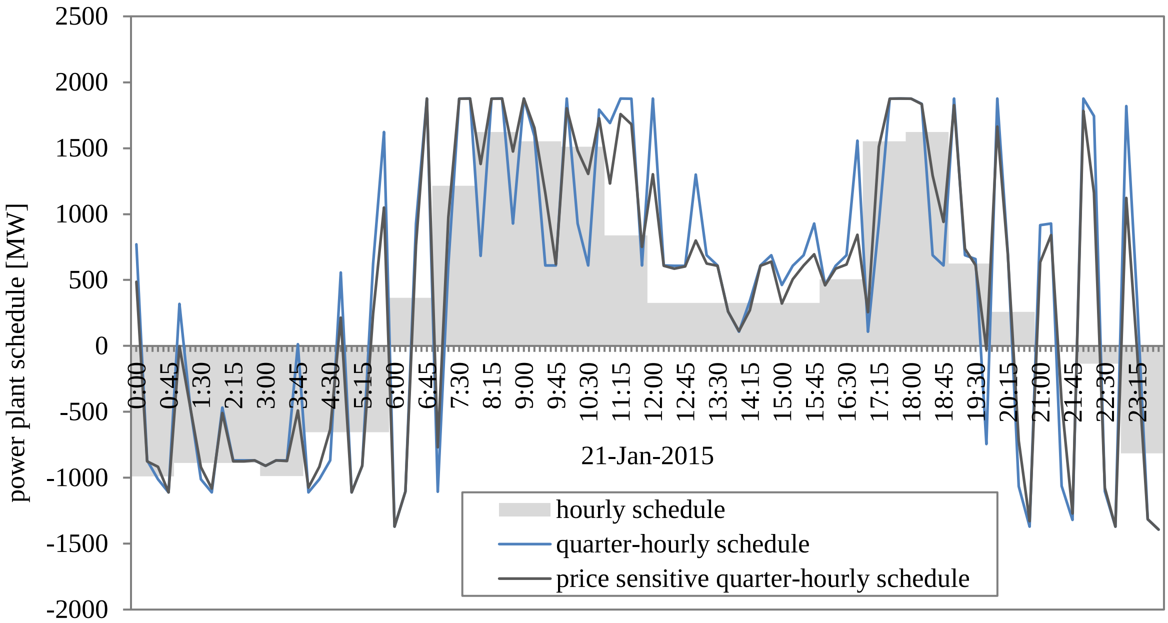


Figure 52 Comparing the production schedule of the first stage optimization for the hourly day-ahead auction (yellow bars) with the production schedule of the post-optimization for the quarter-hourly day-ahead auction (blue line) and considering price sensitive (red dotted line)

As presented in Figure 6, the average quantities traded on the different energy markets vary. Whereas more than 25 GW are medially traded on the hourly day-ahead market just about 0.5 GW are traded on the quarter-hourly day-ahead auction (EPEX Spot, 2017b). Therefore, the limited liquidity is considered, using a price sensitivity discount of 1 €/MWh per 100 MWh traded. The result can be seen in Figure 52 as well. The final production plan considering both markets and price sensitivity is outlined as black dotted line. In some quarter-hours, the quantities traded are significantly reduced in comparison to the calculation without price sensitivity. In other cases when the prices are high enough to compensate the price sensitivity discount all machines are still in the money. Comparing the hourly and the quarter-hourly schedule, a difference in maximum production is apparent. This is due to machines with a low efficiency and therefore high shadow prices which are not used on the hourly market at all.

After presenting the effects of the optimization spanning two markets on the optimal production schedule, the impacts on the water values and the resulting shadow prices need to be discussed. Assuming a perfect market including no-arbitrage and full liquidity as well as unlimited upper and lower reservoirs, the water values for both markets and all-time steps should be the same. None of these assumptions hold true in real-world applications. Nevertheless, the water values from the dual variables of the reservoir balancing equations can be calculated and deliver reasonable results for each of the two markets. The water values for the hourly and the quarter-hourly market deviate about 2 to 5 €/MWh from each other. This might be reasoned with the fluctuating quarter-hourly prices (higher spreads) and limited lower basins, but it reveals also that calculating water values for each market is not practical applicable. Buying in the first market for the hourly shadow price and maybe selling in the second, three hours later due to a lower shadow price, is not acceptable, although the results for each market are reasonable. This problem is solved with the enhanced one-stage optimization algorithm described in chapter 5.3.1. The second case study presents the desired improvements in terms of overarching shadow price based steering parameters in chapter 5.4.2.

Beyond, the consideration of limited liquidity decreases the water values about 0.2 to 1 €/MWh in comparison to the case without liquidity. The latter is reasonable since on the one hand the amount traded is reduced, see Figure 52, and on the other hand the realized price is lower because of the price sensitivity. Although the optimization includes a quadratic term, the calculation of the whole power plant portfolio took just a few minutes using Intel Core i7 CPU and 8 GB memory size and is therefore suitable for real-world operation.

5.4.2. One-Stage Multi-Market Optimization

In comparison to the first case study with one large power plant system with multiple reservoirs and machines demonstrating the feasibility of quadratic quarter-hourly optimization, the intention of the second case study is to understand the effect of storage size or machine power on return and steering parameters as well as the influence of price sensitivity on the hydropower handling.

Model Setup

Three different pumped hydropower storages have been modelled in GAMS, see Table 10. First, the size of the reservoirs corresponds to half-daily, daily and weekly pumped hydropower storages to see the profit gain of additional storage size. And second, the machine capacity varies to regard different market share levels. The scheduling problems are solved with the introduced multistage quadratic optimization approach from chapter 5.3.4. If the price sensitivity is assumed to be null the problem is solved as a linear optimization, in all other cases as a quadratic solution approach applies.

Table 10 Definition of the case study pumped hydropower storages

characteristic	unit	case 1	case 2	case 3
turbine capacity	[MW]	100	600	1,200
turbine flow through	[1000m³/MWh]	1	1	1
pump capacity	[MW]	100	600	1,200
pump flow through	[1000m³/MWh]	0.7	0.7	0.7
full-load hours	[h]	4	8	60

Model Results

The results of the multi-market optimization are presented below focusing on the profit and the shadow price based power plant dispatch. The production schedules generally resemble the already presented in case study one. In Figure 53 an overview on the obtained profits from the different optimizations can be seen. On the left the three different reservoir sizes can be compared using the hourly and the quarter-hourly day-ahead auction. The profit from the latter is significantly higher. Furthermore, the surplus of a sizable reservoir is more important in the hourly market as in the quarter-hourly market, showing also that the difference in profit between both markets decreases with increasing reservoir capacity. This is reasonable since for the quarter-hourly optimization the spreads within hours and days are important, whereas weekly pumped reservoirs balance also longer time periods.

On the right of Figure 53 the influence of price sensitivity on the profit is analyzed. Therefore, different machine sizes are compared since not the full load hours are important but the maximum capacity that is bid into the market. The more capacity is additionally bid into the market the higher the discounts of the profits. The discount on the profit for the 1200 MW machine is about 3 %.

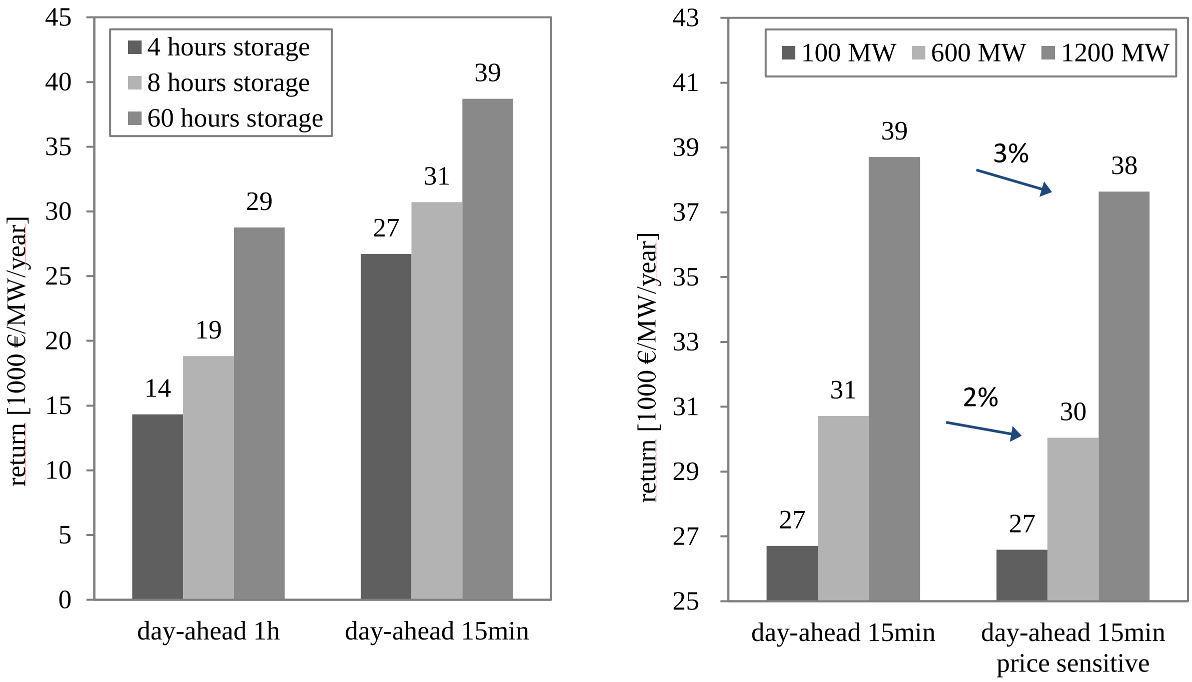


Figure 53 Comparing the profit from the hourly and the quarter-hourly day-ahead market optimization on the left and the influence of price sensitivity on the right.

Furthermore, the revenues are neither evenly distributed over the different markets nor the different months of the year. Figure 54 illustrates that the revenues vary with a factor of two to three between the months. Since this cannot be reasoned with the higher prices in winter, the distribution accentuates that not just the absolute price but especially the spreads are important. The spreads between peak and off-peak as well as within hours are smaller during the summer as in winter. Moreover, the high profits in January, February, May and December result from a few significant price spikes which strongly influence the overall revenue.

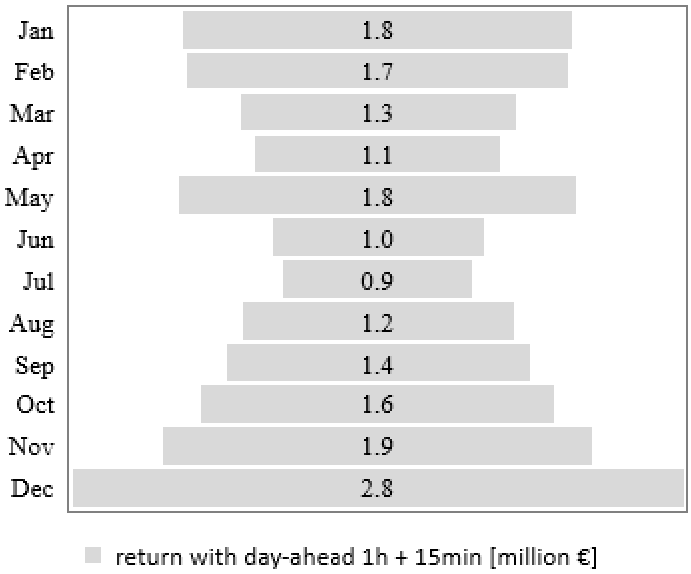


Figure 54 Exemplification of the monthly return form a pumped hydropower storage with 8 full-load hours and 600 MW capacity over the course of the year 2016 taking the quarter-hourly day-ahead auction into account.

A further target of this case study is the analysis of the steering parameters. The steering parameters are important to transfer the calculated results into a real-world dispatch. Many optimization models hamper to provide a sufficient transformation. The week between Monday 18th and Sunday 25th in January 2016 is chosen to present how the reservoirs are planned to be dispatched. Shadow prices are calculated based on the water values of the upper and lower reservoirs, which are results of the reservoir balancing equations dual variables. The derivation of the shadow prices in general is discussed in chapter 3.4.

For the smallest pumped hydropower storage within the case study, with 4 full-load hours of maximum water release and a machine size of 100 MW, the shadow prices are depicted for the respective January week in Figure 55. The black dashed line presents the shadow price just for the hourly market and the grey line for a combination of both markets. Whereas both shadow prices look relatively similar, the difference is still significant, see dotted grey line for the deltas between both shadow prices. It can be seen that the power plant is not used for daily, but for half daily storage cycles. That means one load cycle takes place on a lower price level between the night and the morning peak and the second on a higher price level between the dip at noon and the evening peak. For a 100 MW machine, the consideration of price sensitivity is not presented here.

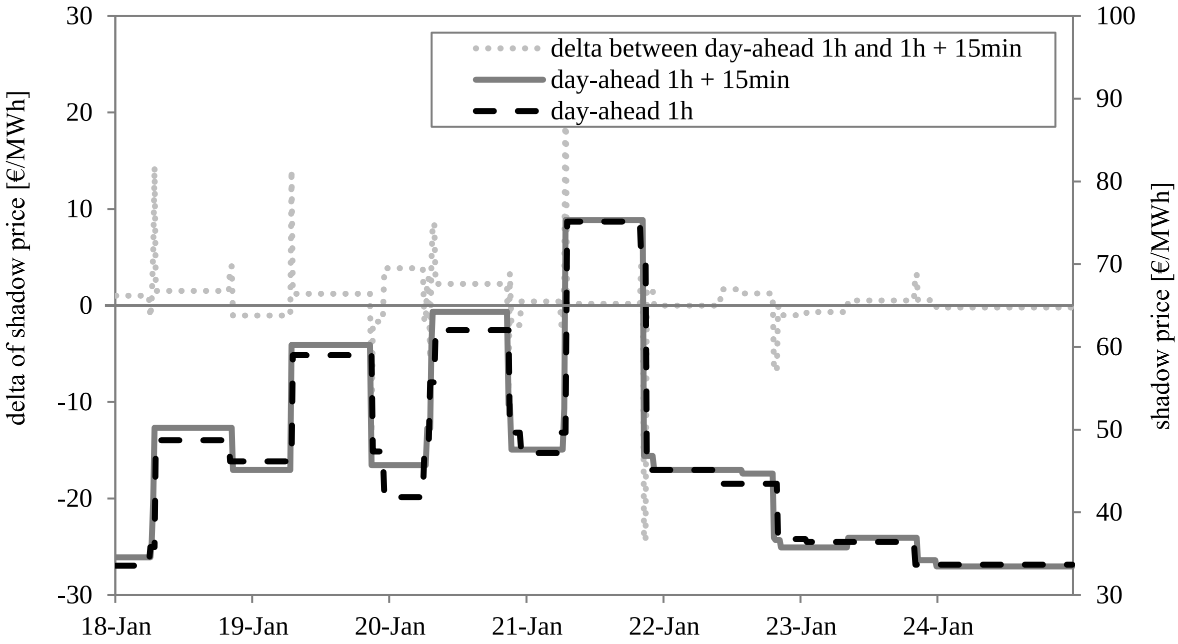


Figure 55 Course of the shadow prices for different short-term energy markets and a reservoir with 4 full-load hours in a January week in 2016.

In Figure 56 the shadow price for the hourly day-ahead market and for a combination of hourly and quarter-hourly day-ahead market with and without sensitivity is presented. Whereas in some parts of the year the difference is significant, large sections reveal nearly no difference. The deltas between considering just the hourly and both markets are illustrated as blue and grey dotted lines. The consideration of price sensitivity, denoted as blue dashed line, seems rather unimportant for the shadow price calculation. This can be reasoned with the symmetric applying price sensitivity. That means the power plant face higher prices for pumping and lower prices for generation. Both leads to a reduced dispatch but not a change in the shadow price. Some examples with an uneven distributed price sensitivity can be found over the course of the year resulting in a difference between the shadow price with and without price sensitive.

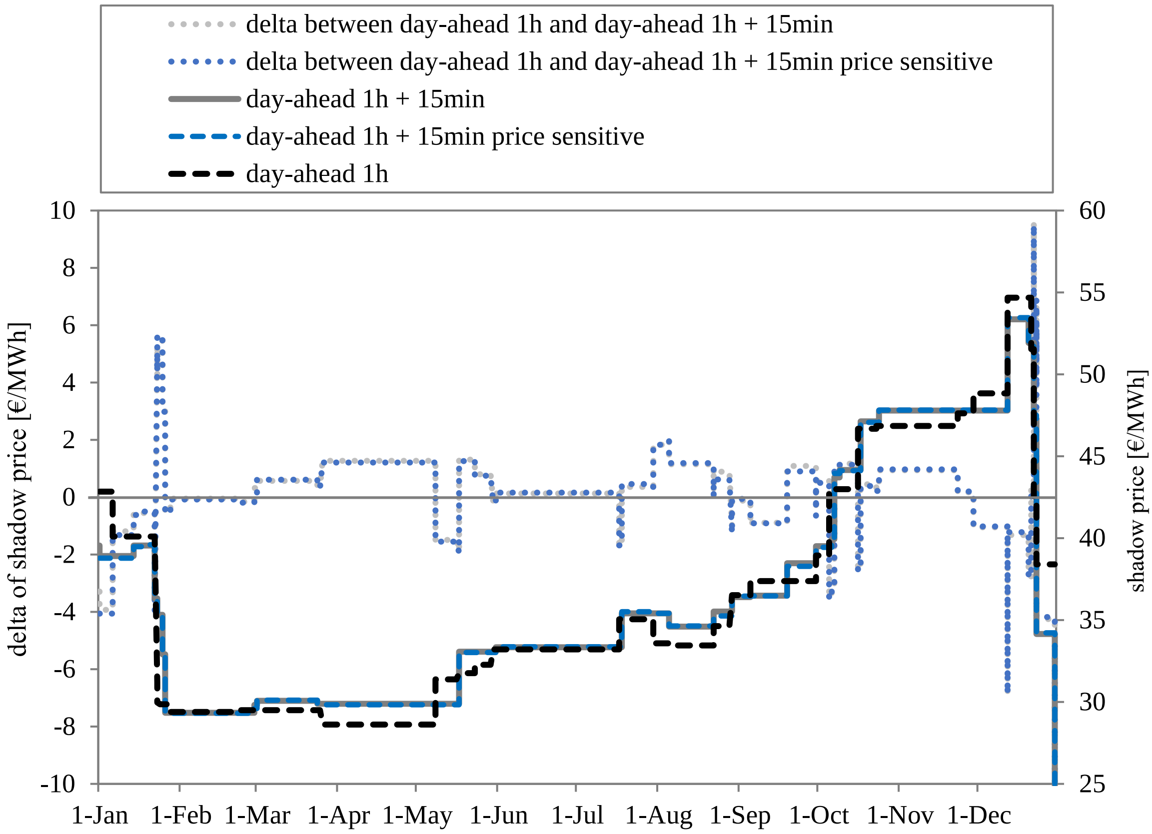


Figure 56 Difference between the hourly day-ahead market based shadow price and the multi-market based shadow price with and without price sensitivity.

5.4.3. Conclusion

The multi-market analysis and optimization of quarter-hourly and hourly day-ahead markets is a novelty and improves profit and reservoir management.

An important part is the price sensitivity calculation on the newly introduced quarter-hourly day-ahead auction in comparison to the already existing hourly day-ahead auction in Germany. The results show that between 100 and 800 MW additional bid and ask quantity the average price sensitivity of the quarter-hourly German day-ahead auction in 2016 (2015) was about 1.3 €/MWh (1.5 €/MWh) per 100 MW shift of supply or demand in comparison to about 0.2 €/MWh (0.25 €/MWh) per 100 MW for the similar shifts on the hourly German-Austrian day-ahead auction. The solar power production as well as gradients of demand and the ramps of thermal power plants have been identified as the main drivers in terms of trading volume and price formation in the quarter-hourly market. Furthermore, the introduction of the second day-ahead market increased the overall short-term trading volume and enables risk mitigation via more adequate products, especially for solar power traders. However, the information on the price sensitivity is also relevant for operators of flexible units on supply and demand side likewise. In addition, the discussion about the zigzag price formation on the quarter-hourly market points out that the two-stage market design is rationale. But, although it might be possible that clearing both markets together could reduce quarter-hourly price fluctuations, the thereby induced reduction of quarter-hourly price spreads is going to lessen investment incentives for new flexibilities that are required to enable even higher shares of RES in the future.

For both, new installations, as well as existing storages, quarter-hourly prices and price sensitivity should be considered in determining the pumped hydropower bidding strategies. An analysis shows that the usage of just hourly day-ahead price based water values in quarter-hourly trading leads to an imprecise steering. This is due to higher fluctuations and sensitive prices on the quarter-hourly day-ahead market. Therefore, multi-market optimization approaches are presented providing optimal power plant production schedules, profits and steering parameters. The results are outlined for different pumped hydropower storages. The case studies present that the consideration of the quarter-hourly market increases the profit for the half-daily pumped hydropower storage by 86 % and for the weekly pumped hydropower storage by 36 %. The influence of price sensitivity consideration in the quarter-hourly market leads to a profit reduction of about 3 % for a 1200 MW machine. Furthermore, it could be shown that the profit highly depends on the infrequent occurring price spikes over the year. The difference in shadow price considering just the hourly market and a combination of both hourly and quarter-hourly market is heterogeneous and in the given example mostly between 1.5 €/MWh and -1.5 €/MWh. The influence of the price sensitivity consideration on the shadow price is negligible. Furthermore, a two-stage approach that determines shadow prices for each market is theoretical correct but practical not feasible. Therefore, it can be advised, integrating all markets in one objective function and to use the overall reservoir filling level equation to retrieve one shadow price for all markets.

Concluding, the consideration of several markets when optimizing the pumped hydropower storage dispatch is crucial for short-term position management. Mainly to determine the day-ahead market profit to be compared with other income possibilities such as balancing power (see chapter 8), but also to provide high-quality shadow prices for pumped hydropower storage steering.

6. Stochastic Optimization of Quarter-hourly Day-ahead Market

The view on today's energy markets has been focused on the short-term. In Germany about 97 GW of RES are installed by now producing a third of 2015 gross electricity demand (187.3 TWh out of 592 TWh) (BMWi, 2016b). The major share of this production depends on the fluctuating primary energy sources sun and wind and is therefore independent to demand and price level. For the latter, variable RES strongly influence the price in the short-term. Weather forecasts do normally not venture outlooks further than one week ahead and a precise production forecast is often just possible a few hours before delivery.

Since demand and production always need to be balanced, the variable RES demand significant amounts of flexibility and storage capacity. Beside the daily PV production and deviations due to the course of the sun, especially the wind production challenges the forecasts. Whereas it can be predicted that a low-pressure area will bring significant amounts of wind power, the exact time when the wind starts to blow can vary within hours and the length of the storm can also be a day less or more. These deviations are often traded in the hourly and quarter-hourly intraday markets, which play an important role for wind power balancing, see chapter 2.2.3. One of the most obvious possibilities to balance wind power production are pumped hydropower storages with good conversion efficiencies, large quantities of potentially stored energy and flexible production units. Especially so called weekly storages with full-load hours in a range of about 48 to 168 hours are highly suitable.

To consider the just described use case in weekly pumped hydropower scheduling and optimization a precise quarter-hourly price forecast including probabilities and the possibility to react on shifts of the planned wind feed-in is needed. Furthermore, as described in chapter 3.3, short-term models with an optimization period of about two weeks are normally modelled with a high level of detail so that the results can be used to generate steering parameters for an optimal risk neutral dispatch. This challenge, stochastic programming with a high level of detail, is approached in this chapter. To consider stochastic prices as well as stochastic inflows the optimization period is limited to a few weeks. The optimization of a hydropower system with cascaded reservoirs is possible as well.

This chapter is based on common work with Judith Vesper. She was a master student I supervised at EnBW in 2016. In the first part, chapter 6.1, SDDP is introduced as a well-known but still state-of-the-art approach to deal with the course of dimensionality. Chapter 6.2 introduces an extension to consider a case with not independent prices. Furthermore, chapter 6.3 presents the numerical results revealing the value of stochastic optimization and steering parameters.

6.1. Stochastic Dual Dynamic Programming

In chapter 4.3.2, a solution to the stage wise independent stochastic optimization problem is given. The intention is to maximize the expected value of the objective function including the nested expected values. The direct solution would require the solution of multiple integrals. Therefore, a dynamic approach for an appropriate approximation of the stochastic processes is applied to avoid solving complex integrals. Although the stochastic problem can be formulated in "scenario tree form" with various linear problems that can be solved with commercial solvers as CPLEX, the marvelousness number of problems is just theoretically solvable. With an increasing scenario tree, the number of variables grows extremely fast and the problem is quickly non-manageable. With a time period of 7 days with quarter-hourly time resolution and three branching's on each stage one receives $3^{(7*96-1)}$ scenarios. For a classification, even with a two-day time period the number exceeds by far the number of protons in the universe which is assumed to be around 10^{89} (Heile, 2012). A possible solution is Benders decomposition, a dynamic approach to cope with the "scenario tree form". But the number of sub problems is still numerous and calculation times are high. Therefore, in this chapter the SDDP method is used. This likewise dynamic approach allows to reduce the number of sub problems even further via sampling, also referred as scenario selection.

In the first part of this chapter 6.1.1, the challenges of stochastic input parameter especially the prices are described. After a literature review in 6.1.2 the already mentioned SDDP algorithm is introduced and explained. A theoretical review in 6.1.4 including a convergence analysis complements this chapter.

6.1.1. Challenges of Independent Prices

In this section, the SDDP method to solve stochastic problems, as defined in chapter 4.3, is applied on a hydropower scheduling problem. Therefore, stochastic inflows as well as stochastic prices are considered; both are assumed to be stage wise independent. A method to deal with stage wise dependent prices is introduced in chapter 6.2. Independency means, $(\Omega_t|\omega_{t-1}) = \Omega_t$ and every combination of realizations is assumed to be possible. Furthermore, the probability vector $\xi_t(\omega_t) = (c_t(\omega_t), b_t(\omega_t))$ is discretized. The stochastic process $\xi_2, \dots, \xi_T$ is replaced by samples $\tilde{\xi}_t^j = (\tilde{c}_{tj}, \tilde{b}_{tj}), j = 1, \dots, N_t, t = 1, \dots, T$ and the distribution $\mathcal{P} = \mathcal{P}_2 \times \dots \times \mathcal{P}_T$ by the empirical distribution $\tilde{\mathcal{P}} = \mathcal{P}_{N_2} \times \dots \times \mathcal{P}_{N_T}$. The stochastic process can then be depicted as a so called recombining tree. Figure 57 presents a possible tree structure.

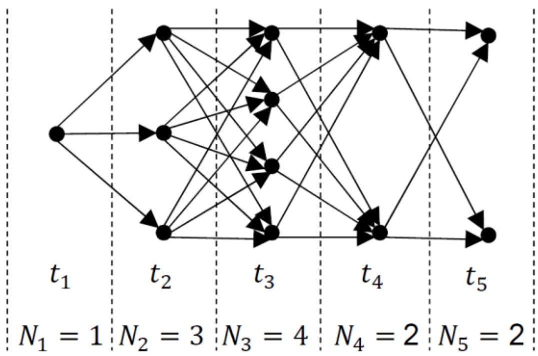


Figure 57 Exemplary recombining scenario tree with $t = 5$

6.1.2. Literature Review

Hydropower scheduling problems are within the most challenging problems in energy planning due to their complexity. It has always been important for practice as well as academia to find solutions to problems including stochastic. The state-of-the-art approach to solve such problems is SDDP and some similar approaches that are widely used mainly for long-term planning between one to five years. A very small but for this chapter most important work on SDDP are compiled below.

- Pereira and Pinto (1991) introduced the SDDP method at first and applied a cost minimization on the Brazil hydropower storage system. The greatest advantage of this new approach is to curb the so called curse of dimensionality (term introduced by Bellman, 1954 when considering problems in dynamic optimization). Hitherto the explosion of the state space when solving with stochastic dynamic programming (SDP) narrowed the number of problems that could be solved. Pereira and Pinto showed that the cost-to-go function is linear in the target variable and therefore can be approximated by a linear function. They further pointed out possibilities for a useful parallelization.
- Shapiro (2011) analyzed SDDP in terms of convergence. He was able to prove convergence for linear multistage problems using sample-average-approximation as discretization of the stochastic process and describes stage wise independent conditions. A further convergence result for SDDP, risk aversion and stopping criterions has been shown and defined by Shapiro.
- Rebennack (2016) looked into advantages and disadvantages of scenario and sampling based SDDP methods. He describes the applications of uncertainties in various models and introduces extensions to classical SDDP including a combination of scenario and sampling based representation of stochastic processes.
- Löhndorf, Wozabal and Minner (2013) apply an approximate dual dynamic programming approach to hydropower systems. It incorporates short-term intraday decisions with a Markov decision process and integrates SDDP with ADP. Prices as well as inflows enter the stochastic input data.
- Agottspon and Andersson (2014) use SDDP for a medium term planning horizon. Therefore, they introduce intra- and inter stages to deal with short-term flexibilities while keeping the decision tree as small as possible. They further describe approaches to integrate balancing power provision. Furthermore, Abgottspon (2015b) characterizes various application possibilities for SDDP in hydropower scheduling and introduces extensions to SDDP to deal with a wider scope of problems. For example, he introduces the method of locally valid cutting planes as an extension to SDDP for to deal with concave value functions and describes multi-horizon planning to solve a combination of stage wise independent and dependent stochastic processes.

6.1.3. Algorithm

This part focuses on the classical SDDP approach introduced by Pereira and Pinto (1991). Input for the algorithm is a discretized stochastic process. Outputs are the expected returns as well as water values for the reservoirs. The water value is given by the change of the objective function due to a marginal relaxation of the restricting reservoir balancing equations; a marginal increase of the right-hand side. In the beginning, with each iteration, K paths of the stochastic process are chosen randomly. Afterwards a forward and a backward step are performed and the first iteration of the algorithm is concluded. During the algorithm, upper and lower bounds for the expected return are calculated and tested for the stopping criterion. Figure 58 presents the operation sequence of the SDDP algorithm schematically.

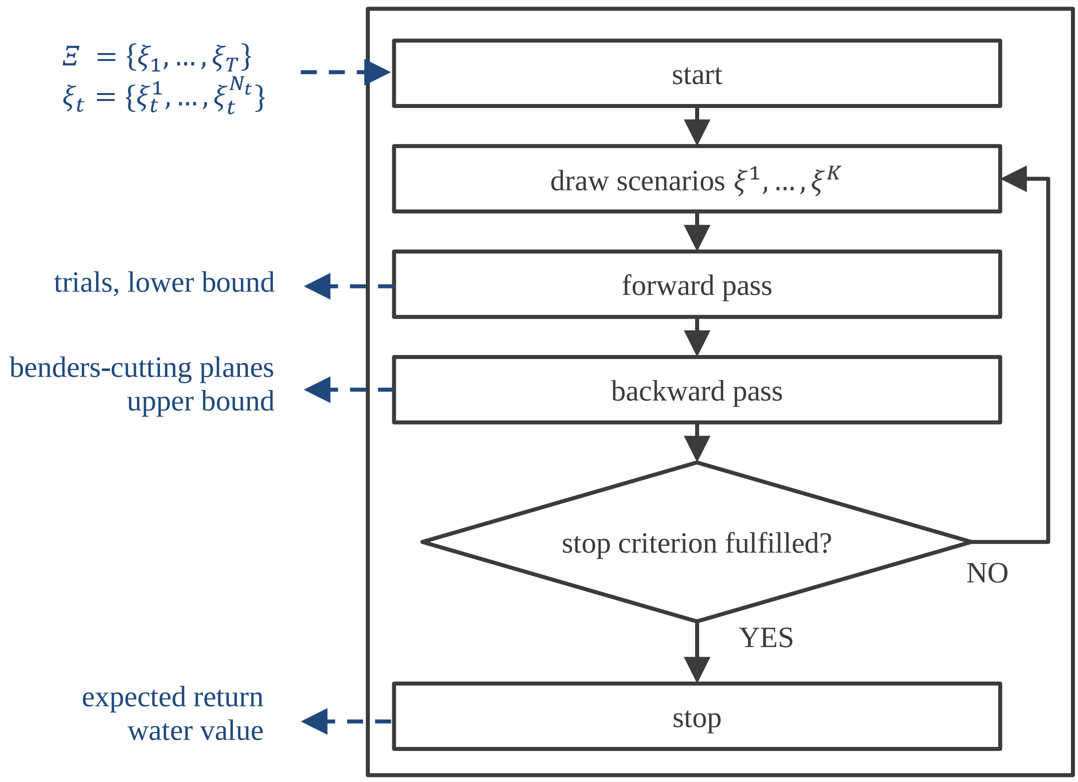


Figure 58 Flow chart stochastic dual dynamic programming

For the forward step test solutions, so called trails, of each path and for every time stage are computed. In the backward step, the corresponding cutting planes of the profit-to-go function are determined. Thereby, with every iteration the profit-to-go function is approximated more accurately. Figure 59 presents a recombining scenario tree with a forward pass on the left-hand side and the backward pass on the right-hand side.

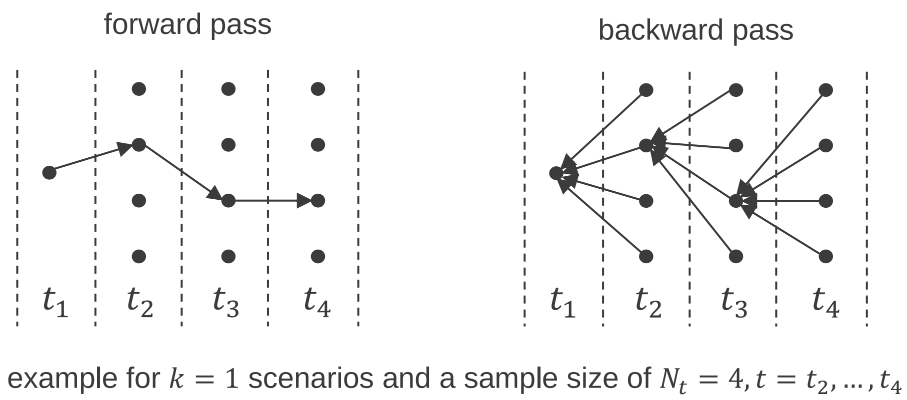


Figure 59 Run through the scenario tree in forward and backward passes

For the forward pass, all time stages beginning with the first and ending with the last time step. The optimal point is handed over to each sub problem at a time. The backward pass starts on the last time step. The result of the penultimate step of the forward pass is fixed and for all possible scenarios of the last stage the returns are calculated. For the backward pass, this procedure is performed for all further time steps going backwards in time until the first time step is reached.

Below, the algorithm is described step by step beginning with the forward pass followed by the backward pass and finally formulated as overall algorithm.

Forward Pass

For the forward pass, all K scenarios of all sub problems, from the first until the last time step, are solved. Henceforth, these problems are stated as master problems. Figure 60 shows the course of the forward step.

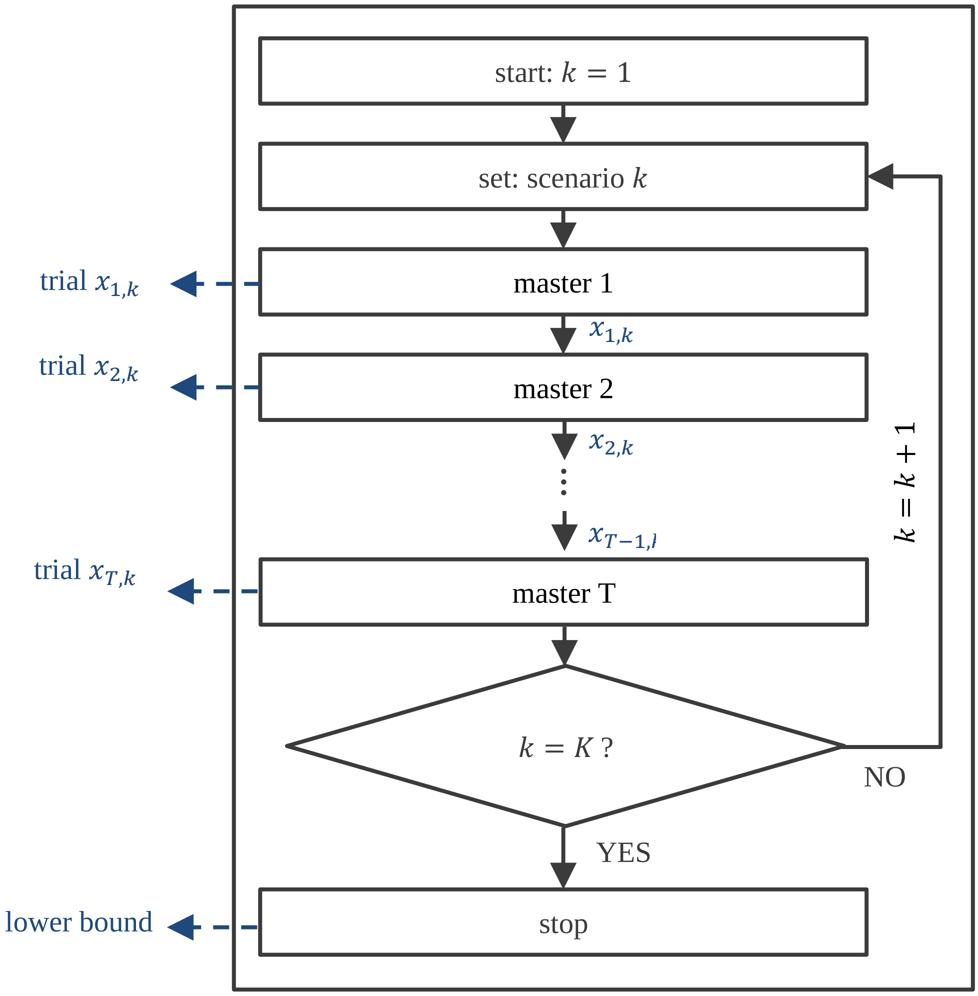


Figure 60 Flow diagram of SDDP algorithm based forward step

In Iteration R of the overall algorithm the master problem of the forward pass for $k = 1, \dots, K$ formulates as below:

Master problem 1:

$$\begin{aligned} \mathcal{C}_1 \quad &= \max_{x_1, \alpha_2} && c_1^T x_1 + \alpha_2 && \\ & \quad s.t. && A_1 x_1 = b_1 && \\ & && \alpha_2 \le \alpha_{2,k}^r - \left(x_1 - x_{1,k}^r\right)^T B_2^T \lambda_{2,k}^r && \begin{matrix} r = 1, \dots, R-1 \\ k = 1, \dots, K \end{matrix} \\ & && x_1 \ge 0. && \end{aligned} \tag{55}$$

Master problem t:

$$\begin{aligned} \mathcal{C}_t(x_{t-1}, \xi_t^s) \quad &= \max_{x_1, \alpha_{t+1}} && (c_t^s)^T x_t + \alpha_{t+1} && \\ & \quad s.t. && A_t x_t = b_t^s - B_t x_{t-1} && \\ & && \alpha_{t+1} \le \alpha_{t+1,k}^r - \left(x_t - x_{t,k}^r\right)^T B_{t+1}^T \lambda_{t+1,k}^r && \begin{matrix} r = 1, \dots, R-1 \\ k = 1, \dots, K \end{matrix} \end{aligned} \tag{56}$$

$$x_t \geq 0$$
$$\text{for } t = 2, \dots, T-1$$

Master problem T:

$$\mathcal{C}_T(x_T, \xi_T^s) = \max_{x_T} \quad (c_T^s)^T x_T \qquad (57)$$
$$s.t. \quad A_T x_T = b_T^s - B_T x_{T-1}$$
$$x_T \geq 0$$

The optimal point on time stage $t-1$ is handed over to the problem at time stage t as a parameter. Results of the forward pass are optimal points

$$x_1^R = \underset{x_1}{\operatorname{argmax}} \, \mathcal{C}_1, x_{t,k}^R = \underset{x_t}{\operatorname{argmax}} \, \mathcal{C}_t\left(x_{t-1,k}^R, \xi^k\right), \qquad (58)$$

and lower bounds for the return and the solution of the overall algorithm that are defined as:

$$\underline{z} = c_1^T x_1^R + \frac{1}{K} \sum_{s=1}^K \sum_{t=2}^T (c_t^s)^T \, x_{t,s}^R. \qquad (59)$$

The cutting planes are derived from the backward passes of the iterations 1 to $R-1$. For each backward pass and each time step K new cutting planes are added. In the first iteration, no cutting planes are available, $\alpha_t = 0, t = 1, \dots, T-1$ are fixed. Alternatively, a start solution can be provided to immediately start with the backward pass.

Backward Pass

In the backward pass all master problems, beginning with the last time step for all realizations of the stochastic process are solved. For that the respective penultimate solution of the forward step is fixed. In doing so, the return is estimated based on the generated reservoir filling levels of the penultimate step considering the different scenarios of the current time step. The sub problems are equal to the ones of the forward step and are solved for all $s = 1, \dots, K$ and $j = 1, \dots, J$. As a major difference in comparison to the forward step, the dual variables $\lambda_{t+1,k}^R$ of the reservoir filling level equations are determined during the backward passes. That is crucial to find the added future revenue that could be generated with one additional unit of water in a specific reservoir. To capture the change of the approximate future revenues as a function of the reservoir filling levels, the dual variables of the cutting planes are multiplied with the change of the reservoir filling levels $\left(x_t - x_{t,k}^R\right)^T$. Further, the expected revenue $\alpha_{t+1,k}^R$ at the reservoir filling level $x_{t,k}^R$ is added for which the dual variable has been determined.

Master problem T:

$$\mathcal{C}_T\left(x_{T-1,s}^R, \xi_T^s\right) = \max_{x_T} \quad (c_T^j)^T x_T \qquad (60)$$

$$
\begin{aligned}
s.t. \quad & A_T x_T = b_T^j - B_T x_{T-1,s}^R \quad [\lambda_{T,k}^j] \\
& x_T \geq 0
\end{aligned}
$$

Master problem t:

$$
\begin{aligned}
\mathcal{C}_t\left(x_{t-1,s}^R, \xi_t^j\right) = \max_{x_t} \quad & (c_t^j)^T x_t + \alpha_{t+1} \\
s.t. \quad & A_t x_t = b_t^j - B_t x_{t-1,s}^R \quad [\lambda_{t,k}^j] \\
& \alpha_{t+1} \leq \alpha_{t+1,k}^r - \left(x_t - x_{t,k}^r\right)^T B_{t+1}^T \lambda_{t+1,k}^r \qquad r = 1, \dots, R \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 1, \dots, K \\
& x_t \geq 0 \\
& \text{for } t = T-1, \dots, 2
\end{aligned} \tag{61}
$$

The master problems of the first stage for the forward and the backward pass are equal. The expected revenue for scenario k of time step t to T in iteration R is

$$\alpha_{t,k}^R = \frac{1}{N_t} \sum_{j=1}^{N_t} \mathcal{C}_t\left(x_{t-1,k}^R, \xi_t^j\right), \tag{62}$$

for $k = 1, \dots, K$ and $t = 2, \dots, T$. The corresponding dual variables for the cutting planes are computed as

$$\lambda_{t,k}^R = \frac{1}{N_t} \sum_{j=1}^{N_t} \lambda_{t,k}^j, \tag{63}$$

with $k = 1, \dots, K$ and $t = 2, \dots, T$. Below, it is shown that the cutting planes are an outer approximation of the profit-to-go function and are therefore the upper bound for the future expected revenues. Therewith, the result of the first stage is the upper bound for the expected revenue which is defined as

$$\overline{z} = \mathcal{C}_t = c_1^T x_1 + \alpha_2, \tag{64}$$

whereby (x_1, α_2) is the optimal point of the backward pass in iteration R. The backward pass process is illustrated in Figure 61.

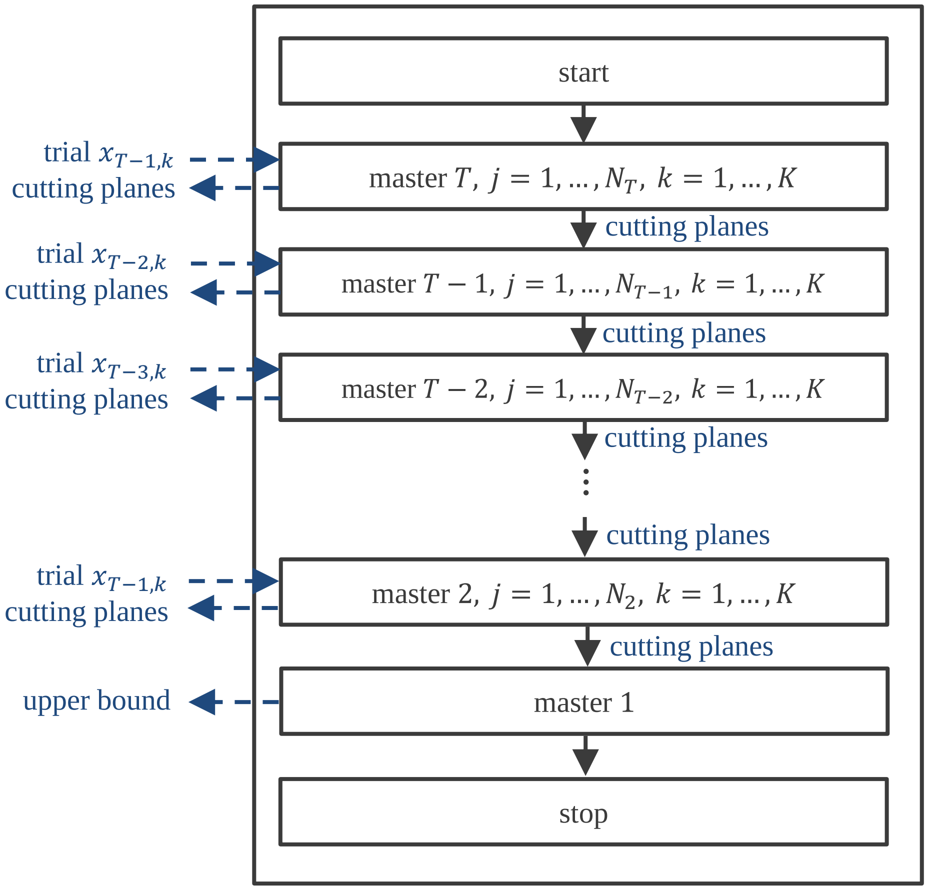


Figure 61 Flow diagram of SDDP based backward pass

Stopping Criterion

Literature on SDDP provides various suggestions for respective stopping criteria. The work of Pinto and Pereira (1991), in which the algorithm has been introduced, recommend to stop the iterating if upper and lower bound are within the confidence interval of 95%. The interval is defined as:

$$\left[\underline{z} - 1.96\ \sigma_z^o\ ,\ \overline{z} + 1.96\ \sigma_z^0\right], \tag{65}$$

with $\sigma_z^0 := \sqrt{\frac{1}{K^2}\sum_{k=1}^{K}\left(\underline{z} - z_k\right)^2}$ and $z_k := c_1^T x_1 + \sum_{t=2}^{T}\left(c_t^k\right)^T x_{t,k}$. Shapiro (2011) took that up and added a modification to the stopping criterion. He states that the difference of the lower confidence bound $\underline{z} - 1.96\ \sigma_z^o \frac{1}{\sqrt{K}}$ and the upper confidence bound $\overline{z}$ shall be located within a specific tolerance limit that can be defined as:

$$\overline{z} - \left(\underline{z} - 1.96\sigma_z^1 \frac{1}{\sqrt{K}}\right) < \epsilon, \tag{66}$$

with $\sigma_z^1 := \sqrt{\frac{1}{K-1}\sum_{k=1}^{K}\left(z_k - \underline{z}\right)^2}$. Further, Abgottspon (2015b) states that in practice the algorithm should be also stopped after a fixed number of iterations or when the lower bound stops improving, which is not considered.

When the stopping criterion is fulfilled the lower bound based on the trail solutions is the best approximation for the expected revenue. The optimal points of the forward pass outline the optimal strategy for the discrete problem. A further result are the water values of the reservoirs that are retrieved form the active cutting planes.

Overall Algorithm

Algorithm 1 provides a clear and stringent overview of the comprehensive SDDP approach in pseudocode. This may help to implement SDDP in various programming languages.

Algorithm 1 Stochastic Dual Dynamic Programming

Input Discretized stochastic process $\Xi = \{\xi_1, \dots, \xi_T\}$ with $\xi_T = \left\{\xi_t^1, \dots, \xi_t^{N_t}\right\}$, $t = 1, \dots, T$ and where $N_1 = 1$. Parameter $K > 1$ the number of paths, $\epsilon > 0$ stopping criterion tolerance.

Iterations index $R = 1$, $x_0 = 1$, $B_1 = 0$, $a_{T+1} = 0$.

while $\overline{z} - \underline{z} + \frac{1.96\sigma}{\sqrt{K}} \geq \epsilon$ **do**

scenario selection:

  Choose of K scenarios $\xi^1, \dots, \xi^K$ with $\xi^s = \xi_1^s, \dots, \xi_T^s$, $s = 1, \dots, K$.

Forward pass

  for $s = 1 : K$ **do**

    for $t = 1 : T$ **do**

      if $R = 1$ **then** $\alpha_{t+1} = 0$ fixed

      end if

$$x_{t,s}^R = \begin{cases} \underset{x_t}{\operatorname{argmax}}\, c_{t,s}^T x_t + \alpha_{t+1} \\ s.t. \begin{cases} \alpha_{t+1} \leq \alpha_{t+1,k}^r - \left(x_t - x_{t,k}^r\right)^T B_{t+1}^T \lambda_{t+1,k}^r \\ r = 1, \dots, R-1 \;\; k = 1, \dots, K \\ A_t x_t = b_{t,s} - B_t x_{t-1,s}^R \\ x_t \geq 0 \end{cases} \end{cases}$$

    end for

    $\underline{z}^s = \sum_{t=1}^{T} c_{t,s}^T x_{t,s}^R$

Algorithm 1 Stochastic Dual Dynamic Programming

```
        end for
        z̲ = (1/K) ∑_{s=1}^{K} z̲^s und σ = sqrt( (1/(K−1)) ∑_{s=1}^{K} (z̲^s − z̲)^2 )
    Backward pass:
        for t = T : 1 do
            for s = 1 : K do
                for j = 1 : N_t do
                    Q_{t,s}^j = { max_{x_t} c_{t,j}^T x_t + α_{t+1}
                                  s.t. { α_{t+1} ≤ α_{t+1,k}^r − (x_t − x_{t,k}^r)^T B_{t+1}^T λ_{t+1,k}^r
                                         r = 1, …, R k = 1, …, K
                                         A_t x_t = b_{t,j} − B_t x_{t−1,s}^R
                                         x_t ≥ o
                end for
                α_{t,s}^R = (1/N_t) ∑_{j=1}^{N_t} Q_{t,s}^j und λ_{t,s}^R = (1/N_t) ∑_{j=1}^{N_t} λ_{t,s}^j
            end for
        end for
        z̄ = (1/K) ∑_{s=1}^{K} Q_{1,s}^1
    end while
```

Derivation of the Two-Stage Model

In this part, the stochastic two-stage model to be solved with the SDDP algorithm is derived from the two-stage model based on the stochastic dynamic formulations (30), (31), (32) introduced in chapter 4.3.

The master problem of the first stage formulates as:

$$P: \quad \max_{x_1} \quad c_1^T x_1 + \mathcal{Q}(x_1) \qquad s.t. \quad A_1 x_1 = b_1 \qquad x_1 \geq 0. \tag{67}$$

The expected value or profit-to-go function is

$$\mathcal{Q}(x_1) = \mathbb{E}_\xi[\mathcal{C}(x_1, \omega)]. \tag{68}$$

Further the sub problem of the second stage is

$$\mathcal{C}(x_1, \omega) \quad = \max_{x_2} \quad c_2(\omega)^T x_2(\omega) \tag{69}$$
$$s.t. \quad B_2 x_1 + A_2 x_2(\omega) = b_2(\omega)$$
$$x_2(\omega) \geq 0.$$

with $x_1, x_2 \in \mathbb{R}^n, A_1 \in \mathbb{R}^{m_1 \times n}, b_1 \in \mathbb{R}^{m_1}, A_2, B_2 \in \mathbb{R}^{m_2 \times n}, b_2 \in \mathbb{R}^{m_2}$ and $c_1, c_2(\omega) \in \mathbb{R}^n$. Below the solvability of all sub problems is assumed to be given.

Discretization of the Probability Distribution

In the first step, the random vector ξ is approximated by the sample $\tilde{\xi}(\omega^j), j = 1, \dots, N$ with the respective empirical distribution P_n. Be that

$$\tilde{Q}(x_1) \quad := \quad \mathbb{E}_{P_N}[\mathcal{C}(x_1, \omega)] = \frac{1}{N} \sum_{j=1}^{N} \mathcal{C}(x_1, \omega^j) \tag{70}$$

whereas $\mathcal{C}(x_1, \omega^j)$ is stated as the maximization problem of stage two, see equation (32), of scenario j. The approximation of the first-stage problem is therefore:

$$\check{P}: \quad \max_{x_1} \quad c_1^T x_1 + \tilde{Q}(x_1) \tag{71}$$
$$s.t. \quad A_1 x_1 = b_1$$
$$x_1 \geq 0.$$

It can be seen, the expected value or profit-to-go function $\tilde{Q}$ is included in the objective function. In the next steps, the summation of the sub problems is reformulated and simplified.

Dualization of the Value Function $\mathcal{C}$

The theorem of strong duality states that if the primal problem has an optimal solution, then the dual problem has an optimal solution as well. Since $\mathcal{C}$ is required to be solvable it follows from the strong duality theorem that $D_{\mathcal{C}}$ is solvable too and the optimal points concur. The dual problem for $\mathcal{C}(x_1, \omega^j)$ is:

$$D_{\mathcal{C}}(x_1, \omega^j) \quad = \min_{\lambda \in \mathbb{R}^{m_2}} \quad \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda \tag{72}$$
$$s.t. \quad A_2^T \geq c_2(\omega^j).$$

The feasible set is defined as:

$$\Lambda^j \quad := \quad \{\lambda \in \mathbb{R}^{m_2} | A_2^T \lambda \geq c_2(\omega^j)\}. \tag{73}$$

As defined, this quantity is a polyhedron (Stein, 2016, Definition 2.1.5). One assumes $Rank(A_2^T) = m_2$.

If this is not the case redundant equations can be deleted with the Gauß elimination (row reduction) method, so that Λ^j is a sharp polyhedron and contains corners (Stein, 2016, Definition 3.1.5). Moreover, it follows from the solvability that the objective function $D_{\mathcal{C}}(x_1, \omega^j)$ is bounded to the feasible set. The corner theorem of linear optimization states that an optimal point is assumed to be in a corner (Stein, 2016, Definition 3.5.3). Therefore, the feasible set to the corners $vert(\Lambda^j)$ can be reduced. Since $vert(\Lambda^j)$ includes a finite number of elements, the non-empty and finite index set K^j is numbered and defined as:

$$vert(\Lambda^j) \quad = \quad \{\lambda^k, k \in K^j\}. \tag{74}$$

The vectors $\lambda^k \in \mathbb{R}^{m_2}, k \in K$ denote the corners of Λ. The dual problem $D_{\mathcal{C}}$ can be displayed as the minimum of a finite number of linear functions:

$$D_{\mathcal{C}}(x_1, \omega^j) \quad = \quad \min_{\lambda \in vert(\Lambda^j)} \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda = \min_{k \in K^j} \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda^k \tag{75}$$

Since $D_{\mathcal{C}}(x_1, \omega^j) = \mathcal{C}(x_1, \omega^j)$ holds, it can be formulated

$$\begin{aligned} \mathcal{C}(x_1, \omega^j) &= \min_{k \in K^j} \quad \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda^k \\ &= \max_{\alpha^j} \alpha^j \quad s.t.\, \alpha^j \le \min_{k \in K^j} \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda^k \\ &= \max_{\alpha^j} \alpha^j \quad s.t.\, \alpha^j \le \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda^k \quad \forall k \in K^j. \end{aligned} \tag{76}$$

Figure 62 presents the visualized result of the equation (76). As can be seen, the value function is pricewise linear and concave.

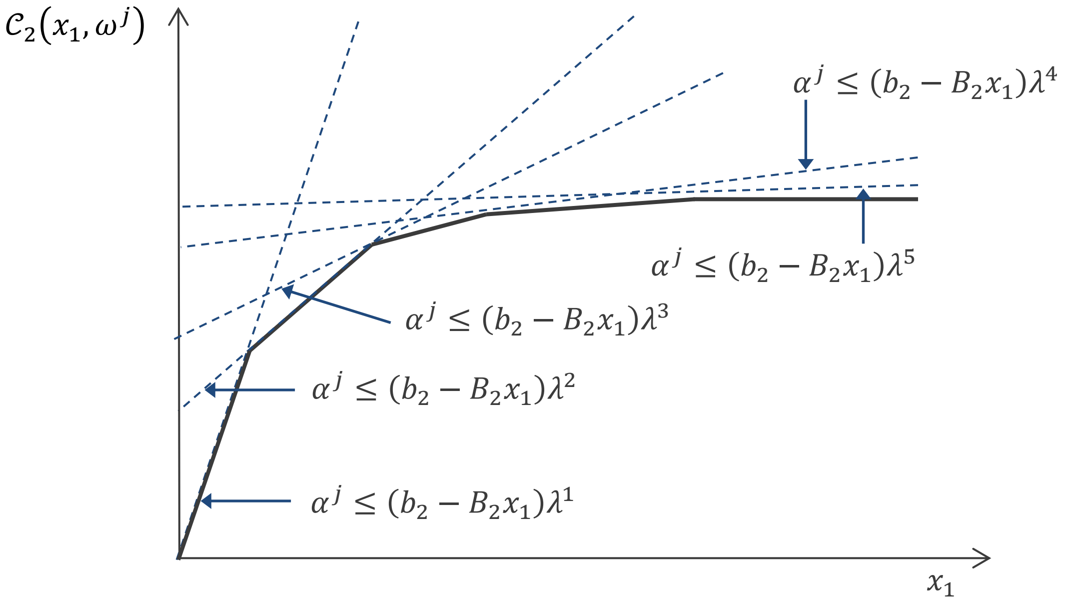


Figure 62 Mapping of the return function with linear functions

The second equality sign in equation (76) holds, since, according to the premises, the minimum is obtained and therefore the lower bound is equal to the minimum. The result is inserted into the value function $\mathcal{C}$:

$$\begin{aligned}
\tilde{\mathcal{Q}}(x_1) \quad &= \quad \tfrac{1}{N}\textstyle\sum_{j=1}^{N} \mathcal{C}\big(x_1, \omega^j\big) \\
&= \quad \tfrac{1}{N}\textstyle\sum_{j=1}^{N} \max\limits_{\alpha^j} \alpha^j \quad s.t.\, \alpha^j \le \min\limits_{k\in K^j} \big(b_2\big(\omega^j\big) - B_2 x_1\big)^T \lambda^k \\
&= \quad \max\limits_{\alpha^1,\dots,\alpha^N} \tfrac{1}{N}\textstyle\sum_{j=1}^{N} \alpha^j \quad s.t.\, \alpha^j \le \min\limits_{k\in K^j} \big(b_2\big(\omega^j\big) - B_2 x_1\big)^T \lambda^k \\
&= \quad \max\limits_{\alpha,\alpha^1,\dots,\alpha^N} \alpha \quad s.t.\, \alpha \le \tfrac{1}{N}\textstyle\sum_{j=1}^{N} \alpha^j \,, \alpha^j \le \min\limits_{k\in K^j} \big(b_2\big(\omega^j\big) - B_2 x_1\big)^T \lambda^k \\
&= \quad \max\limits_{\alpha} \alpha \quad s.t.\, \alpha \le \tfrac{1}{N}\textstyle\sum_{j=1}^{N} \min\limits_{k\in K^j} \big(b_2\big(\omega^j\big) - B_2 x_1\big)^T \lambda^k.
\end{aligned} \tag{77}$$

The first equality sign includes the definition of the value function, the second equality sign is a result of inserting equation (76). The third equality sign holds true since the target variables $\alpha^j, j = 1, \dots, N$ are independent of one another. The forth equality sign results from the epigraph reformulation (Stein, 2016, Exercise 1.3.7). In the last equation, the restrictions are summarized in which case the target variables $\alpha^1, \dots, \alpha^N$ vanish.

Stepwise Calculation of Cutting Planes for the Approximation of the Profit-To-Go Function

For the approximation of $\mathcal{C}$ the corners of the polygons need to be calculated which is a rather difficult problem. Therefore, not all corners are determined at ones but stepwise. Be $\hat{x}_1$the solution of the first stage problem, then $\hat{\lambda}_j$ is defined as follows:

$$\hat{\lambda}_j = \underset{\lambda_j \in \Lambda^j}{\operatorname{argmin}} D_Q(\hat{x}_1, \omega^j) = \underset{\lambda_j \in \Lambda^j}{\operatorname{argmin}} \left\{ \left(b_2(\omega^j) - B_2 \hat{x}_1\right)^T \lambda^k \right\}, \tag{78}$$

for all $j = 1, \dots, N$. For $\hat{x}_1$ it follows:

$$\begin{aligned} \tilde{\mathcal{Q}}(\hat{x}_1) &= \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \mathcal{C}(\hat{x}_1, \omega^j) \\ &= \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \left(b_2(\omega^j) - B_2 \hat{x}_1\right)^T \hat{\lambda}_j \\ &= \max_{\alpha} \alpha \quad s.t.\, \alpha \le \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \min_{k \in K^j} \left(b_2(\omega^j) - B_2 \hat{x}_1\right)^T \hat{\lambda}_j. \end{aligned} \tag{79}$$

The solutions of $D_Q(x_1^r, \omega^j)$ are λ_j^r for $j = 1, \dots, N, r = 1, \dots, R$, with x_1^r as the optimal point of the first stage in iteration r. With that the profit-to-go function is approximated by:

$$\begin{aligned} \tilde{\mathcal{Q}}_{R+1}(x_1) &= \max_{\alpha} \alpha \quad s.t.\, \alpha \le \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \left(b_2(\omega^j) - B_2 x_1\right)^T \lambda_j^r \qquad r = 1, \dots, R \\ &= \max_{\alpha} \alpha \quad s.t.\, \alpha \le \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \mathcal{C}(x_1^r, \omega^j) - (x_1 - x_1^r)^T B_2^T \lambda_j^r \qquad r = 1, \dots, R. \end{aligned} \tag{80}$$

The second equality in the equation (80) holds true for

$$\mathcal{C}(x_1^r, \omega^j) = \left(b_2(\omega^j) - B_2 x_1^r\right)^T \lambda_j^r \text{ and} \tag{81}$$

$$b_2(\omega^j)^T \lambda_j^r = \mathcal{C}(x_1^r, \omega^j) + (B_2 x_1^r)^T \lambda_j^r \tag{82}$$

with $j = 1, \dots, N$ and $r = 1, \dots, R$, if the last equation (82) is applied to the first equation (80).

The $(R+1)^{\text{th}}$ iteration of the master problem (71) of the first stage applying the approximation $\tilde{\mathcal{Q}}_{R+1}$ of (79) formulates as:

$$\begin{aligned} \tilde{P}_{R+1}: \quad & \max_{x_1, \alpha} && c_1^T x_1 + \alpha \\ & s.t. && A_1 x_1 = b_1 \\ & && \alpha \le \tfrac{1}{N} \textstyle\sum_{j=1}^{N} \mathcal{C}(x_1^r, \omega^j) - (x_1 - x_1^r)^T B_2^T \lambda_j^r \qquad \forall r = 1, \dots, R \end{aligned} \tag{83}$$

This problem (83) corresponds to the first stage master problem of the SDDP algorithm.

6.1.4. Theoretical Review and Convergence

In this part, the characteristics of the algorithm are examined. Proposition 1 states that the value function $\mathcal{C}_t(\cdot,\cdot)$ is piecewise linear as well as concave in the first argument and convex in the second argument. It further clarifies that the expected value function $\mathcal{Q}_t(\cdot)$ is also piecewise linear and concave. These characteristics are crucial for the functioning of the algorithm.

Proposition 1: The value function $\mathcal{C}_t(\cdot,\cdot)$ is

a) Piecewise linear and concave in x_t

b) Piecewise linear and convex in c_{t+1}.

If ξ_{t+1} follows a discrete distribution, then the expected value function $\mathcal{Q}_{t+1}(\cdot)$ is piecewise linear and concave.

Proof: see: Birge and Louveaux (2011, Chapter 3, Theorem 2).

Statement a) in proposition 1 can also be reasoned with equation (76) and Figure 62. Hence, it can be followed that also the expected value function $\mathcal{Q}_{t+1}(\cdot)$ is piecewise linear and concave, because $\mathcal{Q}(\cdot)$ is a positive combination of piecewise linear and concave functions. Equation (77) substantiates that and Figure 63 presents an exemplary value function $\mathcal{C}_t(x_{t-1}, c_t)$ for which the pricewise linearity can be seen graphically.

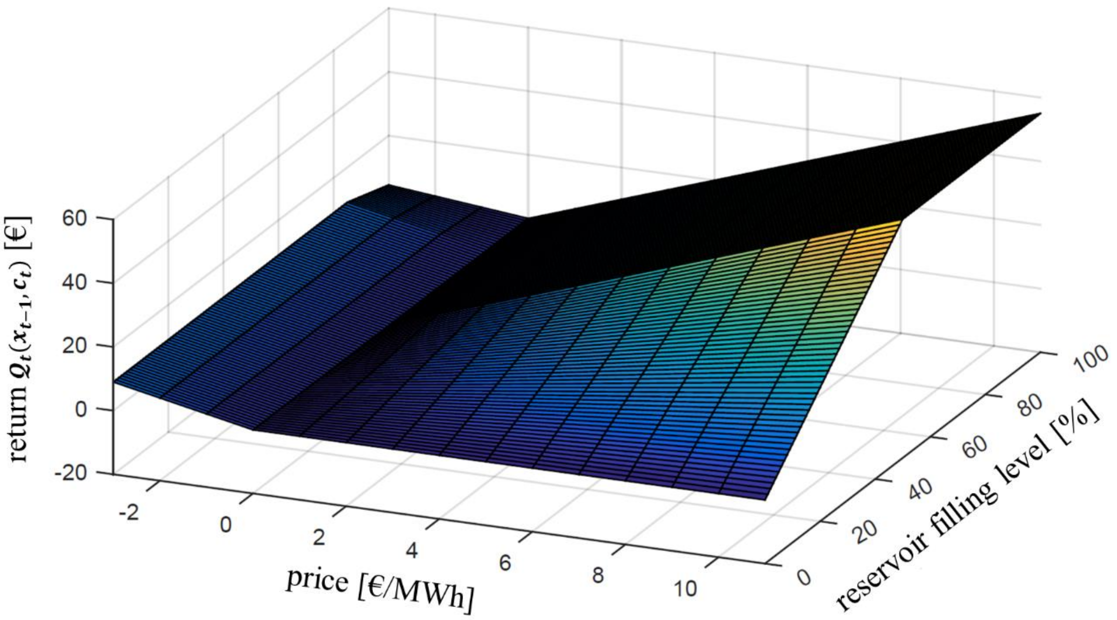


Figure 63 Profit-to-go function $\mathcal{Q}_t(x_{t-1}, c_t)$ mapped onto the reservoir filling level and the market price

Furthermore, Figure 64 illustrates the profit-to-go function $\mathcal{Q}_t(\cdot)$ of the value function $\mathcal{C}_t(x_{t-1}, c_t)$ on a two-dimensional level. On the left, the concavity, with regard to the upper reservoir filling level, shows that the higher the reservoir filling the higher the profits. Nevertheless, the bend at the end makes clear, a reservoir filled to the limit has a lower expected value since no space is available for pumping in times of negative prices. On the right, the convexity in relationship with the market price presents an increasing profit for higher prices. Low prices can be used for pumping reasoning the slope in the beginning.

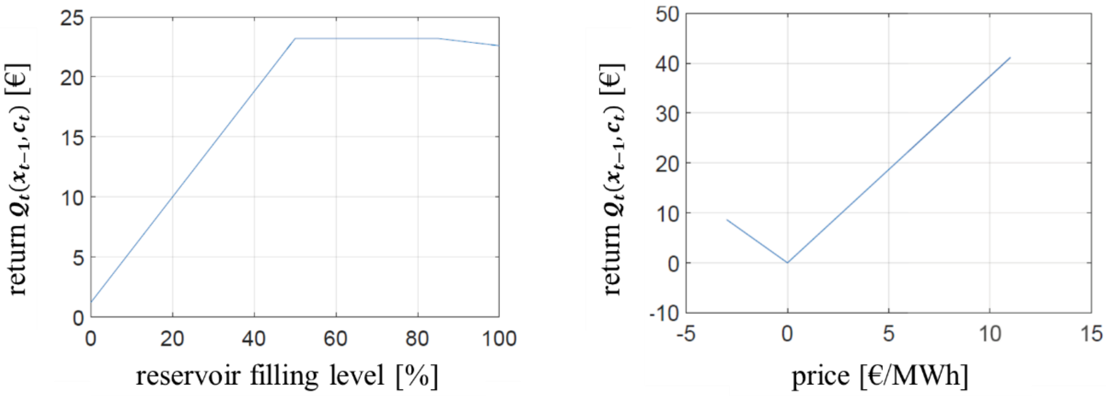


Figure 64 Value function $C_{t+1}(x_t)$. as a function of the reservoir filling level on the left and as a function of the price on the right

The proposition 2 below is needed to justify the stopping criterion.

Proposition 2: *The cutting planes of the SDDP algorithm, equation* (62) *and* (63) *are valid and form an upper bound for the value function on stage* $t + 1$.

Proof: see Birge and Louveaux (2011, Chapter 6, Theorem 1).

Theorem 1 formulates the condition for convergence of the approach.

Theorem 1: *The following conditions be true, if*

a) the sample size of Ξ *is finite,*
b) the data process is pricewise independent (i. e. ξ_{t+1} *is independent of* $\xi_1, \dots, \xi_t$*) and*
c) the sub problems have finite optimal values for all scenarios.

Then the forward step of the SDDP algorithm generates an optimal strategy after a sufficient number of forward and backward steps.

Proof: see Shapiro (2011, Proposition 3.1)

6.2. Multi-Cut Stochastic Dual Dynamic Programming

The original SDDP algorithm is defined to solve stage-wise independent stochastic problems. As for the most optimization problems, a good input is crucial for reliable output and prices and inflows are usually modelled with autoregressive models (AR), autoregressive moving average models (ARMA) or extensions of these which generate linear time discrete time series to describe stochastic processes. In this class of models, at least one input parameter depends on the current and the preceding value and is therefore stated as stage wise dependent.

In this chapter, it is described that the classic SDDP method is not applicable to dependent stochastic input parameters. This includes for example the consideration of dependent prices as presented in 6.2.1. After a literature review in 6.2.2, a further development of SDDP, MCSDDP is presented in part 6.2.3 which considers stochastic dependent input parameters. A similar algorithm were first introduced by the Norwegian researchers Gjelsvik, Belsnes and Haugstad (1999). Furthermore, the model is theoretically reviewed in terms of convergence in 6.2.4.

6.2.1. Challenges of Dependent Prices

The cutting planes of the introduced SDDP algorithm in 6.1.3 are shared over all time steps and for all discretized points $\xi_t^j,\ j = 1, \dots, N_t$ of the random vector ξ_t. This is just possible because the process is assumed to be stage-wise independent und the probability of every future scenario is the same. Geometrically, this can be substantiated with the concavity of the expected returns $\mathcal{Q}_t(\cdot)$ in terms of the state variable x_{t-1} and the reservoir filling levels. Analog to the cutting lines in Figure 62, the cutting planes provide an upper bound for the expected return. This is theoretically stated in the propositions 1 and 8.2 in chapter 6.2.4. Proposition 1 remarks that the return is convex in terms of the market price.

If the realization of the random vector ξ_{t+1} depends on the result of the preliminary step ξ_t, also the expected return can vary for different realization of ξ_{t+1}. This is especially the case when the price scenarios have different price levels. To avoid that the price may jump from the lowest to the highest price scenario the realized outcome of the preliminary step need to be transferred to the problem on the next step. Therefore, the expected return function depends on the decision x_t as well as the event ω_t and is defined as:

$$\mathcal{Q}_{t+1}(x_t, \omega_t) := \mathbb{E}_{(\xi_{t+1}|\omega_t)}[\mathcal{C}_{t+1}(x_t, \omega_{t+1})]. \tag{84}$$

Rebennack (2016, Theorem 4) shows that in this case $\mathcal{Q}_{t+1}(\cdot,\cdot)$ is still concave in x_t but in some special conditions convex in ω_t. Therefore, $\mathcal{Q}_{t+1}(\cdot,\cdot)$ cannot be assumed to be concave in c_t and, because of this, cannot be approximated by overarching cutting planes. In special cases, the expected return $\mathcal{Q}_{t+1}(\cdot,\cdot)$ can be analogously presented as in Figure 63 so that the return function $\mathcal{C}_{t+1}(x_t, c_{t+1})$ which is concave in x_t and convex in c_{t+1} can be seen. Cutting planes of the SDDP algorithm cannot provide proper upper bounds for such functions.

6.2.2. Literature Review

Various approaches exist to solve dependent energy scheduling problems. Below, the focus is on extensions to the SDDP algorithm or similar.

- Shapiro (2011) suggest to introduce a further state variable b_t to consider a possible stage-wise dependent “right hand side”; in his case, he considered dependent reservoir inflows. The expected return Q is dependent on (x_t, b_t). For the new “right hand side” one receives $b_{t+1} - \phi b_t = \epsilon_{t+1}$, with $\epsilon_2, \dots, \epsilon_T$ a stage-wise independent punishing term. The random vector is $\xi_t = (x_t, \epsilon_t)$. This reformulation is possible if b_t is an autoregressive process of the first order. The SDDP algorithm can be applied on the reformulated problem as long as all other stochastic processes are still stage-wise independent.

This work urges to find a solution for the short-term hydropower scheduling problem, focusing on prices rather than inflows since they are expected to have a higher influence on the revenue (Braun, 2015a). Whereas in many hydropower dominated countries the focus is on inflows, in Germany, price dependent pumped hydropower storages are predominant. Furthermore, in such systems, the price can be modelled independently of the inflows and the inflows can be estimated quite good based on historic evaluations. Therefore, below, further literature is reviewed on how dependent stochastic price processes might be included.

- Gjelsvik, Belsnes and Haugstad (1999) map the prices with a discrete Markov model. The generated price paths are ranked into clusters with respective transition probabilities. This problem is solved with a mix of SDDP and SDP. The price uncertainty is covered by the SDP algorithm. Advantage of the model is that the hydropower storages can be still modelled relatively detailed. Drawback is the extended run-time of the model. The combination of SDP and SDDP is also named MCSDDP.
- Furthermore, Gjelsvik, Belsnes and Haugstad (2010) introduced a work on the difference between a price taker and a price maker in local and global situations. The price is assumed to be an exogenous variable. They apply their own SDP-SDDP algorithm (Gjelsvik et al., 1999). They further extended the algorithm to consider water head effects as well as risk management.

6.2.3. Algorithm

In this work the MCSDDP approach of Gjelsvik, Belsnes and Haugstad (2010) is applied, since it fits best for the requirement to solve problems with stage-wise dependent prices in combination with stage-wise independent inflows. The algorithm is explained below.

The MCSDDP algorithm is an extension of the already introduced SDDP algorithm in chapter 6.1. The procedure is also characterized by a forward and a backward step. The price paths are given as discrete scenarios and are sorted in every time step into M discrete clusters. Every cluster has a representative $\zeta_{t,1}, \dots, \zeta_{t,M}$. With c_t as the random variable that describes the price at stage t and

$$\mathbb{P}\left(c_t = \zeta_{t,j} \middle| c_{t-1} = \zeta_{t-1,i}\right) = \phi_{i,j}(t), \qquad \forall i, j \tag{85}$$

as the transition probability of cluster i to cluster j in time stage t. The transition probability $\phi_{i,j}(t)$ is approximated by the share of scenarios in cluster i at time stage $t-1$ which corresponds to cluster j in time stage t. The inflows are modelled stage-wise independent and are approximated by a discrete distribution. The parameters $\delta_{t,1}, \dots, \delta_{t,L}$ describe the discrete inflows and b_t the random variable of the inflows. Be $\psi_{t,l}$ the probability for the inflow $\delta_{t,L}, l = 1, \dots, L, t = 1, \dots, T$. One can write:

$$\mathbb{P}(b_t = \delta_{t,l}) = \psi_l \qquad l = 1, \dots, L. \tag{86}$$

Inflows can be generated from historical data as described in chapter 3.2.3. The expected return function is dependent on the decision variable of the preliminary stage as well as the realization of the prices, as already introduced in chapter 6.2.1. Therefore, overarching cutting planes are not possible anymore. For the MCSDDP approach a set of cutting planes is calculated for every price condition.

In the backward step, it is not needed to solve the sub problem for all M^2 price combinations (c_{t-1}^i, c_t^j with $i, j = 1, \dots, M$). It is sufficient to solve M problems per stage for the prices $c_t^j,\ j = 1, \dots, M$ since the influence of the preliminary step c_{t-1}^i is already considered due to the transition probabilities.

The MCSDDP algorithm resembles the SDDP algorithm and is based on a forward and a backward pass for each iteration as well. In the forward pass trial solutions are computed that are used as lower bounds. In the backward pass, the expected return of every time step is approximated and cutting planes are added. The last result for the first stage determines the upper bound for the expected revenue. The main difference between MCSDDP and SDDP is the calculation and utilization of the cutting planes. This is explained in detail below. The discrete stochastic processes are defined as follows:

$$\Xi^{price} := \{\zeta_1, \dots, \zeta_T\} \text{ with } \zeta_t = \{\zeta_1^1, \dots, \zeta_t^M\} \qquad t = 1, \dots., T \tag{87}$$
$$\Xi^{inflow} := \{\delta_1, \dots, \delta_T\} \text{ with } \delta_t = \{\delta_1^1, \dots, \delta_t^L\} \qquad t = 1, \dots., T. \tag{88}$$

Thereby, $\zeta_1^1 = \zeta_t^M$ and $\delta_1^1 = \delta_t^L$ must apply, because price and inflow can be observed in the first time-step.

Forward Pass

Before the beginning of the forward pass K paths out of the discretized stochastic process are randomly chosen, as for the SDDP approach. These paths are expressed by $(c^k, b^k) \in \Xi^{prices} \times \Xi^{inflows}$ with $c^k = (c_1^k, \dots, c_T^k)$ and $b^k = (b_1^k, \dots, b_T^k),\ k = 1, \dots, K$. In the forward step, for all paths every sub problem is solved and the optimal point (trials) stored. The SDDP flow chart in Figure 60 is also valid for the MCSDDP algorithm. In iteration R of the overall algorithm the master problems of the forward pass for the paths $s = 1, \dots, K$ are:

<u>master problem 1:</u>

$$\begin{aligned} \mathcal{C}_1 \quad = \max_{x_1, \alpha_2} \quad & (c_1)^T x_1 + \alpha_2 \\ s.t. \quad & A_1 x_1 = b_1 \\ & \alpha_2 \le \alpha_{2,k,r} - (x_1 - x_{1,k,r})^T B_2^T \lambda_{2,k,r} \qquad \begin{matrix} r = 1, \dots, R-1 \\ k = 1, \dots, K \end{matrix} \end{aligned} \tag{89}$$

$x_1 \geq 0$

master problem t:

$$\mathcal{C}_t\left(x_{t-1,s,R}, \xi_t^s\right) \quad = \max_{x_t, \alpha_{t+1}} \quad (c_t^s)^T x_t + \alpha_{t+1} \tag{90}$$

$$s.t. \quad A_t x_t = b_t^s - B_t x_{t-1,s,R}$$

$$\alpha_{t+1} \leq \alpha_{t+1,k,r}^i - \left(x_t - x_{t,k,r}\right)^T B_{t+1}^T \lambda_{t+1,k,r}^i \quad \begin{array}{l} r = 1, \dots, R-1 \\ k = 1, \dots, K \end{array}$$

$$x_t \geq 0$$

with $i = \left\{i \in \{1, \dots, M\} \middle| c_t^s = \xi_t^i\right\}$

master problem T:

$$\mathcal{C}_T\left(x_{T-1,s,R}, \xi_T^s\right) \quad = \max_{x_T} \quad (c_T^s)^T x_T \tag{91}$$

$$s.t. \quad A_T x_T = b_T^s - B_T x_{T-1,s,R}$$

$$x_T \geq 0.$$

The optimal point of stage $t-1$ is handed over to the problem of stage t. The results of the forward pass are the optimal points $x_{1,R} = arg\max_{x_1} \mathcal{C}_1, x_{t,k,R} = arg\max_{x_t} \mathcal{C}_t\left(x_{t-1,k,R}, c_t^k, b_t^k\right)$ and a lower bound for the expected revenue. The lower bound is defined as:

$$\underline{z} = c_1^T x_{1,R} + \frac{1}{K} \sum_{s=1}^{K} \sum_{t=2}^{T} (c_t^s)^T x_{t,s,R} \tag{92}$$

The cutting planes are results of the backward pass for the iterations 1 to $R-1$. For every backward pass, in every time-step, K new cutting planes are added. In the first iteration cutting planes are not yet available. Therefore, one sets $\alpha_t = 0, t = 1, \dots, T-1$. Alternatively, as described for the SDDP approach, just for the first iteration a LP over all time steps can be solved for all sub problems and every path. This has the advantage that realistic reservoir filling levels are handed over to the backward pass which might increase the run-time. The first master problem, of every iteration, need to be solved just once since price and inflows of the first time-step are assumed to be known. For the same reason, in the first time-step, no start-cluster need to be separated; all $i = 1, \dots, M$ have the same transition probabilities which is why they can be neglected.

Backward Step

The backward step stands for the solution of the master problems, starting in the last time step, for all realizations of the stochastic process. Therefore, in each case, the trial solution of the forward step of the previous time step is hold fix. It is intended to determine the return in consideration of the different scenarios and the reservoir filling levels. The sub problems are solved for all $s = 1, \dots, K, j = 1, \dots, J$ and $l = 1, \dots, L$. The dual variables $\lambda_{t+1,s}^{j,l}$ of the reservoir filling level equation are issued for every inflow price combination. The dual variables denote the additional return of one further unit in the reservoir. Since the MCSDDP deals with dependent stochastic processes the dual variables are weighted by the probability of occurrence and transition. Further, they are multiplied with the change of the reservoir filling level $\left(x_t - x_{t,s,R}\right)^T$ so that alternations of the future revenue can be approximated depending on the reservoir filling levels. This is done by adding the expected return $\alpha_{t+1,s,R}^{l}$ for the reservoir filling level $x_{t,s,R}$. The profit is weighted with the probability of occurrence and transition. This is done by analogy with the dual variables.

<u>master problem T:</u>

$$\begin{aligned} \mathcal{C}_T\left(x_{T-1,s,R}, \zeta_T^j, \delta_T^l\right) \quad &= \max_{x_T} \quad \left(c_T^j\right)^T x_T \\ &s.t. \quad A_T x_T = b_T^l - B_T x_{T-1} \\ &\qquad\; x_T \geq 0 \end{aligned} \tag{93}$$

<u>master problem t:</u>

$$\begin{aligned} \mathcal{C}_t\left(x_{t-1,s,R}, \zeta_T^j, \delta_T^l\right) \quad &= \max_{x_t, \alpha_{t+1}} \quad \left(c_t^j\right)^T x_t + \alpha_{t+1} \\ &s.t. \quad A_t x_t = b_t^l - B_t x_{t-1,s,R} \\ &\qquad\; \alpha_{t+1} \leq \alpha_{t+1,k,r}^{j} \\ &\qquad\qquad -\left(x_t - x_{t,k,r}\right)^T B_{t+1}^T \lambda_{t+1,k,r}^{j} \qquad r = 1, \dots, R \quad k = 1, \dots, K \\ &\qquad\; x_t \geq 0 \\ &\qquad\; \text{with } t = T-1, \dots, 2 \end{aligned} \tag{94}$$

<u>master problem 1:</u>

$$\begin{aligned} \mathcal{C}_1 \quad &= \max_{x_1, \alpha_2} \quad (c_1)^T x_1 + \alpha_2 \\ &s.t. \quad A_1 x_1 = b_1 \\ &\qquad\; \alpha_2 \leq \alpha_{2,k,r} - \left(x_1 - x_{1,k,r}\right)^T B_2^T \lambda_{2,k,r} \qquad r = 1, \dots, R \quad k = 1, \dots, K \\ &\qquad\; x_1 \geq 0. \end{aligned} \tag{95}$$

The master problem of the first stage is solved just once for every backward pass. The expected return for scenario k of time step t till T in iteration R is composed as follows

$$\alpha_{t,k,R}^{i} = \sum_{l=1}^{L} \sum_{j=1}^{M} \phi_{i,j} \psi_l \mathcal{C}_t\left(x_{t-1,k,R}, \zeta_t^j, \delta_t^l\right) \tag{96}$$

for $k = 1, \dots, K, t = 2, \dots, T$ and $i = 1, \dots, M$. The belonging dual variables for the cutting planes are determined as

$$\lambda_{t,k,R}^{i} = \sum_{l=1}^{L} \sum_{j=1}^{M} \phi_{i,j} \psi_l \lambda_{t,k}^{j,l} \tag{97}$$

for $k = 1, \dots, K, t = 2, \dots, T$ and $i = 1, \dots, M$. On the second time-step there is only one preceding price so that cutting planes are just calculated for $i = 1$; hence the first master problem description leaves the index i aside. As for the SDDP approach the cutting planes build the outer approximation of the profit-to-go function. The result of the first stage corresponds to the upper bound of the expected return

$$\overline{z} = \mathcal{C}_1 = c_1^T x_1 + \alpha_2 \tag{98}$$

with (x_1, α_2) as the optimal point for the backward iteration R.

Stopping Criterion

The stopping criterion of the originally SDDP can be used for the MCSDDP as well, see chapter 6.1.3.

Overall Algorithm

The overall MCSDDP approach is formulated in algorithm 2 as pseudocode to provide a programming language-independent form of the algorithm.

Algorithm 2 Multi-Cut Stochastic Dual Dynamic Programming

Input Discretized stochastic process Ξ^{price}, Ξ^{inflow} Parameter $K > 1$ the number of paths, $\epsilon > 0$ stopping criterion tolerance.

1: Iterations index $R = 1$, $x_0 = 1$, $B_1 = 0$, $a_{T+1} = 0$.

2: **while** $\overline{z} - \underline{z} + \frac{1.96\sigma}{\sqrt{K}} \geq \epsilon$ **do**

scenario selection:

Algorithm 2 Multi-Cut Stochastic Dual Dynamic Programming

Choose of K scenarios $(c^s, b^s) \in \Xi^{price} \times \Xi^{inflow}$ with $c^s = (c_1^s, \dots, c_T^s)$ und $b^s = (b_1^s, \dots, b_T^s)$ and $s = 1, \dots, K$.

Forward pass

for $s = 1 : K$ **do**

  for $t = 1 : T$ **do**

    if $R = 1$ **then** $\alpha_{t+1} = 0$ fixed

    end if

    be $c_t = c_t^s, b_t = b_t^s$ und $i = \left\{ i \in \{1, \dots, M \,|\, c_t^s = \zeta_t^j\} \right\}$

$$x_{t,s}^R = \begin{cases} \underset{x_t, \alpha_{t+1}}{\operatorname{argmax}} c_t^T x_t + \alpha_{t+1} \\ s.t. \begin{cases} \alpha_{t+1} \leq \alpha_{t+1,k,r}^i - \left(x_t - x_{t,k,r}^r\right)^T B_{t+1}^T \lambda_{t+1,k,r}^r \\ r = 1, \dots, R-1 \; k = 1, \dots, K \\ A_t x_t = b_t - B_t x_{t-1,s,R} \\ x_t \geq 0 \end{cases} \end{cases}$$

  end for

end for

$$\underline{z}^s = \sum_{t=1}^T (c_t^s)^T x_{t,s,R},\ \underline{z} = \frac{1}{K} \sum_{s=1}^K \underline{z}^s \text{ und } \sigma = \sqrt{\frac{1}{K-1} \sum_{s=1}^K \left(\underline{z}^s - \underline{z}\right)^2}$$

Backward pass:

for $t = T : 1$ **do**

  for $s = 1 : K$ **do**

    for $j = 1 : M$ **do**

      for $l = 1 : L$ **do**

        be $c_t = \zeta_t^j,\ b_t = \delta_t^l$

$$Q_{t,s}^j = \begin{cases} \max_{x_t, \alpha_{t+1}} c_t^T x_t + \alpha_{t+1} \\ s.t. \begin{cases} \alpha_{t+1} \leq \alpha_{t+1,k,r}^r - \left(x_t - x_{t,k,r}^r\right)^T B_{t+1}^T \lambda_{t+1,k,r}^r \\ r = 1, \dots, R \; k = 1, \dots, K \\ A_t x_t = b_t - B_t x_{t-1,s,R} \\ x_t \geq o \end{cases} \end{cases} \quad [\lambda_{t,s}^{j,l}]$$

      end for

    end for

Algorithm 2 Multi-Cut Stochastic Dual Dynamic Programming

$\alpha_{t,k,R}^{i} = \sum_{l=1}^{L} \sum_{j=1}^{M} \phi_{i,j} \psi_l Q_{t,k}^{j,l} \quad \forall k = 1, \dots, K \; \forall i = 1, \dots, M$

$\lambda_{t,k,R}^{i} = \sum_{l=1}^{L} \sum_{j=1}^{M} \phi_{i,j} \psi_l Q_{t,k}^{j,l} \quad \forall k = 1, \dots, K \; \forall i = 1, \dots, M$

end for

end for

$\overline{z} = \frac{1}{K} \sum_{s=1}^{K} Q_{1,s}^{1,1}$

end while

6.2.4. Theoretical Review and Convergence

In theorem 2 the functioning of the algorithm is substantiated by the ascertain concavity and the peace-wise linearity. Therefore, the expected return can be described by cutting planes.

Theorem 2

The conditional return function

$$\mathcal{Q}_t\left(x_{t-1} \middle| c_{t-1}^{i}\right) := \sum_{l=1}^{L} \sum_{j=1}^{M} \phi_{i,j} \psi_l \mathcal{C}_t\left(x_{t-1}, \zeta_t^{j}, \delta_t^{l}\right) \tag{99}$$

is peace-wise liner, concave and can be represented by hyperplanes. The calculated cutting planes of algorithm 2 construct upper bounds for the expected return function $\mathcal{Q}_t\left(x_{t-1} \middle| c_{t-1}^{i}\right)$.

Proof: see Gjelsvik et al. (1999, Section 4.2)

Because the calculated cutting planes are real upper bounds for the return function, the usage of $\overline{z}$ from equation (98) as an upper bound is reasoned.

6.3. Numerical Results

In this part of the chapter, the methods MCSDDP and SDDP are compared with the deterministic approach based on CPLEX (LP) applied on linear planning problems. Further, the deterministic approach is applied on several linear programs to generate the solution under perfect information. The expected returns, water values, shadow prices and dispatch schedules are used as benchmark for the stochastic solutions. The results are compared and discussed. As motivated in the beginning of this chapter, the methods are applied on a time horizon of two weeks with a quarter-hourly discretization. The two simulated April weeks were characterized by low reservoir inflows and relatively low reservoir filling levels so that the sensitivity on price changes could be sharpened.

6.3.1. Modelling the Stochastic

The modelling of the stochastic is crucial for the result of the program. The quality of the resulting strategy depends on the goodness of the input. The possibilities to model stochastic price paths or inflows is manifold and significant differences exist within the alternatives. Beside a significant and steadily increasing number of models and approaches in literature every company that participates in the energy market has either an own price forward curve or purchases from one of the numerous data providers. Therefore, sophisticated modelling of prices and inflows is out of the scope of this work and is done relatively pragmatically.

Inflows

The modelling of inflows is based on the historic inflows into the respective reservoirs during the last ten years. For an increased comparability, a few exemplary reservoirs are modeled that incur inflows of similar actual reservoirs. EnBW AG provided exemplary but distorted data for the calculation with a daily resolution, see Figure 31. The inflows over the course of one day are assumed to be constant which is non-restricting since the fluctuations within one day are rather limited. Furthermore, the effect of changes in the inflows has been estimated, as relatively low (Braun, 2015a), which is why the historic inflows are not compiled in a recombining tree but further aggregated. Just the historic ten years average, maximum and minimum are considered to cover extreme scenarios. The node with average inflow is weighted by three fifths and the maximum and minimum inflows with one fifth. These are case specific assumption and do not need to apply to every reservoir there is. Especially for countries with large reservoirs as Brazil, Canada or Norway inflows play a more significant role. For this work storages with and without pumps are considered; this aspect may have an impact on the relevance of inflows on the optimization results.

Prices

The price scenarios are generated based on historic price curves. Afterwards, the price curves are shifted to the actual market price level and are further adopted. The price time series of the quarter-hourly EPEX Spot auction form 1st until 14th April 2016 is used (EPEX Spot, 2017b). With this time series 100 price paths are generated by adding random historic price spreads. The price paths are arbitrage free and the average of all prices on every time stage equal the received prices without spreads. These 100 scenarios are the basis for all following calculations. Figure 65 presents the employed reference price paths. The focus is on the exploitation of the quarter-hourly price changes. The quarter-hourly energy markets contain the highest volatility and the largest price spreads. This is because fluctuations of renewable energies, but also thermal power plants are traded into this market (Braun & Brunner, 2018).

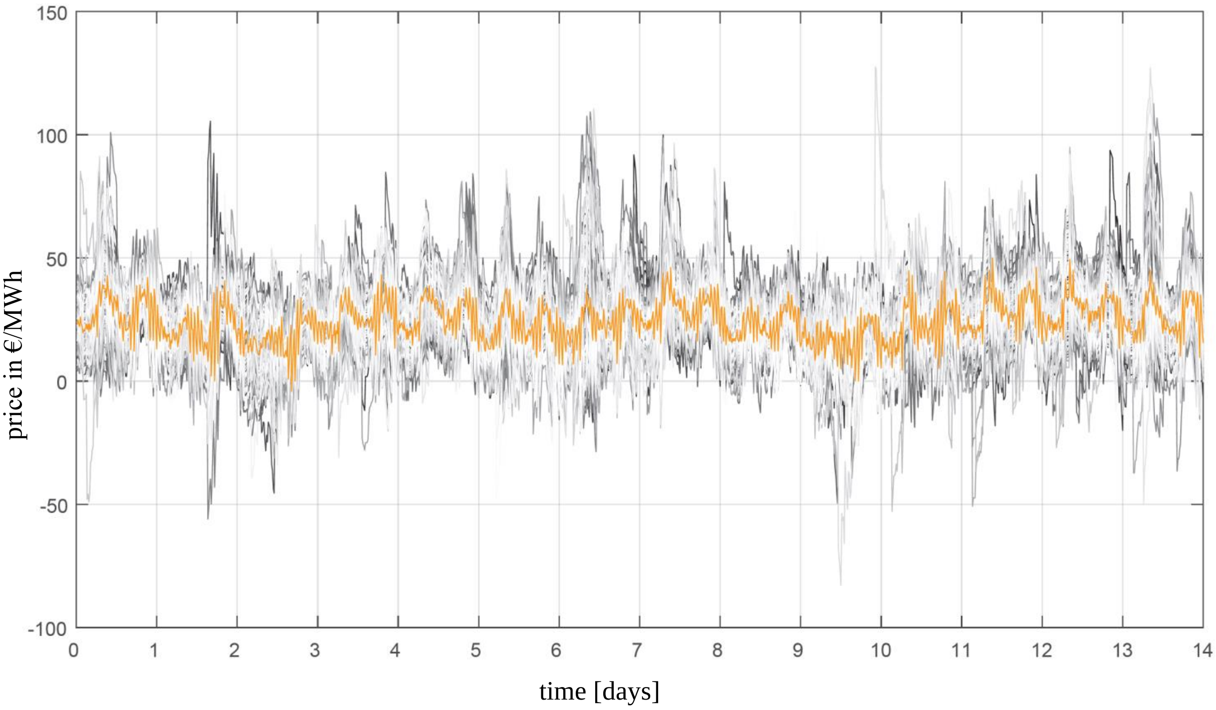


Figure 65 Price scenarios for the calculations, in orange the German quarter-hourly day-ahead Auction results from 1st to 14th April 2016 as reference prices (EPEX Spot, 2017b)

SDDP

The SDDP model requires stage wise independent processes which means that between two time stages the random vector can jump within the whole range of price and inflow scenarios. Based on historic price spreads the span between lowest and highest price path on each time stage differs; for example, at night the price changes are lower as in the morning hours when thermal power plants ramp-up and solar power feed-in increases. Especially during these hours, the price jumps can be significantly. Therefore, the stage wise independent process has drawbacks but due to the limitations to a belt that adapts its width and probability distribution this disadvantage is somehow compensated and the results are auspicious as trends and price levels can be simulated, see Figure 65. Furthermore, it can be assumed that the characteristic zig-zag pattern on the quarter-hourly market is not better mapped with a dependent price process.

The calculation is done for ten representative price samples out of the 100 scenarios. For each time step the prices are sorted according to size and condensed to ten groups. Afterwards, the representative for each group and time step is obtained from the average price of each group.

MCSDDP

MCSDDP considers stage wise independent processes that are based on transition probabilities. According to the SDDP approach, for every time stage, ten price classes are constructed. The transition probability of group i in time step t to group j in time step $t+1$ corresponds to the number of paths that direct from group i in time step t to group j in time step $t+1$.

LP and Perfect Foresight

The LP is based on the average price on each time step over all prices that is identical to the original received price series from the energy exchange. The prefect information model solves the linear deterministic optimization problem for each of the 100 price paths.

6.3.2. Model Setup

The model setup for the introduced algorithms and optimization methods intends to test mainly two things: firstly, the effect of different reservoir sizes on the outputs and secondly the applicability of the algorithm on more complex systems such as cascaded hydropower systems. Four setups of power plant layouts are defined. Three of them are two-reservoir systems connected via a turbine and a pump but with different storage sizes, see Figure 35. Case 1 simulates a classic daily pumped hydropower storage with 8 full load hours, case 2 a weekly pumped storage with 60 full load hours and case three a seasonal reservoir with 1500 full load hours. The last case simulates another weekly pumped storage with 114 and 33 full load hours respectively since it is a three-reservoir system connected with turbines and pumps, see Figure 66. Furthermore, the following assumptions are made:

- start reservoir filling level is half of the maximum filling level
- start and end (target) reservoir filling levels are the same
- minimum filling level is zero
- for consuming energy from the grid (pumping) a grid charge has to be paid amounting to $0.5\frac{€}{MWh}$.
- overflow in the upper reservoir is penalized with $100\frac{€}{1000m^3}$
- overflow in the lowest reservoir in the cascade is not penalized
- not reaching the target reservoir filling level is penalized with $10{,}000\frac{€}{1000m^3}$.

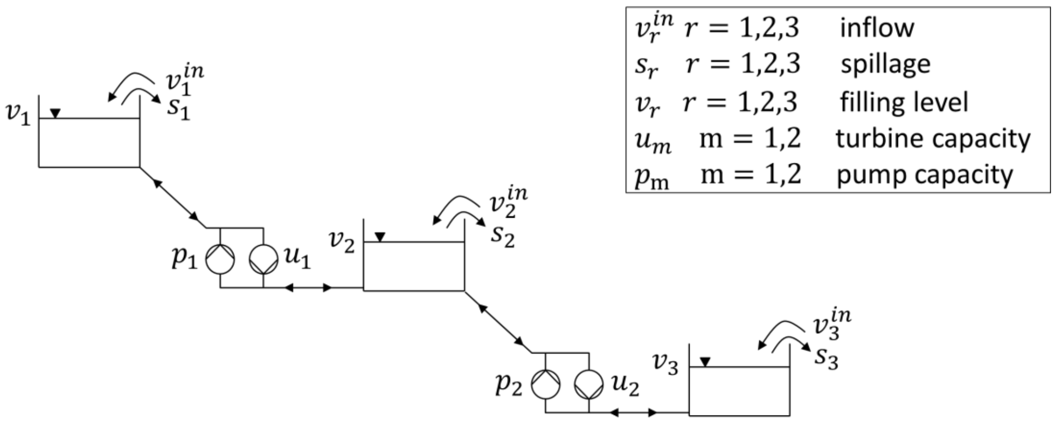


Figure 66 Reservoir cascade with three reservoirs connected with turbines and pumps

The three latter assumption are important measures to ensure target reservoir filling level achievement and to avoid implausible production schedules without risking insolvability. The specific parameters for the different setups are listed in Table 11. When using a quarter-hourly discretization the factors given in hours are divided by four respectively.

Table 11 Parameter definitions for the model calculations

<table>
<tr><th></th><th>characteristic</th><th>case 1</th><th>case 2</th><th>case 3</th><th>case 4</th><th>unit</th></tr>
<tr><td>reservoir 1</td><td rowspan="3">filling level</td><td rowspan="2">1200</td><td rowspan="2">9000</td><td rowspan="2">225000</td><td>40000</td><td rowspan="3">[1000m^3]</td></tr>
<tr><td>reservoir 2</td><td>5000</td></tr>
<tr><td>reservoir 3</td><td colspan="3">-</td><td>1500</td></tr>
<tr><td rowspan="4">machine 1</td><td>turbine capacity</td><td colspan="3">150</td><td>350</td><td rowspan="2">[MW]</td></tr>
<tr><td>pump capacity</td><td colspan="3">100</td><td>300</td></tr>
<tr><td>turbine flow through</td><td colspan="3">1</td><td>1.5</td><td rowspan="2">[1000m^3/ MWh]</td></tr>
<tr><td>pump flow through</td><td colspan="3">0.7</td><td>1</td></tr>
<tr><td rowspan="4">machine 2</td><td>turbine capacity</td><td colspan="3" rowspan="4">-</td><td>150</td><td rowspan="2">[MW]</td></tr>
<tr><td>pump capacity</td><td>100</td></tr>
<tr><td>turbine flow through</td><td>1</td><td rowspan="2">[1000m^3/ MWh]</td></tr>
<tr><td>pump flow through</td><td>0.7</td></tr>
</table>

The optimizations are modeled in GAMS which is an algebraic modeling language for mathematical optimization problems. The linear sub problems solved with the CPLEX solver. It is noted that many more calculations have been performed than discussed here for different storages sizes, time periods and cascades. It is obvious that with increasing complexity of the power plant layout as well as longer optimization periods the convergence of SDDP and MCSDDP method is more computation intensive. Due to a significant number of iterations with new cutting planes the amount of data is high. One strategy could be to delete redundant cutting planes that are not used anymore. To decrease the runtime of the model parallelizing of the algorithms is possible and suggested as well. These runtime improvements were not exploited since it is not focus of this work. The calculation has been performed for the LP, SDDP and MCSDDP. The following data evaluation approach is used:

- assessment, screening and definition of the calculation results
- definition of an optimal strategy
- back testing of the applied strategy by means of the 100 price paths, independently of the model calculations
- evaluation and comparison of the results of the back testing.

6.3.3. Steering Parameters

Although, due to various reasons, models are abstractions of the real-world, at the end, the model results need to be transferred into a real-life dispatch. To transfer the model results into dispatch decisions some assumptions and transformations are needed. For a general introduction into steering parameter see chapter 3.4.

Generally, optimization problems are solved regularly with a rolling horizon, e. g. every day for a period of a week. The latest calculations provide the decision support for the respective next interval. If changes in the input data occur between two calculations these changes are not considered in the dispatch until the next calculation. This has an impact on the dispatch. One question is how important is the influence of changes of the input data on the model output and respectively the dispatch and could the optimization start as soon as changes occur to avoid such gaps. Whereas the latter is a possible solution it is limited by the computational challenges. The former depends on the optimization method.

LP

For the short-term hydropower dispatch, the dual variables of the optimization are suggested, see chapter 3.4.1. The dual variables describe the additional value in the objective function if one additional unit of water is available in the reservoir filling level equation in a specific time step. This additional unit of water can then be used in the time step with the highest possible return that has not been utilized already. Note that water can be stored and shifted within time steps and that the time steps are linked together. The dual variable denotes the marginal cost or the value of the water in the respective reservoir.

For the LP the dual variables can be directly received as the marginals of the reservoir balancing equations as part of the optimization results. Every set of dual variables is linked to the price input of the model due to perfect foresight. If several price paths are solved separately each price path results in an own set of dual variables. A possible overall water value could be calculated taking the mean water value over all price paths for every step or by using shadow price of the input data that fits best to the observed market price.

SDDP

Stochastic optimizations have the advantage to be more insensitive towards changes in the input data since a probability distribution is already considered as input. Due to this advantage, the stochastic optimization model does not provide one unequivocal result but many optimal points for different price and filling levels. Solving the deterministic equivalent of the stochastic model yields to a set of optimal points and decisions for each path and time step. But solving the whole stochastic problem is so time consuming that approximate stochastic optimization approaches are applied such as SDDP or MCSDDP.

To find the optimal dispatch decision for approximate stochastic optimization methods is more challenging since one does not receive optimal points and decisions for each path and time step. The sub problems are just solved for K random scenarios. Nevertheless, the strategy for the K scenarios includes the information about the price expectation of all prices, and therefore, the results are the on average optimal strategy.

The first and simplest possibility to transfer this set of results into a decision support is not to use the optimal points but the optimal values. They represent the expected value of the water in the reservoirs for the observed time periods based on the input parameters. A few possibilities exist to generate shadow prices with SDDP. Abgottspon (2015b) suggests to determine the actual reservoir filling level from the active cutting plane and to use its λ. The active cutting plane is denoted as the one with the lowest value. This approach is relatively strenuous in its application. To calculate the shadow prices beforehand to use them in spot market trading all active cutting planes for all possible combinations of reservoir filling levels need to be computed.

Another approach determines the $\lambda_{t,k}$ of the active cutting planes as the results of the last forward pass and calculates the average of $\lambda_{t,k}$over the K scenarios. This concept avoids the computation of water values for most of the implausible reservoir filling level combinations. It is assumed that the SDDP approach approximates the plausible expected reservoir filling levels. Thus, the expected water values are expected to be a good choice.

MCSDDP

The main difference between MCSDDP and SDDP is that for every price state, instead of overarching cutting planes in SDDP, a particular cutting plane is calculated to take into account that the input prices are dependent. Therefore, it is not possible to use, analogous to SDDP, the active cutting planes. A suggestion is to use the multiplicators $\lambda_{t,k,R}^{j}$ of the added cutting planes in the last iterations (Gjelsvik et al., 2010). Hence, the M different price cluster are used and the average over K scenarios calculated. For every time step and price cluster water value is received. For the dispatch, just the water value of the price cluster is used in which the observed market price is located.

Definition of a Strategy

After receiving the water values for the reservoir these are transferred into shadow prices as described in chapter 3.4.2. In this chapter several dispatch strategies are defined. The first and predominant strategy is, if the shadow price of the turbine is above or equal to the observed market price the turbine is used with full power. The pump is used at full power if the shadow price for the pump is above or equal to the observed market price. Otherwise turbine and pump stand still.

A further subtype strategy is introduced, which is relevant for the back-testing in chapter 6.3.5, to deal with infringements of the reservoir balancing equation. It is analogue to the one above but with the restriction that turbine and pump cannot be used when the maximum and minimum reservoir filling levels are violated. In real-life, this always holds true but for back-testing it is relevant. Grid charges, if needed, can be considered.

6.3.4. Model Results

The two algorithms described in chapter 6.1 and 6.2 are applied on the case studies described in 6.3.2. The algorithms provide, as introduced in chapter 6.3.1, the expected return, optimal schedules, reservoir filling levels and, after the described post-processing in chapter 6.3.3, shadow prices. These results are presented in this chapter and compared with a back-testing approach in the next chapter 6.3.5.

The optimal objective values of the different models: LP, SDDP and MCSDDP applied on small, medium and huge reservoirs can be seen in Figure 67. The optimal values denote the expected return of the respected models. It becomes apparent that the larger the reservoirs the higher the return. This is because the larger storages can realize more and longer load cycles. In this example, the large reservoir cannot take the advantage over the medium reservoir, because the weekly pumped storage can already realize nearly all possible charge cycle options.

Normally the objective value of the perfect foresight model determines the upper bound. As we can see, the return of the SDDP model is slightly higher as the return of the perfect foresight model. This can be explained: The perfect foresight model has calculated the optimal solution for the original 100 price paths. In a back-test against these 100 price paths this model provides necessarily the optimal solution. Nevertheless, the SDDP is performed for all possible combinations of the ten price cluster on each stage. All these possibilities result in 10^{1343} price paths (14 days with 96 quarter-hours) which are further multiplied with the number of inflow combinations that are covered by the overall scenario tree of the SDDP. For a real upper bound, LPs for all combinations would need to be solved which is not possible.

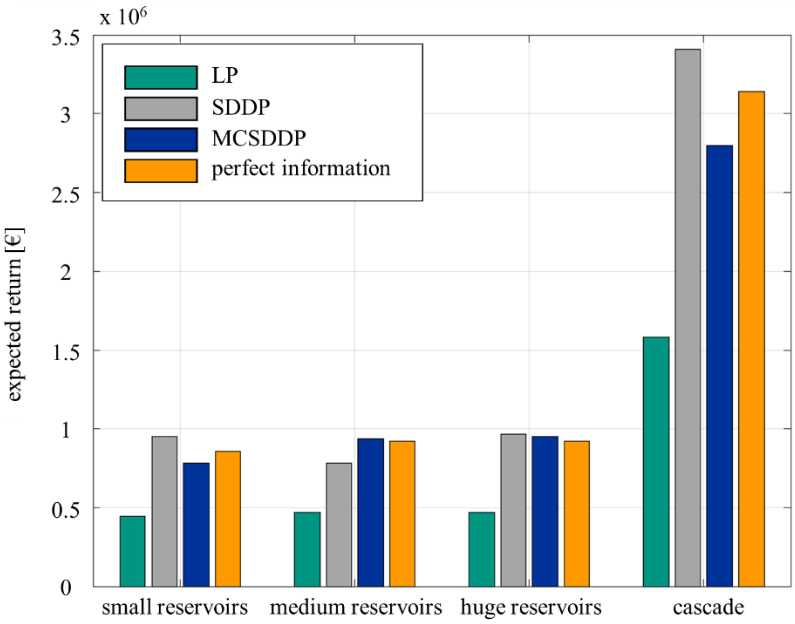


Figure 67 Expected return from the optimal values of the various calculated models and scenarios

The formation of the shadow prices is illustrated in Figure 68 for the LP case. The market prices in €/MWh are featured on the left vertical axis and the reservoir filling levels in 1000m³ on the right vertical axis spreading over a period of two exemplary days. The orange solid line depicts the price curve, the grey solid line the shadow price of the turbine and the grey dotted line the shadow price of the pump. The reservoir filling develops according to the dispatch of the pumped hydropower storages based on the shadow prices. A market price above the turbine price means producing and a price below the pump price means pumping water into the upper reservoir. The algorithms optimize the plants within the limits of the reservoirs. If a reservoir limit takes effect, as seen in the middle of the figure, a new shadow price is given. The next charge cycle is carried out on a new price level. This result can be seen with other optimization methods as well (Braun & Hoffmann, 2016).

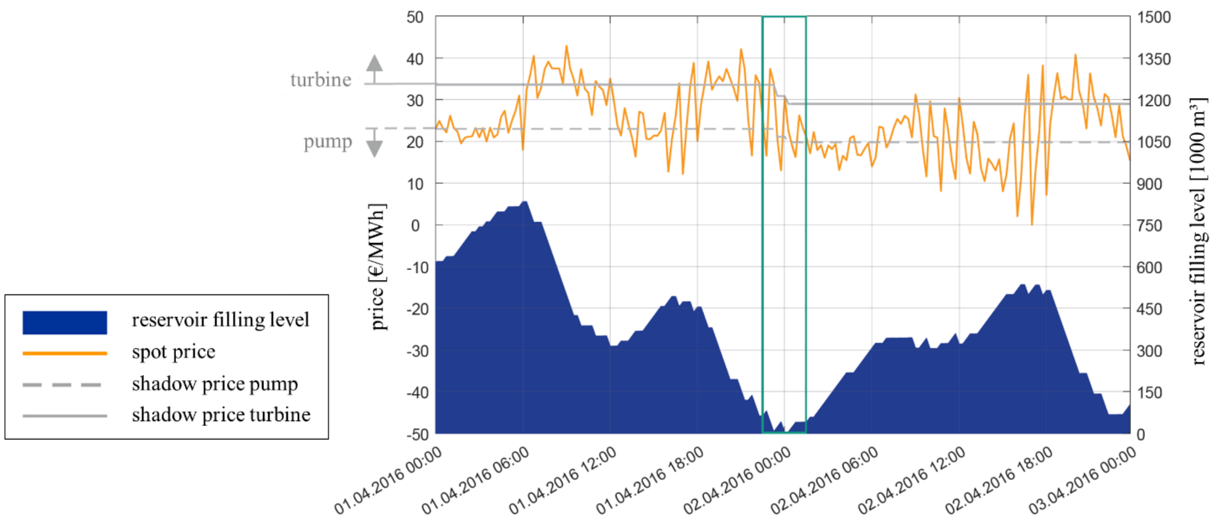


Figure 68 Quarter-hourly day-ahead reference prices (orange) and the results of the LP optimization: shadow prices for pump and turbine (grey) as well as the reservoir filling level (blue)

The LP, SDDP and MCSDDP approaches are applied on different reservoirs to identify the impact of the reservoir size on the behavior of the shadow prices. Figure 69 presents the shadow prices of the turbine for the different reservoir versions over the course of one week. It can be seen that the smaller a reservoir the more the shadow prices of the turbine fluctuate, independent of the chosen method. The medium and the large storages have nearly the same and constant shadow prices since for both storages the reservoir filling level constraints are not violated during this one week. For the LP and SDDP approach one shadow price for each time step is calculated. As a result, of the MCSDDP optimization shadow prices for each of the 10 price clusters are determined since also the dispatch depends on the realized price in each time step. Since the test case with the smallest reservoir shows the reasonably highest fluctuations between the models the focus below will be on this case.

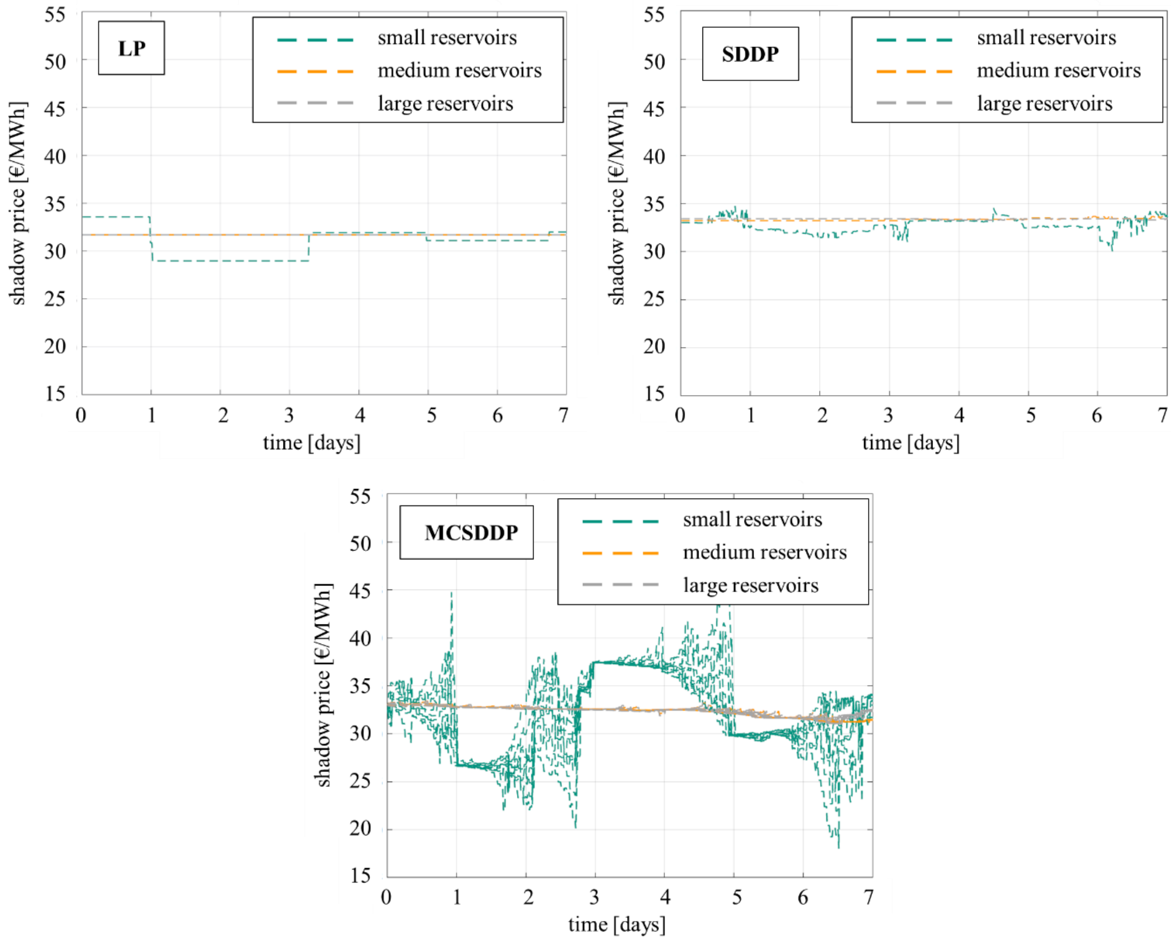


Figure 69 Shadow price for the turbine over the 7-day calculation period for different optimization methods

Figure 70 presents the typical upper and lower bound approximations for the SDDP and the MCSDDP algorithms over the iterations of the algorithms. It can be seen, that the upper bound, calculated in the backward pass, represents a real upper bound. Which means that the upper bound is decreasing with every time step. The upper bound is the sum of the first time-step model result plus the approximation of the future expected value. The approximation improves in every time step while converging to the optimal solution. The lower bound originates form the forward pass and depends on the characteristics of the (randomly) chosen price path. For price paths with higher prices higher profits can be generated as with lower prices. Though, with the continuous improvement of the strategy due to a higher state of information in the profit-to-go function, a trend of the lower bound can be seen clearly.

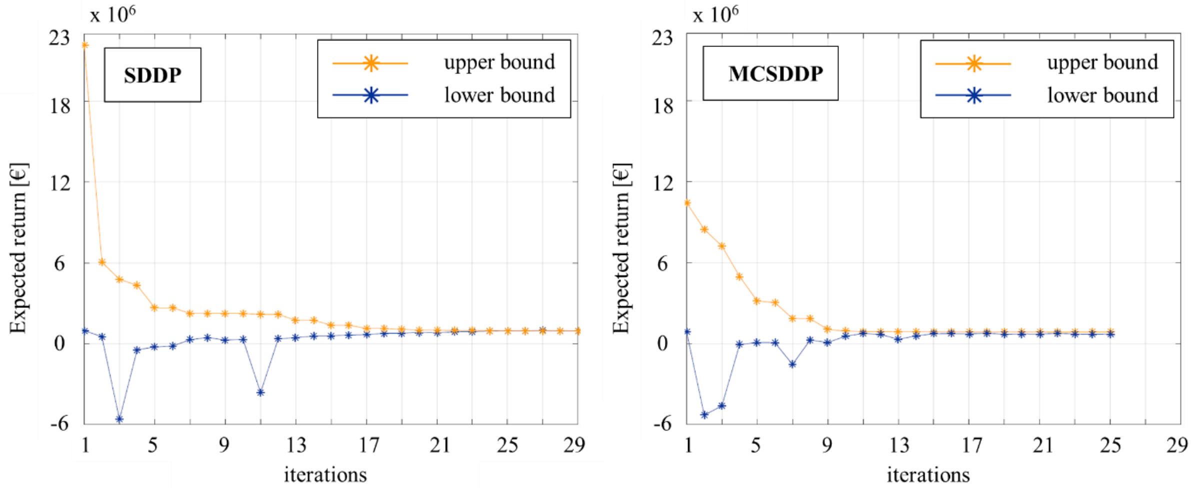


Figure 70 Approximation and convergence of upper and lower bound to the expected return during the iterations of the model

6.3.5. Back-Test Results

In this part, the quality of the model results is analyzed in back-testing simulations using the respective steering parameters of the model to determine the dispatch in the simulation. The general objective is that the steering parameters, here the shadow prices, lead to a similar dispatch in the back-test modulation as intended by the model. This includes a good assessment of prices and inflows as well as steering parameters that can transfer the results of the model into reality. If this is not the case, the whole optimization could be worthless. Below, back-testing strategies are defined, and the results analyzed.

Back-testing strategy no.1 is to use the steering parameters of the model calculations against the 100 original price scenarios. The pumped hydropower storage is switched on into turbine mode if the observed price is above the shadow price of the turbine and switched into pumping mode when the shadow price for the pump is underpriced, see chapter 3.4. With strategy no.1 the reservoir filling level limitations are regularly violated. In practice, a full reservoir cannot be filled and an empty reservoir cannot be emptied anymore. Therefore back-testing strategy no.2 considers that if the maximum upper reservoir filling level is reached the pumps, and if the minimum upper reservoir filling level is reached the turbines, are set as unavailable.

Figure 71 presents the distribution of reservoir end filling levels of the back-tests. The shadow prices of the models were applied in the back-test on 100 price paths. On the left of the graph are the results for strategy no.1 and on the right the results for strategy no.2. The back-testing is presented for the smallest reservoir of the cast study, since the deviations were most significant. Furthermore, the filling refers to the upper reservoir similar to the load level of a normal battery.

The shadow price steering parameters are supposed to lead to a dispatch in the back-testing that adheres the target filling level given to the optimization program beforehand. First, strategy no.1. is analyzed illustrated on the left side of Figure 71. The observation is that all methods are not able to adhere the maximum and minimum filling levels at all time. Although big differences between the LP, SDDP and

MCSDDP can be seen. Comparing the three methods the LP steers more aggressively resulting in a 61% share in which the upper storage is empty or theoretically more than empty. In 32% of the cases the upper reservoir filling level was at more than 100%. This is reasonable since the LP had no knowledge on further price deviations but already reveals the great disadvantage of LP. Nevertheless, the SDDP and MCSDDP algorithms do not perform significantly better, acting to restrained resulting in a theoretical overflow in the upper reservoir in more than half of the cases. Nevertheless, the SDDP method is the most balanced.

For strategy no.2. LP and SDDP perform similar with the LP more aggressive and the SDDP more conservative. The MCSDDP is far too cautious exceeding the target reservoir filling level in 100% of the cases.

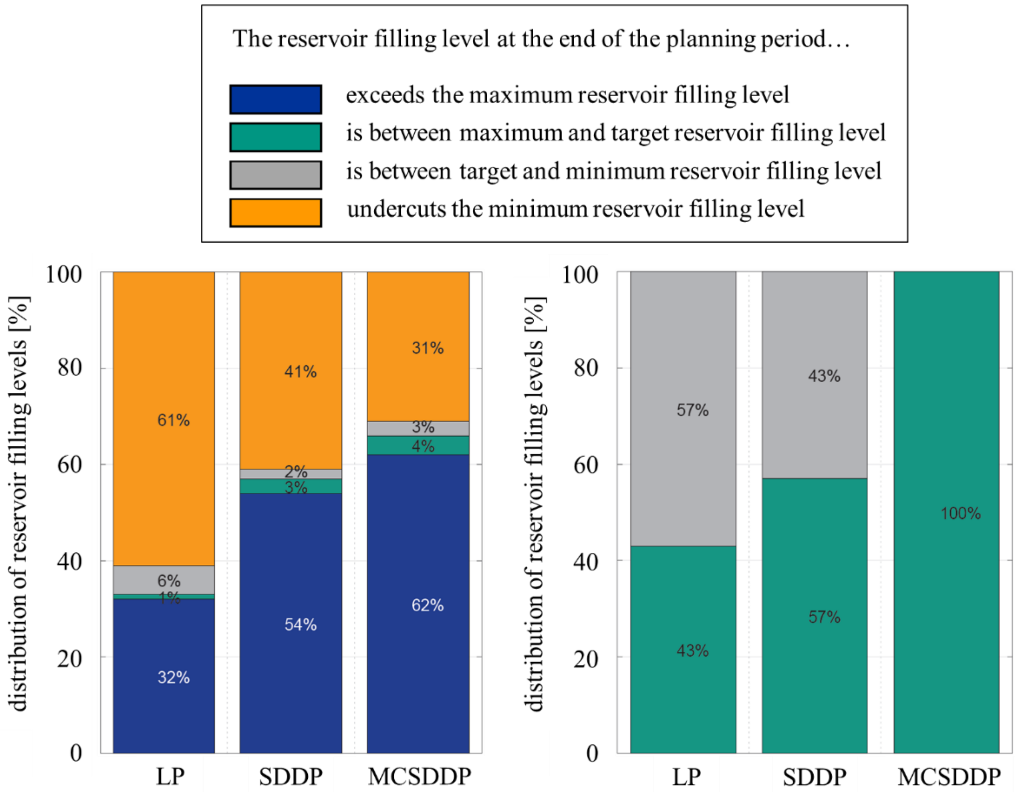


Figure 71 Distribution of the end reservoir filling levels after the back-testing simulation for 100 price paths. On the left the results with strategy no.1 and on the right strategy no.2.

The stochastic algorithms SDDP and MCSDDP present a more conservative approach when it comes to the dispatch of the stored energy. This is an explicitly wished effect since the stochastic models act more restrained to be able to exploit extreme situations. Probability and level of extreme situations is mapped in the stochastic input prices but not in the average price path for the LP.

Nevertheless, in the back-testing, the SDDP and the MCSDDP do not release to less water but pump to much water into the upper reservoir. Exceeding the upper reservoir filling level bounds in the case of the SDDP and the MCSDDP is therefore also a result of the unprecise calculation of the pump shadow price.

Until here, the pump shadow price is calculated as the turbine price minus the cycle efficiency and charges for grid usage. In Figure 72 an approach is presented to find the optimal pumped shadow price also for SDDP and MCSDDP in a back-testing simulation. Therefore, the pump shadow price is proportional shifted downwards. For the SDDP the maximum return can be achieved with a shadow price reduction of about 15%. Then, also the distribution of the reservoir filling levels is more evenly spread between target level exceeded and target level not reached. That means that shadow prices for pumps need to be founded in a back-testing simulation as described above. This is reasonable since SDDP and MCSDDP are more restrained towards water release and therefore have slightly higher shadow prices as the LP model. Calculating the pump price in the same way as for the LP leads to a higher pumped shadow price as well which induces extraordinary quantities of pumped water. Therefore, it is better to decrease the pump shadow price until a balance is reached between water release and pumping.

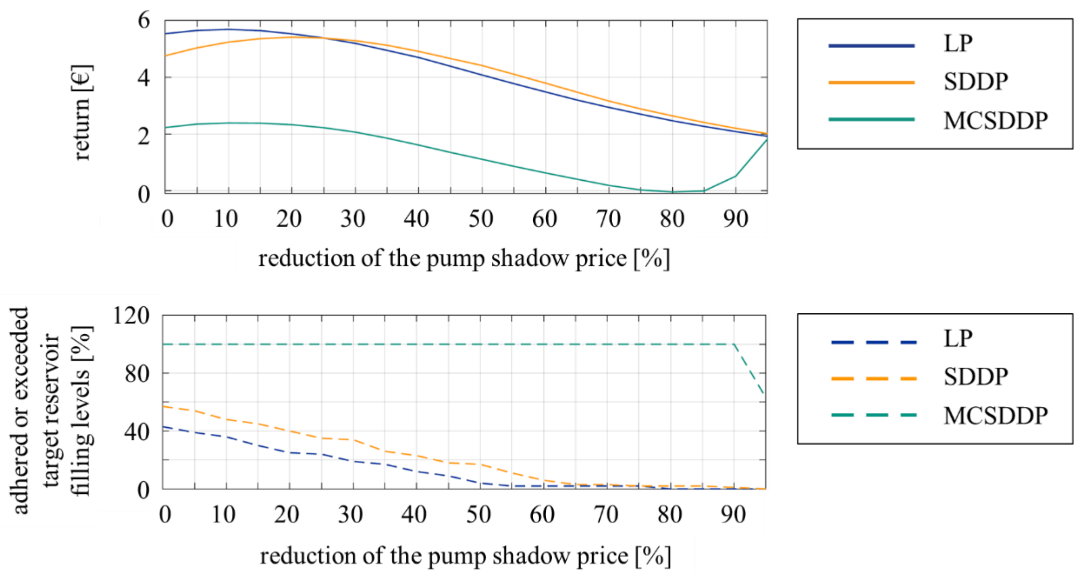


Figure 72 Expected return and the adhering of exceeded target reservoir filling levels as a function of the proportional reduction of the model pump shadow price for the back-testing

It becomes apparent that the consideration of pumps has a great influence on the choice of the optimization model. As presented above, SDDP is generally applicable to short-term optimizations as well as systems with pumps. Drawbacks of the application of stochastic optimization are the more complex generation of pump shadow prices and the limited benefits of stochastic optimization.

This opens up the question why many other papers come to different assessments of SDDP in terms of revenue and adhered reservoir filling levels in comparison to LP. But most of these works have in common to analyze systems in Brazil or Norway with no pumps installed. Other publications do not consider short-term energy markets. Both results in the same effect. The models do not have the option to refill the reservoirs when needed. The reservoir filling solely depends on the unknown hydro inflows. An empty reservoir means no dispatch until the next natural inflow. This makes the dispatch decision more critical. Releasing the water to early can be very expensive in the long run. A pumped hydropower storage and a prevailing liquid energy market do always provide the possibility to adjust the filling level or even correct mistakes.

To show this observation the same pumped hydropower storage system as presented above were calculated, just without pumps. It gets very apparent that in this case the SDDP algorithm outperforms the LP in target filling level compliance as well as revenue, see Figure 73. The upper graphs illustrate the reservoir filling levels over the time in the back-testing with several different price realizations. Furthermore, the SDDP profit is about 10 % higher as for the LP, whereas it undercuts the perfect foresight optimum by about 20 %.

This also means on the one hand side that short-term energy markets in combination with pumped storages do not necessarily need the careful handling of the stochastic optimization rather to be pushed to the limits with a perfect foresight LP. On the other hand side, optimization, steering parameter calculation and back testing of SDDP in combination with systems with pumps reveal further research potential.

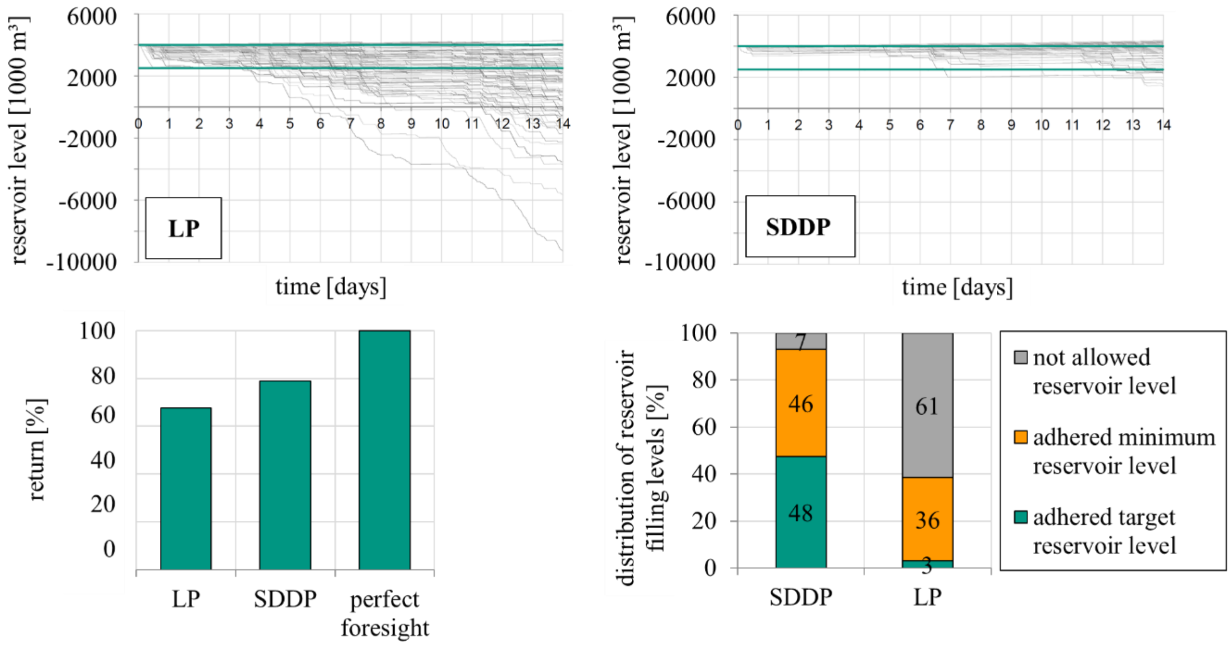


Figure 73 Back-testing of a system without pumps: reservoir filing levels over the course of two weeks for LP and SDDP, the expected return in comparison to perfect foresight and the distribution of the reservoir filling levels

6.3.6. Conclusion

In this part, the stochastic approaches SDDP and MCSDDP are applied on the hydropower scheduling problem. Beside the wide application of the SDDP method in hydropower dominated countries in the long-term, here the application on a short-term pumped hydropower storage system is presented. This includes fore example a weekly pumped hydropower storage that could be used to exploit price changes due to fluctuating wind power generation that is mapped in the stochastic price input data.

In the numeric analysis, the two algorithms are compared with a linear and a perfect information model. With the model results, shadow prices are generated that are used to steer the dispatch in a back-testing against 100 different price paths. Whereas the direct model results of SDDP, MCSDDP and perfect foresight show higher profits as the LP, this cannot be confirmed in the back-testing. Two effects can be seen: First, the SDDP and even more MCSDDP are conservative in terms of water dispatch and therefore exceed the target reservoir filling levels in 57% (SDDP) and 100% (MCSDDP) of the price paths. The LP model is relatively aggressive and undercuts the target reservoir filling level in 57% of the back-tested price paths. Secondly, comparing the revenue of the three models, the stochastic models achieve the planned revenue also in the back test. In comparison, the model calculations for the LP presented an optimal revenue of about half the profit that is calculated with SDDP or MCSDDP. Nevertheless, in the back-test the LP shadow price steering exceeds the revenue of SDDP or MCSDDP.

Concluding, the stochastic models are more cautious and have a better target reservoir filling level compliance, but the LP outperforms the stochastic models in terms of profit. It should be noted that not achieving the target levels is penalized just with the costs of pumping the additional water into the reservoir. With a more rigid penalizing the stochastic models may outperform the LP. Which model to be taken depends therefore on the objective of the power plant operator. The difference between the models decreases for larger reservoirs. Furthermore, these results just apply on systems with pumps. It could be shown that the similar systems without pumps the SDDP significantly outperforms LP. It is therefore recommended to solve short-term pumped hydropower storage systems with pumps and available liquid energy markets with LP and systems without pumps or limited possibilities to flexible trade short-term energy to be solved with stochastic approaches.

7. Continuous Optimization of Quarter-Hourly Intraday Market

In this chapter, an approach to optimize and trade daily pumped hydropower storages during the intraday is presented taking market regulation issues and short-term trading possibilities into account. The fundamentals of this part are based on common work with my college and supervisor Dr. Rainer Hoffmann at EnBW Energie Baden-Württemberg AG that we published in 2016 (Braun & Hoffmann, 2016).

With the so called Energiewende the conditions on the German energy market have changed fundamentally. A lower price level but most notably the flattened regular price spread between peak and off-peak have influenced the profitability of daily pumped hydropower storages. A lot of optimization methods for daily pumped hydropower storages struggle to address the challenges on the energy markets in Germany. Furthermore, new regulatory requirements of the German federal network agency (BNetzA) were introduced (BMWi, 2015). Since 2014 energy producers are committed to frequently report information on planned production and even on provision of balancing energy of each single generator to the transmission system operators. As soon as a deviation in the schedule occurs, the information has to be updated and reported again. These requirements lead to the point where optimization of pumped hydropower storages can no longer be done manually.

In order to fulfil these requirements, an optimization model was developed and a system-based day ahead and intraday asset optimization process has been established that is currently in use at EnBW Energie Baden-Württemberg AG. The optimization problem is formulated as a mixed integer problem which determines the minimum operating cost subject to all technical constraints of a hydrothermal portfolio and covering load (Burger et al., 2004). As a post-optimization of this new intraday optimization system we set up an effective multistage looping optimization algorithm for daily pumped hydropower storages considering e. g. reservoir limits, quarter-hourly prices, grid charges and availabilities.

In the first part 7.1, the reader will find the motivation for the continuous optimization of pumped hydropower storages during the intraday including a short overview on literature, the relevant energy market challenges and the new regulatory requirements. Furthermore, in part 7.2 the implemented intraday optimization model to fulfill the regulatory requirements is introduced. Using the MILP model outputs such as accurate power plant schedules an adopted version of the algorithm introduced by Lu et al. (2004) is outlined in chapter 7.3. A real-world case study is presented and discussed in part 7.4.

7.1. Introduction

7.1.1. Market and Regulatory Environment

The characteristics of the German electricity market have changed significantly over the past decade. The renewable energy act fostered the exploitation of significant amounts of RES that have entered the market in the last years and replaced power generation by fossil fueled power plants. As a consequence, the price at the EPEX Spot Auction decreased between 2012 and 2016 by 10 % per year on average (EPEX

Spot, 2017b). This does not only influence the utilization of fossil fueled power plants but also pumped hydropower storages.

The renewable generation is not equally distributed in time and space. Further, due to limited storage and a lack of sufficient transmission, generation capacity is just gradually leaving the system. Less price fluctuation can be seen than expected with such amounts of RES in the market. In particular, this effect has reduced the average price spread and thereby the profitability of daily pumped hydropower storages that were mainly constructed to balance production and demand.

This market development and the effect on pumped hydropower storages is depicted in Figure 74. The exemplary calculation is made for a daily pumped hydropower storage with 500 MW turbine/pump power, an efficiency of 80 % and grid charges of 4 €/MWh for the consumption of energy. The historic Monday till Friday average hourly day-ahead price and the water values for pumping and water release are plotted. On average, a pumped hydropower storage in 2005 could be operated 9 hours a day in pumping and 7 hours in generating mode with an average spread of 32.21 €.

In the year 2015 the plant could be operated 6 hours in pumping and 5 hours in generating mode taking advantage of an average spread of 21.34 €. Ceteris paribus, the daily contribution margin has been halved and can be seen in Figure 74 as well. Market conditions are changing but since pumped hydropower storages have the advantage of high flexibility, they can be traded on other markets without drawbacks, e. g. in the quarter-hourly intraday. Expecting a market with full liquidity and no arbitrage between the markets, the operating time in pump and generation mode can be increased to 9 and 7 hours, respectively. The average daily contribution margin increases significantly. Nevertheless, in order to exploit these opportunities, optimization and dispatch of pumped hydropower storages need to be adjusted to the new conditions illustrated in Figure 74.

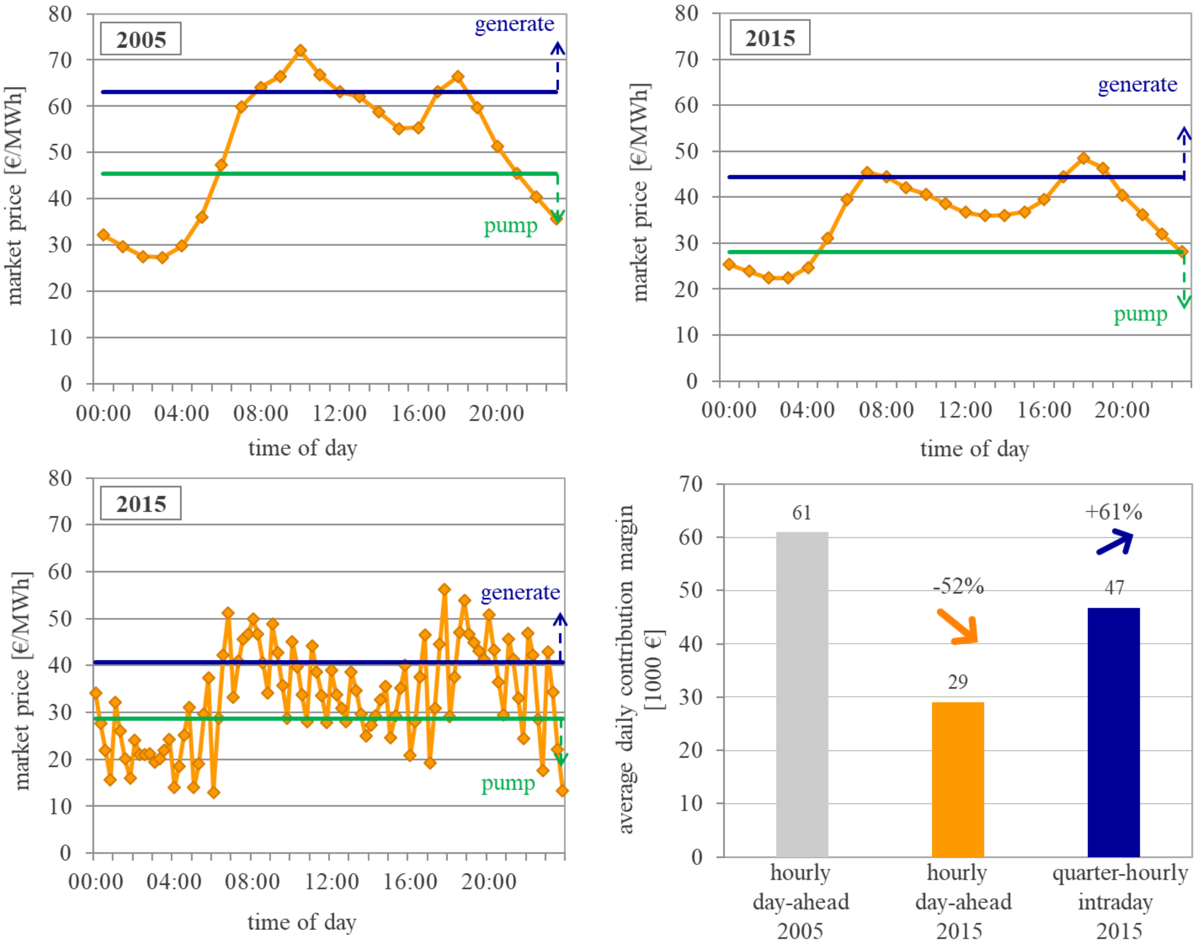


Figure 74 Comparison of the average hourly day-head auction price from Monday to Friday in 2005 with 2015 and the average quarter-hourly intraday price from Monday to Friday in 2015 and the resulting average daily contribution margins, data derived from (EPEX Spot, 2017b)

Beside the changing market conditions, new regulatory requirements have influenced the hydro-thermal dispatch optimization in Germany. The German federal network agency (BNetzA) introduced a resolution in 2014 which commits power plant operators to report extensive information on planned production for the current and the following day (Bundesnetzagentur, 2014). One major requirement is that the data needs to be updated during the day as soon as the planned production changes only slightly. This means that quarter-hourly production schedules, dispatch potential and reserve provision of every single machine has to be sent to the transmission system operator when the planned production changes. This is basically every 15 minutes the case since prices are changing and therefore the planned production. In order to meet this requirement, a model-based intraday optimization of all power plants is necessary. This process is not manually achievable; it is highly challenging to optimize a whole power plant portfolio on the required level of detail several or even 96 times a day.

7.1.2. Literature Review

The literature on solving pumped hydropower storage scheduling problems can be separated into two general categories. On the one hand, literature follows a system economic approach: e.g. Oliveira, McKee, and Coles (1993) solve a mixed integer linear program in a system context and integrate cost-efficient storage capacity. On the other hand, several papers focus on the individual plants and on how to operate a singular or a portfolio of pumped hydropower storages. These approaches are mainly based on using wholesale electricity prices and calculating an optimal control strategy.

The latter approach usually separates the optimization between daily pumped hydropower storages with small reservoirs and seasonal hydropower storages with large reservoirs and relatively small machines in comparison to their reservoir size. Literature that deals, among other things, with the daily pumped hydropower storage scheduling problem are e. g. Thompson, Davison, and Rasmussen (2004). They present a real option approach for pumped hydropower storage operation inspired by financial mathematics. Horsley and Wrobel (2002) use a deterministic continuous price curve and derive valuation methods using duality methods. Lu et al. (2004) suggest an algorithm to determine a bidding strategy for pumped hydropower storages considering reservoir limits. Kanakasabapathy and Swarup (2010) and Zhao and Davidson (2009a) (2009b) expand this idea considering additional aspects such as spinning and non-spinning reserve, storage level-dependent efficiency and random inflows.

Additionally, in Table 12 a short literature overview is given on pumped hydropower storage optimizations considering explicitly continuous traded intraday markets, as highlighted in grey. Nevertheless, just one paper considers quarter-hourly schedules (Braun, 2016b; Braun & Hoffmann, 2016).

Table 12 Literature review on intraday markets considering hydropower optimizations

authors	markets/objective						technology			method/ horizon/ country/ main findings
	day-ahead		intraday		balancing		consideration of			
	price taker	price maker	price taker	price maker	price taker	price maker	hydropower	pumps	inflows	
(Baillo et al., 2006)		x		x		x	x			multistage SP/ 1 day/ Spain/ dominant strategies available, no comparison
(Braun & Burkhardt, 2015)	x		x			x	x	x	x	LP/ 1 year/ Germany/ balancing energy market more profitable as intraday or day-ahead market
(Braun & Hoffmann, 2016)			x	x			x	x	x	LP/ MILP/ 2 days/ Germany/ continuous optimization necessary since intraday market prices changes constantly

authors	markets/objective						technology			method/ horizon/ country/ main findings
	day-ahead		intraday		balancing		consideration of			
	price taker	price maker	price taker	price maker	price taker	price maker	hydropower	pumps	inflows	
(Faria & Fleten, 2011)	x		x				x		x	SP/ LP/ 1 day/ Norway/ including intraday when bidding spot does not increase profit or influence the bids significantly
(Löhndorf et al., 2013)	x			x			x	x	x	ADDP/ 1 year/ Austria/ focus on long-term effects of bidding in day-ahead and intraday, intraday market not very relevant
(Triki, Beraldi, & Gross, 2005)	x		x		x		x			multistage MISP/ 4 hours/ Italy/ no comparison, one-point bids, no price dependent supply curve
(Ugedo et al., 2006)		x		x		x	x			MILP/ 1 day/ Spain/ strategic behavior of competitors, no comparison

7.2. Intraday Optimization of Power Plant Deployment

In consequence of the new regulatory requirements, EnBW has developed its own optimization model and has set up all necessary processes; thereby it established a decision support for an intraday deployment of power plants. The optimization model replaced the manual experience-based process with an automatic model-based system. The major challenges of an intraday optimization are both the development of a mathematical model and the design of new processes.

7.2.1. Portfolio Dispatch Optimization Model

The decision problem is formulated as a mixed integer linear program. The objective function minimizes the costs of the hydro-thermal production. Optionally, the model can use the intraday market to buy or sell energy to meet load requirements. The problem's major constraint is that the load has to be covered. It is further constrained by the technical characteristics of the power plants such as maximum capacity, minimum capacity, load change rates, start-up costs, and availabilities. Furthermore, the model takes into account the prices for fuel and CO2, grid charges, as well as quarter-hourly electricity prices. The sold primary (FCR), secondary (FRR) and tertiary (RR) reserves are distributed among the power plants in a cost optimal way.

The time horizon spans up to two days with quarter-hourly time resolution. The whole problem is solved with a small optimality gap in less than 30 seconds in 95 % of all cases. To improve the runtime and to utilize the system for a short-term intraday optimization the thermal power plants' schedule may be set fixed in periods with prices that do not allow an adjustment of thermal production. Furthermore, the above-mentioned information (quarter-hourly production schedules, unused power plant capacity, dispatch potential, and reserve provision of every single machine) is send frequently to the transmission system operator. The intraday optimization also provides decision support for power plant dispatching and real-time trading. The utilization of the data has therefore great potential for traders and dispatchers that obtain significant support for evaluating orders and assessing power plants deployment strategies.

The power plant portfolio dispatch is optimized and adjusted constantly by processing and displaying relevant power plant and price data.

7.2.2. Modeling Pumped Hydropower Storages

The pumped hydropower storage portfolio deployment is part of the mixed integer optimization. The pumped hydropower storages are modelled on a very high level of detail considering reservoir restrictions, hydraulic short circuit of pumped hydropower storages, water spillage, inflows etc. Most input parameters have a quarter-hourly resolution. It can be distinguished between hard reservoir restrictions such as maximum/minimum filling levels, flow rates, efficiencies, outflow, inflow and restrictions that can be adjusted by the dispatcher, a person that is responsible for the dispatch during the intraday. These include the target reservoir filling levels (set intraday or at the end of the planning horizon), which can be adjusted depending on the trader's or dispatcher's market assessment and experiences on reserve energy activation. Maximum and minimum filling levels can also be adjusted considering security buffers for uncertainties in inflows, prices or high probabilities of outages of thermal power plants. In the first release state, the optimization was performed just for the current day; the planning horizon has afterwards been extended to at least two days. Optimization across several days has the great advantage that start time and length of the storage cycle are more flexible and the potential of the pumped hydropower storages can be better exploited. Target filling levels can always be set by the dispatcher, even within a planning period. Since EnBW's pumped hydropower storages can only operate at full power for a couple of hours until reservoir limits are reached, our experience shows that optimizing across more than two days does not offer any advantage.

7.3. Algorithm for Intraday Trading

The optimization approach is a multistage looping algorithm that runs as a post-optimization after the frequently operated intraday optimization and delivers accurate time dependent water values as well as the planned production which can guide a bidding strategy for the intraday market. The optimization is based on the algorithm presented in Lu et al. (2004) who first presented the approach of variable length storage cycles due to limited reservoir capacity. This fits to the new market conditions where e.g. photovoltaics feed-in during the day causes double hump price curves with a second pumping period at midday, or wind feed-in pushing peak hour prices below the average night prices, resulting in long storage cycles over two or three days.

Following the original idea by Lu et al. (2004) some additional challenges, such as grid charges, power plant availabilities and flat price profiles are additionally accounted for. Furthermore, some shortcomings hindering the practical application of the algorithm are corrected.

The equations and algorithms are defined using the following symbols:

- $\mathcal{G}$: grid charges
- ρ: efficiency of power (pump) machine
- t_u, t_p: number of quarter hours where the unit is in generating/pumping mode
- t_p^{max}: maximum number of quarter hours where unit can be in pumping mode
- λ_u, λ_p: marginal cost of generating/pumping per
- v^{start}, v^{end}: energy level in upper reservoir at beginning/end of planning horizon
- v^{in}, v^{out}: energy in- and outflow
- t: time
- $\tau_{t=0}, \tau'$: limits of the new interval
- T: planning horizon
- u^{max}, p^{min} : maximum power of turbine/pump
- C: price forward curve
- c_t: price in time stage t
- Θ, Θ': prices sorted in ascending/descending order

The reservoir filling level equation depends on the energy level v^{start} at the beginning and v^{end} at the end of the planning horizon T. The assumption by Lu et al. (2004), that initial storage level equals terminal storage level does not hold for intraday operations and is thus not needed anymore.

$$v^{end} = v^{start} + v^{in} - v^{out} \tag{100}$$

The set t with $t = 1, \dots, T$ is sorted in ascending order of the corresponding price forward curve C that consists of the prices in each period c_t so that the period with the lowest price is the first and the period with the highest price is the last element of the set. Denote this set as Θ. The set where periods are sorted in descending order of the corresponding price (the period with the highest price is the first and the period with the lowest price is the last element of the set) is denoted as Θ'. The inflow energy can be calculated based on t_p using the following equation:

$$v^{in}(t_p) = \sum_{\substack{t\in\Theta \\ t\leq t_p}} p_t^{max}\,\rho \tag{101}$$

The outflow energy is

$$v^{out}(t_u) = \sum_{\substack{t\in\Theta' \\ t\leq t_g}} u_t^{max} \tag{102}$$

t_u is defined by:

$$t_u = \frac{v^{start} - p^{max} \cdot t_p - v^{end}}{u^{max}} \tag{103}$$

and t_p^{max} is set as follows:

$$t_p^{max} = \frac{T}{1+\frac{p^{max}.\rho}{u^{max}}} \tag{104}$$

7.3.1. Unconstrained Algorithm

In comparison to the original algorithm by Lu et al. (2004) grid charges are included here because they have a significant impact on the profitability of operating pumped hydropower storages in Germany. Furthermore, if the price curve is very flat and when the terminal energy level deviates from the initial energy level, a spread-based operation of the power plant is not possible. This situation has been explicitly accounted for in algorithm 3.

Algorithm 3 Unconstrainted optimization

1: Obtain a price forward curve C and sort it in an ascending order to receive Θ.

2: Start with $t_p = 1$.

3: Obtain t_u using (103) and find the corresponding λ_p and λ_u from C; **if** $t_u = 0$, set $\lambda_g = \infty$.

4: Check the optimality condition. Is $\lambda_u \leq (\lambda_p + \mathcal{G})/\rho$?

 If the inequality does not hold, set $t_p = t_p + 1$ and go to Step 5.

 If the inequality holds, set $t_p = t_p - 1$, obtain t_u and go to Step 6.

5: **If** t_p is less than t_p^{max}, go back to Step 3. **If** $t_p > t_p^{max}$, **then** stop.

6: **If** $t_p = 0$, $t_u = 0$, and $v^{end} > v^{start}$, set $t_p = (v^{end} - v^{start})/p^{max}/\rho$ and determine λ_p. **Else** find λ_p as well as λ_u from C; **if** $t_p = 0$, set $\lambda_p = -\infty$; **if** $t_u = 0$, set $\lambda_u = \infty$. **Then** stop.

Unlike described in the original algorithm by Lu et al. (2004), the price forward curve is not necessarily monotonous. Thus, slight adaptions of the algorithm were necessary. When the price equals the marginal cost of pumping for the first time, p^{max} is consumed. If the price c_t meets marginal cost again at a later time, no water will be pumped. When price equals marginal cost for generating power for the first time, $(t_u - \lfloor t_u \rfloor) * u^{max}$ is generated. This may be less than u^{max} if the energy comes close to reservoir limits. At any later point when price equals marginal cost for generating, no water will be released. Note that this reasoning works for prices that appear at most twice. If C contains the same price more than two times, water needs to be pumped or released at full power in more than one period where marginal cost equals the price.

7.3.2. Unconstrained Algorithm Accounting for Availabilities

One drawback of the original algorithm is that power plant availabilities are not considered. An example is the atypical grid usage tariffs in Germany. This means that the operator either does not use pumps during predefined hours during the day or has to pay nearly 8 times higher gird charges all the time. The (partial) availability of power plants during the day, grid charges and flat price profiles are addressed in algorithm 4.

Algorithm 4 Unconstrained optimization accounting for availabilities

1: Obtain a price forward curve C and sort it in an ascending order to receive Θ.

2: Start with $t_p = 1$.

3: Obtain v^{in} by using (101) and determine the necessary v^{out} by means of (100). Set $t_u = 1$.

4: Calculate $v^{out}(t_u)$ using (102).

5: **If** $v^{out}(t_u) \geq v^{out}$, set $t_u = t_u - \frac{v^{out}(t_u) - v^{out}}{p_{t=t_u}^{max}}$, and go to Step 6. If $v^{out}(t_u) < v^{out}$, set $t_u = t_u + 1$ and go back to Step 4.

6: Determine λ_p and λ_u from C; **if** $t_u = 0$, set $\lambda_u = \infty$.

7: Check the optimality condition. Is $\lambda_u \leq (\lambda_p + \mathcal{G})/\rho$?

If the inequality does not hold, set $t_p = t_p + 1$ and go to Step 8.

If the inequality holds, set $t_p = t_p - 1$, obtain t_u (Steps 3 – 5, skipping Steps 6 and 7) and go to Step 9.

8: **If** t_p is less than t_p^{max} go back to Step 3. **If** $t_p > t_p^{max}$, stop.

9: **If** $t_p = 0$, $t_u = 0$, and $v^{end} > v^{start}$, obtain t_p from $\sum_{\substack{t \in \Theta \\ t \leq t_p}} p_t^{max} = \frac{v^{end} - v^{start}}{\rho}$. **Then** determine λ_p. **Else** find λ_p as well as λ_u from C; **if** $t_p = 0$, set $\lambda_p = -\infty$; if $t_g = 0$, set $\lambda_u = \infty$. **Then** stop.

7.3.3. Algorithm Accounting for Reservoir Limits

Since the above algorithms may violate reservoir constraints, a further optimization needs to be conducted which is presented in algorithm 5.

Algorithm 5 Constrained optimization accounting for reservoir limits

1: Solve the problem with the unconstrained algorithm 4 with the time interval $[\tau_{t=0}, T]$

2: Check the solution. **If** the energy level is always within the reservoir limits, stop. **If** there are violations of the reservoir constraints, go to Step 3.

3: Subdivide the time interval into $[\tau_{t=0}, \tau']$, where τ' is the quarter-hour where the unconstrained optimization finds the highest or lowest reservoir level (depending on the violated limit). Let v^{end} in τ' be the value that has been violated (upper or lower limit). Then perform the unconstrained algorithm 4.

4: Check the solution. **If** there are more violations, go back to Step 3. If not, set $\tau_{t=0} = \tau'$, let v^{start} be the violated limit and go back to Step 1.

7.4. Numerical Results

In this part, the introduced algorithms are applied on an example from practice. Therefore, 7.4.1 defines the model setup and the steering logic is explained in part 7.4.2. Afterwards the model results are introduced in 7.4.3 and 7.4.4 concludes with a critical discussion.

7.4.1. Model Setup

The most important challenge is to generate a forecast for, or receive the prices form, the intraday market and to consider the characteristics of the continuous trading. This includes mainly two points: the limited depth of the order books, i.e. limited liquidity, see Figure 75 and the continuous changing price over the time, see Figure 76. The first point is addressed with the provided algorithm that can be operated in real-time or at least performed in short time intervals. For the second point, a good market access is important. For the presented case study, the volume weighted average price, based on all executed bids of the last hour, is used. If no or to little trades are available the quarter-hourly day-ahead market auction results were used as best guess for the intraday market.

The innately nimble intraday continuous market is an alternative concept to the unified pricing auctions day-ahead. Especially for automated trading, final orders need to be calculated taking the real-time intraday orderbooks into account. For each quarter-hour, the orderbook can be analyzed in terms of market depth and volume related price, since trading small or huge quantities differs. As an example, the 50 MWh volume weighted average price for product 13-14 would be 51.38 €/MWh analyzing the retrieved orderbook form the exchange in Figure 75. Nevertheless, it can be also seen that the bid-ask-spread is relatively wide with 2 € and inserting an own bid might be more cumbersome but smarter than accepting the already listed offers.

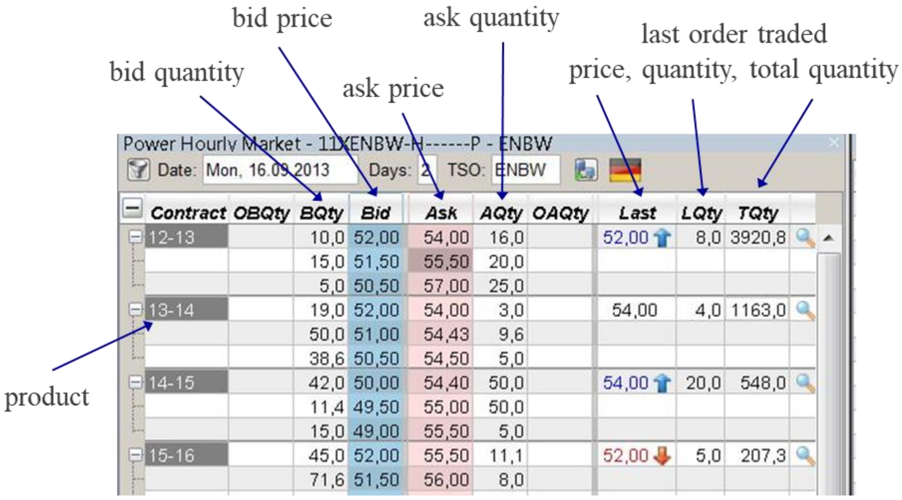


Figure 75 Screenshot of an exemplary intraday orderbook from September 16th, 2013 at 11am including some explanations

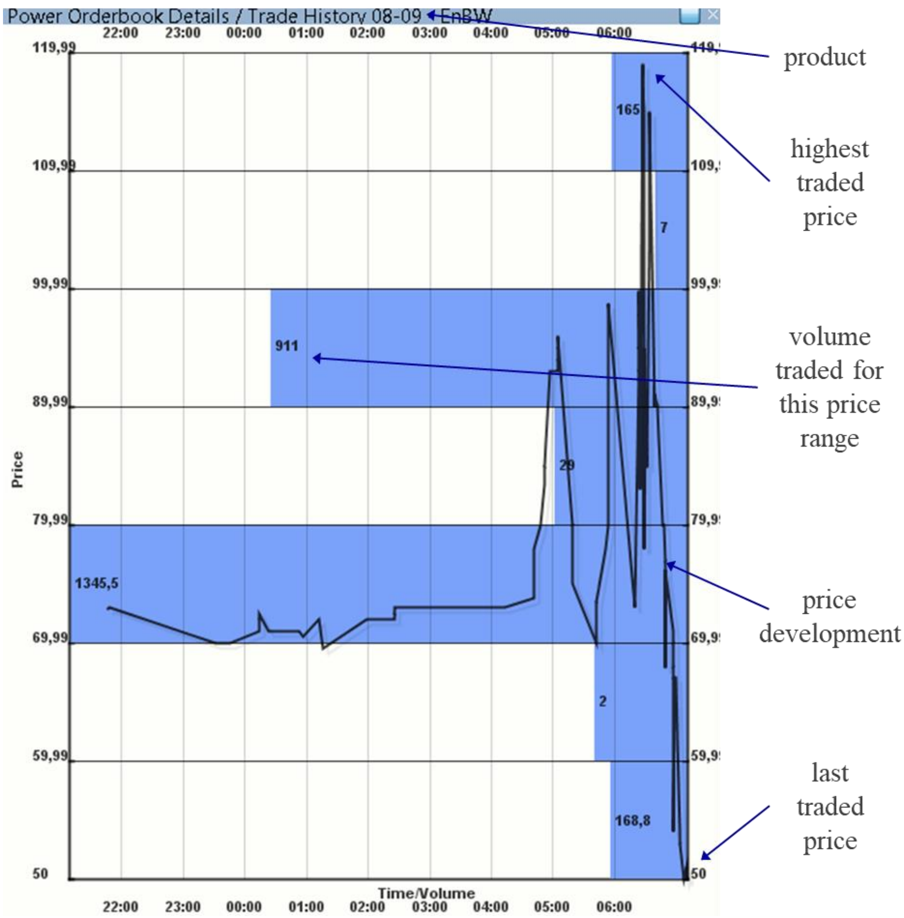


Figure 76 Screenshot of the price development for one product on the continuous intraday market from September 16th, 2013 at 7am including some explanations

7.4.2. Steering Parameters

The result of the algorithm is an optimal dispatch, reservoir filling levels over the time as well as shadow prices for turbines and pumps. The trading of the power plant can be done manually or automated. For the first a table for the dispatcher is provided containing the interval and the shadow prices for pump λ_p and turbine λ_u. In intraday markets, normally just an existing position based on day-ahead market trading is adopted. Therefore, the intraday trading strategy of pumped hydropower storages depends on the intraday price c_t as well as on the already sold/bought energy. In Table 13 the eight possible trading strategies are listed.

Table 13 Different trading strategies for pumped hydropower storages

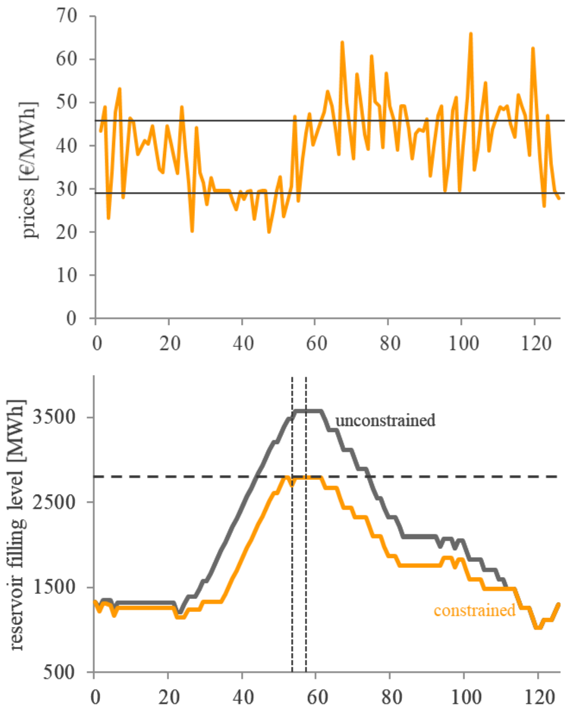
strategy	existing position	shadow price (λ_u, λ_p)	action to be taken
1	turbine sold	$c_t > \lambda_u$	-
2		$c_t < \lambda_u$	buy back turbine position
3	turbine not sold	$c_t > \lambda_u$	sell turbine position
4		$c_t < \lambda_u$	-
5	pump sold	$c_t > \lambda_p$	sell back pump position
6		$c_t < \lambda_p$	-
7	pump not sold	$c_t > \lambda_p$	
8		$c_t < \lambda_p$	buy pump position

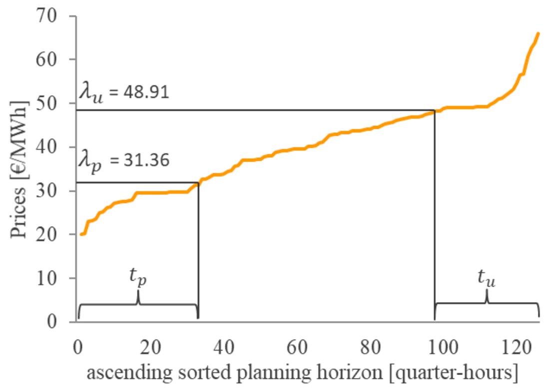
These shadow prices can be displayed together with the storage level of the upper and lower reservoir. This shows at a glance, at which time and at what price the model proposes a corresponding trade direction. In addition, it is clear whether trades lead to a critical area close to the basin limits.

Comparing the planned schedule of the power plant with the orderbook of each quarter-hour leads to the information of the volume weighted price and which quarter-hours are the most profitable. It is therefore suggested to first trade the most profitable quarter-hours and to always trade a buy and a sell order together. That means to buy for example 100 MWh at noon because the price decreased due to additional PV production and to sell 70 MWh in the next hour because the prices increased due a power plant outage. If pump and turbine position are always traded pairwise the risk of not achieving the reservoir target filling levels is zero. The quantity to be traded depends also on the efficiency of the machines. In the just given example ρ equals 70%.

7.4.3. Model Results

The algorithm has been implemented as an extension to the MILP that determines the optimal hydro-thermal power plant deployment for regulatory purposes. This example shows a calculation from August 10th, 2015 4:45pm. For the calculation, a price assumption C is needed for the next day(s). This can be a price forward curve or the actual traded market prices, see Figure 77 top left. The efficiency of the plant is set to 0.75 and grid charges are 1.5 € per MWh consumed. The upper limit of the energy storage level is set to 2800 MWh and the lower bound to 300 MWh. In both directions, a safety buffer has been set. Start level was 1332 MWh and end level was 1300 MWh. Following the introduced algorithm, the prices of the planning horizon are sorted in ascending order, see Figure 77 top left. From both sides, t_p and t_u are calculated stepwise until the optimality criterion $\lambda_u \leq (\lambda_p + \mathcal{G})/\rho$ is reached. The results of this part of the algorithm can be seen in Figure 77 bottom left. The grey dashed line shows how an unconstrainted large reservoir is filled up to nearly 4000 MWh and returned to the end level. In this unconstrained case, all hours are used that exceed the spread of efficiency rate plus grid charges. This operation should be equal to the dispatch of seasonal pumped hydropower storages. A constrained daily pumped hydropower storage, i. e. with a limited reservoir, results in the orange line. In this case the second part of the algorithm applies. The planning period is divided into sub-periods at all points where the reservoir limits are reached. This is for example the case in the first iteration in quarter-hour 42. The new sub-periods are optimized again using the first part of the algorithm to optimally exploit the two sub-periods. In the second sub-period, the reservoir limits are often reached again, as it is the case in this example, in every step until the prices are low enough for water to be released. Therefore, the price for pumping is set to negative infinity, or, so that no water is pumped up into the empty storage in the following periods, see Figure 77 bottom right. The price for generation can be calculated using the efficiently spread but it is also reasonable to set the generation price equal to the one in the next period with a regular price, since the calculation is done so frequently.





time interval [quarter-hours]	λ_p [€/MWh]	λ_u [€/MWh]
[0-52]	29.56	47.82
[53-55]	27.26	46.82
[56-126]	29.68	49.13

Figure 77 Exemplary quarter-hourly intraday continuous price curve from August 10th, 2015 4:45pm until August 11th, 2015 11:45pm on the top left side and the sorted price curve in ascending order on the top right side are depicted. On the down left side is the filling level of the reservoir plotted against the planning horizon. The grey dashed line displays an unconstrained energy reservoir, the orange line a constrained energy reservoir. The down right side shows the table with the respective water values for each period.

7.4.4. Conclusion

In this part, the optimal bidding strategies for daily pumped hydropower storage power plants in a competitive electricity market, considering the perspective of a storage operator and the difficult current market conditions in Germany, are outlined. Starting form a new regulatory requirement that forces power plant operators to submit precise planning data, an intraday optimization model has been set up. Based on this model output an intraday multistage looping algorithm for an intraday pumped hydropower storage optimization has been introduced. Reservoir limits, efficiencies, grid charges and machine availabilities are included in the algorithm. The algorithm is implemented at EnBW and runs as real-world application at a high frequency during the day (in practice at least every 15 minutes). Exemplary results are presented showing the high practicability of the model. The results of the algorithm are further structured to be used for atomized trading. Future work should analyze to include supplemental complexities such as stochastic prices, price sensitivity, balancing power activation and bidding strategies for the continuous intraday market.

This algorithm fits very well in an over all bidding strategy of firstly optimizing the pumped hydro power plants on day-ahead markets as suggested in chapter 5 and 6 and to perform this real-time looping algorithm as post optimization to constantly adopt the existing position to the changing prices during the intraday continuous trading phase. Basically, the pumped hydro power storages are used as a real option from the start of the trading until gate closure 30min before delivery (EPEX Spot, 2017b). This algorithm is therefore also valuable to determine the optionality value of holding flexibility options.

8. Non-Linear Optimization of Energy Only and Balancing Markets

This chapter provides a method to consider both balancing and energy only markets in the hydropower scheduling problem. Although both kind of markets are fundamentally different they are, however, essential for a profit optimal scheduling of pumped hydropower storages. The term energy only markets typifies all kind of spot markets such as day-ahead and intraday in which just a remuneration takes place for the delivered work rather a provision or the like.

Pumped hydropower storages are a major player on the balancing energy markets (Abgottspon & Andersson, 2012). This is due to the ability of many plants to quickly switch between pumping and generating mode as well as to exactly follow a given schedule. This flexibility can be either used for trading on short-term energy only or balancing markets. Therefore, operators of flexible power plants, such as pumped hydropower storages or lithium-ion batteries, face the decision problem on where and how much to bid in the different markets.

This question is difficult to answer in four ways. First, already the energy only multi-market optimization is challenging and subject of chapter 5, 6 and 7 concerning the different day-ahead and intraday markets. Second, the balancing markets auction design is complex and steadily varying as presented in chapter 2.4. The latest market developments show a strong tendency towards more flexible auction designs, especially in terms of shorter product lengths, prequalification requirements as well as more frequently auctions, see the analysis in chapter 2.4.1. Third, the balancing markets are mostly based on a double auction remuneration scheme with pay-as-bid pricing in which the placement of bids in the merit order results in a complex non-linear sorting problem. This is because the received price depends on the own offered quantity as well as the expected balancing energy activation. The fourth challenge results from the combination of energy only and balancing markets and the allocation of the available capacity and energy to these markets. These challenges are further difficult to reconcile keeping the problem computational tractable and applicable to real-world problems.

The interaction between both markets is discussed in chapter 8.1.1, a review on existing literature on balancing power market bidding is given in chapter 8.1.2. Further, in chapter 8.2 the non-linear optimization approach is presented consisting of the energy only 8.2.1 and the balancing problem formulation 8.2.2. The numerical results in chapter 8.3 define the test set and present a generic and a sizable case study. The last part concludes and innervates a critical discussion.

8.1. Introduction

Generally, energy only markets are used to bring supply and demand together to enable physical market clearing and balancing markets have been introduced to secure grid stability (Bhattacharyya, 2011).

Every energy market participant buying, selling and transferring energy is owner of a balancing account that needs to be balanced at all time. The accounts are managed by the balancing account managers. Every account holder books a schedule before feed-in or consumption (Regelleistung.net, 2017c). The sum of all accounts is always balanced. Nevertheless, deviations between the actual feed-in and consumption to the booked schedule can occur in case of power plant outages, forecast errors of variable RES, unprecise consumption prognosis as well scheduling or dispatch mistakes. To balance theses deviations balancing is needed. The activation of balancing energy is penalized with the final balancing energy price (reBAP) (Regelleistung.net, 2017c) that is normally higher than the intraday market price. Therefore, it is generally of interest to keep the balancing account balanced (Regelleistung.net, 2017c). Every power plant that is able to adjust its production or consumption fast enough can provide the demanded flexibility and is able to level out deviations, as long as it is prequalified. Such power plants normally bid into the balancing power markets.

The balancing power markets in Germany (see chapter 2.4.2) and most European countries (see chapter 2.4.1), are based on a three-quality pattern introduced by the European Network of Transmission System Operators for Electricity (ENTSO-E), namely the Frequency Containment Reserve (FCR), the Frequency Restoration Reserve (FRR) and the Replacement Reserve (RR) (ENTSO-E, 2013). The second market, FRR, in Germany also widely known as secondary control reserve, is exemplarily used to illustrate the complexities of the balancing market structure since it includes the already mentioned power as well as work price auction. Therefore, the exemplary modulation of the FRR market typifies all other reserves and is no simplification. Below the terms balancing power market and balancing power price are used to describe the remuneration for the provision of balancing. The auction for the activation of balancing is described as balancing energy market with balancing energy prices.

8.1.1. Interaction of Energy Only and Balancing Markets

Energy only and balancing markets can be separated in terms of auction design, market place, remuneration or market organizer. Nevertheless, an interaction within these markets cannot be denied. One simple coherence is for example that a high energy only market price results in pressure on the negative balancing power prices, because more running capacity is available. A low energy only market price level normally makes the negative balancing power products more expansive and the positive less expansive.

Furthermore, a significant correlation between the quarter-hourly intraday and the final balancing energy price can be seen, e. g. in 2016 with $r = 0.36$. Whereas the average intraday and final balancing energy prices in 2016 with 29.00 €/MWh and 29.21 €/MWh were nearly arbitrage free the standard deviation was 15.41 €/MWh to 47.06 €/MWh. The example from June 28th until 30th, 2017 in Figure 78 illustrates the strong fluctuation of the final balancing energy price and thus on the one hand the high risk for market participants with unbalanced balancing groups and on the other hand the high remuneration for flexibility providers.

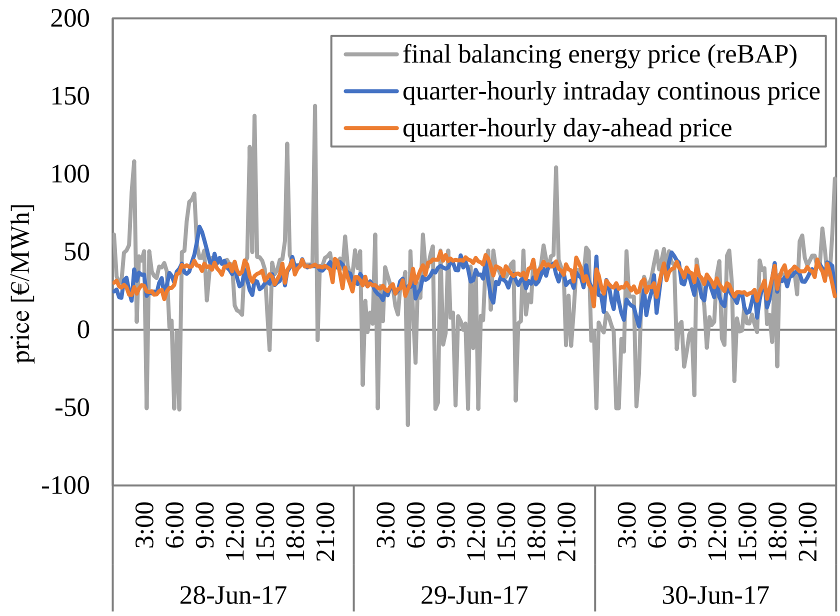


Figure 78 Comparison of quarter-hourly energy only and final balancing energy prices for three exemplary summer days in 2017

The German FRR tendering is a two-stage closed order book pay-as-bid auction. Just an acceptance on the power price auction allows a participation in the work price auction. Whereas the power price auction remunerates the power provision, meaning one fix price for the whole tendering period in €/MW, the work price auction determines the merit order for a possible activation during the tendering period in €/MWh. For the power price auction, the bidder always tries to get the last bids of the merit order to receive the highest possible payment. Depending on the risk aversion of the strategic bidder, one either takes the risk to not get accepted with all bids in return for a better price, or one sorts the bids a little bit lower in the merit order to decrease the risk of not getting accepted on the costs of a lower price. In practice, the auction participants additionally form smaller bids that are spread over the merit order to reduce the risk of not getting accepted.

The second part of the two-stage market is the work price auction. The work price merit order is increasingly sorted by price. In case of activation the participants deliver the requested energy for the pay as bid price. Due to the merit order structure, the lowest bid has the highest probability to be activated and remunerated whereas the last and most expansive bid has the lowest. Prices at the end of the merit order are often speculative and can be very high since a full activation of the whole merit order is extremely rare.

To find the most profitable place in the merit order as a bidder depends on the work price itself as well as the expectation on the activation of the energy and the costs for the provision. On the one hand, if no energy is activated by the TSO, the remuneration is zero. On the other hand, if extreme amounts of energy are activated, the operator need to be able to provide the offered work over a long period of time. The latter, is especially relevant for storages since their energy content is limited. Calculating the respective costs for pumped hydro power storages is therefore very difficult since it might depend on the activated quantity itself. Whereas the variable costs of fossil fueled power plants just depend on coal, gas, oil and CO_2 prices, pumped hydro power storages do not have such variable costs that can be considered constant in the short run. The steering is rather based on a shadow price based dispatch, see chapter 3.4.

Below an example for the complexities of the work price auctioning is given. The historic activation and work price merit orders are presented in Figure 79 and were retrieved for the April week from 17th to 23rd 2017 (Regelleistung.net, 2017a) for the following four products:

- POS HT, Mon.-Fri. 8am-8pm, providing energy
- NEG NT, Mon.-Fri. 8am-8pm, consuming energy
- POS NT, Mon.-Fri. 8pm-8am, Sat., Son. and holidays all day, providing energy
- NEG NT, Mon.-Fri. 8pm-8am, Sat., Son. and holidays all day, consuming energy

For each of the four products the TSOs purchase about 2 GW of FRR for the German market area as can be seen in the example in Figure 79. The blue loops symbolize the respective balancing energy price merit order. For clearness the merit orders are cut at prices of +500 and -500 €/MWh, whereas also high five-digit numbers are regularly bid into the market to wait for extreme activation scenarios. The presented Easter week from 17th to 23rd of April 2017 has 432 NT and 240 HT quarter-hours. In normal weeks, the balancing energy activation (orange loops) barely reaches the 1000 MW bid in each product. Nevertheless, this Easter week shows an extreme situation for the NEG NT product; due to significant deviations, shortly, the whole NEG NT balancing energy merit order was activated.

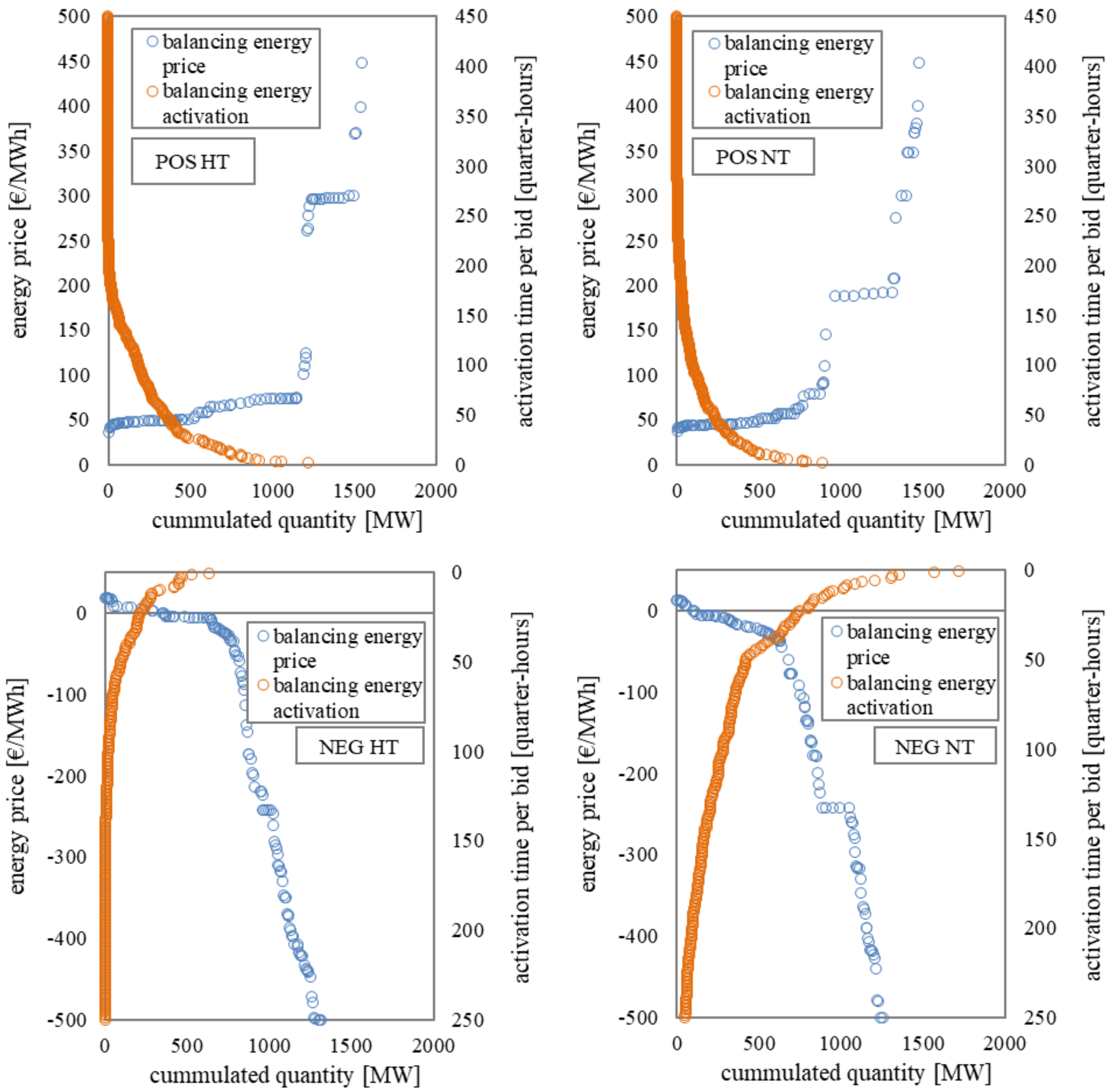


Figure 79 Balancing energy price and balancing energy activation for the different products of the exemplary week from 2017-4-17 to 2017-4-23, Data retrieved from (Regelleistung.net, 2017a).

Assuming perfect foresight and marginal small bids the ex post profit margins are calculated in Figure 80 for three different variable costs levels. In terms of pumped hydro power plants these variable costs levels can be described by shadow prices. The profit margin G for each bid along the merit order is given in €/MWh. The profit margin is the result of the pay-as-bid bid price $AP(i)$ minus the costs of provision $\mathcal{S}$ and multiplied with the activation probability $AA(i)$ at the respective position in the merit order $i = 1MW, \ldots, 2000MW$, see Figure 79. This is calculated over the course of the product length, which is one week for FRR in Germany. Depending on the revenue per bid $G(i)$, see equation (105), the bids from different production technologies should be placed at different places in the merit order.

$$G(i) = (AP(i) - \mathcal{S}(i)) \cdot AA(i) \tag{105}$$

Whereas the two positive products stand for providing energy the negative products characterize energy consumption. Producing and consuming result in different variable costs for the operator. Starting with the positive products the red pluses present the profits for a bidder with no costs, $\mathcal{S} = 0$, the blue dashes costs of $\mathcal{S} = 35\ €/MWh$ and the green rhombs costs of $\mathcal{S} = 45\ €/MWh.$ This means that a bidder aims to bid for the bids with the highest positive profit margin. For example, for the POS HT product, bidder with costs of 35 €/MWh should offer bids between the first MW and 700 MW, whereas the most lucrative ones are located in the beginning of the merit order. The profit margin of the POS NT product is characterized by a high activation in the beginning which slows down relatively fast as well as lower work prices as for the POS HT product. The result is a higher remuneration in the first part of the merit order for the bidder with no costs.

For the negative products, the TSO sells surplus energy to the bidders. Generally, the magnitude of activation is lower as for the positive products. This is because, firstly, power plant outages, as a source of balancing power activation do not play a role in negative products and secondly, in peak times thermal power plant production can be easily reduced avoiding balancing activation of pooled production. The latter is one reason why the provision power price for NEG HT was at 0 €/MWh over the course of the exemplary week (not depicted here).

Furthermore, the higher the production costs of a power plant the higher the profit margins when reducing production. This holds true for gas power plants whereas coal fired plants do normally not realize fuel saving since hot steam is just piped around the turbine to reduce production. In other words, a coal power plant has production costs of about 0 €/MWh and a gas power plant can realize 25 €/MWh for fuel saving. To provide balancing power, a thermal power plant must run over the complete product length and the higher the production costs the higher the risk that the power plant is partly out of the money on the energy only market. This makes the provision with thermal power plants more expansive; especially at night or during weekends with low market prices. The delivery of negative balancing power with pumped hydropower storages is different to fossil fueled power plants since the consumed energy can be stored. The decision on when energy should be stored can be taken based on the shadow price for energy consumption, see chapter 3.4.1. A pumped hydropower storage with a low efficiency and a resulting shadow price of 15 €/MWh and a highly efficient plant with 25 €/MWh is simulated.

The first thing that stands out for the product NEG HT in Figure 80 is the limited amount of activated balancing energy. This underlines how volatile the activation is. The work price merit order is normally structured so that the first bids (here 250 MW) offer a low positive price for consuming energy when activated and the further bids even receive a payment for consuming. For efficient pumped hydropower storage with a pump shadow price of 25 €/MWh the bids between 50 MW and 500 MW are profitable. The less efficient pumped power plant, with a shadow price of 15 €/MWh should offer at least the bids around 50 MW in this particular example.

Due to significant deviations, nearly the whole NEG NT balancing energy merit order was activated. Therefore, all products were in the money in most parts of the merit order. Such activation patterns are seldom in normal weeks but not uncommon in holiday weeks with a low residual load and a high variable RES production. Market participants should forecast the residual load and the renewable energy production to estimate the activation probability and respectively adjust their position in the merit order.

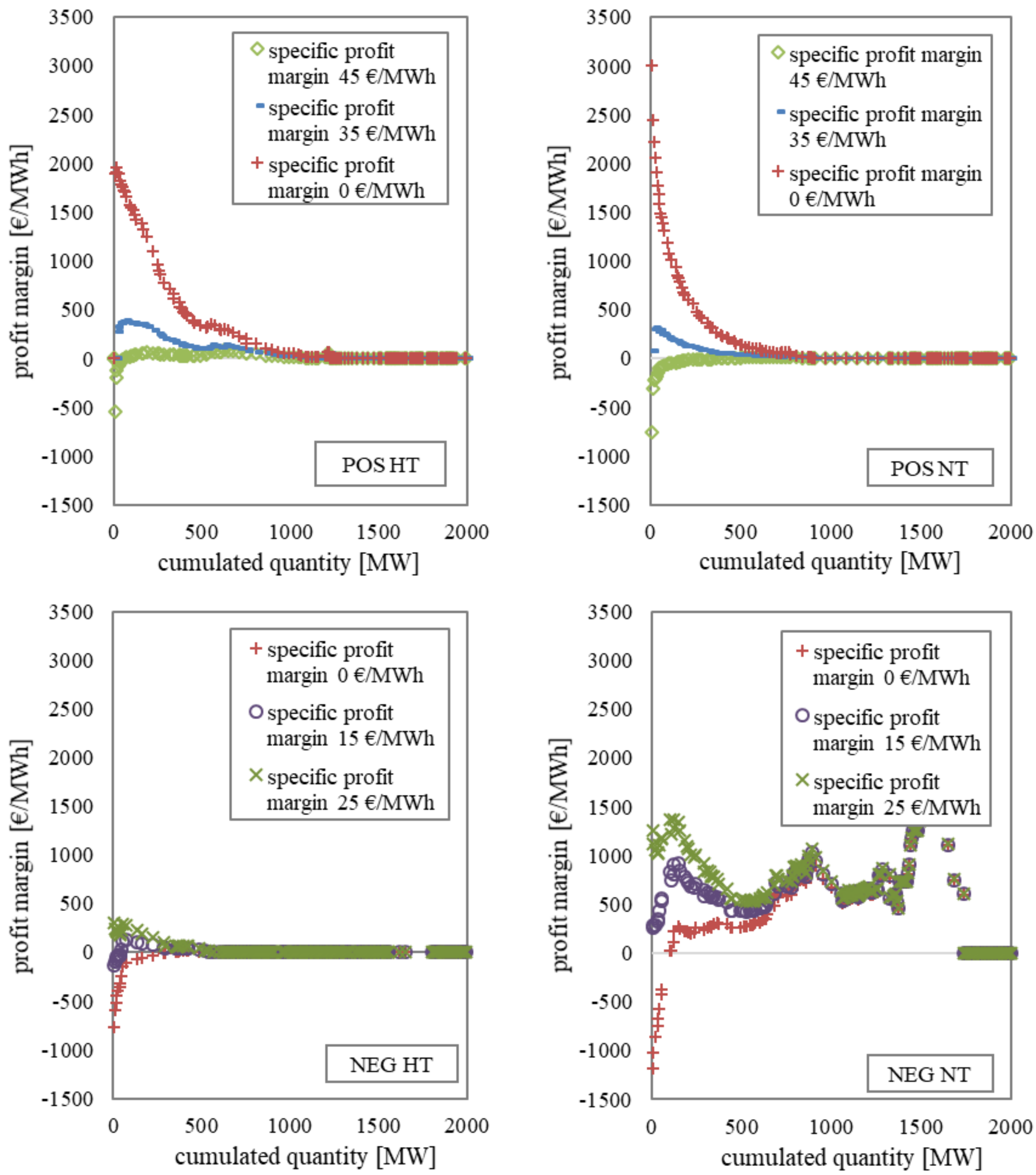


Figure 80 Profit margin for the four different products of the exemplary week from 2017-4-17 to 2017-4-23, Data retrieved from (Regelleistung.net, 2017a).

This example shows that the remuneration from the balancing power markets is heavily varying mainly because of the different activation patters and variable costs. Furthermore, the respective production costs of the operators are crucial for the position in the merit order. For pumped hydropower storages, the dispatch costs, here referred as shadow prices, depend on the energy only markets dispatch, but also on the quantity of offered balancing energy. Offered capacity in machines is reserved and cannot be used in energy only markets. Furthermore, activated energy changes the reservoir filling levels. This means there is a strong relation between balancing and energy only markets. All this motivates a combined approach considering both energy only and balancing power markets to find an optimal allocation of the available capacity.

8.1.2. Literature Review

Bidding on balancing power markets is a very complex task mostly due to the non-linear structure of the resulting optimization problems. Within the approaches in literature one group can be identified trying to present solutions that are applicable to real-world problems and another group converge the problem more theoretically, analyzing the market itself or presenting new optimization methods.

The first practical oriented group mainly suggests using iterative concepts. For example, a two-stage approach that firstly solves a spot market based hydropower scheduling optimization to provide optimal schedules, reservoir filling levels and water values to take the decision on using the water now or later in time and secondly generates optimal bids for the balancing market using the output of the ex-post energy only market optimization as cost structure. In this case, the size and number of bids are set fixed beforehand. Therefore, all problems are normally linear, and no non-linear tie-ins apply. Nevertheless, mixed integer and stochastic formulations can make problems relatively complex. With respective loops, the parameters of the systems can be adjusted towards a local optimum. The main drawback of the iterative approach is the neglection of the back coupling of the balancing power market bids on the energy only markets bids, the global optimum is not found. Furthermore, most approaches do not consider the characteristic two stage market design as suggested by the ENTSO-E (2013) and just regard the balancing power provision problem, rather the balancing work price auctions; possibly due to a complex modulation of activations.

Extended optimizations including case studies that show their applicability to real-world problems with strategic bidding on energy only and ancillary services are given by Ugedo and Lobato (2010) with an one-day example for the Spanish electricity market. They note the importance of stochastic optimization that is still solvable as well as strategic bidding due to the oligopolistic nature of the bidders. Other real-world stochastic linear optimizations are given for Canada (Ladurantaye, Gendreau, & Potvin, 2007), New England (Zhang, Wang, & Luh, 2000) and Norway (Fleten & Pettersen, 2005). A more recent approach including an extensive case study is presentment by Braun and Burkhardt (2015), which is a two stage linear approach considering opportunity costs from the energy only market to generate optimal bids for the reserve market, including strategic bidding and the pay as bid price structure. Nevertheless, none of these real-world examples addresses the non-linear characteristic of the balancing power market directly, rather assuming some factors as given or as results of an ex-ante optimization.

The second identified group in literature addresses many of these challenges on the costs of aggravated practical applicability. One of the first papers considering both energy only and ancillary market was published in the year 2000. Deb (2000) used a heuristic algorithm to compare the income from the energy only and the ancillary services market using water values. Non-linearities were also approached by Swider (2007b) who addressed the non-linear problem structure resulting from the German and European market designs based on a pay-as-bid structure with a limited merit order. He performs a stochastic non-linear optimization for an exemplary power plant portfolio using spot and reserve market whereas he considered strategic bidding behavior of the bidders on the latter. Further approaches exist, as for example Abgottspon (2015a, pp. 127–130), who proposes an agent based model that is solved with a Q-learning framework for short-term ancillary services bidding. He just includes the power provision, not the energy delivery. Furthermore, the highest acceptable price is assumed as known beforehand which results in a perfect foresight problem.

Vardanyan and Hesamzadeh (2014) introduce a multistage stochastic optimization for day-ahead and balancing markets. They use a rolling planning approach that allows re-forecasting and re-dispatching. The stochastic day-ahead price forecasting is modelled with a mean reverting jump diffusion processes and the discrete behavior of the balancing markets with a Markov model. Nevertheless, just three different bidding prices are allowed. Olsen describes a stochastic hydropower planning model that accounts for uncertainty in both day-ahead and balancing market as well focusing on wind power integration (Olsson, 2003; Olsson & Soder, 2003).

A good overview on market design, key drivers and the development of the European balancing markets is given by Ocker, Braun and Will (2016). Further literature on multi-market bidding including balancing markets is listed in Table 14. The focus is on literature that considers either price maker bidding in the balancing market or take the activation of balancing energy into account. No paper regarded both. The overview Table 14 becomes clear about if the authors considered multi-market bidding using additional day-ahead or intraday markets. Furthermore, the column generation technology describes if hydropower storages were taken into account and if essential aspects such as inflows or pumps were considered.

Table 14 Literature review on multi-market bidding considering balancing markets

author	markets/objective							generation technology, consideration of			method/ horizon/ activation technique/ main findings
	day-ahead		intraday		balancing						
	price taker	price maker	price taker	price maker	price taker	price maker	activation	hydropower	inflow	pumps	
(Abgottspon & Andersson, 2012)	x				x		x	x	x	x	SDDP+MILP/ 1year/ fixed percentage/ risk neutral, day-ahead with single scenario, reserve with multiple scenarios
(Black & Strbac, 2007)					x		x	x		x	priority ranking method+LP/ 1 day/ imbalances between actual and forecast wind power is simulated as random walk for balancing activation
(Boomsma et al., 2014)	x	x			x	x		x			multistage SP/ generic example/ - / coordinated bidding increases profit by 2% no market power and 1% assuming market power in the balancing market
(Braun & Burkhardt, 2015)			x			x		x	x	x	LP/ 1 year/ - /balancing energy more profitable as day-ahead or intraday
(Chazarra, Perez-Diaz, & Garcia-Gonzalez, 2014)	x				x		x	x		x	MILP/ 1 day/ fixed percentage/ variable speed operation
(Deb, 2000)	x				x		x	x		x	ad hoc heuristic/ 1 day/ fixed percentage/ single scenario, risk neutral, hedge scenario based

author	markets/objective							generation technology, consideration of			method/ horizon/ activation technique/ main findings
	day-ahead		intraday		balancing						
	price taker	price maker	price taker	price maker	price taker	price maker	activation	hydropower	inflow	pumps	
(Kazempour, Hosseinpour, & Moghaddam, 2009)	x				x		x	x		x	MINLP/ 1 day/ fixed percentage/ single scenario for spinning
(Kazempour, Moghaddam, Haghifam, & Yousefi, 2009)	x				x		x	x		x	MILP/ 1 week and 1 day/ fixed percentage/ variance of forecast errors is used to hedge risk
(Pinto, Sousa, & Neves, 2011)	x				x		x	x		x	MILP/ 1 day/ fixed percentage/ wind integration, single scenario
(Swider, 2007a)	x					x		x			NLP/ 1 day/ - /multiple scenarios for price assumptions
(Varkani, Daraeepour, & Monsef, 2011)	x				x		x	x		x	MILP/ 1 day/ fixed percentage/ wind hydropower system, single scenario

8.2. Non-Linear Programming

The great advantage of the here introduced integrated model is compared to iterative approaches the optimal exploitation of the optionalities between energy only and balancing power market. Nevertheless, in a first step, energy only 8.2.1, balancing power and balancing work auction are modeled separately as standalone optimizations 8.2.2. In a second step, all three problems are composed to the overall integrated optimization model in part 8.2.3.

8.2.1. Energy Only Market Optimization

One part of the overall integrated energy only market optimization is the classical hydropower scheduling problem. This problem takes the here-and-now or wait-and-see decision for the in the reservoir stored water. Optimizing using the expected future energy prices $c(t)$ in time $t = 1, \dots, T$ and taking into consideration all technical restrictions of the power plants such as the maximum turbine u^{max} and pump p^{max} capacities, the reservoir limitations specified by the minimum $v^{min}(r)$ and maximum $v^{max}(r)$ reservoir filling levels and the inflows $v(t,r)$ depending on the reservoir $r = 1, \dots, R$ in the time t. Further important parameters are the efficiencies of turbines $\eta(m)$ and pumps $\rho(m)$ with $m = 1, \dots, M$. The decision variables are $u(t)$ as the turbine schedule, $p(t)$ as the pump schedule and $s(t,r)$ as the spillage. The state variable $v(t,r)$ describes the reservoir filling levels of reservoir r in time t. This is the general problem formulation similar to the ones given in chapter 4.1.2 and chapter 5.3.1. The energy only market model formulates as:

$$
\begin{aligned}
P: \quad G^{EO} = \quad & \max_{u,p,s} \sum_{t=1}^{T} \sum_{m=1}^{M} c(t)\big(u(t,m) - p(t,m)\big) && (106)\\
s.t. \quad & v(t,\mathrm{m},r) = v^{anf}(t,\mathrm{r}) - s(t,\mathrm{r}) + v^{in}(t,\mathrm{r}) + \rho(m)p(t,\mathrm{m}) - \eta(m)u(t,\mathrm{m}) && \forall t,m,r\\
& v^{min}(r) \leq v(t,\mathrm{r}) \leq v^{max}(r) && \forall t,r\\
& 0 \leq p(t,m) \leq p^{max} && \forall t,m\\
& 0 \leq u(t,m) \leq u^{max} && \forall t,m\\
& 0 \leq s(t,r) && \forall t,r.
\end{aligned}
$$

Results of the energy only market problem are the energy only market optimal objective value G^{EO}, power plant schedules, reservoir filling levels as well as the marginal water values from the dual variables of the reservoir filling level equation. The latter can be used as decision support for trading.

8.2.2. Balancing Market Optimization

The balancing problem formulation is the second important part of the multi-market optimization and it splits up into the balancing power auction and the balancing work auction. First, the balancing power optimization is formulated and second, the balancing work optimization is expressed, both based on the formulation of non-linear programs in chapter 4.1. Third, these two problems are formulated as an integrated balancing optimization.

Balancing Power Auction Optimization

For this mixed integer, non-linear optimization the set of bids in the auction is mapped by $l, m, n = 1, \ldots, I$, whereas m and n are needed for the sorting process. The binary variable $z\ (l)$ describes the (accepted) own bids. If just one bid is offered the respective price paid for this bid is $LP(l)$. If more than one own bid is offered the price depends not just on the position of the own bid in the set l but also on the number of already accepted own bids $\sum_l z(l)$. This connection makes the program non-linear and is addressed in the sort function $\alpha(l,m)$ that depends on $z(n)$ for all $n < l$. In the optimization, it is expected that every bid has a bid size of V or can be divided into bids of that size. The power price optimization can be formulated as:

$$
\begin{aligned}
L: \quad G^{LP} = \quad & \max_{z} \sum_l \big(z(l) \cdot V \cdot \sum_m \big(\alpha(l,m) \cdot LP(m)\big)\big) && (107)\\
s.t. \quad & 0 = \alpha(l,m) \cdot (l - m - \textstyle\sum_{n<l} z(n)) && l,m,n = 1,\ldots,I,
\end{aligned}
$$

with the crucial assumption that $\alpha(l,m) = 1$ if $m = l - \sum_{n<l} z(n)$. The results of this optimization are a matrix with the profitable own bids and how they align in the existing set of bids. The sum of all accepted

bids multiplied with the respected bid size gives the overall optimal volume to be bid into the market. Here, no cost term is modelled since in the integrated optimization the opportunity costs result from the interaction with the energy only market optimization. Furthermore, this formulation is related to the respective product length. In order to optimize over a longer period of time an additional time reference must be given.

Balancing Work Auction Optimization

The work price program is formulated similarly to the power price program. With $z(i)$ as the binary decision variable that selects the own bids form all bids $i, j, k = 1, \dots, I$. A similar non-linearity applies because the work price $AP(j)$ depends on the work price at position i as well as the number of already allocated own bids. This connection is taken into account using the sort function $\beta(i,j)$ that depends on $z(k)$ with $k < i$. Furthermore, the remuneration for the work price just applies if the own bids are accepted and if the offered energy is also activated. Therefore, every bid can be assigned with a specific activation probability. Due to pay as bid, the remuneration of every work bid can be estimated by multiplication with the respective expected activation probability of a bid $AA(i)$.

$$A: \qquad G^{AP} = \max_z \sum_i \big(z(i) \cdot V \cdot AA(i) \cdot \sum_j \big(\beta(i,j) \cdot AP(j)\big)\big) \tag{108}$$

$$s.t. \quad 0 = \beta(i,j) \cdot (i - j - \sum_{k<i} z(k)) \qquad i, j, k = 1, \dots, I.$$

Convexity

Generally, both power and work price problems are assigned to the class of non-linear programs. Most of the problems in this class are np-hard, whereas just a few are part of the class of convex problems. A problem is convex when all production and coefficient matrixes are positive semidefinite meaning not one eigenvalue is negative. This can be either tested with the Gaussian elimination (row reduction) or the Cholesky decomposition. The here mentioned optimization problems are for most input parameter non-convex, which complicates the solution process.

Combined Balancing Power and Work Auction Optimization

The balancing market is designed as a two-stage double auction. On the first stage, the power auction with the participation decision as well as the remuneration for the provision of capacity is taken and on the second stage the payment for the actual delivery of energy is defined. Therefore, every market participant has to hand in bids for both auctions at the same time. Bids consist of volume, power price and work price (V, LP, AP). There is no chance to adopt bids after the first auction part and auction participants must be always able to provide the offered bids. Furthermore, from a bidder's perspective the bids should be designed to result in the wished results within some limits and should consider risk mitigation and profit maximization. To generate bids both balancing power and energy auction problems should be composed and solved at ones. A bit that is accepted in the power market will have a place in the work price merit order, whereas a bit that is not allocated in the balancing power market does not participate in the balancing work auction. A not uncommon optimization result is a very low power price bid below provision costs, to ensure participation, combined with a profitable work price with an overall expected positive return. This would just be possible with a combined optimization as can be seen below; all parameters and variables are defined as for the single problems above.

$$L + A: \quad G^{LP+AP} = \max_{z} \sum_i \sum_l \Big(z(i) \cdot z(l) \cdot V \cdot \big(\sum_m \big(\alpha(l,m) \cdot LP(m) \big) + AA(i) \cdot \sum_j \big(\beta(i,j) \cdot AP(j) \big) \big) \Big) \tag{109}$$

$$s.t. \quad 0 = \alpha(l,m) \cdot (l - m - \textstyle\sum_{n<l} z(n)) \qquad l,m,n = 1,\dots,I$$

$$0 = \beta(i,j) \cdot (i - j - \textstyle\sum_{k<i} z(k)) \qquad i,j,k = 1,\dots,I$$

8.2.3. Combined Optimization

In the beginning of this chapter it is worked out that both energy only and balancing markets are linked together in terms of correlating market prices and balancing activation. Furthermore, these markets can be used as well to exploit arbitrage possibilities within both markets. The combination of the above-mentioned balancing power and work as well as the energy only problem leads to the originally aimed problem formulation.

$$P: \quad G^{EO+LP+AP} = \max_{u,p,s} \sum_{t=1}^{T} \sum_{m=1}^{M} c(t)\big(u(t,m) - p(t,m)\big) + \sum_i \sum_l \Big(z(i) \cdot z(l) \cdot V \cdot \big(\sum_l \big(\alpha(i,l) \cdot LP(l) \big) + AA(i) \cdot \sum_j \big(\beta(i,j) \cdot AP(j) \big) \big) \Big) \tag{110}$$

$$s.t. \quad v(t,r) = v^{anf}(t,r) - s(t,r) + v^{in}(t,r) + \rho(m)p(t,m) - \eta(m)u(t,\mathrm{m}) \qquad \forall t,m,r$$

$$v^{min}(r) \leq v(t,r) \leq v^{max}(r) \qquad \forall t,r$$

$$0 \leq p(t,m) \leq p^{max} \qquad \forall t,m$$

$$0 \leq u(t,m) \leq u^{max} \qquad \forall t,m$$

$$0 \leq s(t,r) \qquad \forall t,r$$

$$0 = \alpha(l,m) \cdot (l - m - \textstyle\sum_{n<l} z(n)) \qquad l,m,n = 1,\dots,I$$

$$0 = \beta(i,j) \cdot (i - j - \textstyle\sum_{k<i} z(k)) \qquad i,j,k = 1,\dots,I.$$

This multi-market energy only and balancing market optimization is non-linear and for most input parameter combinations also non-convex. A generic example is given in chapter 8.3. This problem is non-convex because the variables quantity and price are not independent of each other. This means solving this problem is np-hardness. NP-hardness stands for non-deterministic polynomial-time hardness. This means, the problem cannot be solved or if it can be solved not in finite time.

8.3. Exemplary Results

This chapter focuses on the applicability of the model on real-world case studies. The introduced integrated model has the great advantage to be able to exploit arbitrage within energy only and balancing markets. A typical arbitrage possibility to be exploited with such a model could be to fill a reservoir with negative balancing activation and to release the water in times of price spikes on the energy only market. Unfortunately, the non-linearities in the formulations limit the size of the problems to be solved with today's solvers to a minimum. Therefore, a generic example is provided in chapter 8.3.2 that solves the integrated and complex non-linear optimization as given in chapter 8.2.3. This example demonstrates the general functioning and the added value of the model.

Until this integrated model can be solved in large scale, for example with heuristic methods, a practically applicable approach is presented in chapter 8.3.3 as well. The approach leaves the non-linear tie-ins aside and is based on a paper written by Braun and Burkhardt (2015). This gives the possibility to basically solve even large pumped hydropower storage portfolios in a limited amount of time.

8.3.1. Model Setup

Market selection

The here introduced approaches are described regarding pumped hydropower storages. Nevertheless, this is not excluding since the optimizations can be easily adopted to all kind of other storages. Prerequisite is that the storages are suitable for FRR provision.

Until now, balancing and energy only markets are compared in general. For the following analysis the intraday continuous market is used representatively for all existing energy only markets because it is the most fluctuating short-term energy only market. On the one hand, this overestimates the possible profit of a flexible pumped hydropower storage on the energy only markets because of the price sensitivity which is normally addressed with a hedge based strategy using hourly and quarter-hourly day-ahead as well as intraday markets. On the other hand, it underestimates the income by not considering the optionality value over the time. Both aspects can be considered as presented in chapter 5.2 and 5.4.3.

As representative balancing market the FRR is used since it includes both balancing energy and balancing power remuneration and it is the most lucrative market for bidders (Abgottspon & Andersson, 2012). The integration of further energy only markets into the optimization is theoretically just as possible as the integration of further balancing markets. Nevertheless, due to computational problems this seems extremely challenging, since the objective should also be to optimize over time horizon of several weeks or even months.

Input parameters

The input parameters for the energy only market optimization are quarter-hourly intraday prices and inflow expectations as well as the physical layout and technical restrictions of the pumped hydropower storages. Additionally, as input for the balancing market optimization an expectation on power and work price as well as an activation probability is needed. In contrast to day-ahead markets with one MCP for all customers and all bids for a specific product and time, the pricing on the balancing market in Germany is based on a pay as bid market regime. Therefore, the position of the other bidders is important as well to align own orders most profitable into the merit order. Hence, an assumption for all bids, consisting of power price LP, work price AP and volume V, (LP, AP, V), in the market is needed. A precise forecast of the bid structure in the market is extremely difficult since the market is highly volatile, likely shaped by strategic bidding and influenced by power plant availabilities and the prices on the energy only markets.

Output parameters

The aspired outputs are steering parameters for optimal market biddings on both markets. The outputs for the energy only market trading are optimal schedules with the respective capacity and work limits due to balancing power provision as well as water values. For the balancing market bidding the optimal bids (LP, AP, V) itself are needed and calculated. The bids indicate the wished position in the merit order and take the fixed demand and the specific pricing structure of capacity and energy auction into account.

Machine and Reservoir restrictions

In addition to the general pumped hydropower storage optimization, see chapter 4.1.2 and 5.3, further restrictions need to be entailed when considering balancing markets. This is mainly due to the tightening limitations of reservoirs and machines which are illustrated in Figure 81. If no balancing power is provided, the complete reservoir filling can be used. If positive balancing power is provided (orange arrows) some water needs to be kept as a reserve in the upper reservoir and the lower reservoir cannot be filled until top to provide the space needed if in the extreme scenario the residual water from the upper reservoir is released due to a positive balancing power activation. Vice versa, negative balancing power provision (blue arrows) result in a higher minimum water level in the lower and a reduced maximum filling in the upper reservoir to be always able to consume unused electricity from the grid and to pump water from the lower reservoir into the upper one. The buffers that need to be kept in the reservoirs depend on the quantity of balancing offered, the probability and the estimated time of maximum provision. Consensus in industry and literature (Bartelt & Heltmann, 2013) are 4 hours provision, whereas 30 minutes are theoretically enough due to possible intraday trading. For example, with 4 hours provision and 100 MW offered, 400 MWh always need to be kept in the reservoir in case of a long-term activation.

Beside the restrictions of the reservoir filling levels also the machine capacity that is available for the energy only market is limited by the quantities sold on the balancing markets, see Figure 81. Turbine or pump capacity for balancing needs to be reserved at all times and cannot be used for the energy only market, even with no balancing energy activation.

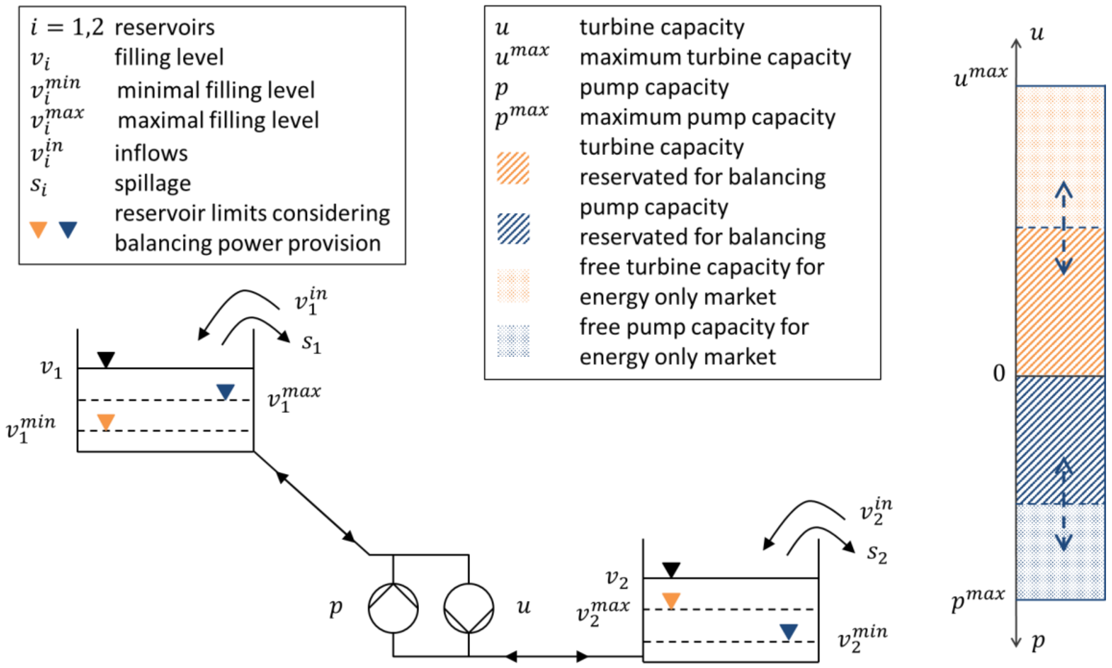


Figure 81 Minimum and maximum reservoir filling levels considering long-term balancing work activation and, on the right, the blocked capacity of turbine and pump for balancing power provision.

As the dotted blue arrows present in Figure 81, the capacity reserved in the machines can vary dependent on how the balancing power is provisioned with pumped hydropower storages. The following four operating modes need to be distinguished.

- Positive activation leads to an increase of turbine production. With positive balancing power provision, the available turbine capacity for energy only is constrained by the maximum turbine capacity minus the quantity offered.
- Negative activation leads to a decrease in turbine production. In this case the turbine produces at least the capacity offered. In case of activation the turbine production can be reduced in the height of the activation signal.
- Positive activation leads to a decrease in pump consumption. This case is very rare. The pump consumes at least the offered capacity and can be reduced by the amount of positive activation. This is just possible with variable speed pumps.
- Negative activation by increasing the pump consumption. There are two ways to realize this strategy. First, if variable speed pumps are present, the pumps consume as much energy as activated and are limited in consumption by the quantity of capacity offered. Secondly, if no variable spread pumps are available as it is the case for most pumps installed, balancing energy can be provided by operating the pump with full load and use a flexible turbine to reduce the consumed energy by the activated amount. This operating mode is called hydraulic short circuit, see chapter 3.1. Both pump and turbine need to block the respective balancing capacity.

8.3.2. Generic Model Results

As mentioned the multi-market optimization problem is non-convex and therefore np-hardness. Nevertheless, generic examples can be constructed which are non-linear but convex. It is important to present such simple and generic examples to check the given optimization approach and to demonstrate a general solvability. In the next chapter the problem is linearized to provide a work around solution for real-world problems as long as the integrated model cannot be solved efficiently.

The following example spans two time-steps, two unrestricted reservoirs, one turbine and one pump, each with a restriction of 10 production units, see Table 15. The energy only market provides a price of 0.1 in the first period and 0.5 in the second. The machines are modeled with no efficiency losses. Furthermore, the complete balancing market contains five offers, each consuming a capacity of one unit. The five positive and negative balancing bids contain work price, power price and the assumed activation. Start and end reservoir filling level are the same.

Table 15 Generic input data for the multi-market energy only and balancing market optimization

positive balancing			
bids	work price	power price	activation
1	2	1	0.8
2	5	2	0.4
3	6	3	0.2
4	6	4	0.1
5	8	4	0.1

negative balancing			
bids	work price	power price	activation
1	2	1	0.8
2	5	2	0.4
3	6	3	0.2
4	6	4	0.1
5	8	4	0.1

energy only	
time	price
1	0.1
2	0.5

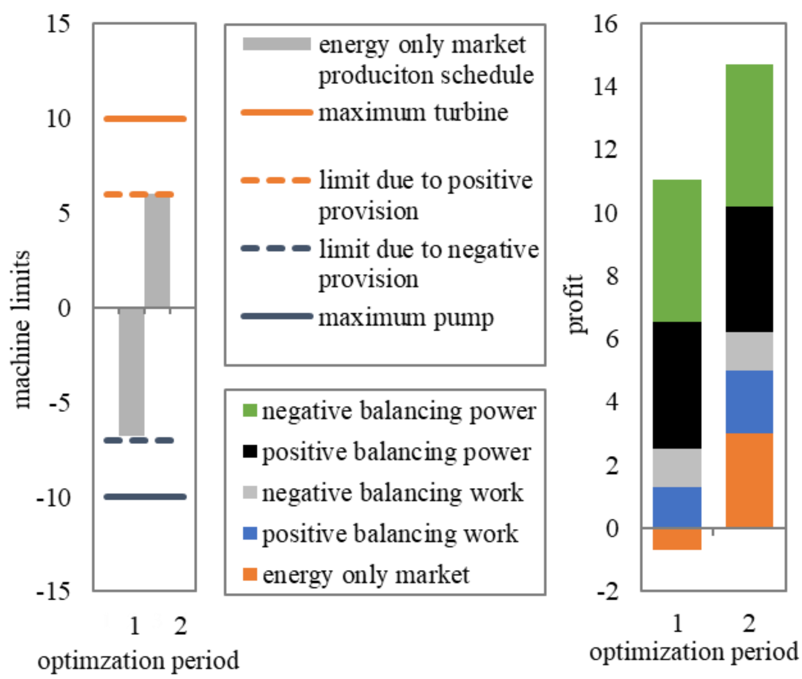
machine characteristics	
maximum turbine	10
maximum pump	10
efficiency	100%

Running the above described algorithm results in an optimal value of 25.72, see Figure 82 adding up the return of stage one and two. It can be seen that the first time step is used for pumping water into the upper reservoir resulting in a negative energy only market revenue contribution (orange bar). The black and green bars symbolize the return from the balancing power market and the blue and grey bars the return from the balancing energy activation. Generally, the energy only market is used by the optimization model to level out the activated balancing energy which is why more energy need to be pumped than generated despite a 100% efficiency. This is the classic arbitrage effect which has been mentioned before.

In Figure 82, the right four tables indicate which bids are allocated by the solver. Every bid is described by two indices i and j. Whereas i numerates all available bids j describes the position of the own allocated bids. All positions in the tables marked with a 1 are bids that are provided by the own power plant. The results of the power price auction are rather optimistic since the algorithm provides all the bids at the end of the merit order, see Figure 82.

Using this given information, the profit can be calculated manually for each product. The work price profit is a result of the multiplication of activation and bid price. For the positive case, this is calculated as $G^{AP} = 5 \cdot 0.4 + 5 \cdot 0.2 + 5 \cdot 0.1 + 5 \cdot 0.1 = 4$.

On the left of Figure 82 the limits of the hydropower machine are presented. The solid orange line marks the maximum turbine and the solid blue line the pump capacities. Furthermore, the optimization assigns this available capacity to the balancing and the energy only markets separated by the orange and blue dashed black lines. This is the crucial point to exploit arbitrage between both markets. Furthermore, the grey bars present the energy only market dispatch schedule which does not exceed the available capacity for the energy only market.



selection positive power bids

j\i	1	2	3	4	5
1		1			
2			1		
3				1	
4					1
5					

selection negative power bids

j\i	1	2	3	4	5
1			1		
2				1	
3					1
4					
5					

selection positive work bids

j\i	1	2	3	4	5
1		1			
2			1		
3				1	
4					1
5					

selection negative work bids

j\i	1	2	3	4	5
1			1		
2				1	
3					1
4					
5					

Figure 82 Distribution of the available machine capacity to energy only and balancing market on the left, the resulting profit form the different markets for the two stages of the optimization in the middle and the selected balancing bids on the right.

For larger problems, the optimization is non-convex and very difficult to solve. As long as computer systems cannot solve such problems in acceptable time no extensive case study can be provided using this approach. Therefore, an adopted linearized optimization approach is presented below which is applicable to large scale problems. Nevertheless, with the progressing flexibilization of the energy only and especially the balancing markets with shorter product lengths and more regular auctions, the solution of the integrated model will be more and more valuable in comparison to the two-stage approach.

8.3.3. Practical Model Results

To solve large scale problems, as it need to be done in real-live dispatch, the above described approach need to be adopted. The introduced optimization problem can be linearized by specifying the quantity to be traded into the balancing market beforehand. This is because one of the two variables quantity and price that are multiplied in the non-linear optimization transforms into an input parameter. Without this tie-in the problem can be split up again into an energy only problem that determines the costs of balancing for provision and a model that determines the incomes from the double auction based balancing market.

To obtain the costs of balancing power provision the opportunity costs of the replaced bids on the energy only market can be calculated by means of blocking certain quantities of power capacities in a hydro power scheduling optimization model. The logic behind this approach is that a hydro power machine can be either used on the energy only markets by realizing a spread or to use the machines for balancing receiving a work and power price.

Model Setup

The cost side is simulated with a linear optimization model, using the 2014 hourly intraday continuous market prices. For the revenue side the historically bids of 2014 as well as the activated balancing energy are used to identify the optimal bidding strategy. The optimization is based on the water values of the hydro scheduling optimization and the publicly available data of the transmission system operators in Germany.

The following characteristics and restrictions are considered in the optimization:

- start reservoir filling level is half of the maximum filling level the
- start and end (target) reservoir filling levels are the same
- minimum filling level is zero
- for consuming energy from the grid (pumping) a grid charge has to be paid amounting to $5\,\frac{€}{MWh}$
- turbine and pump capacity are $100\ MW$
- reservoir capacity is $4000\ MWh$ each
- efficiency of the machine is 70%
- the machines can operate in hydraulic short circle

The inflow input data is set null, because pumped hydro power storages usually do not have significant inflows. Whereas, the crowding out effect is considered in the optimization, it is assumed that an operator, who bids less than 5% of the overall FRR market volume into the market, has no direct market power. This means that since the tendered quantity is fixed, additional quantities of capacity will push other more expensive bidders out of the market and lowers the marginal price. The calculation has been done ex-ante for the year 2014.

Model Results

The results presented in this part, especially in Figure 83, are based on the publication from Braun and Burkhardt (2015). The $100\ MW$ pumped hydropower storage is optimized so that it is partly traded on the hourly intraday continuous as well as the FRR balancing market. Figure 83 shows that the more power is reserved for the balancing market the lower the return from the intraday market. Nevertheless, the income reduction from the intraday market is more than compensated with the incomes from the balancing markets for all products.

Generally, the positive FRR products are completely provided with the turbine, the negative FRR products with a mix of turbine and pumps. This is possible due the hydraulic short circle ability of the machines. It is assumed that in 75% the pump can be used and in 25% the turbine production need to be reduced. This means for the negative products that the turbine needs to produce in base load. Therefore, especially NEG NT is relatively costly. This is also because pumps are normally occupied during low-price periods when used to pump water up and fill the storage with low-priced energy. POS HT can be explained vice versa in terms of blocking turbines during high-price periods. The most lucrative product is POS NT. Generally, it can be said that providing all FRR products at the same time is more efficient than providing one or two single products.

The calculation illustrates that the more FRR is offered to the market the higher the overall profits of the pumped hydropower storages. Although the income from the intraday continuous market would be negative offering 80 MW FRR for all products, it is the most profitable case in this example. Selling more than 80 MW with a 100 MW machine is not possible since at least 20 % of the machine capacity is used to refill or empty the reservoir in times of uneven balancing power activation.

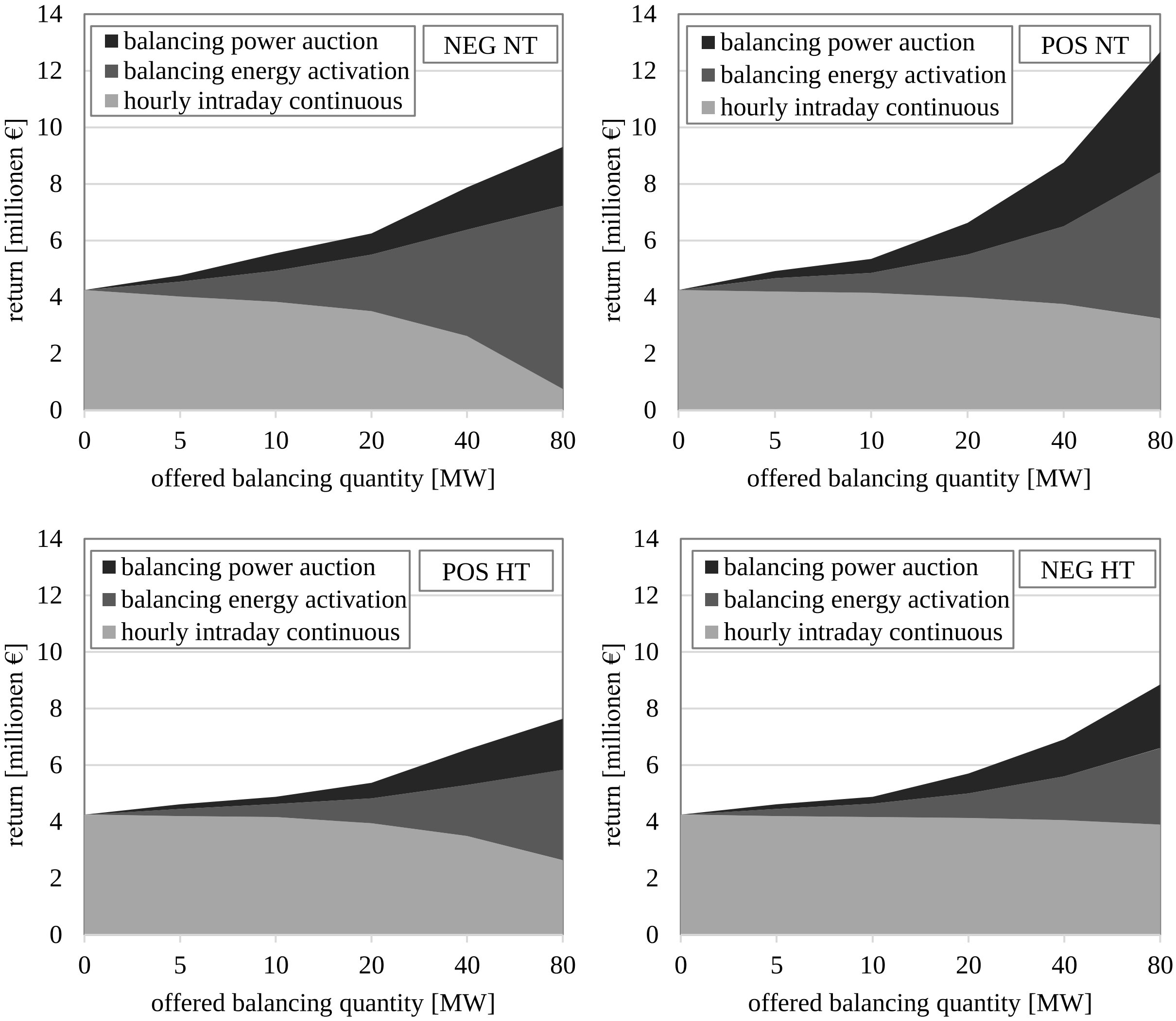


Figure 83 Return from trading a pumped hydropower storage with 100 MW capacity on intraday and balancing markets for the year 2014, (Braun & Burkhardt, 2015)

Critical Discussion

Water values and the cost of FRR provision are assessed using an ex-post intraday price time series in hourly time resolution. To compute the income from FRR market also ex-post historic auction results and historic data for activation are included. This leads to an ex-post optimal analysis which overestimates the income and underestimates the costs.

Furthermore, the optimal position in the activation merit order does not consider all power plants physical limitations. In practice the hydro power plant storages only have a limited quantity of energy. Therefore, a high activation of provided FRR over several hours could be difficult to fulfil. Depending on the risk aversion of the operator this can be an under- or an overestimation of the activation income.

8.3.4. Conclusion

The here introduced approach is an integrated optimization of balancing power and energy only markets. The advantages of the integrated model are clearly the exploitation of arbitrage possibilities between energy only and balancing markets. Nevertheless, the disadvantages are numerous. The resulting dependencies between both markets as well as the non-linear sorting process in the balancing work and power merit orders make the problem highly complex. The resulting MINLP is NP-hardness and therefore nearly unsolvable. Even if these problems could be solved the perfect foresight effect would still be significant. This is because not just the energy only market prices and inflows are taken as given, but also the balancing auction bids of all market participants and the activation pattern.

Although the substantial hindrances the presented generic example is a rebuttal that it is not impossible to solve this complex problem. It could be shown that the optimization is functional, and the problem formulation correct. Future research should focus on solving the formulated problem with evolutionary algorithms for example genetic programming. Latest developments in this field show promising results. The disadvantage of such heuristics is the difficulty to estimate the quality of the solutions.

Instead of solving the overall non-linear and non-convex optimization the problem can be simplified. This is done by separating the energy only market decision from the balancing market bid placement in the merit order and to hold one variable fix to optimize the other. For example, the quantity offered to the balancing power market can be set fixed. With this the problem is not non-convex anymore and can be solved with standard linear solvers. To achieve similar results as with the integrated model an iterative solution approach is suggested (Braun & Burkhardt, 2015). Solving various combinations, with quantities offered to the different markets, helps to approximate a good ratio. Doing this revealed that the balancing markets are more profitable than the hourly intraday market in the given example. Nevertheless, to unlock the described arbitrage within both markets is not possible with this approach.

Beside the application of the optimization model on integrated balancing and energy only market problems the utilization is even broader. In Europe and especially Germany the share of wind power generation increases constantly. As explained in the introduction 8.1, also wind power operators hold a balancing account that need to be balanced to avoid the penalties in the amount of the final balancing price (reBAP). The more wind installed, the higher the discrepancies between planned and actual feed-in.

Wind power operators can use flexible pumped hydropower storages to balance deviations. Since wind farms and pumped hydropower storages are often owned by separate companies, products such as virtual power plants, can be originated to meet this special balancing demand to avoid risk and balancing fees. Such a contract would just come off if it is profitable for both sides, meaning higher incomes for hydropower operator as in the intraday or the balancing markets and lower costs for the wind miller as induced by the fees when being unbalanced. For hydropower operators, this could be calculated using the MINLP. Instead of solving the whole balancing problem, just the balancing work problem need to be computed using the estimated wind power deviations instead of the balancing work activation.

Whether it is reasonable to balance subsystems depends on the influence of the wind power operator on the balancing activation signal. Weber (2010) shows that a wind power producer is exposed to the more risk the more wind energy is in the system. He illustrates that for a higher wind feed-in the correlation increases and the operators pay more, independently of market design and pricing mechanism for their forecast errors. It is assumed that the geographical dispersion of wind power plants reduces the wind forecast error in the beginning of the expansion phase and decreases again with increasing installation in each region, as seen in Germany, Spain and Denmark (Weber, 2010).

Concluding, the optimization problem presented in this chapter is appropriate for a wide field of applications from intraday and balancing markets to virtual power plant based wind forecast balancing. Due to the obstacles of the non-linear approach it is currently not applicable to real-world problems and further research is needed. Regardless, the alternatively provided solution approach has been tested and provides good results as well. Just the arbitrage within both markets cannot be done whereas not even the possible advantage from this can be estimated.

C. Conclusions and Outlook

In this thesis, the optimization and trading of pumped hydropower storages in liberalized electricity markets is analyzed and discussed in detail. Appropriate modelling approaches for various pumped hydropower storage types and the different short-term energy markets are given. It is further analyzed how the results can be used as steering parameters for the dispatch decision. Finally, the models are evaluated and tested using case studies to show real-life applicability.

The main conclusions derived from the analysis is the advantageousness of considering all short-term electricity markets in the marketing process of pumped hydropower storages. The results show that it is not possible to integrate all markets in one optimization due to solvability problems. But a combined optimization of hourly and quarter-hourly markets followed by a rolling intraday optimization provided best possible results as well as significant flexibility.

This part is structured into two parts. Chapter 9 combines and structures the results presented throughout this thesis and suggests future research. This starts with the theoretical overall model. Derived from this and including the results of chapter 5 to 8 a final approach is suggested on how to approach the multi-market hydropower scheduling and bidding problem. The models and results are critically reflected presenting improvements and future research areas. In chapter 10 an outlook is given on the flexibility demand in future electricity systems with more RES, how the pricing mechanisms might work outside the thermal merit order and finally the applicability of the presented methods under such conditions and on a wider range of electricity storage technologies.

9. Findings and Future Research

The core competence of pumped hydropower storages is the provision of flexibility. The higher the demand for flexibility the higher the price. On the existing energy markets, different qualities of flexibility are needed. The quantity of flexibility demand can be generally described by the volatility of the market prices. Looking at the existing electricity markets the volatility increases strongly until delivery. In the long-term this effect is referred to the Samuelson effect (Samuelson, 1965). Samuelson proved the existence of this increasing motion over the time, which is also apparent comparing the volatility of derivatives and spot markets in Germany. Comparing the quarter-hourly day-ahead, intraday and final balancing price (reBAP) in Table 16, the average prices are very similar and the markets are therefore assumed to be arbitrage free. Nevertheless, the volatility represented by the standard deviation increases with the time to delivery. Comparing even the continuously traded quarter-hourly intraday product reveals that the standard deviation over the whole trading period increases. The ID average is lower as the standard deviation of the trades in the last hour before delivery, the so called ID1. This is reasonable since more than 50 % of all quarter-hourly intraday market trades are concluded in the last hour before gate closure (EPEX Spot, 2017b).

Table 16 Volume weighted average market prices and standard deviations of just the quarter-hourly short-term electricity as well as balancing markets for Germany in 2016, data retrieved from (EPEX Spot, 2017b)

	day-ahead [€/MWh]	ID average [€/MWh]	ID3 [€/MWh]	ID1 [€/MWh]	reBAP [€/MWh]
average	28.87	28.65	28.75	28.84	28.37
standard deviation	13.84	16.00	16.45	17.47	49.01

9.1. Theoretical and Suggested Approach

Following the observations in Table 16, the market with the shortest time to delivery is the electricity market with the highest volatility, consequently, with the highest flexibility demand and hence with the best remuneration chances for pumped hydropower storages. Therefore, operators of such must be willing to sell all capacity to this last market. However, due to various reasons such as risk mitigation and hedging most of the electricity is already sold or bought on the foregoing markets (Figure 6 and chapter 2). For example, the liquidity on the hourly day-ahead auction is significantly higher as during intraday trading. As a consequence, flexibility provider bid capacity to the markets with less volatility and higher liquidity.

Beside others, Klæboe and Fosso state that to generate an optimal bidding strategy for one specific market all subsequent markets need to be taken into account when generating bids for the first market

(Klaboe & Fosso, 2013). The results of this thesis generally support this statement. For the short-term markets, this results in a revenue G maximizing the following objective function

$$G^{all} = G^{day-ahead} + G^{intraday} + G^{balancing}. \tag{111}$$

This includes the hourly and quarter-hourly energy only markets as well as the provision and activation of the three balancing markets. Nevertheless, the implementation and optimization of such an overall optimization problem is very difficult. This could be shown in chapter 8 combining energy only and balancing markets in one optimization. The resulting problem is non-linear, non-convex and therefore np-hardness. Future research should focus on solving suchlike complex problems with for example evolutionary algorithms, see chapter 8.

Since the overall and holistic pumped hydropower scheduling problem is basically unsolvable with today's known methods, it is shown that a stage-wise approach is more practical. To find a good solution without solving the overall problem can be done by splitting the problem into smart sub-problems and fitting the results together thereafter. A splitting, exploiting the specific market structure, proved to provide best possible results for the pumped hydropower storage dispatch problem. The general approach is presented in Figure 84.

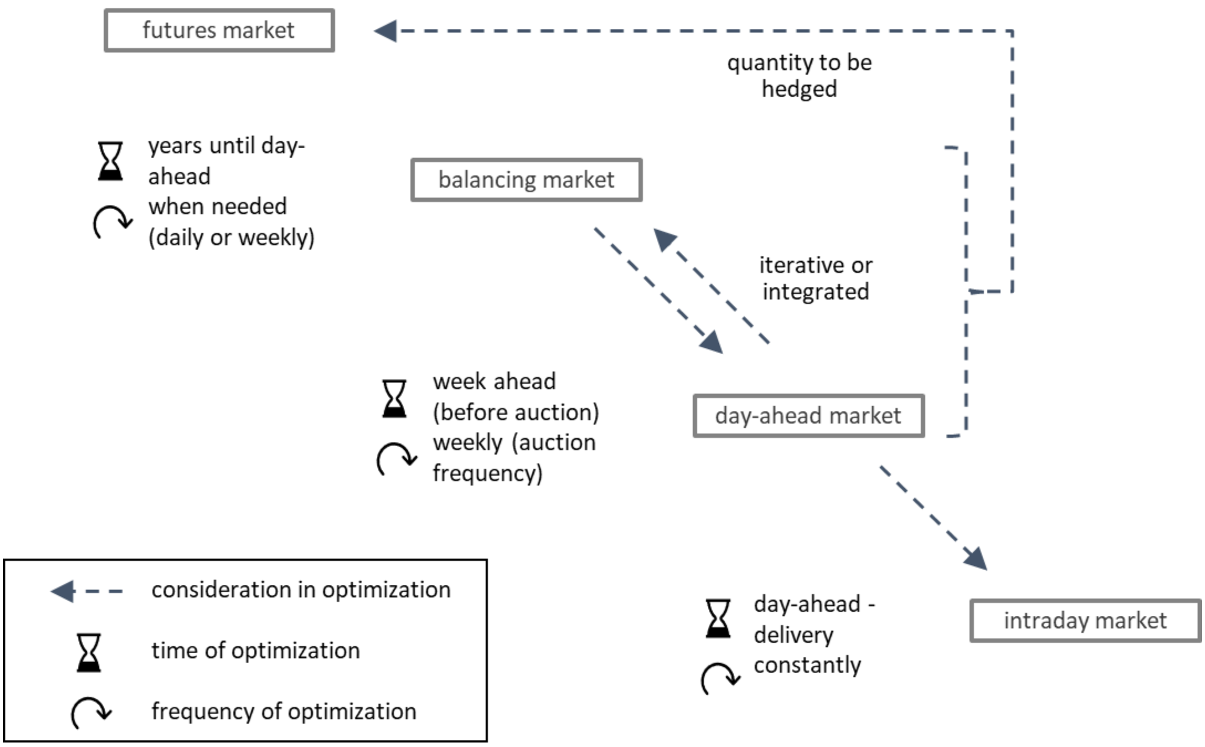


Figure 84 Suggested pumped hydropower storage optimization and scheduling approach

Beginning in the time line with the auctioning of balancing power and energy, independently whether this takes place month-, week- or day-ahead of delivery. For this part, the original statement of considering all subsequent markets for generating bids for the first market holds true, since the power sold to the balancing market is settled after the balancing auction and cannot be reversed. This implicates that the flexibilities cannot be used on the subsequent markets anymore. The quantity traded into the balancing market is therefore dependent on the potential revenue of the subsequent markets.

The question on how much to bid in the balancing market can be solved considering the subsequent markets either iteratively or integrated, see chapter 8. Both approaches are described whereas the integrated approach cannot be solved for sizeable problems. Solving iteratively means that the day-ahead market optimization is performed various times considering that increasing quantities are traded into the balancing market. Additionally, the balancing market problem need to be solved for the different but fixed quantities. This avoids the multiplication of the two variables describing price and quantity so that the balancing problem transforms from a MINLP into a MILP which is smoothly solvable. Comparing the additional incomes of the balancing market with the reduced income of the day-ahead market optimization reveals the break even and the quantity that should be bid into the balancing market. Direct solutions of the balancing market optimization are a set of bids each including quantity, energy price and power price. These bids can be offered to the market.

The question may arise whether the day-ahead market optimization reflects the complete income of all further short-term markets. This is rather not the case but can be alleviated by an integrated hourly and quarter-hourly day-ahead market optimization as introduced in chapter 5. With the consideration of the finer time resolution the volatility of the subsequent electricity markets can be mapped. Furthermore, the trading volume and the liquidity of some short-term energy markets are strongly reduced in some markets. How to consider these effects in the optimizations is described in chapter 5.2.

For a significant number of operators, the consideration of stochastic in inflows or prices are very important. In chapter 6 it is illustrated that a stochastic optimization with SDDP or MCSSP is very valuable for hydropower storages without pumps whereas the advantage is rather limited for pumped hydropower storages. This is because it is always possible to pump released water up again in order to compensate possible dispatch mistakes. No matter if the stochastic (chapter 6) or the deterministic (chapter 5) approach is chosen, the consideration of the finest time resolution to map the volatility of the subsequent markets is crucial to calculate realistic optionalities.

After the balancing auctions took place the day-ahead model is performed again to calculate the expected profit of the day-ahead markets with the remaining capacity. This optimization solution suggests an optimal dispatch for the turbine and pump machines as well as the reservoir filling levels. Based on the dual variables of the deterministic or the cutting planes of the stochastic model water values and shadow prices should be calculated. Due to the integrated multi-market approach, the shadow price for each time stage and machine is applicable to the hourly as well as the quarter-hourly day-ahead auction. This ensures a consistent pumped hydropower steering.

In this approach, the intraday market is explicitly not considered in the day-ahead auction optimization and bidding. The consideration of all subsequent markets when bidding on the day-ahead market is not expedient because of the increasing price sensitivity towards delivery. As already mentioned, with no price sensitivity a flexibility providing pumped hydropower storage would bid all energy into the market with the highest price spreads which is normally the quarter-hourly intraday market. But this would exactly be the same in terms of profit, as firstly bidding to the hourly market and trading the residual quantities in the quarter-hourly market. In the end, the only important part is to bid the residuals on the most volatile market. On the one hand, not bidding into the most volatile market would just mean losing money, but on the other hand, bidding into the foregoing markets does not mean losing opportunities since the energy-only market prices are assumed to be martingale and arbitrage free, see Table 16.

Since every market participant is exposed to price sensitivity, to at least some extant, a hedging approach is reasonable and needs to be applied. This means, market participants bid all quantities into the foregoing markets, if the respective price sensitivity is lower as in the following markets. Afterwards, the planned schedule is adopted by bidding into the next market. If several quarter-hourly markets are available and if the volatilities and the liquidity are similar, then the consideration of one market is sufficient; otherwise all markets should be included.

After the results of the day-ahead market auction are published the intraday continuous market starts. The intraday market prices are highly fluctuating (see Figure 76) over the course of the time influenced by a constant stream of new information about weather and power plant outages. Therefore, also a continuous optimization approach is suggested in chapter 7 calculating a new optimal pumped hydropower storage dispatch as soon as new information is available. This could be for example every five minutes. This approach provides shadow prices as well as the related quantities in MWh to be traded for the respective price. The basic idea is to trade mostly pairwise pump and turbine positions to avoid being exposed to the risk of open positions. A further advantage of the continuous optimization is the consideration of balancing power activation in the trading decision using always up-to-date reservoir filling levels. Such very short-term trading is more and more automated replacing manual trading with algorithmic machine-based trading. Analyzing the structure of the order books and based on quotes from commercial software providers such as Visotech, FIS or Procom, about half of the intraday trades are already based on algorithmic trading by now.

9.2. Exemplary Trading Results

To extant the answer to the research question, of a profit-optimal bidding on short-term electricity markets with pumped hydro power storages, the suggested multistage and multi-market approach that has been introduced in the foregoing part is underpinned with an example below. The results substantiate that the volatility within the different short-term markets varies strongly. This can be seen comparing the quantitative results of a pumped hydropower storage being dispatched to the miscellaneous short-term energy only markets.

In Figure 85 the generated profit of a pumped hydropower storage with a reservoir filling level of about 72,000 MWh and a machine power of 1,200 MW is presented. This equals 60 full-load hours. The optimization is performed for the year 2016. Hourly and quarter-hourly short-term energy markets are considered including intraday average (ID average), the last hour of intraday trading (ID1) and the day-ahead auctions. It becomes apparent that the hourly markets deliver significantly less return in comparison to the quarter-hourly markets. Furthermore, the intraday markets provide generally higher optionalities as the day-ahead markets. The effect of an increasing volatility towards delivery is significant comparing the revenue from the quarter-hourly intraday average and the last hour of intraday trading.

On a yearly average, the last hour of intraday trading provides 12 % higher revenues for flexible pumped hydropower storages in comparison to the intraday average. Firstly, this makes clear how important it is to constantly optimize flexible power plants even until delivery, and secondly, it underpins the need for a hedge-based trading approach due to the limited liquidity.

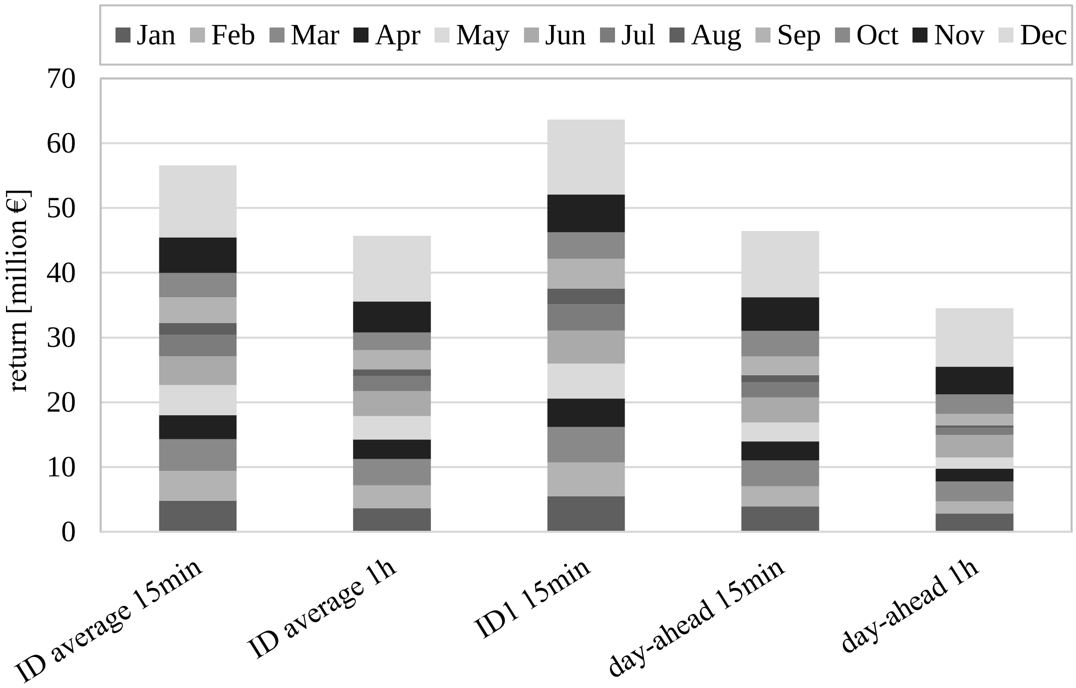


Figure 85 Monthly return of an exemplary pumped hydropower storage with 60 full load hours and 1200 MW capacity over the course of the year 2016 for different German short-term energy markets.

The best way to evaluate the value of different reservoir sizes is to compare the revenue per MW and year for different reservoir sizes and energy markets. The results can be seen in Figure 86. The more inflexible or stable the prices of a market, the more valuable it is to have a larger reservoir. This can be seen for the hourly day-ahead market; the additional revenue with an 8 or a 60 full-load hours reservoir in comparison to 4 full-load hours are 32 % and respectively 102 %. For the very flexible quarter-hourly last hour of intraday trading the advantage of a larger reservoir decreases proportional to 11 % and 25 % additional revenue for 8 or 60 full-load hours reservoirs in comparison to 4 full-load hours.

The difference in income trading the 4 full-load hours reservoir on the hourly day-ahead and the last hour of quarter-hourly intraday trading is about 197%. This difference is large showing again the significance of considering short-term energy markets. Nevertheless, with increasing reservoir size the absolute advantage is constant and decreases relatively. For example, the revenue difference in MW per year in absolute degree between a small 4 and a large 60 full-load hours reservoir for the quarter-hourly intraday trading during the last hour is 10,500 € and for the hourly day-ahead market 14,500 €. Relatively, this means the larger storage adds a value of 25 % in the last hour of intraday trading and even 102 % in the day-ahead. The results suggest that even a very small energy storage is able to exploit significant optionality in the very flexible intraday market but not on the hourly day-ahead market.

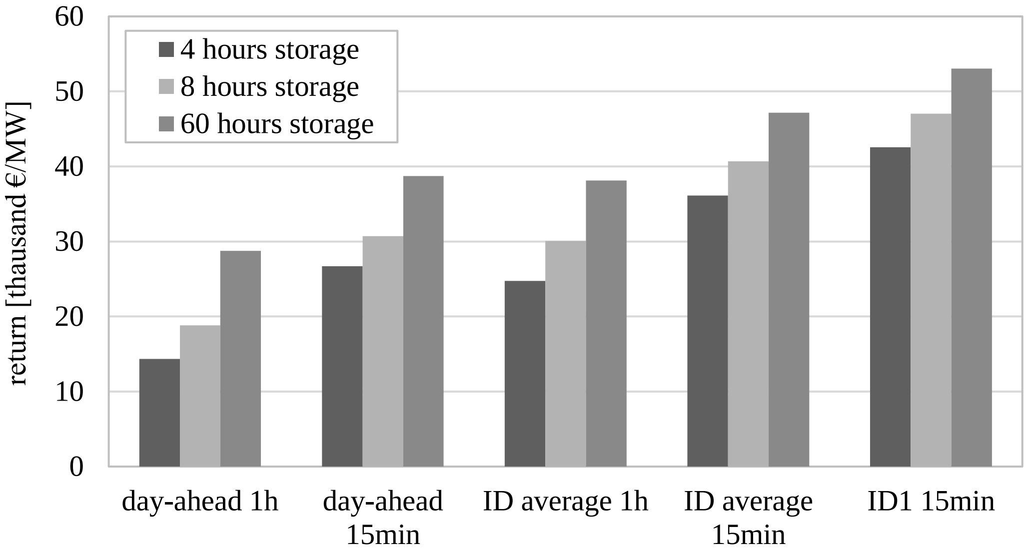


Figure 86 Yearly return form a pumped hydropower storage in the year 2016 for different German short-term electricity markets

The findings clearly present the very short-term electricity markets as important value drivers for pumped hydropower storages. Figure 86 illustrates the revenue losses which are to be expected if the trading on specific markets is left out. An example are pump and turbine machines that are not flexible enough to be switched on and off for quarter-hours. Automatically, the possible revenue decreases to the height of the hourly intraday incomes (ID average 1h). The gap towards the quarter-hourly revenues is a transparent incentive for flexibilization; accepting for example higher deterioration costs in the machines.

Nevertheless, to exploit the whole intrinsic value of e. g. the continuous intraday trading (ID1 15min) as given in the overview Figure 86, the power plant need to be dispatched into the preceding markets as well. This approach is outlined in chapter 9.1. It includes the respective steering parameters and the links to the content chapters for more detailed explanations.

9.3. Future Research

As most of the time, and not just in science, answering one question raises a significant number of new questions. And such questions are the drive of society. The key future research questions that took shape during this work for the different chapters are quoted hereafter followed by some overarching and general questions that have not been reasonably answered so far.

Four main fields for future research in storage scheduling can be seen based on this work:

- transformation of optimization results into a real-world dispatch and trading
- forecasting of the various input factors such as intraday and balancing prices

- improvement of solution algorithms for stochastic and non-linear optimizations
- modification of the existing optimization tools, if the underlying political framework conditions and market terms change

The first three points will be taken up in the next four paragraphs presenting examples of the four content chapters. The last point will be picked up in the last part since it includes more general transformations that could influence every market and therefore applies to basically every chapter.

For the day-ahead auction bidding, explained in chapter 5, the shadow price multi-market storage steering need further research. This includes the problem which storages are suited for the dual variable based water value calculation and which systems are to small and therefore to sensitive for this method. To widen this, it might not just be a question of machine and reservoir size but also dependent on the respective situation. For example, whereas the steering works well in most of the time, some critical situations with high inflows or very low reservoir filling levels are likely to demand special treatment. Furthermore, future research should try to solve this multi-market scheduling problem with stochastic optimization techniques. The challenge will be to keep the course of dimensions under control.

As revealed in chapter 6, most stochastic hydro storage optimizations do not consider systems with pumps. Therefore, the dispatch and steering of pumps, based on stochastic optimization, is not solved. The suggested approach in chapter 6 need to be further developed to find adequate steering parameters for pumps. The challenge is to transfer the model results into an optimal real-life dispatch. Generally, the field of applications for stochastic dynamic programming in short-term optimization is not exhausted, especially in intraday and balancing markets, and should be further analyzed. The additional profit always need to weight up the costs induced by the extra complexity which is not easy to estimate.

In chapter 7 a continuous operating algorithm is presented for intraday trading. Nevertheless, to fully exploit the results of this algorithm the results need to be traded automatized into the market, preferable without a manual interim step. This connection should be subject of future research as well as the integration of more complex reservoir systems into the given algorithm. Special attention should be dedicated to forecast the continuous intraday market prices including not just the volume weighted intraday average but also the intraday price of the last hour or minutes, a sensitivity analysis, a probability range and a tendency of the price development.

Chapter 8, as the last part of section B, revealed significant future research demand in several fields. The solution of the non-linear problem would be a significant step forward but is hardly to be expected soon. Furthermore, to circumvent the perfect foresight problem, balancing power and work price merit orders as well as balancing work activation patterns need to be assumed as random numbers and processed in a stochastic optimization. Since the balancing markets are rather limited in size and auction participants the price maker behavior, as it is already considered in this work, need to be further analyzed. Beside the solution of the optimization problem itself, forecasting all relevant parameters is a significant and unsolved problem especially since efforts are high and the market size is small.

Beside these future research questions which are strongly related to the hydro power scheduling problem, the whole energy sector is still in a transition process which will be a constant source for research questions. This is the point where this work comes full circle. Already the motivation in chapter 1 was based on the energy transition, the introduction of new power markets and a market influencing share of

RES. But, also the outlook and future research are strongly influenced by these fundamental changes. Relevant research questions will be for example: How will future energy systems be organized? What price mechanisms will be used? What is the flexibility demand? Which technologies will be used? Therefore, the next chapter 10 gives an outlook on these questions, especially the future flexibility demand and how the here introduced methods can be applied in future systems.

10. Outlook on Future Flexibility Demand

Over the course of this work the various possibilities to exploit short-term energy markets with flexible pumped hydropower storages are described. This chapter tries to evaluate the further development of flexibility demand in the future and therefore the profitability of pumped hydropower storages as well as other flexibility providing technologies. Historically, as well as in today's electricity markets, pumped hydropower storages are an important source of flexibility. Nevertheless, the electricity sector is changing. How much flexibility is needed in future power systems and what technology might provide this flexibility is uncertain. In this context, news on the shut-down of pumped hydropower storages by Vattenfall in middle Germany (Vattenfall, 2017) stand in contrast to the increasing number of battery electric storages in households and the investments in flexible batteries (IWR, 2015-11-5).

Generally, politics, science and industry agree that with the increasing installation of variable RES the flexibility demand increases as well. Furthermore, to meet the increasing flexibility demand a bouquet of possibilities is available and most likely just a combination will provide an efficient solution, including batteries, demand side management, production flexibilization or sector coupling. Nevertheless, there is no consensus on the overall flexibility demand (Sterner & Stadler, 2014), the costs or the remuneration scheme. Assuming a similar market system as today for the future, the revenues for flexibility options will come from short-term energy only and balancing markets.

First a definition of flexibility demand is given in chapter 10.1 followed by explanatory market models based forecasts concerning future flexibility demand. Chapter 10.2 describes the price formation, in a system with a significant share of RES, outside the thermal part of the merit order and discusses the impact of flexibility on demand and supply side. The last chapter 10.3 concludes with the applicability of the presented optimization tools on a wide range of technologies providing flexibility.

10.1. Definition and Estimation of Flexibility Demand

The International Energy Agency (IEA) defines system flexibility as the general ability to react on deviations of production and consumption (IEA, 2011, p. 35). Such production and consumption changes influence the residual load and finally the electricity prices. Brunner describes the flexibility demand by the overall number of hours with a negative residual load as well as the length of a single and the absolute quantity of maximum and minimum residual load situation (Brunner, 2014a). References for limited flexibility in a market are also high negative and high positive electricity prices (Götz et al., 2014; Nicolosi & Fürsch, 2009) although, both are important elements of the merit order pricing mechanism (Brunner, 2014a). The higher the flexibility demand the higher the incentives to enhance system flexibility.

With the increasing installation of wind and solar based production capacity, regular negative residual load situations will be a natural consequence. Furthermore, the residual load gradients increase. This is the case over the course of a single day considering the PV generation with differences between day and night as well as the heterogeneous distribution of high and low wind times. Nevertheless, the number and level of negative prices over the recent years slightly decreased whereas RES scale up progressed (Götz et al., 2014). This is a clear sign that the market provides more flexibility as a few years ago (BMWi, 2015). Aspects that contributed to more system flexibility are the direct marketing regime (EPEX Spot, 2015b), the reduced RES support in times of negative prices for new installations (EPEX Spot, 2015b) and the modernization of conventional power plants. In a future system with a predominant share of RES additional flexibility on the demand as well as the production side is seen as a key for a successful energy transition. Influencing factors for a sufficient flexible system are the absolute height of electricity demand, a mix of different RES, feed-in priority regulations, conventional power plant designs, inter-sector coupling between heating, transportation and electricity, grid performance, cross border trading and the security of supply level (Brunner, 2014a).

Whereas the flexibility demand is not explicitly mentioned in the RES-Act, it is influenced by the 80% RES-share target in 2050 (BMWi, 2015). More RES in the grid are assumed to increase the flexibility demand which is why the grid development plan (Netzausbauplan) describes a bandwidth for the exploitation of renewable resources and the respective grid extension demand. Studies that focus on the influence of flexibility options in systems with an increasing share of RES are mostly given by research institutions and universities (DLR, Frauenhofer, & IfnE, 2012; Öko-Institut, 2014) or issued on behalf of governmental organizations (consentec & R2B, 2010; ewi, gws, & prognos, 2010). The aim is always to find a general economic maximum and to reveal special effects and problems of respective systems (Brunner & Müller, 2015; Connolly, Lund, Mathiesen, & Leahy, 2010). All models generally show that the introduction of additional flexibility, such as pumped hydropower storages, results in a flattening of the residual load. During times with low residual load energy is stored and in times with a high residual load the stored energy is feed-into the grid again. This results in an increased thermal production for example by lignite power plants and reduces the peak production of more environmentally friendly gas power plants. In other words, the introduction of storage capacity to these systems with a high share of RES probably increases the environmental burden. These results need to be scrutinized, especially since the interaction of spot prices with supply and demand is so sparsely considered. Particularly, in times with a negative residual load, the thermal merit order based pricing, as presented in these models, is not expedient (Brunner, 2014a).

Most of the time, the merit order effect on spot markets is just seen from the supply side. But a separation between flexibility on demand or supply side is nearly impossible. Storages, as major contributors to flexibility provision combine both supply and demand side by definition. Further, even producers of thermal power plants add bids on supply and demand side to the flexibility merit order. This is because the bids do not depend on the absolute production or demand level but just on the deviation between the dispatch schedules. That means, the probability that the prices will fall and the energy of an already marketed power plant need to be bought back or that the prices will rise and a power plant that was hitherto not in the money is now sold into the spot market, is the same.

Ideally, the market finds the optimal equilibrium between flexibility demand and supply itself. In case of an increasing flexibility demand the spot market price elasticity will decrease, the spot price itself varies stronger, the chances for market clearance decrease. Vice versa, with the installation of additional flexibility the spot market price elasticity will increase, the spot price itself will be flattened, the chances for market clearance increase. Each effect generates incentives to invest or disinvest in flexibility so that, following the neoclassic theory of markets (Stoft, 2002), future flexibility demand should be always covered. Nevertheless, whether this mechanism is working efficiently depends on distortions such as installation times, taxes, oligopoly structures or political interventions.

10.2. Price Mechanism Outside the Thermal Merit Order

Because of the low variable costs, RES are sorted in the merit order on the first ranks. This has a strong influence on the price formation in the market. For the Öko institute model this results in many hours with a price of 0 €/MWh (Öko-Institut, 2014). Negative prices are not forecasted, since variable RES are assumed to be switched off. With RES shares of 30% in 2015 and 79% in 2050, in this study, the number of hours with a price close to zero increases significantly to more than 1000 hours per year (Öko-Institut, 2014). Further implications are the shutdowns of thermal base load power plants with low variable but high overall costs. As a result, the spot price increases since more peak load power plants such as gas and oil power plants with higher variable but lower overall costs for the fewer full-load hours are used during these hours with a high residual load (Öko-Institut, 2014).

Generally, it can be scrutinized that for several different fundamental situations the same market price of 0 €/MWh applies. At a market price of 0 €/MWh it might not be reasonable for all market participants to reduce their production, for example due to ramp-up and –down times of large thermal power plants. Furthermore, long periods with prices of about 0 €/MWh might change the behavior of the demand side (Brunner, 2014a; Brunner & Müller, 2015). This could especially be the case for industry customers. Whereas, it needs to be noted that a market price of zero does not necessarily mean that end-customer receive electricity for free. As long as taxes and levies are responsible for the mayor share of the electricity price households cannot be expected to change their behaviors. But substitution of energy carriers in other sectors with for example power-to-gas or power-to-heat are thinkable (BMWi, 2015). Since different end users have different costs providing flexibility, they might also offer different bids to the market. Following the classic micro economic theory of equilibrium prices should therefore adopt the different residual load situations (Kirschen & Strbac, 2004; Stoft, 2002). Situations with prices of exactly 0 €/MWh should be rarer and should reflect similar fundamental situations with similar residual load.

A focus of future research should therefore be on the price formation outside the proven thermal merit order including a significant share of variable RES feed-in. Furthermore, a consideration of flexibility in the demand side merit order might help to provide a more reasonable price formation. On the one hand, in situations with a residual load around zero the flexible demand might be strong enough to provide a MCP above 0 €/MWh. On the other hand, the demand might be reduced during times with strong residual load and reduces extreme high prices (Brunner, Michaelis, & Möst, 2015).

The already introduced merit order with RES, Figure 4 (chapter 0), can be extended including a flexible demand side that considers pumped hydropower storage, e-mobility, power-to-gas and power-to-heat, demand side management, further storage technologies and sector coupling. In Figure 87 a situation with significant generation of variable RES and a low demand is presented. This merit order may represent a sunny hour on a public May holiday. In Figure 88 a situation with very limited variable RES production and significant demand is illustrated, picturing a cold winter hour.

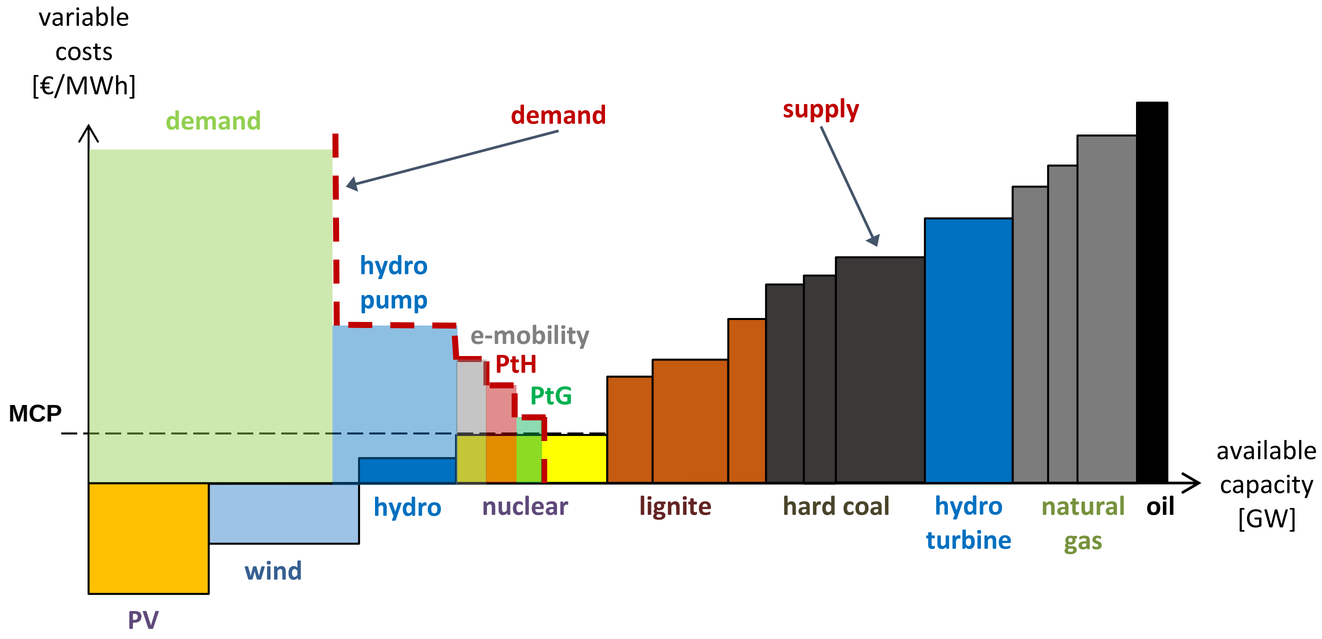


Figure 87 Merit order with flexible demand and supply curves for a day with significant RES production under a fee-in tariff regime and low demand

In Figure 87, instead of a negative price with wind as the marginal technology of the merit order the flexible part of the demand pushes the overall demand into the nuclear production. The market clearing in Figure 88 is determined by the generation technology at the respective level. In this price range, the demand is inflexible and pumped hydropower storages are the marginal technology. A power plant outage would immediately result in a higher MCP.

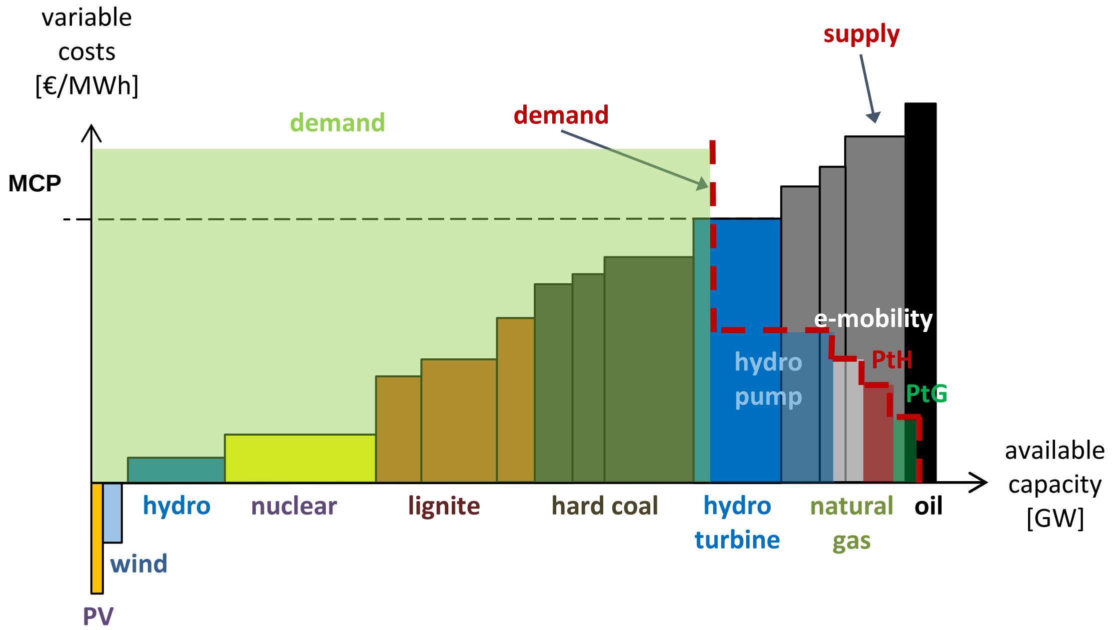


Figure 88 Merit Order with flexible demand and supply curves for a day with low RES production and significant demand

It is not unrealistic that also in the future the pricing will be based on the fundamental merit order mechanism. Nevertheless, a few influencing factors are highly unknown as for example new technologies or the political framework. From today's myopic perspective especially the extension of the grid, end-customer taxation, grid charges for storages and the development of battery technologies seems most relevant.

Whereas the flat electricity price curves with manifested low variance indicate a limited flexibility demand today, many fundamental factors suggest that this effect might reverse due to the nuclear phase out in Germany, reduced production of coal power plants and more weather dependent variable RES production as well as higher demand due to e-mobility and power-to-heat.

10.3. Applicability of the Presented Optimization Tools in the Future

The foregoing qualitative analysis of the future price formation shows that the algorithms and optimization methods developed in this work are no panacea for the future but a significant step forward to match flexible supply and demand. The consideration of short-term energy markets in the optimizations is a crucial and necessary consequence of the increasing demand to level out short-term RES production deviations. Taking into account several markets for the marketing and trading process is significant for an optimal dispatch.

The introduced approaches are not limited to pumped hydropower storages, but can be applied on battery electric vehicles, smart home wall boxes, compressed air storages and so on. This means that the introduced optimizations are generally technology independent and can be applied on all kind of storages and applications. Adding or deleting storage individual constraints is easily possible. The optimization of storages is furthermore not just advantageous for the operator itself but also for a cost-efficient provision of flexibility to the market.

In the recent years, especially the multi-market optimization approaches gained more and more attention. Multi-faceted, open approach thinking in terms of optimization systems is necessary to understand the limitations and possibilities of mathematical methods and real-life complexities. This work brought existing work together and suggests an overall optimization and trading approach for pumped hydropower storages that can be easily applied on any other flexibility providing technology as well. Furthermore, most of the introduced methods have been tested and implemented in practice proving to deliver successful results.

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