# On the Impact of the Mutual Impedance of an Antenna Array on Power and Achievable Rate

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***Abstract*—In this paper, we consider antennas arrays with small mutual resistance, and assess the impact of neglecting both the small mutual resistance and the mutual reactance, on the radiated power and on the achievable rates. This extends our earlier results on antenna arrays with zero mutual resistance, which show that when those antenna arrays are excited by practical radio frequency amplifiers modeled as linear sources, the radiated power depends on the mutual reactance and that there is a gap between capacity and the achievable rate when neglecting the mutual reactance. In this investigation, we consider a Uniform Linear Array (ULA) consisting of $\lambda/2$-dipoles spaced by $\lambda/2$, and compare it to a ULA consisting of hypothetical isotropic radiators spaced by $\lambda/2$. Numerical results are given for the ergodic rates in an independent and identically distributed channel in the up- and downlink, with a base station transmitting to a single antenna mobile. Furthermore, the investigation is extended to a more realistic channel model based on QuaDRiGa.**

## I. Introduction

Antenna arrays are getting more and more important for mobile communications. As the number of antennas in the array increases, it becomes more important to model the antennas accurately. In a recent article [1], the authors show that arrays of hypothetical isotropic radiators also have a mutual reactance, while in signal processing and wireless communications literature, usually only the mutual resistance is considered. They furthermore consider a base station with an antenna array of canonical minimum scattering antennas [2] and model the array using an impedance matrix $\boldsymbol{Z}_{\mathrm{BS}}$. In addition, it is assumed that the distances between the antennas in the array are such that the mutual resistance is zero. In this case, the radiated power is independent of the mutual reactance, if the array is excited by ideal current sources [1]. However, practical radio frequency amplifiers are not ideal current sources, but can often be modeled as linear sources. In this case, the radiated power does depend on the mutual reactance [1]. Then they consider the impact of neglecting the mutual reactance of the antenna array on the radiated power and on the ergodic rates for an independent and identically distributed (i.i.d.) channel. They consider two types of antenna arrays: a Uniform Linear Array (ULA) consisting of hypothetical isotropic radiators spaced by $\lambda/2$, where $\lambda$ is the wavelength of the carrier, and a Uniform Circular Array (UCA) consisting of three lossless and infinitely thin $\lambda/2$-dipoles.

In this paper, we extend these results to antenna arrays that only have a small mutual resistance. As in [1], the analysis is based on the Multiport Communications Theory [3], [4] and in particular on the simplified model from [5] that does not consider a matching network.

*Notation*: Lowercase bold letters denote vectors, uppercase bold letters matrices. $\boldsymbol{A}^T, \boldsymbol{A}^H$ correspond to the transpose and Hermitian. $\mathbf{0}$ and $\boldsymbol{I}$ denote all-zero vector and identity matrix, $\mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{R})$ a complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{R}$, $\operatorname{diag}(\boldsymbol{A})$ the matrix whose diagonal elements are equal to those of $\boldsymbol{A}$, and whose other entries are zero, and $\mathrm{E}[\cdot]$ the expectation operator.

## II. Antenna Arrays with Small Mutual Resistance

As in [1], we start with lossless antenna arrays consisting of $N$ identical antenna elements with resistance $R_{\mathrm{r}}$ and reactance $X_{\mathrm{r}}$, whose mutual resistance is zero, at the base station. That means that its impedance matrix $\boldsymbol{Z}_{\mathrm{BS}} \in \mathbb{C}^{N\times N} \cdot \Omega$ fulfills [1]

$$\boldsymbol{Z}_{\mathrm{BS}} = R_{\mathrm{r}}\boldsymbol{I} + j\boldsymbol{X}_{\mathrm{BS}}, \quad \operatorname{diag}(\boldsymbol{X}_{\mathrm{BS}}) = X_{\mathrm{r}}\boldsymbol{I}, \tag{1}$$

where $\boldsymbol{X}_{\mathrm{BS}}$ is the mutual reactance matrix. Then the (instantaneous) radiated power when the array is excited by ideal current sources $\boldsymbol{i}$, can be computed as [1], [4]

$$P_{\mathrm{T,i}} = \mathrm{Re}\left(\boldsymbol{i}^H\boldsymbol{u}\right) = \boldsymbol{i}^H\,\mathrm{Re}\left(\boldsymbol{Z}_{\mathrm{BS}}\right)\boldsymbol{i} = R_r\|\boldsymbol{i}\|_2^2. \tag{2}$$

For example this is true for a ULA consisting of hypothetical isotropic radiators spaced by $\lambda/2$ [1], but note that isotropic radiators are hypothetical since isotropic sources of coherent electromagnetic radiation do not exist [6, Sec. 1.13]. Therefore in practice, other types of antennas need to be used, but in general arrays consisting of these antennas do not have zero mutual resistance, but some (small) mutual resistance since the zeros of the mutual resistance are not equidistant in general. Thus in this paper, we want to consider more practical antenna arrays with small mutual resistance. The following analysis also applies if it is not small, but that would lead to different simulation results. Then impedance matrix of the array fulfills

$$\boldsymbol{Z}_{\mathrm{BS}} = \boldsymbol{R}_{\mathrm{BS}} + j\boldsymbol{X}_{\mathrm{BS}}, \tag{3}$$

where $\boldsymbol{R}_{\mathrm{BS}}$ is the mutual resistance matrix, which is symmetric positive semidefinite.

In this case, the (instantaneous) radiated power is

$$P_{\mathrm{T,i}} = \boldsymbol{i}^H\,\mathrm{Re}\left(\boldsymbol{Z}_{\mathrm{BS}}\right)\boldsymbol{i} = \boldsymbol{i}^H\boldsymbol{R}_{\mathrm{BS}}\boldsymbol{i}, \tag{4}$$

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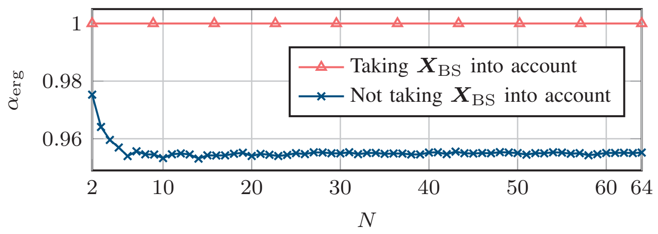


Fig. 1. Ergodic ratio $\alpha_{\text{erg}}$ of true and predicted power for a ULA consisting of hypothetical isotropic radiators spaced by $\lambda/2$ in an i.i.d. channel [1].

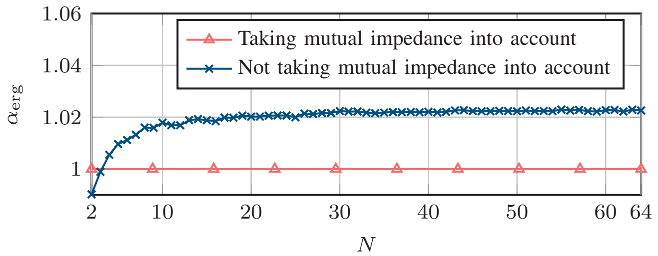


Fig. 2. Ergodic ratio $\alpha_{\text{erg}}$ of true and predicted power for a ULA consisting of $\lambda/2$-dipoles spaced by $\lambda/2$ in an i.i.d. channel.

when the antennas in the array are excited by ideal current sources $\boldsymbol{i}$. Note that due to the mutual resistance, $P_{\text{T,i}}$ is still directly proportional to $\|\boldsymbol{i}\|_2^2$, but contrary to (2), the proportionality factor is not constant and different from $R_{\text{r}}$ in general.

If more realistic linear power amplifiers are used to excite the antenna array, which can be modeled [1] as a linear voltage source $\boldsymbol{u}_{\text{G}}$ with inner resistance $\boldsymbol{R} = R\boldsymbol{I}$, the (instantaneous) radiated power can be computed as [1], [3], [5]

$$P_{\text{T,i}} = \frac{\boldsymbol{u}_{\text{G}}^H \boldsymbol{B} \boldsymbol{u}_{\text{G}}}{R}, \tag{5}$$

$$\boldsymbol{B} = R \left(\boldsymbol{Z}_{\text{BS}} + \boldsymbol{R}\right)^{-H} \boldsymbol{R}_{\text{BS}} \left(\boldsymbol{Z}_{\text{BS}} + \boldsymbol{R}\right)^{-1}, \tag{6}$$

where $\boldsymbol{B}$ is the so-called power coupling matrix. If the base station neglects the mutual impedance, i.e., it assumes that $\boldsymbol{Z}_{\text{BS}}$ is diagonal, it predicts the (hypothetical instantaneous) radiated power via [1]

$$P_{\text{T,p,i}} = \frac{\boldsymbol{u}_{\text{G}}^H \hat{\boldsymbol{B}} \boldsymbol{u}_{\text{G}}}{R}, \quad \hat{\boldsymbol{B}} = R \frac{R_{\text{r}}}{(R_{\text{r}} + R)^2 + X_{\text{r}}^2} \boldsymbol{I}. \tag{7}$$

Then the true and predicted radiated power are defined as [1]

$$P_{\text{T}} = \text{E}[P_{\text{T,i}}], \tag{8}$$

$$P_{\text{T,p}} = \text{E}[P_{\text{T,p,i}}]. \tag{9}$$

At the receiver side, the load voltage $\boldsymbol{u}_{\text{L}}$ is measured across the input impedance $R$ of the low noise amplifier (LNA) in each RF chain [1]. Let us assume the underlying physical model is [4]

$$\boldsymbol{u}_{\text{L}} = \boldsymbol{D}\boldsymbol{u}_{\text{G}} + \sqrt{R}\boldsymbol{\eta}, \quad \boldsymbol{\eta} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}\sqrt{\text{W}}, \boldsymbol{R}_\eta), \tag{10}$$

where $\boldsymbol{D}$ is the input-output relation of the voltages and $\boldsymbol{\eta}$ is additive noise with variance $\boldsymbol{R}_\eta$ [5]. In the simulations, we will use the noise parameters from [7], but with input impedance $R$ of the LNA.

According to the Multiport Communications Theory, we can obtain the information theoretic model [4]

$$\begin{aligned} \boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\vartheta}, \quad \boldsymbol{\vartheta} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}\sqrt{\text{W}}, \sigma_\vartheta^2 \boldsymbol{I}), \\ \boldsymbol{H} = \sigma_\vartheta \boldsymbol{R}_\eta^{-1/2} \boldsymbol{D} \boldsymbol{B}^{-H/2} \end{aligned} \tag{11}$$

with the receive vector $\boldsymbol{y}$, the transmit vector $\boldsymbol{x}$ and the information theoretic channel $\boldsymbol{H}$. Similarly when neglecting the mutual impedance [1], [5], we can obtain

$$\boldsymbol{y} = \hat{\boldsymbol{H}}\hat{\boldsymbol{x}} + \boldsymbol{\vartheta}, \quad \hat{\boldsymbol{H}} = \sigma_\vartheta \boldsymbol{R}_\eta^{-1/2} \boldsymbol{D} \hat{\boldsymbol{B}}^{-H/2}. \tag{12}$$

## III. Simulation Results for the I.I.D. Channel

We will compare the results for the ULA consisting of hypothetical isotropic radiators spaced by $\lambda/2$ with one consisting of $\lambda/2$-dipoles. For the latter,

$$\max_{\substack{k,l \\ k \neq l}} |[\boldsymbol{R}_{\text{BS}}]_{k,l}| \approx 0.171\, [\boldsymbol{R}_{\text{BS}}]_{k,k} \tag{13}$$

holds for $N = 2, \ldots, 64$, and the maximum is attained for $|k - l| = 1$, i.e., the mutual resistance is indeed small.

In this section, we compare the ergodic capacities, rates and the ratio of (hypothetical) predicted radiated power and radiated power by a Monte Carlo simulation of 1000 channel realizations of the i.i.d. channel

$$\boldsymbol{z} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}\,\Omega, \sigma_z^2 \boldsymbol{I}), \quad \sigma_z \approx 0.019\,085\,\Omega, \tag{14}$$

$$\boldsymbol{D} = \frac{R}{R_{\text{r}} + jX_{\text{r}} + R} \boldsymbol{z}^T (\boldsymbol{Z}_{\text{BS}} + \boldsymbol{R})^{-1}, \tag{15}$$

and assume that $R = R_{\text{r}}$, which is exactly the same setup as in [1]. This choice of $R$ leads to power matching for the hypothetical isotropic radiators at the transmitter and receiver, and to a matching of the resistance for the $\lambda/2$-dipoles, a heuristic [1].

The impedance $Z_{\lambda/2}$ and the mutual impedance of infinitely thin $\lambda/2$-dipoles can be found in [6, Sec. 13.4]. For the isotropic radiators, we assume that their resistance is $R_{\text{r}} = \text{Re}(Z_{\lambda/2})$ and their reactance is $X_{\text{r}} = 0\,\Omega$ [1].

### A. Downlink

In the downlink, we consider the transmission of the base station to a mobile with a single $\lambda/2$-dipole as an antenna. Then $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$ become row vectors. We define $\boldsymbol{h}^H = \boldsymbol{H}$, $\hat{\boldsymbol{h}}^H = \hat{\boldsymbol{H}}$. First, consider the ratio of true and predicted radiated power [1]

$$\alpha_{\text{erg}} = \exp(\ln(2)\, \text{E}[\log_2 \alpha]), \tag{16}$$

$$\alpha = \frac{P_{\text{T}}}{P_{\text{T,p}}} = \frac{\hat{\boldsymbol{h}}^H}{\|\hat{\boldsymbol{h}}\|_2} \hat{\boldsymbol{B}}^{-1/2} \boldsymbol{B} \hat{\boldsymbol{B}}^{-H/2} \frac{\hat{\boldsymbol{h}}}{\|\hat{\boldsymbol{h}}\|_2}. \tag{17}$$

Let us compare Figs. 1 and 2. Please note that not taking $\boldsymbol{X}_{\text{BS}}$ into account and not taking the mutual impedance into account is the same for the ULA consisting of hypothetical isotropic radiators, since its mutual resistance is zero. We see that for this type of array, $\alpha_{\text{erg}}$ saturates around 95.5 % already at about $N = 7$, but with $\lambda/2$-dipoles, it saturates for larger

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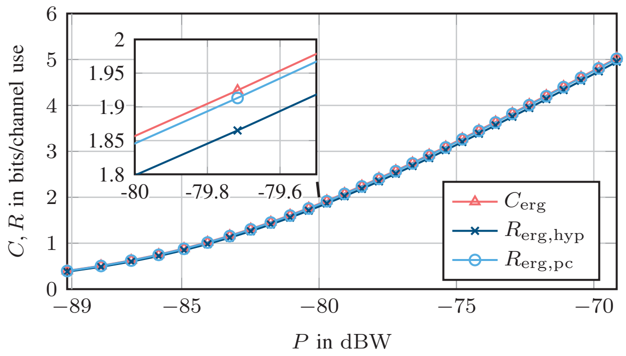


Fig. 3. Ergodic downlink rates of a ULA consisting of 64 isotropic radiators spaced by $\lambda/2$ with and without neglecting the mutual impedance in an i.i.d. channel [1].

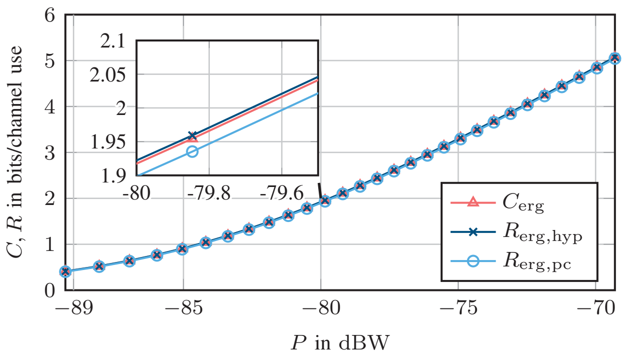


Fig. 4. Ergodic downlink rates of a ULA consisting of 64 $\lambda/2$-dipoles spaced by $\lambda/2$ with and without neglecting the mutual impedance in an i.i.d. channel.

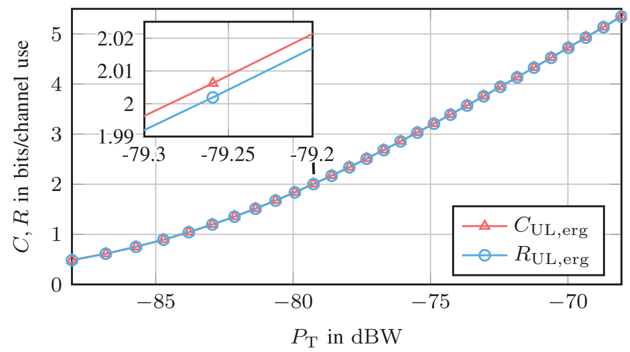


Fig. 5. Ergodic uplink rates of a ULA consisting of 64 isotropic radiators spaced by $\lambda/2$ with and without neglecting the mutual impedance in an i.i.d. channel [1].

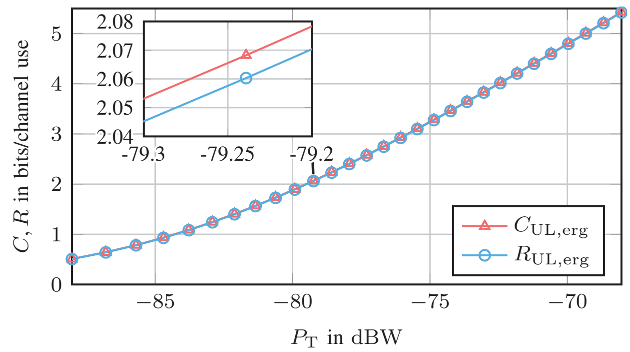


Fig. 6. Ergodic uplink rates of a ULA consisting of 64 $\lambda/2$-dipoles spaced by $\lambda/2$ with and without neglecting the mutual impedance in an i.i.d. channel.

$N$ and reaches about $102.3\,\%$. That means, for larger $N$, on average a slightly different amount of power than predicted is radiated by the arrays – about $0.2\,$dB less for the former [1] and about $0.099\,$dB more for the latter.

Second, consider the ergodic capacity $C_{\text{erg}}$, the hypothetical rate $R_{\text{erg,hyp}}$ and the power corrected ergodic rate $R_{\text{erg,pc}}$ for a given power $P$, which are defined as [1]

$$C = \log_2\left(1 + \|\boldsymbol{h}\|_2^2 P/\sigma_\vartheta^2\right) \quad \text{for } P_{\text{T}} = P, \tag{18}$$

$$R_{\text{hyp}} = \log_2\left(1 + \|\hat{\boldsymbol{h}}\|_2^2 P/\sigma_\vartheta^2\right) \quad \text{for } P_{\text{T,p}} = P, \tag{19}$$

$$C_{\text{erg}} = \text{E}\left[C\right], \quad R_{\text{erg,hyp}} = \text{E}\left[R_{\text{hyp}}\right], \tag{20}$$

$$R_{\text{pc}}(P) = R_{\text{hyp}}(P/\alpha), \quad R_{\text{erg,pc}} = \text{E}\left[R_{\text{pc}}\right]. \tag{21}$$

There are two losses when the mutual impedance is neglected: the one due to the suboptimal beamforming vector and the one due to the base station using $P_{\text{T,p}}$ instead of $P_{\text{T}}$, i.e., a different than predicted power is radiated, as we have seen in Figs. 1 and 2. $R_{\text{hyp}}$ contains both losses, while $R_{\text{pc}}$ only contains the one due to the suboptimal beamforming vector.

Consider the ULAs with $N = 64$ antennas. Fig. 3 corresponds to the one consisting of hypothetical isotropic radiators and shows that $R_{\text{hyp}}$ is $0.081\,$bpcu (bits per channel use), smaller than $C_{\text{erg}}$ at high SNR, where using $P_{\text{T,p}}$ results in a smaller average radiated power that accounts for $0.066\,$bpcu, which is exactly the gap between $R_{\text{erg,hyp}}$ and $R_{\text{erg,pc}}$ at high SNR [1]. That means, here the influence of the smaller average radiated power dominates over the suboptimal beamforming vector. Let us compare that to the ULA consisting of $\lambda/2$-dipoles, see Fig. 4. Here $R_{\text{erg,hyp}}$ is slightly larger than $C_{\text{erg}}$ – $0.0061\,$bpcu at high SNR – due to the larger average radiated power. The gap between $R_{\text{erg,pc}}$ and $C_{\text{erg}}$ is $0.025\,$bpcu, i.e., the effect of the larger radiated power and the one of the suboptimal beamforming vector are about the same scale.

The gap between $C_{\text{erg}}$ and $R_{\text{erg,pc}}$ and between $R_{\text{erg,pc}}$ and $R_{\text{erg,hyp}}$ is a bit larger for the ULA consisting of $\lambda/2$-dipoles than for the one consisting of isotropic radiators, but still on the same order of magnitude. However, neglecting the mutual impedance leads to a larger instead of a smaller radiated power and $R_{\text{erg,hyp}}$ for the former.

### *B. Uplink*

In the uplink, the mobile transmits over the transposed channel. Then $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$ become the column vectors $\boldsymbol{h}$ and $\hat{\boldsymbol{h}}$. We consider the same base station arrays as in the downlink and compute the ergodic capacity [1]

$$C_{\text{UL,erg}} = \text{E}\left[\log_2\left(1 + \|\boldsymbol{h}\|_2^2 P_{\text{T}}/\sigma_\vartheta^2\right)\right] \tag{22}$$

and the ergodic rate neglecting the mutual impedance

$$R_{\text{UL,erg}} = \text{E}\left[\log_2\left(1 + P_{\text{T}}\|\hat{\boldsymbol{h}}\|_2^4/(\hat{\boldsymbol{h}}^H \boldsymbol{R}_{\hat{\vartheta}}\hat{\boldsymbol{h}})\right)\right] \tag{23}$$

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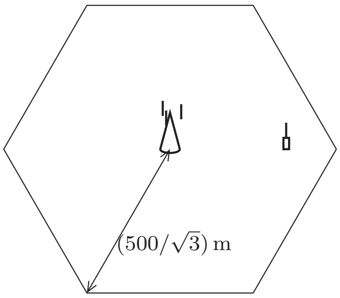


Fig. 7. A base station serving a mobile in a hexagonal cell.

for a single antenna mobile with a $\lambda/2$-dipole.

When we compare the results for both arrays (see Figs. 5 and 6), the rates are almost the same. In both cases, $R_{\mathrm{UL,erg}}$ is a little bit smaller than $C_{\mathrm{UL,erg}}$, about $0.0058\,\mathrm{bpcu}$ with the isotropic radiators and $0.010\,\mathrm{bpcu}$ with the $\lambda/2$-dipoles at high SNR. The gap between them only results from a suboptimal equalizer that does not take into account the noise correlations between the antennas, introduced by the mutual impedance. The predicted and (true) radiated power are the same, as the mobile only has a single antenna. That means, also in the uplink, the gap for the ULA consisting of hypothetical isotropic radiators and the one for the ULA consisting of $\lambda/2$-dipoles are on the same order of magnitude.

So far, we have only considered an i.i.d. channel. In the next section, we will consider a more realistic channel model.

## IV. Simulation Results with QuaDRiGa

QuaDRiGa [8], [9] implements the 3GPP channel models defined in TS 38.901 [10], which are valid from $500\,\mathrm{MHz}$ to $100\,\mathrm{GHz}$ carrier frequency, and more realistic than an i.i.d. channel model. Consider a single base station site in the urban macrocell (UMa) model (without mobility), which assumes a regular hexagonal pattern of cells with base station sites on certain corners of the hexagons. Let us assume that this site is not sectored, i.e., there is a single base station at this site. When it serves all mobiles that are closest to it, it serves a hexagon whose edge length is $(500/\sqrt{3})\,\mathrm{m}$, as the inter site distance is $500\,\mathrm{m}$, see Fig. 7. The base station is located at the center of this hexagon at an altitude of $35\,\mathrm{m}$. For each channel realization, a mobile is placed in the hexagon according to a uniform distribution, except for a circle with radius $35\,\mathrm{m}$ around the base station, which does not contain any mobiles. Their altitude is computed according to [10]. We only consider stationary mobiles that have got a vertically oriented $\lambda/2$-dipole as an antenna, as for the i.i.d. channel model. QuaDRiGa assumes that isotropic radiators are vertically polarized. The orientation of the ULA at the base station is such that it is parallel to the ground, centered and perpendicular to one of the long diagonals of the hexagon.

For each channel realization, QuaDRiGa computes a channel impulse response in base band in continuous time

$$\boldsymbol{z}(t) = \sum_{i=1}^{N_{\mathrm{path}}} \delta(t - t_i)\boldsymbol{z}_i, \tag{24}$$

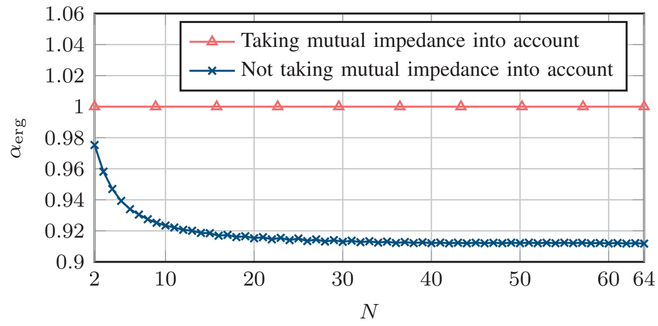


Fig. 8. Ergodic ratio $\alpha_{\mathrm{erg}}$ of true and predicted power for a ULA consisting of isotropic radiators spaced by $\lambda/2$ in a UMa channel.

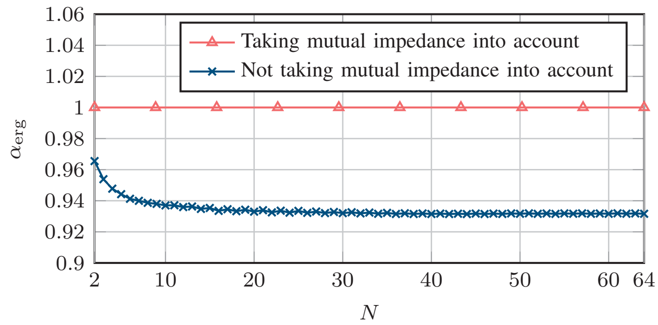


Fig. 9. Ergodic ratio $\alpha_{\mathrm{erg}}$ of true and predicted power for a ULA consisting of $\lambda/2$-dipoles spaced by $\lambda/2$ in a UMa channel.

consisting of $N_{\mathrm{path}}$ paths described by a Delta distribution $\delta$, with a delay $t_i$ and coefficients $\boldsymbol{z}_i$. To use this $\boldsymbol{z}(t)$ in our model, we need to assume a transmit and a receive filter with a certain symbol rate $\Delta f$. We use $\Delta f = 15\,\mathrm{kHz}$ similar to a subcarrier in LTE. Regarding the noise, we assume that we can scale its covariance matrix by $15/740$ to obtain the same noise power per bandwidth as in Section III. An analog root-raised cosine transmit and receive filter with roll-off factor $1$ is used, which does not introduce any temporal noise correlations. We assume that we can approximate the system to be frequency flat at the center frequency $3.5\,\mathrm{GHz}$, as the relative bandwidth is only about $0.000\,43\,\%$. The channel in discrete time then is

$$\boldsymbol{z}[l] = \sum_{i=1}^{N_{\mathrm{path}}} h_{\mathrm{RC}}\left(\frac{l}{\Delta f} - t_i\right)\boldsymbol{z}_i, \tag{25}$$

where $h_{\mathrm{RC}}(t)$ is the impulse response of a raised cosine filter with symbol rate $\Delta f$ and roll-off factor 1. We also assume this channel to be frequency flat, so it is sufficient to consider the channel at the base band frequency $\nu = 0\,\mathrm{Hz}$. To evaluate it there, we compute the discrete-time Fourier transform of $\boldsymbol{z}[l]$, which can be simplified using the Poisson summation formula

$$\sum_{l=-\infty}^{\infty} h_{\mathrm{RC}}\left(\frac{l}{\Delta f} - t_i\right) e^{-j\omega l/\Delta f} = \Delta f \sum_{k=-\infty}^{\infty} e^{-j2\pi(k\Delta f+\nu)t_i} H_{\mathrm{RC}}(k\Delta f + \nu), \tag{26}$$

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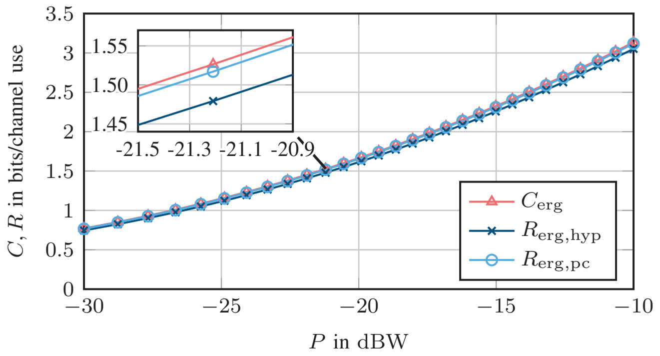


Fig. 10. Ergodic downlink rates of a ULA consisting of 64 isotropic radiators spaced by $\lambda/2$ with and without neglecting the mutual impedance in a UMa channel.

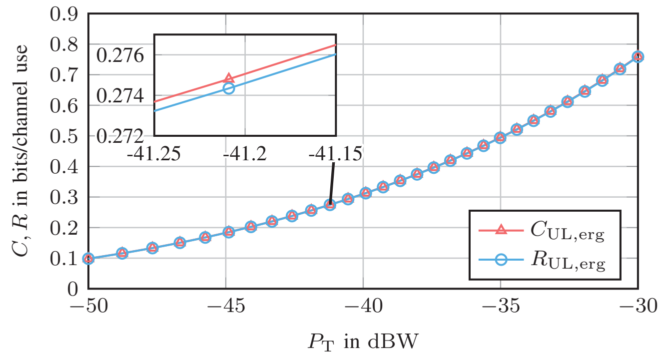


Fig. 12. Ergodic uplink rates of a ULA consisting of 64 isotropic radiators spaced by $\lambda/2$ with and without neglecting the mutual impedance in a UMa channel.

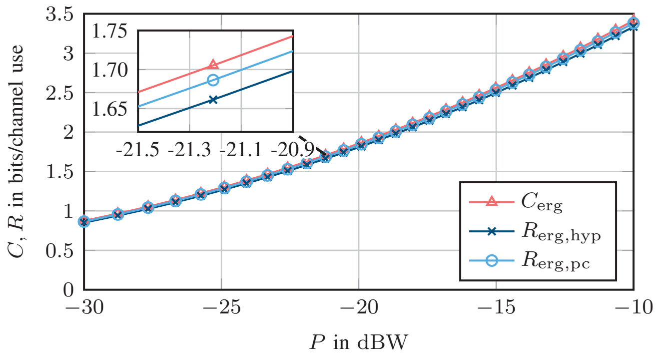


Fig. 11. Ergodic downlink rates of a ULA consisting of 64 $\lambda/2$-dipoles spaced by $\lambda/2$ with and without neglecting the mutual impedance in a UMa channel.

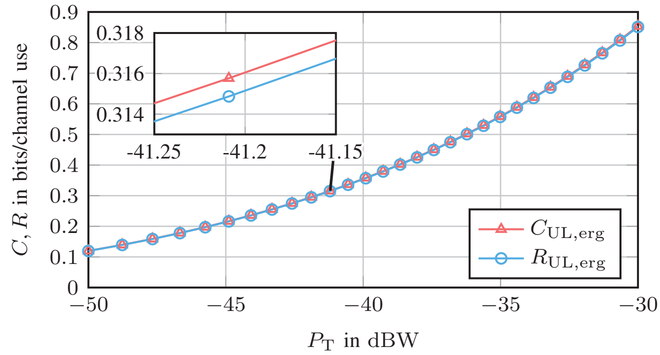


Fig. 13. Ergodic uplink rates of a ULA consisting of 64 $\lambda/2$-dipoles spaced by $\lambda/2$ with and without neglecting the mutual impedance in a UMa channel.

as inspired by [11], where $H_{\text{RC}}(f)$ is the Fourier transform of $h_{\text{RC}}(t)$. As the raised cosine pulse is sufficiently frequency-limited, for $\nu = 0\,\text{Hz}$ only the summand with $k = 0$ is non-zero, and the discrete-time Fourier transform evaluated at this $\nu$ is

$$\boldsymbol{z} = \sum_{i=1}^{N_{\text{path}}} H_{\text{RC}}\left(0\Delta f\right)\boldsymbol{z}_i = \sum_{i=1}^{N_{\text{path}}} \boldsymbol{z}_i. \tag{27}$$

In the following, we will compare the two ULAs we already compared for the i.i.d. channel: one consisting of isotropic radiators and one consisting of $\lambda/2$-dipoles. All antenna elements are oriented vertically.

In Figs. 8 and 9, we can see that compared to Figs. 1 and 2, $\alpha_{\text{erg}}$ saturates at lower values, $91\,\% \approx -0.41\,\text{dB}$ and $93\,\% \approx -0.32\,\text{dB}$. That means for more realistic channels it may be more important not to neglect the mutual impedance than in an i.i.d. channel. Interestingly for the ULA of $\lambda/2$-dipoles, $\alpha_{\text{erg}} < 1$ in the UMa channel model, but $\alpha_{\text{erg}} > 1$ for $N \geq 4$ in the i.i.d. channel model, see Figs. 2 and 9.

Let us assume that the base station radiates $10\,\text{W}$ in a channel bandwidth of $20\,\text{MHz}$. If the power is equally spread among this bandwidth, $7.5\,\text{mW} \approx -21\,\text{dBW}$ are radiated on one subcarrier with $15\,\text{kHz}$ bandwidth. Similarly a transmit power of the mobile of $100\,\text{mW}$ corresponds to $75\,\mu\text{W} \approx -41\,\text{dBW}$.

Compared to the i.i.d. channel, significantly more power is needed to achieve the same ergodic capacity or rate, as $\sigma_z$ for the i.i.d. channel (see (14)) at $3.5\,\text{GHz}$ center frequency corresponds to a line of sight channel in free space between two parallel $\lambda/2$-dipoles spaced roughly $85.7\,\text{m}$. In the UMa model however, there is additional attenuation since mobiles can be indoor or have non-line of sight reception. Also, the mobiles can be further away from the base station, between $35\,\text{m}$ and about $289\,\text{m}$. This leads to a large variation in the rates, see Fig. 14 for the cumulative distribution function of the downlink capacity and rates for $N = 64$. Please note that this distribution is only for one narrowband subcarrier and as the channel is frequency selective due to multi-path propagation, for one channel realization, other subcarriers may support a significantly higher or smaller rate, so a good resource allocation is needed.

When we compare Figs. 10 and 11, and 12 and 13, we can see that the ergodic rates and capacities are higher with the $\lambda/2$-dipoles at the base station, which means that the directivity of the $\lambda/2$-dipoles is beneficial in the UMa channel model.

In the downlink the gap between $C_{\text{erg}}$ and $R_{\text{erg,hyp}}$ at high SNR is significantly larger than in the i.i.d. channel model – $0.16\,\text{bpcu}$ for the isotropic radiators and $0.17\,\text{bpcu}$ for the $\lambda/2$-dipoles compared to $0.081\,\text{bpcu}$ and $0.0015\,\text{bpcu}$ respectively.

 

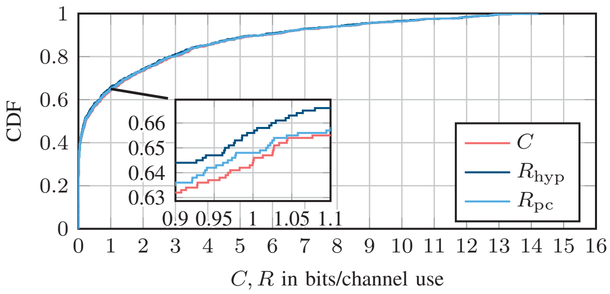


Fig. 14. Cumulative distribution function (CDF) of the downlink rates of a ULA consisting of 64 $\lambda/2$-dipoles spaced by $\lambda/2$ with and without neglecting the mutual impedance in a UMa channel for $P = 7.5\,\mathrm{mW}$.

In power, at $P = 7.5\,\mathrm{mW}$, this translates to a loss of $0.43\,\mathrm{dB}$ and $0.37\,\mathrm{dB}$, which is not negligible.

In the uplink, the gap between $C_{\mathrm{UL,erg}}$ and $R_{\mathrm{UL,erg}}$ at high SNR is larger as well, $0.0095\,\mathrm{bpcu}$ for the isotropic radiators and $0.014\,\mathrm{bpcu}$ for the $\lambda/2$-dipoles compared to $0.0058\,\mathrm{bpcu}$ and $0.010\,\mathrm{bpcu}$ respectively. At $P_{\mathrm{T}} = 75\,\mu\mathrm{W}$, this translates to a loss of $0.017\,\mathrm{dB}$ and $0.030\,\mathrm{dB}$ respectively, which might be negligible in practice.

## V. Conclusions

We have extended the evaluation of the impact of neglecting the mutual impedance on the radiated power and on the ergodic rates from antenna arrays with zero mutual resistance to antenna arrays with small mutual resistance. In particular, we have considered a ULA consisting of $\lambda/2$-dipoles. The effect of neglecting its mutual impedance on the ergodic rates is a bit larger, but still on the same order of magnitude compared to the one for a ULA consisting of hypothetical isotropic radiators.

Furthermore, simulation results for a more realistic channel model without mobility, based on the TS 38.901 UMa channel model in QuaDRiGA, show that the effect of neglecting the mutual impedance is larger there, than in an i.i.d. channel, and that the losses in the downlink are non-negligible. Also the losses with the $\lambda/2$-dipoles are larger than with hypothetical isotropic radiators.

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## References

[1] T. Laas, J. A. Nossek, S. Bazzi, and W. Xu, "On the impact of the mutual reactance on the radiated power and on the achievable rates," *IEEE Trans. Circuits Syst. II: Exp. Briefs*, doi: 10.1109/TCSII.2018.2796643.

[2] W. K. Kahn and H. Kurss, "Minimum-scattering antennas," *IEEE Trans. Antennas Propag.*, vol. 13, no. 5, pp. 671–675, Sep. 1965.

[3] M. T. Ivrlač and J. A. Nossek, "Toward a circuit theory of communication," *IEEE Trans. Circuits Syst.*, vol. 57, no. 7, pp. 1663–1683, Jul. 2010.

[4] ——, "The multiport communication theory," *IEEE Circuits Syst. Mag.*, vol. 14, no. 3, pp. 27–44, Aug. 2014.

[5] T. Laas, J. A. Nossek, S. Bazzi, and W. Xu, "On reciprocity of physically consistent TDD systems with coupled antennas," in *Proc. 21st Int. ITG Workshop Smart Antennas (WSA)*, Berlin, Germany, Mar. 2017, pp. 377–382.

[6] S. A. Schelkunoff and H. T. Friis, *Antennas, Theory and Practice*. Wiley: New York, Chapman & Hall: London, 1952.

[7] B. Lehmeyer, M. T. Ivrlač, and J. A. Nossek, "LNA characterization methodologies," *Int. J. Circuit Theory Appl.*, vol. 45, no. 9, pp. 1185–1202, Sep. 2017.

[8] S. Jaeckel, L. Raschkowski, K. Börner, L. Thiele, F. Burkhardt, and E. Eberlein, "QuaDRiGa – quasi deterministic radio channel generator, user manual and documentation," Fraunhofer Heinrich Hertz Institute, Tech. Rep. Version 2.0.0-664, Aug. 2017.

[9] S. Jaeckel, L. Raschkowski, K. Börner, and L. Thiele, "QuaDRiGa: A 3-D multi-cell channel model with time evolution for enabling virtual field trials," *IEEE Trans. Antennas Propag.*, vol. 62, no. 6, pp. 3242–3256, Jun. 2014.

[10] "Study on channel model for frequencies from 0.5 to 100 GHz," 3GPP, TS 38.901, Jun. 2017, Version 14.1.0.

[11] S. Gault, W. Hachem, and P. Ciblat, "Joint sampling clock offset and channel estimation for OFDM signals: Cramér–Rao bound and algorithms," *IEEE Trans. Signal Process.*, vol. 54, no. 5, pp. 1875–1885, May 2006.

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