

Enhanced Electromagnetic Corrections to the Rare Decay $B_{s,d} \rightarrow \mu^+ \mu^-$

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We investigate electromagnetic corrections to the rare B -meson leptonic decay $B_{s,d} \rightarrow \mu^+ \mu^-$ from scales below the bottom-quark mass m_b . Contrary to QCD effects, which are entirely contained in the B -meson decay constant, we find that virtual photon exchange can probe the B -meson structure, resulting in a “nonlocal annihilation” effect. We find that this effect gives rise to a dynamical enhancement by a power of m_b/Λ_{QCD} and by large logarithms. The impact of this novel effect on the branching ratio of $B_{s,d} \rightarrow \mu^+ \mu^-$ is about 1%, of the order of the previously estimated nonparametric theoretical uncertainty, and four times the size of previous estimates of next-to-leading order QED effects due to residual scale dependence. We update the standard model (SM) prediction to $\bar{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.57 \pm 0.17) \times 10^{-9}$.

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Rare leptonic decays $B_q \rightarrow \ell^+ \ell^-$ of neutral B mesons ($q = d, s$ and $\ell = e, \mu, \tau$) provide important probes of flavor-changing neutral currents, since the decay rate in the standard model (SM) is predicted to be helicity and loop suppressed. Both suppressions can be lifted, for example, in models with extended Higgs sectors, in which case the leptonic decays constrain the scalar masses far above current direct search limits.

Only the muonic decay $B_s \rightarrow \mu^+ \mu^-$ has been observed to date [1,2]. The most recent measurement of the LHCb experiment for the untagged time-integrated branching ratio finds $\bar{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{LHCb}} = (3.0^{+0.7}_{-0.6}) \times 10^{-9}$ [3], compatible with the SM prediction [4]

$$\bar{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9}. \quad (1)$$

With higher experimental statistics and improvement in the knowledge of SM parameters, the accuracy of both results is expected to increase in the future, eventually providing one of the most important precision tests in flavor physics.

The neutral B -meson leptonic decays are, indeed, well suited for precision physics, because long-distance strong-interaction (QCD) effects, which cannot be computed with perturbative methods, are under exceptionally good control. This follows from the purely leptonic final state and the fact that the decay is caused by the effective local interaction

$$Q_{10} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{q} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu \gamma_5 \ell), \quad P_L \equiv \frac{1 - \gamma_5}{2}. \quad (2)$$

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The strong interaction effects are, therefore, confined to the matrix element

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | \bar{B}_q(p) \rangle = i f_{B_q} p^\mu, \quad (3)$$

which defines the B -meson decay constant. f_{B_q} can be computed nonperturbatively with few percent accuracy within the framework of lattice QCD [5].

In this Letter, we report on an investigation of electromagnetic (QED) quantum corrections to the leptonic decay which even at the one-loop order reveals a surprisingly complex pattern. As a consequence, the suppression of the correction due to the small electromagnetic coupling is partially compensated by a powerlike enhancement in the ratio of the B -meson mass $m_B \approx 5$ GeV and the strong interaction scale $\Lambda_{\text{QCD}} \approx 200$ MeV. While logarithmic enhancements due to collinear and soft radiation are well-known in QED and also appear in the process under consideration, the powerlike enhancement arises due to a dynamical mechanism that, to our knowledge, has not been observed before. A virtual photon exchanged between the final-state leptons and the light spectator antiquark \bar{q} in the \bar{B}_q meson effectively acts as a weak probe of the QCD structure of the B meson. The scattering “smears out” the spectator- b -quark annihilation over the distance $1/\sqrt{m_B \Lambda_{\text{QCD}}}$ inside the B meson, as opposed to the local annihilation through the axial-vector current in Eq. (3). This provides power-enhancement and also shows that, at first order in electromagnetic interactions, the strong interaction effects can no longer be parametrized by f_{B_q} alone. Our calculation below shows that the effect is of the same order as the nonparametric theoretical uncertainty previously assumed to obtain Eq. (1).

Before discussing the main result, we briefly review the computations and theoretical uncertainties entering Eq. (1),

referring to Ref. [4] for further details. The general framework employs the effective weak interaction Lagrangian, which generalizes the Fermi theory to the full SM, includes all short-distance quantum effects systematically by matching, and sums large logarithms between the scale m_W of the W -boson mass and $\mu_b \sim m_b$ of the order of the bottom-quark mass, m_b . The SM prediction (1) includes next-to-leading order (NLO) electroweak [6] and next-to-next-to-leading order QCD [7] corrections and the resummation of large logarithms $\ln(\mu_W/\mu_b)$ due to QCD and QED radiative corrections by means of the renormalization-group evolution [8,9] down to μ_b at the same accuracy. Relevant to this work is the observation that unlike QCD effects, which are contained in f_{B_q} to any order, QED corrections below the bottom mass scale μ_b have not been fully considered even at NLO.

The largest uncertainties in the SM prediction are of parametric origin: 4% from the B_s meson decay constant f_{B_s} , 4.3% from the quark-mixing element V_{cb} [10], and 1.6% from the top-quark mass. These uncertainties will reduce as lattice QCD calculations and measurements of SM parameters improve. Nonparametric uncertainties are due to the omission of higher-order corrections α_s^3 , α_{em}^2 , $\alpha_s\alpha_{\text{em}}$ in the QCD and QED couplings α_s and α_{em} , respectively, and also m_b^2/m_W^2 from higher-dimension operators in the weak effective Lagrangian. Altogether, the nonparametric uncertainties have been estimated to be about 1.5% [4]. Among these, the renormalization scale dependence of $\bar{B}(B_q \rightarrow \ell^+\ell^-)$ due to higher-order QED corrections accounts for only 0.3%. In view of such extraordinary precision, it is necessary to exclude the existence of unaccounted theoretical effects at the level of 1%.

Although NLO electromagnetic effects above the b -quark mass scale μ_b are completely included in Eq. (1), this is not the case for photons with energy or virtuality below this scale. Since the decay involves electrically charged particles in the final state, only a suitably defined decay rate $\Gamma(B_q \rightarrow \ell^+\ell^-) + \Gamma(B_q \rightarrow \ell^+\ell^- + n\gamma)_{\text{cut}}$ including photon radiation and virtual photon corrections is infrared finite and well-defined. Energetic photons are usually vetoed in the experiment and accordingly neglected on the theory side. Soft-photon emission from the final-state leptons is accounted for by experiments [1–3]. Initial-state soft radiation has been estimated to be very small based on heavy-hadron chiral perturbation theory [11]. The quoted measured branching fraction is corrected for soft emission and actually refers to the nonradiative branching ratio [12],

as does Eq. (1). For the purpose of the SM prediction [4], it was assumed that other NLO QED corrections below μ_b can not exceed the natural size of $\alpha_{\text{em}}/\pi \sim 0.3\%$. However, as we discuss now, the true size of so far neglected QED effects is substantially larger and, in fact, of the same order as the nonparametric theoretical uncertainty of 1.5%.

The primary challenge of NLO QED computations below μ_b consists in the reliable computation of nonlocal matrix elements. For example, a virtual photon connecting the spectator quark with one of the final-state leptons involves the QCD matrix element

$$\langle 0 | \int d^4x T \{ j_{\text{QED}}(x), \mathcal{L}_{\Delta B=1}(0) \} | \bar{B}_q \rangle, \quad (4)$$

where $j_{\text{QED}} = Q_q \bar{q} \gamma^\mu q$ is the electromagnetic quark current and $\mathcal{L}_{\Delta B=1}$ denotes the (QCD part of the) weak effective Lagrangian for $\Delta B = 1$ transitions. This matrix element bears close resemblance to the hadronic tensor that contains the strong-interaction physics of $B^+ \rightarrow \ell^+ \nu_\ell \gamma$ decay, which is known to be highly nontrivial (for example, Ref. [13]) despite its apparently purely nonhadronic final state.

The two-body kinematics and QCD dynamics of $B_q \rightarrow \ell^+\ell^-$ also determine the scales relevant for the QED corrections. The lepton energies scale as $m_b \gg m_\ell$, Λ_{QCD} , and the QCD bound-state dynamics implies that the momentum of the light quark scales like Λ_{QCD} . In the following, we focus on the muonic final state $\mu^+\mu^-$, which allows us to count the muon mass m_μ and spectator quark mass m_q as $m_\mu \sim m_q \sim \Lambda_{\text{QCD}} \ll m_b$. We have analyzed the complete NLO electromagnetic corrections below the bottom mass scale μ_b and employ the hierarchy of the above scales allowing us to organize the calculation in an expansion in Λ_{QCD}/m_b . Then, we find that the electromagnetic correction to the decay amplitude is enhanced by one power of m_b/Λ_{QCD} compared to the pure-QCD amplitude. In the following, we discuss only this formally dominant power-enhanced contribution, leaving the analysis of the complete QED correction to a separate publication. Note that the standard collinear and soft electromagnetic logarithms belong to these further, non-power-enhanced terms, and are, therefore, not discussed here.

We, then, find that the leading-order $\bar{B}_q \rightarrow \ell^+\ell^-$ decay amplitude plus the electromagnetic correction can be represented as

$$\begin{aligned} i\mathcal{A} &= m_\ell f_{B_q} \mathcal{N} C_{10} \bar{\ell} \gamma_5 \ell \\ &+ \frac{\alpha_{\text{em}}}{4\pi} Q_\ell Q_q m_\ell m_B f_{B_q} \mathcal{N} \bar{\ell} (1 + \gamma_5) \ell \times \left[\int_0^1 du (1-u) C_9^{\text{eff}}(um_b^2) \int_0^\infty \frac{d\omega}{\omega} \phi_{B^+}(\omega) \left(\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right) \right. \\ &\left. - Q_\ell C_7^{\text{eff}} \int_0^\infty \frac{d\omega}{\omega} \phi_{B^+}(\omega) \left(\ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right) \right] + \dots, \end{aligned} \quad (5)$$

where the overall factor

$$\mathcal{N} = V_{tb} V_{tq}^* \frac{4G_F \alpha_{\text{em}}}{\sqrt{2} 4\pi} \quad (6)$$

contains Cabibbo-Kobayashi-Maskawa quark-mixing elements, the Fermi constant G_F , and $Q_\ell = -1$, $Q_q = -1/3$ denote the lepton and quark electric charge, respectively. We use the short-hands $\bar{\ell} = \bar{u}(p_{\ell^-})$, $\ell = v(p_{\ell^+})$ for the external lepton spinors. Omitted terms are power suppressed. The two terms in the electromagnetic correction in the above equation arise from the four-fermion operator $Q_9 = (\alpha_{\text{em}}/4\pi)(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell)$ and the electric dipole operator Q_7 in the effective weak interaction Lagrangian

$$\mathcal{L}_{\Delta B=1} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i Q_i + \text{H.c.}, \quad (7)$$

with the effective operators Q_i as defined in Ref. [14]. The effective short-distance coefficients [15,16]

$$C_7^{\text{eff}} = C_7 - \frac{C_3}{3} - \frac{4C_4}{9} - \frac{20C_5}{3} - \frac{80C_6}{9}, \quad (8)$$

$$C_9^{\text{eff}}(q^2) = C_9 + Y(q^2), \quad (9)$$

account for the quark-loop induced contributions. The relevant Feynman diagrams are shown in Fig. 1.

An important observation on Eq. (5) is that the non-perturbative strong-interaction physics is no longer contained in the B -meson decay constant f_{B_q} alone. Rather, the exchange of an energetic photon between the lepton pair and the spectator antiquark \bar{q} probes correlations between the constituents in the B meson separated at large but lightlike distances. The corresponding strong-interaction physics is parametrized by the inverse moment of the B -meson light-cone distribution amplitude (LCDA) λ_B , introduced in Ref. [17],

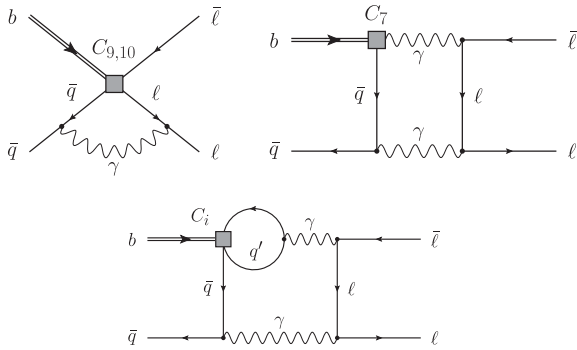


FIG. 1. Feynman diagrams that contain the power-enhanced electromagnetic correction. Symmetric diagrams with order of vertices on the leptonic line interchanged are not displayed.

$$\frac{1}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_{B^+}(\omega, \mu), \quad (10)$$

$$\frac{\sigma_n(\mu)}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\mu_0}{\omega} \phi_{B^+}(\omega, \mu), \quad (11)$$

and the first two inverse-logarithmic moments, which we define as in Ref. [13] with fixed $\mu_0 = 1$ GeV. These parameters have frequently appeared in other exclusive B -meson decays. In the numerical analysis below, we shall adopt [13] $\lambda_B(1 \text{ GeV}) = (275 \pm 75) \text{ MeV}$, $\sigma_1(1 \text{ GeV}) = 1.5 \pm 1$, and $\sigma_2(1 \text{ GeV}) = 3 \pm 2$. The nonlocality of $\bar{q}b$ annihilation due to the photon interaction removes a suppression factor of the local annihilation process. The enhancement of the electromagnetic correction by a factor m_B/Λ_{QCD} in Eq. (5) arises from

$$m_B \int_0^\infty \frac{d\omega}{\omega} \phi_{B^+}(\omega) \ln^k \omega \sim \frac{m_B}{\lambda_B} \times \sigma_k. \quad (12)$$

There is a further single-logarithmic enhancement of order $\ln m_B \Lambda_{\text{QCD}}/m_\mu^2 \sim 5$ for the C_9^{eff} term, and even a double-logarithmic enhancement of the C_7^{eff} term.

We obtained Eq. (5) in two different ways. First, from a standard computation of QED corrections to the four-point amplitude with two external lepton lines, one heavy-quark and one light-quark line, and second, from a method-of-region computation [18] in the framework of soft-collinear effective theory (SCET) [19,20]. The second method is instructive as it reveals the origin of the enhancement from the hard-collinear virtuality $\mathcal{O}(m_B \Lambda_{\text{QCD}})$ of the spectator-quark propagator. A further single-logarithmic enhancement arises from the contribution of both hard-collinear and collinear (virtuality $\Lambda_{\text{QCD}}^2 \sim m_\ell^2$) photon and lepton virtuality. The double logarithm in the C_7^{eff} term is caused by an endpoint singularity as $u \rightarrow 0$ in the hard-collinear and collinear convolution integral for the box diagrams, whereby the hard photon from the electromagnetic dipole operator becomes hard collinear. The singularity is canceled by a soft contribution (virtuality $\Lambda_{\text{QCD}}^2 \sim m_\ell^2$), where the leptons in the final state interact with each other through the exchange of a soft lepton. The relevance of soft-fermion exchange is interesting by itself since it is beyond the standard analysis of logarithmically enhanced terms in QED. Therefore, we shall return to a full analysis within SCET in a detailed separate paper.

Now, we proceed to the numerical evaluation of the power-enhanced QED correction. Let us denote m_B times the bracket in Eq. (5) by Δ_{QED} . Since the scalar $\bar{\ell}\ell$ term in the amplitude \mathcal{A} does not interfere with the pseudoscalar tree-level amplitude, the QED correction can be included in the expression for the tree-level $B_s \rightarrow \ell^+ \ell^-$ branching fraction [21],

$$\frac{\tau_{B_q} m_{B_q}^3 f_{B_q}^2}{8\pi} |\mathcal{N}|^2 \frac{m_\ell^2}{m_{B_q}^2} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}} |C_{10}|^2, \quad (13)$$

by the substitution

$$C_{10} \rightarrow C_{10} + \frac{\alpha_{\text{em}}}{4\pi} Q_\ell Q_q \Delta_{\text{QED}}. \quad (14)$$

We calculate the Wilson coefficients $C_i(\mu_b)$ entering Δ_{QED} at the scale $\mu_b = 5$ GeV at next-to-next-to-leading logarithmic accuracy in the renormalization-group evolution from the electroweak scale, evaluate the convolution integrals in Eq. (5) with $m_b = 4.8$ GeV, and express them in terms of $\lambda_B(1 \text{ GeV})$, $\sigma_1(1 \text{ GeV})$, $\sigma_2(1 \text{ GeV})$ specified above. We then find

$$\Delta_{\text{QED}} = (33 - 119) + i(9 - 23), \quad (\ell = \mu), \quad (15)$$

where the large range is entirely due to the independent variation of the poorly known parameters of the B -meson LCDA. In this result, the total effect is reduced by a factor of 3 by a cancellation between the $C_9^{\text{eff}}(q^2)$ and C_7^{eff} terms. With $C_{10} = -4.198$, this results in a (0.3 – 1.1)% reduction of the muonic $B_s \rightarrow \ell^+ \ell^-$ branching fraction. We update the SM prediction to

$$\bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.57 \pm 0.17) \times 10^{-9}, \quad (16)$$

which supersedes the one from Eq. (1). To obtain this result, we proceeded as in Ref. [4] and used the same numerical input except for updated values of the strong coupling $\alpha_s^{(5)}(m_Z) = 0.1181(11)$ and $1/\Gamma_H^s = 1.609(10)$ ps [23], $f_{B_s} = 228.4(3.7)$ MeV ($N_f = 2 + 1$) [5], $|V_{tb}^* V_{ts}/V_{cb}| = 0.982(1)$ [24] and the inclusive determination of $|V_{cb}| = 0.04200(64)$ [25]. The parametric (± 0.167) and nonparametric non-QED (± 0.043) uncertainty and the uncertainty from the QED correction ($^{+0.022}_{-0.030}$) have been added in quadrature [26]. Quite surprisingly, the QED uncertainty (which itself is almost exclusively parametric, from the B -meson LCDA) is now almost as large as the nonparametric non-QED uncertainty.

The generation of a scalar $\bar{\ell}\ell$ amplitude in Eq. (5) leads to further interesting effects. The time-dependent rate asymmetry for B_s decay into a muon pair $\mu_\lambda^+ \mu_{\bar{\lambda}}^-$ in the $\lambda = L, R$ helicity configuration is given by

$$\frac{\Gamma(B_s(t) \rightarrow \mu_\lambda^+ \mu_{\bar{\lambda}}^-) - \Gamma(\bar{B}_s(t) \rightarrow \mu_\lambda^+ \mu_{\bar{\lambda}}^-)}{\Gamma(B_s(t) \rightarrow \mu_\lambda^+ \mu_{\bar{\lambda}}^-) + \Gamma(\bar{B}_s(t) \rightarrow \mu_\lambda^+ \mu_{\bar{\lambda}}^-)} = \frac{C_\lambda \cos(\Delta M_{B_s} t) + S_\lambda \sin(\Delta M_{B_s} t)}{\cosh(y_s t / \tau_{B_s}) + A_{\Delta\Gamma}^2 \sinh(y_s t / \tau_{B_s})}, \quad (17)$$

where all quantities are defined in Ref. [22]. For example, the mass-eigenstate rate asymmetry $A_{\Delta\Gamma}^2$ equals exactly +1, if only a pseudoscalar amplitude exists, and is, therefore,

assumed to be very sensitive to new flavor-changing interactions, with essentially no uncertainty from SM background. Now, we see that the SM itself generates a small ‘‘contamination’’ of the observable, given by

$$A_{\Delta\Gamma}^2 = 1 - r^2 |\Delta_{\text{QED}}|^2 \approx 1 - 1.0 \times 10^{-5}, \quad (18)$$

$$C_\lambda = -\eta_\lambda 2r \text{Re}(\Delta_{\text{QED}}) \approx \eta_\lambda 0.6\%, \quad (19)$$

$$S_\lambda = 2r \text{Im}(\Delta_{\text{QED}}) \approx -0.1\%, \quad (20)$$

where $r \equiv (\alpha_{\text{em}}/4\pi)[(Q_\ell Q_q)/C_{10}]$ and $\eta_{L/R} = \pm 1$. Present measurements [3] set only very weak constraints on the deviations of $A_{\Delta\Gamma}^2$ from unity, and C_λ , S_λ have not yet been measured, but the uncertainty in the B -meson LCDA is, in principle, a limiting factor for the precision with which new physics can be constrained from these observables.

The power-enhanced QED correction reported here may also appear relevant to the leptonic charged B -meson decay $B^+ \rightarrow \ell^+ \nu_\ell$, but cancels due to the $V - A$ nature of the charged current. While we discussed only the case $\ell = \mu$ above, the other leptonic final states $\ell = e, \tau$ are also of interest. However, whereas the muon mass is numerically of the order of the strong interaction scale, the much larger mass of the tau lepton, and the much smaller electron mass imply that the results are not exactly the same. Therefore, we conclude that the systematic study of hitherto neglected electromagnetic corrections to exclusive B decays reveals an unexpectedly complex structure. Its further phenomenological and theoretical implications are currently under investigation.

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