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Analytics for Shared Mobility and Transportation Systems

Advances in Methods and Insights

Szymon Janusz Albiński
Master of Science, Master of Science in Engineering

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Vorsitzender: Prof. Dr. Maximilian Schiffer

Prüfer der Dissertation:
1. Prof. Dr. Stefan Minner
2. Prof. Teodor Gabriel Crainic, Ph.D., FRSC, Université du Québec à Montréal, Montréal, Québec, Kanada

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To my parents

Abstract

Shared mobility and transportation systems hold potential for a sustainable change in the way people and goods are moved. This thesis uses analytics to address open questions in these innovative systems, namely bike-sharing, car-sharing and truck platooning. The goal is to advance methods and to provide new insights for decision makers.

For a hybrid bike-sharing system, a customer-based performance analysis is developed. Censored demand observations in the data are corrected with a data-driven imputation method. An analysis with real booking data from Munich reveals that there is high excess demand in the free-floating areas. Based on the results, recommendations for reducing unsatisfied customer requests are given.

When rebalancing vehicles in one-way car-sharing systems, providers should consider that the presence of a competitor might influence the demand due to customer substitution. This problem is formulated as a single-period, non-cooperative game with two players. It is proven that this game has a unique Nash-equilibrium, which can be derived from the Lagrangian multipliers. A computational study shows that customer behavior and the fleet's initial distribution mainly influence the players' reactions. A case study with real car-sharing data from Munich demonstrates that ignoring the presence of a competitor comes at high cost. Moreover, repositioning increases under competition.

Truck platooning is an innovative, shared transportation concept where several trucks drive in close succession to save fuel. To get a better understanding of a central coordination of platoons, a planning process is designed. Next, the day-before platooning planning problem is formulated as a mixed-integer linear program on a time-space expanded network. This formulation is extended in such way that driving time regulations can be considered in the planning. A pre-processing procedure, which allows to reduce the input size of the model, is developed. A computational study demonstrates the efficiency of the method. Moreover, it shows that a central coordination of platoons can reduce costs, even under driving time regulations.

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List of Abbreviations

ACEA	European Automobile Manufacturers' Association.
CO_2	Carbon dioxide.
CRG	Competitive Rebalancing Game.
CRG-ab	Competitive Rebalancing Game under availability-based substitution.
CRG-st	Competitive Rebalancing Game under stock-out-based substitution.
CRP	Competitive Rebalancing Problem.
CRP-ab	Competitive Rebalancing Problem under availability-based substitution.
CRP-st	Competitive Rebalancing Problem under stock-out-based substitution.
EU-RTP	Restricted Truck Platooning Problem under European driving time regulations.
GHG	Greenhouse gases.
KKT	Karush-Kuhn-Tucker.
MaaS	Mobility as a Service.
MRP	Monopolistic Rebalancing Problem.
MRP-C	Monopolistic Rebalancing Problem with fixed competitor inventory.
MVG	Münchner Verkehrsgesellschaft.
PER	Platoon Exploitation Rate.
RTP	Restricted Truck Platooning Problem.
tkm	Tonne-kilometers.

Chapter 1

Analytics for Shared Mobility and Transportation Systems

Transportation directly and indirectly influences our everyday life, with all its advantages and disadvantages: On the one hand, it enables connections between places, on the other hand, it consumes resources like space and energy and causes emissions. This conflict has been intensified by current trends like the ongoing urbanization and globalization. Urbanization leads to growing cities, both in size and population density, and thus, more people and goods have to be moved on lesser space. Globalization causes an increase in traffic and greater distances, as transportation becomes easier and more affordable due to reduced regulations. Consequently, new forms of transportation have been developed in response to the changing needs, often supported by advances in technology.

In the last decade, the evolution of location-based services, mobile communication and digitization have facilitated the peer-to-peer sharing of goods and services, the so-called *sharing economy*. This new way of "collaborative consumption" (Hamari et al., 2016) has fostered the development of *shared mobility* and *shared transportation* concepts. First experiences have shown that these concepts hold potential for a sustainable change in the way people and goods are being moved. Due to this potentially high impact, the concepts should be implemented in such way that all players involved enjoy maximal benefit. This goal can be achieved through the use of *analytics* that provides a quantitative basis for finding the optimal solution to decision problems (Hillier and Lieberman, 2014, p. 3). Although there has been substantial progress in this field, there are still knowledge gaps how to optimally plan and manage new shared mobility and transportation systems. This thesis addresses several of the research gaps by applying analytics to selected processes in shared mobility and transportation systems. The goal is to advance methods and to generate new insights in order to use the transportation resource as efficiently as possible.

1.1 Shared mobility and transportation systems

Following the sharing economy's paradigm of "using instead of owning", *shared mobility* has the potential to make the possession of privately owned vehicles redundant. Cohen and Shaheen (2018) define shared mobility as a "[...] transportation strategy that enables users to have short-term access to a mode of transportation on an as-needed basis." Therefore, less space is needed for traffic and parking. Especially in cities, where space is scarce, this can help towards repurposing areas that were previously reserved for private vehicles. The mode of transportation can be *vehicle sharing* (e.g. cars, bikes or scooters), *on-demand ride-services* or *ride-sharing*. While in car-sharing, customers have to drive by themselves, on-demand ride-services transport one ore more persons between origins and destinations selected by the passengers. *Demand-adaptive-transit systems* (DAS) are a particular subclass of on-demand services. In these systems, buses are driving on pre-defined routes with fixed stops. However, customers can request transportation between optional stops, which might also cause detours for the bus (Crainic et al., 2012). Ride-sharing describes the idea of matching drivers and passengers with similar origins and destinations such that they share the car journey. This *car-pooling* helps to increase the utilization of cars.

In literature, vehicle sharing is distinguished into *station-based* and *free-floating* systems (Cohen and Shaheen, 2018). In free-floating services, the customers can pick-up and return the vehicles within the whole operating area. Station-based services can be either *roundtrip* based, i.e. customers have to return the vehicles to the collecting point, or *one-way* based, i.e. customers can select another station for the return of their vehicle. The meaning of the term "station" depends on the mode of transportation: In car-sharing, it usually refers to a facility where the cars can be parked. However, some car-sharing providers define city districts as stations. In bike-sharing systems, a station is typically a facility with docks where the bikes are placed into. In recent years, some bike-sharing providers begun to combine station-based and free-floating services into so-called *hybrid services*. That is, providers partition their operating area into free-floating and station-based areas, or place bike stations within free-floating areas. Vehicle sharing systems, like bike-sharing, can help to solve the "last-mile problem" in public transportation. This term describes the short distances between the passenger's origin or destination and the public transport station that are too long for walking (Shaheen et al., 2010). Addressing the last-mile problem increases the attractiveness of public transport systems, which in turn leads to a reduction of traffic burden in cities.

Shared transportation describes the idea of a short-term co-operation of shippers or carriers. The aim is to increase the efficiency of moving goods through a joint usage of existing transportation resources. Two prominent examples can be found in the field of road transportation: *freight platforms* and *truck platooning*. Similar to ride-sharing, freight platforms match shippers and freight carriers in order to improve the utilization of truck capacities by sharing the trucks' journeys. In truck platooning, vehicles drive in close succession in order to reduce their air drag and thus their fuel consumption. That is, the trucks have shared itineraries.

1.2 Challenges in shared mobility and transportation

McKinsey & Company (2019) estimated that shared mobility services will generate up to 2.0 trillion US dollars in revenues in the United States by 2030. In 2017, the generated revenues amounted to 70 million US dollars. In the trucking industry, shared transportation will lead to profound changes. In case of Europe, for instance, the consulting company Strategy& (2018) estimates that the trucks' times spend on the road will increase from 29% in 2018 to 78% by 2030, thanks to new forms of co-operation between the market players and higher automation. Another study estimates the volume of the platooning market at 2.72 billion US dollars in 2030, with Europe being the fastest growing region (Research and Markets, 2018).

Shared mobility and transportation give rise to new questions and challenges that affect all three players involved: policy makers, providers and customers. Policy makers have to define legal frameworks for these systems. Simultaneously, they can incentivize the development and usage of those mobility and transportation concepts that increase public welfare. Within the given legal bounds, providers can develop new business models that exploit the potential of these emerging markets. This requires strategic, tactical and operational decision making, which is complex and will greatly influence business success. Customers can make use of innovative transportation modes, which in turn will affect their travel and shipping behavior. Naturally, there is an interaction between all stakeholders: For instance, if a provider develops a new business model, policy makers might need to adapt regulations; or unexpected customer behavior might force the provider to change its business strategy. Furthermore, the three parties involved pursue different goals, which in turn will lead to conflicts of interest. While policy makers aim at maximizing public welfare, providers strive towards maximizing their profits, whereas customers seek the most efficient way of traveling or shipping.

These challenges can be summarized as follows: *How can shared mobility and transportation systems be planned and managed in such a way that the benefit is maximal for all players?*

1.3 The role of analytics and contribution

To answer this question, quantitative decision support is required. This can be provided by *analytics*, which "[...] is the scientific process of transforming data into insights for making better decisions" (Hillier and Lieberman, 2014, p. 4). In this thesis, analytics are applied to the following three shared mobility and transportation systems: bike-sharing, car-sharing and truck platooning.

In vehicle sharing systems, the demand underlies spatial and temporal variations. Therefore, providers need to reposition the vehicles to keep these systems economically viable. The first step in this process is assessing the future demand, which, in a second step, is then used to plan the repositioning of vehicles. When estimating demand, one has to keep in mind that booking data only represents the observed demand, whereas the real demand might be higher as lost customer requests are not recorded in this data. Ignoring these so-called censored demand observations can lead to an underestimation of the actual need of vehicles and thus result in a decreasing customer satisfaction.

Another important question is how to measure customer satisfaction in such systems. In literature, the customer satisfaction of bike-sharing systems has been measured based on the total number of trips. This approach does not allow a quantification of neither, the potential of demand nor the lost bookings due to missing bikes. Therefore, important aspects of customer satisfaction are neglected. This effect is enhanced when censored demand observations are used. Especially in hybrid-bike sharing systems, where providers need to find the right balance between the size of the free-floating area and the number and positions of their stations, a correct assessment of customer satisfaction might be a key factor when it comes to making the right decisions.

When rebalancing vehicle sharing systems, providers should keep in mind that the presence of a competitor influences the expected demand as customers might be willing to substitute. From a public point of view, especially repositioning of car-sharing vehicles means a lot of empty rides that cause additional traffic and pollution in cities. If companies react to the presence of a competitor, the number of empty rides might even increase. Thus, city planners should consider an obligatory cooperation between providers who would then have to give access to their cars to their competitors' customers. This cooperation potentially reduces repositioning and thus the traffic load. Consequently, this increases the efficiency and the acceptance of such systems.

In contrast to vehicle sharing, truck platooning represents a new technology for which, so far, no business model exists. It is still an open question how truck platoons will be organized and what the planning process will look like. A possible answer could be the development of a platform that centrally coordinates the routing and formation of individual trucks into platoons; a coordination process that has not been discussed in literature yet. Furthermore, it is unclear how legal requirements such as driving time regulations or a limited platoon size or technical factors such as fuel-savings will affect this planning. So far, these questions could only be answered to a limited extent since all mathematical models that have been proposed in literature were too complex to be solved on instance of large scale.

As trucks typically cover long distances, the formation of platoons will be affected by driving time regulations that apply in many countries. These additional restrictions might hinder the platoon formation or reduce the time span during which trucks can drive in a platoon. On the other hand, mandatory pauses might be scheduled in such way that trucks can be synchronized into platoons. In addition, if the driving times of platoon followers were counted as rest times, trucks could cover distances in shorter time. This would result in an increase in trucking capacity, which in turn would be a remedy to overcrowded truck parking lots and a shortage of drivers.

The previous examples show that there are still open questions how to optimally plan and manage shared mobility and transportation systems. This thesis addresses several of those questions by advancing methods from the following fields of analytics: inventory control, constrained non-linear optimization and mixed-integer linear programming. This work contributes to literature by tackling four methodological challenges:

1. How to set up a framework for the performance analysis of a hybrid bike-sharing system and how to correct censored demand observations?

2. How to model the problem of rebalancing vehicles in one-way car-sharing under competition and how to solve it efficiently?

3. How to model the day-before truck platooning planning problem and solve it efficiently?

4. How to model and solve the truck platooning problem under driving time regulations?

The methodological advances provide new insights into the decision making by answering the following research questions:

1. How to increase the customer satisfaction in hybrid bike-sharing systems?
2. How does competition influence the optimal rebalancing plan in one-way car-sharing and how should a provider react to the presence of a competitor?
3. How can the planning process of a platform that centrally coordinates truck platoons be designed and what is the benefit of such a platform?
4. How do European driving time regulations affect the coordination of truck platoons?

The following paragraph explains how we address these research questions and summarizes our contributions:

Chapter 2 reviews the literature on inventory control, which is used as a methodological foundation for the work on bike-sharing and car-sharing. Furthermore, related work on bike-sharing, one-way car-sharing, truck platooning and the scheduling and routing of trucks under driving time regulations is summarized.

In Chapter 3, we conduct a performance analysis of a hybrid bike-sharing system. We use descriptive analytics to identify spatial and temporal booking patterns within a real data set that contains bike-sharing trips which occurred in the given time period. To correct censored demand observations, we introduce a data-driven imputation method. To measure the performance from a customer-oriented point of view, we determine the non stock-out probabilities and fill rates in this system. This chapter has been made available in Albiński et al. (2018), which is co-authored by Pirmin Fontaine and Stefan Minner.

In Chapter 4, we consider the problem of rebalancing vehicles in a one-way car-sharing system under competition. We combine inventory transshipment with inventory competition to formulate a Nash game between car-sharing providers with substitutable cars. Both providers operate in the same business area, which is divided into multiple zones. We use Lagrangian multipliers to transform the resulting convex optimization problem into an unconstrained problem. From the Karush-Kuhn-Tucker conditions, we can derive that these multipliers indicate if a zone has an overage or an underage of cars. This finding allows us to develop an efficient solution method. To reflect different customer behaviors, we consider both stock-out-based and availability-based substitution. We prove that the resulting Nash equilibrium is unique in both cases. We evaluate a case study with self-collected car-sharing booking data. Furthermore, we examine the case of full pooling. That is, the companies grant their competitor's customers access to their cars. This chapter has been made available in Albiński and Minner (2019).

In Chapter 5, we structure the process of a platform that orchestrates single trips of trucks into platoons. We divide this planning process into four steps and discuss the underlying decision problems. We identify the day-before problem as an important step of the planning process. We model this problem as a mixed-integer linear program on a time-space expanded network. We use an arc-based formulation that allows to minimize the problem size so that instances of a reasonable size can be solved on standard commercial solvers. To this end, we develop a pre-processing procedure that exploits the trucks' time limits. Furthermore, we provide a starting solution and fix non-basic variables. From a computational study, we provide insights into the savings potential of platooning (both cost and emission of CO_2). Furthermore, we discuss the value of centralized planning of truck platoons. This chapter has been made available in Albiński et al. (2019a), which is co-authored by Teodor Gabriel Crainic (Université du Québec à Montréal) and Stefan Minner (Technical University of Munich).

Truck platooning is mainly beneficial on long-haul trips, where legal regulations on driving times, break times and daily rest periods play an important role. So far, there is no literature that studies the impact of driving time regulations on the planning of truck platoons. We tackle this problem in Chapter 6 by extending the mixed-integer linear programming formulation introduced in Chapter 5. Thereby, we include the option that the working times of drivers in trailing trucks are partially counted as rest times. This option allows to skip breaks or daily rest periods and thus influences the scheduling of platoons. Although this research project is work-in-progress, it is sufficiently advanced to show that the developed methodology is capable to incorporate the issues induced by driving time regulations. This chapter has been made available in Albiński et al. (2019b), which is co-authored by Stefan Minner (Technical University of Munich) and Teodor Gabriel Crainic (Université du Québec à Montréal).

Chapter 7 presents a summary of the methodological contributions and main insights. Moreover, limitations of the work and ideas for further research are stated and a final conclusion is drawn.

Chapter 2

Literature Review

This chapter presents literature from three relevant fields: *(i)* Inventory control in retail with a focus on censored demand observations, inventory competition and inventory transshipment (Section 2.1). These are the main methodological concepts that we use for our two studies in shared mobility. *(ii)* Rebalancing in one-way vehicle sharing systems, namely bike-sharing and car-sharing (Section 2.2). *(iii)* Routing and scheduling problems in trucking, with focus on platooning and driving time regulations (Section 2.3).

2.1 Inventory control in retail

In inventory control, the prevention of stock-outs is one of the main objectives. In retail, the customers' reaction to a stock-out in a store can be one out of the following:

1. Wait until the product is replenished. That is, the excess demand is backordered.
2. Substitute the product with another one that is at stock.
3. Leave the store and buy the product somewhere else or not at all.

Corsten and Gruen (2005) conducted a study that shows that 15% of the customers will backorder, 45% will substitute the missing product with another one, 31% will visit another store and the remaining 9% will not buy any product at all.

In cases two and three, the retailer faces a lost sale of the missing item. This leads to a censored demand observation, since the real demand corresponds to the observed sales plus lost sales. If the retailer relies on the censored observations for his estimations, he risks an underestimation of the future demand, which results in lower inventory stocks and thus even more lost sales (Nahmias, 1994). Therefore, correcting censored demand observations is crucial when determining optimal order quantities. We review the literature on non-parametric estimation approaches in Section 2.1.1. Section 2.1.2 summarizes the literature on inventory control under customer substitution. If retailers have several stores, they can transship items between the stores to prevent stock-outs. This is also called inventory rebalancing and has been thoroughly studied (see Section 2.1.3).

2.1.1 Non-parametric censored demand estimations

The literature on estimating lost sales can be partitioned into parametric and non-parametric approaches. Non-parametric approaches do not rely on assumptions about the underlying demand distributions. Lau and Lau (1996) introduce a non-parametric approach that estimates lost sales and predicts the true demand in the stock-out periods. This is achieved by extrapolating the sales patterns for censored periods from uncensored periods in the preceding hours. Huh et al. (2011) propose a non-parametric, data-driven algorithm for multi-product inventory systems with censored demand. For periods with censored demand observations, they set the order-up-to levels to those of the previous period. Huber et al. (2019) develop a data-driven solution method for the newsvendor problem. They correct censored demand observations in the input data by interpolating the lost sales from days when the item was not out-of-stock. Sachs and Minner (2014) formulate a data-driven model as linear programming problem. Their distribution-free formulation estimates the parameters of an inventory function directly from the sales data, which are corrected with regards to censored demand observations. Jain et al. (2014) propose to include information on the timing of sales observations to mitigate the problem of censored demands. Shi et al. (2016) derive a non-parametric, data-driven policy for the distribution-free newsvendor model with censored demands. The authors emphasize that parametric maximum likelihood estimators show the risk of underestimating the demand when the initial inventory is not sufficient for covering all the initial demand.

2.1.2 Inventory competition

Parlar (1988) was the first to study the problem of inventory competition. He investigates the optimal policies for two newsvendors under stock-out-based substitution, i.e. a certain portion of a retailer's excess demand flows to the competitor, and shows that there exists a unique Nash equilibrium for this non-cooperative game. Silbermayr (2019) defines this kind of non-cooperative game, where retailers have to consider the competitors' inventory levels when deciding on their inventory, as horizontal interaction. Lippman and McCardle (1997) use splitting rules to allocate the initial industry demand to the retailers. They assume that the single demands are correlated and show that competition can lead to an increased industry inventory as well as to decreasing profits. Caro and Martínez-de-Albéniz (2010) extend this work by studying the effect of quick responses on stock-outs and unequal reordering capabilities of the two competitors. Their findings show that asymmetric competition can be desirable for both players. Netessine and Rudi (2003) generalize Parlar's model and prove the existence of a Nash equilibrium for the case of n newsvendors. Their findings support the results of Lippman and McCardle (1997). Jiang et al. (2011) study the case of inventory competition under asymmetric demand information and show that there exists a unique Nash equilibrium. They show that having better information about the own demand distribution than the competitor does not necessarily give an advantage. Güler et al. (2018) prove that, under asymmetric cost information, there exists a unique Nash equilibrium as well. From their comparative statics, they conclude that the total inventories increase under competition only if the firms are identical.

Ryzin and Mahajan (1999) propose a Multinomial Logit approach for modeling assortment-based substitution. That is, they assume that each customer associates a certain utility with buying the product at a retailer's store and that this utility can be directly influenced by the retailer's assortment and inventory levels. Wang and Gerchak (2001) argue that, in retail, the shelf space assigned to a product influences the demand. They study the cases of single and multiple players by considering two cases of demand allocation. Either the entire demand depends on the aggregated inventory on display and is then split according to the retailers' inventory, or customers are split among the retailers and the retailers' individual inventory levels influence the assigned customers' decisions. Gaur and Park (2007) discuss inventory competition under consumer learning. That is, the underlying consumers' choice model is updated based on the availability of the products. Readers who are interested in further details might want to take a look at the detailed literature reviews by Kök et al. (2015) and Silbermayr (2019).

2.1.3 Inventory transshipment

The problem of inventory rebalancing was initially addressed by Allen (1958). He discusses the redistribution of stock between n zones in a single period with the objective of minimizing the total stock-out plus transportation cost. He shows that the zones can be ordered according to their shortage probability and thus, one can partition the zones into senders and receivers. This observation reduces the computational effort significantly. Additionally, if the transportation cost is equal for all links, one can easily determine the type of each zone (Allen, 1961, 1962).

The problem of rebalancing, also called inventory transshipment, has been discussed under various aspects in the retail context. An extensive review is presented by Paterson et al. (2011). The authors classify the existing literature, among others, according to proactive and reactive transshipment, periodic and dynamic inventory reviews and if transportation times are negligible or not. Furthermore, they point out that in recent years more attention has been given to transshipment between competing retailers. Thereby, it is assumed that the retailers are willing to partially pool their inventories at a certain transshipment cost. Competition can be either on prices or on inventories (i.e. service levels), or on both. Zou et al. (2010) study the effects of transshipments between two competing newsvendors. The authors use substitution rates to take the switching of customers from one provider to another into account. The authors show that the benefits of transshipments depend on the customers' switching rates. Çömez et al. (2012) confirm these results by computing optimal holdback inventories for competitors with substitution rates. Gavirneni (2001) studies the value of co-operation in a supply chain and shows that this value increases with the number of participating retailers.

2.2 Rebalancing one-way vehicle sharing systems

Temporal and spatial demand volatility can lead to shortages of vehicles in one-way vehicle sharing systems and consequently unsatisfied customers. Therefore, service providers have to reposition the vehicles to keep the systems profitable (Jorge et al., 2014). The repositioning can be done operator-based or by incentivizing customers (Angelopoulos et al., 2018; Waserhole and Jost, 2012). The Operations Research community has paid particular attention to the rebalancing problems in bike-sharing and car-sharing systems (Laporte et al., 2018). Even though both systems have many similarities, they differ in the way the vehicles are relocated. While bikes are typically collected by trucks and then transported in batches, cars are usually moved by drivers individually. Section 2.2.1 gives an overview of the existing literature on demand estimation and rebalancing in bike-sharing systems. Section 2.2.2 summarizes the work on rebalancing problems in one-way car-sharing systems.

2.2.1 Demand analysis and relocation problems in bike-sharing

DeMaio (2009) and Fishman et al. (2013) give overviews of the literature on bike-sharing, which can be divided into two main streams: demand analysis including forecasting and the redistribution of bikes.

Demand analysis

Froehlich et al. (2009) present a spatio-temporal analysis for a station-based bike-sharing system in Barcelona. In an analysis of the number of bikes at stations, they identify different demand patterns over the day and behavioral patterns in different areas by applying clustering techniques. The results are used for predicting the number of available bikes at each station. For the same system, Kaltenbrunner et al. (2010) use regression analysis to predict the number of bikes at stations several minutes before the occurrence. The dependency of demand on weather conditions is analyzed for the bike system in Lyon by Borgnat et al. (2011). Moreover, they cluster bike flows between stations to identify spatial patterns. For the bike-sharing system in Vienna, Vogel et al. (2011) identify five different types of stations, depending on the demand over the day. O' Brien et al. (2014) use data-mining techniques to investigate the usage patterns in 38 bike-sharing systems.

Faghih-Imani and Eluru (2016a,b) and Faghih-Imani et al. (2014) give insights into such influencing factors as socio-demographic characteristics, the bicycle infrastructure, and land-use characteristics for arrival and departure rates at bike stations based on data from Barcelona, Montréal and New York City. Li et al. (2019) analyze the impact of introducing a free-floating bike-sharing system on a station-based system. Using data from London, they show that the usage of the station-based bicycles reduces by six percent on average. The data show that mainly trips with a length of up to three kilometers or a duration of up to 15 minutes are preferably undertaken with free-floating bikes.

Redistribution of bikes

Regue and Recker (2014) present a four step approach for repositioning bikes proactively. This includes demand forecasting, solving an inventory model for the stations, determining a redistribution plan and computing routes for the service shuttles. Schuijbroek et al. (2017) address the rebalancing problem in a station-based system by applying a clustering heuristic. The clustering considers service level requirements and routing costs of the redistributing trucks. Ho and Szeto (2014) propose Tabu Search for solving the rebalancing problem in a station-based system. Dell'Amico et al. (2018) study the rebalancing problem of a station-based system with stochastic demands. Datner et al. (2017) set target inventory levels for bike-stations and develop a local search algorithm to achieve these levels through rebalancing. Their study shows that the inventory levels of neighboring stations interact and that ignoring this effect can come at high cost. Çelebi et al. (2018) develop a method that assigns the demand to the stations in such way that service level requirements are met. This can help towards obtaining a balanced bike-sharing systems, as a case study with real data from Istanbul Technical University's campus bike-sharing system shows.

Pfrommer et al. (2014) develop a model that combines operator-based and customer-based repositioning of bikes in London. The prices are computed dynamically and the aim is to incentivize customers towards returning bikes to nearby under-used stations. These relocations are complemented by operator-based repositioning. Waserhole and Jost (2012) use a fluid approximation model for controlling the balance of vehicle sharing systems exclusively through the adaptation of prices. Similarly, Haider et al. (2018) develop a pricing method to incentivize customers towards rebalancing bikes in a station-based system. The authors classify the stations according to the expected demand and define service level bounds for the stations. A computational study with data from a station-based system in Washington DC demonstrates how dynamic pricing can help with reducing operator-based repositionings.

For free-floating bike-sharing systems, Reiss and Bogenberger (2015, 2016a,b) use the data of Munich's "Call a Bike" operated by the German Railway. They analyze temporal patterns and combine operator-based and user-based relocation strategies. Pal and Zhang (2017) introduce a hybrid nested large neighborhood search for a large-scale free-floating bike-sharing system.

Albiński et al. (2018) study the performance of a hybrid bike-sharing system under censored demand observations. The results show that service levels are overestimated when unobservable, censored demand effects are ignored. Furthermore, the service levels show considerable discrepancies between free-standing and station based bikes. For station based systems, Negahban (2019) develops a method that corrects censored demand observations based on simulation results. This simulation is based on a non-parametric bootstrap test and a subset selection procedure that aims at estimating the underlying demand distribution. A controlled computational study and tests with data of the Citi bike-sharing system in New York City confirm that censored demand observations can lead to suboptimal decisions.

2.2.2 Rebalancing problems in one-way car-sharing

Research on the application of Operations Research methods to the field of car-sharing has been steadily increasing in the last few years (see Brandstätter et al. (2016), Ferrero et al. (2015), and Illgen and Höck (2019) for reviews). Using a queuing-based discrete-event simulation, Barth and Todd (1999) investigate the influence of different parameters on the performance of a car-sharing system. According to their study, the fleet size and the relocation strategy have the greatest impact on the attractiveness of the system. Almeida Correia and Antunes (2012) propose a mixed-integer linear programming formulation for selecting the optimal locations in station-based systems in order to reduce repositioning. They evaluate this model using data from Lisbon. The authors also ask if repositioning makes a system economically viable. An answer to this question is provided by Jorge et al. (2014), who emphasize the importance of repositioning to make one-way car-sharing systems profitable. Their assessment is based on the results of a simulation study that evaluates an exact model and on real-time relocation policies on the Lisbon data.

Weikl and Bogenberger (2015) propose a two-step algorithm for positioning cars optimally in a free-floating system. They partition the business area into honeycombs and suggest rule-based intra-zone and inter-zone repositioning. Both, Boyacı et al. (2015) and Bruglieri et al. (2017), use mixed-integer linear programming to solve the relocation problem in one-way car-sharing. Nair and Miller-Hooks (2011) consider the problem under demand uncertainty, using stochastic programming. They aim at generating vehicle distribution plans with the objective of minimizing the repositioning cost by continuously satisfying all near-term demand scenarios.

Lu et al. (2017) propose a two-stage stochastic model to solve the strategic planning problem how to reserve parking lots for car-sharing vehicles under uncertain demand. The goal is to optimize the provider's profit and the service quality. To overcome computational complexity, Lee and Park (2014) suggest the partitioning of the zones into sending and receiving zones, depending on their current inventory status. Based on this partition, one can determine the repositioning flows between the sending zones and the receiving zones. Nourinejad et al. (2015) observe that repositioning vehicles can lead to imbalances in the positioning of the staff. They study the joint optimization of vehicle relocation and staff rebalancing in one-way car-sharing. Boyacı and Zografos (2019) consider this joint optimization problem in the context of shared electric vehicles. They propose a heuristic to solve the problem. In addition, they assess the value of the customers' temporal and spatial flexibility when booking or returning a vehicle. Their results show that both flexibility levels can help to increase the profitability, whereby spatial flexibility has a greater impact. Gambella et al. (2018) address the repositioning of shared electric vehicles by taking the battery levels and recharging processes into account. Folkestad et al. (2020) propose a model for combining the repositioning with the recharging of the electric cars.

To our knowledge, the only study that studies the influence of competition on the repositioning of car-sharing vehicles is by Balac et al. (2019). The authors use a simulation to investigate the case of two competing free-floating car-sharing providers, assuming that customers are willing to use both systems. Among other results, their analysis shows that the competitor might profit if only one provider is doing repositioning. The reason is that repositioning makes cars temporarily unavailable. In addition, the removal of cars from an area increases the dominance of the competitor and thus the number of cars that are rented. When both providers reposition, competition increases the number of cars that are moved.

2.3 Routing and scheduling problems in trucking

Road transportation requires a thorough planning to ensure that the routing and scheduling of trucks is done optimally. The emergence of truck platooning opens new questions how to synchronize the trucks and which routes are the best under the option of driving in such convoys. Section 2.3.1 gives an overview over the growing body of literature in this field. When covering long distances, truck drivers have to take breaks and rests according to the country's laws. The optimal planning of truck routes under driving time regulations has been intensively studied. We summarize the literature in Section 2.3.2.

2.3.1 Truck platooning

Larsson et al. (2015) were the first to introduce a mixed-integer linear programming formulation for the problem of routing trucks under the option of platooning. The authors call it the Unlimited Platooning Problem (UPP) since, in this model, trucks have no latest arrival time and the platoon size is not restricted. As the UPP belongs to the class of $\mathbb{NP}$-hard problems, the authors propose two construction heuristics for the design of platoons: the Best Pair heuristic and the Hub heuristic. The former identifies the combinations of trucks with the highest savings potential. In the latter, the trucks are partitioned into groups based on the edge-similarity of their corresponding shortest paths. Each group is then assigned to hub nodes and platoons can be built within these groups only.

In a follow-up, Larson et al. (2016) use different characteristics of the platooning problem to reduce the problem size. Among other things, they show that there exists a bound on the maximal detour length within which the fuel savings through platooning exceed the additional fuel expenses for every truck. They use this bound to set decision variables, which would lead to infeasible or sub-optimal solutions, to zero. This pre-processing allows a reduction of the number of possible paths for each truck while simultaneously solving the platooning problem. With this approach, they solve instances for up to 25 trucks in a 10×10 grid with unit distance to optimality. Moreover, the authors apply their method to the Chicago highway network with 100 trucks. Out of the 4,553 nodes in this network, the authors select five node pairs, each of them representing the origin and destination for 20 trucks. In most cases, the instances can be solved within a one percent optimality gap in between 100 and 300 seconds.

Van De Hoef et al. (2015) allow for different speed profiles. In their heuristic, platoons are formed based on the shortest path and then a speed level is assigned to them. Following the idea of multiple speed levels, Luo et al. (2018) propose a mixed-integer linear problem that uses different speed profiles in order to improve the formation of platoons by means of waiting times. For bigger instances, the authors propose a cluster-first-route-second decomposition approach. Hoef et al. (2018) introduce the idea of a centralized platoon coordinator that determines routes in such way that the trucks arrive at their destination in time while fuel savings through platooning are also exploited. Due to the high complexity of the problem, the authors suggest a heuristic that schedules the trucks on fixed routes with the platoon size limited to two vehicles.

Zhang et al. (2017) consider uncertain travel times in the planning of truck platoons. They conclude that delays of a single truck can be propagated by the formation of platoons and thus make truck platoons unattractive. Boysen et al. (2018) come to a similar conclusion that, due to tight schedules, delays will outweigh the benefits of the fuel savings. However, the authors point out that if wages could be reduced, e.g. by driver-less platoon followers, platoons might become profitable even with penalties for delays. In their literature review, Bhoopalam et al. (2017) mention the platoon formation under restrictions such as a limited platoon size could be an interesting future research direction. Scherr et al. (2018) show that, besides saving fuel, the technology of truck platooning can also be used to guide autonomously driving trucks through areas that require a truck driver. The authors describe a scenario in city logistics where autonomous trucks serve customer requests in certain districts. However, to get to and from the depot outside the city, the trucks are merged into a platoon where the leading truck is manned. The results of the controlled computational study indicate that this concept can render considerable cost savings.

Larsen et al. (2019) take mandatory breaks into account and study the hypothetical case that the platoon followers' driving time can be counted as rest time and that, the drivers can skip breaks. The authors assume that all trucks travel on fixed routes that pass one single hub. At this hub, a platooning service provider coordinates the formation of platoons, forcing the trucks to wait if necessary. The results of their computational study with data from a European highway network show that rest options make trips profitable. Besides, the trucks' waiting times increase with the distance they drive.

Freight train operators often consolidate cars with the same destination into blocks to reduce the handling effort at intermediate stops. Truck platoons can be seen as "road trains", where trucks correspond to the cars and a platoon can be seen as a block. In their review section, Barnhart et al. (2000) summarize the works of many authors who all have addressed the problem of railway blocking by using network design approaches. Zhu et al. (2014) propose a mixed-integer linear model that integrates the selection and scheduling of services (the trains), the classification and blocking of trains and the routing of the individual cars. To this end, the authors introduce a three-layer time-space network that allows them to track the movements of the trains, blocks and cars, including delays and waiting times. Chardaire et al. (2005) use a time-space expanded network to solve the Convoy Movement Problem. It describes the problem of having to move several convoys in a network under the limitation that certain convoys are not allowed to meet during their movement. The authors propose a Lagrangian relaxation to solve the problem. Their computational study shows that, with this approach, they can achieve small optimality gaps within half an hour for instances with 17 convoys.

2.3.2 Driving time regulations

To the best of our knowledge, there is no work that includes driving time regulations in connection with the routing and scheduling of truck platoons. However, motivated by the introduction of *Regulation (EC) No 561/2006*, Goel (2009) studied the Vehicle Routing Problem with Time Windows (VRPTW) under those EU restrictions that affect the working times between two weekly rest periods. Due to the complexity of this problem, the author uses a Large Neighborhood Search. To insert new candidates into the routes, the author proposes a multilabel method that reduces the number of feasible solutions through dominance criteria. Computational experiments on Solomon's instances show the advantage of the multilabel method over a naive insertion. Furthermore, the study shows that pauses scheduled before the maximum driving time limit can help to meet narrow time windows at subsequent customers. Kok et al. (2010) extend this research direction by including all the regulations named in *Regulation (EC) No 561/2006* plus working time restrictions defined in *Directive 2002/15/EC*. They adapt the restricted dynamic programming heuristic to subsequentially add customers to tours and check the feasibility with all driving and working time regulations. The check is done by help of a break-scheduling-method. Their computational results show that *Directive 2002/15/EC* has a high impact on the VRPTW solutions. Goel (2010) studies an open TSP for a weekly schedule under *Regulation (EC) No 561/2006*, which also allows splitting break and rest times into two parts. The author identifies this splitting as a main challenge. Therefore, he proposes an iterative scheme that combines pseudo-feasible schedule solutions based on truck driver scheduling under driving time regulations. A key insight from the computational study is that the splitting of breaks increases the solution time of the heuristic, whereas split breaks do not lead to significantly better schedules.

Rancourt et al. (2013) study long-haul trips under the United States hours-of-service (HOS) regulations. They formulate this problem as a Vehicle Routing Problem with multiple time-windows and develop a tabu search heuristic as solution method. From the computational study they conclude that the splitting of breaks can lead to better working schedules. Xu et al. (2003) study the impact of HOS on the transportation problem, where a driver has to visit a certain sequence of locations. Due to the multiple time-windows, the problem is $\mathbb{NP}$-hard and therefore the authors propose a column generation approach. Studying a similar problem with single time windows, Archetti and Savelsbergh (2009) develop a polynomial runtime algorithm (cubic in the input size) to create schedules that are feasible with regard to the HOS regulations. Goel and Kok (2012) introduce an algorithm that creates a feasible schedule with a runtime that is quadratic in the input size.

Chapter 3

Performance Analysis of a Hybrid Bike Sharing System

In this chapter, we investigate a hybrid bike-sharing system. To this end, we carry out a usage pattern and demand analysis on booking data from Munich and include the effects of censored demand in a service level analysis. Service levels are used as meaningful measures for evaluating the customer-oriented performance of bike-sharing systems. Our results show that service levels are overestimated when ignoring unobservable, censored demand effects. Furthermore, there are significant differences in the usage of free-standing and station-based bikes. Based on these results, an adjusted incentive and repositioning policy could increase the booking number of free-standing bikes and thus customer satisfaction as well as the system's profitability.

3.1 Introduction

The increasing urbanization overstrains the traffic networks and leads to congestion. Therefore, more and more cities promote the use of bikes as an alternative transportation mode. In a city with a well-developed bike infrastructure, the bike can even be the fastest option to travel (e.g., Faghih-Imani et al., 2017; Leth et al., 2017). Time savings, health aspects and reduced emissions are other key benefits of increasingly popular bike-sharing systems (DeMaio, 2009; Fishman et al., 2013).

While the available infrastructure facilitates cycling, the boom of biking can also be attributed to the growing number of bike-sharing systems around the world. The first bike-sharing system, the "white bikes of Amsterdam", was introduced as early as in 1965 (DeMaio, 2009). 52 years later, in 2017, more than 1,200 cities were operating bike-sharing systems (The Bike-sharing World Map, 2017) and the number is growing as cities want to earn a reputation of being green and environment-friendly. In bike-sharing, there are different usage groups: For the local population, it provides an additional option to get to work or to the places of their leisure activities, whereas tourists use these bicycles spontaneously for exploring the city.

Many systems in Europe and North America are station-based systems where bikes are rented at predefined stations and also returned to a station after usage. Free-floating systems offer a higher degree of flexibility to the customers as bikes can be parked anywhere inside the operating area and no stations exist. In the last years, hybrid models, too, e.g. Norisbike in Nuremberg or the MVGRad in Munich, were introduced. In these systems, the bikes can be parked and rented everywhere inside the operating area. Additionally, several stations where bikes can be rented and returned are placed throughout the city. The stations have two key functions: (1) The battery of the bike's radio module can be recharged if the bike is connected to a station. Since the battery can only be loaded during a ride, this may become a necessity for those bikes that have been sitting idle for a longer period. (2) The positioning of such stations at public transport stops or points of interest guarantees a better availability of bikes at highly frequented locations. This makes the system more attractive to customers, as the search for bikes can be time-consuming even if smartphone apps indicate the position of bikes on a map.

The success of a bike-sharing system depends on many factors. For the users, the availability of functional bikes and the price are the key aspects. The operators, on the other hand, want to maximize their revenues. Typically, the demand varies for different areas of the city. Also, the inflows into and outflows from several areas are often imbalanced. To maintain availability, the operators thus have to reposition bikes by using shuttles. The planning of repositioning operations is based on the analysis of the customers' demand patterns. Since repositioning is cost-intensive but fosters the number of rentals, the operator has to continuously determine the optimal number of repositioning operations in a dynamically changing system.

While for station-based bike-sharing systems the literature on demand analysis and repositioning operations is broad (see, e.g., DeMaio, 2009; Fishman et al., 2013), hybrid systems have not been studied so far. Compared to a station-based system, the operational costs of a hybrid system can be higher if too many bikes need to be collected and returned to stations for recharging. Using a data set from MVGRad, we analyze demand patterns in this hybrid system and identify important strategic decisions in such a system. We show different demand behaviors dependent on the location in the city and whether or not the demand originates from a station or from a free-standing bike.

So far, the performance of a bike-sharing system is measured by the total number of trips. We use service level approaches from supply chain management to quantify both, the potential of demand as well as the performance of the system in terms of stock-outs, since these measures better reflect customer satisfaction (Tempelmeier, 2000). Moreover, existing literature neglects the fact of censored demand observations. If a station is out of stock, true demand might be higher than observed demand in which case forecasting models will underestimate the actual demand. We address this gap by including censored demand in our analysis.

Our contribution is: (1) conducting a performance analysis of a hybrid bike-sharing system based on service level measures and using real data, (2) evaluating the effect of censored demand observations that leads to service level overestimation if neglected, (3) suggestions for operators on how to increase the profitability of hybrid bike-sharing systems by combining the advantages of the station-based and free-floating bikes.

The remainder of this chapter is structured as follows: In Section 3.2, we describe the hybrid bike-sharing system and the used data. Section 3.3 introduces the performance measures and shows how censored demand observation were corrected. The results of our analysis are presented in Section 3.4. Based on these results, we deduce several managerial insights in Section 3.5. In Section 3.6, we summarize our findings and give an outlook for future research.

3.2 System and data description

The MVGRad system (Münchner Verkehrsgesellschaft mbH, 2017a), owned by Munich's public transport provider MVG, is a combination of a station-based and a free-floating bike-sharing system. The MVG is a subsidiary of the Stadtwerke München GmbH (Munich City Utilities), a company owned by the city of Munich. In February 2017, 1,200 bikes were in service and 73 bike rental stations existed all over the operating area of 110 square kilometers. Bikes can be rented and returned to those stations. In addition, they can also be parked and rented anywhere in the operating area, which relates to a a free-floating system. Three of the 73 stations are located outside the free-floating area. An important function of all stations is the recharging of the radio modules while the bike is not rented out.

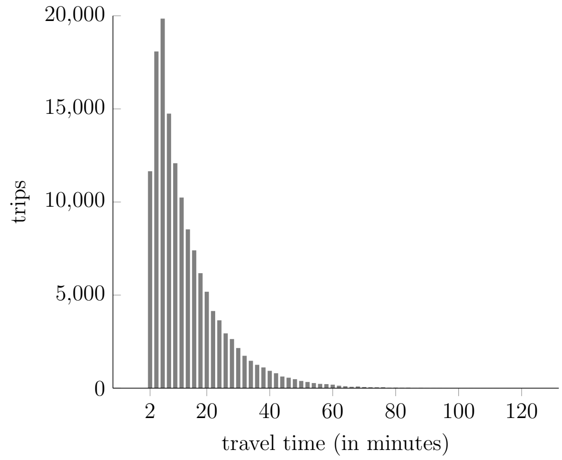
The usage fee is a combination of fixed and variable costs. Depending on the subscription, between 0.05 Euro and 0.08 Euro per minute have to be paid. Annual subscription further allows the rider 30 free minutes per day. Moreover, if the bike is returned to a station, the cyclist gets free bonus minutes. The number of bonus minutes equals the usage time and is capped at 10 minutes. Parking outside of the operating area and not at a station is penalized with a fee. This fee depends on the distance to the operating area.

The position data of the MVG bikes was scraped from the web between June 23, 2016 and July 31, 2016 and between October 10, 2016 and February 07, 2017. The gap in the data results from a system change and different data availability. Thus, we omitted that period for the purpose of comparability. A data set with all positions of all bikes was stored every other minute. Each data set contains the following information:

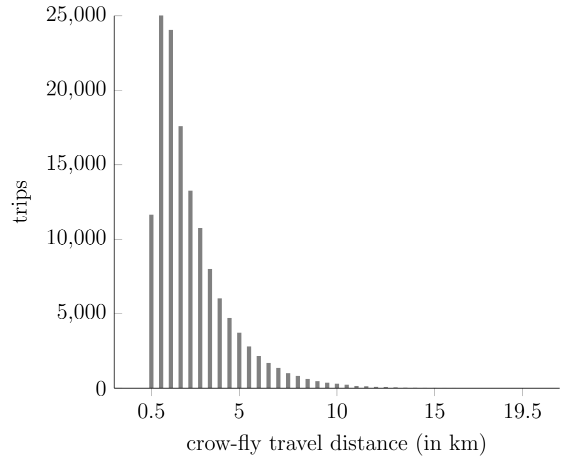
- bike id,
- latitude and longitude of the position,
- station id (if parked in a station),
- number of bikes at the station (if parked in a station),
- time stamp.

Bikes can be reserved for 15 minutes free of charge. Therefore, a bike that disappears at a certain time stamp and re-appears within 15 minutes at the same location is counted as a reservation. 33% of all disappearing bikes were recognized as reservations and removed when the trip matrix was generated. That means a trip is defined as a movement of a bike between two GPS coordinates. Trips out of or into the operating area were also removed from the data set. This results in 140,879 trips and 46,765 reservations. For the trips, the average crow-fly distance between origin and destination is 2.25 km with an average travel time of 13.54 minutes and every bike was rented 115 times on average over the whole time of the data set. Figure 3.1 further shows the detailed travel time and travel distance patterns. These patterns are in line with findings in the literature (e.g., Borgnat et al., 2011). Most users take the bike for short distance trips. More than 50% of the rentals are less or equal to 10 minutes and within a cycling radius of 2 kilometers. 4

From now on, we will refer to the collection of trips and reservations as the booking data. A trip will be categorized as station-based if the bike used for the trip is taken from a station. Bikes not parked in a station are considered as free-standing bikes. This also means that a bike can change its category if it is rented as a free-standing bike and returned to a station or vice versa.



(a) Travel time patterns in 2 minute intervals.



(b) Distance patterns in 0.5 km intervals.

Figure 3.1: Distance and travel time pattern analysis of the MVGRad system.

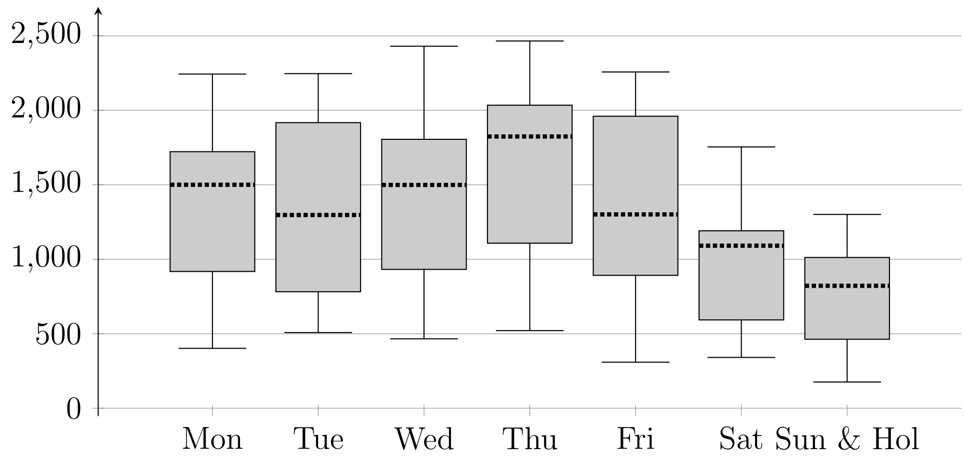


Figure 3.2: Number of bookings in the MVGRad system per day of the week.

While evaluating the number of bookings per day of the week, we observed that the number of bookings vary (see Figure 3.2). On average, 1,266 bookings were recorded per day with a standard deviation of 277. Monday and Wednesday, as well as Tuesday and Friday, show pairwise similar numbers, while most bookings happen on Thursday. During the weekend, significantly less bikes are requested.

The whole observation period includes six holidays, which form a too small data basis for separate analysis. For that reason, we merged holidays with Sundays to obtain a sufficient number of historical periods.

In the data set, it was not possible to distinguish between customer bookings and repositioning operations executed by the service provider. Consequently, every collection of a free-standing bike is counted as a booking. However, we learned from a company's representative that the collection of bikes does not follow any specific strategy. In the free-floating area, the main reasons for collecting bikes are recharging the battery modules and maintenance. Therefore, we assume that free-standing bikes are collected uniformly in the operating area. In contrast to that, the stations are continuously monitored and kept at a certain fill-level by adding or removing bikes.

3.3 Service level computation and censored demand

Service levels allow for a customer-oriented evaluation of the bike-sharing system's performance. To obtain meaningful results, we partition the operating area into multiple zones in the shape of hexagons. As service levels measure how many requests were served with the available bikes, the zones should be as large as customers are willing to walk to pick up a bike. We assume that this distance is 150 meters. We cover the whole operating area with these hexagons, starting in the North-Western corner. To prevent biased results, we only consider hexagons with at least ten bookings over the complete observation period. Furthermore, several hexagons with bookings were located outside the operating area. We assume that these bookings result from the collection of bikes that were left outside the operating area. Thus, we do not consider these hexagons in our analysis, either. Consequently, we obtain 996 hexagons that contain 93% of all bookings.

Because reserved bikes are also unavailable to customers, bikes that are not available due to a reservation are not excluded from the service level analysis. For each of the hexagons, we determine the service levels. To account for temporal changes, we took a time interval of ten minutes, that is 1,008 periods per week. We assume that customers who cannot be served immediately will not wait. Hence, we presume lost sales.

The α-service-level describes the availability of the system. That is, the probability that all customer requests in a zone can be served within a ten minute time interval with the existing stock of bikes:

$$\alpha := \frac{\textit{number of time intervals where all requests are satisfied}}{\textit{number of all time intervals}}. \tag{3.1}$$

This KPI is event-oriented, as it determines how often all requests could be met. Another KPI is the number of requests that could be satisfied from available bikes. This is measured by the quantity-oriented β-service-level, or fill rate. It reflects the share of requests that can be satisfied in a zone in a certain time interval:

$$\beta := 1 - \frac{\textit{unsatisfied requests}}{\textit{all requests}} = 1 - \frac{\max\{\textit{all requests} - \textit{bikes available}; 0\}}{\textit{all requests}}. \tag{3.2}$$

Hence, the service levels can be used for determining the potential for renting out bikes, which in turn provides a good basis for planning repositioning operations.

While computing the service levels, one has to keep in mind the effect of censored demand observations. That is, demand within a hexagon can only be observed as long as there are bikes available and real demand may be higher than observed demand (cf. Lau and Lau (1996)). If this effect is not taken into account, censored demand can lead to an underestimation of the real demand and consequently an overestimation of the realized service levels. This can mislead the provider to reduce the number of bikes in such areas, which further reduces the number of satisfied requests while the real demand still remains higher. To overcome this estimation error, we apply a data-driven approach to determine the expected demand similar to Sachs and Minner (2014). The authors define sales patterns based on historical demand observations. These patterns are used for estimating the demand for censored periods based on periods with real demand observations. As pointed out by Lau and Lau (1996), the advantage of the non-parametric approach is that no prior knowledge of the demand distribution is required. Consequently, estimation errors that stem from wrong assumptions on the underlying distribution or fitting errors from maximum likelihood estimations, are avoided. The numerical results reported by Lau and Lau (1996) or Huh et al. (2011) show that these data-driven methods prove to be powerful tools that outperform previously known methods.

We introduce set $\mathcal{T}$ that comprises all intervals over the complete observation period. $\mathcal{H}$ denotes the set of the 996 hexagons. To compute the service levels, we calculate $b_{h,t}$, the number of bookings in hexagon $h \in \mathcal{H}$ in period $t \in \mathcal{T}$, and $r_{h,t}$, the number of returns. Then, we define, $n_{h,t}$, the accumulated number of bikes as follows: $n_{h,t} := n_{h,t-1} + r_{h,t} - b_{h,t}$. Thus, when $n_{h,t} = 0$, we assume that a stock-out occurred in this period and consequently demand observations were censored.

Due to the day-dependent variations of the demand (see Figure 3.2), we only consider bookings from the corresponding weekday. Nevertheless, we can only observe the number of bookings. That is, if the net inventory level $n_{h,t} = 0$, the true demand $d_{h,t}$ may be higher than $b_{h,t}$. To impute the true demand, we take the mean of the corresponding historical observations. Therefore, we proceed as follows:

1. Determine $n_{h,t}$ for all $h \in \mathcal{H}$ and $t \in \mathcal{T}$.

2. If $n_{h,t} > 0$, set $d_{h,t} = b_{h,t}$, else add the tuple (h,t) to the set $\mathcal{C}$.
3. For every tuple $(h,t) \in \mathcal{C}$ determine $\mathcal{T}_{h,t} := \{\tau \in \mathcal{T} : n_{h,\tau} > 0 \wedge \tau \textit{ same period as } t\}$. Set $d_{h,t} = \frac{1}{|\mathcal{T}_{h,t}|} \sum_{\tau \in \mathcal{T}_{h,t}} b_{h,\tau}$.

3.4 Data analysis

In this section, we analyze the booking data to get more insights into the demand patterns that originated from station-based and free-standing bikes. First, we investigate the difference between these two categories, then we compare the performance of the system in highly used areas to the performance in little used areas. Finally, we define service levels in the context of bike-sharing systems and analyze them for the MVGRad system.

3.4.1 Comparison between station-based and free-standing bikes

A high availability of bikes is one of the primary goals of a bike-sharing system. Customers typically use their smartphone app to find a bike by checking the current system status and are unsatisfied if no bike is available within walking range. The high number of reservations in the data set (33%) shows that the perceived availability is influenced by the users' pre-rental behavior. The fact that 90% of the trips started with a free-standing bike but 99% of the reservations were made for them further underlines this statement.

From these numbers, we can deduce two behavioral patterns. First, since typically several bikes are available at bike stations, users do not see the need to reserve a bike at those stations. However, for free-standing bikes, this is different. The users want to book the bike at the perfect location for starting their trip. Second, the high share of trips from free-standing bikes indicates that, in most of the cases, customers select a free-standing bike that is perhaps closer to them than the next station.

3.4.2 Geographical patterns

In this subsection, we compare the efficiency of the system in the different districts of Munich, which can be seen in Figure 3.3. The dark blue districts have the highest population density while the light blue are the ones with low population density. We use the area size of each of the districts to evaluate the density and fluctuation of bikes in all

Figure 3.3: Political districts of Munich.

the districts (Landeshauptstadt München, 2017). For districts that are not completely covered by the operating area of the MVGRad, the corresponding share of the area is used (Münchner Verkehrsgesellschaft mbH, 2017b). In total, 21 districts were used. Two of them only allow stations, which is why no area was assumed for them.

Since five districts showed a significantly higher number of trips per square kilometer than all others, we divided the operating area into highly used districts (HU, with more than 2,000 trips per square kilometer in total) and districts with low utilization (LU, fewer than 2,000 trips per square kilometer in total). In Figure 3.3, the HU districts are marked with orange borders. It shows that the utilization is not only based on the population. The district with the second highest population density has only 1,343 trips per square kilometer, whereas in the city center, which has a medium population density, the utilization of bikes is very high. This is caused by the many tourist attractions and shopping malls. The other HU districts are also not only districts with a high population: The two main universities of Munich are located in these areas and also the cultural life is very active there.

The analysis shows that 52.7% of the trips are made from high utilization (HU) areas and 47.3% from low utilization (LU) areas. However, these HU districts have only a share of 16.5% of the total operating area. Moreover, 69.9% of the trips that originated in the HU areas also stay in the HU areas and only 30.1% leave to an LU area. For the LU areas, the share is even higher: 72.1% stay in the LU areas. Even for the 21 districts, still 39.2% of the trips start in the same district in which they end. These findings can also be explained by the distance patterns of Figure 3.1. Bikes are mostly used for short distance trips.

Moreover, 9.7% of the parking time of all bikes is generated in the low utilization areas, which results in significantly longer parking times of the bikes. Figure 3.4 shows the parking time of each bike when located in the LU or the HU area. In both areas, the first and second quartile are still very similar; still, the parking time is lower in the HU area. However, for the third and fourth quartile, the parking times are much longer in the LU area. Since the demand is lower in LU areas and yet many bikes are sitting in those areas, this result is not surprising. However, this leads to a much higher risk of a low bike battery in the LU area. Since the LU areas are much larger, the costs for collecting those bikes and park them in a station for recharging are also much higher.

This opens the potential for location-specific incentive systems. While, in the HU areas, returning a bike to a station does not seem to be necessary, incentivizing customers to return bikes in LU areas to stations could reduce the operating costs in those areas. Also, incentivizing the usage of specific bikes, could further reduce the operating costs. This could either be done by deactivating specific bikes or by giving a reward similar to the one granted for returning bikes to a station.

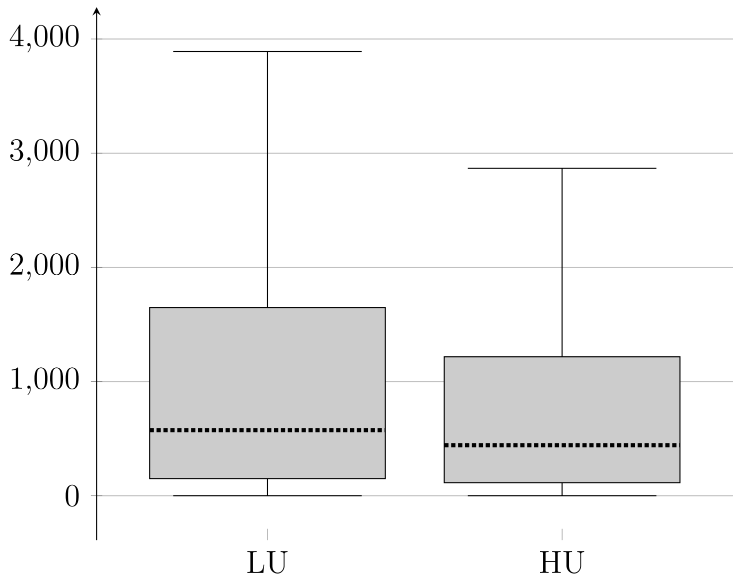
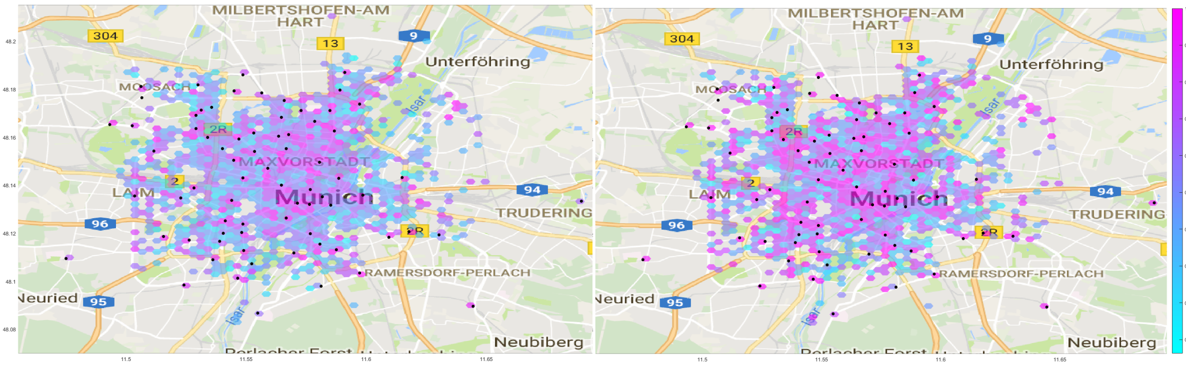
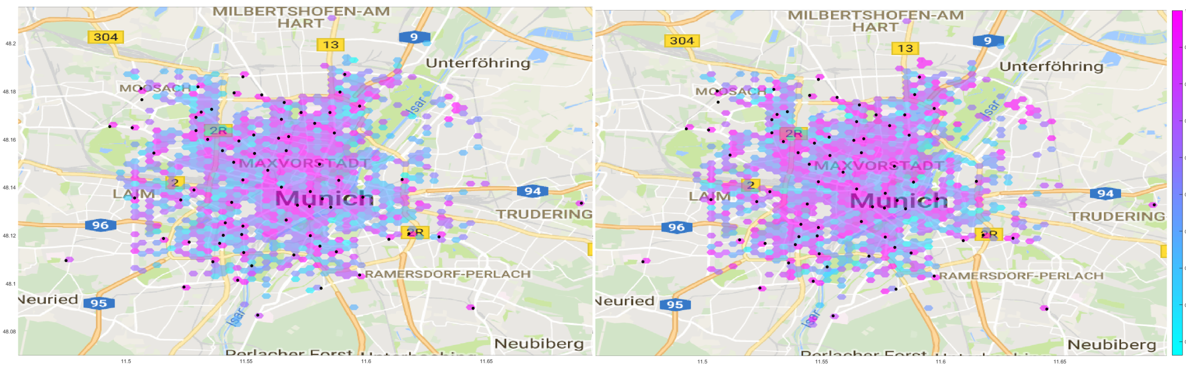


Figure 3.4: Parking time (in minutes) depending on the area.



(a) Considering corrected, uncensored demand. **(b)** Under observed, censored demand.

Figure 3.5: Heatmap of the average α-service levels, plotted in Google Maps (2017). The stations are represented by black dots.



(a) Considering corrected, uncensored demand. **(b)** Under observed, censored demand.

Figure 3.6: Heatmap of the average β-service levels, plotted in Google Maps (2017). The stations are represented by black dots.

3.4.3 Service level analysis

Figure 3.5 (a) shows a heatmap with the average of all ten-minutes α-service-levels over the complete period. It considers all those hexagons where at least ten bookings were registered during the whole observation period. Additionally, the 73 bike stations are marked with a dot.

The mean availability of bikes in the operating area is 61% with a standard deviation of 26%. The availability at the stations is significantly higher (at a significance level of 95%) with a mean value of 91% and a standard deviation of 15%. The median lies at 94%. The analogous heatmap of the β-service levels (cf. Figure 3.6 (a)), shows a similar pattern. That is, the fill rates are high at the stations: 93% on average with a standard deviation of 17% and the median at 96%. The mean of the fill rates over the whole operating area is 63% with a standard deviation of 28% and a median of 69%. An explanation for this difference could be the fact that we cannot distinguish between customer bookings and repositioning operations executed by the service provider. Hence, a collection of free-standing bikes is counted as a booking and demand is overestimated. This may lead to an underestimation of service levels in the plane.

We show in Table 3.1 that there are substantial differences between the service levels in the HU and LU areas. The availability and fill rates in the HU districts (cf. Figure 3.3) are higher than in the LU areas. To be precise, the mean α-service-level is 75% with a median of 77%. The average β-service-level in the HU districts is 82% and the median lies at 86%. In the LU areas, one can see several clusters, often around a rental station, where the service levels are higher (see Figure 3.5 (a) and Figure 3.6 (a)). In hexagons without a station, the service levels are substantially lower. The numbers also show that on average, the α-service-level in the LU areas lies at 48% (median at 53%), while the β-service-level is 50% with a median of 54%.

The HU area covers the central station, the main campuses of Munich's two universities, as well as districts with a high density of tourist attractions, shops, restaurants and other points of interest. In these areas, people leave their bikes next to these points of interest, thus maintaining a higher bike availability. Nevertheless, we observe several station-free hexagons with significantly lower fill rates in this area which means that a noticeable number of booking requests could not be served since not enough bikes were available. This relates to the low availability of bikes in the corresponding hexagons, as the α-service levels indicate. In the peripheral area (LU), the density of offices and public locations is lower. Consequently, bikes are returned less frequently to points where other people start their trips. Therefore, high fill rates and availability are only achieved around stations and in certain higher frequented hexagons.

Weather conditions have a considerable influence on the usage of the bikes and thus on the service levels. We use historical data of the German Meteorological Office (Deutscher Wetterdienst, 2018) to examine the effects of different weather conditions. The two sample t-test shows that significantly less trips were taken when it was windy (more than 4 kph) or raining. At a significance level of 95%, the p-levels lie at $3.4 \cdot 10^{-12}$ for wind and $2.2 \cdot 10^{-16}$ for rain, respectively. Since less bikes are used, the service levels under such adverse weather conditions are higher while good weather conditions lead to lower service levels. As one can see from Table 3.1, this effect poses a problem mainly in the free-floating area: If there is no wind, the mean α-service-level and the mean β-service-level at the stations lies at 90% and 92%, while with wind the values increase to 94% and 96%. In the whole operating area, the mean α-service-level for no wind lies at 58%, the mean β-service-level at 60%, whereas with wind the values are at 67% and 69%, respectively. Rain has a similar influence. That is, the service levels at the stations are satisfying with values above or close to 90%, even when there is no rain. In the whole operating area, good weather conditions and the resulting higher demand cause a drop in the average service levels to values around 60%.

3.4.4 Effect of censored demand

To demonstrate the effect of censored demands, we computed the service levels, assuming that the bookings correspond to the observed demand. This leads to an overestimation of the service levels since the number of requests is reduced (cf. equations (3.1) and (3.2)). Table 3.1 shows that the mean availability under censored demand in the operating area increases substantially to 71%. The median under censored demand observations lies at 79%. If the demand is corrected, the mean availability in the operating area is at 61%, the median lies at 68%. Considering the stations only, the statistical measures show small differences between the use of censored demand observations and the corrected, uncensored demand observations (cf. Table 3.1).

For the β-service-level, we observe similar characteristics. The results in Table 3.1 show that the estimated mean fill rate and median are significantly higher under censored demand: the mean is 74% (63% under uncensored demand) and the median lies at 79% (69% under uncensored demand). On the other hand, Table 3.1 shows that at the stations mean, median, and quartiles are almost identical for censored and uncensored demand. The reason is that stock-out situations are rare at the stations because there is typically a sufficient number of bikes available. Consequently, the number of bookings normally corresponds to the actual demand and the results under censored demand show no significant deviations.

The risk of overestimating becomes particularly apparent when we look at different weather conditions. As Table 3.1 shows, the service levels at the stations differ only slightly. This is due to the fact that there are typically enough bikes sitting idle at the stations. Thus, stock-out events rarely occur and therefore the risk of overestimation is small. In contrast, the differences are substantial in the free-floating areas. When the weather is good, that is no wind and no rain, censored demand observations lead to differences in the mean α-service-level of 13 percentage points when there is no wind or 10 percentage points when there is no rain. For the mean β-service-level, the differences are 14 percentage points for the case of no wind and 11 percentage points if there is no rain. On the other hand, when there is wind or rain, the demand is lower and therefore less stock-out cases happen. Hence, the differences between censored demand observations and corrected, uncensored demand observations are 3 percentage points (wind) and 4 percentage points (rain) for the mean α-service-level and 4 percentage points (wind) and 5 percentage points (rain) for the mean β-service-level.

Similar differences can be observed when we distinguish between the LU and HU areas (cf. Table 3.1). That is, at the stations there are almost no differences between censored and corrected, uncensored demand observations. This is different for the whole operating area. Looking at the mean α-service-level in the HU areas, the difference lies at 7 percentage points and the mean β-service-level at 6 percentage points. For the LU areas, the discrepancy is even higher: 12 percentage points for the mean α-service-level and the mean β-service-level. Figure 3.5 (b) and Figure 3.6 (b) visualize this estimation errors under censored demand. This can be seen particularly well in the outer, western area on the map. The fact that service levels are overestimated by a higher degree in LU areas, results from most of the bike-sharing traffic occurring in the HU areas. Hence, less bikes are parked in the LU area, which is also larger than the HU area. Consequently, the number of periods with no bikes available is higher than in the inner districts.

Our findings show that stock-out events lead to a substantial overestimation of service levels due to censored demand. This effect increases with an increased demand (for example due to good weather) or with less bikes offered in an area. Thus, the bike-sharing operator might underestimate the number of unsatisfied requests and therefore not take suitable countermeasures.

Table 3.1: Comparison of the α-service-level and β-service-level under corrected, uncensored demand (uncens.) as well as under censored demand observations (cens.). The operating area includes the stations as well. All values are given in percentages.

		α-service-level				β-service-level			
		station		operating area		station		operating area	
		uncens.	cens.	uncens.	cens.	uncens.	cens.	uncens.	cens.
total	Mean	91	92	61	71	93	93	63	74
	Stdv	15	15	26	28	17	18	28	28
	Q3	96	98	81	88	97	98	80	88
	Q2	94	95	68	79	96	96	69	79
	Q1	88	91	44	57	91	91	42	56
LU	Mean	90	92	48	60	92	92	50	62
	Stdv	14	15	22	25	17	17	21	23
	Q3	96	98	76	86	97	98	75	86
	Q2	94	95	53	61	95	96	54	68
	Q1	86	90	34	45	89	90	32	44
HU	Mean	92	92	75	82	93	93	82	88
	Stdv	15	15	28	30	17	18	29	30
	Q3	96	98	86	91	97	98	85	91
	Q2	95	95	77	86	96	96	86	89
	Q1	88	91	70	72	91	91	67	71
no wind	Mean	90	91	58	71	92	92	60	74
	Stdv	14	15	25	28	16	18	27	28
	Q3	95	97	77	88	96	97	76	88
	Q2	92	95	65	73	94	96	66	73
	Q1	86	91	44	56	89	91	41	55
wind	Mean	94	95	67	70	96	96	69	73
	Stdv	14	15	27	27	16	18	27	29
	Q3	97	98	89	90	98	98	86	88
	Q2	95	96	74	79	97	97	76	79
	Q1	89	92	48	57	92	92	53	56
no rain	Mean	89	91	60	70	91	92	62	73
	Stdv	15	16	24	27	17	19	26	27
	Q3	94	97	80	87	95	97	79	87
	Q2	91	93	65	77	93	94	66	77
	Q1	86	89	39	56	89	89	38	55
rain	Mean	94	95	67	71	96	96	69	74
	Stdv	13	14	27	28	15	17	29	28
	Q3	97	98	89	89	98	98	88	89
	Q2	96	97	76	80	98	98	77	80
	Q1	90	93	49	58	93	93	47	57

3.4.5 Limitations

In the following, we want to mention three limitations to our data analysis. First, different from Sachs and Minner (2014), who extrapolate the demand patterns for censored periods from uncensored periods in the previous hours, we select all demand observations in the same zone in the same period and at the same time of day when there was no stock-out. The reasoning for this different approach is that the number of uncensored demand observations in our data set may be too small for a thorough pattern estimation. Therefore, we decided to use all available periods. Consequently, the imputed demands do not factor in seasonal effects or the influence of the weather. However, the effects of different times and types of day are considered. Furthermore, our regression analysis did not reveal a statistically significant increase in the riding trend. Therefore, we use the average for imputing censored demand.

Second, from our data set, we cannot differentiate between a user booking and a repositioning by the company. As previously mentioned, the company had not established a systematic repositioning. Thus, we assume a uniform collection of the bikes and hence the distortion in our analysis is negligible.

Third, by assigning demand to a particular zone, the possibility of spillover effects due to stock-outs in neighboring zones is not considered. We set the hexagons size to the distance we assume customers are willing to walk. This should help to reduce the effect. Furthermore, zone-partitioning has been previously used in bike-sharing literature (e.g. Pal and Zhang, 2017; Reiss and Bogenberger, 2016b).

3.5 Summary of insights

The outcome of our combined analysis of service levels indicates that the MVG bike-sharing system has been widely adopted by customers such that the demand for bikes often exceeds the supply. This shows the value but also the challenge of the flexibility provided by the free-floating system. Repositioning bikes at the right time to the right location is more complex if not only stations but the whole operating area are potential demand locations. The fact that 90% of the trips were started from free-standing bikes, along with the comparably low fill rates, indicates that more free-standing bikes could be rented out if they were available. This is also supported by the results for periods with good weather conditions, i.e. when there is no rain or wind. Then, the high demand leads to low service levels in the free-floating areas. Consequently, offering more free-standing bikes could further increase the attractiveness of the system.

On February 7th, 2017, the city of Munich decided to more than double the number of bikes by adding 2,000 new bikes and to simultaneously increase the number of stations by 52 to 125 in total (Sueddeutsche Zeitung, 2017). The new stations were located both in the station-based area and in the free-floating area. Since the bike stations have a typical capacity of 10 to 30 bikes, a substantial portion of the bikes will not fit and thus will have to be parked outside the stations. Based on our findings, we can conclude that these additional bikes will help to increase the service levels in the free-floating area. As our analysis showed, the service levels at the stations are significantly higher than in the free-floating area. Thus, a denser network of stations in the free-floating area will also contribute a lot to higher overall service levels.

Adjusting the two components of the provider's repositioning strategy also has significant potential for increasing service levels. The first component, customer based relocation, incentivizes customers who return their bikes to a station. This is mainly done for recharging the radio module battery. Thus, only offering incentives for those bikes with a battery level below a certain threshold could decrease operational costs. A similar idea applies to the second component, the operator based repositioning: Preferably bringing bikes with a low battery level back to stations and taking fully charged bikes out of stations to hexagons with a low β-service-level should be considered.

These findings are in line with the results for the free-floating bike-sharing system "Call a bike" (Reiss and Bogenberger, 2015). Although the area covered by this system is smaller than the MVGRad operating area, we see similar behavioral patterns. That is, in the HU (zones with orange borders in Figure 3.3), the number of bookings is high and the corresponding idle times are typically low. In the outer districts, there are many free-floating bikes with idle times of more than ten days. Furthermore, the analysis of Reiss and Bogenberger (2015) shows that in the outer areas more trips end than start. The authors recommend a rebalancing strategy that collects bikes in the outer districts (approximately the light blue districts in Figure 3.3) and brings them back to the city center. Naturally, this strategy comes at high costs as the outer districts are relatively big and thus collecting the scattered bikes is time consuming. Such costs can be reduced in a hybrid system by offering a station-based system in districts with low demand. This has two advantages: First, demand is concentrated at the stations and thus utilization of the bikes increases. Second, search and rebalancing costs are reduced because only stations have to be called in the outer districts.

To summarize, we provide three main insights for bike-sharing operators:

1. When ignoring censored demand, the real demand is underestimated and the service level is overestimated. This is especially true when demand is high and more stock-outs occur, which misleads providers to assessing the bike coverage as sufficient, while in reality customers are lost. In the long run, this can lead to losses in revenues and lower customer satisfaction.

2. Operators of a hybrid bike-sharing system should carefully select free-floating areas in order to achieve customer-friendly service levels in these areas.

3. Operators of a free-floating bike-sharing system may consider the introduction of a hybrid bike-sharing system. That is, offering a free-floating system in areas with high demand and a station-based system in the other areas. This can help to reduce costs in low utilization areas.

3.6 Conclusion

We analyzed a real data set of a hybrid bike-sharing system based on several key figures, such as the number of trips taken and reservations from station-based and free-standing bikes, the distribution of area size, parking times and number of trips between high-utilization areas and low-utilization areas, as well as the α-service-level and the β-service-level. To prevent an overestimation of the service levels due to censored demands, we applied a data-driven imputation method to correct the demand. The results show that 90% of the trips are made from free-standing bikes and therefore the free-floating option makes the system more attractive. The service levels underline this observation, since the service levels at the stations are significantly higher than for free-standing bikes. When demand is high (e.g. when the weather is good) or the number of free bikes offered in a zone is low (e.g. in LU zones), the service levels in the free-floating areas are not satisfying. These cases also show the effect of censored demands since higher demand leads to more stock-outs and thus more lost sales. If demand observations are not corrected for lost sales, the service levels in the high demand periods (or lower supplied areas) are overestimated; our results showed that the mean α-service-level would be overestimated by up to 13 percentage points. As a consequence, the provider is not aware of the additional customers he could have served with more bikes. The reservation behavior reveals that customers are aware of the high demand for free-standing bikes. 99.5% of the reservations, which are free of charge, were done for free-standing bikes and 33% of the reservations were not followed by a ride. These figures show that the operator could increase the bookings by making more free-standing bikes available. The downside of free-standing bikes are higher operating costs, since the batteries for the radio modules discharge while bikes are not parked at a station. To increase the bookings of bikes, and therefore customer satisfaction, while simultaneously reducing operating cost, the provider could introduce a flexible incentive system that depends on the position and battery status of both, the bikes, and the overall demand status in the system. This could also include a fee for reservations of free-standing bikes that were not followed by a ride. In addition, the provider could reposition the bikes depending on the current service and inventory levels of the individual hexagons. Both, the customer-based and the operator-based repositioning, have potential for future research. Consideration of customer flows in between adjacent hexagons and substitution to other modes of transport pose interesting directions for future research. Moreover, the design of the free-floating area should be addressed thoroughly, for example, within a network design problem.

Chapter 4

Competitive Rebalancing in One-Way Car-Sharing

We consider the problem of two competing car-sharing providers who determine the optimal car distribution in order to maximize their profits. Since the vehicles of the providers are substitutable among each other, the presence of a competitor's cars influences the provider's demand. Thus, both companies have to reposition their vehicles while anticipating the competitor's reaction. This can be seen as inventory transshipment under competition. Both problems, inventory transshipment and inventory competition, have been intensively studied by its own. This work is the first that combines both research streams. To cover different customer behaviors, we consider the problem under stock-out-based as well as availability-based substitution. Based on a new model for determining the optimal vehicle distribution of a monopolistic provider, we analyze a single-period, non-cooperative Nash game between two competing providers. We prove that there exist a unique Nash equilibrium under both, stock-out-based and availability-based substitution, and develop an efficient solution algorithm that makes use of Lagrangian multipliers. A controlled computational study demonstrates how the customers' substitution behavior influences the reactions of the providers. Under availability-based substitution, competition between the providers is intense, as both aim at increasing their market shares. Stock-out-based substitution typically leads to the situation that providers share the market. A case study with booking data from Munich shows that ignoring the presence of a competitor comes at a high cost and that competition leads to a higher number of cars that are rebalanced.

4.1 Introduction

Car-sharing services have grown considerably in recent years. The number of car-sharing members worldwide showed a 14-fold increase from 346,000 in 2006 to 4.8 million in 2014 (TSRC Berkeley, 2016). For 2014 alone, the authors report a growth rate of 65%. Simultaneously, the worldwide fleet increased from 11,501 to 104,125 vehicles in these eight years (TSRC Berkeley, 2016). In an outlook towards the future, the Boston Consulting Group predicts about 35 million car-sharing users worldwide by 2021, generating global revenues of EUR 4.7 billion (The Boston Consulting Group, 2016). Figures also clearly illustrate that the popularity of car-sharing has been accelerated particularly by the increasing offer of one-way services. In Germany, the number of one-way car-sharing users grew from less than 50,000 in 2012 to 1.81 million by the beginning of 2019. In 2019, this service has been able to attract a total of 350,000 users, i.e. 17% more than in the year before (German Carsharing Association, 2019).

The most significant advantage of one-way car-sharing over the two-way system is the higher flexibility, as customers do not have to return the vehicles to the original pick-up point. A further enhancement of the latter system is the free-floating model, where customers are allowed to leave the cars at any parking spot within the business area. Such car-sharing systems have the potential to cover the needs for individual mobility and thus to replace privately owned cars. According to a study with data from London, 35% of the free-floating car-sharing users decided not to buy a new car or to dispose an existing one (Le Vine and Polak, 2019).

For car-sharing providers like the flexibility of one-way systems often leads to an uneven distribution of the cars that results in lower customer satisfaction and less usage of the vehicles. Consequently, car-sharing operators have to rebalance their cars regularly to keep the system profitable. For that purpose, the providers partition their business area into several zones and move cars from zones with an expected overage to zones with an expected underage. Many authors have addressed the problem of determining optimal repositioning strategies, considering different user-related factors like adoption rates, booking behavior or time and day of the week (Ferrero et al., 2015).

All publications up to now have the one assumption in common that the car-sharing provider is operating in a monopoly. In real life, however, there are many cities where two or more providers offer their services. For example, the two free-floating providers *ShareNow* and *Sixt share* competed in three German cities in 2019. As findings from retail management show, companies should not neglect competition as it has a direct influence on the customers' demand (McGillivray and Silver, 1978). One may argue that most of the customers will not switch to a competitor's car as easily as they will go to another retailer whenever a product is out of stock at the first place. The reason is that, as opposed to the retail sector, the usage of a car-sharing service typically requires a one-time registration at the corresponding provider and the payment of a registration fee. However, the new concept of Mobility as a Service (MaaS) may intensify the competition in the car-sharing sector, as in MaaS different mobility options like public transport, car-sharing or bike-sharing can be used with one access card. This card also allows customers to use car-sharing vehicles of different companies. First MaaS field trials are already under way, for instance in Finland, Germany and the Netherlands (The Economist, 2017). Thus, it is likely that car-sharing customers will be able to choose among different operators. This motivates an important research question: *How does competition influence the optimal rebalancing plan and how should the companies react to the presence of a competitor?*

While inventory based competition and inventory rebalancing by themselves have been studied very well (Paterson et al., 2011; Silbermayr, 2019), rebalancing under competition has not been addressed directly so far. In this article, we develop a method for determining the optimal rebalancing under competition for a given pool of stock. We then apply our model to the case of car-sharing.

In retail, one distinguishes between two types of competition: stock-out-based and assortment-based competition (Kök et al., 2015). In the first case, one assumes that a portion of those customers who cannot find a car of a company will substitute with a competitor's car. In that case, it may be advantageous for a provider to position more cars in zones that are sparsely covered by the competitor, thus he can collect the competitor's unsatisfied customers. In the latter case, when customers make their choice based on the observed availability in a zone, it may be better for a provider to locate cars in zones that are densely covered by the competitor to attract more customers to his service.

Independent of such assumptions, the decision to reposition more cars to one zone inevitably reduces the coverage in other zones as the total fleet size is fixed and replenishment is not possible at short hand. Consequently, the players have to find the most profitable redistribution of their cars among the zones, at the same time, taking into account the competitor's reaction.

To study rebalancing under competition, we consider two players who act as newsvendors with a given stock that they can distribute over several zones. Under stock-out-based substitution, we assume that a certain portion of users will substitute with the competitor's cars if, and only if, they cannot find an available car of their preferred provider in a zone. Under availability-based substitution, the customers' decision is taken after he observed the availability of the cars in the zone.

We state the player's individual repositioning problem by maximizing the expected profit minus the repositioning cost. Using Lagrangian multipliers, we transform this constrained optimization problem into an unconstrained one. We use this to formulate a non-cooperative game where both players solve their repositioning problems simultaneously. We prove that there exists a unique Nash equilibrium for both assumptions on customer behavior.

To derive an efficient solution algorithm, we make use of the characteristics of the Lagrangian multipliers. These multipliers allow us the partitioning of the zones into sending, dormant and receiving zones. Thus, we can derive the optimal target inventories without explicitly solving the underlying optimization problem.

We contribute to the literature by presenting a new mathematical model for transshipment under competition, including the consideration of stock-out-based and availability-based substitution. Moreover, we show that there exists a unique Nash solution to the resulting newsvendor game. To solve the competitive rebalancing problem, we introduce an efficient method based on Lagrangian relaxation. This research is motivated by the observation that in many cities car-sharing providers operate in direct competition with other providers. Therefore, the objective was to study the optimal rebalancing policies under the presence of a competitor. The controlled numerical design gives insights into how fleet size, demand parameters and initial positioning of the fleet influence the optimal distribution. The results of our case study show that ignoring the presence of a competitor can come at a high cost. From the customer's point of view, the availability of car-sharing vehicles could be substantially increased if the providers made their cars accessible to the competitor's customers. This would also help to considerably reduce the number of overall repositionings.

The remainder of this chapter is structured as follows: The problem of optimal rebalancing under competition without replenishment is introduced and discussed in Section 4.2. The computational study in Section 4.3 provides useful insights into the effects of competition. In Section 4.4, we evaluate the results on real car-sharing booking data from Munich. We conclude in Section 4.5.

4.2 Model formulation

We consider two players with a given fleet in a non-cooperative, single-period inventory game under stock-out-based and availability-based substitution. In order to show the influence of competition, we introduce the Monopolistic Rebalancing Problem for one player. Afterwards, we extend it to the competitive case. We conclude with comparative statics on both cases of substitution. At the beginning of this section, we introduce the notation and discuss our assumptions.

4.2.1 Notation and assumptions

In the following, we consider a car-sharing company (in the following denoted as *player*) that offers its services in n zones. $\mathcal{N} := \{1, \ldots, n\}$ describes the set of all zones. y_i^0 denotes the initial stock at zone i, $y^0 = (y_1^0, \ldots, y_n^0)$ is the initial car-stock vector. The player faces a demand D_i in each zone i, expressed by a random variable that follows a probability density function f_i and a cumulative probability function F_i. The inventory levels and demands are assumed to be continuous. This is justified by the fact that the typical fleet size of a provider is between 500 and 1,500 cars. Hence, the inventory levels are sufficiently high for a continuous approximation. The margin for one car in zone i is s_i. To maximize the expected revenues, the player can rebalance the cars between the zones before demand is realized. The cost of repositioning one car is c and does not depend on the distance. This is motivated by what we learned from a car-sharing company that pays a fixed amount per car to a service provider. Without loss of generality, we assume $s_i \geq c > 0$. The additional car stock received at zone i is denoted by x_i. The vector $y = (y_1, \ldots, y_n)$ describes the stock levels after rebalancing.

Many car-sharing providers maintain a standardized fleet in order to reduce upkeep costs and to achieve better purchase prices. However, some car-sharing companies offer mixed fleets to better meet customer demands. As we are interested in effects caused by the presence of a competitor, we presume that each provider owns a homogeneous fleet. Consequently, there is only one product in our model and thus assortment-based substitution becomes availability-based substitution (Collado and Martínez-de-Albéniz, 2014).

The prices for renting a car-sharing vehicle are given. This is in accordance with the business model of many providers that use fixed pricing. However, car-sharing providers typically charge a subscription fee, which means that users are only registered with one provider. We presuppose that those customers who are willing to substitute are registered with both providers. As studies show, a certain portion of customers is registered with more than one car-sharing provider (Teamred, 2015). For the case of stock-out-based substitution, we can factor in this portion by adapting the substitution rate. Under availability-based substitution, this is not possible. Nevertheless, in a MaaS scenario customers will have access to both providers anyway.

Observe that we study a single-period problem. In doing so, and in line with the common practice of many car-sharing providers, we consider several hours as one period and assume that the providers do the rebalancing for the next period. Additionally, we assume that both players have perfect information about the positions of the competitor's cars. This assumption is based on the ability to extract such data in real-time by using web scraping methods (Balac et al., 2017).

4.2.2 Monopolistic rebalancing problem

We state the player's individual repositioning problem, maximizing the expected profit minus the repositioning cost. The player's expected revenues are

$$\pi(y) = \sum_{i=1}^{n} s_i \cdot \left(\int_0^{y_i} d \cdot f_i(d) \mathrm{d}d + \int_{y_i}^{\infty} y_i \cdot f_i(d) \mathrm{d}d \right) = \sum_{i=1}^{n} \underbrace{s_i \cdot \left(y_i - \int_0^{y_i} F_i(d) \mathrm{d}d \right)}_{:=\pi_i(y_i)} . \quad (4.1)$$

The *Monopolistic Rebalancing Problem* (MRP) is defined as:

$$\max \ \Pi(y,x) = \textstyle\sum_{i=1}^{n} \left(\pi_i(y_i) - c \cdot x_i \right) \quad (4.2)$$

$$s.t. \quad x_i \geq y_i - y_i^0 \qquad i = 1, \ldots, n \quad (4.3)$$

$$\textstyle\sum_{i=1}^{n} y_i^0 \geq \sum_{i=1}^{n} y_i \quad (4.4)$$

$$x_i, y_i \geq 0 \qquad i = 1, \ldots, n \quad (4.5)$$

The player's objective (4.2) is to maximize his expected profit, which is the rebalancing cost minus the revenues. This corresponds to the newsvendor equation without shortage or salvage cost, summed over all zones. Inequality (4.3) ensures that the received stock corresponds to the increase in inventory in this zone. Constraint (4.4) limits the total stock after rebalancing to the initial amount. Since the expected revenues are maximized, both constraints can be stated as inequalities. This allows us to introduce Lagrangian multipliers $\lambda_i \geq 0$ and $\alpha \geq 0$ for constraints (4.3) and (4.4). λ_i is the marginal value of a received unit for zone i, α is the marginal value of repositioning. By aid of these multipliers, we transform the MRP into the following unconstrained problem:

$$\max \ \mathcal{L}(y,x,\lambda,\alpha) := \Pi(y,x) - \sum_{i=1}^{n} \lambda_i \cdot (y_i - x_i - y_i^0) - \alpha \cdot \sum_{i=1}^{n}(y_i - y_i^0). \tag{4.6}$$

For (4.6), we can use the Karush-Kuhn-Tucker (KKT) conditions to describe the optimal solution solely by the Lagrangian multipliers.

Lemma 4.1 (Optimal solution characteristics of the MRP)**.** *(a) The target inventory levels y_i are decreasing in λ_i and α. (b) The Lagrangian multipliers partition $\mathcal{N}$ into three sets: $\mathcal{R} := \{i \in \mathcal{N} : x_i > 0$ and $y_i^0 < y_i\}$, the set of receiving locations, $\mathcal{D} := \{i \in \mathcal{N} : x_i = 0$ and $y_i^0 = y_i\}$, the set of dormant locations and $\mathcal{S} := \{i \in \mathcal{N} : x_i = 0$ and $y_i^0 > y_i\}$, the set of sending locations. For λ_i, the marginal values of a received unit of stock, it holds that $\lambda_i = c_i$ if $i \in \mathcal{R}$ or else $\lambda_i = \pi_i'(y_i^0) - \alpha \in (0; c_i)$ if $i \in \mathcal{D}$ or else $\lambda_i = 0$ if $i \in \mathcal{S}$.*

Proof: see Section 4.6.

Consequently, it suffices to compute the Lagrangian multipliers λ_i and α if we wish to determine the optimal solution to MRP.

Computation and existence of a unique solution

Theorem 4.1 (Existence and uniqueness of a solution for the MRP)**.** *If the revenue function $\pi(y)$ is continuous and strictly concave, then there exists a unique solution (y^*, x^*) to the MRP. That is, $\Pi(y^*, x^*) \geq \Pi(y,x) \ \forall (y,x) \in \mathbb{R}^n \times \mathbb{R}^n :$ y and x fulfill* (4.3) - (4.5).

Proof: see Section 4.6.

Our algorithm starts with an arbitrary value of $\alpha \in (0; \min_{i \in \mathcal{N}} s_i)$, then determines the corresponding λ_i and classifies the zones as stated in Lemma 4.1. $\delta(y) := \sum_{i=1}^{n}(y_i - y_i^0)$, the sum of the new inventory levels minus the total initial inventory, is decreasing in α (cf. proof of Theorem 4.1). Thus, when $\delta(y)$ is greater than zero, we increase the value of α, whereas we decrease it when $\delta(y)$ is smaller than zero. A solution to the MRP is found when $\delta(y)$ equals zero. Algorithm 4.1 summarizes this procedure.

Corollary 4.1 shows that Algorithm 4.1 always converges to the optimal solution.

Corollary 4.1 (Convergence to the optimal solution of the MRP)**.** *If π fulfils the conditions of Theorem 4.1, i.e. if it is strictly concave, the corresponding $\delta(\alpha)$ is strictly monotone in α. Thus, Algorithm 4.1 converges to the unique solution.*

The revenue function $\pi(y)$ is continuous and strictly concave if the demand distribution f is continuous and positive on its support. This is true for many of the commonly

Algorithm 4.1 Solution algorithm for the MRP

```
Initialization α ∈ (0; min_{i=1...n} s_i)
while δ(y) := Σ_{i=1}^n (y_i − y_i^0) ≠ 0 do
    for i = 1, ..., n do
        if π'_i(y_i^0) − α ≤ 0 then
            zone i is a sender, λ_i = 0, y_i solution to π'_i(y_i) − α = 0
        else if π'_i(y_i^0)) − α ≥ c then
            zone i is a receiver, λ_i = c, y_i solution to π'_i(y_i) − c − α = 0
        else
            zone i is dormant, λ_i = π'_i(y_i) − α ∈ (0; c), y_i = y_i^0
        end if
    end for
    if δ(y) > 0 then
        increase α
    else
        decrease α
    end if
end while
```

applied demand distributions, for instance, the (log-) normal, the exponential, the logistic, the gamma and the power distribution (Banciu and Mirchandani, 2013). The profit function $\Pi(y)$ is a linear translation of $\pi(y)$. Hence, Theorem 4.1 and Corollary 4.1 also hold for $\Pi(y)$.

4.2.3 Competitive rebalancing game

We extend the monopolistic rebalancing problem stated in Section 4.2.2 to the *Competitive Rebalancing Problem* (CRP). For that purpose, we now consider two players who offer their car-sharing services in the same n zones. To distinguish between the two players, we introduce a second index k. This also allows us to assume player-dependent revenues s_{ik} and repositioning costs c_k. If we refer to a vector that includes all zones $i \in \mathcal{N}$, we only use index k (e.g. y_k denotes the vector of all inventory levels y_{ik} of player k).

In the following, we take player k's view and denote the competitor as player l. In contrast to the monopolistic case, k has to take the influence of l's inventory levels on his own revenue in each zone i into account. Thus, the revenue function $\pi_{ik}(y_{ik}, y_{il})$, as well as the profit function, now also depends on the competitor's inventory level

$$\Pi_k(y_k, x_k, y_l) := \underbrace{\sum_{i=1}^{n} \pi_{ik}(y_{ik}, y_{il})}_{:=\pi_k(y_k, y_l)} - \sum_{i=1}^{n} c_k \cdot x_{ik} \ . \tag{4.7}$$

However, constraints (4.3) - (4.5) remain unaffected for the CRP. Analogously to the monopolistic case, we transform CRP to an unconstrained problem using the Lagrangian function:

$$\max\ \mathcal{L}_k(y_k, x_k, y_l, \lambda_k, \alpha_k) := \Pi_k(y_k, x_k, y_l) - \sum_{i=1}^{n}\left(\lambda_{ik}\cdot(y_{ik} - x_{ik} - y_{ik}^0)\right) - \alpha_k \cdot \sum_{i=1}^{n}(y_{ik} - y_{ik}^0). \quad (4.8)$$

As k has perfect information about l's inventory levels, we assume that y_l is arbitrary but fixed. Then, k's profit function Π_k only depends on his own inventory level y_k, and the CRP transforms into an MRP denoted by MRP-C. By virtue of Theorem 4.1, MRP-C has a unique solution if the revenue function $\pi_k(y_k, \cdot)$ is continuous and strictly concave. Consequently, CRP has a unique solution, which can be found by Algorithm 4.1, replacing $\pi_i'(y_i)$ with $\pi_{ik}'(y_{ik}) = \frac{\partial \pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il})$.

So far, we have shown that player k can determine his unique optimal inventory levels given the inventory levels y_l of competitor l. However, l also solves a CRP with respect to k's inventory levels. Consequently, we get a non-cooperative game where both players solve the CRP simultaneously. We denote this game as the *Competitive Rebalancing Game (CRG)*.

Naturally, the question arises if there exists a solution $(y_k^*, x_k^*, y_l^*, x_l^*)$ to CRG where no player can gain a better solution by changing his inventory level and if this solution is unique. In other words, does the CRG posses a unique Nash equilibrium? In Theorem 4.2 we state sufficient conditions for such a unique solution to the CRG.

Theorem 4.2 (Existence of a Nash equilibrium). *Let $\mathcal{I}_{ik}(y, x) := \frac{\partial \mathcal{L}_k}{\partial y_{ik}}(y, x, \lambda_k, \alpha_k) = 0$ be the reaction curve of player k in zone i and l the competitor.*

$$z_{ik} = -\frac{\partial \mathcal{I}_{ik}}{\partial y_{ik}} / \frac{\partial \mathcal{I}_{ik}}{\partial y_{il}}, \quad z_{il} = -\frac{\partial \mathcal{I}_{il}}{\partial y_{ik}} / \frac{\partial \mathcal{I}_{il}}{\partial y_{il}}$$

are the corresponding implicit differentials. If $\pi_{ik}(y_{ik}, \cdot)$ is a twice continuously differentiable and strictly concave function such that the limits $\lim_{y_{il}\to 0} \mathcal{I}_{ik}(y, x) \in \mathbb{R}$, $\lim_{y_{il}\to\infty} \mathcal{I}_{ik}(y, x) \in \mathbb{R}$ exist, then there exists a Nash equilibrium $(y_k^*, x_k^*, y_l^*, x_l^*)$ to (4.8), such that $\Pi_k(y_k^*, x_k^*, y_l^*) \geq \Pi_k(y_k, x_k, y_l^*)$ and $\Pi_l(y_l^*, x_l^*, y_k^*) \geq \Pi_l(y_l, x_l, y_k^*)$ for all y_l, x_l, y_k, x_k feasible to CRP. If $z_{ik} < z_{il} < 0$ holds, the solution is unique.

Proof: see Section 4.6.

The fact that $\mathcal{L}_k$ is decreasing in y_{il} allows us to develop Algorithm 4.2 as an iterative method that determines the Nash equilibrium in the CRG. Due to Theorem 4.2, Algorithm 4.2 will always converge. To cope with varieties of customer behavior, we look at

Algorithm 4.2 Solution algorithm for the CRG

Initialization initial inventories y_1^0, y_2^0; $y_1^1 := 0$, $y_2^1 := 0$; $m = 1$
while $y_1^m \neq y_1^{m-1} \wedge y_2^m \neq y_2^{m-1}$ **do**
 Compute the optimal inventory $\hat{y}_k$ by solving the CRP for player k with y_k^0 and y_l^{m-1} given.
 Set $y_k^m = \hat{y}_k$.
 Update $m = m + 1$.
end while

stock-out-based as well as availability-based substitution. In the former case, customers only substitute in a zone when the preferred provider is out of stock, while in the latter case, customers make their choice based on the observed availability. These two different approaches allow us to incorporate two logics for allocating demand to the players as pointed out by Lippman and McCardle (1997). Either the players' independent demands form the total demand for car-sharing vehicles, or the total demand is split among the players by some rule. The stock-out-based case follows the logic of individual demands, as the players have individual demand assessments in every zone. As opposed to this, we use the ratio of stocked cars to split the total demand in each zone between the two players in the availability-based case.

Stock-out-based substitution

We model stock-out-based substitution following the idea of Parlar (1988): If a player is out of stock, a certain percentage of his unsatisfied customers will switch to the competitor, whereas the remaining customers leave the zone without using a car. Consequently, a player should try to satisfy the *real demand*, that is the demand of his own customers plus the overflow of unsatisfied customers from the competitor.

To consider this overflow in our model, we introduce the *substitution rate* $b_{ik} \in [0;1]$. It denotes the share of player l's unsatisfied demand that player k can attract in zone i. $B_{ik}(d_k, y_{il}) := \frac{y_{ik}-d_k}{b_{ik}} + y_{il}$ is the maximum demand stemming from l that k can serve (Parlar, 1988). Reformulating the revenue function yields:

$$\pi_{ik}(y_k, y_l) = s_{ik} \cdot \Big(y_{ik} - \int_0^{y_{ik}} F_{ik}(d_k) \mathrm{d}d_k + \int_0^{y_{ik}} \int_{y_{il}}^{B_{ik}(d_k, y_{il})} b_{ik} \cdot (d_l - y_{il}) \cdot f_{il}(d_l) f_{ik}(d_k) \mathrm{d}d_l \mathrm{d}d_k + \\ + \int_0^{y_{ik}} \int_{B_{ik}(d_k, y_{il})}^{\infty} (y_{ik} - d_k) \cdot f_{il}(d_l) f_{ik}(d_k) \mathrm{d}d_l \mathrm{d}d_k \Big). \tag{4.9}$$

By plugging (4.9) into the profit function $\Pi_k(y_k, x_k, y_l)$, we obtain the *Competitive Rebalancing Problem under stock-out-based substitution* (CRP-st). The profit function fulfills the conditions of Theorem 4.1 as stated in Corollary 4.2 (a). By virtue of this corollary, we can deduce that, in the stock-out-based case, player k can determine his optimal car stocks given competitor l's car balance and vice versa. This results in a Nash game that we denote CRG-st. As shown in Corollary 4.2 (b), the stock-out-based revenue function (4.9) fulfils the conditions of Theorem 4.2.

Corollary 4.2 (Existence and uniqueness of CRP-st and CRG-st)**.** *(a) There exists a unique solution to CRP-st. (b) There exists a unique Nash equilibrium solution to CRG-st.*

Proof: see Section 4.6.

Hence, using Algorithm 4.2 in combination with Algorithm 4.1, CRG-st can be solved.

Availability-based substitution

We consider the case where each potential customer chooses between both players in zone i. I.e., there is a certain total demand for car-sharing services in each zone i. Following the idea of availability-based demand allocation by Cachon (2003, p. 272), we split the demand between the two players proportionally to their inventory levels in the corresponding zone. The share gained by player k with y_{ik} inventory in zone i is defined as

$$q_{ik}(y_{ik}, y_{il}) := \frac{y_{ik}}{y_{ik} + y_{il}}. \tag{4.10}$$

Note that q_{ik} constitutes a rate that determines the portion of customers that will choose player k's cars. Let D_i be a random variable that describes the total demand in zone i, and f_i and F_i the corresponding probability density and cumulative density functions. The demand that player k expects in zone i is $q_{ik}(y_{ik}, y_{il}) \cdot D_i$. Thus, a stock of y_{ik} suffices to cover a total demand of $y_{ik}/q_{ik}(y_{ik}, y_{il}) = y_{ik} + y_{il}$. This is reflected in the revenue function:

$$\begin{aligned}\pi_{ik}(y) &= s_{ik} \cdot \Big(\int_0^{y_{ik}+y_{il}} q_{ik}(y_{ik}, y_{il}) \cdot d_i \cdot f_i(d_i) \mathrm{d}d_i + \int_{y_{ik}+y_{il}}^{\infty} y_{ik} \cdot f_i(d_i) \mathrm{d}d_i \Big) = \\ &= s_{ik} \cdot \Big(y_{ik} - \int_0^{y_{ik}+y_{il}} q_{ik}(y_{ik}, y_{il}) \cdot F_i(d_i) \mathrm{d}d_i \Big) \ . \end{aligned} \tag{4.11}$$

Inserting (4.11) into the profit function $\Pi_k(y_k, x_k, y_l)$ yields the *Competitive Rebalancing Problem under availability-based substitution* (CRP-ab). Given competitor l's inventory levels, player k can determine a unique solution to CRP-ab. As Corollary 4.3 proves, such a unique solution exists.

We dub the resulting game the *Competitive Rebalancing Game under availability-based substitution* (CRG-av). The corresponding revenue function (4.11) fulfils the conditions of Theorem 4.2 as shown in Corollary 4.3.

Corollary 4.3 (Existence and uniqueness of CRP-ab and CRG-ab)**.** *(a) There exists a unique solution to CRP-ab. (b) There exists a unique Nash equilibrium solution to CRG-ab.*

Proof: see Section 4.6.

Thus, we can apply Algorithm 4.2 to determine the unique solution of CRG-av.

4.2.4 Comparative statics

Making use of the fact that y^*_{ik}, the optimal target inventory in each zone, is determined by solving

$$\frac{\partial \pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il}) - \lambda_{ik} - \alpha_k = 0, \tag{4.12}$$

we discuss some qualitative properties of the objective functions for the two cases of substitution. During the analysis, we always assume that we make changes to only one parameter at the same time.

An observation that holds for MRP and CRP is the influence of the cost on the repositioning policy made by Çömez et al. (2012). The costs affect the inventory levels of all receiving zones r, as $\lambda_{rk} = c_k$ in (4.12). A higher c_k leads to a lower $\hat{y}_{rk}$ and therefore to more cars in the sending zones. Thus, our model complies with a fundamental assumption: the higher the repositioning costs are, the less willing a provider is to move cars.

Stock-out-based case

For the stock-out-based case, the derivative of the profit function is

$$\frac{\partial \pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il}) = s_{ik}\Big(1 - \int_0^{y_{ik}} f_{ik}(d_k) \cdot F_{il} \cdot \big(B_{ik}(d_k, y_{il})\big)\, \mathrm{d}d_k\Big) \tag{4.13}$$

with $B_{ik}(d_k, y_{il}) = \frac{y_{ik}-d_k}{b_{ik}} + y_{il}$.

$\frac{\pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il})$ is decreasing in B_{ik}. Thus, if y_{il} is increasing, then y^*_{ik} decreases. That means, if $k's$ competitor l increases the number of cars in a zone, player k reduces the number of cars in this zone. As a consequence, k increases the number of cars in those zones j where y_{jl} was reduced due to the increase in i.

As B_{ik} is inversely proportional to b_{ik}, a reduction of b_{ik} leads to a reduction of y^*_{ik}. This can be explained by the fact that the smaller the substitution rate b_{ik} is, the fewer of $l's$ customers are willing to substitute with k's cars. Hence, k's real demand is reduced. In the extreme case of no substitution, i.e. $b_{ik} = 0$, $F_{il}(B_{ik}(d_k, y_{il})) = 1$ holds and player k solves the MRP, as he does not expect any demand overflow from l. When l's expected demand grows, $F_{il}(B_{ik}(d_k, y_{il}))$ becomes smaller and thus y^*_{ik} bigger. That means, when l cannot react to increasing demand in a zone, k will increase his number of cars in this zone to skim some of l's unsatisfied demand.

Based on these observations, we can conclude that, in the stock-out-based case, each provider tends to serve his primary demand first. If it is profitable, he will distribute the cars in such a way that he can skim the competitor's demand overflow.

Availability-based case

From the derivative of the availability-based profit function,

$$\frac{\partial \pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il}) = s_{ik} \cdot \Big(1 - F_i(y_{ik} + y_{il}) + \frac{y_{il}}{(y_{ik} + y_{il})^2} \int_0^{y_{ik}+y_{il}} d_i f_i(d_i) \mathrm{d}d_i\Big), \tag{4.14}$$

we can deduce that player k reacts differently to changes of l's parameters. When y_{il} increases, a bigger y^*_{ik} is required to satisfy (4.12). Hence, a higher number of l's cars will also motivate k to increase the number of cars in the respective zone. Analogously, when l reduces the number of cars in a zone, k will also reduce the number of cars down to the point where he can still serve the now bigger market share.

Summarizing, availability-based demand allocation motivates the players to compete for customers.

4.2.5 Rebalancing with fixed-cost

So far, we have considered the single-period case. In the multi-period case, the inventory levels in period t can influence the demand and repositioning in the following periods $t + 1, t + 2, \ldots$. From the perspective of a multi-period setting, solving the single-period

MRP or CRP is myopic since the inventory levels can influence customer behavior and the demand might be time-dependent. However, assuming that the car-sharing operators are doing the repositioning on their own hand, the cost of repositioning can be regarded as fixed cost (in contrast to the service provider model, where the operator has to pay c_k for each single car). Under this assumption we can set $c_k = 0$. Then the myopic, single-period model, is also optimal for the multi-period case since the operators can establish in each period the optimal car distribution at no additional cost. Therefore, planning with foresight is not necessary and thus we can state the following proposition.

Proposition 4.1 (Myopic policy is optimal when cost are zero)**.** *If* $c_m = 0 \; \forall \; m \in \{k, l\}$, *then the optimal policy is to solve the MRP or CRP, respectively, in each period.*

4.3 Computational study

In this section, we present results of the computational study we designed to understand the behavior of the two players (dubbed *P1* and *P2*) in the CRG. Our three primary goals in this section are: (1) to study the different reactions of the players under stock-out-based and availability-based substitution, (2) to show the differences to the monopolistic case, and (3) to examine the sensitivity of the model to different parameters.

To highlight the effects of competition on the optimal car distribution, the special case of two zones is considered since changes in the optimal number of cars in one zone directly influence the choice in the other zone.

We apply Algorithm 4.2 to obtain the optimal solutions and compare the results to the solutions of the MRP (Algorithm 4.1), which are evaluated under the respective profit functions of the competitive cases. For the evaluation of the results, the following three key performance indicators are used: profit, number of cars rebalanced and availability. This allows us to quantify the effects of ignoring the presence of a competitor. To model the demand, we choose the gamma distribution for the probability density function.

Availability is measured by computing the β-service-levels. For the case of availability-based substitution, we can use the traditional β definition. However, in the case of stock-out-based substitution we have to use a *customer-oriented* β-service-level as the effect of substitution has to be taken into account.

Remark 4.1 (Customer-oriented β-service-level)**.** *Let* D_m *be the demand that player* $m \in \{k, l\}$ *faces,* S_m *the stock available and* b_m *the substitution rate to player* m*. The*

Table 4.1: Overview of the six scenarios used in the computational study.

Scenario	1	2	3	4	5	6
Charac-teristic	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	demand in zone 1 is twice as high	P2's coefficient of variation is 0.5	P1's fleet 3 times bigger than P2's

customer-oriented β-service-level is defined as

$$\beta_k^C := \frac{\min\{D_k, S_k\} + \min\{b_k(D_k - S_k)^+, (S_l - D_l)^+\}}{D_k} \ . \tag{4.15}$$

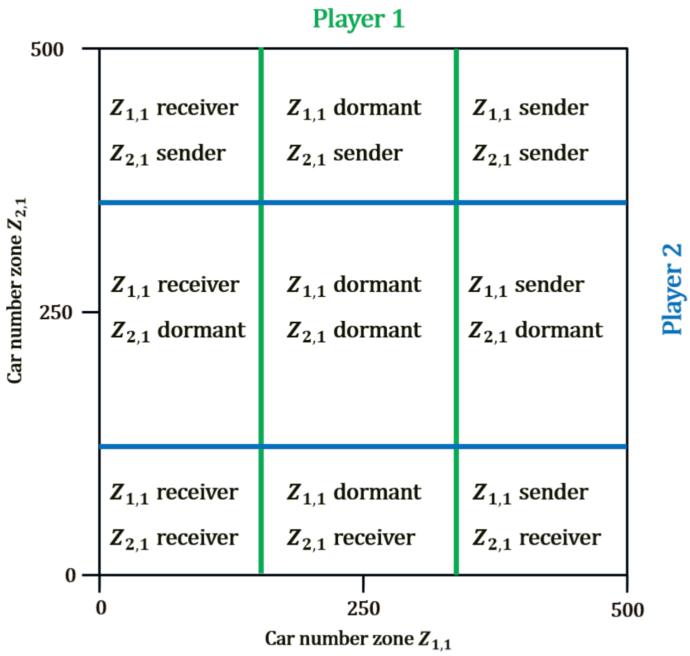
That is, β_k^C measures the demand assigned to player k that can be served either from k's inventory or by (partial) substitution from l's excess inventory, if available.

4.3.1 Experimental set-up

To understand the effects of different parameters and settings in the two-zone case, we consider six scenarios. In the reference scenario 2, each player owns a fleet of 500 cars and the mean demand in every zone is 250 cars. That is, the mean demand equals the capacity of each player. The coefficient of variation is set to 0.3. The profit per car is 4 and the cost of rebalancing is 1. The substitution rates are set to 50%. Note that, in the case of availability-based demand allocation, we sum up the players' demands to obtain the total industry demand (cf. Lippman and McCardle (1997)). Since the effect of varying rebalancing cost has already been studied by Çömez et al. (2012), we forego this analysis.

In the first three scenarios, the parameters for both players and both zones are identical and we only vary $\rho := \frac{\textit{mean demand}}{\textit{fleet size}}$, the ratio of the mean demand to the fleet size. To study the effect of different demands in the zones, we set the mean demand in scenario 4 twice as high for zone 1 as for zone 2 while both players are identical. As customer demand may vary more, we increase the coefficient of variation for P2 in scenario 5 to 0.5. The case where one player has a bigger fleet than his competitor is reflected in scenario 6, where we assume that P1's fleet consists of 750 vehicles, whereas P2 has 250 cars. Table 4.1 gives an overview of the characteristics of the six scenarios. The initial distribution of the cars influences the reaction of the players. To quantify these reactions, we study two different instances that reflect rather extreme cases, but in return provide

Figure 4.1: Example of a barrier graph that visualize the inventory levels at which player 1 (green) and player 2 (blue) change the status of zone 1



meaningful insights. In the first instance, P1 has 80% of his fleet located in zone 1 whereas P2 has 20% of his fleet positioned there. In the second instance, both players have 20% of their initial fleet located in zone 1.

4.3.2 Barrier graphs

In addition to the quantitative analysis, we introduce *barrier graphs* to visualize the players' reactions. Without loss of generality, we draw the barrier graphs for zone 1. These graphs show the initial car levels at which the players change the state of the zone. That is, from *receiving* to *dormant* and from *dormant* to *sending* (and vice versa). Thereby, we make use of the special case of having only two zones: a zone can only be sending when the other one is receiving. Hence, the zones are complementary to each other. Since there are three states for zone 1 for both players, each barrier graph is partitioned into nine sections to show the simultaneous reactions of the players. Thus, the barrier graphs provide at one glance the players' reactions at different initial starting levels. Figure 4.1 gives an example of a barrier graph. Player 1 is colored in green, player 2 in blue.

Table 4.2: Changes in the profit under stock-out based (st) and availability-based (ab) substitution, relative to the monopolistic case. All values are given in percentages.

Scenario		1		2		3		4		5		6	
Player		P1	P2	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
Instance 1	st	0	0	4	4	7	7	2	2	5	5	25	3
	ab	5	5	12	12	8	8	15	9	9	19	45	4
Instance 2	st	1	1	3	3	2	2	1	1	3	4	21	5
	ab	1	1	4	4	1	1	1	1	0	10	39	9

Table 4.3: Changes in the number of repositioning rides under stock-out based (st) and availability-based (ab) substitution, relative to the monopolistic case. All values are given in percentages.

Scenario		1		2		3		4		5		6	
Player		P1	P2	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
Instance 1	st	0	0	-6	-6	-100	-100	-4	0	-9	-6	5	-100
	ab	-48	-48	-93	-93	-100	-100	-100	-18	-98	-98	-8	-100
Instance 2	st	7	7	3	3	-22	-22	3	3	3	5	54	-24
	ab	26	26	-3	-3	-55	-55	-2	-2	-10	4	60	306

4.3.3 Varying the mean-demand-fleet ratio

The difference between scenarios 1, 2 and 3 lies in ρ. That is, in the first scenario $\rho = 0.5$ for the players. Both of them have more cars than required to cover all demand. In the second scenario, we set $\rho = 1$ and in the third scenario $\rho = 1.5$.

The monopolistic case shows how the different ρ's influence the players' behavior. If the ratio is low, the players can only rent out a part of their fleet. Therefore, they reposition a minimum number of cars to cover possible shortages in a zone. If the ratio equals 1, the players relocate more cars as no car will be sitting idle if brought to the right zone. If the demand exceeds the fleet size, the players need to reposition fewer cars than otherwise. If we assume stock-out-based substitution, we can observe a different behavior. In Figure 4.2(b), we can see the case with $\rho = 0.5$. Comparing these graphs to the monopolistic case, one can see that the barriers changed. The kinks show that, whenever one player initially dominates a zone, the competitor is more reluctant to move more cars to the zone and thus the players avoid fierce competition. On the other hand, when both players start with similar imbalances, they reposition more cars to serve some

Table 4.4: Changes of the β-service-levels under stock-out based (st) and availability-based (ab) substitution, relative to the monopolistic case. All values are given in percentages.

Scenario		1		2		3		4		5		6	
Player		P1	P2	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
Instance 1	st	0	0	2	2	0	0	29	9	1	4	0	50
	ab	0	0	5	5	0	0	29	9	5	9	-1	98
Instance 2	st	0	0	1	1	0	0	9	9	1	1	0	50
	ab	0	0	-1	-1	0	0	9	9	-3	1	-6	89

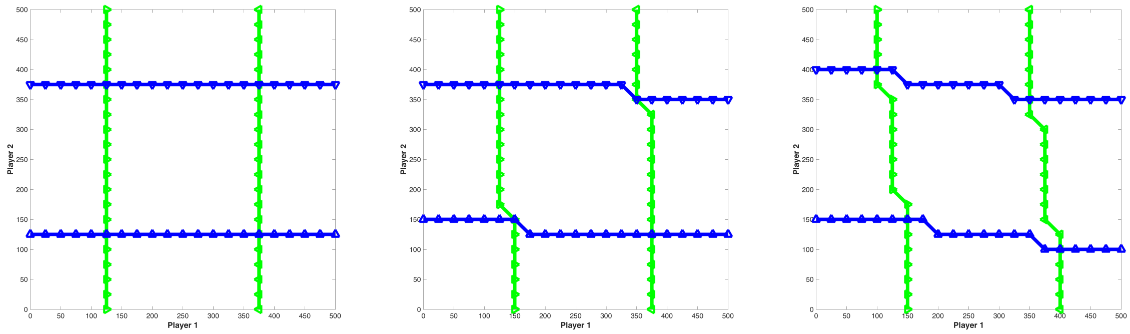
of the competitor's unsatisfied demand. The competitor will follow the same strategy and thus competition increases. These two different strategies can also be seen in the results for instance 1 and 2. In instance 1, the players react as in the monopolistic case, in instance 2, the growth in the number of repositionings is 7.4%. Thus, although the revenues increase due to substitution, the profit increase is reduced by the higher rebalancing cost (1%).

For $\rho = 1$, we observe reactions similar to those as outlined in the aforementioned case (cf. Figure 4.3 (a)). That is, when one player initially dominates a zone, the players are even more reluctant to compete and thus share the market by skimming the competitor's unsatisfied demand. Thus, in instance 1 the repositionings are reduced by 6%. This helps also to increase profit by 4%. Conversely, in instance 2, the rivals intensify competition, moving more cars to zone 1. This results in 3% more movements. However, as the β-service-level increase by 5% shows (cf. Table 4.4), it helps to serve more customers. This results in a profit increase of 3%.

In scenario 3, where the demand exceeds the fleet size, the players make use of the effect of substitution. That is, they drastically reduce the repositionings because they know that unsatisfied customers from the competitor will also use their fleet. This can also be seen from Figure 4.4 (b): the more cars a competitor has initially positioned in a zone, the lower is the level at which a player stops bringing or removing cars from the zone. In instance 2, the players reduce the repositionings by 22%, in instance 1 there are no repositionings at all. As one can see, the service levels do not improve. But thanks to the savings from fewer repositionings, the profits increase by 7% and 2%, respectively.

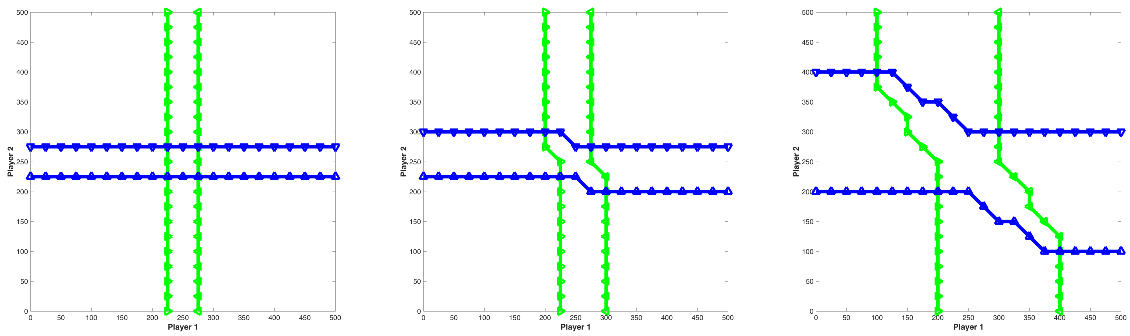
When the demand allocation is availability-based, the players are even more sensitive to the competitor's reaction. In general, the barrier graphs (Figures 4.2 (c), 4.3 (c) and 4.4 (c)) show a similar behavior as under stock-out-based substitution. If both

Figure 4.2: Scenario 1: The cumulated mean demand falls below the fleet size.



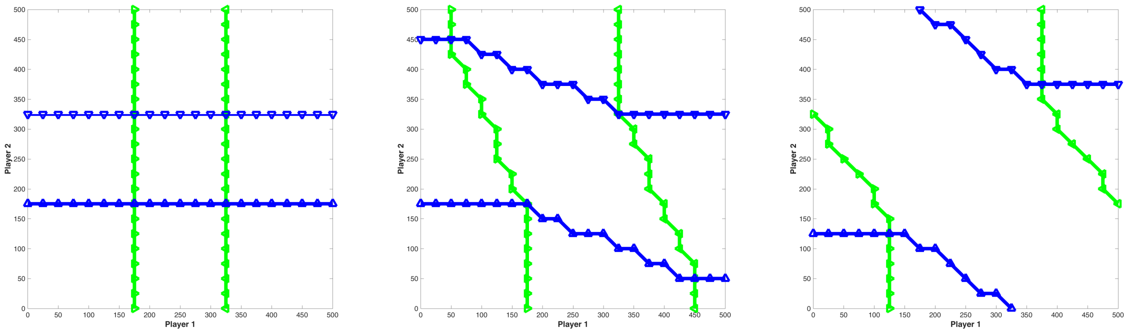
(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

Figure 4.3: Scenario 2: The cumulated mean demand equals the fleet size.



(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

Figure 4.4: Scenario 3: The cumulated mean demand exceeds the fleet size.



(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

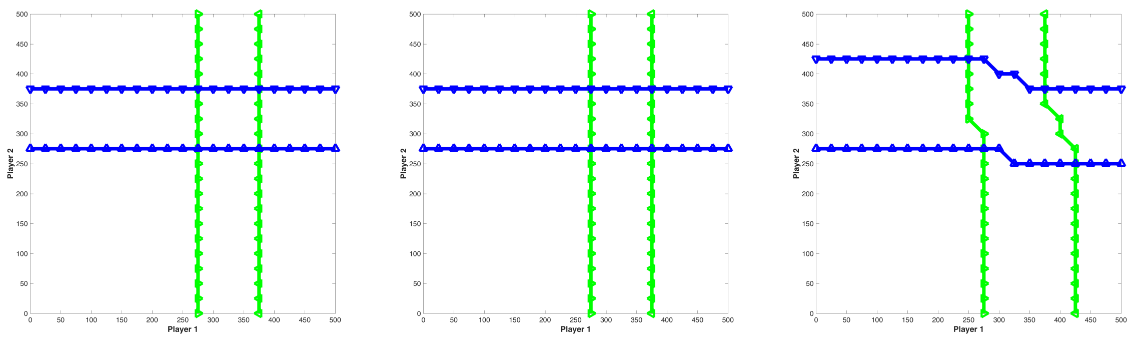
players initially face similar imbalances, they compete in both zones, whereas whenever one player dominates a zone, they share the market. An interesting reaction can be observed in scenario 3 where ρ is 1.5 (Figure 4.4 (c)). If the competitor initially has more than 325 cars in a zone, the player will not reposition any cars to this zone but instead focus on the other zone. This is because the player would have to reposition many cars to gain a sufficiently high market share in that zone. However, as there is enough demand for the player in the other zone, the two players share the market. Generally, the sometimes substantial profit increases in the first three scenarios can be attributed to the significant decreases in rebalancing. In scenario 2, instance 2, the players even accept a lower β-service-level (decreases by 1%) in exchange for fewer repositionings. An exception from this observation is scenario 1, instance 2: here, the players mutually incite each other to move more cars to zone 1 and thus to increase the revenues.

4.3.4 Higher demand in zone 1

As the mean demand in zone 1 is two times higher than in zone 2, the players move more cars to zone 1 in the monopolistic case.

Under stock-out-based substitution, the players also focus more on zone 1. The barrier graph (Figure 4.5 (b)) is identical to the monopolistic case. In instance 1, P1 slightly reduces the number of rebalancings, whereas P2 does not change anything. That is, both players make use of the substitution. In instance 2, where both players have a low number of cars in zone 1, they both increase the repositioning by 3% to cover all demand in zone 1 and thus to avoid customer losses due to substitution.

Figure 4.5: Scenario 4: The demand in zone 1 is twice as high as in zone 2.



(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

In the case of availability-based demand allocation, the barrier graph (Figure 4.5 (c)) shows that the players also focus more on zone 1. In instance 1, P1 refrains from repositioning any cars, whereas P2 keeps a few more cars in zone 2 to make use of his dominance there. In instance 2, both players send enough cars to zone 1 to attract half of the demand there.

4.3.5 Higher uncertainty

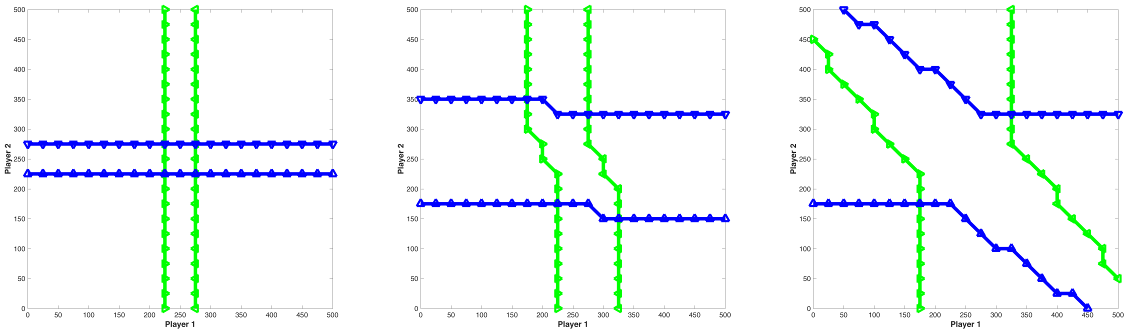
Higher uncertainty leads to a reduction of the rebalancing operations under stock-out-based substitution. P2 makes use of the substitution to increase his profits. That is, instead of relocating cars to serve his own more uncertain demand, P2 keeps cars to serve parts of P1's more certain demand. P1 reacts to this strategy by also moving fewer cars.

When the demand allocation is availability-based, both players share the demand. Hence, they both face a higher standard deviation and therefore react in the same way, as Figure 4.6 (c) shows. The reason for the different values reported is the different basis of comparison since in the monopolistic case P2 had a higher coefficient of variation.

4.3.6 Big and small player

The case of a big and a small player demonstrates how beneficial it can be for both providers to take customer behavior into account. In scenario 6, P1 owns a fleet that is three times bigger than P2's fleet. P1's ρ is 0.5, whereas P2's ρ is 1.5.

Figure 4.6: Scenario 5: P2 faces higher uncertainty.



(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

Under stock-out-based substitution, P1 exploits the situation that P2 can only partially satisfy his demand. Therefore, P1 increases his repositioning activities: in instance 1 by 5% and in instance 2 even by 54%. P2, on the other hand, reduces his repositionings substantially as P2's demand exceeds the number of cars available; he does almost no repositionings, which makes the reaction similar to the case of ignorance.

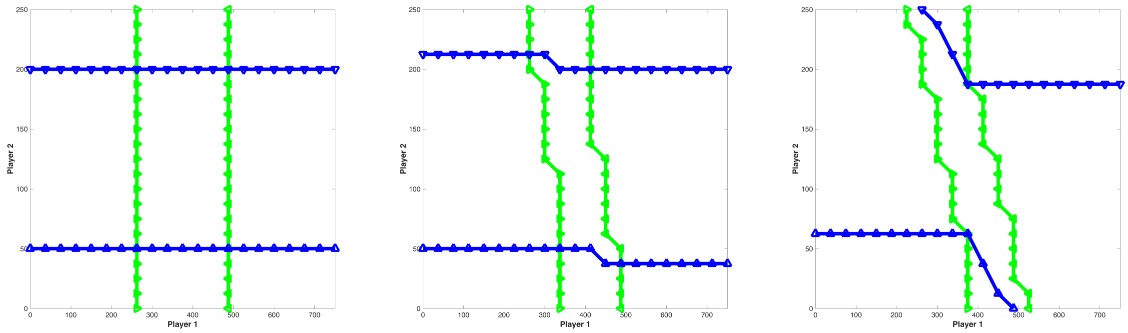
When the customers' decision is availability-based, P1 reacts differently in instance 1. That is, he repositions 8% fewer cars, thus increasing his dominance in zone 1. P2 does not reposition any cars. In instance 2, P2 moves four times as many cars, whereas P1 moves 60% more. That is, P2 tries to increase his share in zone 1. P1's decreased service level in both scenarios stems from the fact that, due to the availability-based allocation, more demand is assigned to P1.

The barrier graphs (Figures 4.7) show how P1 increases his activity, whereas P2 is more reluctant about rebalancing cars. This is particularly obvious for the availability-based case in Figure 4.7 (c), where P2 does not react at all if P1 initially has 500 or more cars in that zone.

4.3.7 Different substitution rates

In the previous sections, we studied the stock-out-based case under the assumption of a substitution rate of 0.5 for both players. To get better insights into the influence of the substitution rates, we now consider four additional scenarios with different substitution rates and a ρ of 1 (cf. Table 4.5). When the substitution rate is set to 0.01%, the relative changes over the monopolistic case are 0%. This is due to the fact that the

Figure 4.7: Scenario 6: The big P1 faces a ρ of 0.75, the small P2's ratio is 2.



(a) monopolistic case **(b)** stock-out-based subst. **(c)** availability-based subst.

Table 4.5: Relative changes of the profit, number of rebalancing rides and β-service-levels when increasing the substitution rate in the stock-out-based case. All values are given in percentages.

Substitution rate		0.01%		25%		50%		75%		100%	
	Player	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
Instance 1	Profit	0	0	2	2	4	4	5	5	6	6
	Reba	0	0	-2	-2	-6	-6	-11	-11	-17	-17
	β-SL	0	0	1	1	2	2	3	3	5	5
Instance 2	Profit	0		2		3		3		4	
	Reba	0		2		3		3		4	
	β-SL	0		1		1		1		1	

CRP-st approximates the MRP when the substitution rate tends towards zero, as the upper limit of the integral in the revenue function (4.9) becomes

$$\lim_{b_{ik}\to 0} B_{ik}(d_k, y_{il}) = \lim_{b_{ik}\to 0} b_{ik}^{-1}(y_{ik} - d_k) + y_{il} = \infty.$$

As one would expect, the revenues increase with increasing substitution rates, since each player can serve more of the unsatisfied demand from his competitor. The decrease of repositionings in instance 1 and the increase in instance 2 (see Table 4.5) are also in line with our previous findings, since the players can serve a bigger market. When the substitution rate is at 100%, the demand is completely pooled among the two players and the players can increase their profits by 4% and 6.3%, respectively. Additionally, the service level increases by up to 5%.

4.3.8 Summary of insights

From the controlled computational study, we deduce three main findings:

1. Stock-out-based and availability-based substitution lead to different competitive behavior. Under availability-based substitution, the number of cars in a zone directly influences the players' shares. Thus, they send only the required number of cars to serve the demand share. On the other hand, this leads to more competition as a higher number of competitor's cars forces a player to increase the number of his cars available in that zone.

   Under stock-out-based substitution, the players focus on serving their own demand and additionally skim the competitor's unserved demand. Therefore, the deviations in the rebalancing are typically lower than in the availability-based case. However, if a player initially dominates a zone, the Nash equilibrium suggests to make use of the substitution instead of rebalancing cars.

2. The initial distribution of the fleet is the main driver for an increase or decrease in the number of repositionings. If one player initially dominates a zone, the competitor will focus on serving the player's unserved demand in the other zone. In contrast, when both players have few cars in a zone, the number of repositioned cars increases.

3. Substitution is typically beneficial for operators and customers. The clients enjoy a higher availability of car-sharing vehicles, while the companies increase their profits. In certain cases, substitution can even help to reduce the number of repositionings.

4.4 Case study

In our case study, we use real booking data from Munich, Germany, to quantify the extent to which car-sharing providers could increase their profits by taking consumer behavior into account.

4.4.1 Data description and current situation

Until February 2019, the two car-sharing providers *car2go* and *DriveNow* were competing in Munich with a free-floating system with almost identical business areas. Over a period of one year (May 2016 to May 2017), we recorded a total of 711,686 bookings of DriveNow cars and 560,748 bookings of car2go vehicles. During this time, DriveNow operated a fleet of 880 cars, car2go had 490 vehicles in use. On average, a DriveNow vehicle was booked for 58 minutes, a car2go car for 39 minutes. Thus, we estimate that in these twelve months DriveNow generated a revenue of EUR 14.0 million in Munich, while car2go's revenue was approximately EUR 6.4 million.

To find out if the two companies took the presence of their direct competitor into account, we looked at the service levels of both companies. To this end, we divided the shared business area into 24 zones and determined the α-service-levels and β-service-levels for both providers over the entire period. Then, we looked at the correlation between these levels and their evolution over time. In busy zones, the performance of the two providers are highly correlated. However, in most of the zones, a moderate and varying correlation (between 0.3 and 0.6) occurred. Since the smallest correlation value reported was -0.046, a systematic sharing of the market similar to the one shown in the computational study was not observed. In sum, these findings indicate that the two providers ignored the presence of a competitor in the observed time period.

4.4.2 Results of the case study

To demonstrate by how much car2go and DriveNow could increase their profits if they took competition into account, we solved the multi-period CRG, using the approximation discussed in Section 4.2.5 for both providers. For that purpose, we computed the average demand and variance for each zone of the 24 zones as well as the average trip rates ω_{ij}. Furthermore, we determined the revenue for renting out a car in zone i by multiplying the average travel time for a trip starting in zone i with the price per minute charged by the corresponding provider. As we learned from a practitioner, the providers reviewed the optimal positioning of the cars four to six times a day. Thus, we assumed a time interval of four hours and determined the maximum likelihood estimates for the parameters of the gamma distribution for this interval. The resulting fleet-demand-ratio ρ for the four hour interval was 0.65 for car2go and 0.51 for DriveNow.

Analogously to the computational study, we compared the profit, the number of repositionings and the β-service-level of the rebalancing under stock-out-based substitution, and availability-based demand allocation to the monopolistic case (see Table 4.6). The results show that both providers could increase their profits if they considered the presence of a competitor in their planning. DriveNow, which owned almost twice as many cars as car2go, could have raised its profits by 9% under availability-based substitution

Table 4.6: Results of the case study relative to the monopolistic case. Absolute numbers are given in brackets.

	DriveNow		car2go	
	st	ab	st	ab
Profit	6% (842,000 EUR)	9% (1,236,000 EUR)	4% (253,000 EUR)	5% (317,000 EUR)
Rebalancing	8% (4,319 cars)	32% (15,243 cars)	3% (1,784 cars)	8% (4,841 cars)
β-service-level	2% (87%)	1% (86%)	5% (97%)	3% (95%)

and by 6% under stock-out-based substitution. This corresponds to more than EUR 1.3 million and EUR 0.8 million in additional earnings in that year, respectively. car2go could have increased its profits by 4% (EUR 253,000) under stock-out-based substitution and by 5% (EUR 317,000) if we assume availability-based demand allocation.

We observe that under availability-based demand allocation, both providers increase the number of cars in those zones that promise high revenues; either due to high demand or due to high prices. This strategy leads to a significant increase in the repositionings: DriveNow moves 32% more cars and car2go 9% more cars. Under stock-out-based substitution, we observe that the two providers tend to share the market and thus reduce competition. This is due to the reason that the fleet-demand-ratio is below one for both providers. Hence, the probability that a provider is out of stock in a zone is small. Nevertheless, rebalancing increases by 8% for DriveNow and 3% for car2go.

Competition is advantageous for customers by means of the β-service-level. Table 4.6 shows that especially car2go's customers can benefit. This is due to the fact that DriveNow tries to serve car2go's unsatisfied customers (in the substitution-based case) or that due to the higher availability of DriveNow cars, customers choose one of those (availability-based case). Thus, the β-service-level for car2go customers increases by 5% for the stock-out-based case and by 3% for the availability-based case. For DriveNow customers, the increases are smaller (2% and 1%, respectively).

4.4.3 Merger of car2go and DriveNow

In February 2019, the BMW Group and Daimler announced the merger of their car-sharing subsidies DriveNow and car2go into the new company *ShareNow* (Moovel Group

GmbH, 2019). Since then, ShareNow has been working on the technical details of this step, while the two brands continued to operate side-by-side. However, since the announcement, customers of DriveNow can access car2go cars and vice versa. For the near future, it is expected that ShareNow will coordinate the rebalancing of both brands. That is, ShareNow would solve a monopolistic rebalancing problem. We quantify the additional value of a joint rebalancing on our data set. The results show that the profits of ShareNow would increase by 17%, this corresponds to an additional profit of EUR 2.55 million per year. Simultaneously, the overall β-service-level would increase from 89% to 93%. Furthermore, thanks to the complete pooling of the cars, the number of repositioning operations would decrease by 23%. This would correspond to 48 fewer repositionings per day, thus 17,500 fewer journeys in one year.

4.5 Conclusion

In this chapter, we formulated a model for two providers who reposition substitutable one-way car-sharing vehicles. The model allows us to study the influence of competition on the repositioning strategies. The model is motivated by the observation that, in many cities, more than one provider offers a car-sharing service. As optimized repositioning plans are vital when it comes to making one-way car-sharing services profitable (Jorge et al., 2014), a growing body of literature has been dedicated to the rebalancing of car-sharing vehicles. However, to our best knowledge, the presence of a competitor has not been modeled so far.

Our model combines inventory transshipment with inventory competition. By applying Lagrangian transformation and deriving the corresponding Karush-Kuhn-Tucker conditions, we show that it suffices to determine the status of the zones for solving the problem. This finding allows us to derive an efficient solution method for the model. To study different customer behaviors, we consider both stock-out-based and availability-based substitution. We prove that there exists a unique solution for the resulting Nash game in both cases.

We show that a provider can increase his profits by taking the presence of a competitor into account. This is mainly due to the awareness that substituting customers can be attracted. We find that the customers' behavior influences the level of competition. In the case of stock-out-based substitution, the providers focus on fulfilling their competitors' unfulfilled demand. Thus, the providers tend to avoid fierce competition. Under availability-based substitution, the providers vie with each other for market shares, which leads to more competition. A major driver for the increase or decrease in the number of repositionings under competition is the initial distribution of the fleets. Especially in cases of large imbalances, the providers make use of the customers' willingness to substitute and therefore share the market, thus reducing repositionings.

Car-sharing is often considered part of public transportation or at least part of the mobility mix. Thus, city planners are interested in a high availability of car-sharing vehicles. One way would be to oblige car-sharing providers to grant customers of the competitor access to their cars. That is, to set the substitution rate to 100%. As our computational study shows, this complete pooling also has the potential to reduce the number of repositionings and thus empty runs.

Some car-sharing providers think about price differentiation in car-sharing. Adapting our approach to such a price setting and analyzing the resulting rebalancing policies seems to be a valuable task for further research. Furthermore, using the model for determining the optimal fleet size constitutes a challenging future research challenge.

The Competitive Rebalancing Game presented in this chapter addresses the question of the optimal rebalancing of car-sharing vehicles under competition. Another interesting application of this model can be the fast-fashion industry, where clothing is moved at short notice between stores to prevent stock-outs and to attract more customers. Due to the growing competition in this industry, planners may also take the reaction of competing fashion chains into account. This example shows that there are different applications for the Competitive Repositioning Game, which closes the gap between inventory transshipment and inventory competition.

4.6 Proofs

Karush-Kuhn-Tucker conditions

$$\frac{\partial \mathcal{L}}{\partial x_i}(y,x,\lambda,\alpha) = -c + \lambda_i \le 0 \qquad i = 1,\dots,n \tag{4.16}$$

$$x_i \cdot (\lambda_i - c) = 0 \qquad i = 1,\dots,n \tag{4.17}$$

$$\frac{\partial \mathcal{L}}{\partial y_i}(y,x,\lambda,\alpha) = \pi_i'(y_i) - \lambda_i - \alpha \le 0 \qquad i = 1,\dots,n \tag{4.18}$$

$$y_i \cdot \big(\pi_i'(y_i) - \lambda_i - \alpha\big) = 0 \qquad i = 1,\dots,n \tag{4.19}$$

$$\lambda_i \cdot \big(y_i^0 - (y_i - x_i)\big) = 0 \qquad i = 1,\dots,n \tag{4.20}$$

$$y_i^0 \ge y_i - x_i \qquad i = 1,\dots,n \tag{4.21}$$

$$\alpha \cdot \Big(\sum_{i=1}^{n} y_i^0 - y_i\Big) = 0 \tag{4.22}$$

$$\sum_{i=1}^{n} (y_i^0 - y_i) \ge 0 \tag{4.23}$$

with $\pi_i'(y_i) = s_i \cdot (1 - F_i(y_i))$.

Proof of Lemma 4.1

a) Due to (4.18), the target inventory levels y_i are decreasing in λ_i and α. From (4.16) it follows that $\lambda_i \le c$. Similarly, α is limited by $\max_{i\in\mathcal{N}} \pi_i'(0)$, as one can deduce from (4.19).

b) For a receiving zone $i \in \mathcal{R}$ it holds that $x_i > 0$. Thus, from (4.17) it follows that $\lambda_i = c$. For sending zones $i \in \mathcal{S}$ it holds that $y_i^0 > y_i$ and $x_i = 0$. Thus, from condition (4.20) the equality $\lambda_i = 0$ follows. For dormant zones $i \in \mathcal{D}$, the marginal cost have to lie in the range $0 < \lambda_i = \pi'(y_i) - \alpha < c$ to fulfill condition (4.17) and inequality (4.19). □

Proof of Theorem 4.1

Consider an arbitrary but fixed zone i. We have to solve $L_i(y_i) := \pi_i'(y_i) - \alpha = 0$ to determine if the zone is receiving, sending or dormant. L_i is a continuous function and the first derivative exists and is also continuous: $L_i'(y_i) = \pi_i''(y_i)$.

Let $s := \max_{i\in\mathcal{N}} s_i$ and choose $\alpha \in (0; s)$. Define $w_i(\alpha, r) := \{y_i : L_i(y_i) - \alpha = r\}$, with $r \in \mathbb{R}_{\ge 0}$ arbitrary but fixed. Since L_i is bijective, it follows from the inverse function theorem (Rudin, 1976, pp. 221) that the inverse L_i^{-1} is continuous and thus w_i is also continuous. Furthermore, it holds by definition of π_i that $L_i' < 0$. Consequently, $w_i(r, \alpha)$ is strictly decreasing in α and r and w_i is bijective.

Next, define the monotonously decreasing function

$$\mathrm{y}_i(\alpha) \coloneqq w_i^{\alpha,c} \ \textit{if } c \leq L_i(y_i^0) - \alpha, \ \mathrm{y}_i(\alpha) \coloneqq y_i^0 \ \textit{if } 0 < L_i(y_i^0) - \alpha < c, \ \mathrm{y}_i(\alpha) \coloneqq w_i^{\alpha,0} \ \textit{if } L_i(y_i^0) - \alpha \leq 0,$$

which determines the optimal inventory based on the initial inventory level y_i^0 (cf. Algorithm 4.1).

Consider the function $\delta(\alpha) \coloneqq \sum_{i=1}^{n} \left(\mathrm{y}_i(\alpha) - y_i^0 \right)$. It holds that $\delta(s) < 0$ and due to the strict monotonicity of w_i there exists an $s > \bar{\alpha} > 0$ with $\delta(\alpha) \geq 0$ for all $\bar{\alpha} \geq \alpha$. δ is monotonously decreasing and, as we will show below, continuous. Thus, due to the intermediate value theorem, δ has at least one root. If this root is not unique, all roots lie in an interval $[\bar{\alpha} + \varepsilon'; \bar{\alpha} - \varepsilon']$, $\varepsilon' > 0$ and all zones are dormant for those α. That is, the solution is unique in the inventory levels y^0.

It remains to show that y_i is continuous. As w_i is a continuous function, it suffices to show the continuity on the boundaries of the three subdomains: Since $0 < c < s$, there exists an $\check{\alpha}$, $0 < \check{\alpha} < s$ such that $w^{\check{\alpha},c} = y_i^0$ and consequently $\mathrm{y}_i(\check{\alpha})$. Since $c > 0$, there exists an $\varepsilon > 0$ with $\mathrm{y}_i(\check{\alpha} + \varepsilon) = y_i^0$. On the other hand, $\mathrm{y}_i(\check{\alpha} - \varepsilon) = w_i^{\check{\alpha}-\varepsilon,c} > y_i^0$ as $w_i^{\alpha,c}$ is strictly decreasing. Consider a sequence $\check{\alpha}_m \to \check{\alpha}$ for $m \to \infty$. Partition the sequence into two sub-sequences $\check{\alpha}_{m_1}$ and $\check{\alpha}_{m_2}$ with $\check{\alpha}_{m_1}$ containing all elements $\check{\alpha}_m > \check{\alpha} - \varepsilon$ and $\check{\alpha}_{m_2}$ all $\check{\alpha} + \varepsilon > \check{\alpha}_m$. Then, $\mathrm{y}_i(\check{\alpha}_{m_2}) = y_1^0 \ \forall m$ and thus $\mathrm{y}(\check{\alpha}_{m_2}) \to \mathrm{y}_i(\check{\alpha})$, $m_2 \to \infty$. For all m_1, it holds that $\mathrm{y}_i(\check{\alpha}_{m_1}) = w^{\check{\alpha}_{m_1},c}$. Since w_i is continuous, $w_i^{\check{\alpha}_{m_1},c} \to w^{\check{\alpha},c} = y_i^0$ and thus $\mathrm{y}_i(\check{\alpha}_{m_1}) \to \mathrm{y}(\check{\alpha})$ for $m_1 \to \infty$. Hence, $\mathrm{y}(\check{\alpha}_m) \to \mathrm{y}(\check{\alpha})$ for $m \to \infty$, which shows the continuity of y_i in $\check{\alpha}$. The continuity in $\hat{\alpha}$ $(0 < \hat{\alpha} < \check{\alpha})$ with $y^0 = w^{0,\hat{\alpha}}$ can be shown analogously. Therefore, y_i is continuous. □

Proof of Theorem 4.2

Consider the Lagrangian function for player k as stated in (4.8). The influence of a zone's inventory level on the inventory levels of all zones of player k is measured by λ_{ik} and α. Thus, it is possible to decouple the system of equations and show the existence of a Nash equilibrium for an arbitrary (but fixed) zone $i \in \mathcal{N}$. As stated in Parlar (1988), the player's reaction curve $\mathcal{I}_{ik}$ can be obtained by partially differentiating the Lagrangian (4.8) with respect to the player's index. $\mathcal{I}_{ik}(y, x) \coloneqq \frac{\partial \mathcal{L}_k}{\partial y_{ik}} = \frac{\partial \pi_{ik}}{\partial y_{ik}} - \lambda_{ik} - \alpha$. The second partial derivatives of $\mathcal{I}_{ik}$ with respect to the players' reactions are (for given $i \in \mathcal{N}$) $\frac{\partial \mathcal{I}_{ik}}{\partial y_{ik}}(y, x) = \frac{\partial^2 \mathcal{L}_k}{\partial y_{ik}^2} = \frac{\partial^2 \pi_{ik}}{\partial y_{ik}^2} < 0$. Thus, $\mathcal{L}_{ik}$ is a strictly concave function in y_{ik}. To show that $\mathcal{I}_{ik}$ is strictly concave in the (y_{ik}, y_{il}) plane, we implicitly differentiate $y_{il}(y_{ik})$ by considering $\mathcal{I}_{ik}$. This gives $z_{ik} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{ik}}{\partial y_{ik}} / \frac{\partial \mathcal{I}_{k}}{\partial y_{il}} < 0$. We obtain a similar result when considering $\mathcal{I}_{ik}$, the competitor's indifference function. Implicitly differentiating $y_{ik}(y_{il})$ with respect to this function, we get $z_{il} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{il}}{\partial y_{ik}} / \frac{\partial \mathcal{I}_{l}}{\partial y_{il}} < 0$, thus showing that $\mathcal{I}_{ik}$ is also strictly decreasing in the (y_{ik}, y_{il}) plane.

To prove that the solution exists, we use the property of $\mathcal{I}_{ik}$ that for any value y_{ik} there is a finite upper and lower bound to the reaction functions: $\lim_{y_{il}\to 0} \mathcal{I}_{ik}(y,x) \in \mathbb{R}$, $\lim_{y_{il}\to\infty} \mathcal{I}_{ik}(y,x) \in \mathbb{R}$. Uniqueness is given if player k's reaction curve is strictly smaller than player l's reaction curve (Parlar, 1988). This holds as $z_{ik} < z_{il}$. □

Proof of Corollary 4.2

(a) The profit function $\pi_{ik}(y_k, y_l)$ is differentiable and thus continuous:

$$= s_{ik}\Big[1 - \int_0^{y_{ik}} f_{ik}(d_k) F_{il}(B_{ik}(d_k, y_{il}))\mathrm{d}d_k\Big] \; .$$

The second derivative

$$\frac{\partial^2 \pi_{ik}}{\partial y_{ik}^2} = -s_{ik}\Big(\int_0^{y_{il}} b_{ik}^{-1} f_{ik}(d_k) f_{il}(B_{ik}(d_k, y_{il}))\mathrm{d}d_k + f_{ik}(y_{ik}) f_{il}(B_{ik}(y_{ik}))\Big) < 0$$

shows that $\pi_{ik}(y_k, \cdot)$ is strictly concave. Hence, the necessary conditions of Theorem 4.1 are fulfilled.

(b) We have already shown in (a) that $\pi_{ik}(y_k, \cdot)$ is twice continuously differentiable and strictly concave. Consider the reaction function

$$\mathcal{I}_{ik}(y,x) := \frac{\partial \mathcal{L}_k}{\partial y_{ik}}(y, x, \lambda_k, \alpha_k) = s_{ik}\Big(1 - \int_0^{y_{ik}} f_{ik}(d_k) F_{il}(B_{ik}(d_k, y_{il}))\mathrm{d}d_k\Big) - \lambda_{ik} - \alpha_k \; .$$

The limits

$$\lim_{y_{il}\to 0} \mathcal{I}_{ik}(y,x) = s_{ik}\Big(1 - \int_0^{y_{ik}} f_{ik}(d_k) F_{il}\big(\frac{y_{ik} - d_k}{b_{ik}}\big)\Big) - \lambda_{ik} - \alpha_k \geq$$
$$\geq s_{ik}\Big(1 - \int_0^{y_{ik}} f_{ik}(d_k)\Big) - \lambda_{ik} - \alpha_k = \lim_{y_{il}\to\infty} \mathcal{I}_{ik}(y,x)$$

are finite. The implicit differentials give

$$z_{ik} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{ik}}{\partial y_{ik}} \Big/ \frac{\partial \mathcal{I}_{ik}}{\partial y_{il}} = -\frac{1}{b_{ik}} - \frac{f_{ik}(y_{ik})F_{il}(y_{il})}{\int_0^{y_{ik}} f(d_k) f_{il}(B_{ik}(d_k, y_{il}))\mathrm{d}d_l} < 0$$

and

$$z_{il} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{il}}{\partial y_{ik}} \Big/ \frac{\partial \mathcal{I}_{il}}{\partial y_{il}} = -\frac{\int_0^{y_{il}} f_{il}(d_l) F_{ik}(B_{il}(d_l, y_{ik}))\mathrm{d}d_l}{\int_0^{y_{il}} b_{il}^{-1} f_{il}(d_l) F_{ik}(B_{il}(d_l, y_{ik}))\mathrm{d}d_l + f_{il}(y_{il})F_{ik}(y_{ik})} < 0 \ .$$

From

$$z_{il} = -\underbrace{b_{il}}_{\leq 1} \frac{\overbrace{\int_0^{y_{il}} f_{il}(d_l) F_{ik}(B_{il}(d_l, y_{ik}))\mathrm{d}d_l}^{=:A}}{\underbrace{\int_0^{y_{il}} f_{il}(d_l) F_{il}(B_{il}(d_l, y_{ik}))\mathrm{d}d_l}_{=A} + \underbrace{b_{il} f_{il}(y_{il}) F_{ik}(y_{ik})}_{>0}} \geq -1 > \underbrace{-\frac{1}{b_{ik}}}_{-1\leq} \underbrace{-\frac{f_{ik}(y_{ik})F_{il}(B_{ik}(d_k, y_{il}))}{\int_0^{y_{ik}} f_{ik}(d_k) f_{il}(B_{ik}(d_k, y_{il}))\mathrm{d}d_k}}_{<0} = z_{ik}$$

we can see that the indifference functions $\mathcal{I}_{ik}$ and $\mathcal{I}_{il}$ are decreasing in the (y_{ik}, y_{il}) plane. Thus, the Nash equilibrium solution exists. Since z_{ik} is strictly smaller than z_{il}, the solution is unique. □

Proof of Corollary 4.3

(a) The profit function $\pi_{ik}(y_k, y_l)$ is differentiable and thus continuous:

$$\frac{\partial \pi_{ik}}{\partial y_{ik}}(y_{ik}, y_{il}) = s_{ik}\Big(1 - \frac{y_{ik}}{y_{ik} + y_{il}} F_i(y_{ik} + y_{il}) - \frac{y_{il}}{(y_{ik} + y_{il})^2} \int_0^{y_{ik}+y_{il}} F_i(d_i)\mathrm{d}d_i\Big).$$

The second derivative

$$\frac{\partial^2 \pi_{ik}}{\partial y_{ik}^2} = -s_{ik}\Big(\frac{y_{ik}}{y_{ik} + y_{il}} f_i(y_{ik} + y_{il}) + \frac{2y_{il}}{(y_{ik} + y_{il})^3} \int_0^{y_{ik}+y_{il}} d_i f_i(d_i)\mathrm{d}d_i\Big) < 0$$

shows that $\pi_{ik}(y_k, \cdot)$ is strictly concave. Hence, the necessary conditions of Theorem 4.1 are fulfilled.

(b) We have shown in (a) that $\pi_{ik}(y_k, \cdot)$ is twice continuously differentiable and strictly concave. Consider the reaction function

$$\mathcal{I}_{ik}(y,x) := \frac{\partial \mathcal{L}_k}{\partial y_{ik}}(y,x,\lambda_k,\alpha_k) = s_{ik}\Big(1 - F_i(y_{ik}+y_{il}) + \frac{y_{il}}{(y_{ik}+y_{il})^2}\int\limits_0^{y_{ik}+y_{il}} d_i f_i(d_i)\mathrm{d}d_i\Big) - \lambda_{ik} - \alpha_k \ .$$

The limits

$$\lim_{y_{il}\to 0}\mathcal{I}_{ik}(y,x) = s_{ik}\Big(1 - F_i(y_{ik})\Big) - \lambda_{ik} - \alpha_k \geq -\lambda_{ik} - \alpha_k = \lim_{y_{il}\to\infty}\mathcal{I}_{ik}(y,x)$$

are finite. The implicit differentials give

$$z_{ik} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{ik}}{\partial y_{ik}} \Big/ \frac{\partial \mathcal{I}_{ik}}{\partial y_{il}} =$$

$$= \frac{\overbrace{f(y_{ik}+y_{il})(1-\frac{y_{il}}{y_{ik}+y_{il}}) + \frac{2y_{il}}{(y_{ik}+y_{il})^3}\int\limits_0^{y_{ik}+y_{il}} d_i f_i(d_i)\mathrm{d}d_i}^{:=-A_k}}{\underbrace{f(y_{ik}+y_{il})(\frac{y_{il}}{y_{ik}+y_{il}}-1) - \frac{2y_{il}}{(y_{ik}+y_{il})^3}\int\limits_0^{y_{ik}+y_{il}} d_i f_i(d_i)\mathrm{d}d_i}_{:=A_k} + \underbrace{\frac{1}{(y_{ik}+y_{il})^2}\int\limits_0^{y_{ik}+y_{il}} d_i f_i(d_i)\mathrm{d}d_i}_{:=B_k}} =$$

$$\frac{\overbrace{-A_k}^{>0}}{\underbrace{A_k + B_k}_{\in(A_k;0)}} < -1$$

and

$$z_{il} := \frac{\mathrm{d}y_{il}}{\mathrm{d}y_{ik}} = -\frac{\partial \mathcal{I}_{il}}{\partial y_{ik}} \Big/ \frac{\partial \mathcal{I}_{il}}{\partial y_{il}} =$$

$$= \frac{\overbrace{f(y_{il}+y_{ik})(\frac{y_{ik}}{y_{il}+y_{ik}}-1) - \frac{2y_{ik}}{(y_{il}+y_{ik})^3}\int_0^{y_{il}+y_{ik}} d_i f_i(d_i)\mathrm{d}d_i}^{:=A_l} + \overbrace{\frac{1}{(y_{il}+y_{ik})^2}\int_0^{y_{il}+y_{ik}} d_i f_i(d_i)\mathrm{d}d_i}^{:=B_l}}{\underbrace{f(y_{il}+y_{ik})(1-\frac{y_{ik}}{y_{il}+y_{ik}}) + \frac{2y_{ik}}{(y_{il}+y_{ik})^3}\int_0^{y_{il}+y_{ik}} d_i f_i(d_i)\mathrm{d}d_i}_{:=-A_l}} =$$

$$-1 - \underbrace{\frac{B_l}{A_l}}_{\in(-1;0)} > -1 \ .$$

Since $\lim_{y_{il}\to 0} \mathcal{I}_{ik}(y,x) > \lim_{y_{il}\to\infty} \mathcal{I}_{ik}(y,x)$ and $0 > z_{il} > z_{ik}$ holds, there exists a unique intersection of the indifference curves in the (y_{ik}, y_{il}) plane. Thus, a unique Nash equilibrium solution exists. □

Chapter 5

The Day-before Truck Platooning Planning Problem

Advances in autonomous driving technology have fostered the idea of truck platooning. This cooperative transportation mode allows for fuel savings, which help to reduce transportation costs and pollution. In this chapter, we define and discuss the process steps that are required for a central coordination of platoons. We identify the *day-before planning problem* for routing and scheduling trucks into platoons as an important part of the process. For this problem, we introduce a novel mixed-integer linear programming formulation that is defined on a time-expanded two-layer network. Thereby, we assume limits to the maximal platoon size and fixed time-windows. We develop a pre-processing procedure that allows us to significantly reduce the problem size of the input. In our computational study, we present insights into the efficiency of this approach, the savings potential through truck platooning and the sensitivity to different parameters of the model. The results show that truck platooning can lead to considerable fuel-savings. Furthermore, they indicate that trucks will rarely deviate from their shortest paths as the detour costs exceed the fuel savings. Thus, the main advantage of a central coordination origins from the scheduling of the trucks.

5.1 Introduction

Truck platooning is a form of shared transportation, where several trucks drive in close succession in order to reduce their air resistance in the predecessor's slipstream and thus their fuel consumption. In a platoon of size three, this allows for fuel savings up to 16% for the followers and even the leader can save up to 7.5%, thanks to less vorteces behind the truck (Tsugawa et al., 2011). Other authors report lower savings of 10%

for the followers and 6% for the leader (Bhoopalam et al., 2017). The reduced safety distance within a platoon is enabled by truck-to-truck communication and autonomous driving technology. This allows the following trucks to mimic the braking, acceleration and steering commands of the leading truck.

Almost all major truck manufacturers have started first field trails, for instance the *PATH* project in California (UC Berkeley, 2018) or the *European Truck Platooning Challenge* (European Truck Platooning, 2018). However, after test evaluations, *Daimler* concluded that the fuel savings achieved through platooning are modest, even under perfect conditions. Therefore, the company announced in January 2019 to abandon the development of platooning, arguing that the fuel cost savings are disproportional to the research cost (Truck News, 2019a).

Nevertheless, a reduced fuel consumption results in less air pollution. Between 1990 and 2016, transport greenhouse gas (GHG) emissions in the European Union (EU) increased by 18% to 931 million tonnes of CO_2 equivalent (European Commission, 2018). This represents 21% of all European GHG emissions, making the transport sector the second largest producer of GHG. In addition, this is the only sector where GHG emissions have risen beyond the value of 1990. In 2011, the EU Commission decided to reduce Europe-wide GHG emissions by 80% by 2050 (European Commission, 2011). The transport by road carries the main burden of freight transport in Europe with a share of 75% (European Union, 2017). Consequently, the European Union identified road freight transportation as one of the pillars of its concepts towards achieving the GHG goals. In February 2019, the European Parliament and the European Commission agreed on a bill that forces truck manufacturers to reduce the average CO_2 emissions of newly built trucks by 15% until 2025 and 30% by 2030, compared to the status in 2019 (Reuters, 2019). Similar emission limits have already been established in the United States and Canada. Since current Diesel engines are already highly efficient, truck manufactures have to develop and combine new technologies to meet such directives (Reuters, 2019). Obviously, the introduction of hybrid or electric trucks will play a crucial role. Nevertheless, combining several trucks into platoons offers a further promising approach for saving energy and thus reducing GHG emissions and meeting the goals.

Besides a reduced fuel consumption and less CO_2 emissions, truck platooning bears the potential to increase space utilization and thus improve traffic flow on highways (Van Arem et al., 2006). Furthermore, platooning can help towards improving road safety (Janssen et al., 2015). Therefore, companies are continuing their efforts to realize this form of collaborative trucking. In July 2019, Peloton Technology (2019) announced the release of a new platooning technology that allows the platoon followers to be driverless. This so-called "Level 4 automation" can help to reduce the room for human errors like inattention, distraction or fatigue (Truck News, 2019b). In 2007, 85% of all road accidents were a result of human errors and 21% of all truck accidents happened while driving in a convoy (International Road Transport Union and European Commission, 2016).

In line with the sustained growth of interest in truck platooning, research on the application of Operations Research to this new form of shared transportation has been steadily increasing (Bhoopalam et al., 2017). Larsson et al. (2015) formulated the *Truck Platooning Problem* (*TPP*) as a mixed-integer linear program and showed that it is $\mathbb{NP}$−hard. It describes the combinatorial problem of routing and scheduling trucks under the possibility of platooning. From then on, the literature can be divided into two streams: First, extensions and solution methods for the TPP (Larsen et al., 2019; Larson et al., 2016; Luo et al., 2018; Scherr et al., 2018; Van De Hoef et al., 2015). Second, the identification and analysis of factors that may complicate the formation of platoons (Boysen et al., 2018; Zhang et al., 2017).

The *European Automobile Manufacturers' Association* (*ACEA*) pursuits the goal to introduce *multi-brand platooning* in the European Union until 2025 (European Automobile Manufacturers' Association, 2018). That is, platoons can be formed out of trucks of different brands or companies. In July 2019, *Continental* and *Knorr Bremse* presented a platooning system that shows that multi-brand platooning is possible from a technical point of view (Continental AG, 2019). To foster the realization and admission of truck platooning in the European Union, the ACEA "EU Roadmap for Truck Platooning" lists steps that are required to introduce multi-brand platooning in the European countries by 2025. Besides further necessary technical developments and changes in regulations and legislation, these steps comprise:

1. The introduction of a multi-brand platooning platform that coordinates individually driving trucks into platoons.

2. Development of incentives like reduced tolls and taxes, CO_2 bonuses or flexibility in driving time.

Both points have only been partially discussed from the perspective of Operations Research. Hoef et al. (2018) describe the process of a central *platoon coordinator* that determines for carriers such routes that their trucks arrive at their destination within the desired time-window while simultaneously exploiting fuel savings through platooning. However, no platform-based business model has been mentioned in the literature so far. Moreover, there is a lack of methodology that allows a platform to perform a day-before planning of the routes and schedules with a large number of trucks and to return this information fast to the carriers. In this chapter, we close this research gap by introducing the idea of a platform-based business model for multi-brand platooning and by proposing a model for the day-before planning problem.

That is, we assume that there exists a platform where different carriers can register their trips up to a certain cutoff date. Based on this information, the platform creates platoons and returns to the carriers information about whether or not a trip has been accepted and what the savings will be. Furthermore, the platform provides the carriers with individual routes and the corresponding schedules. A portion of the savings is kept by the platform as a reward for planning. Since the platform takes over the routing and scheduling of the trucks, it has to return the information to the carriers with a certain lead time such that the carriers can instruct their truck drivers in time. We assume that this lead time is one day. Consequently, we have to formulate and solve the day-before planning problem, which is the main research question addressed in this chapter.

We use a mixed-integer linear programming formulation and assume hard time-windows and a limit on the platoon length. We call this problem the *Restricted Truck Platooning Problem* (*RTP*) and define it on a time-space expanded network. This network uses two layers: in the *truck layer*, we keep track of individual movements of the trucks, whereas in the *platoon layer*, only platoons are routed. The two layers are connected with *interlayer arcs* that model the formation and disbanding of platoons. To reduce the input size, we exploit the trucks' temporal restrictions to create a low density time-space network, which includes only feasible arcs. Furthermore, we reduce the solution space by generating a starting solution and fixing non-basic variables.

In our computational study, we use large instances to evaluate the performance of our solution approach. Furthermore, we assess the sensitivity of the RTP solution to different parameters like the fuel-savings factor, the maximal allowed platoon size or the impact of varying buffer times and study the effect of the network structure on the platoon formation. In addition, we quantify the value of a central coordination of platoons.

Our contribution is three-fold:

1. We structure and define the process of a multi-brand platooning platform.
2. We propose a new mathematical model for the day-before planning of truck platoons under platoon-size limits and temporal restrictions. In our computational study, we demonstrate that the pre-processing procedure allows us to solve large instances of this $\mathbb{NP}$−hard problem with off-the-shelf solvers in reasonable size. Consequently, our model is ready to use in industry without further implementation effort.
3. From our computational study, which is conducted on instances of realistic size, we provide insights into the savings potential of platooning (both cost and CO_2). Furthermore, we discuss the value of a centralized planning of truck platoons.

The remainder of this chapter is structured as follows. In Section 5.2, we discuss a platform-based business model. In Section 5.3, we introduce the two-layer time-space expanded network, present and explain our model and propose a pre-processing procedure that reduces the input size. In Section 5.4, we describe our computational study and present the results. In Section 5.5, we summarize and discuss the gained insights. Section 5.6 concludes the chapter.

5.2 A platform-based business model for truck platooning

The market of road freight transportation in Canada is highly segmented with 66,751 companies operating in December 2016 (Government of Canada, 2016). A similar situation exists in Europe: 550,000 companies offer trucking transportation and more than 80% of the companies employ less than the EU average of 5.2 people per company (European Commission, 2017). Consequently, in both regions a platform that centrally coordinates all truck platoons would lead to the highest cost savings. We propose the following business model for a multi-brand platform (or simply *platform*) and describe the process as follows:

1. Carriers register their trips for a certain time period $T = [T^s; T^e]$ (e.g. one week) until the registration deadline $T^r < T^s$. Thereby, they provide information about the origin and destination as well as earliest possible start and latest possible

arrival times. To achieve the highest potential savings through truck platooning, trucks may deviate from the shortest route in order to travel together with other trucks. Therefore, all carriers who enter their trips agree to the terms that their trucks may deviate from the shortest paths as long as the time windows are met and the traveling costs are not higher than what would be if every truck drove by itself.

2. After T^r, the platform informs the carriers if the trips can be integrated in a platoon and by how much the travel cost can be reduced.

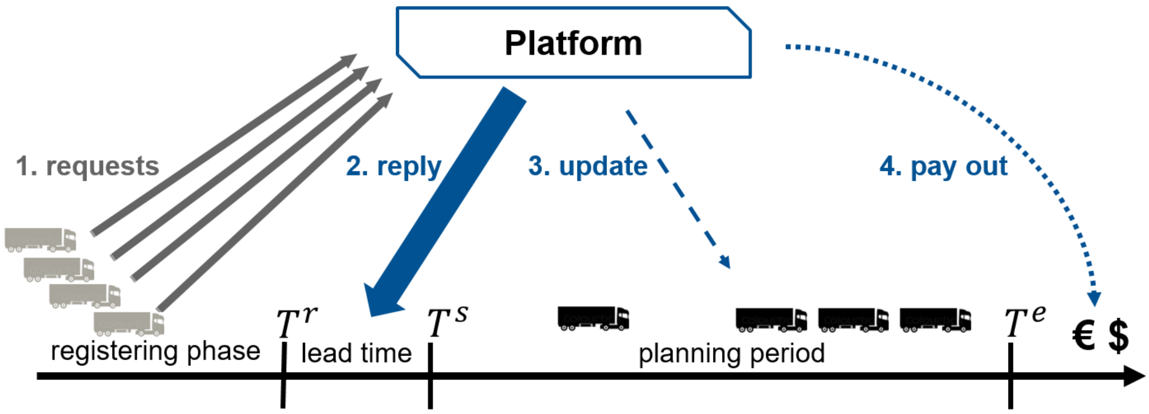
3. The trucks realize the trips according to the routes and schedules provided by the platform. These may change due to delays or no-shows of trucks.

4. As soon as all trips are finished, the promised savings are payed to the carriers. To determine the savings, the platform needs to determine for every truck the fuel consumption under the assumption that the truck was driving individually. Obviously, the actual consumption depends on different factors like the topography of the road or the traffic situation. However, since all trucks are equipped with autonomous driving technology, it is reasonable to assume that enough data is collected during the ride to obtain a good estimate on the consumption. In the long run, the platform can also build up a data base to use historical data to improve those estimates. Once the consumption is known, the total platoon savings are determined. Then, the platoon followers are charged the portion of the fuel-savings that the platoon leader missed plus a reward for the platform. The platoon leader is payed-out the savings minus the reward.

Figure 5.1 illustrates the time line of that process. In order to give the carriers an acceptance note and to grant some lead time, the platform has to reply in step two before the planned period starts. This can be done by solving a *day-before planning problem.* Since travel times may be influenced by external factors like the traffic situation or weather conditions, there is a chance that trucks do not show up at meeting points or parking lots are overcrowded. Thus, a re-optimization during the planning period might be then required. This can be done by solving an *online optimization problem* during the driving phase. The results of both steps can then be used in step four to determine the savings that will be shared.

The objective is to reduce the total traveling cost of all registered trucks through platooning. Naturally, this can lead to sub-optimal solutions for the individual trucks.

Figure 5.1: Visualization of the coordination process with the platform.



However, these additional expenses are compensated through the distribution of the savings among all platoon members. Due to the limit on the platoon size, it might happen that several platoons are travelling simultenoeusly in the same time interval. In that case, the cost savings are evenly distributed among all trucks traveling in one of those platoons. Otherwise, trucks in the smaller platoon would be discriminated as far as their costs are concerned. Other redistribution schemes, partially based on Game Theory, can be found in Bhoopalam et al. (2017).

Platoons are only formed on roads that provide a highly isolated environment. This is motivated by the observation that it is easier to implement the technology of autonomously driving trucks in such environments. Highways meet these requirements, which is why we focus on this type of road while acknowledging that there are also other types of roads where platoons can be formed (e.g. federal highways in Germany or roads in Canada's Northwest Territories). We assume that all trucks are traveling at the same, constant speed. Consequently, we say that a platoon formation or disbanding can only happen at parking lots that are next to highways. Usually, the origins and destinations of the trucks are located away from the highways. Thus, we assume that the trucks have to drive on the first and last miles individually.

To calculate the potential savings, the following parameters must be given:

- Full cost of operating a truck per time unit, including expenses like fuel, repair cost and wear and tear (Wittenbrink, 2014).
- Wages of the truck drivers per time unit.

- Travel times on the road segments.
- Fuel reduction factor in a platoon.

Presumably, carriers will be reluctant when it comes to revealing their cost structure to a third-party provider. However, knowing the truck model and type and weight of the truck-trailer combination, the costs can be estimated with high accuracy. The travel times are based on forecasts from the moment when the corresponding optimization problems were solved. After the platform has been operational for a longer time, it can also use historical data to improve or correct the travel time forecast. The same applies to the fuel reduction factor that depends on different factors like speed, topography and the number of trucks participating. One approach would be to start with an average value for all road segments, and then to refine the data after sufficient learning from historical data.

Within the platoons, there is a rotation scheme such that every truck takes the leading role for the same amount of time. This rotation is done while driving and requires the leader to change the lane, decelerate and then accelerate to join the platoon as taillight. Apart from these rotation operations, trucks drive at the maximum allowed speed.

To calculate the potential savings, the following parameters must be given:

- Full cost of operating a truck per time unit, including expenses like fuel, repair cost and wear and tear (Wittenbrink, 2014).
- Maximal platoon size, defined by legislation.
- Travel times on the road segments.
- Fuel reduction factor in a platoon.

Presumably, carriers will be reluctant when it comes to revealing their cost structure to a third-party provider. However, knowing the truck model and type and weight of the truck-trailer combination, the costs can be estimated with high accuracy. Due to the severe competition in the trucking market, wages should also be comparable among the different drivers. The travel times are based on forecasts from the moment when the corresponding optimization problems were solved. After the platform has been operational for a longer time, it can also use historical data to improve or correct the travel time forecast. The same applies to the fuel reduction factor that depends on different factors like speed, topography and the number of trucks participating. One approach would be to start with an average value for all road segments, and then to refine the data after sufficient learning from historical data.

5.3 The Restricted Truck Platooning Problem

In this section, we show how the day-before planning (Stage 2, cf. Section 5.2) can be modelled by using mixed-integer linear programming. We call this problem the *Restricted Truck Platooning Planning Problem* (*RTP*). First, we characterize the problem setting and discuss our basic assumptions. This is done in Section 5.3.1. In Section 5.3.2, we adopt the idea of Zhu et al. (2014) by introducing a two-layer time-space expanded network. Then, we introduce in Section 5.3.3 the mixed-integer linear formulation of the RTP. In Section 5.3.4, we explain the pre-processing steps to reduce the size of the solution space. Finally, we discuss in Section 5.3.5 how we can manipulate the input data such that we obtain the maximum on the optimal solution value of the RTP.

5.3.1 Problem description and assumptions

We introduce a node set $\mathcal{V}$ that includes (i) the origins and destinations of the trucks, (ii) the parking lots along the highways and (iii) intersection nodes that represent the highway entrances and exits. We call the last two types of nodes *waypoints* and collect them in the subset $\mathcal{V}^W \subsetneq \mathcal{V}$. The set $\mathcal{A}$ contains all (physically) feasible connections between the nodes. We call $\mathcal{G} := (\mathcal{V}, \mathcal{A})$ the *supporting network*. τ_{ij} denotes the usual travel time between nodes i and j.

Carriers register their trips for time period $T = [T^s; T^e]$ at the platform up to a registration time $T^r < T^s$. Since every trip is associated with a truck, we refer in the following to trucks and denote the set of all trucks by $\mathcal{K}$. The following information is given: (i) origin and destination, (ii) earliest possible start and latest possible arrival time, e_k and l_k, (iii) c_k^o, operating cost of truck k per time unit. The fuel reduction for the following trucks in a platoon is denoted by η^f, the one for the platoon leader by η^l. Both factors are assumed to be the same for every truck. Platoons can be of different sizes up to a limit of ς. The set $S := \{2, \ldots, \varsigma\}$ includes all size options. We assume that there is enough highway capacity for several platoons – potentially of the same size – to travel simultaneously on a highway section.

5.3.2 Time expanded two-layer network

To synchronize the schedules of the trucks, we divide the planning period T into h finer intervals of equal length. Hence, we use a time-space network where we expand each node $v \in \mathcal{V}$ h times. Let i be a node of this time-space network. Then, the functions

$f : V \to \mathcal{V}$ and $g : V \to T$ determine the node and the time interval, which is represented by i. That is, $f(i) = v$ and $g(i) = t$. As mentioned in Section 5.2, the travel times might depend on the time-period $t \in T$. Therefore, $t_{ij} = \tau_{ij}(t)$ denotes the travel time between $i, j \in \mathcal{V}$ at time-step $t \in T$. The travel cost can then be determined by multiplying the cost with the travel time: $c_{ijk} = c_k^o \cdot t_{ij}$. The network consists of two layers: the truck layer and the platoon layer.

Truck layer

The node set V_K contains the time-expanded *truck nodes*, that is: $|V_K| = h \cdot |\mathcal{V}|$. $V_K^W \subsetneq V_K$ represents all waypoint nodes in the time-space network. We introduce A_K, the set of truck arcs. An arc $(i, j) \in A_K$ connects two nodes $i, j \in V_K$ when $g(j) - g(i) = \tau_{f(i),f(j)}$. The costs correspond to the traveling cost in the supporting network, $c_{ij} = c_{f(i),f(j)}$. To model the waiting option, we use *waiting arcs* that connect those nodes of the same waypoint node that differ in one time interval. $c_{ij} > 0$, $i, j \in V_K^W$ with $f(i) = f(j)$ and $g(j) = g(i) + 1$, describes the cost of waiting. We call the resulting graph $G_K := (V_K, A_K)$ the *truck layer*.

Platoon layer

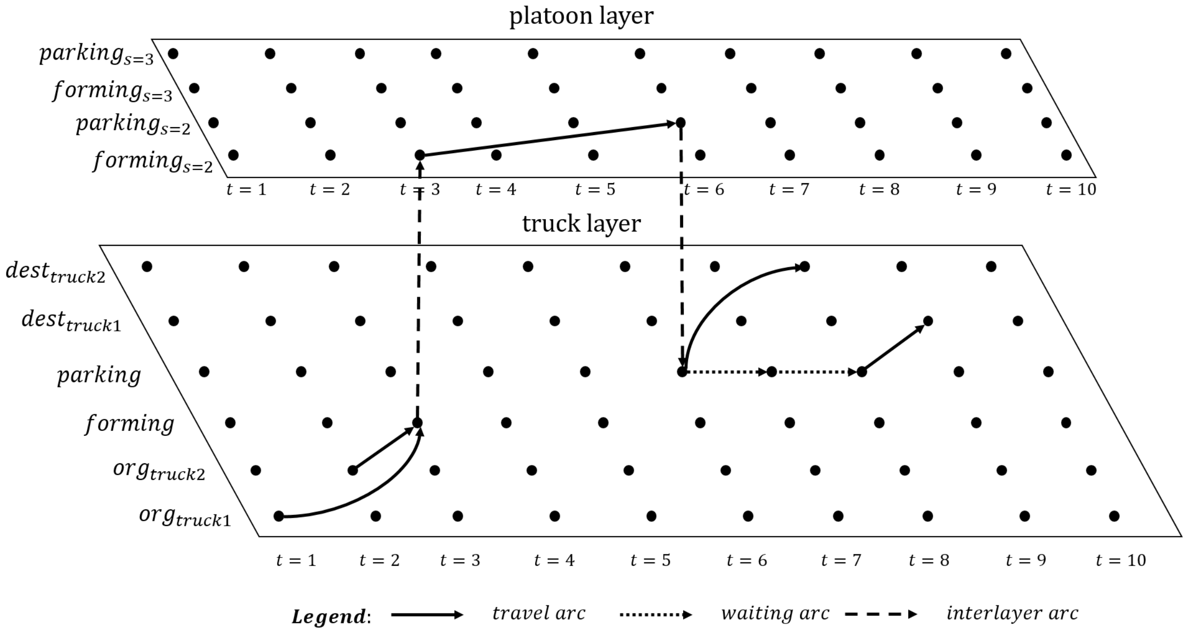
The platoons move within the *platoon layer* $G_P := (V_P, A_P)$. V_P represents the time-expanded nodes where platoons can be formed. They are the waypoint nodes and therefore the *platoon nodes* are a copy of V_K^W. Let $i \in V_K^W$ be a waypoint node in the time-expanded truck layer, then i_P denotes its copy in the platoon layer. The cost of traveling between two nodes $i_P, j_P \in V_P$ also correspond to the traveling costs within the supporting network: $c_{i_P,j_P} = c_{ij}$.

Two-layer network

To model the forming and disbanding of platoons, we introduce *interlayer arcs* $A_I = A_I^+ \cup A_I^-$. That is, A_I^+ contains all arcs (i, i_P), that represent the formation of a platoon at a waypoint node, whereas A_I^- comprise those arcs (j_P, j) that describe the disbanding of a platoon. We assume that the formation or dissolution of a platoon creates a cost for every truck $c_{i_P,i}, c_{j_P,j} > 0$. The whole *layer network* $G := (V, A)$ is formed by the nodes $V := V_K \cup V_P$ and the arcs $A := A_K \cup A_P \cup A_I$.

Figure 5.2 shows an example of two trucks traveling in this two-layer time-space network with $\varsigma = 3$. Truck 1 heads from its origin at interval $t = 1$ to waypoint 1 in

Figure 5.2: Example of the iterinaries of two trucks in the time-space expanded two-layer network with size-limit $\varsigma = 3$. Truck 2 leaves its origin at $t = 2$ and then forms a platoon of size $s = 2$ with truck 1 at parking lot 1. The platoon is disbandoned at parking lot 2, where truck 1 stays for two time steps before continuing to its final destination.



interval three where, together with truck 2, it forms a platoon of size $s = 2$ that drives to waypoint 2 – where the platoon is disbanded – inside three time intervals. After a rest period of two time steps, truck one arrives at its destination at $t = 9$. Truck two starts its tour by waiting one time step in order to synchronize with truck one. Truck two arrives at its final destination at $t = 8$.

5.3.3 Mathematical model

We propose a mixed-integer linear program that is defined on $G = (V, A)$ and uses three decision variable sets. We introduce superscripts o and d to indicate the origin or destination of an arc $a \in A$. That is, we write $a = (a^o, a^d)$. The binary decision variable x_{ak} is set to one if truck k uses arc $a \in A_K \cup A_I$. The integer decision variable y_{as} counts the number of platoons of size s traveling on arc $a \in A_P$. To keep track of the trucks in the platoons, we use the binary decision variable z_{aks}. It is set to one when truck k joins a platoon of size $s \in S$ on arc $a \in A_P$. Table 5.1 summarizes the notation.

Table 5.1: Sets, functions, parameters and decision variables used in the RTP model.

Sets and functions	
$\mathcal{K}$	set of all trucks
T	set of all time periods
$S = \{2, \dots, \varsigma\}$	set of all platoon sizes
$V = V_K \cup V_P$	set of all time-expanded nodes, which can be partitioned into nodes on the truck level and platoon level
$\mathcal{V}$	set of nodes in the supporting network,
$\mathcal{V}^W$	set of way-point nodes in the supporting network
$A = A_K \cup A_I \cup A_P$	set of all arcs, which can be partitioned into truck arcs, interlayer arcs and platoon arcs
f, g	functions mapping $V \to \mathcal{V}$ and $V \to T$
Parameters	
o_k, d_k	origin and destination of truck k
e_k, l_k	earliest departure and latest arrival of truck k
t_{ij}	travel time between node i and node j in period t
c_{ak}	cost of truck k traveling on arc a
η^l, η^f	fuel reduction factor of a platoon leader and platoon follower
ς	maximum size of a platoon
Binary decision variables	
x_{ak}	if truck k uses arc a
z_{aks}	if trucks k travels on arc a in a platoon of size s
Integer decision variable	
y_{as}	number of platoons of size s traveling on arc a

The *Restricted Truck Platooning Planning Problem* (*RTP*) reads as follows:

$$\min \sum_{k\in\mathcal{K}} \Bigg(\sum_{a\in A_K\cup A_I} c_{ak}\cdot x_{ak} + \sum_{a\in A_P}\sum_{s\in S} c_{ak}\cdot \Big(1-\frac{\eta^l+(s-1)\cdot\eta^f}{s}\Big)\cdot z_{aks}\Bigg) \tag{5.1}$$

$$s.t. \quad \sum_{\substack{a\in A_K:\\ f(a^o)=o_k}} x_{ak} = 1 \qquad \forall k\in\mathcal{K} \tag{5.2}$$

$$\sum_{\substack{a\in A_K:\\ f(a^d)=d_k}} x_{ak} = 1 \qquad \forall k\in\mathcal{K} \tag{5.3}$$

$$\sum_{\substack{a\in A_K:\\ f(a^d)=i\wedge g(a^d)=t}} x_{ak} = \sum_{\substack{a\in A_K:\\ f(a^o)=i\wedge g(a^o)=t}} x_{ak} \qquad \forall k\in\mathcal{K}, i\in\mathcal{V}^W, t\in T \tag{5.4}$$

$$\sum_{s\in S}\sum_{\substack{a\in A_P:\\ f(a^o)=i\wedge g(a^o)=t}} z_{aks} = \sum_{\substack{a\in A_I^+:\\ f(a^o)=i\wedge g(a^o)=t}} x_{ak} \qquad \forall k\in\mathcal{K}, i\in\mathcal{V}^W, t\in T \tag{5.5}$$

$$\sum_{s\in S}\sum_{\substack{a\in A_P:\\ f(a^d)=j\wedge g(a^d)=t}} z_{aks} = \sum_{\substack{a\in A_I^-:\\ f(a^o)=j\wedge g(a^o)=t}} x_{ak} \qquad \forall k\in\mathcal{K}, j\in\mathcal{V}^W, t\in T \tag{5.6}$$

$$\sum_{k\in\mathcal{K}} z_{aks} = s\cdot y_{as} \qquad \forall a\in A_P, s\in S \tag{5.7}$$

$$\sum_{s\in S}\sum_{\substack{a\in A_P:\\ f(a^o)=i}} z_{aks} \le 1 \qquad \forall i\in\mathcal{V}^W, k\in\mathcal{K} \tag{5.8}$$

$$x_{ak}, z_{aks}\in\{0;1\},\; y_{as}\in\mathbb{N}_0 \qquad \forall a\in A, k\in\mathcal{K}, s\in S \tag{5.9}$$

We minimize the objective function (5.1) that can be divided into two parts: The first term describes the total cost of an individually driving truck, while the second term includes all the costs induced by traveling in a platoon. For the platoon traveling cost, we credit the fuel savings factor η^f for the $(s-1)$ platoon followers, and η^l for the platoon leader. Equalities (5.2) and (5.3) ensure that each truck leaves its origin and enters its destination. (5.4) conserves the flow for all other nodes in the truck layer. Hereby, the inflow and outflow can also originate from the platoon nodes via the interlayer arcs. Equations (5.5) and (5.6) state that when a truck uses an interlayer arc to or from a platoon node, the truck has to join or to leave a platoon at this waypoint. Since we assume that platoons offer a direct service between two nodes, platoons are disbanded as soon as they arrive at a waypoint. Nevertheless, trucks can join another platoon at the same waypoint in the same time step by choosing the corresponding interlayer arc. (5.7) ensures that the correct number of trucks is traveling in a platoon of size s between two waypoint nodes. (5.8) states that each truck can only join one platoon at a waypoint.

Observe that all trucks are routed due to (5.2) and (5.3), although some may not join a platoon. Those correspond to the trucks that will be not accepted by the platform. They can be identified in a post-processing step.

Valid inequalities

We introduce the following two valid inequalities, which help to tighten the formulation:

$$\sum_{\substack{a\in A_K:\\ f(a^d)=i\wedge g(a^d)=t}} x_{ak} \le \sum_{\substack{a\in A_K:\\ f(a^o)=i\wedge g(a^o)=t}} x_{ak} \qquad \forall k\in\mathcal{K}, i\in\mathcal{V}^W, t\in T \tag{5.10}$$

$$\sum_{\substack{a\in A_K:\\ f(a^o)=i\wedge f(a^d)=j}} x_{ak} + \sum_{s\in S}\sum_{\substack{a\in A_P:\\ f(a^o)=i\wedge f(a^d)=j}} z_{aks} \le 1 \qquad \forall i,j\in\mathcal{V}: i\ne j, k\in\mathcal{K} \tag{5.11}$$

(5.10) states that a truck can only travel from a node after its arrival at this node. The second inequality (5.11) states that every truck can enter any physical node $j \in V$ at most once, either on the truck layer or on the platoon layer.

5.3.4 Pre-processing

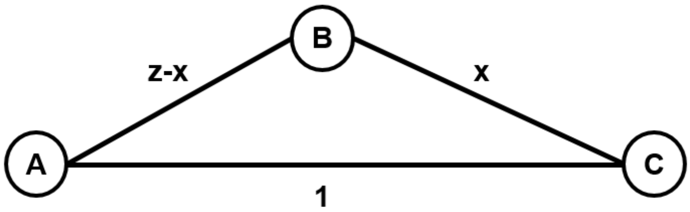
To reduce the problem size of the RTP, we introduce a pre-processing procedure where we only create feasible arcs. In doing so, we exploit the RTP's specific characteristics.

Bounded paths

In their Lemma 2.2, Larson et al., 2016 state that – when only considering the fuel cost – no truck is willing to drive more than $1 + \eta^f \cdot \frac{\varsigma-1}{\varsigma}$ of its shortest path distance to join a platoon. However, this is only true if the total savings are not split afterwards, as Example 5.1 shows.

Example 5.1 (Detour optimum with platooning)**.** *Consider two trucks traveling in the network given in Figure 5.3, where the triangle inequality holds. Truck 1 travels from* A *to* C*, while truck 2 goes from* B *to* C*. Let the distance between* A *and* C *be 1, and the path from* A *to* C *via* B *have length z. x denotes the distance between B and C, where the trucks are platooning. Furthermore, let be $\eta^l = 0$ and $c^l_{ak} = 0$. For simplicity, we assume $c^f_{ak} = 1$. If the trucks go individually, the total travel costs are $\gamma^{ind} = 1 + x$. If the trucks platoon between* B *and* C*, the total travel costs are $\gamma^{plt} = z + (1 - \eta^f) \cdot x$. Thus, the total savings are $\delta = \gamma^{ind} - \gamma^{plt} = 1 - z + \eta^f \cdot x$. Hence $x > \frac{z-1}{\eta^f} \Leftrightarrow \delta > 0$. Without loss of generality, truck 1 acts as platoon leader during the entire route, that is, has travel*

Figure 5.3: Example of a detour that lies beyond the threshold defined by Larson et al., 2016 but still leads to overall cost savings.



costs of z while truck 2 has costs of $(1-\eta^f)\cdot x$. If the savings are equally split, truck 1 receives a payment of $\frac{1}{2}\cdot(-1+z+\eta^f\cdot x)$, while truck 2 has to pay $\frac{1}{2}\cdot(1-z+3\cdot\eta^f\cdot x)$. As a result, both trucks benefit from platooning as truck 1 pays $1-\frac{\delta}{2}$ for its tour and truck 2 pays $x-\frac{\delta}{2}$.

Example 5.1 shows that, even if $z > 1+\frac{1}{2}\cdot\eta^f$, both trucks can reduce their travel cost as long as the platooning distance x is sufficiently long enough. As a consequence, the only bound that we can use, is the maximal traveling time of truck k, which is defined by $\theta_k := l_k - e_k$

To determine $\mathcal{P}_k^\theta$, the set of all θ_k-*bounded paths*, we use a search for every truck $k \in \mathcal{K}$ that relies on the bound θ_k. We initialize the search by determining $shortest_{i,d_k}$, all shortest paths from every node $i \in \mathcal{V}\backslash\{d_k\}$ in the supporting network to the truck's destination. $\mathcal{N}_i$ denotes all neighbors of node i and $\mathcal{N} = \bigcup_{i\in\mathcal{V}\backslash\{d_k\}}\mathcal{N}_i$ collects all neighbors. Information about the path are saved in the tuple $path = (path_1, path_2)$. $path_1$ is again a tuple that saves the order of the visited nodes. $path_2$ saves the current length of the path from o_k to $leaf$, which is the current last node of the path. All tuples $path$ are saved in the candidate set $\mathcal{C}$.

In the search, we select a candidate $path \in \mathcal{C}$ and explore all neighbors j of $leaf$. If the sum of the travel time to j plus $path_2$ plus the travel time from j to d_k is less or equal to θ_k, we update $path$ and select a new candidate. If $leaf = d_k$, we add $path_1$ to $\mathcal{P}_k^\theta$ and continue with the next candidate. The algorithm stops when $\mathcal{C}$ is empty. Algorithm 5.3 summarizes the search procedure, which can be accomplished in polynomial time as Lemma 5.1 shows.

Lemma 5.1 (Polynomial runtime of Algorithm 5.3)**.** *Algorithm 5.3 can be accomplished in $\mathcal{O}\Big(|\mathcal{K}|\cdot\big(|\mathcal{A}|\cdot|\mathcal{V}|+|\mathcal{V}|^2\cdot log(|\mathcal{V}|)\big)\Big)$.*

Algorithm 5.3 Bounded paths algorithm

Initialization: Compute $shortest_{i,d_k}$, $i \in \mathcal{V}\backslash\{d_k\}$, $\forall k \in \mathcal{K}$. Set $\mathcal{C} = \{\}$, $\mathcal{N} = \{\}$
for $k \in \mathcal{K}$ **do** $\mathcal{P}_k^\theta = \{\}$, $leaf = o_k$, $path = ((o_k), 0)$ $\mathcal{C} = \{path\}$,
  while $\mathcal{C} \neq \{\}$ **do**
    $\mathcal{C} = \mathcal{C}\backslash\{path\}$
    if $leaf = d_k$ **then** $\mathcal{P}_k^\theta = \mathcal{P}_k^\theta \cup path_1$
    else
      Determine all neighbors of $leaf$
      for $j \in \mathcal{V}\backslash\{path\}$: $\theta_k \geq t_{leaf,j} + path_2 + shortest_{j,d_k}$ **do**
        $\mathcal{N} = \mathcal{N}_{leaf} \cup \{j\}$
      end for
      Update candidate list
      for $i \in \mathcal{N}$ **do**
        $\mathcal{C} = ((path_1, i), path_2 + t_{leaf,j})$
      end for
    end if
    Select new candidate and set leaf: $path \in \mathcal{C}$, $leaf = path_1^{end}$
  end while
end for

Proof:
Computing all shortest paths from a node to d_k requires a runtime of $\mathcal{O}\Big(|\mathcal{V}| \cdot \big(|\mathcal{A}| + |\mathcal{V}| \cdot log(|\mathcal{V}|)\big)\Big)$. The selection of the candidates can be accomplished in $\mathcal{O}(|\mathcal{V}| + |\mathcal{A}|)$. Since the procedure has to be repeated for every truck, the total runtime is $\mathcal{O}\Big(|\mathcal{K}| \cdot \big(|\mathcal{A}| \cdot |\mathcal{V}| + |\mathcal{V}|^2 \cdot log(|\mathcal{V}|)\big)\Big)$. □

Feasible paths

When considering driving time regulations, the mandatory pauses extend the travel times on some paths $p \in \mathcal{P}_k^\theta$ to such extent that trucks violate their latest arrival l_k. Therefore, we exclude all those paths p, where mandatory break times and daily rests extend the travel times to such an extend that truck would arrive too late. We denote this set of driving-time-regulation-feasible paths, or simply *feasible paths* as $\mathcal{P}_k^f$. However, this is only true for stage 1. Due to the rest-while-trailing option in stage 2, trucks might not need to take a break or rest at all. Consequently, we cannot limit $\mathcal{P}_k^\theta$ to driving-time-regulation-feasible paths only.

Earliest arrival and latest departure

Having determined all bounded or feasible paths, for every truck we can determine its earliest arrival period and latest departure period at every node that is included in one of its feasible paths. We denote these values as $earliest_{ik}$ and $latest_{ik}$. Based on these values, we create the following arcs:

- **Truck arcs:**
  - Moving arcs: if for any two nodes $i, j \in \mathcal{V}$ and time period $t \in T$ there exists at least one truck k such that $earliest_{ik} \leq t \leq latest_{jk} - t_{ij}$.
  - Waiting arcs: if for any node $i \in \mathcal{V}$ and time period $t \in T$ there exists at least one truck k such that $earliest_{ik} \leq t \leq latest_{ik} - 1$.
- **Interlayer arcs**: if for any node $i \in \mathcal{V}$ and time period $t \in T$ there exist at least two trucks k and l such that $earliest_{ik} \leq t \leq latest_{ik}$ and $earliest_{il} \leq t \leq latest_{jl}$.
- **Platoon arcs**: if for any two nodes $i, j \in \mathcal{V}$ and time period $t \in T$ there exists at least one truck k such that $earliest_{ik} \leq t \leq latest_{ik} - t_{ij}$, $earliest_{jk} \leq t \leq latest_{jk} - t_{ij}$.

We denote these reduced arc sets as A'_K, A'_P and A'_I and their union as A'.

Fixing non-basic decision variables

For every truck k there may exist a subset of arcs in A' that cannot be traversed. Thus, we can set the following decision variables to zero:

- x_{ak}: if the travel-period lies beyond k's time window, i.e. $latest_{ik} < g(i)$ or $g(j) < earliest_{jk}$.
- w_{aks}, z_{aks}: if the travel-period lies beyond k's time window, i.e. $latest_{ik} < g(i)$ or $g(j) < earliest_{jk}$, $s \in S$.

Providing a starting solution

We know that an upper bound to the RTP is the total cost of all trucks driving individually on their shortest paths without platooning. Thus, we can provide an initial starting solution to the solver as follows: Let $short_k \subsetneq \mathcal{A}$ denote truck $k's$ shortest path in the network $\mathcal{V}$ and $short_k^{e_k} \subsetneq A_K$ the collection of arcs in the time-expanded network when starting at the earliest possible departure time e_k. By setting $x_{ak} = 1$ for all arcs $a \in short_k^{e_k}$ and all other decision variables to zero, we obtain a starting solution.

Remarks

1. Carriers may insist on excluding certain routes or parking lots for their truck for various reasons. This can be incorporated into the model by setting the corresponding decision variables for those arcs (traveling or waiting arcs) to zero. This is beneficial for the solution time as more arcs can be excluded upfront.

2. By allowing arcs to connect to a destination node i even after $latest_i$ we can incorporate soft time windows into our model. To incur a penalty, we add delay cost to these arcs, which can increase at every time step. Similarly, we can allow earlier departures adding a penalty for the earlier start to the truck travel cost. However, soft time windows come with a wider corridor. That is, less arcs can be excluded upfront and thus, a slower runtime is to be expected for the solution of the model.

5.3.5 Opportunistic truck platooning

In Section 5.2, we discussed a business model of a platform that does centrally coordinate the formation of platoons through routing and scheduling the trucks. Bhoopalam et al. (2017) denote this concept as *scheduled platooning planning*, whereas *opportunistic platooning* describes the idea of trucks forming platoons on the fly, without any prior planning. We can obtain the solution to the *Opportunistic Platooning Problem* (*OPP*) by forbidding any detours or waiting in the RTP. That is, for every truck k, we set $x_{ak} = 0$ for all $a \in A_K \backslash short_k$. However, for all arcs $a \in A_I$, x_{ak} is not pre-set such that trucks can form platoons on the fly.

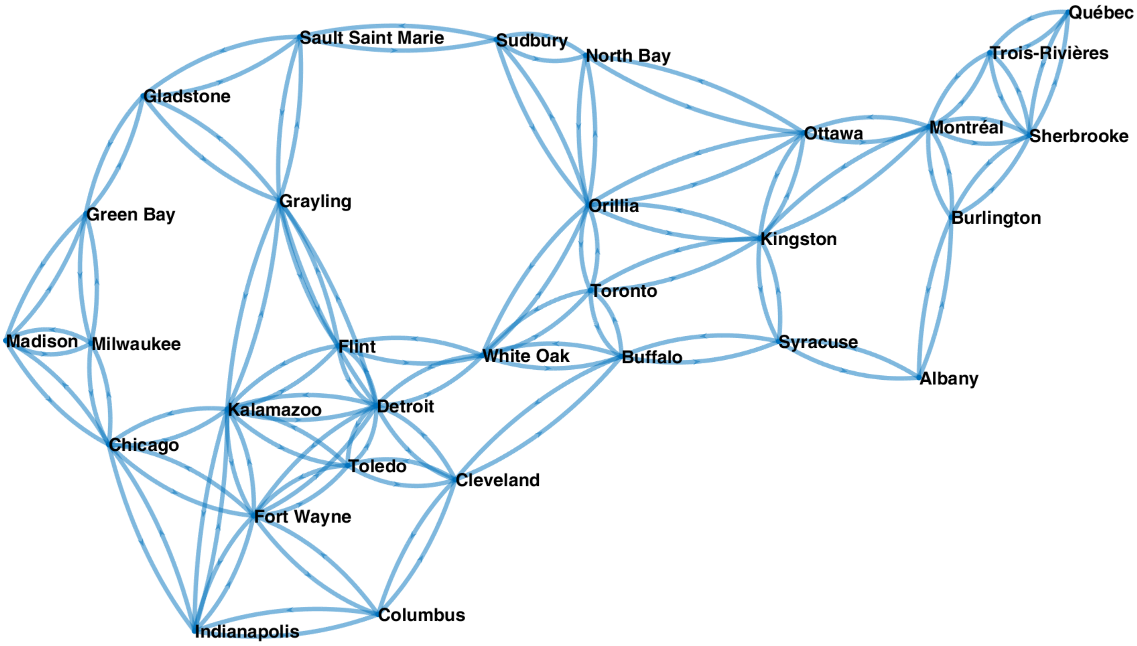
Let denote $optsol(MIP)$ the optimal solution value of a MIP then it holds that

$$addval := optsol(OPP) - optsol(RTP) \geq 0 \ , \tag{5.12}$$

where *addval* measures the *additional value* that a central platform generates, which is the maximal premium the platform can charge from the carriers.

Observe that we implicitly assume in the OPP that all trucks that are on the same arc a in period t will join a platoon. Depending on the length of t, this might be unrealistic since the different trucks might not even see each other. Thus, the optimal solution of the OPP might overestimate the savings through opportunistic platooning.

Figure 5.4: Thirty cities in the area of the Great Lakes (Canada and U.S.) form the first network for our computational study.



5.4 Computational study

The goals of the computational study are three-fold. First, we want to evaluate the computational performance of our solution approach. Second, we conduct a sensitivity analysis where we assess the impact of different platooning parameters and of the network structure on the optimal solution. Third, we determine the added value that is generated through a central coordination of platoons. That is, we compare our solution to the one obtained through opportunistic platooning.

5.4.1 Experimental set-up

For our computational study, we use two different highway networks. While the first one contains 30 cities in the region of the Great Lakes in the upper mid-east region of North America (Figure 5.4), the second network is formed out of 29 cities in the larger region of the Ruhr in the west of Germany and Venlo, a Dutch border town (Figure 5.5). In the following, we refer to them as "Great Lakes network" and "Ruhr network".

The differences in the two networks lie in the distances and in the shapes. The Great Lakes network has a total length of 27,432 km and contains 120 edges. It resembles a

Figure 5.5: Thirty cities in the region of the Ruhr form the second network for our computational study.

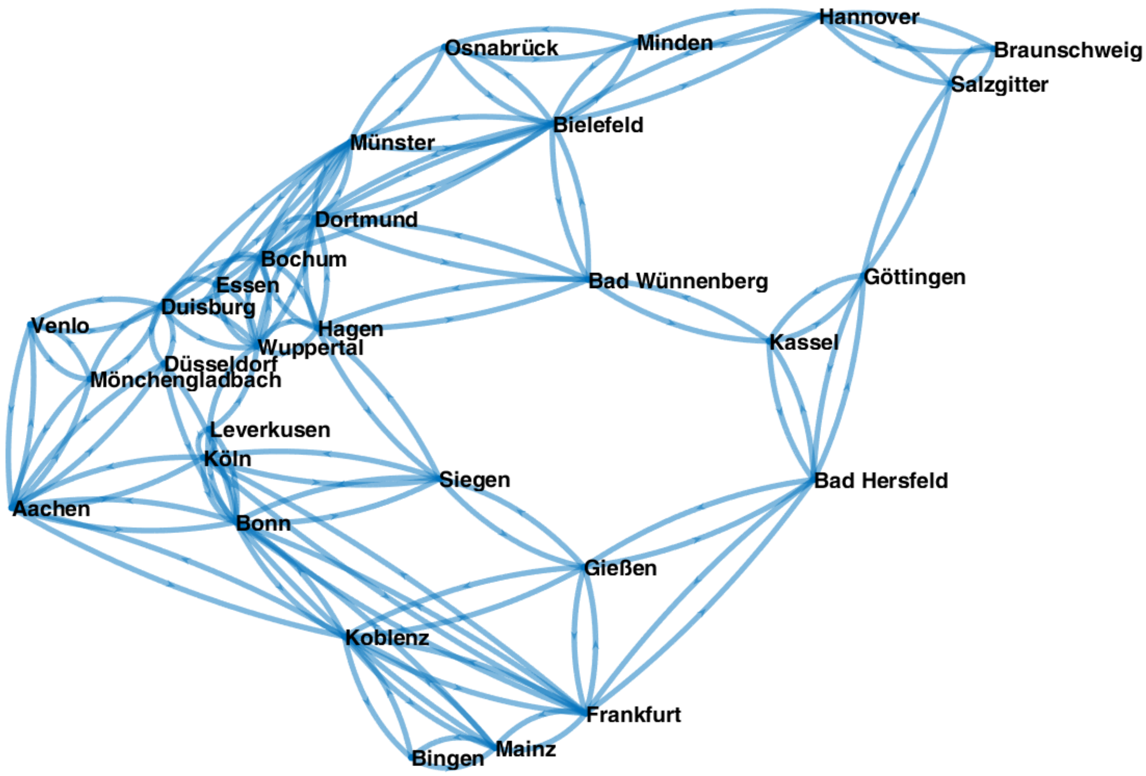


Table 5.2: Statistics for the pre-processing procedure on the Great Lakes and the Ruhr network. The number of arcs that were generated are distinguished by the two layers (truck, platoon). The runtimes are reported in seconds.

	Great Lakes			Ruhr		
	arcs		runtime	arcs		runtime
	truck	platoon	[sec]	truck	platoon	[sec]
Mean	3,992.00	1,489.05	21.83	4,714.65	1,640.60	43.62
Stdv	214.49	118.09	0.78	146.54	80.95	3.44
Max	4,372.00	1,684.00	23.17	4,975.00	1,794.00	50.81
Q3	4,149.75	1,575.75	22.37	4,807.50	1,709.00	45.76
Q2	3,989.00	1,510.00	21.93	4,709.33	1,644.00	43.54
Q1	3,830.75	1,397.75	21.15	4,646.25	1,597.75	41.39
Min	3,658.00	1,270.00	20.51	4,385.00	1,462.00	38.46

corridor, where the detour lengths are rather long. In contrast to that, the distances in the Ruhr network are shorter: the whole network contains 134 edges with a total length of 10,006 km. Thus, the average edge length in the Ruhr, 74.67 km, is three-times smaller than in the Great Lakes, where it lies at 228.60 km. Furthermore, the Ruhr network shows a grid-like structure with more possibilities for detours. This can be also seen from the number of arcs that were created during the pre-processing procedure, which we report in Table 5.2. The total, average number of arcs in the Ruhr is 6,355.25 and thus 16% higher than in the Great Lakes network with a total average number of 5,481.05 arcs.

In every network, the 30 cities serve as waypoint nodes. Other waypoints like rest areas along the highway are not considered. We set the number of trucks to 150 and assume that their origins and destinations are off the highway network. Therefore, we select their entry and exit point in the highway network from the 30 locations. In the following, we refer to these points as *entries* and *exits*. To reflect the real world situation, where the incoming and outgoing freight volumes of the cities are different, the random generation of the entries and exits follows a multinomial distribution. We report the self-chosen probabilities in Table 5.3. To get statistically sound results, we use these probabilities to create for both networks 20 instances with randomly selected origins and destinations.

The earliest possible start time is set equally for every truck k to $t = 0$, whereas truck

Table 5.3: Probabilities (Prob.) for the multinomial sampling of entries and exits out of 30 cities in the regions of the Great Lakes and of the Ruhr.

Region	City	Prob.	City	Prob.	City	Prob.
Great Lakes	Albany	0.01	Grayling	0.01	Ottawa	0.03
	Buffalo	0.03	Green Bay	0.03	Québec	0.02
	Burlington	0.01	Indianapolis	0.10	Sault Saint Marie	0.02
	Chicago	0.15	Kalamazoo	0.01	Sherbrooke	0.02
	Cleveland	0.02	Kingston	0.01	Sudbury	0.01
	Columbus	0.05	Madison	0.03	Syracuse	0.01
	Detroit	0.11	Milwaukee	0.03	Toledo	0.01
	Flint	0.01	Montréal	0.10	Toronto	0.10
	Fort Wayne	0.01	North Bay	0.01	Trois-Rivières	0.02
	Gladstone	0.01	Orillia	0.01	White Oak	0.01
Ruhr	Aachen	0.05	Düsseldorf	0.06	Leverkusen	0.03
	Bad Hersfeld	0.10	Essen	0.01	Mainz	0.02
	Bad Wünnenberg	0.02	Frankfurt	0.10	Minden	0.04
	Bielefeld	0.05	Gießen	0.01	Mönchengladbach	0.01
	Bingen	0.01	Göttingen	0.01	Münster	0.01
	Bochum	0.02	Hagen	0.02	Osnabrück	0.03
	Bonn	0.03	Hannover	0.05	Salzgitter	0.04
	Braunschweig	0.02	Kassel	0.01	Siegen	0.01
	Dortmund	0.05	Koblenz	0.01	Venlo	0.09
	Duisburg	0.07	Köln	0.01	Wuppertal	0.01

k's latest arrival time $late_k$ is calculated as

$$late_k(\varphi) = min\{(1+\varphi) \cdot earliest_k; latest_k\} \ , \tag{5.13}$$

where $earliest_k$ denotes the travel time on the shortest path and $latest_k$ the travel time on the longest path. The traveling time on the first and last mile is randomly drawn from the interval $[45; 90]$. Consequently, trucks arrive at different points in time at the highway nodes. Therefore, the random assignment of traveling times on the first mile can be interpreted as assigning different start times to the trucks.

In the Great Lakes network, we discretize the time by using intervals of 45 minutes. Consequently, we have to round the travel times to multiples of 45 minutes, which can lead to deviations by up to 22.5 minutes. Since the distances in the Ruhr network are smaller, we use time intervals of 15 minutes. Consequently, the maximal deviation due to rounding lies at 7.5 minutes.

We set the cost per time unit driven to 1, independent of truck k. The waiting costs are set to 0.05 per time step and joining or leaving a platoon costs 0.001.

The pre-processing is done with *MATLAB R2016b* on a Windows 10 PC with 4 Intel Core Xeon CPU (2.60 GHz) and 12 GB of RAM. To solve the model, we use *FICO Xpress 8.6* on a Linux 4.4 server with 16 Intel Core Xeon CPU (2.60 GHz) and 72 GB of RAM. We consider a problem to be solved to optimality when the optimality gap, i.e., the relative difference between the lower bound and current best solution, falls below 0.1%. For the evaluation, we use the following key performance indicators:

- Runtime for solving the problem to optimality.
- Relative fuel savings achieved by platooning, i.e., comparing the cost of the optimal solution to the cost of all trucks driving individually on their shortest route (that is, $\eta^l = 0\%$ and $\eta^f = 0\%$).
- The *Platoon Exploitation Rate* (*PER*), which quantifies the share of the overall travel time in a platoon as follows:

$$PER := \frac{\sum_{k \in K} \textit{total time traveled by truck } k \textit{ in a platoon}}{\sum_{k \in K} \textit{total time travelled by truck } k} \in [0;1] \; . \tag{5.14}$$

  A PER of one means that all trips in the network are done in platoons while a value of zero means that there is no platoon formation at all. Due to the fact that the trucks drive on the first and last mile individually, the PER is strictly smaller one in our computational study.

- The total waiting time, which might be necessary to synchronize with other trucks. The value is measured in time steps, that is 45 minutes.
- Relative difference in the travel time (TT_{diff}), compared to the shortest path. That is:

$$TT_{diff} := \frac{\sum_{k \in K} (\textit{travel time truck } k - \textit{shortest path truck } k)}{\sum_{k \in K} \textit{shortest path truck } k} \; . \tag{5.15}$$

  TT_{diff} measures the detours that trucks take in order to platoon with others.

- The number of platoons of size $s \in S$ that are formed. Observe that a truck may join several platoons during its trip.

The computational study is divided into two parts: In Section 5.4.2, we evaluate the computational performance of the pre-processing procedure and the runtimes for solving the RTP. In Section 5.4.3, we conduct a sensitivity analysis to assess the parameters' influence on the total fuel savings.

5.4.2 Computational performance

In Table 5.2, we report the statistics of the pre-processing procedure. For the Great Lakes network, the average runtime lies at 21.83 seconds. In the Ruhr network, where the total average number of arcs created is 16% higher than for the Great Lakes network, the average runtime is 43.62 seconds, which is more than twice as high than for on the Great Lakes network. This can be caused by the fact that in the Ruhr network, there are more feasible paths that need to be evaluated by the procedure. In total, the results show that the pre-processing procedure can produce the required arcs in a very short time.

To evaluate the computational performance for solving the RTP on both networks, we set $\varphi = 10\%$ and $\eta^l = 0\%$, whereas the platoon followers' fuel savings factor and the maximal platoons size are varied as follows: $\eta^f \in \{5\%, 10\%, 15\%\}$ and $\varsigma \in \{3, 5\}$. We report the results in Table 5.4. As one can see, there are substantial differences between the performance on both networks. On the Great Lakes network, the runtimes vary between 8 and 18 minutes. Thereby, the main driver for longer computational times is the fuel savings factor η^f. This can be caused by the fact that a higher η^f increases the number of detours that are still economical to drive and thus, more options need to be evaluated. The maximal platoon size ς, on the other hand, has no noticeable influence on the runtimes. Since the pre-processing procedure created more arcs for the Ruhr network (see Table 5.2), we observe longer runtimes on those instances. Here, the average runtimes lie between 14 and 53 minutes. What is of interest, is the fact that for $\varsigma = 5$, the average runtimes are lower than for $\varsigma = 3$. An explanation to this might be symmetries in the solution, which occur when more than three trucks are platooning on the same arc. Then, more than one platoon has to be formed and every distribution of the trucks yields the same cost. Analogous to the Great Lakes network, a higher η^f increases the runtimes, since higher savings increase the number of detours with a positive fuel savings potential.

In sum, the results show that our approach is capable to solve large instances on both networks within one hour, which is an acceptable response time for the platform. The runtimes are influenced by the network structure and platooning parameters.

Table 5.4: Runtimes (in seconds) for solving the RTP on the Great Lakes network and the Ruhr network.

	Great Lakes network						Ruhr network					
	$\varsigma = 3$			$\varsigma = 5$			$\varsigma = 3$			$\varsigma = 5$		
η^f	5%	10%	15%	5%	10%	15%	5%	10%	15%	5%	10%	15%
Mean	469	1,012	1,124	426	788	1,028	1,035	2,484	3,130	813	859	1,313
Stdv	295	257	20	258	350	269	1,016	1,036	957	795	826	1,235
Max	1,147	1,150	1,167	1,176	1,188	1,202	4,035	4,077	4,063	4,233	4,112	4,191
Q3	475	1,129	1,137	389	1,163	1,180	733	3,362	3,816	722	741	1,165
Q2	318	1,115	1,121	329	626	1,157	581	2,691	3,487	644	603	737
Q1	285	1,100	1,109	295	466	1,087	499	1,907	2,890	549	474	598
Min	238	362	1,087	254	334	406	409	596	714	422	366	393

5.4.3 Sensitivity analysis

In the first part of the sensitivity analysis, we examine the influence of the platoon followers' fuel-savings factor and the maximal platoon size. Next, we vary the trucks' buffer times by increasing φ in $late_k(\varphi)$ (cf. formula (5.13)). In the third part of the sensitivity analysis, we study the case when the platoon leader saves fuel as well.

Varying the fuel-savings and maximal platoon size

We use the same parameter setting as for the evaluation of the computational study. That is, $\varphi = 10\%$, $\eta^l = 0\%$, $\eta^f \in \{5\%, 10\%, 15\%\}$ and $\varsigma \in \{3, 5\}$. In Table 5.5, we display the relative fuel savings achieved by platooning and the PER. Figure 5.6 visualizes the average fuel savings, which lie in the range of 2.13% to 7.44%. As one can see, the main driver for the fuel savings is η^f, whereas the platoon size limit ς has a smaller effect. The average PER lies in the range of 70% (cf. Table 5.5) and there are only small differences between the cases: the maximum difference between the average PER's lies at 1.03 percentage points. This nearly constant PER results from the fact that the trucks do almost never a detour to platoon with other trucks. This can be seen from the small deviations in the travel times in Table 5.6. In the maximum, the travel times are 0.12% longer than without platooning. This indicates that the expenses for a detour exceed the fuel savings. Thus, the modest increase in the PER origins from the fact that for a higher η^f, the fuel-savings exceed the cost of waiting. Therefore, the waiting times increase with a higher η^f (see Table 5.6). A larger ς allows more trucks to exploit a reduced air drag and therefore to achieve higher fuel savings. Therefore we observe

Table 5.5: Relative fuel savings and PER (in percentages) in the Great Lakes network.

	Fuel savings [%]						PER [%]					
	$\varsigma = 3$			$\varsigma = 5$			$\varsigma = 3$			$\varsigma = 5$		
η^f	5%	10%	15%	5%	10%	15%	5%	10%	15%	5%	10%	15%
Mean	2.13	4.30	6.49	2.44	4.94	7.44	69.26	69.66	70.11	69.47	70.10	70.39
Stdv	0.08	0.17	0.26	0.10	0.19	0.28	2.44	2.63	2.49	2.42	2.31	2.14
Max	2.28	4.59	6.94	2.65	5.38	8.09	73.42	74.19	74.93	74.02	74.99	74.48
Q3	2.17	4.39	6.61	2.49	5.03	7.56	70.77	71.00	71.44	71.35	71.45	71.60
Q2	2.14	4.33	6.54	2.44	4.96	7.47	69.53	69.94	69.88	69.88	69.99	70.34
Q1	2.06	4.14	6.26	2.36	4.79	7.25	67.05	67.07	68.47	67.28	68.13	68.49
Min	1.98	3.98	6.03	2.25	4.58	6.89	64.92	64.98	66.03	65.96	66.82	67.46

Table 5.6: Relative travel time difference TT_{diff} (in percentages) and waiting (in time steps) in the Great Lakes network.

	TT_{diff} [%]						Waiting [time steps]					
	$\varsigma = 3$			$\varsigma = 5$			$\varsigma = 3$			$\varsigma = 5$		
η^f	5%	10%	15%	5%	10%	15%	5%	10%	15%	5%	10%	15%
Mean	0.00	0.01	0.04	0.00	0.01	0.03	404	498	551	347	412	420
Stdv	0.00	0.02	0.04	0.00	0.02	0.04	159	152	216	117	128	150
Max	0.00	0.05	0.12	0.00	0.04	0.12	820	920	1,260	600	800	680
Q3	0.00	0.04	0.04	0.00	0.01	0.04	520	560	620	425	460	530
Q2	0.00	0.00	0.04	0.00	0.00	0.04	400	490	540	360	400	430
Q1	0.00	0.00	0.00	0.00	0.00	0.00	300	390	435	255	355	350
Min	0.00	0.00	0.00	0.00	0.00	0.00	140	320	240	160	140	140

higher fuel savings for $\varsigma = 5$.

Concluding, the results demonstrate that trucks do rarely deviate from their shortest paths. An explanation to this observation can be the shape of the Great Lakes network, which offers limited possibilities to drive a detour. Therefore, we solve the RTP on the Ruhr network, where the distances are shorter and more detour options are given. Hereby, we limit the factorial design to $\varsigma = 5$.

In Table 5.7, we report the results for the Ruhr network. The average fuel-savings that are achieved lie between 1.60% and 4.83%, which is considerably lower than in the Great Lakes network. Figure 5.7 visualizes this by displaying the average fuel savings that were achieved in the two networks with $\varsigma = 5$. The average PER, reported in Table 5.7, lies between 47.37% and 48.06%, which is substantially lower than in the Great Lakes network with values of 69.47% and 70.39% (cf. Table 5.5). This shows that

Figure 5.6: Average fuel savings achieved through platooning in the Great Lakes network.

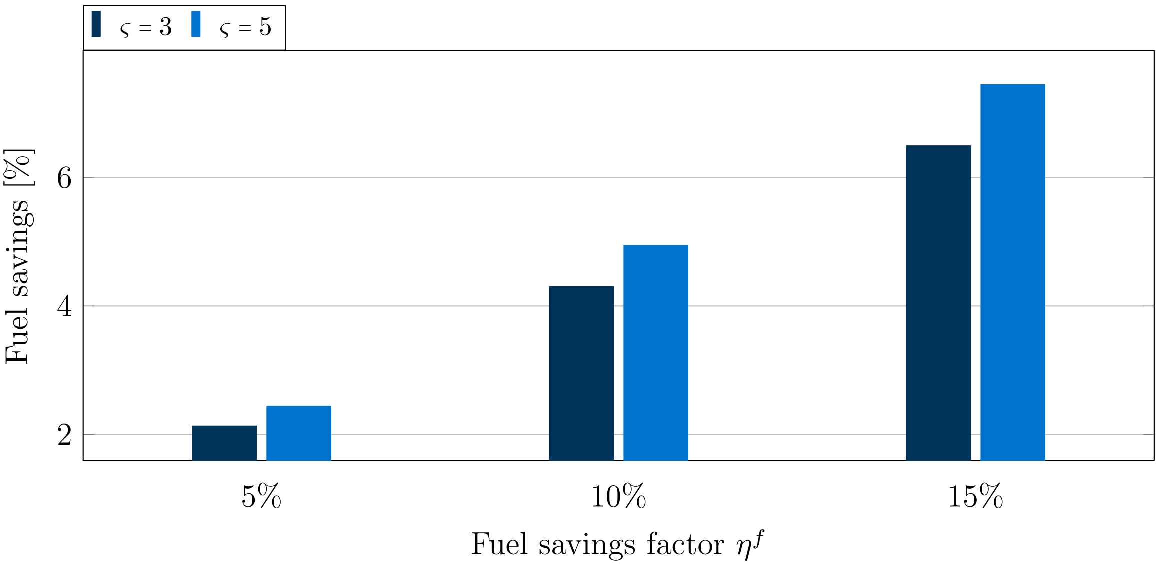
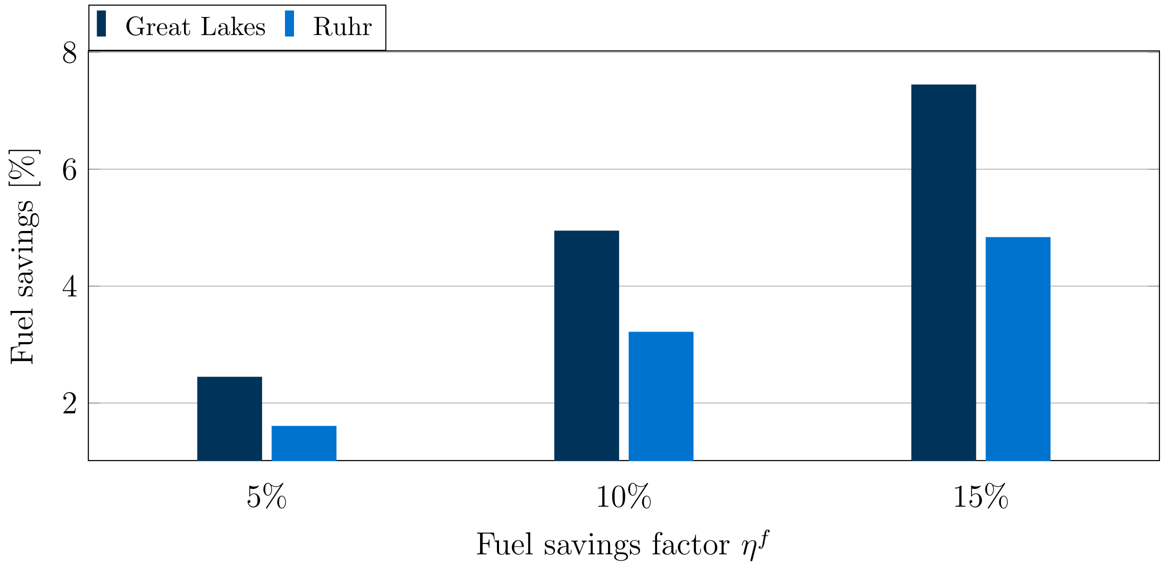


Table 5.7: Relative fuel savings, PER, relative travel time difference TT_{diff} (all three in percentages) and waiting times (in time-steps) in the Ruhr network.

	Fuel savings [%]			PER [%]			TT_{diff} [%]			Waiting		
η^f	5%	10%	15%	5%	10%	15%	5%	10%	15%	5%	10%	15%
Mean	1.60	3.21	4.83	47.37	47.59	48.06	0.00	0.01	0.06	3	7	11
Stdv	0.06	0.13	0.19	1.58	1.45	1.71	0.00	0.02	0.03	2	3	4
Max	1.71	3.43	5.16	49.91	50.05	51.87	0.00	0.07	0.14	6	11	20
Q3	1.65	3.32	5.00	48.57	48.55	48.93	0.00	0.00	0.07	4	9	13
Q2	1.58	3.17	4.77	47.25	47.44	47.80	0.00	0.00	0.05	3	9	12
Q1	1.55	3.12	4.71	46.21	46.80	47.11	0.00	0.00	0.04	1	6	9
Min	1.46	2.94	4.43	44.31	44.49	44.51	0.00	0.00	0.00	0	2	5

Figure 5.7: Average fuel savings achieved through platooning in the Great Lakes network and around the Ruhr network ($\varphi = 10\%$, $\varsigma = 5$).



less distances are covered in platoons.

As one can see from TT_{diff}, reported in Table 5.7, the deviations from the shortest paths are twice as high as in the Great Lakes network, but still marginal. Hence, also in the Ruhr network trucks stick mostly to their shortest paths and since the network contains more edges, less truck paths do overlap. Therefore, the PER and consequently the fuel-savings are lower than in the Great Lakes network.

In sum, we have seen that in a network with more and shorter edges, trucks drive longer detours but the deviations from the shortest path are still marginal.

Varying the buffer time

So far, we fixed the trucks' time slack with $\varphi = 10\%$. To see how the buffer times influence the optimal solution, we vary $\varphi \in \{0\%, 10\%, 20\%\}$. Hereby, the fuel savings factor is set to $\eta^f = 10\%$ and the platoon size limit to $\varsigma = 5$. We report the resulting KPIs in Table 5.8, which shows that the buffer time has a significant impact on the fuel savings. When granting no slack, the average fuel savings are 3.63% whereas the PER lies at 54.41% (cf. Table 5.8).

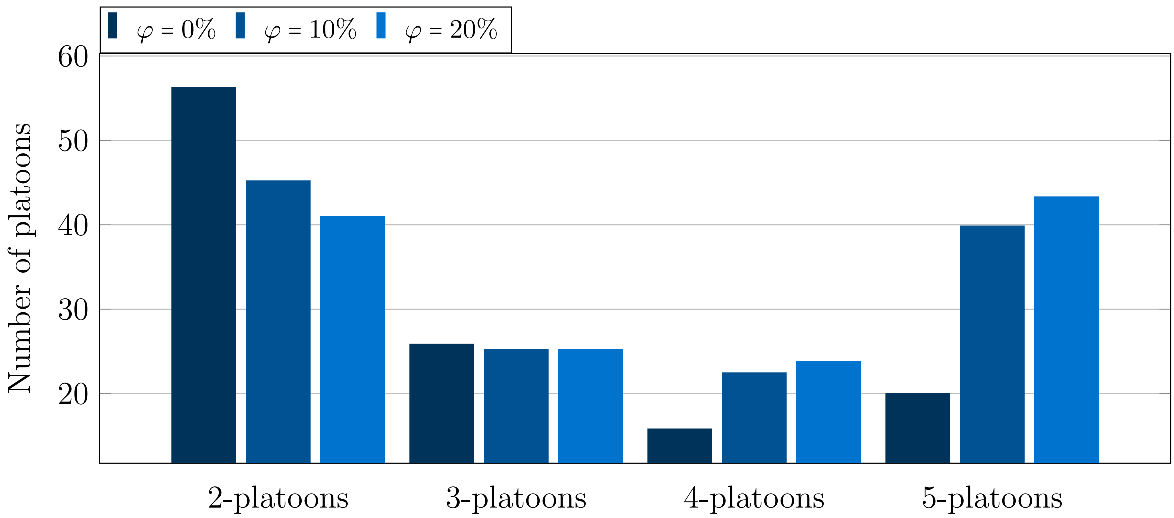
When increasing φ to 20%, the average fuel savings are 5.17%. Simultaneously, we can observe a higher PER of 72.60%. However, a larger buffer does not lead to more detours driven: for $\varphi = 20\%$ and $\varphi = 10\%$, the quartiles of TT_{diff} are identical and those deviations are very small, lying in the order of 0.01 percentage points. From

Table 5.8: Relative fuel savings, PER, relative travel time difference TT_{diff} (all three in percentages) and waiting times (in time steps) for varying buffer time ($\varsigma = 5$).

φ	Fuel savings [%] 0%	10%	20%	PER [%] 0%	10%	20%	TT_{diff} [%] 0%	10%	20%	Waiting 0%	10%	20%
Mean	3.63	4.94	5.17	54.41	70.10	72.60	0.00	0.01	0.01	0	412	830
Stdv	0.28	0.19	0.18	3.30	2.31	2.09	0.00	0.02	0.02	0	128	301
Max	4.08	5.38	5.55	60.08	74.99	77.01	0.00	0.04	0.04	0	800	1,3000
Q3	3.87	5.03	5.20	56.54	71.45	73.23	0.00	0.01	0.01	0	460	1,085
Q2	3.65	4.96	5.14	54.67	69.99	72.34	0.00	0.00	0.00	0	400	890
Q1	3.39	4.79	5.08	52.21	68.13	71.49	0.00	0.00	0.00	0	355	600
Min	3.06	4.58	4.90	48.10	66.82	69.28	0.00	0.00	0.00	0	140	340

that we conclude that the buffer time is not spent for driving longer detours. Instead, the additional time is spent for waiting for other trucks to form larger platoons along the shortest paths. Therefore, the average waiting time is more than twice as high for $\varphi = 20\%$, compared to $\varphi = 10\%$ (see Table 5.8). The grater the buffer time, the larger platoons are formed. This can bee seen from Table 5.9, where we report the statistics for the absolute numbers of platoons of size $s \in \{2, \ldots, 5\}$ that were formed. Figure 5.8 visualizes the average numbers.

Figure 5.8: Average number of platoons of size s that are formed under varying buffer time in the Great Lakes network.



To conclude, larger buffer times are not exploited to drive longer detours but to wait for other trucks to form larger platoons. This allows for higher fuel savings.

Table 5.9: Number of platoons of size s that are formed under varying buffer time ($\eta^f = 10\%$) in the Great Lakes network.

s	2			3			4			5		
φ	0%	10%	20%	0%	10%	20%	0%	10%	20%	0%	10%	20%
Mean	56	45	41	26	25	25	16	22	24	20	40	43
Stdv	11	9	6	4	6	6	4	3	6	6	5	5
Max	80	64	51	33	39	34	24	30	43	34	52	54
Q3	65	52	44	28	30	30	19	25	25	23	42	45
Q2	52	43	41	26	25	26	15	23	23	21	40	43
Q1	48	38	37	22	21	21	14	20	20	16	36	40
Min	38	26	31	20	13	15	8	17	17	10	30	34

Fuel-savings for the platoon leader

According to Tsugawa et al. (2011), the platoon leader can reduce its fuel consumption by up to 7.5% due to reduced vortexes. In the following, we set $\eta^l = 5\%$, $\eta^f = 10\%$, $\varphi = 10\%$ and vary $\varsigma \in \{3, 5\}$. Table 5.10 summarizes the results. It shows that the average fuel-savings lie at 5.70% and 6.02%, respectively. These values are, compared to the case of $\eta^l = 0\%$, obviously higher since the fuel-savings potential of the whole platoon increases. However, a part of the increase in the savings stems from a slightly higher PER; the average values go up by two percentage points to 71.62% and 71.80%. This is a result of slightly more detours that are driven. This can be seen from the average TT_{diff} which is three times higher than in the basic case with $\eta^l = 0\%$. What is of interest is the fact that the maximum deviation lies at 0.12% and 0.17%, respectively. These results indicate that with sufficiently high fuel-savings, trucks are willing to drive more and longer detours.

5.4.4 Value of centralized planning

As discussed in Section 5.2, one way of generating revenues for the platform is charging the carriers a premium. In (5.12), we specified this value as *addval*, which is the additional value that is generated through the central coordination. We solve the OPP on the Great Lakes instances with $\eta^l = 0\%$, $\eta^f = 10\%$, $\varphi = 10\%$ and $\varsigma \in \{3, 5\}$. We determine the absolute difference between the solution of the *RTP* and the *OPP*, using the KPIs relative fuel-savings and PER. The values are reported in Table 5.11

A central coordination increases the average fuel-savings by between 0.07 and 0.23 percentage points, in the best case by 0.41 percentage points. The difference in the

Table 5.10: Relative fuel savings, PER, relative travel time difference TT_{diff} (all three in percentages) and total number of platoons that were formed for $\eta^l = 5\%$, $\eta^f = 10\%$ and $\varphi = 10\%$ in the Great Lakes network.

	Savings [%]		PER [%]		TT_{diff} [%]		Platoons	
ς	3	5	3	5	3	5	3	5
Mean	5.70	6.02	71.69	71.80	0.03	0.03	183	145
Stdv	0.20	0.20	2.22	2.12	0.03	0.04	11	8.26
Max	6.09	6.41	76.14	75.87	0.12	0.17	204	161
Q3	5.80	6.11	72.96	72.83	0.04	0.04	190	149
Q2	5.73	6.02	71.51	72.04	0.00	0.00	186	146
Q1	5.52	5.88	70.40	70.67	0.00	0.00	174	141
Min	5.37	5.70	68.33	68.05	0.00	0.00	164	125

Table 5.11: Absolute difference between the solutions of the RTP and the OPP, evaluated on the Great Lakes network with $\eta^l = 0\%$, $\eta^f = 10\%$, $\varphi = 10\%$ and $\varsigma \in \{3, 5\}$.

	Fuel savings [%]						PER [%]					
	$\varsigma = 3$			$\varsigma = 5$			$\varsigma = 3$			$\varsigma = 5$		
η^f	5%	10%	15%	5%	10%	15%	5%	10%	15%	5%	10%	15%
Mean	0.07	0.13	0.23	0.07	0.15	0.23	2.36	2.14	2.44	1.72	1.68	1.85
Stdv	0.02	0.06	0.08	0.03	0.06	0.09	0.94	0.92	0.79	0.84	0.95	0.88
Max	0.11	0.26	0.41	0.12	0.26	0.41	4.28	4.34	3.95	3.56	3.73	3.43
Q3	0.09	0.17	0.26	0.09	0.19	0.30	3.07	2.43	3.05	2.27	2.06	2.86
Q2	0.06	0.12	0.23	0.07	0.16	0.25	2.34	2.28	2.37	1.65	1.58	1.65
Q1	0.05	0.08	0.16	0.05	0.10	0.13	1.64	1.68	1.95	1.09	1.13	1.20
Min	0.03	0.05	0.13	0.02	0.04	0.10	0.76	0.31	0.94	0.45	0.19	0.44

average PER lies between 1.72 and 2.44 percentage points. Since the optimal solution to the RTP contains only a few detours, the advantage of a central coordination stems from the fact that additional waiting times can be scheduled. An explanation to this small gain through a central planning can be the relatively high number of journeys compared to the network size. Thus, there is a certain "density" that allows trucks to find enough platooning partners, even without planning in advance. Another contributing factor is the multinomial distribution of origin-destination pairs, which creates certain major links in the network.

However, as mentioned in Section 5.3.5, we use discrete time-steps and therefore the solution of the OPP tends to overestimate the real objective value. Furthermore, opportunistic platooning also requires a central instance, which assures and coordinates a fair split of the savings. Thus, enhancing the platform with the possibility to do a central planning increases the margin that the platform can charge from the carriers. Furthermore, planning, particularly when there are good numbers of vehicles, with many diverse origin-destination definitions and significant distances that can be covered together but with a number of path alternatives, takes out part of the uncertainty for the carriers.

Finally, even though percentages seem small, the additional savings potential through a central coordination is significant. For example, in 2017, the total number of tonne-kilometers (tkm) driven in the 28 countries of the European Union were 1,913,116 million (Eurostat, 2017). Assuming that all trucks that drove more than 150 km were using highways, this number reduces to 1,488,812 million tkm (Eurostat, 2017). The average fuel consumption of a truck in Europe lied at this time around 2.9 liters/tkm (Statistisches Bundesamt, 2018), the average CO_2 emissions around 61 g/tkm (Institut für Energie- und Umweltforschung Heidelberg GmbH, 2014). Using the added value that we determined, the central coordination of trucks into platoons could have saved approximately between 3,022 ($\eta^f = 5\%$) and 9,930 million liters ($\eta^f = 15\%$) of fuel in Europe in 2017. This corresponds to 0.06 to 0.2 million tons less CO_2 emissions. Although these numbers represent an estimate, they show the scale of the fuel and CO_2 emissions savings potential through a multi-brand platform.

5.5 Summary of insights

The outcome of our computational study shows that, in most of the cases, trucks do not deviate from their shortest paths when forming platoons. This result is quite indicative as we observed it on both networks and might be the effect of the relatively small saving through platooning, which make only few detour economically viable. This hypothesis is supported by the fact that, if we assume that the platoon leader saves fuel as well and thus the total platoon savings increase, more detours are driven. Therefore, we expect that advances in the platooning technology, which would lead to higher fuel savings, would increase the number of viable detours.

The sensitivity analysis also showed that increasing the parameters for the fuel-savings factor, the platoon size or the buffer time leads to more and larger platoons, and thus higher savings. Comparing the results of the Great Lakes network with the Ruhr network, we could see that networks with a corridor-like structure foster the formation of platoons as more trucks share their shortest paths. Thus, the main challenge when solving the RTP is to schedule the trucks' departures and waiting times in such way that the trucks can platoon on their shortest paths. Under the current savings potential through truck platooning, this scheduling is also the additional value that a central platform generates.

5.6 Conclusion

In this chapter, we focused on the central coordination of truck platoons through a multi-brand platform. First, we defined the individual process steps that are required to organize platoons. Then, we took a deeper look on the second process step. It describes the planning of the platoons with the information that the carriers provided to the platform before the trucks start their journeys. Since this planning has to be done with some lead time, we identified it as the day-before truck platooning planning problem. We introduced a novel mixed-integer linear programming formulation for this problem. The formulation combines the routing and scheduling of trucks under the option of forming platoons and includes restrictions on the maximal size of the platoons and on the trucks' time-windows. To reduce the problem size, we developed a pre-processing procedure that exploits the fact that the model is defined on a time-expanded network and that the trucks have time-windows. Thus, it is possible to exclude infeasible arcs in advance. In addition, we use the time-windows to fix infeasible decision variables and furthermore provide a starting solution, which is based on the trucks' shortest paths.

The computational study shows the efficiency of the pre-processing procedure, which allows us to solve large instance within reasonable time. From our sensitivity analysis, we saw that in only few cases trucks will drive a detour to join a platoon. The reason is that the additional fuel expenses extend the fuel savings. Therefore, the additional value of a central coordination of platoons is small, compared to the unorganized, on-the-fly formation of platoons. However, if further advances in the platooning technology allow for higher savings, more detours become economically viable and then, the added value further increases. In addition, a multi-brand platform is always required as an intermediary that redistributes the savings among the platoon members and is trusted by competing carriers. Solving the day-before planning problem, increases the benefit of such a platform.

Summarizing, we formalized the process of a central organization of truck platoons, proposed a mathematical formulation and a pre-processing method and evaluated our model. Our methodological contribution goes well beyond truck platooning and can find application in the general field of consolidation-based transportation.

Chapter 6

Truck Platooning under Driving Time Regulations

Truck platooning is currently discussed not only from a fuel savings perspective, but also with respect to a bridging technology for the development towards autonomous trucking. It is presumed that this transition phase will happen in three steps, where in the first two steps the trucks need to be manned. Consequently, the routing and scheduling of trucks under the platooning option will be affected by driving time regulations. We consider the impact of driving time regulations on the day-before platooning planning problem by formulating a mixed-integer linear program that includes the regulations on mandatory rest periods and breaks. Moreover, we anticipate a higher degree of autonomous driving by including a *rest-while-trailing* option. That is, the driving times of those drivers who are trailing with their trucks in a platoon are only partially counted, or not at all. The results of a controlled computational study with European driving time regulations show that these regulations have a substantial impact on the routing and scheduling of the trucks and thus, the fuel savings through platooning decrease. However, trucks can exploit mandatory breaks to wait for other trucks to form platoons. Furthermore, we demonstrate that the rest-while-trailing option can substantially lower the total transportation costs, mainly due to reduced labor cost. In addition, trucks drive under the rest-while-trailing option longer and more detours since the savings in labor cost exceed the additional fuel expenses for detours.

6.1 Introduction

Truck platooning is based on the idea that several trucks drive in close succession. Being digitally connected and using autonomous driving technology, the platoon followers automatically steer, break and accelerate with the manually driven first truck, the platoon leader. Thus, the trucks in this train-like formation can travel with a smaller safety distance, which can help with reducing fuel consumption and thus CO_2 emissions. It is also a step towards fitting more trucks on the road, and potentially improving the safety on the street (Janssen et al., 2015). Moreover, platooning can be used as a bridging technology towards fully autonomously driving trucks.

In 2018, the European Automobile Manufacturers' Association (2018) (*ACEA*) published an "EU Roadmap for Truck Platooning" with the intention to introduce this innovative technology in the European Union by 2023. The ACEA assumed that drivers are only required as long as trucks cannot drive fully autonomously. Already allowing the truck drivers of the trailing trucks to (partially) rest would increase the time of trucks on the road. This would help to address the problems of overfilled parking lots along highways (Deutsche Welle, 2018) and the lack of truck drivers (International Road Transport Union, 2018).

According to the ACEA roadmap, the transition phase will happen in three stages, which we call in the following *ACEA stages*, or simply *stages*:

- *Stage 1*: All drivers need to be attentive while driving in a platoon.
- *Stage 2*: The drivers of the trailing trucks can (partially) rest while driving in a platoon.
- *Stage 3*: Trucks are driving fully autonomously, no drivers are needed or all drivers rest.

Larsson et al. (2015) formulated the Truck Platooning Problem as mixed-integer linear problem. This formulation describes the combinatorial problem of routing and scheduling trucks in a graph under the option of jointly traversing arcs and thus saving fuel cost. Temporal aspects like time windows were not considered.

Since trucks typically cover long distances, the formation of platoons in stages 1 and 2 will be affected by driving time regulations that apply in many countries. In the European Union, the EU social legislation *Regulation (EC) No 561/2006* (European Union, 2006) on driving times, breaks and rest periods, as well as *Directive 2002/15/EC*(European Union, 2002), define the framework for the truckers' working times. Similar legislation can also be found in many other countries, for instance in Canada (*Commercial Vehicle Drivers Hours of Service Regulations*) or in the United States (*hours-of-service regulations*), and several authors have shown that, even for individual trucks, such regulations make the computation of cost-efficient tours a challenging task (Goel and Vidal, 2013).

The only work that relates driving time regulations to platooning is the one by Larsen et al. (2019). The authors consider stage 2, where the driver of a trailing truck can skip a break since he is resting-while-trailing. This is formulated under the assumptions that the trucks drive on fixed routes and platoons can only be formed at a single hub node. According to the consulting firm Strategy& (2018), stage 2 will be reached in 2025 and stage 3 in 2030. The ACEA expects that truck platooning in stage 1 will be fully possible across Europe by 2023, after the roll-out started in 2020. Consequently, driving time regulations will be a key factor in the successful implementation of truck platooning. In Chapter 5, we already showed how the day-before truck platooning planning problem in ACEA stage 3 can be modeled and solved. This motivates us to formulate the following research question:

How can the day-before truck platooning planning problem be modeled and solved in ACEA stages 1 and 2 and what are the optimal solutions to this problem in all three stages?

To answer this question, we focus on the European driving time regulations and leave other legislation like the Canadian or U.S. driving time regulations for future research. We include the basic regulations on driving times, breaks or daily rest periods in the European Union as an extension to the mixed-integer linear programming formulation of the *Restricted Truck Platooning Problem* (*RTP*, see Section 5.3). We call this problem the *European Restricted Truck Platooning Problem* (*EU-RTP*). The implementation of the driving time regulations is achieved by using clock time variables that keep track of the drivers' working hours. Furthermore, these variables can be used for imposing required breaks and rest periods. In addition, we allow trucks to delay their departure or to wait for other trucks at meeting points in order to enable the formation of platoons. When calculating the costs, we consider fuel costs and truck driver wages. To model stage 2 of the ACEA roadmap, we assume that the drivers of the following trucks "rest-while-trailing". That is, they do not need to be fully attentive. In that case, we have to adapt the EU-RTP in such way that only the driving times of the leading trucks are fully considered.

We extend the pre-processing procedure introduced in Section 5.3.4 in such way that paths where trucks would violate the driving time regulations in stage 1 are excluded. This helps to further reduce the number of created arcs.

The remainder of this chapter is structured as follows: In Section 6.2, we describe the current legislation on working hours for truck drivers in the European Union, Canada and the United States. In Section 6.3, we show how the EU-RTP can be formulated as an extension to the RTP. Furthermore, we discuss adaptions to the pre-processing procedure. In Section 6.4, a controlled computational study is used to evaluate the performance of solving the EU-RTP and the RTP. Moreover, we assess the impact of the European driving time regulations and the rest-while-trailing option on the formation of platoons and summarize our insights. Section 6.5 concludes the chapter.

6.2 Legal framework for truck driver scheduling in different countries

The driving and working times of truck drivers in the European Union are regulated by *Regulation (EC) No 561/2006* and *Directive 2002/15/EC*. While the former defines the maximum driving times along with minimum breaks and rest periods for different time horizons (European Union, 2006), the latter legislation formulates the general working conditions for truck drivers (European Union, 2002).

According to *Regulation (EC) No 561/2006*, a break of at least 45 min has to be taken after a maximum driving period of 4.5 h. As soon as the maximum daily driving time of 9 h is reached, the driver needs to take a rest period for at least 11 h. Furthermore, a rest period has to be completed within 24 h after the end of the last rest period. In the course of one week, a driver must not drive more than 56 h, while the limit for the total number of driving hours in two weeks is 90 h. Between two weeks of working, a driver has to rest at least 45 h. To give the fleet operators more flexibility, the legislator introduced so-called *splitting rules*. According to these rules, a break can be divided into two parts, where the first one has to be at least 15 min and the second 30 min. Similarly, a daily rest can be divided into two parts with a minimum of 3 h for the first part and 9 h for the second part. This means that drivers are granted an additional hour if they split up a daily rest period. In addition to these *splitting rules*, driving times can be extended and rest periods can be reduced under certain conditions. However, such extensions have to be compensated afterwards by additional rest periods and thus should only be used in unforeseeable events.

Similar to the European Union, other countries have established comparable rules. In Canada, the *Commercial Vehicle Drivers Hours of Service Regulations* (Government of Canada, 2005) grant the carriers more flexibility: they require that a driver has to take a daily rest of 8 h after having driven for 13 h. In addition, a total of 2 h of breaks has to be taken during these 13 h with each brake lasting at least 30 min. Similar to the European rules, the splitting of breaks and rest periods is allowed. Due to the large overlaps between the Canadian and the European driving time regulations, our findings should hold in a similar way for Canada.

In the United States, the *hours-of-service regulations* (*HOS*) define the legal framework for truck drivers. Among other things, these regulations require that after 11 h of driving, a 10 h rest has to be taken and that a driver can drive a maximum of 14 h after the last rest has been taken (Goel and Kok, 2012). Since the HOS do not include obligatory breaks, trucks have to take voluntary pauses to synchronize with other trucks and thus the waiting cannot be absorbed into breaks. However, the option to rest-while-trailing might allow the drivers to prolong their tours to up to 14 h of daily driving time.

In this chapter, we study the influence of driving time regulations on the day-before truck platooning problem, which we addressed in Chapter 5 and where we determine the optimal routes and schedules of the trucks. In contrast to Larsen et al. (2019), the trucks are flexible in the route choice and do not have to pass through a single hub node. Furthermore, we allow that rest-while-trailing is partially counted as driving time and that several breaks or rest periods can probably be skipped.

6.3 The European Restricted Truck Platooning Problem

In this section, we extend the Restricted Truck Platooning Problem (see Section 5.3) to include the European regulations on the truck drivers' weekly working hours. In addition, we introduce the option for platoon followers to rest-while-trailing. In Section 6.3.3, we summarize which variant of the models represents the corresponding ACEA stages. Finally, we adapt the pre-processing procedure from Section 5.3.4 such that those paths where driving time regulations are violated are excluded and infeasible decision variables are set to zero.

6.3.1 Problem description and assumptions

A set of K trucks $\mathcal{K} = \{1, \ldots, K\}$ is traveling on a *supporting network* $\mathcal{G} := (\mathcal{V}, \mathcal{A})$. The travel times between the nodes are given by τ_{ij}. The subset $\mathcal{V}^W \subsetneq \mathcal{V}$ represents the parking lots along the highways, where platoons can be formed, and the highway entrances and exits. We assume that platoons are only allowed on highways. Therefore, we say that the first mile after the origin and the last mile before the destination are driven individually. The planning is done for some time period T, which is discretized into h intervals.

We define the EU-RTP on the two-layer time-space network $G = (V, A)$, which was introduced in Section 5.3. G comprises the graph $\mathcal{G}$, which we expand to h time intervals. The network consists of two layers, the truck layer and the platoon layer. The node set $V := V_K \cup V_P$ is partitioned accordingly into *truck nodes* and *platoon nodes*. Since we assume that platoons can only be formed on highways, V_P contains exclusively time-expanded copies of the waypoint nodes $\mathcal{V}^W$. The arc set $A := A_K \cup A_P \cup A_I$ consists of truck arcs and platoon arcs, which connect the truck nodes and the platoon nodes, respectively. The third subset, $A_I = A_I^+ \cup A_I^-$, comprises the *interlayer arcs*; A_I^+ contains all arcs that lead from the truck layer to the platoon layer. These arcs represent the formation of a platoon, whereas A_I^- comprises all arcs from the platoon layer to the truck layer. This set of arcs models the disbanding of a platoon. Trucks are allowed to wait at nodes. This is modeled by *waiting arcs*, which connect nodes that represent the same physical location but differ in one time interval.

Every truck has an earliest starting time e_k at its origin o_k and a latest arrival time l_k at its destination d_k. When trailing in a platoon, the fuel consumption is reduced by η^f for the followers and by η^l for the leader. $\rho \in [0, 1]$ defines the share of the driving time that is not counted in the rest-while-trailing option. That is, if $\rho = 1$, the driving time of the drivers in the following trucks is not credited at all. The maximal size of a platoon is limited by ς and the set $S := \{2, \dots, \varsigma\}$ comprises all size options. We assume that several platoons can travel simultaneously on the same highway. Since the travel times between nodes may vary over the planning period T, we denote the travel time between nodes i and j, $i, j \in \mathcal{V}$, at time point $t \in T$ by t_{ij}.

While *Regulation (EC) No 561/2006* sets a European framework for driving times, *Directive 2002/15/EC* extends the temporal rules to restrict night work and working times on and off the vehicle (Goel, 2018). Off-vehicle duties are not related to driving and comprise tasks like administration, freight handling, maintenance or waiting times at customer sites. Consequently, they get more relevant if we wish to optimize delivery tours like Vehicle Routing Problems with Time Windows. Thus, we consider only *Regulation (EC) No 561/2006*, which mainly influences long-distance tours. Similarly to Goel (2009, 2010), we focus on those parts of *Regulation (EC) No 561/2006* that affect the driving time between two weekly rest periods. This is motivated by the consideration that our planning horizon for the platoons is one week. Since the aim of this chapter is to provide a general proof of concept of the model and to evaluate the impact of driving time regulations, the splitting rules are dropped. However, it is a straightforward task to incorporate these rules into the basic model and this extension will be left for future work.

We assume that all truck drivers start their tours completely rested and that breaks, rest periods and additional waiting times can only be taken at the waypoints. Furthermore, truck drivers are assigned to the same truck throughout the whole planning period. Consequently, the driving times of a driver correspond to those of a truck. After driving for a period with a length of at most D^b (4.5 h), a minimum break time B (45 min) has to be taken. After a total driving time of 9 h (D^r), every driver has to rest a minimum time of 11 h (R). Furthermore, after having finished the last rest period, the driver has to complete the next rest period inside a 24-hour-window (D).

c^f_{ak} describes the fuel cost of truck k when it travels on arc a, whereas c^l_k describes the wages that have to be paid. For waiting arcs, we set $c^f_{ak} = 0$, whereas c^l_k corresponds to the wages of one time interval. When drivers take a mandatory break or rest, or when they rest-while-trailing, we assume that no wages are paid. However, for additional pauses, where the trucks wait voluntarily, wages are paid. This can be seen as imposing a cost of waiting. Joining or leaving a platoon creates a cost c^p, which is independent of the truck or location.

Our objective is to minimize the total travel cost of all trucks through platooning. That is, we assume that the overall savings are fairly distributed such that no truck in the platoon is worse off, i.e. any allocation belonging to the core could be a feasible distribution of savings. In case of a rest-while-trailing, the platoon leaders also need to be compensated for personnel cost. However, if the platoon leaders are paid out a fair portion of those savings, there is - from a cost perspective - no disadvantage for being a platoon leader.

6.3.2 Mathematical model

Our model is an extension of the arc-based formulation on $G = (V, A)$, as introduced in Section 5.3. Superscripts o and d indicate the origin or destination of an arc $a \in A$. That is, $a = (a^o, a^d)$. The sets of decision variables can be divided into two groups. The first group is defined on the arcs A as follows: The binary decision variable x_{ak} is set to one if truck k uses arc $a \in A_K \cup A_I$. The integer decision variable y_{as} counts the number of platoons of size s traveling on arc $a \in A_P$. The binary decision variable z_{aks} is set to one if truck k is traveling in a platoon of size s on arc a and the binary decision variable w_{aks} is set to one if k leads a platoon.

The second group of decision variables is related to the driving time regulations. Since these variables capture the time, it is sufficient to define them on $\mathcal{V}$, the node set of the supporting network. So-called *clock variables* keep track of the trucks' driving times by measuring the time after the last break or daily rest. u_{ik}^{in} saves truck k's driving time since the last break after its arrival at node $i \in \mathcal{V}$, u_{ik}^{out} safes the driving time when the truck leaves i. Similarly, v_{ik}^{in} and v_{ik}^{out} measure the driving time after the last daily rest. u_{ik}^{out} is reset when a truck takes a break or daily rest at i, while v_{ik}^{out} can only be reset after a daily break. To reset the breaks, the binary variables b_{ik} and r_{ik} indicate whether or not truck k takes a break or daily rest at node $i \in \mathcal{V}^W$.

To formulate the model, it is necessary to identify which node $v \in \mathcal{V}$ in the supporting network and which time interval $h \in T$ are represented by a node $i \in V$ of the time-space expanded node set. To this end, we use the following two functions $f : V \to \mathcal{V}$ and $g : V \to T$. The image of f is the corresponding node in the supporting network and the image of g the corresponding time interval: $f(i) = v$ and $g(i) = t$. Table 6.1 gives an overview of the notation.

The total costs consist of the following three cost components:

(i) *fuel costs :=*

$$\sum_{k\in\mathcal{K}}\Big(\sum_{a\in A_K} c^f_{ak}\cdot x_{ak} + \sum_{a\in A_P}\sum_{s\in S}\big(c^f_{ak}\cdot\big(1-\eta^f\cdot\frac{(s-1)}{s}\big)\cdot z_{aks} + c^f_{ak}\cdot(1-\eta^l)\cdot w_{aks}\big)\Big).$$

When trucks platoon, the $s-1$ follower's fuel costs are reduced by η^f, whereas η^l reduces the leader's fuel expenses.

(ii) *personnel costs:=*

$$\sum_{k\in\mathcal{K}}\Big(\sum_{a\in A_K} c^l_{ak}\cdot x_{ak} + \sum_{a\in A_P}\sum_{s\in S}\big((1-\rho)\cdot c^l_{ak}\cdot z_{aks} + \rho\cdot c^l_{ak}\cdot w_{aks}\big)\Big) - \sum_{i\in\mathcal{V}^{\mathcal{W}}} c^w\cdot(B\cdot b_i + R\cdot r_i)\,.$$

If the rest-while-trailing option is allowed ($\rho > 0$), the follower's drivers have to be paid for $1-\rho$ of the driving time, whereas the driver of the leader has to be paid for the full time. Since we assume that drivers are not paid for mandatory breaks or rests, we subtract these costs.

Table 6.1: Sets, parameters, functions and decision variables used in the EU-RTP model.

Sets, parameters and functions	
$V = V_K \cup V_P$	set of all time-expanded nodes, which can be partitioned into nodes on the truck level and platoon level
$\mathcal{V}$	set of nodes in the supporting network,
$\mathcal{V}^W$	set of way-point nodes in the supporting network
$A = A_K \cup A_I \cup A_P$	set of all arcs, which can be partitioned into truck arcs, interlayer arcs and platoon arcs
$\mathcal{K}$	set of all trucks
o_k, d_k	origin and destination of truck k
e_k, l_k	earliest departure at origin and latest arrival at destination of truck k
T	set of all time periods
t_{ij}	travel time between node i and node j
f, g	functions mapping $V \to \mathcal{V}$ and $V \to T$
ς	maximum size of a platoon
$S = \{2, \dots, \varsigma\}$	set of all platoon sizes
c^f_{ak}, c^l_{ak}	fuel cost and labor cost of truck k on arc a
c^w, c^p	waiting cost, cost of joining or leaving a platoon
η^l, η^f	fuel reduction factor of platoon leader and platoon follower
ρ	rest-while-trailing factor
D^b, D^r	maximum driving time until next break or daily rest
B, R, D	duration of a break, daily rest or full day
Binary decision variables	
b_{ik}	if truck k takes a break at node i
r_{ik}	if truck k takes a daily rest at node i
x_{ak}	if truck k uses arc a
w_{aks}	if trucks k leads a platoon of size s on arc a
z_{aks}	if trucks k travels on arc a in a platoon of size s
Integer decision variable	
y_{as}	number of platoons of size s traveling on arc a
Continuous decision variables	
u^{in}_{ik}	truck k's driving time after the last break or daily rest when entering node i
u^{out}_{ik}	truck k's driving time after the last break or daily rest when leaving node i
v^{in}_{ik}	truck k's driving time after the last daily rest when entering node i
v^{out}_{ik}	truck k's driving time after the last daily rest when leaving node i

(iii) *forming costs* :=

$$\sum_{k\in\mathcal{K}}\sum_{a\in A_I} c^p \cdot x_{ak}.$$

The cost for joining or leaving a platoon.

The EU-RTP reads as follows:

$$\begin{aligned} \min \quad & \Big(\textit{fuel costs} + \textit{personnel costs} + \textit{forming costs}\Big) && (6.1)\\ s.t. \quad & (5.2) - (5.8) \end{aligned}$$

In the objective function, we minimize the total costs, which are the sum of the fuel costs, personnel costs and forming costs. Constraints (5.2) - (5.8) of the RTP are also required for the correctness of the EU-RTP and explained in Section 5.3.3.

Constraint (6.2) ensures that every platoon has one leader. This can be relaxed to an inequality since platoon leaders have higher costs than followers, which means that the optimal solution will always contain the minimal number of platoon leaders. Inequality (6.3) states that a truck can only lead a platoon if it is also part of the platoon.

$$\sum_{k\in\mathcal{K}} w_{aks} \geq y_{as} \quad \forall a \in A_P, s \in S \tag{6.2}$$

$$z_{aks} \geq w_{aks} \quad \forall a \in A_P, s \in S, k \in K \tag{6.3}$$

The following constraints formulate the European driving time regulations. Inequality (6.4) assures that the break clock is increased by the driving time if and only if a truck traverses an arc on the truck level. Similarly, (6.5) increases the driving time for a truck that moves on the platoon level. If the rest-while-trailing option is activated by choosing $\rho > 0$, the incoming break clock increases for platoon followers by the share $1-\rho$. If the truck is a platoon leader, the factor is increased to one by adding ρ. Whenever a truck takes a break or a rest, the outgoing break clock is reset by (6.6). Since the movements of the trucks are defined on the arcs of the time-expanded network, but the breaks and rests are related to the supporting network, we use the functions f and g to map the time-expanded nodes on the corresponding supporting nodes and time period, respectively.

$$u_{jk}^{in} \geq t_{ij} + u_{ik}^{out} - D^b \cdot (1 - x_{ak}) \qquad \forall k \in \mathcal{K}, a \in A_K : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.4)$$

$$u_{jk}^{in} \geq u_{ik}^{out} - D^b \cdot (1 - \sum_{s \in S} z_{aks}) +$$
$$t_{ij} \cdot \big(\rho \cdot \sum_{s \in S} w_{aks} + (1 - \rho) \cdot \sum_{s \in S} z_{aks}\big) \qquad \forall k \in \mathcal{K}, a \in A_P : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.5)$$

$$u_{jk}^{out} \geq u_{ik}^{in} - D^b \cdot (b_{ik} + r_{ik}) \qquad \forall k \in \mathcal{K}, a \in A_K : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.6)$$

Analogously to the previous three constraints, (6.7) and (6.8) increase the incoming rest clock variable and (6.9) resets the outgoing rest clock variable. Observe that v_{jk}^{out} is only reset after a daily rest. Inequality (6.10) states that a daily rest has to be taken 24 hours (D) after the last rest was taken.

$$v_{jk}^{in} \geq t_{ij} + v_{ik}^{out} - D^r \cdot (1 - x_{ak}) \qquad \forall k \in \mathcal{K}, a \in A_K : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.7)$$

$$v_{jk}^{in} \geq v_{ik}^{out} - D^r \cdot (1 - \sum_{s \in S} z_{aks}) +$$
$$t_{ij} \cdot \big(\rho \cdot \sum_{s \in S} w_{aks} + (1 - \rho) \cdot \sum_{s \in S} z_{aks}\big) \qquad \forall k \in \mathcal{K}, a \in A_P : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.8)$$

$$v_{jk}^{out} \geq v_{ik}^{in} - D^r \cdot r_{ik} \qquad \forall k \in \mathcal{K}, a \in A_K : f(a^o) = i, f(a^d) = j \in \mathcal{V} \quad (6.9)$$

$$D \geq v_{ik}^{in} + D^r \cdot r_{ik} \qquad \forall k \in \mathcal{K}, i \in \mathcal{V} \quad (6.10)$$

To preclude that breaks and daily rests are offset against each other, (6.11) forbids that breaks and rests are taken at the same location. (6.12) and (6.13) limit the maximum driving time after the last break or rest period, respectively. The minimum duration of a break or daily rest is enforced by (6.14) and (6.15), which requires trucks to “traverse” B or R waiting arcs, respectively. Due to the flow balancing equality (5.4), it is guaranteed that these waiting arcs are consecutive.

$$1 \geq b_{ik} + r_{ik} \qquad \forall i \in \mathcal{V}^W, k \in \mathcal{K} \quad (6.11)$$

$$D^b \geq u_{ik}^{in} \qquad \forall i \in \mathcal{V}^W, k \in \mathcal{K} \quad (6.12)$$

$$D^r \geq v_{ik}^{in} \qquad \forall i \in \mathcal{V}^W, k \in \mathcal{K} \quad (6.13)$$

$$\sum_{a \in A_K} x_{ak} \geq B \cdot b_{ik} \qquad \forall k \in \mathcal{K}, a = (i,j) \in A_K : f(i) = f(j) \quad (6.14)$$

$$\sum_{a \in A_K} x_{ak} \geq R \cdot r_{ik} \qquad \forall k \in \mathcal{K}, a = (i,j) \in A_K : f(i) = f(j) \quad (6.15)$$

$$x_{ak}, w_{aks}, z_{aks} \in \{0;1\}, \;\; y_{as} \in \mathbb{N}_0 \qquad \forall a \in A, k \in \mathcal{K}, s \in S \quad (6.16)$$

$$b_{ik}, r_{ik} \in \{0;1\}, \;\; u_{ik}^{in}, u_{ik}^{out}, v_{ik}^{in}, v_{ik}^{out} \in \mathbb{R}_0 \qquad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (6.17)$$

6.3.3 The Restricted Truck Platooning Problem in all three ACEA stages

We give a brief overview on how the Restricted Truck Platooning Problem can be formulated in the respective ACEA stages:

- *Stage 1*: EU-RTP with $\rho = 0$. All drivers need to be attentive and follow the driving time regulations. Platooning allows the trucks to save fuel cost.
- *Stage 2*: EU-RTP with $\rho \in (0, 1]$. Only the leading truck driver's time is fully counted. The drivers of the trailing trucks can perform other tasks or (partially) rest. Platooning allows the trucks to save personnel cost (trailing trucks) and fuel cost.
- *Stage 3*: RTP as defined in Section 5.3.3. Since the trucks are unmanned, personnel cost do not need to be considered. Platooning allows the trucks to save fuel cost.

6.3.4 Adaptations to the pre-processing procedure

In Section 5.3.4, we presented a pre-processing procedure that allowed us to create the minimal arc set A. This procedure can be adapted to stage 1 as follows:

- When computing $\mathcal{P}_k^\theta$, we do not consider mandatory break or rest times. These pause times can extend the travel times on some paths $p \in \mathcal{P}_k^\theta$ to such an extent that trucks violate their latest arrival l_k. Therefore, we test for all p if mandatory break times and daily rests are required and exclude all those paths where a truck would arrive too late. We denote this set of *driving-time-regulation-feasible* paths as $\mathcal{P}_k^f$.
- For every truck k, there may exist a subset of arcs in A' that cannot be traversed. Thus, we can set the following decision variables to zero:
  - x_{ak}: if the travel-period lies beyond k's time window, i.e. $latest_{ik} < g(i)$ or $g(j) < earliest_{jk}$.
  - w_{aks}, z_{aks}: if the travel-period lies beyond k's time window, i.e. $latest_{ik} < g(i)$ or $g(j) < earliest_{jk}$, $s \in S$.
  - b_{ik}, r_{ik}: if node i is not included in any of k's feasible paths $p \in \mathcal{P}_k^f$.

Observe that, due to the rest-while-trailing option, trucks might not need to take a break or rest at all. Consequently, we cannot limit $\mathcal{P}_k^\theta$ to driving-time-regulation-feasible paths in stage 2.

6.4 Computational study

The goal of this controlled computational study is twofold. First, we assess the computational performance of the Restricted Truck Platooning Problem in the European Union in all three ACEA platooning stages is solved. Second, we evaluate the optimal solutions in the corresponding stages and assess the cost savings potential.

6.4.1 Experimental set-up

To study the effect of a varying rest-while-trailing factor, we solve the EU-RTP for the following values $\rho \in \{0, 0.25, 0.5, 0.75, 1\}$. $\rho = 0$ corresponds to ACEA stage 1, whereas all cases with $\rho > 0$ describe different scenarios in stage 2, where the driving times of the trailing truck drivers are (partially) counted as rest times. In stage 3, the trucks are driving unmanned such that there are no personnel costs or driving time regulations that need to be considered. Consequently, we solve the RTP as defined in Section 5.3.3.

We use a self-chosen network with twelve nodes, which is depicted in Figure 6.1. The number of trucks is set to 60 and their origins and destinations are randomly selected from the twelve locations in the network.

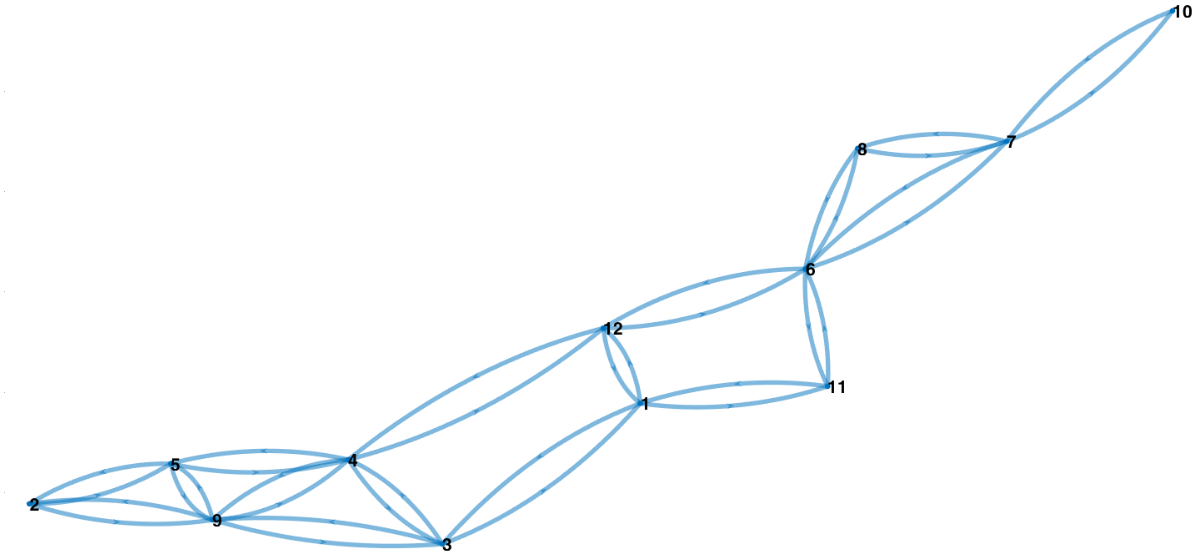
The earliest possible start time is set to be equal for every truck k to $t = 0$, whereas truck k's latest arrival time $late_k$ is calculated as

$$late_k = min\{1.1 \cdot earliest_k; latest_k\} \ ,$$

where $earliest_k$ denotes the travel time on the shortest path and $latest_k$ the travel time on the longest path. Note that the duration of the shortest path and the longest path are shorter in stage 3 since driving time regulations do not need to be considered. Due to the rest-while-trailing option in stage 2, $earliest_k$ corresponds to the shortest path without driving time regulations and $latest_k$ corresponds to the path with driving time regulations. This results in more arcs that need to be created, since driving-time-infeasible paths cannot be removed if $\rho > 0$ (cf. Section 6.3.4).

The time is discretized into time-steps with a length of 45 minutes. The fuel cost is given by EUR 1.15 per liter (Statista, 2018) and we base our calculations on an equal

Figure 6.1: Twelve nodes form the network for our controlled computational study.



fuel consumption of 24 liters per hour. Thus, the fuel cost is EUR 20.70 per time-step. The personnel cost is fixed at EUR 30 per hour (Comite National Routier, 2016) and therefore EUR 22.50 per time-step. Joining or leaving a platoon costs EUR 1. We assume that the platoon followers save $\eta^f = 15\%$ fuel, whereas the platoon leader enjoys no fuel savings ($\eta^l = 0\%$). The platoon size limit is set to $\varsigma = 5$. We assume that trucks have to travel on the first and last mile individually with a travel time that is randomly drawn from the interval $[45; 90]$.

We create 15 instances with randomly selected origins and destinations. For the evaluation, we use the following key performance indicators:

- *Savings*: Relative changes in costs (total, fuel and personnel) for the optimal solution with the platooning option, as opposed to the costs of the optimal solution without the platooning option (that is, $\eta^f = 0$).

- *PER*: The *Platoon Exploitation Rate* (*PER*), which quantifies the share of the overall travel time in a platoon and is defined as follows (cf. Section 5.4.1):

$$PER := \frac{\sum_{k \in K} \textit{total time traveled by truck k in a platoon}}{\sum_{k \in K} \textit{total time travelled by truck k}}\ . \tag{5.14}$$

- TT_{diff}: Relative difference in the travel time, compared to the shortest path (cf. Section 5.4.1):

$$TT_{diff} := \frac{\sum_{k\in K}(travel\ time\ truck\ k - shortest\ path\ truck\ k)}{\sum_{k\in K} shortest\ path\ truck\ k} \ . \tag{5.15}$$

  TT_{diff} measures the detours that trucks take in order to platoon with others.

- b_{diff}: relative difference in the number of breaks between the optimal solution with the platooning option and the optimal solution without the platooning option (that is, $\eta^f = 0$):

$$b_{diff} := \frac{no.\ of\ breaks\ with\ platooning - no.\ of\ breaks\ without\ platooning}{no.\ of\ breaks\ without\ platooning} \ . \tag{6.18}$$

- r_{diff}: relative difference in the number of rest periods between the optimal solution with the platooning option and the optimal solution without the platooning option (that is, $\eta^f = 0$):

$$r_{diff} := \frac{no.\ of\ rests\ with\ platooning - no.\ of\ rests\ without\ platooning}{no.\ of\ rests\ without\ platooning} \ . \tag{6.19}$$

b_{diff} and r_{diff} are used to see, how the number of breaks and rest periods that are taken changes if platooning (without and with the rest-while-trailing option) is allowed.

6.4.2 Computational performance

The pre-processing procedure is done with *MATLAB R2016b* on a Windows 10 PC with 4 Intel Core Xeon CPU (2.60 GHz) and 12 GB of RAM. The resulting number of arcs for every layer and the runtimes of the procedure are reported in Table 6.2. The smallest number of arcs is created for stage 1, as driving time regulations allow to exclude several paths beforehand. The highest number of arcs is created for stage 2, which is a result of the larger time windows due to the rest-while-trailing option (see Section 6.3.4). The arcs are created quickly, on average as few as 12 and up to 21 seconds.

To solve the model, we use *FICO Xpress 8.6* on a Linux 4.4 server with 16 Intel Core Xeon CPU (2.60 GHz) and 72 GB of RAM. We consider a problem solved to optimality as soon as the optimality gap falls below 0.1%. Table 6.3 summarizes the corresponding runtimes.

Table 6.2: Statistics for the pre-processing procedure in all three stages. The number of arcs that were generated are distinguished by the two layers (truck, platoon). The runtimes of the pre-processing procedure are reported in seconds.

	Stage 1			Stage 2			Stage 3		
	truck	platoon	time	truck	platoon	time	truck	platoon	time
Mean	3,044	857	12.06	8,260	2,721	21.38	5,357	1,459	16.48
Stdv	206	90	3.35	365	174	3.37	273	104	3.48
Max	3,405	1,047	26.12	9,299	3,111	24.90	5,838	1,627	21.88
Q3	3,181	892	11.99	8,288	2,874	22.42	5,506	1,520	19.06
Q2	3,061	858	11.23	8,391	2,739	21.33	5,350	1,469	16.91
Q1	2,897	806	10.94	8,053	2,509	20.28	5,201	1,406	14.30
Min	2,684	700	9.42	7,629	2,193	18.85	4,693	1,264	9.63

As expected, the RTP is the one that, with an average runtime of 20 minutes, takes the least time to solve, whereas the EU-RTP requires more runtime. For $\rho = 0\%$, the instances are solved on average within 52 minutes. For $\rho > 0\%$, the runtime increases even further to average values between 71 to 83 minutes. This is caused by the larger input size due to more arcs and by the increased number of options due to the rest-while-trailing option. The runtimes of the EU-RTP exhibit a high deviation: certain instances require more than 100 minutes to be solved to optimality (cf. Table 6.3).

Table 6.3: Runtimes (in seconds) for solving the EU-RTP with varying values of ρ and the RTP.

	EU-RTP					RTP
ρ	0%	25%	50%	75%	100%	
Mean	3,141.02	4,806.33	4,999.18	4,802.11	4,275.28	1,163.62
Stdv	1,805.48	1,297.17	1,292.53	1,000.61	1,651.01	493.77
Max	5,943.13	6,183.98	6,239.73	6,075.58	6,314.00	1,811.86
Q3	4,878.01	5,892.67	6,015.85	5,653.01	5,631.48	1,629.90
Q2	2,135.51	4,935.03	5,169.88	4,885.93	4,898.61	1,151.75
Q1	1,496.05	4,046.89	4,541.35	3,964.36	1,868.02	824.84
Min	875.31	1,275.01	1,029.41	3,158.89	1,119.59	220.51

6.4.3 Truck platooning under European driving time regulations

In the following, we analyze the optimal solutions to the EU-RTP for varying values of ρ, organized according to the stages. Table 6.4 summarizes the resulting KPIs.

Table 6.4: *Cost savings*, *PER*, TT_{diff}, b_{diff} and r_{diff} for solving the EU-RTP with varying values of ρ. All values are given as percentages.

		Savings			PER	TTdiff	pauses	
ρ		Total	Fuel	Pers			breaks	rests
0%	Mean	2.46	5.13	0.00	47.28	0.00	8.59	0.00
	Stdv	0.10	0.21	0.00	1.74	0.00	10.00	0.00
	Max	2.62	5.47	0.00	50.69	0.00	28.23	0.00
	Q3	2.52	5.27	0.00	48.01	0.00	16.32	0.00
	Q2	2.48	5.17	0.00	47.17	0.00	10.74	0.00
	Q1	2.39	4.99	0.00	46.34	0.00	-0.77	0.00
	Min	2.27	4.74	0.00	43.38	0.00	-5.93	0.00
25%	Mean	6.74	5.00	11.62	46.57	0.00	3.29	-2.06
	Stdv	0.24	0.18	0.51	2.06	0.00	7.26	2.52
	Max	7.12	5.29	12.38	49.63	0.00	20.75	0.00
	Q3	6.94	5.15	12.06	48.30	0.00	8.83	0.00
	Q2	6.73	5.00	11.73	47.00	0.00	0.87	-2.44
	Q1	6.58	4.88	11.23	44.99	0.00	-3.14	-2.90
	Min	6.31	4.68	10.72	42.97	0.00	-4.41	-9.52
50%	Mean	11.24	5.04	24.02	48.21	0.06	0.91	-5.99
	Stdv	0.89	0.41	1.60	3.21	0.10	7.34	6.52
	Max	12.53	5.59	25.82	51.74	0.36	14.15	0.00
	Q3	11.83	5.34	25.10	50.28	0.11	6.70	0.00
	Q2	11.43	5.10	24.59	49.27	0.00	-0.78	-4.55
	Q1	10.94	4.84	23.38	46.93	0.00	-5.02	-8.20
	Min	8.92	4.03	19.56	39.20	0.00	-11.11	-21.21
75%	Mean	19.00	6.03	42.64	57.08	0.19	1.11	-7.86
	Stdv	1.07	0.37	2.33	3.12	0.16	6.02	7.28
	Max	20.81	6.51	46.59	62.46	0.51	12.90	0.00
	Q3	19.62	6.31	43.86	58.84	0.29	5.00	0.00
	Q2	19.39	6.15	43.73	58.40	0.21	0.83	-7.14
	Q1	18.44	5.79	41.99	56.24	0.05	-4.05	-14.39
	Min	16.69	5.27	37.47	50.21	0.00	-7.35	-20.69
100%	Mean	31.53	7.39	73.64	73.95	0.77	8.92	-16.12
	Stdv	1.30	0.43	2.26	2.26	0.33	8.26	12.23
	Max	33.42	7.84	77.51	77.84	1.52	23.97	0.00
	Q3	32.12	7.69	74.63	74.96	0.95	13.59	-6.93
	Q2	31.93	7.45	73.77	74.14	0.85	7.87	-15.15
	Q1	31.29	7.29	72.87	73.16	0.53	5.04	-24.72
	Min	28.70	6.11	68.91	69.10	0.23	-8.59	-36.36

Stage 1

In stage 1, where all drivers have to be attentive (i.e., $\rho = 0\%$), the average total cost savings are 2.46%, with a maximum of 2.62% and a standard deviation of 0.1%. As Table 6.4 shows, these savings are a result of a lower fuel consumption; on average, 5.13% less fuel is burned if the trucks can platoon. The PER is rather low, namely in the range of 43.38% to 50.69% with a mean of 47.28%. The personnel costs do not change, although trucks might have to wait for others to form platoons. However, this waiting is counted as a break since drivers are not paid during these periods, whereas they receive wages when they have to wait additionally. Consequently, the number of breaks that are taken is on average 8.59% higher than without platooning, whereas the number of rest periods remains unchanged.

Stage 2

The rest-while-trailing option in stage 2 leads to considerable cost savings, which are directly proportional to ρ. Moreover, in most of the cases, the rest-while-trailing option helps to reduce the number of rest periods taken. As Table 6.4 shows, r_{diff} is in the maximum 0% and on average, between 2.06% and 16.12% less rest periods are scheduled, compared to the case without platooning option.

For $\rho = 25\%$, the total average savings are 6.74%. The main factor are savings in the personnel costs, which are on average reduced by 11.62%. Interestingly, the PER and the fuel savings are slightly lower than in stage 1. This stems from the fact that less additional breaks are taken; on average 3.29% more, compared to 8.29% in stage 1. As a consequence, trucks are waiting less and thus less platoons are formed. Similar reasoning also applies to the scenario with $\rho = 50\%$, where the total average cost savings are 11.24%. Here, the number of additional breaks that are scheduled is even less, 0.91%. However, TT_{diff} exhibits that trucks sometimes drive detours. If one further increases ρ, the detour length increases as well, as Table 6.4 shows.

For the extreme case of $\rho = 100\%$, the total cost are by 31.51% lower than for the case without platooning (see Table 6.4). This is mainly due to the substantial reduction in the personnel cost of 73.64% on average. In addition, the fuel savings are higher than in stage 1, with an average value of 7.39% and even the minimum of 6.11% lying above the maximal fuel savings that can be achieved in stage 1. It needs to be mentioned that the average PER is 56% higher for $\rho = 100\%$ than for $\rho = 0\%$. However, the average fuel savings are 44% higher. The reason for this lower number is the fact that the average

Table 6.5: *Cost savings*, *PER* and TT_{diff} for solving the RTP. All values are given as percentages.

	savings	PER	TT_{diff}
Mean	8.91	74.01	0.10
Stdv	0.33	2.12	0.14
Max	9.59	76.69	0.53
Q3	9.15	75.75	0.11
Q2	8.79	73.90	0.11
Q1	8.63	73.19	0.00
Min	8.53	69.31	0.00

deviation from the shortest path is 0.85%, with a maximum of 1.52%. This shows that trucks drive detours to reduce more personnel cost, since the savings in wages exceed the additional fuel expenses.

6.4.4 Truck platooning in stage 3

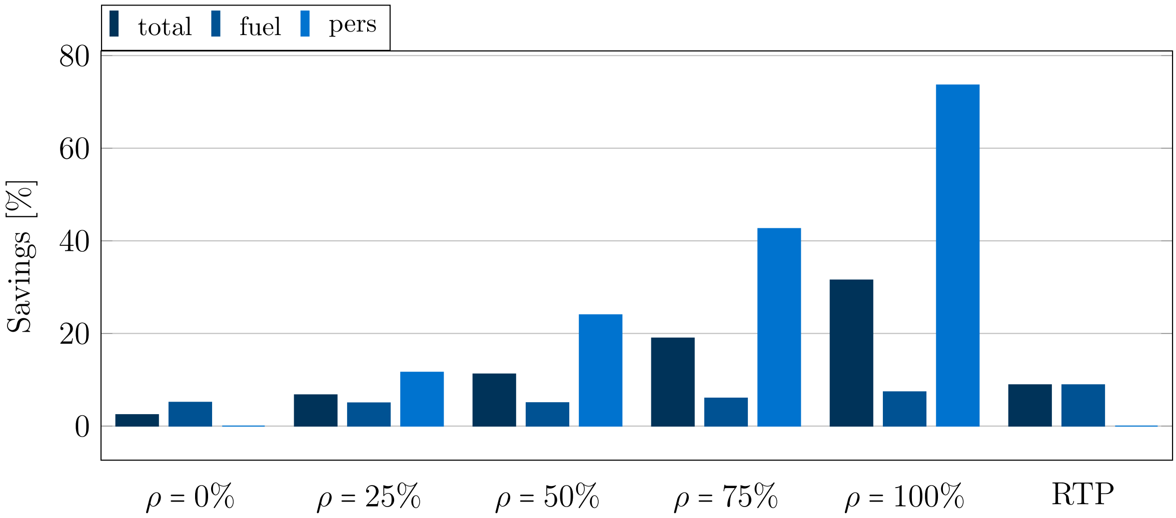
In stage 3, all trucks drive autonomously and since there are no drivers, no driving time regulations need to be considered, neither wages need to be paid. Consequently, only the fuel savings influence the decision on the routing and scheduling of the trucks. As Table 6.5 shows, the platooning option leads to average fuel cost savings of 8.75% and a mean PER of 73.15%. The reason why the fuel savings are higher than in stage 2 while the average PER remains almost unchanged (72.03%, cf. Table 6.4) is that less detours are driven. The mean difference in travel times is 0.09%. This is due to the fact that the trucks are unmanned and thus cannot exploit personnel cost savings like in stage 2. Thus, similar to stage 1, the trucks do mostly stay on their shortest paths since the additional fuel expenses for detours exceed the savings.

6.4.5 Summary of insights

Figure 6.2 gives an overview of the average cost savings (total costs, fuel costs and personnel costs) for the EU-RTP with varying ρ and the RTP. This figure shows that truck platooning can reduce transportation costs in all three ACEA stages. Consequently, truck platooning can be seen as a promising bridging technology towards fully autonomously driving trucks.

In summary, we gain the following three managerial insights:

Figure 6.2: Average cost savings (total, fuel, personnel) for the EU-RTP with $\rho \in \{0\%, 25\%, 50\%, 75\%, 100\%\}$ and the RTP. All values are given as percentages.



1. Mandatory breaks can be exploited as waiting times for platoon partners.
2. The rest-while-trailing option in stage 2 helps to substantially reduce the total costs, mainly due to lower personnel cosst. Moreover, the number of rest periods decreases. The higher the rest-while-trailing factor ρ is set, the longer are the detours the trucks are driving.
3. In stage 1 and 3, where trucks can only save fuel through platooning, trucks stay on their shortest path.

The choice of $\eta^f = 15\%$ and $\varsigma = 5$ renders rather ambitious estimations on the saving potential, as other authors estimate the fuel reduction factor at 10% (Bhoopalam et al., 2017). In addition, we assumed that drivers are not paid for breaks and rest periods, which resulted in an increase of breaks taken in stages 1 and 2. Thus, it might be difficult to persuade the drivers of such a schedule. Similarly, drivers might not agree that they are not paid while trailing in a platoon. Including a cost for breaks and rest periods would reduce the number and size of platoons, since the cost for waiting might exceed the fuel and personnel cost savings and thus less trucks would be willing to wait.

6.5 Conclusion

In this chapter, we studied the optimal solutions to the day-before truck platooning planning problem in all three stages defined by the *European Automobile Manufacturers' Association*. To this end, we extended the mixed-integer linear formulation of the Restricted Truck Platooning Problem with constraints that model the basic European driving time regulations according to *Regulation (EC) No 561/2006*. By including the option to rest-while-trailing, we modeled the case where only the driver of the leading truck needs to be fully attentive, whereas the drivers of the following trucks can perform other tasks or rest.

The results of our computational study show that truck platooning offers a great potential towards reducing the fuel consumption in long-haul trucking in all three ACEA stages. In view of the present discussion about reducing green house gas emissions in the European trucking market (Reuters, 2019), this aspect might gain even more importance.

The high labor cost savings that can be achieved through platooning in stage 2 lead to detours that the trucks are driving. This changes the traffic flows in the network, which might result in undesirable effects like congestion or unused network capacities. Consequently, further research on the impact of such effects and possible counter-measures from a network-design point-of-view are required.

Chapter 7

Advances in Methods and Insights

The prevalence of the sharing economy and technological advances have fostered the development of shared mobility and transportation systems. This thesis aimed at optimizing two problems in shared mobility systems and two variants of a problem in shared transportation systems by using analytics.

7.1 Contributions and insights

First, the methodological contributions are summarized by showing how the four methodological challenges, which were identified in Chapter 1, were solved. Then, the new insights that were gained are provided by answering the four research questions, which were posed in Chapter 1.

Methodological contributions

How to set up a framework for the performance analysis of a hybrid bike-sharing system and how to correct censored demand observations?
We developed a framework for evaluating the performance of a hybrid bike-sharing system based on booking data. We adapted a data-driven imputation method to correct censored demand observations that came as a result of unobservable lost sales due to missing bikes in a zone. Next, we extracted booking patterns by means of trip duration, trip distance and temporal aspects before identifying high and low utilization districts. We suggested the use of α-service-levels and β-service-levels as performance measures. These service levels allow the evaluation of the system from a customer-oriented point of view and to reveal unsatisfied demand.

How to model the problem of rebalancing vehicles in one-way car-sharing under competition and how to solve it efficiently?
We formulated the rebalancing problem as a convex optimization problem (called MRP) and showed under which conditions there exists a unique solution to the problem. Furthermore, we proved that these conditions do also hold in case of competition. To consider different customer behaviors, we formulated the players' revenue function under stock-out-based and availability-based substitution. From the Karush-Kuhn-Tucker conditions of the MRP, we derived a solution algorithm for the problem that finds the optimal solution based on Lagrangian multipliers. We extended the MRP to a non-cooperative game with two players who compete in several locations with an identical product and who can transship cars between the locations to prevent stock-outs. We proved that there exists a unique Nash-equilibrium solution to this single-period game and extended the solution algorithm of the MRP. Since it is based on bisection, the algorithm converges quickly to the Nash equilibrium.

How to model the day-before truck platooning planning problem and solve it efficiently?
We formulated the day-before truck platooning planning problem as a mixed-integer linear program, defined on a time-space expanded two-layer network. In the first layer, the movements of the trucks are tracked, whereas the second layer models the movements of the platoons. Arcs between the layers describe the formation and disbanding of platoons. Since this formulation considers limitations on the maximal platoon size and the temporal constraints, we named it the Restricted Truck Platooning Problem. To minimize the input size of the problem, we chose an arc-based formulation and developed a pre-processing procedure that exploits the trucks' temporal restrictions and thus only generates feasible arcs. To further accelerate the solution process, we set infeasible decision variables to zero and provided the solver with a starting solution that is based on the trucks' shortest paths. Our computational study showed that with this approach, we can solve instances with 30 nodes and 150 trucks to optimality within one hour.

How to model and solve the truck platooning problem under driving time regulations?
We introduced clock-variables, which enabled us to formulate the necessary constraints that extend the Restricted Truck Platooning Problem by the European driving time regulations. In addition, we included a rest-while-trailing option, which allows the consideration of parts of the platoon followers' driving times as rest times. Furthermore, we demonstrated how driving time regulations can be exploited in the pre-processing

procedure to reduce the input size of the problem. The computational study confirmed that we can solve instances with 12 nodes and 60 trucks to optimality within one hour.

New insights

How to increase the customer satisfaction in hybrid bike-sharing systems?
Our analysis of booking data from a hybrid bike-sharing system in Munich revealed that the free-floating option is very popular among the customers, which leads to a high excess demand. As a reaction, customers use the complimentary reservation option almost exclusively for free-standing bikes. To increase customer satisfaction, providers could incentivize the return of free-standing bikes, increase the number of bike stations in the free-floating area and introduce a fee for no-show reservations.

How does competition influence the optimal rebalancing plan in one-way car-sharing and how should a provider react to the presence of a competitor?
The comparative statics and a controlled computational study showed that customer behavior influences the companies' reactions as follows: Under stock-out-based substitution, companies tend to share the market, whereas under availability-based substitution, the rivals compete for market shares. If rebalancing cost are charged, the initial distribution of the cars also influences the providers' reactions since the repositioning costs might exceed the additional revenues. Therefore, substitution can help with the reduction of the number of rebalancing operations in certain cases while simultaneously increasing customer satisfaction and the providers' profits. If the providers were mutually willing to grant their competitor's customers access to their cars, less vehicles would need to be rebalanced and thus the companies' profits would increase. A joint rebalancing planning would lead to even higher profits and less empty rides due to rebalancing.

How can the planning process of a platform that centrally coordinates truck platoons be designed and what is the benefit of such a platform?
We suggested a planning process, which is divided into the following four steps: First, carriers register the trips with the platform until a certain cut-off date. Second, the platform gives a response to the carriers that contains the routes and schedules of the trucks in such way that the overall fuel costs are reduced through platooning. Third, updates on the trucks' routes and schedules are given during the driving phase if delays or other distortions occur. Fourth, as soon as all trucks have reached their destination, the savings are fairly distributed. Our computational study with real-size instances showed

that trucks mostly stay on their shortest paths, even under the platooning option. The main advantage of a central platform comes from the possibility to schedule the trucks in such way that the number and size of platoons is maximized.

How do European driving time regulations affect the coordination of truck platoons? Our computational study showed that driving time regulations reduce the savings potential through platooning by two thirds of its value. This is mainly a result of rest periods that need to be taken and thus reduce the trucks' flexibility to synchronize with other trucks. However, breaks can be scheduled in such way that they are used as waiting times for other trucks. Therefore, the fuel savings potential is still remarkable. If the driving times of drivers in trailing trucks are only partially counted, or not at all, the total cost savings considerably increase due to substantial reductions in the personnel costs. Therefore, trucks might drive detours in a platoon to save personnel cost.

7.2 Directions for future research

The methodological challenges and practical problems that were addressed in this thesis open several avenues for further research.

The imputation method for correcting censored demand observations we proposed in Chapter 3 currently relies on data of the same zone. One could increase this data basis by considering all zones with similar booking patterns. Such zones could be identified, for example, with clustering algorithms like k-nearest-neighbors.

In Chapter 4, we studied the competitive rebalancing game in one-way car-sharing for the case of two players. Similar to Netessine and Rudi (2003), it would be interesting to generalize the game to n players. Moreover, we formulated the rebalancing game as a single-period problem and showed that the single-period solution is also optimal in the multi-period case if rebalancing costs are considered as fixed costs. Thus, it is an interesting research direction to study the optimal solution in the multi-period case with variable rebalancing cost.

In Chapter 5, we considered the day-before planning truck platooning problem under a deterministic setting. Knowing that trucks might be delayed through changing traffic conditions or other factors, it would be worth to study the planning problem under uncertain travel times or arrival times. This could be addressed with a non-parametric, data-driven approach that estimates the travel times based on historical data and different features. Furthermore, we discretized the planning period, which leads to inevitable rounding errors. Thus, it might be a valuable research direction to relax this assumption to a continuous time horizon. Moreover, multi-brand platooning requires the collaboration of competing companies. Hence, it would be interesting to examine this case of *co-opetiton* (Battista and Giovanna, 2002) and to develop a framework for the platform that helps with the structuring of the process of coordinating rivaling carriers who all want to achieve a joint target, namely the cost-reduction through truck platooning.

In Chapter 6, we studied the day-before platooning problem under European driving time regulations. It is left for future research to evaluate the impact of other countries' driving time regulations (e.g. Canada or the United States) on the formation of truck platoons. Furthermore, one could assume that the following trucks are unmanned. Then the question arises, how to optimally plan the schedules of the drivers of such "road trains" and for driving manually on the first or last mile.

Another technology that might substantially change the long-haul trucking industry are battery electric trucks (Earl et al., 2018). Since the recharging of these trucks will take considerably more time than refueling conventional trucks, transportation costs will probably increase due to higher idle times. Truck platooning can help with the reduction of the number of rechargings required for battery electric trucks since less energy is consumed and thus greater distances can be covered. Consequently, it is worth to further investigate truck platooning with battery electric trucks.

7.3 Final conclusion

This thesis developed quantitative methods that can answer open questions in the optimal planning and management of several shared mobility and transportation systems. Along with the new insights that were provided, these methodological advances help towards increasing the efficiency in the use of the transportation resource.

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