

Georeferencing in the Context of Building Information Modelling

Štefan Jaud^{a,*}, Andreas Donaubaue^b, Otto Heunecke^c, André Borrmann^{a,**}

^a*Chair for Computational Modeling and Simulation, Technical University of Munich, Munich, Germany*

^b*Chair of Geoinformatics, Technical University of Munich, Munich, Germany*

^c*Institute of Geodesy, Universität der Bundeswehr München, Munich, Germany*

Abstract

An important element of metadata for any building information modelling (BIM) model is its location and orientation on the Earth. In most cases, engineering design is based on Cartesian coordinate systems. However, as facilities are placed in a geospatial context, discrepancies result from the transformation from the Earth's curved surface to the orthogonal coordinate system and engineers and developers must take this into account. With this in mind, the dimensions of a model may not correspond to those in the real world, but are rather distorted according to the used coordinate reference system (CRS).

We provide a thorough background of geospatial and BIM models to define and illustrate the problem at hand. We introduce three possibilities for spatial interpretation of the geometries and their locations within a BIM model. Option A sees the model as a true-to-scale representation of the asset, option B interprets the model distorted in the same manner as the underlying CRS, and option C is a combination of the former. We explore each option with a case study and visual clues. We show that, while Option A is the most prevalent interpretation in the literature, experts from the infrastructure field prefer Option C, whose underlying rationale is explained in detail. We find that introducing infrastructural concepts to BIM methods requires the systematic resolution of georeferencing. We propose a workflow for the correct handling of any BIM geometries for construction projects. Additionally, we provide a decision diagram to help project stakeholders determine when the distortions of a CRS can be knowingly neglected.

Keywords: building information modelling, georeferencing, coordinate reference system, geodetic distortions, geometry representation

*Corresponding author, ORCID: 0000-0003-0387-3440

**Principal corresponding author, ORCID: 0000-0003-2088-7254

Email addresses: stefan.jaud@tum.de (Štefan Jaud), andreas.donaubaue@tum.de (Andreas Donaubaue), otto.heunecke@unibw.de (Otto Heunecke), andre.borrmann@tum.de (André Borrmann)

1. Introduction

1.1. Motivation

The architecture, engineering and construction (AEC) domain is experiencing a transition from two-dimensional (2D) planning to three-dimensional (3D) object-based modelling. Buildings and infrastructure assets are no longer described with drawings and accompanying documents; instead, they are represented by a digital model composed of parametric geometry representations in conjunction with a range of semantic information. The underlying concept of building information modelling (BIM) is steadily gaining importance supplanting conventional computer-aided design (CAD) practices and being implemented in every aspect of the very complex software and stakeholder landscape [16]. The methods and digital processes of BIM promise to entirely remove the need for physical documents altogether, with different actors exchanging digital models throughout the project. In order for this to work, these exchanges need to be clearly defined and the models must be interpreted indifferently and consistently by everyone involved [5].

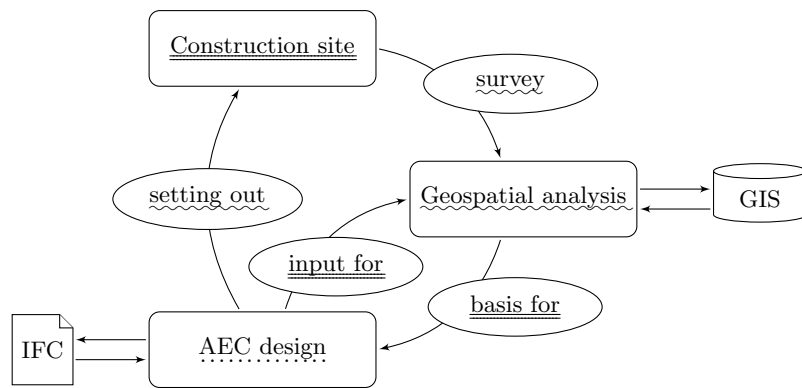
Figure 1 shows an abstraction of the flow of data over the lifetime of a construction project [cf. 11]. The process chain starts with a survey of the construction site and its surroundings (whether an empty lot or an existing structure) and the production of an *as-is* model. The obtained geospatial data is usually stored, managed and merged with existing data from geographic information systems (GISs) during geospatial analysis. This data represents the initial state model (basis) for the AEC design processes. As the project proceeds, the models are iteratively evaluated, improved and exchanged between different actors, often using the non-proprietary Industry Foundation Classes (IFC) data format.

The *as-designed* model can then be returned directly to the geospatial analysis process, both to visualise the planned object in its geographical context and to test its impact and interaction with the environment (e.g. through noise or flooding simulations). As construction begins, the model needs to be realised in the material world and foundations at the construction site must be set out¹. The progress of the construction is surveyed and *as-built* models are produced and joined with the *as-is* models during geospatial analysis. This cycle is repeated iteratively until completion of the project.

With the introduction of digital methods, an implicitly managed problem reappeared: the placement and true dimensions of the asset on the curved surface of the Earth. The selection and use of the project coordinate system (PCS) require close attention. *Georeferencing* refers to the process of specifying a geolocation (the placement of an asset on the surface of the Earth), as aptly described by Clemen and Görne [11]. In addition, it includes the definition of the parameters of the underlying coordinate system (CS), and thus the consequences of geodetic transformations (described in detail in Section 3.1).

If we are to change the established processes and design submissions, the problem of georeferencing needs to be adequately addressed [35].

¹Also called a ‘stake-out’.



Legend:

Responsibility: AEC expert

Surveyor

Both

Possible change of CS: Yes

No

Data flow: \longrightarrow

Figure 1: A conceptual data flowchart for the AEC industry. The digital model lies in a coordinate system that may be transformed during transitions between different stages.

44 *1.2. Discrepancies*

45 Digital engineering design is performed in the PCS, which is a Cartesian CS
46 [48]. In this CS (X, Y, Z) , the Earth’s surface is modelled as an infinite, flat
47 surface. All Z -planes of the PCS are equipotential surfaces² with any geo-
48 graphic elements, like mountains or trenches, deviating from the base height
49 [34]. This corresponds to the intuitive human understanding of horizontality
50 and of elevations on Earth, where it *seems* that the Earth is locally flat. In order
51 to transform the curved surface of the Earth to a flat representation, some
52 distortions are introduced through map projection.

53 All geospatial data comes in a well-defined compound coordinate reference sys-
54 tem (CRS)³. We need to consider that projected CRSs possess a certain well-
55 known discrepancy from the real world, which inherently includes a distortion in
56 lengths, angles, and/or areas. For example, Mercator projection distorts lengths
57 but preserves angles between any two points⁴.

58 These distortions apply only along horizontal axes, are location-dependent and
59 vary significantly in different CRSs. Since the map projection distorts lengths,
60 the dimensions and position of objects as well as their shape are subject to
61 discrepancies. They need to be accounted for when setting out the construction
62 site. The scale factor m between a geospatial distance and its real-world
63 dimension is defined as

$$m \stackrel{\text{def}}{=} \frac{\text{geospatial}}{\text{real world}} = 1 + 10^{-6}\Delta, \quad (1)$$

64 and is often conveyed as the difference factor Δ in parts-per-million (ppm).
65 A scale in which $m < 1$ or $\Delta < 0$ means that the model is smaller than its
66 real-world counterpart, while $m > 1$ or $\Delta > 0$ indicate the opposite.

67 This discrepancy has conventionally been handled by surveying engineers (Fig-
68 ure 1), and the problem itself was at least partly hidden from other stakeholders
69 in the project, such as design engineers. However, with the introduction of BIM
70 methods, it needs to be addressed appropriately, especially when large, linear
71 objects are being modelled. There, the aspects of CRS and the distortions they
72 imply have a much more significant impact on dimensions than when considering
73 buildings with comparatively limited extents [10, 48].

74 *1.3. Problem Definition*

75 As BIM practices mature, the benefits they provide are spurring increased inter-
76 est in the infrastructure sector [7, 13]. Additionally, as prefabrication directly
77 from BIM models is gaining popularity, neglecting the scale factor can become
78 a serious issue [e.g. 31, 51]. The discrepancies between the real world and the
79 (projected) geospatial data may easily reach up to a few cm at a distance of
80 100 m, as explained in detail in Section 3.1.5 [20]. Projects that fail to account

²*Equipotential* surfaces are surfaces where any fluid without viscosity would not flow.

³The phrase *coordinate reference system* is an established term that refers to a *geodetic* coordinate reference system [24], as conveyed by the word *reference*.

⁴This is explained in more detail in Section 3.1.5. Since Mercator projections are used extensively in the AEC domain, this paper focuses only on these projections.

81 for this reality may operate under false interpretations and incur delays and
82 increased costs.

83 This is why a clear understanding of the PCS specification used in the BIM
84 model is needed. The main issue is the correct application of CRS to BIM data,
85 or a lack thereof. This applies to models of building or infrastructure assets
86 which are in either open formats (like IFC) or proprietary formats from software
87 vendors. This paper explores the correct understanding and consideration of the
88 scale factor from Equation (1) in the context of BIM.

89 The contributions of this paper are as follows:

- 90 1. It aims to serve as a knowledge bridge between AEC and geodetic experts,
91 and between the BIM and geospatial worlds through the field of georeferencing.
92 We achieve this by explaining the concepts from both fields, which may be
93 familiar to experts in one but not the other.
- 94 2. We present three possible interpretations of geometries and locations within a
95 BIM model as being a A) true or B) distorted representation of the asset or C) a
96 combination of the two. We provide the reasoning behind these interpretations
97 and evaluate their applicability to BIM models. They are discussed and depicted
98 with practical examples to facilitate a better understanding of the problem. The
99 goal is to familiarise the community with the subject and avert future mistakes
100 [e.g. 12].
- 101 3. We provide a thorough analysis of all three options mentioned above and the
102 rationale behind the most prominent proposed solution. We present a process
103 map that can guide users to the correct geospatial interpretation of the contents
104 of a BIM model. It can help software vendors implement correct algorithms and
105 also enable project stakeholders to circumvent any problems derived from false
106 understandings.

107 *1.4. Structure of the Paper*

108 We start the paper with an introduction, where we frame the problem, present
109 our motivations and declare the objectives of this paper. Related works dealing
110 with georeferencing of BIM models are presented in Section 2.

111 An in-depth explanation of geodetic as well as BIM models is given in Section 3.
112 We present the well-known problems of mapping the curved surface of the Earth
113 to a Cartesian CS, and vice versa in Section 3.1. If the reader's background is
114 in geodesy, these topics are likely redundant. The BIM philosophy is described
115 in Section 3.2, where the different geometric representations and positioning
116 possibilities are highlighted. Readers from the AEC field are likely to already
117 be familiar with these topics.

118 Section 4 describes the main contribution of this paper: the three interpretations
119 of a BIM model, each illustrated with a case study. We evaluate these options
120 with an extensive discussion in Section 5. We present the cost of ignorance of
121 georeferencing in Section 5.2 and formulate recommendations for correct usage
122 of georeferenced data in Section 5.3. We conclude this paper in Section 6 and
123 offer a look forward.

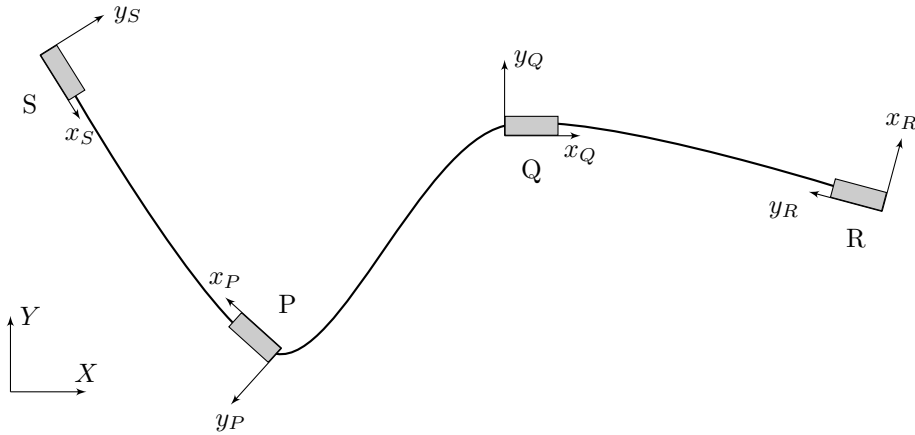


Figure 2: A hypothetical railway line with stations in cities S, P, Q, and R. Each station is a project on its own with its own $PCS_i = (x_i, y_i, Z), \forall i \in \{S, P, Q, R\}$. The railway track itself is planned in the global CS, thus $PCS_{\text{rail}} = (X, Y, Z)$. All CSs are right-handed [author redrawn from 10].

124 2. Related work

125 With the introduction of long objects from the infrastructure sector into the
 126 BIM context, georeferencing has become a topic of interest within the build-
 127 ingSMART International (bSI) community [9]. Its members have addressed
 128 this issue in a recent publication called *Model Setup IDM* [10]. As the name
 129 suggests, the focus of the report was the model set-up process at the beginning
 130 of a BIM design project. The main goal of the report was to enable clear and
 131 concise model coordination for the duration of a project, which allows for mean-
 132 ingful clash detection. This is especially complex when many construction sites
 133 are included in a larger project [10].

134 They set up a hypothetical railway project as an example, in which they de-
 135 signed a new track with four stations (Figure 2). Each component was handled
 136 separately because they were managed by different design companies, so five
 137 models were passed to the coordinator: one for the railway line and one for
 138 each of the stations. The authors of the study considered the PCS used for the
 139 railway and each station, and demonstrated how to correctly define them and
 140 the relationships among them [10].

141 Their proposed solution was to use the established entities in the IFC data
 142 format to denote the underlying CRS. For transformations between different
 143 PCSs, they used 2D-Helmert transformations⁵. Additionally, they provided a
 144 guideline for the software implementers of the IFC schema for a currently wide-
 145 spread version, IFC2x3, and the latest official release, IFC4 [23]. Although the
 146 report provided a deep analysis of the geodetic background, it did not state
 147 whether or not a BIM model should be considered distorted or not.

148 Wunderlich and Blankenbach [50] denoted in their publication that a BIM model

⁵The Helmert transformation is thoroughly explained in Section 3.1.2.

149 is a true representation of the structure on site and should thus be considered
150 not distorted. The importance of CRSs for the success of a BIM project was
151 noted by Kaden and Clemen [27]. The authors expressed a recognition that
152 “*their correct understanding is crucial especially in the infrastructure sector*”
153 [27], but that “*most CAD data is created without this consideration*” [27]. They
154 walked through an example study where they linked the CRS and the CS of the
155 BIM model through an additional, intermediate CRS. This idea of using a local
156 surveying CS, which represents the PCS, was presented by bSI as well [10].

157 In their latest study Clemen and Görne [11] looked at the different options
158 provided by the IFC schema to specify the position of the model on Earth. They
159 defined five Levels of GeoReferencing (LoGeoRef) and asserted that only the
160 highest level provides enough information for precise surveying work. However,
161 they made no mention of the *definition* of a PCS in terms of a CRS, describing
162 only how the geolocation of the BIM model can be modelled within the IFC
163 schema, and thus stored in IFC files.

164 A pair of recent studies from Ugglå and Horemuz [48, 49] argued that a BIM
165 model should be viewed as a “*1:1 representation . . . at the construction site*” [49].
166 In their view, IFC data is not georeferenced, per se, and must be transformed
167 to georeferenced data accordingly. They considered the horizontal plane of the
168 PCS in the BIM model to be tangent to the Earth at point of origin (POO),
169 and thus it deviates from Earth’s curved surface as it moves farther away from
170 the POO. As such, the PCS’s agreement with the real world stays within given
171 tolerances only within a small area around the POO⁶.

172 In order to transform the coordinates from the PCS to a CRS of choice, Ugglå
173 and Horemuz [48] proposed three options. The first option is drawn from Liebich
174 et al. [32] and Borrmann et al. [4], and the other two are their own developments.
175 They calculated the disagreements between the real world and the models for
176 each of the three options and provided visual representations. However, no
177 clear instructions or decision-making tools were provided for choosing the best
178 option for future use. They concluded that “*the current implementation in*
179 *the IFC schema is suitable for infrastructure design in areas where sufficiently*
180 *accurate well-known map projections are available, and which are not too high*
181 *above the reference ellipsoid*” [49]. They called for support of “*object specific*
182 *map projections*” [49] and “*separate scale factors for different axes*” [49].

183 Ohori et al. [39] considered integrating BIM and GIS geometry by looking at the
184 most prominent open standard formats from each world: IFC and CityGML,
185 respectively. They demonstrated a best practice approach on several example
186 buildings from the Netherlands. They paid close attention to correct geoloca-
187 tion, but they did not consider any geodetic distortions while converting the
188 geometries. Liu et al. [33] provided a good overview of the state of the art of
189 BIM and GIS integration. We disagree with their assertion that *The major*
190 *difference between GIS and other information systems is that the GIS data are*
191 *geo-referenced* [33] as shown within this study.

192 In our previous work, we have critically evaluated the current official IFC4

⁶In geodesy, this is called a ‘topocentric system’. It is explained in detail in Section 3.1.2.

193 schema and its capability to store geospatial metadata [34]. With the intro-
194 duction of BIM systems and the exchange of digital models this metadata must
195 be handled as part of the process and correctly incorporated into the models.
196 We found that the IFC4 version provides sufficient support for the typical case
197 occurring in the majority of projects. However, based on two recent real-world
198 infrastructure projects, the current schema is rendered as insufficient. We pro-
199 posed two new IFC entities which would provide support for grid-shift parameter
200 data-sets [34].

201 In another study, we looked at the Brenner Base Tunnel project in detail, in
202 cooperation with the project team [35, 26]. The project-specific CRS was de-
203 signed to minimize geodetic distortions, with the CRS and the PCS seen as
204 identical. However, the PCS was defined in a way that accentuated the need
205 for an expansion of the IFC schema. Even the simple inclusion of well-known
206 text (WKT) strings would provide the needed flexibility to support even such
207 peculiarities [25]. The proposal was tested on the custom CRS of the Brenner
208 Base Tunnel, with positive results [26].

209 None of the studies above considered the research questions addressed in this
210 paper; instead, each followed a single interpretation. We have only identified
211 two studies which acknowledge the problem of different interpretations of BIM
212 models.

213 Brenner et al. [8] produced guidelines for bridge design which included both
214 distorted and undistorted interpretations of the BIM model. They published
215 a process map that defined the points of transformations from distorted to
216 undistorted modelling when exchanging information between different trades
217 like road and bridge design. Their findings have been incorporated into this
218 study in Section 5.2.3.

219 In one of our previous studies, Heunecke [20] considered the question of the
220 interpretation of the PCS of BIM models. He presented the three possible
221 interpretations discussed within this paper in Section 4. It was not his goal to
222 propose one of these as the final solution.

223 An overview of the discussed literature and some additional studies is provided
224 in Table 1, where BIM model interpretation is marked according to our under-
225 standing of each paper.

226 **3. Theoretical Background**

227 *3.1. Geodesy*

228 The geospatial data used in a design process is saved in a well-defined CRS,
229 which is composed of two independent systems: the location and the height
230 reference. The former is composed of the geodetic datum and a map projection
231 (which flattens the curvature of the Earth's) and the latter is based on a vertical
232 datum. Together, this is a compound CRS [24]. It is important to fully under-
233 stand these models if we are to answer the research questions at hand. To this
234 end, the following subsections provide a detailed overview, and the rationale
235 behind, the geodetic models relevant to this study. More detailed explanations
236 can be found in the ISO 19111:2019 [24] standard.

Table 1: An alphabetical overview of the related studies, with their interpretation of the spatial contents of BIM models marked with an ‘X’. The column categories correspond to the three possible interpretations explored in detail in Section 4.

Interpretation	undistorted	distorted	combination
Borrmann et al. [4]		unclear	
Brenner et al. [8]			X
buildingSMART International [10]		unclear	
Clemen and Görne [11]	X		
Heunecke [20]	X	X	X
Jaud et al. [26]		X	
Kaden and Clemen [27]	X		
Liebich et al. [32]		unclear	
Markič et al. [34]		X	
Markič et al. [35]		X	
Ohuri et al. [39]	X		
Uggla and Horemuz [48]	X		
Uggla and Horemuz [49]	X		
Wunderlich and Blankenbach [50]	X		
Option (see also Section 4)	Option A	Option B	Option C

237 3.1.1. Model of Earth

238 In a global sense, a Cartesian CS (X, Y, Z) can be anchored to the Earth’s centre
 239 of mass. This is an Earth-centred, Earth-fixed (ECEF) CRS in geodesy and is
 240 defined as follows (Figure 3) [10]:

- 241 • The Z -axis spans from the geocentre through the international reference pole
 242 (IRP) and coincides with the Earth’s rotational axis.
- 243 • The X -axis is the intersection of the international reference meridian (IRM)
 244 and the mean equatorial plane (MEP).
- 245 • The Y -axis is then defined so that it spans a right-handed orthogonal CS.

246 The unit of measurement (UoM) is metres on all axes. This model is exten-
 247 sively used in satellite-based positioning. However, such a model is not really
 248 useful in the AEC industry, since it does not provide support for an intuitive
 249 understanding of *horizontality*, as described in Section 1.2. When AEC ex-
 250 perts discuss about the form of the Earth, they implicitly refer to the form of
 251 the equipotential gravity field. Here, geographical elements like mountains and
 252 trenches deviate from the base height⁷, $H = 0$.

253 Figure 4 depicts a mathematical and physical model of the Earth. Since the
 254 Earth is basically a sphere that is squashed at the poles due to centrifugal forces,
 255 a very strong mathematical approximation is a rotational ellipsoid⁸, which is an
 256 ellipse rotated around its minor axis (Figure 4a) [24]. In a geodetic context, it is

⁷This base height is often set to mean sea level. However, a CRS can define other reference surface(s) if this brings enough benefits to justify the deviation from the convention [35].

⁸An excellent explanation of the error between an ellipsoid and a sphere can be found through StackExchange [45].

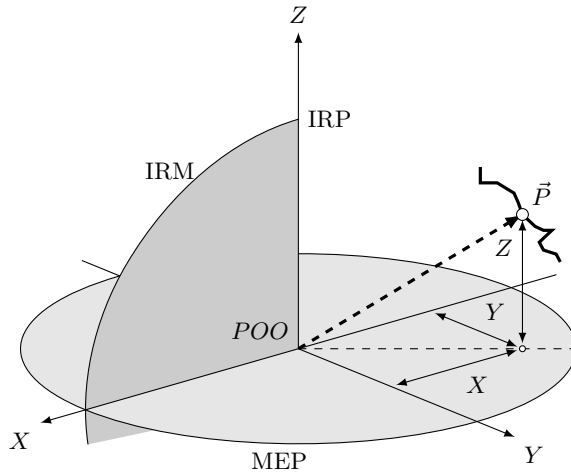


Figure 3: ECEF CRS with the POO defined as Earth’s center of mass, IRP defining the Z axis, MEP defining the (X, Y) plane, and IRM defining the X axis. The point \vec{P} is uniquely defined by its three coordinates (X, Y, Z) [author redrawn from 10].

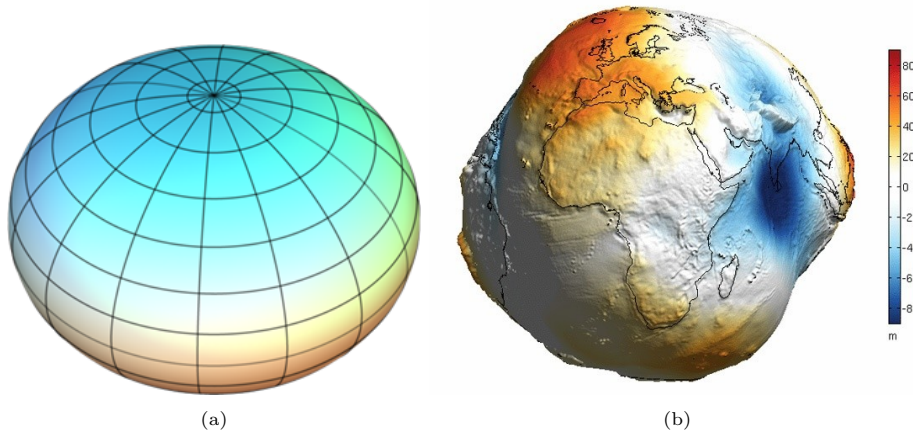


Figure 4: Images of the models of the Earth: the mathematical model (a) is a rotational ellipsoid [14], and the physical model (b) is a geoid or “potato” [21]. The surfaces represent base surface, $h = 0$, and the equipotential surface, $H = 0$, respectively. The differences on the geoid are exaggerated and shown relative to the ellipsoid WGS84.

257 common to define additional variables to simplify the calculations as presented
 258 in Table 2 [17, 29]. This table also includes exemplary values for widely used
 259 ellipsoids from the World Geodetic System 1984 (WGS84) and the Geodetic
 260 Reference System 1980 (GRS80) [17].

261 Gravimetric measurements can determine the natural equilibrium form of the
 262 Earth – the so-called geoid. This physical model of the equipotential surface
 263 of Earth is often dubbed ‘the potato’ (see Figure 4b). These two forms define
 264 separate reference surfaces for measuring elevation. The height of a specific
 265 point above the ellipsoid is called the ellipsoidal height, h , which must be strictly
 266 distinguished from the physical height, H , which is relative to the geoid. The

Table 2: The parameters of a rotational ellipsoid and their interdependency with exemplary values for the WGS84 and GRS80 ellipsoids. The ellipsoid is described by two independent parameters, normally the major axis, a , and the inverse flattening, f^{-1} , in the geodetic context [17, 24, 29].

Variable	Parameter	Equation	WGS84	GRS80
a	major axis	$a = b/(1 - f)$	6 378 137.0 m	6 378 137.0 m
b	minor axis	$b = a(1 - f)$	6 356 752.314 245 m	6 356 752.314 140 m
e	eccentricity	$e = \sqrt{a^2 - b^2}/a$	0.081 819 191	0.081 819 191
e'	second eccentricity	$e' = \sqrt{a^2 - b^2}/b$	0.082 094 438	0.082 094 438
f	flattening	$f = (a - b)/a$	1/298.257 223 563	1/298.257 222 101
n	third flattening	$n = (a - b)/(a + b)$	0.001 679 220	0.001 679 220

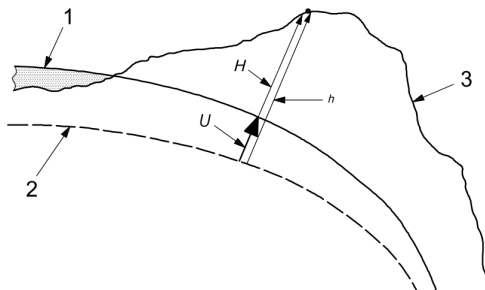


Figure 5: The ellipsoid (2) and geoid (1) heights of Earth (3) are h and U (denoted N in the literature), respectively. H is the gravity-related height measured along the direction of gravity [24, 47].

267 differences between them are called *undulations*, U^9 (see Figure 5), which are
 268 defined as

$$h = H + N \stackrel{def}{=} H + U, \quad (2)$$

269 and can amount to up to $U = \pm 100$ m from the WGS84 ellipsoid (Figure 4b). At
 270 this point, a strict distinction must be made between the geoid (with orthometric
 271 heights) and the quasi-geoid (with normal heights). This results from the way
 272 the equilibrium figure is calculated and will not be discussed here [see 47, for
 273 more details].

274 3.1.2. Topocentric System

275 The simplest example of a CRS on Earth's surface is a topocentric system. A
 276 local Cartesian CS (x, y, z) is defined at a chosen POO on the Earth's surface
 277 (hence the prefix *topo*). The z -axis coincides with the negative direction of the
 278 gravity pull at the POO, while the x and y axes are chosen to form a Cartesian
 279 CS in the plane perpendicular to that z -axis. Note that such a topocentric CRS
 280 is defined as a left-handed system in surveying and as a right-handed system in
 281 the AEC domain.

⁹Undulation is usually given the symbol N in the literature [24]. We have decided to use U in this paper in order to differentiate it from the Northing coordinate, N .

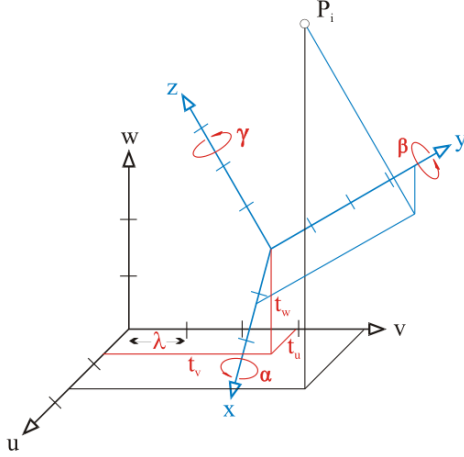


Figure 6: 7-parameter Helmert transformation from Equation (3). The originating coordinate system is (x, y, z) , the transformed is (u, v, w) . The 7 parameters $(t_u, t_v, t_w, \lambda, \alpha, \beta, \gamma)$ are also shown.

282 In order to transform the coordinates between two right-handed Cartesian CSs
 283 (for example from ECEF to a topocentric system – though this is not done in
 284 practice), a relationship between the originating (x, y, z) and transformed coordi-
 285 nates (u, v, w) needs to be established [48]. A so-called 7-parameter Helmert
 286 transformation for point $P_i = [u_i, v_i, w_i]^T = f([x_i, y_i, z_i]^T)$ is defined as (Fig-
 287 ure 6)

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} t_u \\ t_v \\ t_w \end{bmatrix} + \lambda R(\alpha, \beta, \gamma) \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \quad (3)$$

where

$$R(\alpha, \beta, \gamma) = \begin{bmatrix} c\gamma c\beta & c\gamma s\beta s\alpha + s\gamma c\alpha & -c\gamma s\beta c\alpha + s\gamma s\alpha \\ -s\gamma c\beta & -s\gamma s\beta s\alpha + c\gamma c\alpha & s\gamma s\beta c\alpha + c\gamma s\alpha \\ s\beta & -c\beta s\alpha & c\beta c\alpha \end{bmatrix}, \quad (4)$$

288 with c and s standing for *cos* and *sin* functions, respectively.

289 As noted by Uggla and Horemuz [48], a topocentric CRS only represents the
 290 equipotential surface of Earth well in close proximity to the POO. The horizon-
 291 tal plane of the CS deviates increasingly from the curved surface of Earth with
 292 increasing distance from POO. As such, following the curved Earth during con-
 293 struction while interpreting the model as is (i.e. flat) induces a steadily growing
 294 error as we move farther away from the POO. For this reason, this CRS is only
 295 useful for small construction sites where these discrepancies remain negligible.
 296 For projects with a greater extent, a CS needs to be defined which allows for
 297 clear transformations while still retaining the human understanding of horizon-
 298 tality as described in Section 1.2. The exact values distinguishing small and big
 299 extents are discussed in Section 5.2.3.

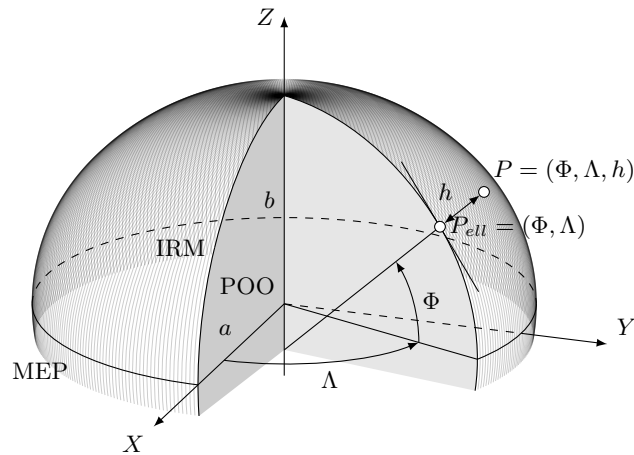


Figure 7: Latitude and longitude (Φ, Λ) uniquely define a point, P_{ell} , on an ellipsoid with major and minor axes a and b , respectively (only the top half is shown). Together with the ellipsoidal height, h , measured perpendicular to the ellipsoid surface, any point P can be referenced by its three coordinates as $P = (\Phi, \Lambda, h)$ (cf. Figure 3).

300 3.1.3. Geodetic Datum

301 As shown above, an ellipsoid is a good mathematical approximation of the shape
 302 of Earth. A geodetic datum relates a specific ellipsoid to the Earth by providing
 303 its form, position and orientation in space. The use of spherical coordinates
 304 offers a way of referencing points on ellipsoidal and thus Earth's surface as
 305 shown on Figure 7. The latitude, Φ , and longitude, Λ , denote the angles from
 306 the reference lines (IRM and MEP, respectively). A pair of angles (Φ, Λ) defines
 307 a unique location on the ellipsoid.

308 Throughout history, many ellipsoids have been defined and used with different
 309 areas of best fit. The *best fit* objective is to minimise the differences between
 310 the geoid and the ellipsoid in a specific area or in a global context. In the case of
 311 a *local* geodetic datum, the coordinates of reference points on Earth are defined
 312 to be identical with the coordinates on the ellipsoid, and the Earth's surface
 313 normals (plumb lines) should coincide as closely as possible with the surface
 314 normals on the ellipsoid at these reference points. In the case of a *global* geodetic
 315 datum, the centre of the ellipsoid is set to be identical with the Earth's centre of
 316 gravity and its minor rotational axis coincides with the Earth's mean rotational
 317 axis (through the IRP). The European Terrestrial Reference System (ETRS)
 318 is a three-dimensional (x, y, z) geodetic reference system, which is affixed to
 319 the Eurasian continental plate and was identical to the International Terrestrial
 320 Reference System (ITRS) in 1989. While the coordinates of the ITRS have
 321 had to be redefined repeatedly, especially due to global plate tectonics, the
 322 coordinates of the ETRS used in Europe are largely time-constant. For the
 323 specification of ellipsoidal coordinates, the parameters of the GRS80 are used
 324 within the ETRS.

325 3.1.4. Vertical Datum

326 There are several possible definitions of elevation on Earth. One of them is to
 327 define verticality on Earth's surface as parallel with direction of Earth's gravity

328 pull, so that the vertical axis, H , follows the plumb line. In this way, water does
 329 not flow between two points with the same elevation, and this corresponds with
 330 the human notion of elevation and is very practical in construction. For easier
 331 notation, the coordinate value is usually given as a distance to some reference
 332 surface and not to the POO [24]. This reference surface is one of Earth’s gravity
 333 field equipotentials and is set as the zero orthogonal height, $H = 0$. The surface
 334 most commonly chosen is mean sea level [17]. An example of a vertical datum is
 335 the Deutsches Haupthoehennetz 2016 (DHHN 2016), used mainly in Germany.

336 Knowing a point’s orthogonal height, H , and the undulation between the chosen
 337 geodetic and the vertical datum, U , any point can be uniquely referenced as

$$P = (\Phi, \Lambda, h) = (\Phi, \Lambda, H - U) . \quad (5)$$

338 This is reflected in Equation (2) and Figures 5 and 7.

339 3.1.5. Projected CS

340 The horizontal Cartesian plane (X, Y) of a PCS is obtained by projecting the
 341 curved ellipsoidal surface onto a plane using one of many types of map projec-
 342 tion [44]. First, the ellipsoidal coordinates (Φ_i, Λ_i) get mapped to the geodetic
 343 coordinates. For example, with the widely used Universal Transverse Mer-
 344 cator (UTM) projection, these are Easting and Northing (E_i, N_i) , which are
 345 used hereinafter. A comprehensive procedure is provided in Appendix A for
 346 both the transformation $(E_i, N_i) = f_{\text{proj}}(\Phi_i, \Lambda_i)$ and the reverse transformation
 347 $(\Phi_i, \Lambda_i) = f_{\text{proj}}^{-1}(E_i, N_i)$. The Cartesian coordinates (X_i, Y_i) at the construc-
 348 tion site are then connected to geodetic coordinates using the 2D version of
 349 Equation (3) with $t_w = \alpha = \beta = 0$:

$$\begin{bmatrix} E_i \\ N_i \\ 0 \end{bmatrix} = \begin{bmatrix} E_{POO} \\ N_{POO} \\ 0 \end{bmatrix} + \lambda R(0, 0, \gamma) \begin{bmatrix} X_i \\ Y_i \\ 0 \end{bmatrix} , \quad (6)$$

350 where (E_{POO}, N_{POO}) represent the geodetic coordinates of the PCS’s POO¹⁰.

351 Projecting the curved surface of an ellipsoid onto a plane without any deforma-
 352 tion is geometrically impossible [44]. Hence, a map projection can preserve
 353 only one of the following: angles, distances or surface areas. The compromise
 354 most frequently chosen in large-scale topographic applications or cadastral sur-
 355 veying is to preserve angles by using a conformal map projection. The two most
 356 commonly used map projections are the Gauss-Kruger (GK) and the UTM pro-
 357 jections [34].

358 To keep the distance and surface area distortions within an acceptable range,
 359 strips of the ellipsoid are defined and projected onto a cylinder with an elliptical

¹⁰Many geodetic CSs are left-handed, including (N, E, H) . Notice that Northing is the first coordinate. In this paper we knowingly neglect this fact in lieu of simplicity and consider all CSs to be right-handed, thus switching the order of the coordinates in Equations (3) and (6). This is also the basis for ISO 10303-42:2019 [22], upon which the IFC data format bases: *All geometry shall be defined in a right-handed rectangular Cartesian coordinate system with the same units on each axis.*

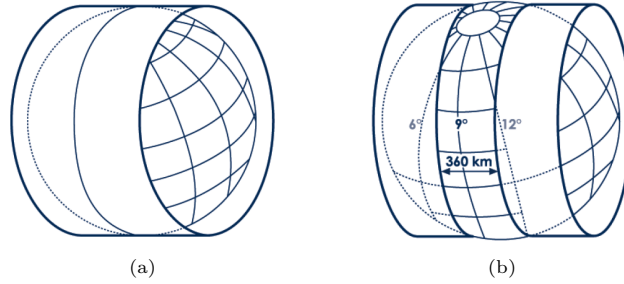


Figure 8: Different ways of projecting an ellipsoid on a cylinder: (a) GK projection and (b) UTM projection [modified from 19].

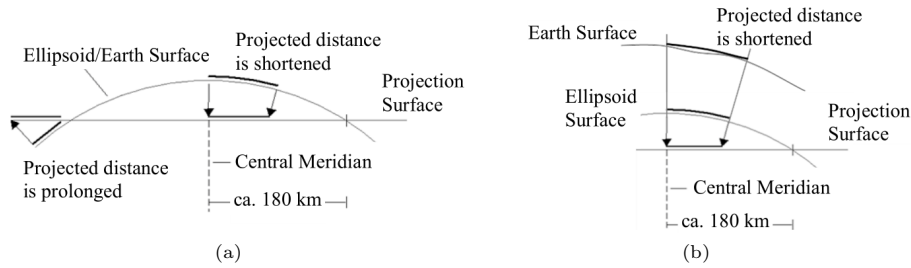


Figure 9: The distortions induced by (a) the UTM projection and (b) height reduction [reproduced with permission from 27].

360 cross section (Figure 8). Note that the borders of the strips are defined by
 361 meridians on the ellipsoid; thus, the strips are tapering with increasing distance
 362 from MEP. This can be clearly seen on Figure 8b, where the distance between
 363 the intersections of the projection cylinder with the ellipsoid is constant, whereas
 364 the distance between the meridians on the ellipsoid gets shorter from the equator
 365 towards the poles.

366 The GK projection uses a cylinder that touches the ellipsoid at a meridian
 367 (Figure 8a). Therefore, only the distances along the meridian are not distorted
 368 ($\Delta_{0,\text{GK}} = 0$ ppm) and the distortion increases for locations farther away from
 369 the meridian. This is why the strips of the projection have a width of 3° , where
 370 the maximum scale is $\Delta_{\text{border,GK}} \approx 150$ ppm [24, 27].

371 In the UTM projection, the cylinder intersects with the ellipsoid approximately
 372 180 km east and west of the central meridian of a specific zone, which has a
 373 width of 6° (Figure 8b). Thus, the central meridian is shortened with $\Delta_{0,\text{UTM}} =$
 374 -400 ppm and the borders lengthened for $\Delta_{\text{border,UTM}} \approx 160$ ppm. This keeps
 375 the distortions in an acceptable range, even at the borders of the zone [24, 27].

376 A cross section of the UTM projection cylinder and the ellipsoid can be seen in
 377 Figure 9 [27]. All geospatial data lie on the chosen ellipsoid and must first be
 378 projected there from their elevations. This process causes additional distortion
 379 in the distances as shown in Figure 9. The factors of these distortions are
 380 explained thoroughly in Appendix A.

381 *3.1.6. Compound coordinate reference system*

382 In summary, a CRS defines the underlying CS of all geospatial data and is com-
383 posed of multiple parts. The choice of ellipsoid’s size, position and orientation
384 with regard to the Earth together with the elevation reference define the geode-
385 tic and vertical datums, respectively. The chosen map projection defines the
386 method by which the double-curved surface of the ellipsoid is mapped to the
387 Cartesian CS.

388 The map projection, together with the geodetic datum, is called a *projected*
389 CRS. When combined with a vertical CRS, the reference system is called a
390 *compound* CRS [24]. The European Petroleum Survey Group (EPSG) database
391 contains a comprehensive collection of these references, systems, and their com-
392 binations; nearly 6000 CRSs from around the world are currently listed together
393 with datum definitions and transformations [17].

394 The distortions ($[\text{cm}/100 \text{ m}] = 100 \text{ ppm}$) induced by the chosen ellipsoid, map
395 projection, and height above the projection surface are illustrated in Figure 10
396 for the state of Bavaria, Germany [15]. Figure 10a shows the distortions induced
397 by the UTM map projection from the ETRS geodetic datum. The UTM zone 32
398 (UTM32) has its central meridian at 9° E , which goes through the western tip of
399 Bavaria. The scale there is therefore $\Delta_{\text{UTM32};9} = -4 \text{ cm}/100 \text{ m}$. Although the
400 UTM32 stops at 12° E (where zone 33 begins), the scale is still at a manageable
401 $\Delta_{\text{UTM32};12} = 1.6 \text{ cm}/100 \text{ m}$, so the Bavarian state decided to extend zone 32 for
402 reasons of simplicity. This way, all Bavarian geospatial data is within the same
403 zone and complicated calculations at the zone change can be omitted. However,
404 this means that the distortions at the far eastern edge of Bavaria reach a non-
405 negligible $\Delta_{\text{UTM32};13.8} = 11.7 \text{ cm}/100 \text{ m}$, which corresponds to $m_{\text{UTM32};13.8} =$
406 1.00117 .

407 Figure 10b shows the distortions due to the vertical distance from the projection
408 surface as seen on Figure 9, right. Here, the distortions induced by using the
409 ETRS geodetic datum with the underlying ellipsoid GRS80 and the DHHN 2016
410 vertical datum is shown.

411 To determine the scaling factor induced by all of these concepts at a given point,
412 the distortions caused by projection m_{proj} and height reduction m_h from Equa-
413 tions (A.5) and (A.14) need to be multiplied together (provided in Appendix A)

$$m_{CRS}(\Phi, \Lambda, h) = m_{proj}(\Phi, \Lambda) \cdot m_h(\Phi, h), \quad (7)$$

414 where h is defined in Equation (2). This scale function only applies to the
415 horizontal axes (E and N) and does not influence the vertical values, despite
416 being dependant on both the location and elevation values. Note that m_{CRS}
417 is a function of location as well as the chosen CRS, and it can be set to an
418 approximated constant value only within a limited range.

419 *3.2. Building Information Modelling*

420 BIM stems from CAD methods, where 3D object-oriented modelling has found
421 its use in parametric design and semantically rich data. Instead of drawing lines
422 with multiple geometric constraints on different layers that convey semantics,
423 objects are contained in a BIM model which inherently provides all kinds of

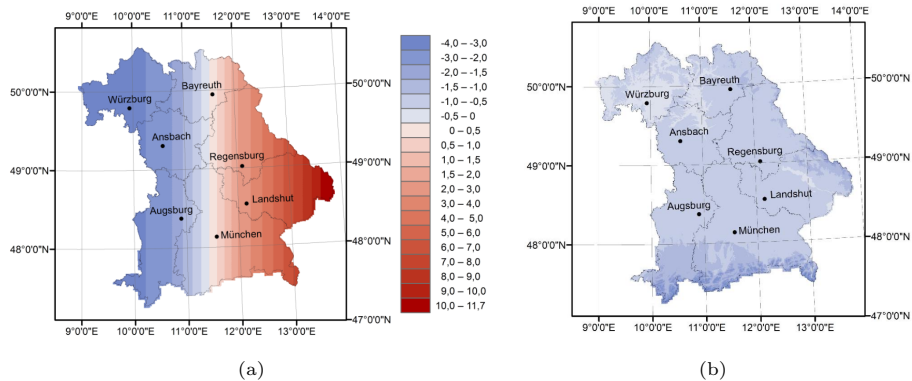


Figure 10: The values of length distortions [cm/100 m] from Figure 9 shown using Bavaria, Germany, as an example. (a) Distortions due to the UTM projection from GRS80 ellipsoid (see also Figures 8b and 9a). Note: Per definition, there should have been a zone change at 12° from UTM32 to UTM33; however, Bavaria considered to have all its geospatial data in one zone. (b) Distortions due to height reduction $H \approx h \neq 0$ to GRS80 ellipsoid (see also Figures 5 and 9b) [15].

424 constraints and rich semantic data. Resources like drawings, bills of quantities,
 425 time tables and more can be automatically derived from a BIM model, and so
 426 only the model needs to be exchanged between stakeholders. Additionally, BIM
 427 models support other use cases, such as clash detection, automatic prefabrica-
 428 tion and construction simulation. Potential errors identified can thus be averted
 429 even before construction has started.

430 One of the foundations of BIM is the parametric 3D modelling of objects that
 431 is supported by the majority of software vendors. With the introduction of
 432 constructive solid geometry (CSG), complex geometries previously depicted on
 433 a blueprint could be implicitly described according to their construction steps.
 434 In the next subsections, we cover the principal concepts of 3D modelling in BIM:
 435 i) the positioning and CSs involved in placement, and ii) different geometry
 436 representations. Both concepts are influenced by the different interpretations
 437 of the underlying CS and are therefore important to understand in the scope
 438 of this research¹¹. While there are many different placements and geometry
 439 representations available in software products, we focus in our paper on those
 440 present in the IFC standard [23].

441 3.2.1. Positioning

442 Each element's geometry resides within a local CS with a well-defined POO and
 443 orientation of the coordinate axes. This CS is usually a right-handed Cartesian
 444 CS (x, y, z) , as this is intuitive and simple [22]. The coordinate axes scale with
 445 each other, by definition, and have the same UoM.

446 The position of this CS is specific to each object and is always defined relative

¹¹Strictly speaking, all properties attached to an object containing values for lengths, areas, or volumes are influenced too, and should have been considered. However, we assume that the geometry and location are depicted with the corresponding representations, and we can therefore exclude these quantities from consideration.

447 to another CS. This can be the global CS of the model (*absolute placement*)
448 or another local CS of an element higher in the hierarchical structure (*relative*
449 *placement*). There are multiple possibilities for relative placement, which are
450 described in the following paragraphs. These options can be combined freely
451 and used recursively, *ad libitum*. To simplify the notation, we use (x, y, z) for
452 the originating CS and (u, v, w) for the object's CS.

453 *Local Placement.* The definition of a CS using local placement is common prac-
454 tice in building design. This is done using Equation (3) with $\lambda = 1$ (Figure 6).

455 For example, a wall is placed within a building storey with a POO at (t_u, t_v, t_w) .
456 The u axis is defined to run along the wall's centreline, the w axis functions as
457 the z axis of the storey ($\alpha = \beta = 0^\circ$) and $v = w \times u$ such that it defines a
458 right-handed Cartesian CS.

459 *Grid Placement.* Multiple elements can be placed in a grid-like constellation.
460 The base point, the distance step(s) and the reference direction(s) uniquely
461 define the POO of each element. The (u, v, w) axes are additionally defined
462 once for all elements and may diverge from the reference directions (x, y, z) .

463 A simple example is the raster-like representation of a digital terrain model
464 (DTM). In this case, only the first point has its (x_0, y_0) coordinates and the
465 steps d_x and d_y are given. The heights are represented with a 2D array of
466 values in the parent's CS, $w_{i,j} = z_{i,j}$, where i and j represent the indices of the
467 point $P_{i,j}$ in the array. The coordinates of the point are thus $P_{i,j} = (u, v, w)_{i,j} =$
468 $(x_0 + id_x, y_0 + jd_y, z_{i,j})$.

469 *Linear Placement.* Another possibility is to define the CS of an element relative
470 to a curve. This so-called 'linear placement' is common practice in infrastructure
471 design. The POO is placed in a point, which is placed a certain distance along
472 the basis curve (directrix).

473 For the definition of the coordinate axis, there are two possibilities which are
474 differentiated during the definition of the w axis.

475 I) The u axis follows the tangent of the curve at the POO in the direction of
476 increasing distance from the beginning of the curve. The v axis is then defined
477 to lie on the (x, y) plane to the left of the curve, with the w axis being the cross
478 product of the two: $w = u \times v$. This yields a right-handed Cartesian CS.

479 II) A slightly different possibility is to keep the direction of the w axis the same
480 as the z axis. The u axis is then defined as the projected tangent direction of
481 the curve at the POO on the (x, y) plane and the v axis is the cross product of
482 the other two: $v = w \times u$. This again yields a right-handed Cartesian CS.

483 An example of the first option is the placement of cross sections in a tunnel dug
484 with a tunnel boring machine (TBM). Here, the plane of the front of TBM, and
485 thus the scans and profiles, always lies perpendicular to the base curve of the
486 tunnel. An example of the second option is the placement of a typical supporting
487 column on the railing of a staircase. It is placed parallel to the staircase's main
488 directrix, but the vertical direction remains that of the risers.

489 *3.2.2. Geometry*

490 There are multiple possible representations of 3D geometry in a BIM model [43].
491 These can be split in two categories: implicit and explicit representations. The
492 former are *top-down* representations, where the geometry is described with the
493 required steps and parameters from its design process. For example, a cube can
494 be uniquely described by its centre point, its side length, and the orientation of
495 two of its axes in space. The latter representations take a *bottom-up* approach to
496 building the geometry, with 3D vertices that are connected to higher-dimension
497 geometries like edges, surfaces, and, eventually, solids. Again, a cube can be
498 described by its 8 vertices in space, the 12 sides connecting them, and the 6
499 surfaces bounded by these elements, which enclose and complete the cube.

500 In the following subsections, we depict individual geometries with an exemplary
501 geometry: a frustum of a right circular cone. Such a geometry is often used
502 at bridge approaches for embankments if the abutment was not designed with
503 retaining walls. Figures 11 and 12 show its explicit and implicit representations,
504 respectively.

505 *Boundary representation.* An efficient and flexible explicit geometry represen-
506 tation is the boundary representation (BRep) depicted in Figure 11a. Here, 3D
507 vertices (represented by points) make up the basis for higher-dimension geome-
508 tries. These are linked to form (potentially curved) edges, such as non-uniform
509 rational B-splines (NURBS), arcs, or simple polylines. Solids are bordered by
510 (potentially curved) faces bounded by the lines.

511 The example in Figure 11a was built up from eight vertices. These are connected
512 by nine edges: seven lines and two B-splines, which approximate a circular arc.
513 Each arc is defined by three vertices as described by Abedallah [1]. In turn, the
514 edges border five faces, which enclose the volume.

515 *Tessellated Geometry.* The tessellated representation is a special type of BRep
516 that strives for simplicity. It consists of an array of 3D points connected in
517 loops; hence the name ‘tessellated’. The loops represent polygonal planar sur-
518 faces, most commonly triangles. In order to achieve its simplicity, tessellated
519 geometries make a trade-off between precision and data required to encode them.
520 This is clear for curved geometries, which could theoretically be split into an
521 infinite number of polygons.

522 The example in Figure 11b consists of ten points. The B-splines from Figure 11a
523 have been approximated by four points and the curved surface by three trape-
524 zoids. The figure also shows the tessellation that would be used if the only
525 shape allowed for a face were a triangle, as is the case in some 3D systems. This
526 could be refined infinitely if higher precision is needed, but the amount of data
527 grows as well. Each new point that is added improves the approximation of the
528 arc, but it adds an additional trapezoid (or pair of triangles) to the slope, for a
529 total of four additional triangles that must be defined and stored in memory (1
530 top, 1 bottom and 2 on the slope).

531 *Sweep.* A swept geometry is an implicit geometric representation that is based
532 upon a shape (cross section) that is extrapolated (extruded) along a basis curve
533 (directrix). This implicit representation is often used in infrastructure, e.g. a
534 rail profile that is swept along the railway’s alignment. If the cross section

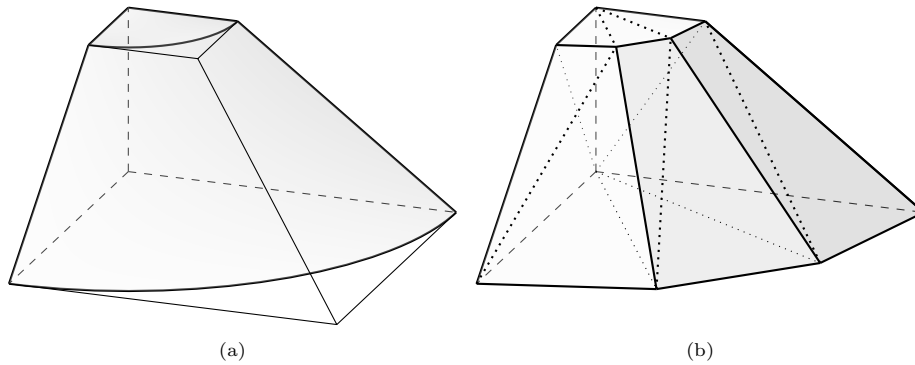


Figure 11: Example of explicit geometry representations in BIM models: a frustum of a right circular cone. (a) A boundary representation. The front face is a NURBS surface bordered by two lines and two B-splines. (b) Tessellated geometry. Dotted lines show how the faces would be defined when only triangles must be used.

535 never changes along the line, the operation is called *sweeping*; otherwise it is
 536 *lofting*. If the basis curve is a straight linear segment, the operation is called
 537 *extrusion*. The cross section may be a single line, a NURBS or a closed curve
 538 that is empty or full. The results of extruding these cross sections would be the
 539 creation of a plane, a curved surface, a hollow pipe-like object and a solid body
 540 like a cylinder, respectively.

541 The example in Figure 12a shows a vertical full profile being extruded along a
 542 quarter-circle directrix. The profile has a trapezoidal form, which could have
 543 been defined by four points or lofted from the bottom to the top line in the
 544 vertical direction. The path itself can take many forms; in this case it is an arc,
 545 defined by its centre on the vertical line of the profile, with the radius being the
 546 width of the trapezoid at that height.

547 *Constructive solid geometry*. Constructive solid geometry (CSG) representation
 548 uses the principles of point set theory to define elementary building blocks and
 549 join them using Boolean operators. In this way, the geometry is described by
 550 the operations that must be applied to construct it. The base blocks may be
 551 simple geometric bodies, like cubes or spheres defined by their parameters or
 552 geometries of other geometric representations. The possible Boolean operators
 553 are union ($A \cup B$), intersection ($A \cap B$) and subtraction ($A - B$).

554 Figure 12b shows the example geometry with two base forms used to produce
 555 it. The right cone and the cuboid are positioned relative to one another in such
 556 way that the cone's centre and cuboid's side coincide. The desired geometry is
 557 achieved by intersecting these two primitives.

558 4. Spatial Interpretation of BIM Models

559 This section handles the main question as posed in Section 1.3. With the back-
 560 ground knowledge provided in Section 3, it can be reformulated as follows: *What*
 561 *is the functional connection between the project scale, m_p , of a BIM model and*

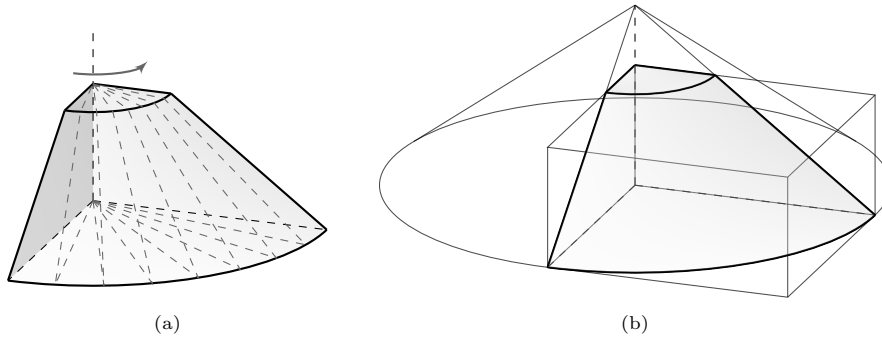


Figure 12: Examples of implicit geometry representations in BIM models: a frustum of a right circular cone. (a) Sweep of the trapezoid along a quarter-circle directrix defines the frustum. (b) Constructive solid geometry (CSG): the primitives cuboid and right circular cone intersect to produce the frustum.

562 *the scale of the underlying geospatial data, m_{CRS} ?* That is, how to properly
 563 define this function, which is a variable locus function:

$$m_p \stackrel{def}{=} f(m_{CRS}) . \quad (8)$$

564 Since the value of m_{CRS} varies continuously from point to point, complex distortions
 565 are not uncommon (e.g. straight lines can become curved lines in another
 566 CRS).

567 We stress again that this spatial interpretation only applies to the horizontal,
 568 and not the vertical, extent of the BIM model (Equations (7) and (8)).
 569 Whereas m_{CRS} is mostly dependant on longitude, elevation, lastly latitude
 570 (Equation (7)), m_p can be set to a constant value with certain assumptions
 571 (discussed in detail in Section 5). This text uses the words ‘*factor*’ and ‘*func-*
 572 *tion*’ interchangeably for m_p .

573 Additionally, m_p is not the same as the scale factor of a drawing, instead it
 574 indicates the distortion of lengths between the model and on-site representation
 575 due to the underlying CRS features.

576 In the following subsections we present the three options of defining the function
 577 in Equation (8) as well as a case study for each option to better depict their
 578 approach to the interpretation of a BIM model.

579 4.1. Options

580 We have identified three interpretations of the spatial extents of a BIM model
 581 (Figure 13).

582 **Option A:** We consider a BIM model to be a true representation of a real
 583 asset. With $m_p = 1 + 0m_{CRS}$, the dimensions and relative positioning of the
 584 objects in the model correspond to those in nature.

585 However, the underlying geospatial data used during the design process first
 586 needs to be transformed back to its true dimensions by using the inverse of its
 587 underlying CRS, i.e. by m_{CRS}^{-1} (Figure 13a). This means that the true nature

Table 3: An overview over the case studies presented in Section 4.2.

Section	Case study	Interpretation	Reference
4.2.1	<i>Embankment</i>	Option A	Section 3.2.2
4.2.2	<i>Road</i>	Option B	Equation (7)
4.2.3	<i>Railway</i>	Option C	Figure 2

of the curved Earth is represented in the model itself, and that the Z planes *no longer* correspond to equipotential surfaces. If this assumption is still held, agreement is only maintained in a small area around the POO [48].

Option B: We consider the models to be distorted by the scale function $m_p = m_{\text{CRS}}$ grounded in the compound CRS of the underlying geospatial data used for design. This option is the opposite of Option A (Figure 13b).

When setting out the construction site, the model must be transformed back to true dimensions by m_{CRS}^{-1} . Knowing this, the design norms need to be adjusted before use, e.g. if a building edge must stand d_{norm} away from the street, the correct value used for the design is $d_{\text{design}} = m_{\text{CRS}} d_{\text{norm}}$.

Option C: This option is a combination of Options A and B (Figure 13c). We consider the models for buildings and infrastructure objects separately, interpreting their spatial extents according to Option A and Option B, respectively. In this case, *buildings* are all structures with limited extents in horizontal dimensions (e.g. houses, skyscrapers and short bridges and tunnels), while *infrastructure objects* include all elongated objects (e.g. roads and railways).

4.2. Case Studies

To better visualise the differences and consequences of spatial reference, we provide three simple theoretical case studies, each establishing a different scenario (Table 3).

1) The first case study, *Embankment*, was introduced in Section 3.2.2. It provides an overview of the possible geometry representations and their influence on the computed volumes. This influence is shown in Section 4.2.1 in order to distinguish it to the influence of geodetic distortions on the volume depicted in Section 4.2.3.

2) The second case study, *Road*, is a long object with a simple geometric representation, where a rectangular cross section is swept along a predetermined route. It shows the difficulties of the location-dependent distortions of a CRS from Equation (7).

3) The third case study, *Railway*, considers the railway and its stations from Figure 2. Here, the different spatial references of the BIM objects, as well as the difficulties of their synchronisation, are highlighted.

4.2.1. Embankment

The embankment from Figures 11 and 12 is taken as our first case study, with which we highlight the different geometry representations listed in Section 3.2.2.

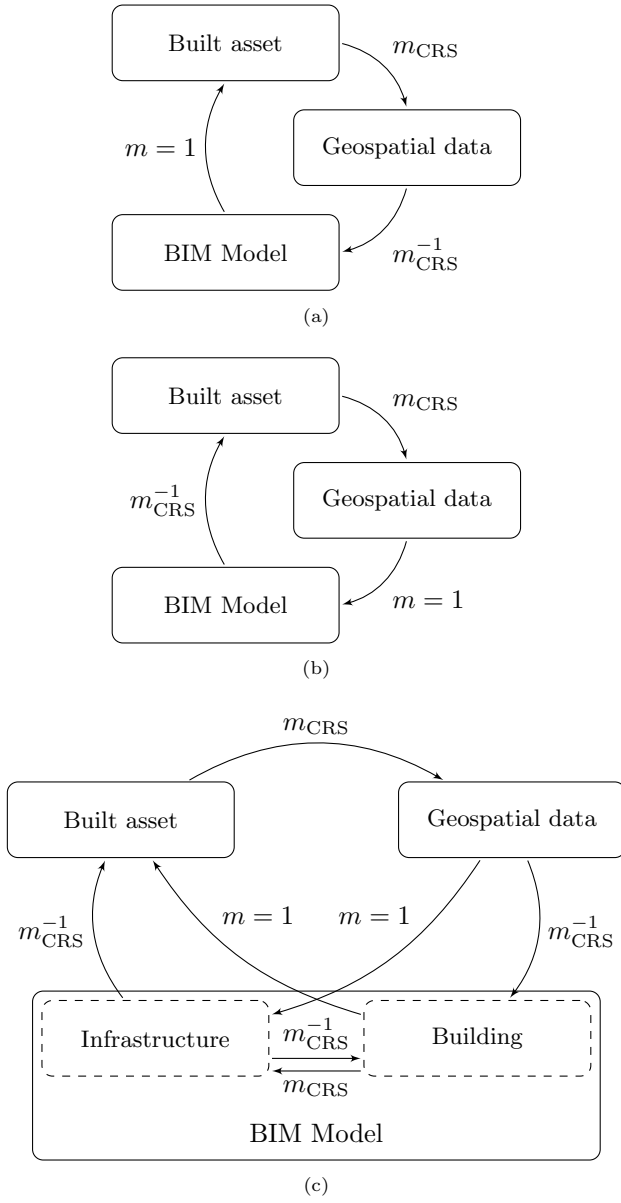


Figure 13: Different options for transforming the data following the information flow from Figure 1: (a) Option A with $m_p = 1$, (b) Option B with $m_p = m_{CRS}$, and (c) Option C as a combination of Options A and B. These are the different interpretation of the project scale, m_p , of BIM models discussed within this paper. Options A and B differentiate in the step where m_{CRS}^{-1} is applied. Option C is a combination of the two, where buildings are considered according to Option A and infrastructure objects according to Option B. Notice the necessary transformation of data between the BIM models from different fields in Option C.

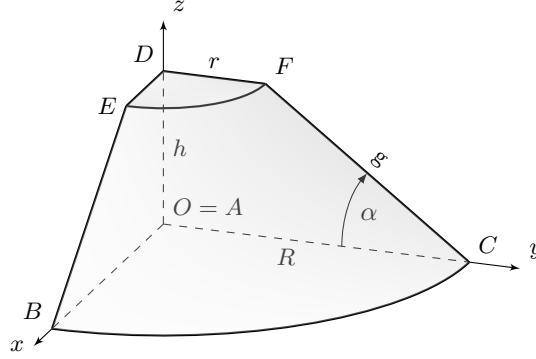


Figure 14: The dimensions of the embankment with marked POO and coordinate axes (not to scale).

623 The expected result on the construction site should have the required dimensions
 624 (Figure 14):

- 625 • the top radius, $r = 1$ m,
- 626 • the height of the embankment, $h = 10$ m, and
- 627 • the slope of the embankment, $1 : 2$, i.e. $g = 50\%$ or $\angle ABE = \angle ACF = 26.6^\circ$.

628 From these parameters others can be derived. For example, the radius at the
 629 bottom of the embankment is $R = r + h/g = 21$ m.

630 Consider an engineer who would like to produce a bill of quantities, and thus
 631 needs an exact calculation of the volume of the material needed for the construction of such an embankment. The volume of the frustum in Figure 12 can
 632 be calculated using the parametric formula:
 633

$$V = \frac{1}{4} \frac{\pi h}{3} (R^2 + Rr + r^2) \quad (9)$$

$$= \frac{\pi 10}{12} (21^2 + 21 \cdot 1 + 1^2) = 1212.131 \text{ m}^3. \quad (10)$$

634 On the other hand, the volume of any manifold, non-intersecting and triangulated polytope without borders (e.g. the dotted lines in Figure 11b) can be
 635 calculated using the 3D version of the Shoelace formula [6, 38]:
 636

$$V = \frac{1}{6} \sum_{i=1}^N \det(P_1^i, P_2^i, P_3^i) = \frac{1}{6} \sum_{i=1}^N \begin{vmatrix} x_1^i & x_2^i & x_3^i \\ y_1^i & y_2^i & y_3^i \\ z_1^i & z_2^i & z_3^i \end{vmatrix}, \quad (11)$$

637 where N is the number of triangles and P_j^i represents the coordinates of the i^{th}
 638 triangle's j^{th} vertex and is defined by

$$P_j^i = [x_j^i, y_j^i, z_j^i]^T \quad \forall i \in \{1, \dots, N\}, \forall j \in \{1, 2, 3\}. \quad (12)$$

Table 4: The calculated volumes of the different geometric representations of the frustum in Figure 14 following Section 3.2.2. Its correct volume is derived in Equation (10).

	Geometric representation				Unit
	explicit		implicit		
	Tessellation	BRep	Extrusion	CSG	
Volume	1157.500	1200.649	1212.131	1212.131	m ³
	95.49	99.05	100.00	100.00	%
Figure	11b	11a	12a	12b	

639 The vertices in each triangle need to be provided in counter-clockwise order, as
 640 seen from the inside of the solid.

641 Table 4 shows the differences in volume between the different geometric repre-
 642 sentations.

643 4.2.2. Road

644 Since the geodetic distortions depend on the location of the asset on the Earth,
 645 we consider a 10 m wide road going through four different locations within the
 646 state of Bavaria. The road begins just shy of the western border of Bavaria, at
 647 the southern-most point of the Lake *Gustavsee*, near the city of Seligenstadt.
 648 It runs to the east until it crosses the town of Bayreuth, where it turns south
 649 on the campus of the University of Bayreuth. In the centre of Munich, in front
 650 of the main entrance to the Technical University of Munich, it turns eastwards
 651 again. The road ends at the eastern border of the state of Bavaria, where
 652 Germany, Austria and the Czech Republic meet. The corresponding indices of
 653 the prominent locations are *west*, *byu*, *muc* and *east*, respectively.

654 Because of the huge difference in the sizes of the Earth and the road, a true-to-
 655 scale representation would exceed the precision of a pixel on printed medium.
 656 This is why, Figure 15 shows the road only schematically, where the road is
 657 shown on the ellipsoid and in the Cartesian PCS. The geodetic parameters for
 658 transformation between the curved and projected representations are provided
 659 in Tables 5 and 6.

660 As is usually the case in infrastructure projects, the PCS and the CRS coincide,
 661 $(X, Y) = (E, N)$, and all geometry representations have ‘big’ coordinate values,
 662 i.e. geodetic coordinates¹². The distortions induced by UTM map projection
 663 at a particular location on the (X, Y) plane (i.e. the (E, N) plane) can be
 664 calculated according to Equation (A.10).

665 The road can be treated if it were split into three segments¹³. The first

¹²Many (architectural) modelling software suites chose to only allow models to reside close to the POO of the PCS. Reasoning behind this is the computational stability of digital representations of decimal numbers when only single precision arithmetic is used [see also 9]. As such, these programs encounter problems when BIM models are imported with – although valid – “big” coordinates. However, as far as modelling as a concept is concerned, this does not represent any obstacle.

¹³Euclidean geometry holds in BIM models per definition of the PCS. On Earth, it holds in

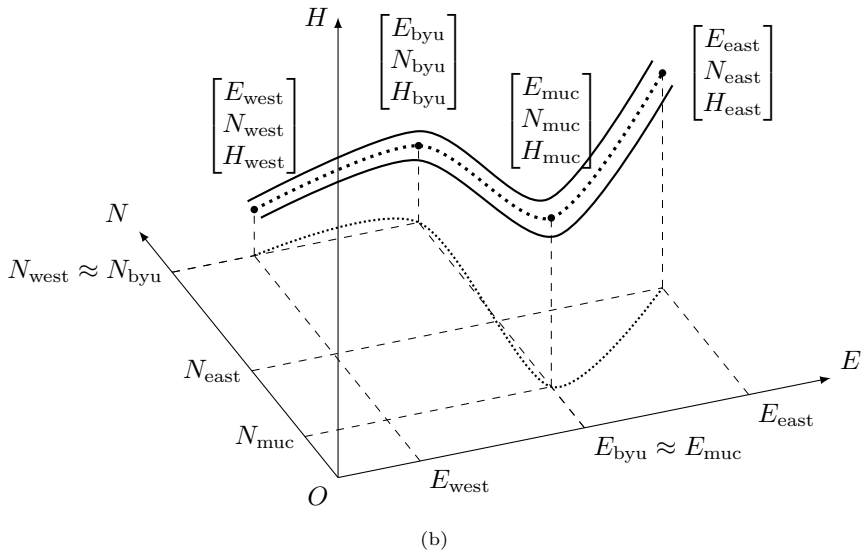
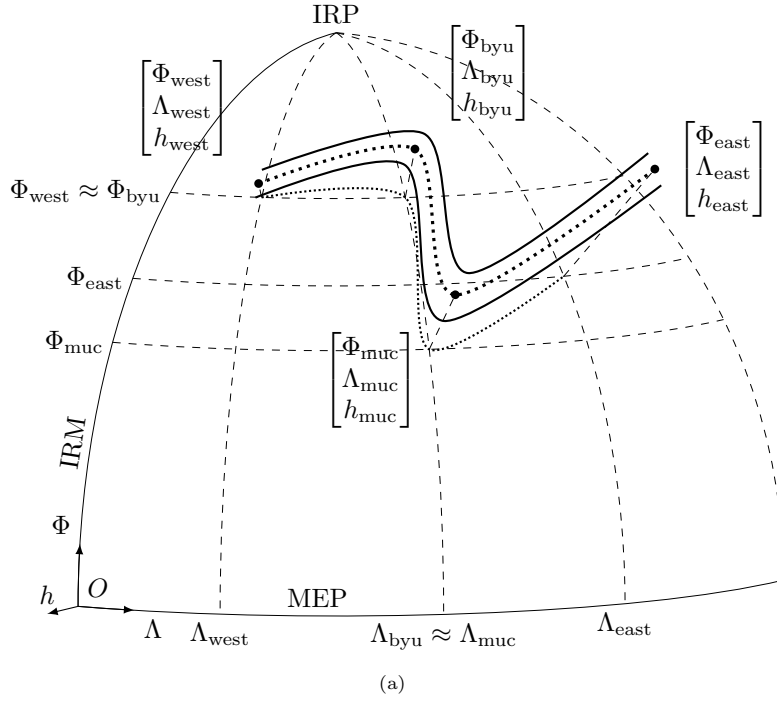


Figure 15: The road example with schematic representation of the four location (not to scale): (a) on the ellipsoid and (b) in the PCS. The exact values of coordinates are provided in Table 6.

Table 5: The parameters of the CRS for all four locations needed for geodetic transformations.

Parameter	Value
Geodetic datum	ETRS89
EPSG code	6258
Ellipsoid	GRS80 1980
EPSG code	7019
R_{major}	6 378 137.0 m
f^{-1}	298.257 222 101
Vertical datum	DHHN 20162016
EPSG code	1170
Map projection	UTM
zone	32N
Λ_0	9°0'0"
EPSG code	25832
E_0	500 000 m
N_0	0 m

Table 6: The parameters of the different locations needed for geodetic transformations. m_{CRS} , m_{proj} and m_h are calculated according to Equations (7), (A.10) and (A.14), respectively.

Parameter	<i>west</i>	<i>byu</i>	<i>muc</i>	<i>east</i>
Ellipsoidal coordinates				
Φ	N50°03'02.770"	N49°55'43.693"	N48°08'54.899"	N48°46'17.541"
Λ	E08°59'29.545"	E11°35'09.047"	E11°34'06.888"	E13°50'22.080"
Projected coordinates and scale				
E^{a}	499 394.35 m	685 582.84 m	691 046.73 m	855 531.47 m
N	5 544 275.53 m	5 533 920.52 m	5 336 005.65 m	5 413 364.53 m
m_{proj}	0.999 600 0	1.000 023 0	1.000 048 5	1.001 152 9
Elevation coordinates and scale				
h	104.0 m	352.0 m	515.0 m	1321.0 m
H	56.15 m	305.37 m	469.34 m	1274.00 m
U	47.85 m	46.63 m	45.66 m	47.00 m
m_h	0.999 983 7	0.999 944 8	0.999 919 3	0.999 793 0
CRS scale at (Φ, Λ, h)				
m_{CRS}	0.999 583 7	0.999 967 8	0.999 967 7	1.000 945 6
Δ_{CRS}	-416.3 ppm	-32.1 ppm	-32.3 ppm	945.6 ppm

^a In some cases, these coordinates are prepended with the zone number, in our case that would be 32xxxxxx.xx.

666 segment, *west-byu*, lies between the meridian and the intersection of the UTM
667 projection cylinder with the ellipsoid which lies approximately at Bayreuth.
668 Here, the projection is shortening the real distances (observe m_{proj} in Table 6).
669 The second segment, *byu-muc*, runs almost along the intersection line, where
670 very little distortion occurs. The third and last segment, *muc-east*, lies between
671 the intersection line and the border, extending over the usual boundary between
672 zone 32 and zone 33 (at 12°) to 13.8°. Here, the UTM projection lengthens the
673 real distances (Figures 9 and 10a).

674 In AEC design, the distance between two points is often important. On the
675 one hand, the distance between two points $|P_1P_2|_{\angle}$ in the projected CS can be
676 calculated as

$$|P_1P_2|_{\angle} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \quad (13)$$

677 On the other hand, the distance between two points $|P_1P_2|_{\circ}$ on the ellipsoid
678 (also called the geodesic) can only be calculated iteratively in the general case.
679 The calculation of the inverse geodesic is complex [described in full by 29]:

$$k = e' \cos \alpha_0 \quad (14)$$

$$I(\sigma) = \int_0^{\sigma} \sqrt{1 + k^2 \sin^2 \sigma'} d\sigma', \quad (15)$$

$$|P_1P_2|_{\circ} = \frac{I(\sigma_2) - I(\sigma_1)}{b}, \quad (16)$$

680 where α_0 denotes the azimuth of the geodesic at the Equator and σ_1 and σ_2
681 are the spherical arc lengths on the auxiliary sphere from the intersection of the
682 geodesic with Equator to P_1 and P_2 , respectively [29, Figure 2].

683 The comparison of the distances from Equations (13) and (16) for each segment
684 is shown in Table 7. As expected, the difference between the on-site and ellip-
685 soidal distances corresponds to the height reduction from Section 3.1.4, while
686 the difference between the ellipsoidal and projected distances corresponds to
687 the properties of the UTM projection from Section 3.1.5. In the first segment,
688 *west-byu*, each meter in the model is on average shorter by about 0.30 mm com-
689 pared to on-site measurements, whereas in the third segment, *muc-east*, each
690 meter in the model is on average roughly 0.40 mm longer than that on the site.
691 These values are the averages along the whole length of that segment; in reality,
692 they change with varying m_{proj} and m_h , as calculated from Equations (A.5)
693 and (A.14), respectively. For example, compare averages of Δ_{CRS} from Table 6
694 with the corresponding differences $d_{\text{proj}}/d_{\text{true}}$ from Table 7.

695 4.2.3. Railway

696 The last case study is presented in Figure 2, where a new railway line with
697 four stations is planned. As presented in Section 4, there are three possibilities

close proximity of the observer as noted in Section 1. This is why a split into segments does not infer any modelling obstacles.

Table 7: The comparison of the 2D distances between the on-site locations in Table 6 (true distance), on the ellipsoid using Equation (16); and those in the projected CS using Equation (13). The differences are calculated following Equation (1).

Distance	Expression	<i>west-byu</i>	<i>byu-muc</i>	<i>muc-east</i>	Unit
on site ^{a,b}	d_{true}	186 531.285	197 996.654	181 695.134	m
ellipsoidal ^a	d_{ell}	186 524.617	197 983.198	181 668.986	m
difference	$d_{\text{true}} - d_{\text{ell}}$	6.668	13.456	26.148	m
	$d_{\text{ell}}/d_{\text{true}} - 1$	-35.747	-67.961	-143.911	ppm
projected	d_{proj}	186 476.218	197 990.277	181 768.056	m
difference	$d_{\text{ell}} - d_{\text{proj}}$	48.399	-7.079	-99.070	m
	$d_{\text{proj}}/d_{\text{ell}} - 1$	-259.476	35.757	545.330	ppm
combined	$d_{\text{true}} - d_{\text{proj}}$	55.067	6.377	-72.922	m
difference	$d_{\text{proj}}/d_{\text{true}} - 1$	-295.214	-32.207	401.340	ppm

^a Calculated using the GeodSolve calculator [30].

^b True distances have been calculated using the average elevation between the two points.

698 for interpreting this BIM model. Recall that Option C is a combination of
699 Options A and B, where some objects are interpreted according to the first and
700 others according to the second option. This is the case in this project, where the
701 stations were developed according to Option A and the railway line according
702 to Option B.

703 *Railway Line.* For the railway line, any chosen POO would be equally sub-
704 optimal. The m_p cannot be neglected anywhere, as the railway line runs through
705 all four stations and it changes continuously according to Equations (A.5)
706 and (A.14). Historically, this is the reason that the POO of long infrastruc-
707 ture objects has usually been left as the POO of the projected CRS, which
708 would be $(X_0, Y_0) = (E_0, N_0)$ in our case. With the usage of ‘big’ coordinate
709 values, all project stakeholders are typically aware of the equality of CRS =
710 PCS [35].

711 When calculating the set out values, m_{CRS}^{-1} needs to be applied. When exchang-
712 ing the data between the models of stations and of the railway, it needs to be
713 converted accordingly, using m_{CRS} or m_{CRS}^{-1} as marked in Figure 13c. Should
714 this be omitted (forgotten or knowingly neglected) at any point in the process,
715 it could introduce errors that are hard to discover and remediate. This problem
716 is discussed in detail in Section 4.3.

717 *Stations.* The four stations are located at the four locations described in Sec-
718 tion 4.2.2 as vertices in the *road* case study, and their locations on Earth are
719 presented in Table 6. Since the station’s overall size is small compared to Earth,
720 the deviation of a plane from the curved surface is very small (< 1.25 cm at
721 0.4 km distance from POO). Thus, the Earth can be assumed flat in the vicinity
722 of the station’s origin.

723 Before the design, the geospatial data was transformed back to its true form as
724 shown in Figure 13c. Therefore, we can interpret the geometries according to
725 Option A. When the reading software interprets the geometry and produces the

726 set-out values, or calculates the bill of quantities, the horizontal dimensions do
 727 not need to be scaled by the corresponding value of m_p to obtain the dimensions
 728 on Earth, but can be used as-is, instead. This approach introduces a certain
 729 discrepancy at the borders between the railway line and the stations as discussed
 730 in Sections 4.3 and 5.

731 4.3. Negligence of the Scale Function

732 If any of the factors is not applied correctly when exchanging data during the
 733 project (especially when exchanging BIM models with Option C), certain mea-
 734 surable discrepancies will be introduced between the model and the reality that
 735 it models. We show two possible consequences based on the *railway* case study
 736 from Section 4.2.3, both depicting the problem which emerges from the lack of
 737 (back-)transformation.

738 4.3.1. Incorrect Volumes

739 Let us assume that at each of the stations from Section 4.2.3, some sort of
 740 embankment for a bridging structure, like the one presented in Section 4.2.1,
 741 would be needed. The geospatial data was not transformed by m_{CRS}^{-1} as shown
 742 in Figure 13c; rather, it was used as-is in the design process (i.e. distorted).
 743 The designer was aware of that and followed Option B when designing the em-
 744 bankments using scaled horizontal dimensions $[rR]_{i,\text{design}}^T = m_{\text{CRS},i} [rR]^T; \forall i \in$
 745 $\{\textit{west, byu, muc, east}\}$.

746 Following the design phase, an engineer would like to produce a bill of quantities
 747 and would thus need an exact calculation of the volume of material needed for
 748 the construction. However, he interpreted the model according to Option A
 749 and thus deducted wrong quantities as presented in Table 8. As shown in
 750 Section 4.2.1, different geometric representations imply different volumes. The
 751 volumes for all 16 combinations (4 representations \times 4 locations) are presented
 752 in Table 8, where the clear influence of the geodetic transformations can be seen.
 753 The correct volumes of the different geometric representations are provided in
 754 Table 4, as a reference.

755 To calculate the volumes in Table 8, we assumed a constant underlying scale
 756 m_p , because of a very small change in the scaling factor $|\Delta_l - \Delta_{l+20}| < 0.1$ ppm.
 757 We applied $\lambda = m_p = m_{\text{CRS}}$ from Table 6 to the horizontal axes of the PCS.
 758 For that, Equation (3) cannot be used as is, but needs to be changed to the
 759 following (notice the position of λ):

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} t_u \\ t_v \\ t_w \end{bmatrix} + R(0, 0, \gamma) \begin{bmatrix} \lambda x_i \\ \lambda y_i \\ z_i \end{bmatrix}. \quad (17)$$

760 4.3.2. Broken Continuity

761 Consider the railway with a station from Section 4.2.3, whose design process
 762 and resulting (mis-)interpretations are shown in Figure 16. The real-world grid
 763 of distances on the construction site is shown in Figure 16a, together with two
 764 compulsory points denoting the existing railway with its outgoing directions.
 765 First, the construction area was surveyed and the current state of the site was
 766 scaled down, as seen in Figure 16b. For the sake of simplicity, the m_{CRS} was

Table 8: The calculated volumes of the geometric representations from Figures 11 and 12 for all four locations, where they were designed following Option B, but interpreted according to Option A. The base-line references are presented in Table 4 and Equation (10). Note, that m_{CRS} is only applied to the horizontal axes.

		Geometric representation				
		explicit		implicit		
		Tessellation	BRep	Extrusion	CSG	
Location	<i>west</i>	1156.536	1199.649	1211.122	1211.122	m ³
		95.41	98.97	99.92	99.92	%
	<i>byu</i>	1157.425	1200.572	1212.053	1212.053	m ³
		95.49	99.05	99.99	99.99	%
	<i>muc</i>	1157.425	1200.571	1212.053	1212.053	m ³
		95.49	99.05	99.99	99.99	%
	<i>east</i>	1159.690	1202.921	1214.425	1214.425	m ³
		95.67	99.24	100.19	100.19	%

767 assumed to be constant for the whole construction site¹⁴. At the projection
768 plane in UTM coordinates, for example, the railway line was designed using
769 Option B, with a representative (mean) scaling factor $m_p < 1$.

770 The line sections were used *as-is* to design each station. However, the cor-
771 responding engineer misinterpreted the model to be in the scale $m_p = 1$ (i.e.
772 according to Option A; Figure 16c). Following Figure 13c, the surroundings of
773 the stations were shortened by the underlying CRS to the model and thus the
774 railway platform was designed on shortened base data. The platform, in turn,
775 used prefabricated elements derived directly from the BIM model, still following
776 the Option A. The surveyor was instructed of that fact and set out the station
777 to be $m_p = 1$ with the model in order not to interfere with the prefabrication
778 processes. Thus, the railway line within the station does not match the design
779 intent of the railway engineer; instead, it deviates from it by a factor of m_{CRS}
780 (Figure 16c).

781 Misinterpretation due to neglect or lack of understanding causes divergences
782 from the design, which may cause a break in the continuity and/or smoothness
783 of geometries. If we keep the tangent direction of the railway track and the
784 neighbouring railway platform the same, then the track *leaving* the station will
785 not connect to the track of the free route *leading into* the station without a
786 measurable discontinuity. If the surveyor were to set out the free route of the
787 railway strictly following m_{CRS}^{-1} , then the two tracks would not meet at the
788 station's edges (Figure 16d). Thus, it is necessary to introduce a transition zone
789 between the two spatial references, which results in additional considerations
790 during the design and construction processes. The extent of this zone and
791 parameters used are decided by the surveyor responsible for setting out and, to
792 the authors' best knowledge, currently cannot be represented in any BIM model
793 or standard data formats like IFC.

¹⁴Following that, it is irrelevant, where the identical point resides; depicted is the middle of the station. In the general case, none of these assumptions holds true; see Equation (7).

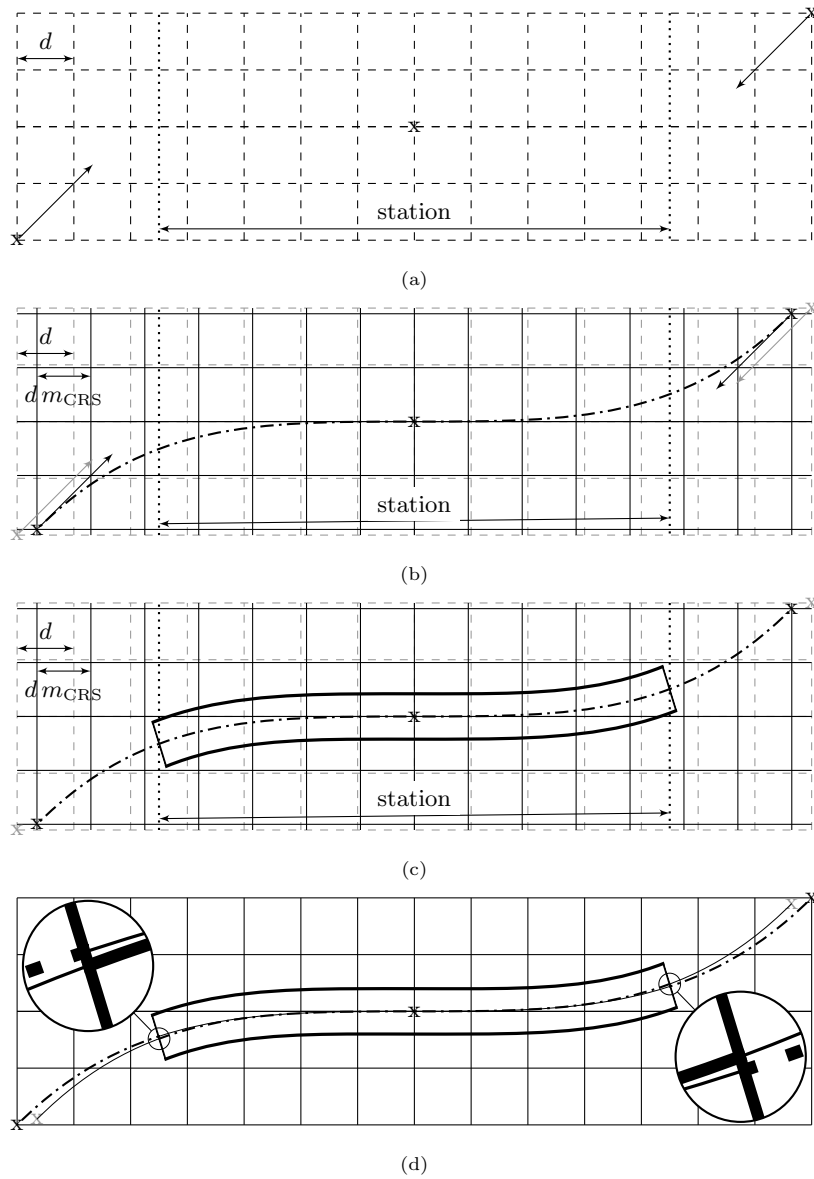


Figure 16: Visualisation of missing transformations from Figure 13c between infrastructure and building models, and their consequences on the example of a station from Figure 2. (a) Grid of the nature with cell length d together with marked design parameters for a railway station. The compulsory points (X) and track directions at the border of the construction site are marked, as well as desired station's length l_{station} . (b) The construction site is surveyed and the grid scaled by CRS is produced (dm_{CRS}) with its centre at the central compulsory point (surveyor's decision, it could have been anywhere else as well). The scaling factor is set to $m_{\text{CRS}} = 0.95$ and assumed constant for the whole construction site in order to better visualize the scenario. Here, the track engineer then designs the track alignment as shown with dash-dot line. (c) The station engineer now takes the alignment without scaling it back with m_{CRS}^{-1} and designs the railway station parallel to it. (d) Setting out the station as designed by option A and the track as option B results in discontinuity of the main alignment at station's ends (see zoom-ins at the edges). Not accounting for this when setting out the free route track would result in a *jump* in the track, which is not acceptable. This is why a transition zone needs to be introduced where the track is additionally distorted to fit into the station.

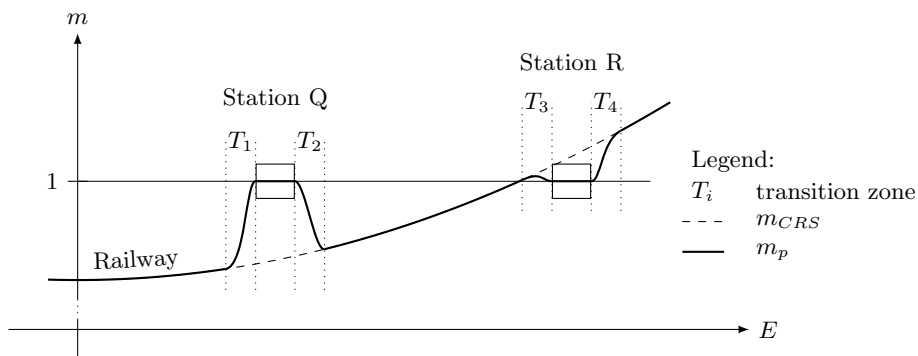


Figure 17: The BIM model from Figure 2, interpreted according to Option C. The stations are interpreted according to Option A and thus have a constant scale, $m_p = 1$, for the whole structure. The railway line is interpreted according to Option B with a continuously changing scale, $m_p = m_{CRS}$ (the representation is simplified to be only dependent on the Easting coordinate from UTM). The disagreement induced by this needs to be smoothed out in the transition zones (marked with T_i) between the two interpretations. Here, the transition follows a twice-derivable function for steadiness reasons.

794 Figure 17 depicts this problem for our case study. The project scale, m_p , used at
 795 stations Q and R, is depicted together with the one used throughout the railway
 796 track as an example. As mentioned above, the BIM models of the stations are
 797 interpreted according to Option A, with $m_p = 1$. The track is interpreted
 798 according to Option B, and thus the project's scale follows the underlying scale
 799 of CRS, $m_p = m_{CRS}$. Between the two interpretations, the transition zones, T_i ,
 800 need to be defined where m_p follows some function of m_{CRS} :

$$\forall T_i : m_p = f_i(m_{CRS}). \quad (18)$$

801 For example, the transition zones in Figure 17 show these functions to be C^2
 802 continuous.

803 With this constellation, the BIM models of the stations have been designed us-
 804 ing distorted geospatial data, but they are interpreted according to Option A.
 805 The BIM model of the railway line has been designed with an Option B inter-
 806 pretation. This induces a discontinuity in the transition areas, which requires
 807 some accommodation. The strategy most commonly used is to steadily 'brush
 808 off' the differences, as shown in Figure 17. The line and the stations in the Fig-
 809 ure lie purely in the East-West direction. Of course, circumstances can be more
 810 complicated in the nature, where, in addition to the North-South direction,
 811 differences in ellipsoidal heights also play an important role (see Equation (7)).

812 5. Discussion

813 5.1. The Root of the Problem

814 We discussed in Section 3.1 how geodesy models the Earth. In Section 3.2, we
 815 described how the AEC domain bases the model on the design steps needed
 816 to produce it. Both approaches yield information models, but on a different
 817 basis. These different approaches are rooted in the historical development and
 818 base philosophies of the two disciplines [11, 28]. This difference in modelling

819 paradigm can be best addressed with two questions that are derived from Fig-
820 ure 1.

821 The geodetic world primarily answers the question: ‘*what can be seen there*
822 *and how can it be captured?*’ It tries to encode the state of the Earth, and
823 thus the construction site, as it resides in the present. Surveyors model what
824 can be measured on the Earth in a *bottom-up* manner. The objects’ top sur-
825 faces are reconstructed from observations of individual points (*bottom*), which
826 are then connected to form increasingly rich semantic entities, geometries and
827 relationships (*up*).

828 The BIM world primarily answers the question: ‘*how should that object be de-*
829 *signed and how can that information best be transferred?*’ That is, the designers
830 imagine objects that do not yet exist, and thus still need to be transposed from
831 concept to reality. AEC experts design in a *top-down* manner, and that is how
832 the model is constructed. The idea of the structure is, at first, only vaguely
833 represented with simple geometries and semantics (*top*) [2]. The model is then
834 refined in later design stages (*down*), with increasing levels of detail until it is
835 finally erected at the construction site from a complete model [28].

836 Both worlds have developed their own methods and models, which require care
837 to merge and clear definition to transform. The core problem is the completely
838 different definitions of the (E, N) and H coordinate axes in the geospatial world,
839 which is contradictory to the equality of the Cartesian axes (X, Y, Z) in the
840 BIM world [36]. This leads to a deceptively simple question at the intersection
841 of geodesy and design: ‘*How should we set out the objects on the construction*
842 *site?*’ This difference must be adequately addressed by the whole community in
843 order to minimise the possibility for misunderstandings and potentially costly
844 errors.

845 5.2. Interpretation

846 In Section 4, we presented three options for how to interpret digital models,
847 and Table 9 shows how these models are usually interpreted. Clearly, anything
848 surveyed is transformed with m_{CRS} of the underlying CRS in the geospatial
849 domain. The interpretation of a BIM model is not as straight-forward as also
850 implied by Table 1.

851 On the one hand, Option A sees the model as being a 1:1 copy of the real asset,
852 as is prevalent in the building sector where the structure’s extents are rather
853 small (Figure 13a). Table 1 shows that BIM models are mostly interpreted in
854 this manner in the literature. In our opinion this stems from the fact that BIM
855 had its roots in building design and has only recently been introduced to the
856 infrastructure sector [e.g. 3]. As such, the vast majority of research has been
857 focused on buildings.

858 On the other hand, Option B interprets the model as distorted according to the
859 underlying CRS, which is prevalent in the infrastructure sector [e.g. 34] and is
860 the *de-facto* norm in GIS [33]. If this is the correct choice for a project, then
861 combining GIS and BIM data is easily achieved as long as both are in the same
862 CRS. Considering that different kinds of geospatial data, like DTM or property
863 borders, must be referenced during the alignment design, there is no realistic
864 alternative to Option B for any kind of infrastructure project.

Table 9: The prevalent interpretations of buildings and infrastructure objects on blueprints or in models within the corresponding disciplines (Table 1).

Object	BIM	Geospatial
Building	Option A	Option B
Infrastructure	Option B	Option B
Combination	Option C	–

865 Option C combines both approaches, interpreting parts of the model according
 866 to Option A, while other parts follow Option B. This option is prevalent for
 867 models of small infrastructure objects (like short bridges or railway stations),
 868 where the structure itself is *small* (Option A), but is integrated into a larger
 869 system (Option B).

870 5.2.1. Why does it Matter?

871 What are the consequences of falsely interpreting a distorted model as undis-
 872 torted, or vice-versa? What happens if there was no back-transformation of
 873 geospatial data before the design process started for buildings, as presented in
 874 Figures 13a and 13c? Under these conditions, the designs (Option A) were based
 875 on still-distorted data and thus cannot be set out in the real world without in-
 876 troducing some discrepancy between the objects, as shown in Figure 16. For
 877 example, direct prefabrication from a BIM model can lead to finished products
 878 that do not exactly fit in their prescribed places. In the same manner, imagine
 879 a contractor ordering $(100 \pm 5)\%$ of the material needed, as shown in Table 8,
 880 just by using the volumes directly from the design model, and not accounting
 881 for the distortions introduced by the underlying CRS. This kind of mistake can
 882 be hard to spot, as it is an absence of a step in the process, rather than an
 883 included step that is wrong.

884 Another example is a norm that prescribes at least d_{norm} clearance between
 885 the façade of a building and the centreline of the railway track. Because the
 886 designers know the geospatial data used in the design possesses a certain scale
 887 factor, $m_{\text{CRS}} < 1$, they use the scaled-down value for this distance ($d_{\text{norm,CRS}} =$
 888 $m_{\text{CRS}} d_{\text{norm}}$) following the interpretation in Option B. They then design the
 889 building as close as possible to the railway line, with $d_{\text{design}} = d_{\text{norm,CRS}}$. Code-
 890 compliance checking software considers the BIM model according to Option A
 891 and reports an error because $d_{\text{design}} < d_{\text{norm}}$. As shown above, that is of course
 892 not the case, but pin-pointing the source of this discrepancy could prove to be
 893 very cumbersome.

894 Similarly, imagine the radius of a railway curve, $R_{\text{design}} = 1000$ m, resulting in
 895 an $R_{\text{real}} = 1000.5$ m at the construction site. The discrepancies in geometry be-
 896 tween the design coordinates and the set-out values (because of the application
 897 of m_{CRS}^{-1}) influence the driving dynamics insignificantly. However, the change
 898 in position could result in a violation of a compulsory point’s margin, such as a
 899 railway platform’s edge, if not handled correctly [20]. If the railway line model
 900 were to be interpreted according to Uggla and Horemuz [48], the discrepancies

901 would be $\Delta = 81$ ppm at 140 km from the POO¹⁵.

902 5.2.2. Which Option to Use?

903 Option A considers the models to conform to $m_p = 1$ with the real asset. If
904 geospatial data was not used as input for the model, or if it was used and
905 scaled back to its natural dimensions by m_{CRS}^{-1} according to its underlying CRS
906 (Figure 13a), this option can still hold true.

907 Additionally, if the model has been designed based on only one spatial reference
908 point (e.g. one cadastral point with its elevation), then this option is viable.
909 The set-out can be based on this point, with the dimensions provided by the
910 BIM model. This represents the topocentric system described in Section 3.1.2,
911 which is the interpretation used by Uggla and Horemuz [48]. In this case, the
912 POO of the PCS represents the POO of the topocentric (right-handed) CRS.
913 This is, however, only valid within a small area around the POO, as described
914 previously.

915 Option B considers the models distorted according to the underlying CRS, as
916 $m_p = m_{CRS}$ (Figure 13b). In this case, the geometries in the BIM model are,
917 in principle, not equivalent to their real-world counterparts in either size, shape
918 or relative position. If the geodetic parameters, as described in Section 3.1, are
919 chosen correctly, this equality may be achieved to a certain extent. For example,
920 the custom CRS of the Brenner Basis Tunnel was designed so that the meridian
921 of the projection runs along the main alignment of the tunnel, thus making the
922 scaling factor of the projection $m_{proj} \approx 1$ within the project's area. As the zero
923 height level was set to the mean terrain height, it yielded a value of $m_h \approx 1$,
924 and thus $m_p = m_{CRS} \approx 1$ following Equation (7). After defining such a custom
925 CRS, any geospatial data relevant for the project must be transformed into the
926 newly defined CRS [35].

927 This discrepancy between the model and the real world may be hard to grasp,
928 but is very important, especially if code-compliance checking or direct prefabrication
929 systems use the BIM model as their primary source of design data [e.g.
930 31]. Additionally, this needs to be properly addressed if data from BIM and
931 GIS models are to be linked or transformed between each other [e.g. 39, 42].
932 For that, the dimensions and rules need to be carefully adjusted using the scale
933 factor induced by the underlying CRS.

934 Option C combines the other two options, applying option A to buildings and
935 Option B to infrastructure objects (Figure 13c). This option is ambiguous,
936 because a cut-off must exist to answer the question, '*which objects are to be*
937 *interpreted according to Option A and which to Option B?*' Heunecke [20] notes
938 that this limit depends on the precision requirements and tolerances of the
939 construction of the 'small' structures involved. A more specific phrasing of this
940 question would be '*what degree of distortion induced by CRS can be neglected*
941 *while still achieving the requested results?*' We provide an answer to these
942 questions in the following section.

¹⁵However, their calculations do not take into account the non-spherical shape of the Earth, or that locations on the ellipsoid have different rates of curvature depending on location (see Equations (A.11) and (A.12)), which would complicate their calculations.

943 *5.2.3. Correct Borderline Interpretation*

944 Sections 4.3 and 5.2.1 discussed some of the consequences of a false interpreta-
945 tion of a BIM model. These depend on the required construction precision and
946 the specific geospatial data used during the design. Brenner et al. [8] produced
947 a nomograph (Figure 18) that allows for a determination of the maximal hori-
948 zontal extent of a structure, depending on the distortion Δ_{CRS} of the underlying
949 CRS and the prescribed construction precision. This reflects false interpreta-
950 tion *by choice* and needs to be addressed adequately by all stakeholders within
951 a project.

952 Consider the 10 m wide road from Section 4.2.2 constructed to a precision of
953 1 cm. Its maximal horizontal dimensions cannot exceed about 22 m, 330 m,
954 330 m and 11 m at the four locations, respectively (given the CRS defined in
955 Table 5 and the distortions provided in Table 6). This means that when setting
956 out the lateral extent of the road, one can knowingly neglect the discrepancies
957 between the model and the reality, since its width is smaller in all four cases.
958 However, when setting out the longitudinal extent of the road, one *cannot* use
959 BIM data without (back-)transformation.

960 *5.3. Recommendations*

961 *5.3.1. Process of Interpretation*

962 Generally speaking, if project participants transformed their data strictly ac-
963 cording to Figure 13, any option presented in this paper can be viable. Ad-
964 ditionally, interpreting any BIM model according to option B will always be
965 correct, bearing in mind that the dimensions in the real world may not be the
966 same as in the model (perhaps longer; perhaps shorter). Figure 18 provides the
967 means to determine the maximal horizontal extent of the model below which
968 the distortions of the CRS can be neglected.

969 The flow chart in Figure 19 shows the interpretation process of a BIM model,
970 where the intended usage is uncertain. First, a distinction is drawn if the BIM
971 geometry has been produced on the basis of any (projected) geospatial data. If
972 there were multiple reference points taken into account (e.g. DTM, neighbouring
973 buildings, streets or geodetic reference points), then the design model needs to
974 be interpreted according to Option B, i.e. being distorted by the underlying
975 CRS of the reference data with m_{CRS} . If such a model were set out according
976 to Option A, some reference points would lie differently compared to the model,
977 so the distance to some reference points may be shortened or elongated. The
978 other option is a geometry based on only one 3D-point – using fewer points
979 would be building castles in the sky.

980 Next, the maximum horizontal extent of the model needs to be determined from
981 Figure 18, using the project scale factor, Δ_{p} , and the prescribed construction
982 precision. If the dimensions in the model exceed the maximum horizontal extent,
983 the model needs to be handled according to Option B. Otherwise it *may* (though
984 not always) be interpreted as Option A.

985 *5.3.2. Transition Zones*

986 The necessity for transition zones emerges because of the lack of (back-)trans-
987 formation between infrastructure and building BIM models (cf. Figure 13c).
988 For illustration, consider a bridge constructed to a precision of 1 cm in Munich,

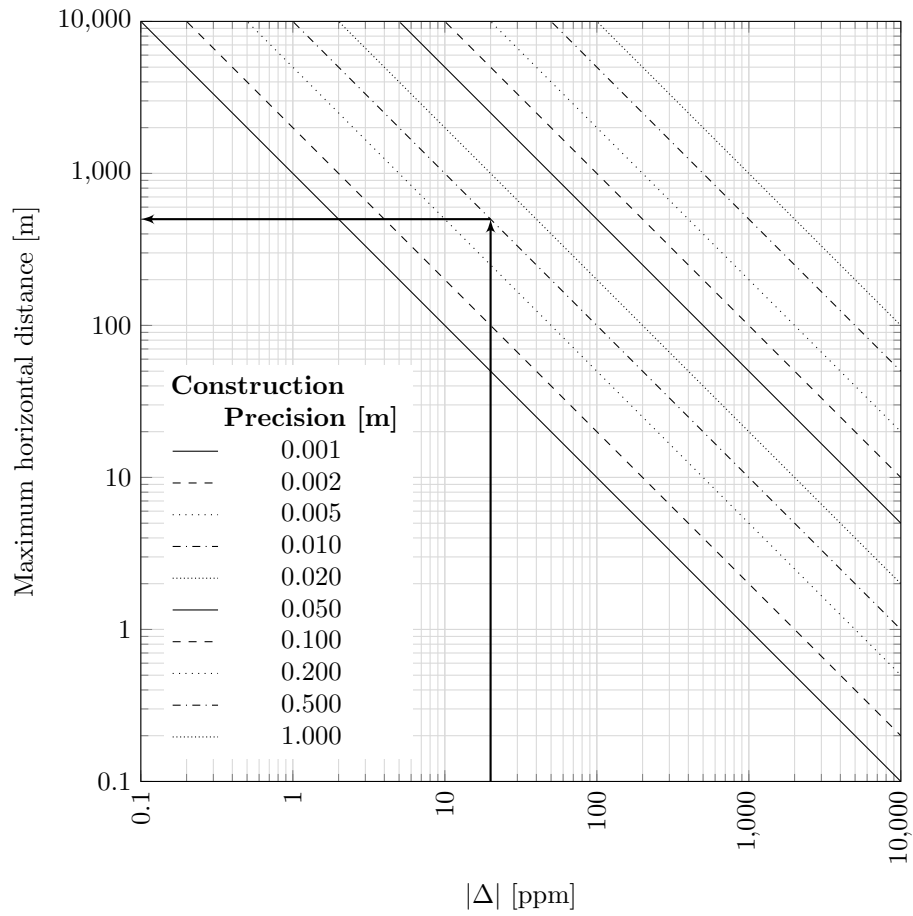


Figure 18: A nomograph to determine the maximum horizontal extents of a structure, depending on the distortion of the underlying CRS, Δ , and the required level of precision of the end structure [redrawn with permission from 8]. For example, the maximum horizontal extent for a construction site with $\Delta = 20$ ppm and a required construction precision of 1 cm is 500 m, as indicated with black arrows.

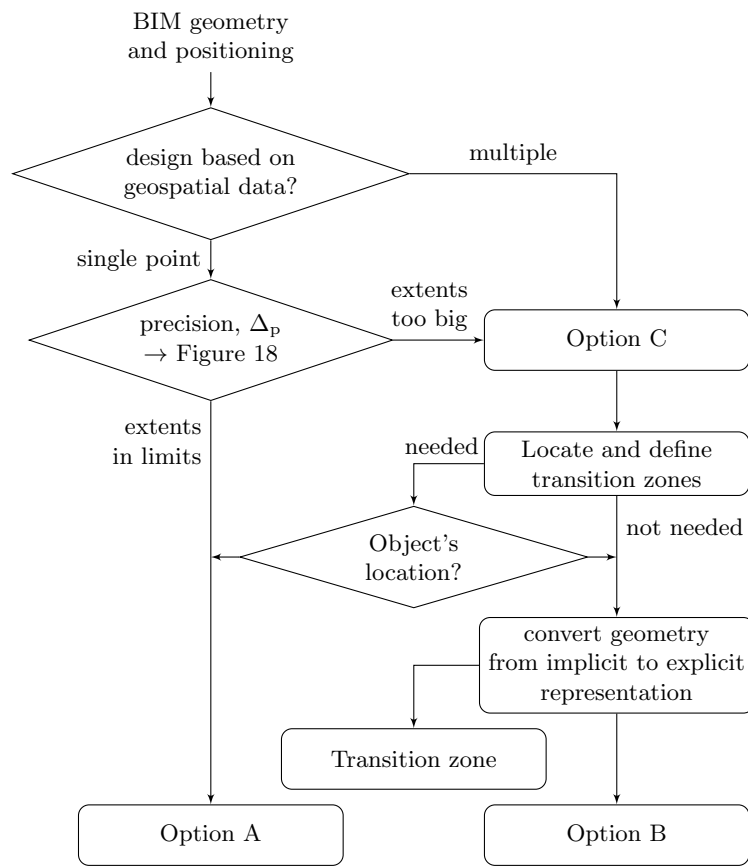


Figure 19: A proposed decision tree to determine the correct interpretation of the geometry within a BIM model.

989 Germany. Its maximal horizontal dimensions cannot exceed about 350 m, given
990 the CRS defined in Table 5. Up to this extent, engineers could knowingly ne-
991 glect the implications of the CRS within the BIM model, and still be within
992 the given tolerances. However, the road (or railway) alignment running along
993 the bridge would be incorrectly distorted. Instead of calculating the dimen-
994 sions using the m_{CRS}^{-1} , they would be interpreted as $m_p = 1$ from the distorted
995 model, thereby inducing an additional discrepancy between the model and the
996 real world at bridge’s ends. These discrepancies would need to be smoothed out
997 in the transition zones, as shown in Figure 17.

998 We note, that if the BIM data had been handled correctly, there would be
999 no need for transition zones. If the maximum horizontal dimensions of the
1000 structure are close to, or overshoot, the maximum allowance, then the transition
1001 zones need that much more attention and clear definition among the project
1002 partners. If the model should be interpreted according to Option B, but with
1003 some exceptions handled by Option A then transition zones should be clearly
1004 defined.

1005 We stress again that this disagreement is induced by interpreting the stations,
1006 embankments and other objects with small extents according to Option A, al-
1007 though based on scaled geospatial data, while correctly interpreting the rest of
1008 the extensive model according to Option B [20]. Here, the position and the ex-
1009 tent of the asset play an important role in the approach taken its ramifications.
1010 The project scale, m_p , and thus the mismatches in location and geometry, de-
1011 pends on the location of the asset on the Earth. If these models must be merged
1012 for clash detection, one needs to be converted to the other in some fashion. Such
1013 calculations are complex and are often beyond the scope of clash detection soft-
1014 ware suites.

1015 5.3.3. Conversion of Geometry

1016 Before interpreting the model according to Option B, however, an additional
1017 step must be taken. If there are any implicit geometry representations in the
1018 BIM model, these need to be made explicit. Because Equation (7) transforms
1019 the coordinates of individual points, only the geometries based on points can be
1020 correctly georeferenced and thus transformed back during the set-out process.
1021 This comes from the different interpretation of the horizontal and vertical axes
1022 in the geospatial world and the equality of all three axes in the BIM world, as
1023 noted previously.

1024 In order to be able to scale geometries in only two dimensions, they need to be
1025 discretised into points. As mentioned above, geodetic transformations of points
1026 result in their different relative position and therefore in a possible change of
1027 geometrical shape. Collinear points in a CRS may not be collinear anymore
1028 and thus even straight lines need to be tessellated. The density of newly added
1029 points on lines can be determined following Figure 18 and is dependent on the
1030 required precision as well as on the local properties of the CRS. The maximum
1031 horizontal extent determines in this case the maximum distance between any
1032 two neighbouring points in the geometric representation.

1033 6. Conclusions

1034 In this paper, we have investigated the geospatial context of BIM models; their
1035 PCS and how it is located on the *curved* Earth. We wish to raise awareness in
1036 the AEC community about this problem, which has previously been addressed
1037 entirely within the geospatial domain. We hope that this paper serves as a
1038 bridge between the geospatial and BIM worlds by bringing together the back-
1039 ground information of these two fields into a single document. We identified
1040 three options for BIM model interpretation in the spatial context: the model
1041 is either distorted according to the underlying geospatial data or not, or it is a
1042 combination of the two. We have presented a case study for each of these options
1043 in Section 4.2, which depicted the philosophy behind each interpretation.

1044 As long as construction crews use any levelled instruments at the construction
1045 site to determine equal elevation (e.g. spirit levels), we need to account for
1046 the fact that the water follows the equipotential surface around the Earth. As
1047 shown in Section 2 and Table 1, georeferencing systems have been indecisive
1048 with no clear consensus in the literature for how models should be designed
1049 and realised. Accurate and unambiguous georeferencing is critical for correct
1050 quantity take-offs, automated building and other processes [40]. We stress that
1051 a BIM model can represent any construction project, whether in the building
1052 or infrastructure sectors. Additionally, this applies to all BIM models, whether
1053 they are stored in open data formats like IFC or CityGML, or vendor-specific
1054 proprietary models.

1055 We argue that all three options are viable in the AEC industry and none of the
1056 options in Table 9 need to be changed. Countless projects have been executed
1057 successfully with this approach, and it need not be changed. The BIM models,
1058 which are still maturing, need to account for the different possibilities occurring
1059 in the industry and not the other way around. They should provide a clear data
1060 format in order to circumvent any future disputes about the meaning of saved
1061 information. Following that, a recommendation was formulated in Section 5.3.
1062 Our primary conclusions are:

- 1063 1. We join the call of buildingSMART International [10] for increased com-
1064 munication between all participants and the free sharing of information. The
1065 processes in Figure 1 need to be adequately understood by all stakeholders.
- 1066 2. There needs to be a clear distinction between a distorted BIM model (Op-
1067 tion B) and one that is not (Option A). If this is not known, Figure 19 helps
1068 distinguish *small* and *big* BIM models and objects.
- 1069 3. Interpreting BIM models with a varying factor following Equation (8) needs
1070 to be researched more deeply. We have limited our view on individual points
1071 and volumes; however, the distortions can be more complex (see Section 5.2).
- 1072 4. We need to consider the definition of the transition zones from Equation (18),
1073 if we are to allow for interpretations as described in Section 4.3 and Figure 16.
1074 The data formats (e.g. IFC) need to develop support for the inclusion of tran-
1075 sition zone information. The authors are not aware of any BIM format that
1076 would allow for such information, at the moment.

1077 Of course, if the project data were handled correctly, there would be no need for
1078 transition zones as presented in Sections 4.3 and 5.3.2. That is, if the data were
1079 handled following one of the options shown in Figure 13 strictly (no negligence
1080 or misinterpretation). However, this is often not the case in practice.

1081 5. Georeferencing is very important for data conversions and data linking be-
1082 tween different data formats [e.g. 46]. If all the relevant information were
1083 saved in the same ECEF CRS, data exchanges and linking of data would func-
1084 tion without a problem, as all geometries would lie within the same global CS.
1085 As this is not the case, all data must include additional metadata about the un-
1086 derlying CRS. The linking approaches have to adapt, as well, as the geometries
1087 from different models do not necessarily lie in the same CRS. The integration
1088 of the various partial models in a project would work seamlessly, at least from
1089 a spatial point of view, if every object were provided in the same (ECEF) CRS.

1090 6. We call for fast implementation of CRS transformation abilities in software
1091 suites and their wide adoption by the industry. BIM-ready software solutions
1092 currently do not model information like changing scale factor and transition
1093 zones, to the best of our knowledge. These should offer transformation func-
1094 tionality to the user – there are already some free libraries available that would
1095 support this end [e.g. 18, 37, 41].

1096 *Acknowledgements*

1097 This research was funded by the *Bundesministerium für Verkehr und digitale*
1098 *Infrastruktur* (Federal Ministry of Transport and Digital Infrastructure, Ger-
1099 many), *Obermeyer Planen+Beraten GmbH* and *ProVI GmbH*. The authors
1100 gratefully acknowledge their support.

1101 *Conflict of Interest*

1102 The authors declare no potential conflicts of interest.

1103 **A. Mapping**

1104 This section describes the mathematics behind the geodetic projections and
1105 distortions described in Section 3.1.5. Only individual points can be transformed
1106 and thus any curved geometry needs to be discretised to a tessellated geometry
1107 before this process can be performed (see Section 3.2.2). Some variables used
1108 in the formulae have been defined in Table 2.

1109 *A.1. Mapping $(\Phi, \Lambda) \rightarrow (E, N)$*

1110 To convert from ellipsoidal coordinates (Φ, Λ) to geodetic coordinates (E, N) ,
1111 the following formulae can be used [44]. All distances are in kilometres and
1112 all angles are in radians. These transformations are valid for both GK and
1113 UTM projections, which are differentiated only in terms of their value of scale

1114 at meridian m_0 (see Section 3.1.5 and Equation (1)). First, some intermediate
 1115 values that depend on the chosen ellipsoid are calculated:

$$\begin{aligned}
 A &= \frac{a}{1+n} \sum_{i=0}^{\infty} \left(\frac{n}{2}\right)^{2i} , \\
 \alpha_1 &= \frac{1}{2}n - \frac{2}{3}n^2 + \frac{5}{16}n^3 , \\
 \alpha_2 &= \frac{13}{48}n^2 - \frac{3}{5}n^3 , \\
 \alpha_3 &= \frac{61}{240}n^3 .
 \end{aligned} \tag{A.1}$$

1116 Additional intermediate variables are defined for easier notation:

$$\begin{aligned}
 t(\Phi) &= \sinh \left(\tanh^{-1} \sin \Phi - \frac{2\sqrt{n}}{1+n} \tanh^{-1} \left(\frac{2\sqrt{n}}{1+n} \sin \Phi \right) \right) , \\
 \xi'(\Lambda) &= \tan^{-1} \left(\frac{t}{\cos(\Lambda - \Lambda_0)} \right) , \\
 \eta'(\Phi, \Lambda) &= \tanh^{-1} \left(\frac{\sin(\Lambda - \Lambda_0)}{\sqrt{1+t(\Phi)^2}} \right) , \\
 \sigma(\Phi, \Lambda) &= 1 + \sum_{j=1}^3 2j\alpha_j \cos(2j\xi'(\Lambda)) \cosh(2j\eta'(\Phi, \Lambda)) , \\
 \tau(\Phi, \Lambda) &= \sum_{j=1}^3 2j\alpha_j \sin(2j\xi'(\Lambda)) \sinh(2j\eta'(\Phi, \Lambda)) .
 \end{aligned} \tag{A.2}$$

1117 The final formulae for Easting, E , Northing, N and scale, m_{proj} , depending on
 1118 the chosen ellipsoid, map projection and ellipsoidal coordinates (Φ, Λ) , are:

$$E(\Phi, \Lambda) = E_0 + m_0 A \left(\eta'(\Phi, \Lambda) + \sum_{j=1}^3 \alpha_j \cos(2j\xi'(\Lambda)) \sinh(2j\eta'(\Phi, \Lambda)) \right) , \tag{A.3}$$

$$N(\Phi, \Lambda) = N_0 + m_0 A \left(\xi'(\Lambda) + \sum_{j=1}^3 \alpha_j \sin(2j\xi'(\Lambda)) \cosh(2j\eta'(\Phi, \Lambda)) \right) , \tag{A.4}$$

$$m_{\text{proj}}(\Phi, \Lambda) = \frac{m_0 A}{a} \sqrt{\left\{ 1 + \left(\frac{1-n}{1+n} \tan \Phi \right)^2 \right\} \frac{\sigma^2(\Phi, \Lambda) + \tau^2(\Phi, \Lambda)}{t^2(\Phi) + \cos^2(\Lambda - \Lambda_0)}} , \tag{A.5}$$

1119 where E_0 and N_0 denote the false Easting and false Northing, respectively, and
 1120 Λ_0 is the longitude of the meridian.

1121 *A.2. Inverse Mapping $(E, N) \rightarrow (\Phi, \Lambda)$*

1122 To convert from geodetic coordinates (E, N) to ellipsoidal coordinates (Φ, Λ) ,
 1123 the following formulae can be used [44]. Again, all distances are in kilometres
 1124 and all angles are in radians, and these equations are valid for both GK and
 1125 UTM projections. First, some intermediate values that depend on the chosen
 1126 ellipsoid are calculated (see also Equation (A.1)):

$$\begin{aligned}
 \beta_1 &= \frac{1}{2}n - \frac{2}{3}n^2 + \frac{37}{96}n^3, \\
 \beta_2 &= \frac{1}{48}n^2 + \frac{1}{15}n^3, \\
 \beta_3 &= \frac{17}{480}n^3, \\
 \delta_1 &= 2n - \frac{2}{3}n^2 - 2n^3, \\
 \delta_2 &= \frac{7}{3}n^2 - \frac{8}{5}n^3, \\
 \delta_3 &= \frac{56}{15}n^3.
 \end{aligned} \tag{A.6}$$

1127 Additional intermediate variables are defined for easier notation:

$$\begin{aligned}
 \lambda_0 &= \begin{cases} UTM : & \text{Zone} \times 6^\circ - 183^\circ \\ GK : & \text{Zone} \times 3^\circ \end{cases}, \\
 \xi(N) &= \frac{N - N_0}{m_0 A}, \\
 \eta(E) &= \frac{E - E_0}{m_0 A}, \\
 \xi'(E, N) &= \xi(N) - \sum_{j=1}^3 \beta_j \sin(2j\xi(N)) \cosh(2j\eta(E)), \\
 \eta'(E, N) &= \eta(E) - \sum_{j=1}^3 \beta_j \cos(2j\xi(N)) \sinh(2j\eta(E)), \\
 \sigma'(E, N) &= 1 - \sum_{j=1}^3 2j\beta_j \cos(2j\xi(N)) \cosh(2j\eta(E)), \\
 \tau'(E, N) &= \sum_{j=1}^3 2j\beta_j \sin(2j\xi(N)) \sinh(2j\eta(E)), \\
 \chi(E, N) &= \sin^{-1} \left(\frac{\sin \xi'(E, N)}{\cosh \eta'(E, N)} \right),
 \end{aligned} \tag{A.7}$$

1128 where *Zone* denotes the zone of the projection (e.g. UTM32 / UTM33 or GK
 1129 2 / 3 / 4, for Germany).

1130 The final formulae are:

$$\Phi(E, N) = \chi(E, N) + \sum_{j=1}^3 \delta_j \sin(2j\chi(E, N)) , \quad (\text{A.8})$$

$$\Lambda(E, N) = \Lambda_0 + \tan^{-1} \left(\frac{\sinh \eta'(E, N)}{\cos \xi'(E, N)} \right) , \quad (\text{A.9})$$

$$m_{\text{proj}}(E, N) = \frac{m_0 A}{a} \sqrt{\left\{ 1 + \left(\frac{1-n}{1+n} \tan \Phi(E, N) \right)^2 \right\} \frac{\cos^2 \xi'(E, N) + \sinh^2 \eta'(E, N)}{\sigma'^2(E, N) + \tau'^2(E, N)}} . \quad (\text{A.10})$$

1131 *A.3. Vertical Reduction of Horizontal Distances*

1132 Because geospatial data is mapped from an ellipsoidal surface to a flat surface,
 1133 it first needs to be referenced to it and not the geoid. The data is projected on
 1134 the ellipsoid first which induces additional dimensional distortions (Figure 9)
 1135 [24, 35]. The scale, m_h , used to reduce a horizontal distance at the terrain
 1136 elevation $h = H + U$ to the ellipsoid can be calculated as follows [20]:

$$R_M(\Phi) = \frac{a^2 b^2}{\sqrt{(a^2 \cos^2 \Phi + b^2 \sin^2 \Phi)^3}} , \quad (\text{A.11})$$

$$R_N(\Phi) = \frac{a^2}{\sqrt{a^2 \cos^2 \Phi + b^2 \sin^2 \Phi}} , \quad (\text{A.12})$$

$$R_G(\Phi) = \sqrt{R_M(\Phi) R_N(\Phi)} , \quad (\text{A.13})$$

$$m_h(\Phi, h) = 1 - \frac{h}{R_G(\Phi)} , \quad (\text{A.14})$$

1137 where $R_M(\Phi)$ and $R_N(\Phi)$ are the meridional (north-south) and prime vertical
 1138 (east-west) radii of curvature, respectively; and $R_G(\Phi)$ is the Gaussian radius
 1139 of curvature of the ellipsoid [20]. Points on the ellipsoid ($h = 0$) do not get
 1140 scaled. However, to get the necessary ellipsoidal heights, h , we must know the
 1141 undulations, U , and thus the geoid. The use of H instead of h in Equation (A.14)
 1142 leads to an incorrect scale factor, m_h (see also Figure 5).

1143 **References**

- 1144 [1] Abedallah, R., 2016. The best uniform quadratic approximation of circular
 1145 arcs with high accuracy. *Open Mathematics* 14, 118–127. doi:10.1515/
 1146 math-2016-0012.
- 1147 [2] Abualdenien, J., Borrman, A., 2019. A meta-model approach for formal
 1148 specification and consistent management of multi-LOD building models.
 1149 *Advanced Engineering Informatics* 40, 135 – 153. doi:10.1016/j.aei.
 1150 2019.04.003.
- 1151 [3] Barazzetti, L., Banfi, F., 2017. BIM and GIS: When parametric
 1152 modeling meets geospatial data. *ISPRS Annals of Photogrammetry,
 1153 Remote Sensing & Spatial Information Sciences* 4. URL:

- 1154 [https://www.isprs-ann-photogramm-remote-sens-spatial-inf-sci.](https://www.isprs-ann-photogramm-remote-sens-spatial-inf-sci.net/IV-5-W1/1/2017/)
1155 [net/IV-5-W1/1/2017/](https://www.isprs-ann-photogramm-remote-sens-spatial-inf-sci.net/IV-5-W1/1/2017/). accessed: 2020-03-29.
- 1156 [4] Borrmann, A., Amann, J., Chipman, T., Hyvärinen, J., Liebich, T., Muhič,
1157 S., Mol, L., Plume, J., Scarponcini, P., 2017a. IFC Infra Overall Ar-
1158 chitecture Project: Documentation and Guidelines. Technical Report.
1159 buildingSMART International. URL: [https://www.buildingsmart.org/
1160 standards/bsi-standards/standards-library/](https://www.buildingsmart.org/standards/bsi-standards/standards-library/). accessed: 2020-03-29.
- 1161 [5] Borrmann, A., König, M., Koch, C., Beetz, J., 2017b. Building Information
1162 Modeling – Technology Foundations and Industry Practise. Springer Inter-
1163 national. doi:10.1007/978-3-319-92862-3. ISBN: 978-3-319-92861-6.
- 1164 [6] Braden, B., 1986. The surveyor’s area formula. The College Mathematics
1165 Journal 17, 326–337. doi:10.2307/2686282.
- 1166 [7] Bradley, A., Li, H., Lark, R., Dunn, S., 2016. BIM for infrastructure: An
1167 overall review and constructor perspective. Automation in Construction
1168 71, 139 – 152. doi:10.1016/j.autcon.2016.08.019.
- 1169 [8] Brenner, J., Mitchell, A., Maier, F., Yockey, M., Pulikanti,
1170 S., 2018. Development of 3D and 4D Bridge Models and
1171 Plans. URL: [https://www.michigan.gov/mdot/0,
1172 4616,7-151-9622_11045_24249-485483--,00.html](https://www.michigan.gov/mdot/0,4616,7-151-9622_11045_24249-485483--,00.html). accessed: 2020-03-29.
- 1173 [9] buildingSMART International, 2019. Article about IFC
1174 coordinate reference systems, and Revit [forum discus-
1175 sion]. URL: [https://forums.buildingsmart.org/t/
1176 article-about-ifc-coordinate-reference-systems-and-revit/](https://forums.buildingsmart.org/t/article-about-ifc-coordinate-reference-systems-and-revit/).
1177 accessed: 2020-03-29.
- 1178 [10] buildingSMART International, 2020. User Guide for Geo-referencing
1179 in IFC. URL: [https://www.buildingsmart.org/standards/
1180 bsi-standards/standards-library/](https://www.buildingsmart.org/standards/bsi-standards/standards-library/). accessed: 2020-03-29.
- 1181 [11] Clemen, C., Görne, H., 2019. Level of Georeferencing (LoGeoRef) using
1182 IFC for BIM. Journal of Geodesy, Cartography and Cadastre 2019, 15–
1183 20. URL: [https://jgcc.geoprevi.ro/docs/2019/10/jgcc_2019_no10_
1184 3.pdf](https://jgcc.geoprevi.ro/docs/2019/10/jgcc_2019_no10_3.pdf). accessed: 2020-03-29.
- 1185 [12] Conchúir, D.O., 2016. Great miscalculations: the importance of validating
1186 project management assumptions. URL: [http://www.engineersjournal.
1187 ie/2016/06/28/gmiscalculation-laufenburg-bridge/](http://www.engineersjournal.ie/2016/06/28/gmiscalculation-laufenburg-bridge/). accessed: 2019-
1188 09-08.
- 1189 [13] Costin, A., Adibfar, A., Hu, H., Chen, S.S., 2018. Building Information
1190 Modeling (BIM) for transportation infrastructure – Literature review, ap-
1191 plications, challenges, and recommendations. Automation in Construction
1192 94, 257 – 281. doi:10.1016/j.autcon.2018.07.001.
- 1193 [14] Derbyshire, Sam, 2012. OblateSpheroid.PNG. URL: [https://commons.
1194 wikimedia.org/w/index.php?curid=209084](https://commons.wikimedia.org/w/index.php?curid=209084). accessed: 2020-03-29.

- 1195 [15] Donaubauer, A., Kolbe, T.H., (Eds.), 2017. Leitfaden Bezugssystemwechsel
1196 auf ETRS89/UTM: Grundlagen, Erfahrungen und Empfehlungen [Guide-
1197 lines for change of reference system to ETRS89/UTM: Basics, Expe-
1198 riences and Recommendations]. URL: [https://rundertischgis.de/
1199 publikationen/leitfaeden.html](https://rundertischgis.de/publikationen/leitfaeden.html). accessed: 2020-03-29, [in German].
- 1200 [16] Eastman, C., Teicholz, P., Sacks, R., Liston, K., 2008. BIM Handbook:
1201 A Guide to Building Information Modeling for Owners, Managers, De-
1202 signers, Engineers and Contractors. Wiley Publishing. doi:10.1002/
1203 9780470261309. ISBN: 978-0-470-18528-5.
- 1204 [17] EPSG, 2020. European Petroleum Survey Group. URL: <http://epsg.io/>.
1205 accessed: 2020-03-29.
- 1206 [18] Evers, K., Knudsen, T., 2017. Transformation pipelines for PROJ.4, in:
1207 Technical Programme and Proceedings of the FIG Working Week 2017,
1208 Helsinki, Finland. ISBN: 978-87-92853-61-5, contribution 9156.
- 1209 [19] GeoBremen, 2015. UTM Abbildung – Informationen und Hand-
1210 lungsempfehlungen [UTM Projection – Information and Recommended
1211 Actions]. Geoinformation Bremen Landesamt, Hansestadt Bre-
1212 men. URL: [http://www.geo.bremen.de/sixcms/media.php/13/UTM_
1213 Abbildung_Info_Handlungsempfehlung_150611.pdf](http://www.geo.bremen.de/sixcms/media.php/13/UTM_Abbildung_Info_Handlungsempfehlung_150611.pdf). accessed: 2019-05-
1214 31, [in German].
- 1215 [20] Heunecke, O., 2017. Planung und Umsetzung von Bauvorhaben mit
1216 amtlichen Lage- und Höhenkoordinaten [Design and Realization of Con-
1217 struction Works with Official Plane and Vertical Coordinates]. zfv 142,
1218 180–187. doi:10.12902/zfv-0160-2017. [in German].
- 1219 [21] Ince, E.S., Barthelmes, F., Reißland, S., Elger, K., Förste, C., Flechtner, F.,
1220 Schuh, H., 2019. ICGEM – 15 years of successful collection and distribution
1221 of global gravitational models, associated services, and future plans. doi:10.
1222 5194/essd-11-647-2019.
- 1223 [22] ISO 10303-42:2019, 2019. Industrial automation systems and integra-
1224 tion – Product data representation and exchange – Part 42: Integrated
1225 generic resource: Geometric and topological representation. Standard. In-
1226 ternational Organization for Standardization. Geneva, CH. URL: [https:
1227 //www.iso.org/standard/78579.html](https://www.iso.org/standard/78579.html). accessed: 2020-03-29.
- 1228 [23] ISO 16739-1:2018, 2018. Industry Foundation Classes (IFC) for data shar-
1229 ing in the construction and facility management industries – Part 1: Data
1230 schema. Standard. International Organization for Standardization. Geneva,
1231 CH. URL: <https://www.iso.org/standard/70303.html>. accessed: 2020-
1232 03-29.
- 1233 [24] ISO 19111:2019, 2019. Geographic information – Referencing by coordi-
1234 nates. Standard. International Organization for Standardization. Geneva,
1235 CH. URL: <https://www.iso.org/standard/74039.html>. accessed: 2020-
1236 03-29.

- 1237 [25] ISO 19162:2019, 2019. Geographic information – Well-known text repre-
1238 sentation of coordinate reference systems. Standard. International Orga-
1239 nization for Standardization. Geneva, CH. URL: [https://www.iso.org/
1240 standard/76496.html](https://www.iso.org/standard/76496.html). accessed: 2020-03-29.
- 1241 [26] Jaud, Š., Donaubaueer, A., Borrmann, A., 2019. Georeferencing within
1242 IFC: A Novel Approach for Infrastructure Objects, in: Cho, Y.K., Leite,
1243 F., Behzadan, A., Wang, C. (Eds.), Computing in Civil Engineering 2019:
1244 Visualisation, Information Modelling, and Simulation. American Society
1245 of Civil Engineers, Atlanta, Georgia, USA, pp. 377–384. doi:10.1061/
1246 9780784482421.
- 1247 [27] Kaden, R., Clemen, C., 2017. Applying Geodetic Coordinate Reference
1248 Systems within Building Information Modeling (BIM), in: Technical Pro-
1249 gramme and Proceedings of the FIG Working Week 2017, Helsinki, Finland.
1250 ISBN: 978-87-92853-61-5, contribution 8967.
- 1251 [28] Kaden, R., Clemen, C., Seuß, R., Blankenbach, J., Becker, R., Eichhorn, A.,
1252 Donaubaueer, A., Kolbe, T.H., Gruber, U., (Eds.), 2017. Leitfaden Geodäsie
1253 und BIM [Guidelines Geodesy and BIM]. URL: [https://rundertischgis.
1254 de/publikationen/leitfaeden.html](https://rundertischgis.de/publikationen/leitfaeden.html). accessed: 2020-03-29, [in German].
- 1255 [29] Karney, C.F.F., 2013. Algorithms for geodesics. *Journal of Geodesy* 87,
1256 43–55. doi:10.1007/s00190-012-0578-z.
- 1257 [30] Karney, C.F.F., 2015. Online geodesic calculations using the Geod-
1258 Solve utility. URL: [https://geographiclib.sourceforge.io/cgi-bin/
1259 GeodSolve](https://geographiclib.sourceforge.io/cgi-bin/GeodSolve). accessed: 2020-03-29.
- 1260 [31] Li, X., Shen, G.Q., Wu, P., Yue, T., 2019. Integrating building information
1261 modeling and prefabrication housing production. *Automation in Construc-
1262 tion* 100, 46 – 60. doi:10.1016/j.autcon.2018.12.024.
- 1263 [32] Liebich, T., Amann, J., Borrmann, A., Chipman, T., Hyvärinen, J.,
1264 Muhič, S., Mol, L., Plume, J., Scarponcini, P., 2017. IFC Align-
1265 ment 1.1 Project: IFC Schema Extension Proposal. Report. build-
1266 ingSMART International. URL: [http://www.buildingsmart-tech.
1267 org/downloads/ifc/ifc5-extension-projects/ifc-alignment/
1268 ifcalignment-conceptualmodel-cs](http://www.buildingsmart-tech.org/downloads/ifc/ifc5-extension-projects/ifc-alignment/ifcalignment-conceptualmodel-cs). accessed: 2018-09-29.
- 1269 [33] Liu, X., Wang, X., Wright, G., Cheng, J.C.P., Li, X., Liu, R., 2017. A State-
1270 of-the-Art Review on the Integration of Building Information Modeling
1271 (BIM) and Geographic Information System (GIS). *ISPRS International
1272 Journal of Geo-Information* 6. doi:10.3390/ijgi6020053.
- 1273 [34] Markič, Š., Donaubaueer, A., Borrmann, A., 2018. Enabling Geodetic Co-
1274 ordinate Reference Systems in Building Information Modelling for Infras-
1275 tructure, in: Proceedings of the 17th International Conference on Comput-
1276 ing in Civil and Building Engineering, Tampere, Finland. URL: [http:
1277 //programme.exordo.com/icccb2018/delegates/presentation/373/](http://programme.exordo.com/icccb2018/delegates/presentation/373/).
- 1278 [35] Markič, Š., Windischer, G., Glatzl, R.W., Hofmann, M., Borrmann, A.,
1279 Bergmeister, K., 2019. Requirements for geo-locating transnational in-
1280 frastructure BIM models, in: Peila, D., Viggiani, G., Celestino, T. (Eds.),

- 1281 Tunnels and Underground Cities. Engineering and Innovation Meet Archae-
 1282 ology, Architecture and Art. Proceedings of the WTC 2019 ITA-AITES
 1283 World Tunnel Congress (WTC 2019). 1 ed.. CRC Press, pp. 972–981.
 1284 doi:10.1201/9780429424441.
- 1285 [36] Nagel, C., Stadler, A., Kolbe, T.H., 2009. Conceptual Requirements
 1286 for the Automatic Reconstruction of Building Information Models from
 1287 Uninterpreted 3D Models, in: Kolbe, T.H., Zhang, R., Zlatanova, S.
 1288 (Eds.), Proceedings of the Academic Track of the Geoweb 2009 - 3D
 1289 Cityscapes Conference in Vancouver, Canada, 27-31 July 2009, ISPRS. pp.
 1290 46–53. URL: [http://www.isprs.org/proceedings/xxxviii/3_4-c3/
 1291 paper_geow09/paper26_nagel_stadler_kolbe.pdf](http://www.isprs.org/proceedings/xxxviii/3_4-c3/paper_geow09/paper26_nagel_stadler_kolbe.pdf). accessed: 2020-03-
 1292 29.
- 1293 [37] NGA, 2019. Mensuration Services Program (MSP) Geographic Transla-
 1294 tor (GeoTrans). National Geospatial-Intelligence Agency. URL: [http://
 1295 //earth-info.nga.mil/GandG/update/index.php](http://earth-info.nga.mil/GandG/update/index.php). accessed: 2020-03-29.
- 1296 [38] Nürnberg, R., 2013. Calculating the volume and centroid of a polyhedron
 1297 in 3d URL: <http://www2.imperial.ac.uk/~rn/centroid.pdf>. accessed:
 1298 2020-03-29.
- 1299 [39] Ogori, K.A., Diakité, A., Krijnen, T., Ledoux, H., Stoter, J., 2018. Process-
 1300 ing BIM and GIS Models in Practice: Experiences and Recommendations
 1301 from a GeoBIM Project in The Netherlands. ISPRS International Journal of
 1302 Geo-Information 7, 311. doi:10.3390/ijgi7080311.
- 1303 [40] Olsson, P.O., Axelsson, J., Hooper, M., Harrie, L., 2018. Automation of
 1304 Building Permission by Integration of BIM and Geospatial Data. ISPRS In-
 1305 ternational Journal of Geo-Information 7, 307. doi:10.3390/ijgi7080307.
- 1306 [41] PROJ contributors, 2019. PROJ coordinate transformation software li-
 1307 brary. Open Source Geospatial Foundation. URL: <https://proj.org/>.
 1308 accessed: 2020-03-29.
- 1309 [42] Sani, M., Abdul Rahman, A., 2018. GIS AND BIM INTEGRATION
 1310 AT DATA LEVEL: A REVIEW. ISPRS - International Archives of the
 1311 Photogrammetry, Remote Sensing and Spatial Information Sciences XLII-
 1312 4/W9, 299–306. doi:10.5194/isprs-archives-XLII-4-W9-299-2018.
- 1313 [43] Saygi, G., Agugiaro, G., Turan, M., Remondino, F., 2013. Evaluation of
 1314 GIS and BIM roles for the information management of historical buildings,
 1315 pp. 283–288. doi:10.5194/isprsannals-II-5-W1-283-2013.
- 1316 [44] Snyder, J.P., 1987. Map Projections: A Working Manual. Technical Report.
 1317 Washington, D.C. URL: <https://pubs.usgs.gov/pp/1395/report.pdf>,
 1318 doi:10.3133/pp1395. accessed: 2020-03-29.
- 1319 [45] StackExchange, 2012. How accurate is approximating the Earth as a
 1320 sphere? URL: <https://gis.stackexchange.com/questions/25494>. ac-
 1321 cessed: 2020-03-29.

- 1322 [46] Stouffs, R., Tauscher, H., Biljecki, F., 2018. Achieving Complete and Near-
1323 Lossless Conversion from IFC to CityGML. *ISPRS International Journal*
1324 *of Geo-Information* 7, 355. doi:10.3390/ijgi7090355.
- 1325 [47] Torge, W., 1991. *Geodesy*. Walter de Gruyter & Co. ISBN: 3-11-012408-4.
- 1326 [48] Uggla, G., Horemuz, M., 2018a. Geographic capabilities and limitations of
1327 Industry Foundation Classes. *Automation in Construction* 96, 554 – 566.
1328 doi:https://doi.org/10.1016/j.autcon.2018.10.014.
- 1329 [49] Uggla, G., Horemuz, M., 2018b. Georeferencing Methods for IFC, in: *Pro-*
1330 *ceedings of the 2018 Baltic Geodetic Congress, BGC–Geomatics 2018*, In-
1331 *stitute of Electrical and Electronics Engineers Inc.*. pp. 207–211. doi:10.
1332 1109/BGC-Geomatics.2018.00045.
- 1333 [50] Wunderlich, T., Blankenbach, J., 2017. Building Information Model-
1334 ing (BIM) und Absteckung [Building Information Modeling (BIM) and
1335 Setting-out], in: *Ingenieurvermessung 2017*, TU Graz. URL: [http://](http://mediatum.ub.tum.de/doc/1435852/1435852.pdf)
1336 mediatum.ub.tum.de/doc/1435852/1435852.pdf. accessed: 2020-03-29,
1337 [in German].
- 1338 [51] Yin, X., Liu, H., Chen, Y., Al-Hussein, M., 2019. Building information
1339 modelling for off-site construction: Review and future directions. *Automa-*
1340 *tion in Construction* 101, 72 – 91. doi:10.1016/j.autcon.2019.01.010.