3D Coupled Simulations of Dynamic Rupture, Elastic, Acoustic and Tsunami Wave Propagation

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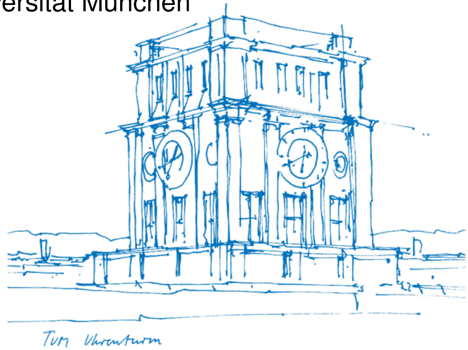
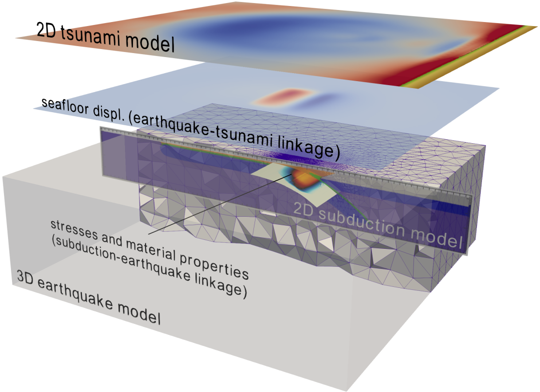
ChEESE
Center of Excellence for Exascale in Solid Earth

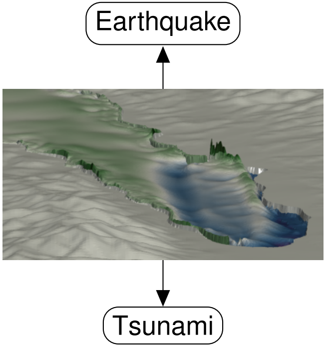


Earthquake-Tsunami Coupling Workflows

One-way Linking[1]



Fully Coupled



[1] E. H. Madden et al. "Linked 3-D modelling of megathrust earthquake-tsunami events: from subduction to tsunami run up". In: *Geophysical Journal International* 224.1 (2021)

One-way linking vs 3D coupling

Using shallow water equations for tsunami has disadvantages:

- No dispersion (if not using Boussinesq approximation)
- No acoustic waves (i.e. assuming incompressible ocean)
- No vertical momentum transfer
- Only works in shallow water limit

Fully-coupled elastic-acoustic model solves entirely new class of earthquake-tsunami problem
Compares well with one-way linking given simplifying assumptions
Seismic and acoustic waves can be dominant in data recorded by offshore instruments.

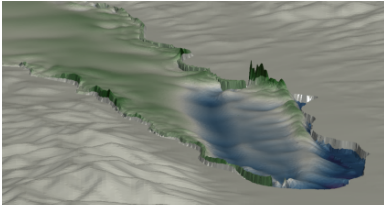
Detailed model comparison work in progress[2]

[2] L. S. Abrahams et al. "Comparison of techniques for coupled earthquake and tsunami modeling". In: *AGU Fall Meeting Abstracts* (2020).

SeisSol

What

- (An)**Isotropic elastic** seismic wave propagation
- **Acoustic** wave propagation
- Viscoelastic wave propagation
- **Off-fault plasticity**
- **Dynamic earthquake rupture**



How

- Numerics: ADER-DG
- Unstructured tetrahedral meshes with local time-stepping
- Optimized Hybrid MPI + OpenMP Parallelization

Available (**open-source**) at `https://github.com/SeisSol/SeisSol/`.

The ADER-DG Approach[3]

Solve linear hyperbolic equations of the form

$$\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{A}\frac{\partial \boldsymbol{q}}{\partial x} + \boldsymbol{B}\frac{\partial \boldsymbol{q}}{\partial y} + \boldsymbol{C}\frac{\partial \boldsymbol{q}}{\partial z} = 0 \tag{1}$$

with $\boldsymbol{q}$ vector of variables, $\boldsymbol{x} = (x, y, z)$ position, t time, $\boldsymbol{A}(\boldsymbol{x})$, $\boldsymbol{B}(\boldsymbol{x})$, $\boldsymbol{C}(\boldsymbol{x})$ flux matrices.

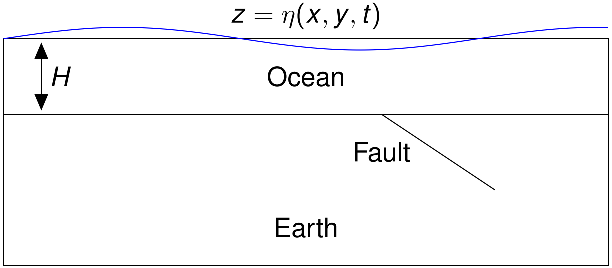
Discontinuous Galerkin (DG) divides domain into disjoint elements, approximates solutions by **piecewise-polynomials**.
Elements are connected by solving the **Riemann** problem.

ADER-Approach uses **element-local Taylor expansion** for time integration instead of Runge-Kutta procedures.
Advantages: **One-step scheme**, **arbitrary order in time and space**

[3] V. A. Titarev and E. F. Toro. "ADER: Arbitrary High Order Godunov Approach". In: *Journal of Scientific Computing* 17.1 (Dec. 2002).

Two-Way Elastic-Acoustic Coupling

2D sketch of our model



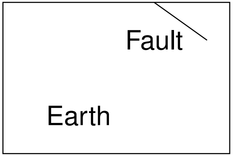
Based on 2D model of Lotto and Dunham[4]
We present first 3D implementation of this model!

With:

- $\eta(x, y, t)$ sea surface height
- H height of the ocean
- Figures in 2D for illustration, all simulations are **3D**

[4] G. C. Lotto and E. M. Dunham. “High-order finite difference modeling of tsunami generation in a compressible ocean from offshore earthquakes”. In: *Computational Geosciences* 19.2 (2015)

Earth Model



$$\frac{\partial \sigma_{ij}}{\partial t} - \lambda \delta_{ij} u_k \frac{\partial}{\partial x_k} - \mu \left(\frac{\partial}{\partial x_j} u_i + \frac{\partial}{\partial x_i} u_j \right) = 0, \qquad (2)$$

$$\rho \frac{\partial}{\partial t} u_i - \frac{\partial}{\partial x_j} \sigma_{ij} = 0$$

δ_{ij} Kronecker delta, summation implied

- Isotropic elastic medium
- Velocity-stress formulation
- u, v, w velocities
- $\boldsymbol{\sigma}$ stress tensor
- ρ density, (μ, λ) Lame parameters
- Dynamic rupture earthquake source (here: non-linear rate & state friction)

Ocean Model[5]

Modeled as **linear acoustic** medium, $\boldsymbol{q} = (u, v, w, p)$
Treated as special case of elastic wave equation with $\mu = 0$.
Pressure p sum of background pressure p_0 (in **hydrostatic equilibrium**) and perturbation p'.

$$\begin{aligned} p &= p_0 + p'(x, y, z) \\ p_0 &= p_a + \rho g(-z) \end{aligned} \tag{3}$$

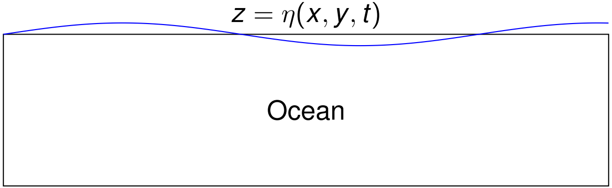
with atmospheric pressure p_a and $g = 9.81\,\mathrm{m/s^2}$.
Pressure at some point is:

$$p(x, y, z, t) = p_a + \rho g(-z) + p'(x, y, z, t) - \rho g u_z(x, y, z, t) \tag{4}$$

with z-displacements u_z.

[5] G. C. Lotto and E. M. Dunham. “High-order finite difference modeling of tsunami generation in a compressible ocean from offshore earthquakes”. In: *Computational Geosciences* 19.2 (2015).

Ocean Model: Free Surface



Physical free surface boundary condition at sea surface height η:

$$p(x, y, \eta) = 0 \tag{5}$$

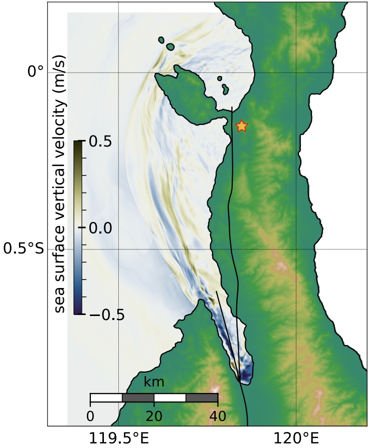
Typically solved by moving mesh.
Expensive, instead use **linearization** and **hydrostatic background pressure**:

$$\begin{aligned} p(x, y, z = 0) &= -\rho g \eta(x, y) \\ \frac{\partial \eta}{\partial t} &= u \end{aligned} \tag{6}$$

Important to use u at boundary (solution of Riemann problem), otherwise unstable!

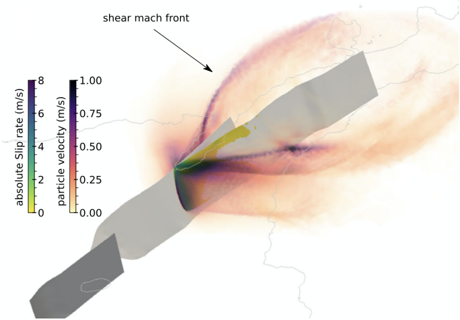
Example: Palu, Sulawesi September 2018

- $M_w 7.5$ strike-slip earthquake
- Propagation at supershear speed crossing narrow Palu bay
- Followed by unexpected and localized tsunami
- Complicated geometry (bath-tub like bay, very shallow water)



Our setup

- Added water layer to existing earthquake model[6].
- Fully coupled model (including plasticity, dynamic earthquake rupture)[7]
- Two meshes: **M** (89 million elements), **L** (518 million elements)
- Poly. Order 5, 46 and 261 billion degrees of freedom
- **M** took 5.3 hours on 1000 nodes of SuperMUC-NG for 100s simulated time
- **L** took 5.5 hours on 3072 nodes of SuperMUC-NG for 30s simulated time



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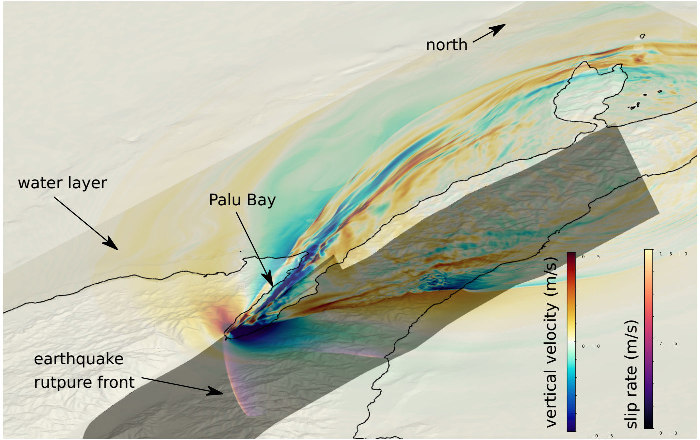
[7] T. Ulrich et al. "Coupled, Physics-based Modelling Reveals Earthquake Displacements are Critical to the 2018 Palu, Sulawesi Tsunami". In: *Pure and Applied Geophysics* (2019)

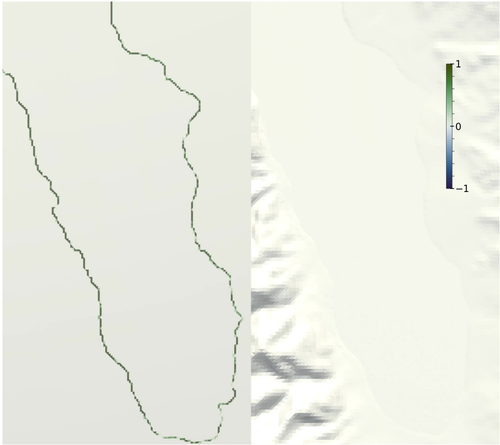
[7] L. Krenz et al. "3D Acoustic-Elastic Coupling with Gravity: The Dynamics of the 2018 Palu, Sulawesi Earthquake and Tsunami". In: *Proceedings of the international conference for high performance computing, networking, storage and analysis*. accepted. 2021

Palu: 3D View at 15s



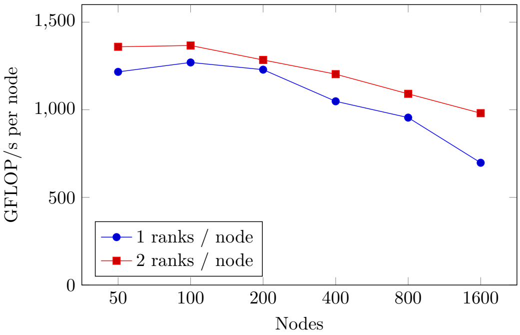
Particle velocity (slip rate) across faults, vertical sea-surface/Earth velocity at 15 s

Comparison with One-Way Linking

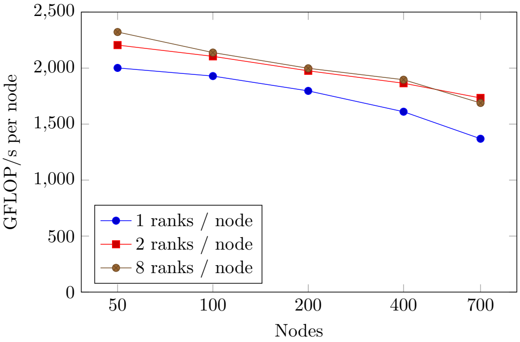


Left: One-way linking, **Right: fully-coupled**

Strong Scaling



On SuperMUC-NG (Intel Skylake)



On Mahti (AMD Rome)

Strong scaling of our production scenario (89 million elements).
Parallel efficiency of 72% percent

Coupling Strategies

Combining strengths of 3D coupled and one-way linking
3D coupled is **powerful yet expensive**
One-way linking is **cheap**, supports inundation and (with **simplifying assumptions**) compare quite well to fully coupled model

Idea: Run 3D coupled model to capture acoustic waves. Switch over to cheap shallow water solver
How to initialize Shallow Water Solver?

1. Use sea floor displacement from 3D fully coupled as initial condition/forcing term for SWE solver
2. Average velocity from 3D wavefield, use together with sea surface height as initial condition

We currently do 1) but are working on 2)

Conclusion

- Fully coupled elastic-acoustic simulations capture more effects than typical two-step strategies
- Linearization of free surface boundary conditions efficient way of tracking sea surface height
- Results for Palu scenario are very promising
- Further work on numerics, performance and scenarios

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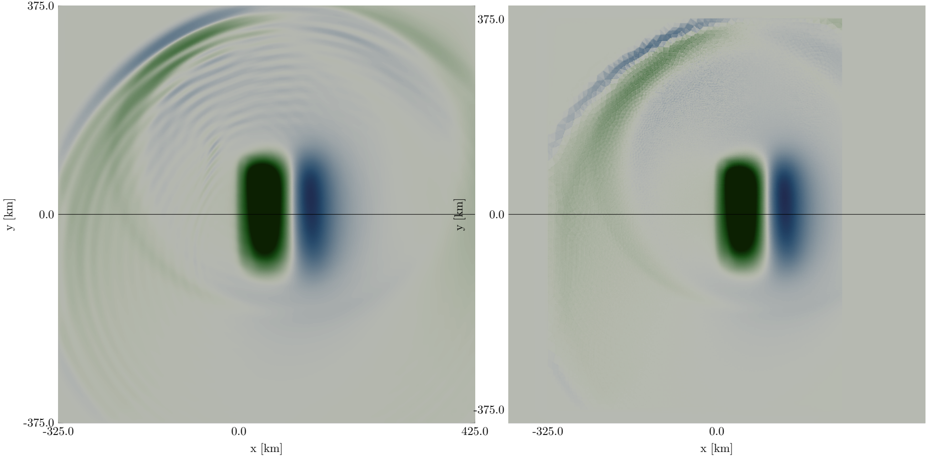
Tsunami Benchmark Scenario

- Added water layer to existing setup[8]
- Relatively simple scenario: no topography/bathymetry
- Linear slip-weakening friction law
- Compared standard one-way linking with fully-coupled model[9]

[8] E. H. Madden et al. "Linked 3-D modelling of megathrust earthquake-tsunami events: from subduction to tsunami run up". In: *Geophysical Journal International* 224.1 (2021).

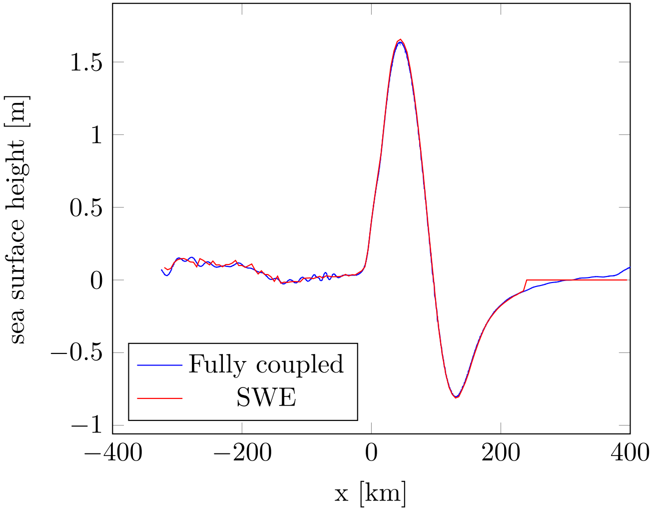
[9] L. Krenz et al. "3D Acoustic-Elastic Coupling with Gravity: The Dynamics of the 2018 Palu, Sulawesi Earthquake and Tsunami". In: *Proceedings of the international conference for high performance computing, networking, storage and analysis*. accepted. 2021.

Comparison Free Surface After 120s

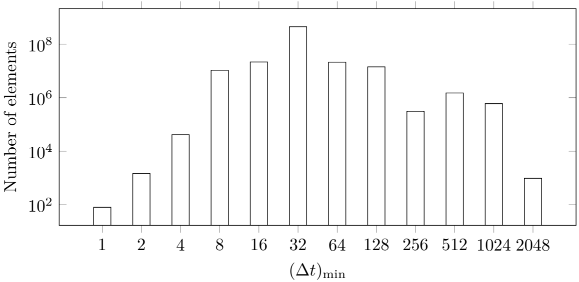


Left: Fully-coupled, right: One-way linking

Comparison Slice After 120s



Effect of Local Time-Stepping



Effect of local time-stepping on our largest (**L**) mesh with 518 million elements
More than 86% of all elements fall within the cluster with timestep $32(\Delta t)_{\mathrm{min}}$.
Local time-stepping hence crucial for time-to-solution!