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Omni-channel Assortment Planning and Store Operations

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List of Abbreviations

<i>Avg</i>	Average
<i>BIP</i>	Binary-Integer Program
<i>BOPS</i>	Buy online, pick-up in store
<i>DAE</i>	Digital assortment extension
<i>DC</i>	Distribution center
<i>DIY</i>	Do it yourself
<i>ED</i>	Exogenous demand
<i>FAS</i>	Fashion
<i>JO CIAO</i>	Joint Omni-Channel Inventory and Assortment Optimization
<i>Max</i>	Maximum
<i>MC</i>	Multi-channel
<i>Min</i>	Minimum

<i>MIP</i>	Mixed-Integer Program
<i>MNL</i>	Multinomial logit
<i>NFC</i>	Near-field communication
<i>OC</i>	Omni-channel
<i>OC – ASPI</i>	Omni-Channel Assortment, Space, Position and Inventory Optimization
<i>OOA</i>	Out of Assortment
<i>OOS</i>	Out of Stock
<i>OPT</i>	Optical
<i>OR</i>	Operations Research
<i>ROPS</i>	Reserve online, pick-up and pay in store
<i>RQ</i>	Research question
<i>SC</i>	Single-channel
<i>SFS</i>	Ship from store
<i>VAR</i>	Various
<i>VRP</i>	Vehicle routing problem

1 Introduction

This thesis addresses new challenges in omni-channel (OC) retailing, a business model that describes the integration of e-commerce and bricks-and-mortar stores. Novel OC concepts emerge that particularly impact assortment planning and store operations. In this dissertation, these areas of research are addressed as assortments are optimized across channels (i.e., bricks-and-mortar stores and webshops), demand effects are analytically examined, and challenges, solution approaches, and imperative research fields for store operations are laid out.

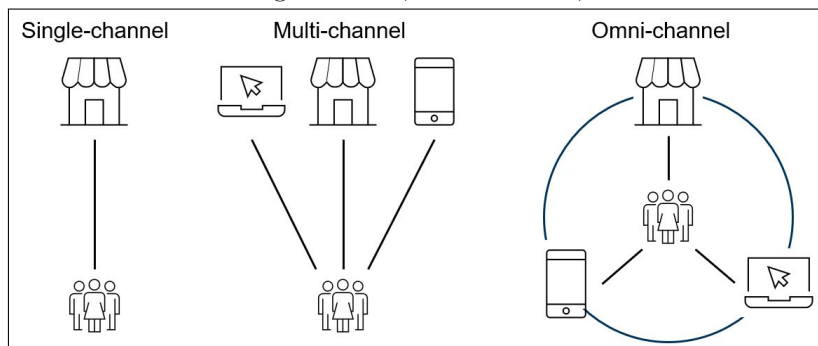
In this first chapter the notion and importance of OC retailing and the underlying OC delivery concepts (Section 1.1) are outlined. Next, it presents substantial planning questions that are impacted by OC retailing (Section 1.2). It first introduces the assortment planning problem and then describes challenges, decision support systems, and research opportunities for store operations.

The remainder is organized as follows. Chapter 2 details the scope, main contributions, authors, and status of publication of the three articles that compose the main body of the dissertation. Chapter 3 to Chapter 5 each contain one of the three articles. Lastly, Chapter 6 summarizes the findings and outlines areas of future research.

1.1 Omni-channel retail

Definition and relevance The concept of OC is revolutionizing the retail landscape and corporate planning. Many traditional retailers with bricks-and-mortar store concepts invest heavily in online sales channel while online pure players start building their offline presence through physical stores (e.g., Wollenburg et al. (2018b); Caro et al. (2020)). Hence, single-channel (SC) and multi-channel (MC) retailers that used to operate a single or multiple isolated sales channel are now establishing OC presences (Rooderkerk and Kök, 2019). OC has been defined as the full integration of a retailer's channels, operations, information, and assortments to provide a seamless shopping experience for customers (Brynjolfsson et al., 2009; Verhoef et al., 2015). Figure 1.1 illustrates the different retailing concepts. From both a customer and a retailer perspective, processes and channels are converging, making OC a fundamental part of retailing. Customers these days expect effortless switching between channels and the vast majority of shoppers have already used various channels in their shopping journey (e.g., Sopadjieva et al. (2017); iVend Retail (2019)). Sopadjieva et al. (2017) further underline the relevance of this concept for retailers as they find out that OC shoppers are more loyal, have bigger shopping baskets, have more repeat shopping trips, and are more likely to endorse OC retailers.

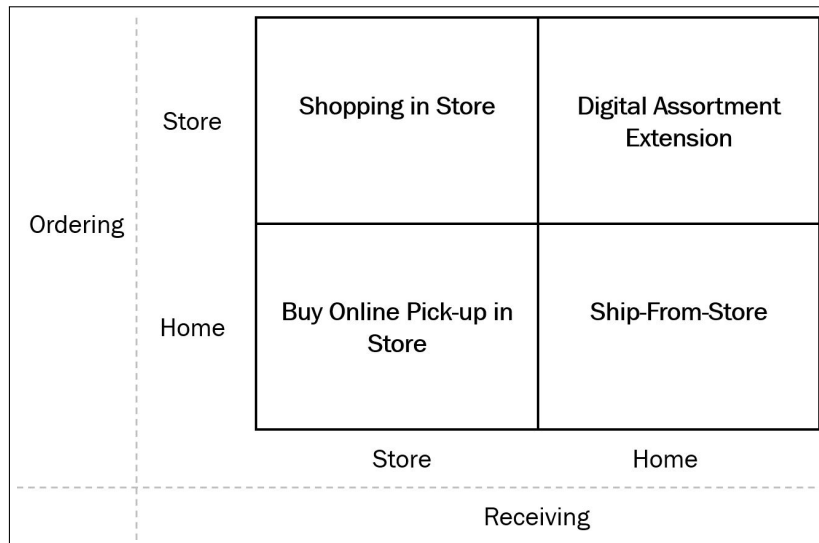
Figure 1.1: Illustration of single-channel, multi-channel, and omni-channel retailing



OC delivery concepts Integrated operations and connected assortments and inventories across channels enable new OC delivery concepts such as buy online, pick-up in store (BOPS, also called click & collect), ship from store (SFS), digital assortment extensions (DAE), or showrooms (Gallino and Moreno, 2014; Hübner et al., 2016b; Bell et al., 2018a). BOPS describes the process, where customers can view the availability of in-store products, buy the products online, and subsequently pick them up in the stores. Using SFS, retailers fulfill online orders through store inventory instead of stock from the distribution center (DC). DAE is used to offer store customers access to the online assortment (e.g., additional products or varying colors) through digital point of sales like tablets. BOPS, SFS, and DAE are depicted in Figure 1.2. Lastly, showrooms represent a special case of stores without inventory. Products are only showcased but can be purchased in the store with home delivery or pick-up in store. As an example for OC delivery concepts, the German supermarket chain REWE offers BOPS and SFS through an online cookbook. Ingredients can be ordered right on the website for home delivery or pick-up in store. Outdoor retailer Timberland applies an experimental design for DAE using touch walls and tablets in its stores. The touch walls are used to display online-only inventory and allows the customer to create shopping lists via a user profile. Tablets show further information on products that are equipped with near-field communication (NFC) technology as well as personalized product recommendations. In both cases, the retailer is adding value for the customer through convenience and time savings while collecting valuable information.

1.2 Important planning problems in omni-channel retailing

In order to satisfy customers and exploit the potential of OC retailing, integrated and coordinated channels are required that give birth to novel planning issues (e.g. Verhoef et al. (2015); Hübner et al. (2016b)). Cus-

Figure 1.2: Illustration of omni-channel delivery concepts

tomers are not bound to one channel within their customer journey anymore and raise the bar for seamless OC shopping experiences and integrated assortments across channels (Verhoef et al., 2015).

1.2.1 Omni-channel assortment planning

It becomes increasingly crucial to offer the right products, in the right quantity, in the right channel, and at the right time to the end-consumers. This requires the integration of relevant customer behavior within and across channels. For example, customer demand for certain products can be increased by a favorable, prominent positioning of those products in store shelves (e.g., Chandon et al. (2009)) or web pages (e.g., Atalay et al. (2012)). Also, if the product of choice is unavailable, customers may chose to substitute for another product within the same channel or across channels (Kök et al., 2015; Dzyabura and Jagabathula, 2018). Chapter 3 and

Chapter 4 primarily cover this field of research called assortment planning and the following research questions:

- i) How can we find the optimal assortment and inventories across channels to meet customer demand?
- ii) What is the potential of OC delivery concepts?
- iii) What are relevant demand effects in OC assortment planning?
- iv) What is the risk of ignoring relevant OC customer behavior when planning OC assortments?

To answer the questions, a model for OC assortment optimization is developed and solved through a specialized heuristic, which aims to find the most profitable assortment, inventory, and positioning of products on store shelves and web pages. The solutions are evaluated and practical insights on the potential, risks, solution structures, and demand effects are derived.

Mathematical optimization To take such business decisions in an efficient manner, analytical approaches from the field of Operations Research (OR) are utilized. Besides assortment planning, applications can include deciding on the most appropriate locations for new retail stores, assigning Uber drivers to customers, or bidding on space on Google Ads. Such decisions are dictated by an objective (e.g., maximizing profit for new stores) and underlying constraints (e.g., location-dependent costs and revenues). The real-world decisions are therefore reduced to its most essential characteristics and subsequently translated into a mathematical model, which depicts the objective and constraints (Domschke et al., 2015).

An OR-concept that is particularly applicable to assortment and inventory management is the newsvendor model. In its original form it describes a newsboy selling newspapers over a day. The newsboy has one chance at the beginning of the day to buy newspapers while facing uncertain demand. Unsold newspapers will be worthless at the end of the day (i.e., overstock)

and unmet demand is punished (i.e., stock out). Hence, he needs to decide how many newspapers to buy to meet demand x and optimize profit π . To formulate a mathematical model and apply a solution algorithm, the independent per unit variables revenue R , purchase cost C , salvage cost V , and shortage cost S are introduced. Salvage cost can be interpreted as inventory holding or disposal costs, but can also serve as a residual value. Decision variable Q defines the order quantity. As only either situations of overstock or stock out can occur, the profit per period is calculated through Objective Function (1.1). If the newsboy does not sell all newspapers, the revenue is defined through realized demand x multiplied with selling price R . Costs are incurred through left-over newspapers $(Q - x)$ multiplied with their salvage cost V and order quantity Q multiplied with purchasing cost C . If the newsboy does not satisfy all the demand, the revenue is given through the multiplication of selling price R with quantity Q while purchasing costs C for each unit as well as penalty costs S for each unit of unsatisfied demand have to be deducted.

$$\pi = \begin{cases} Rx - V \cdot (Q - x) - CQ & \text{if } x \leq Q \\ (R - C) \cdot Q - S(x - Q) & \text{if } x \geq Q \end{cases} \quad (1.1)$$

In situations where demand is unknown (stochastic) and a high number of possible order quantities exist, it can be assumed that demand x is a continuous random variable with mean μ and standard deviation σ . This facilitates the usage of the probability distribution of the demand $f(x)$ to calculate expected profit $E(\pi)$ through Equation (1.2). The five parts of (1.2) resemble Equation (1.1) as they represent the unit costs for each order item (part 1), the expected revenue for overstock (part 2) and stock out (part 4), the expected salvage costs (part 3), and the expected shortage costs (part 5). The simplified problem at hand is constrained through integer solution values for order quantity Q as the newsboy can only buy integer number of newspapers.

$$\begin{aligned}
E(\pi) = & -C \cdot Q + R \int_0^Q x f(x) dx - V \int_0^Q (Q - x) f(x) dx \\
& + R \int_Q^\infty Q f(x) dx - S \int_Q^\infty (x - Q) f(x) dx
\end{aligned} \tag{1.2}$$

For simplified models as the one given, closed form solutions are able to find order quantity Q (i.e., the optimal solution) that leads to the maximal profit (i.e., maximum objective value). Yet, the newsvendor problem can be extended in various directions. For example, different objectives, varying prices, knowledge about the underlying demand, multiple products, or multiple periods can be considered. Such extensive problems can be solved efficiently by commercial solvers such as CPLEX or Gurobi. For a more detailed overview see Choi (2012). In Chapter 3 and Chapter 4 we focus in particular on multiple products, multiple locations with constrained space, and complex demand structures including substitution between products. While this increases the validity of the model and helps to model abstractions closer to the real world, the problem at hand becomes more complex and creates particularly high computational efforts. Therefore alternative approaches like integer programming techniques or intelligent search heuristics are developed and applied.

Heuristics Such heuristic procedures usually contain rules and steps that simplify the decision-making process and reduce the range of possible solutions. Hence, heuristics often provide a good but not necessarily optimal, solution to the problem at faster run times. Knowledge about the problem and characteristics of the mathematical model are leveraged for the development of the heuristic. Often, possible solutions are generated and tested. For instance, instead of calculating the objective value for all possible integers of Q , a heuristic could calculate the objective value for $Q = 10$ and $Q = 11$. Depending on which solution provides the higher objective value, the heuristic continues to calculate the objective value for smaller or larger order quantities of newspapers until the objective value starts

to drop. In case of a linear solution structure this heuristic could provide the optimal solution, in other cases it may only compute the best-known but not optimal solution. For each of our contributions in Chapter 3 and Chapter 4 we develop and apply a specialized heuristic that solves problems which cannot be solved in a reasonable time through commercial solvers.

1.2.2 Omni-channel store operations

Store operations also become more relevant as the store itself lies at the heart of new OC delivery concepts. Originally being primarily a customer shopping area, stores can now additionally serve as a fulfillment center, a pick-up station, an experience center, or a point of order acceptance (e.g., Erik Brynjolfsson et al. (2013); Gallino et al. (2017); Gallino and Moreno (2019); Janjevic et al. (2020)). In particular the trend towards fulfillment centers was fueled by the COVID19 pandemic when stores were closed down. To comply with its new tasks and responsibilities, agile, integrated, and efficient store operations are needed. For example, fulfilling orders from stores requires deciding on the subset of all stores that are generally enabled for store fulfillment as well as deciding on the particular store that fulfills a certain online order. Similarly, questions on demand forecasting, assortment, inventories, and replenishment require to be approached and solved through advanced models and OR concepts. We therefore set out to answer the following research questions:

- i) What are store-related planning issues in OC operations?
- ii) Which of these planning issues are already sufficiently supported by decision support systems?
- iii) Which planning issues constitute further research opportunities?

In Chapter 5 we cover this topic as we use a triangulation approach to derive operational issues in OC store operations and provide an up-to-date

analyses of current literature with OR applications and decision support systems.

Triangulation approach To approach such a nascent topic, it is reasonable to apply a multi-method approach (e.g., Boyer and Swink (2008); DeHoratius and Rabinovich (2011)). The advantages of a variety of research methods can be utilized and help to view the problem at hand from different perspectives. At the same time, the scarcity of existing contributions covering the topic can be compensated.

First and foremost, a conceptual framework is developed and drawn upon. It constitutes the theoretical basis for the analysis of such an arising research field (Webster and Watson, 2002). The overview is focused on relevant OC delivery concepts (i.e., BOPS, SFS, and DAE) and thereby defines the scope of the following steps. Following the recommendation of Edmondson and Mcmanus (2007) for nascent topics, semi-structured expert interviews are conducted to identify the most relevant issues in store operations and complement the theoretical foundation with practical insights. Since we give voice to experts who share knowledge they have acquired from positions within relevant firms (Flynn et al., 1990; Creswell, 2009; Trautrim et al., 2012), external validity is ensured as well as practical relevance. To complete the approach, a systematic literature review is carried out that analyzes the latest and most relevant contributions from the OR domain. Joining these sources within a cross-method triangulation approach enables us to structurally identify and discuss relevant issues.

2 Contributions

This chapter introduces the three articles (Chapter 3 to Chapter 5) that compose the main body of the doctoral thesis. For each of the articles, it gives an overview on the topics, research questions, and contributions and guides the reader to areas of particular interest. Table 2.1 lists the title, authors and respective contribution of each author and states the current status of publication.

Table 2.1: Status of publications (as of March 2022)

Title	Authors and contribution	Status
1 Assortment optimization in omni-channel retailing	Hense, Jonas (66%); Hübner, Alexander(33%)	Accepted for publication in European Journal of Operational Research on 29.09.2021
2 An analytical assessment of demand effects in omni-channel assortment planning	Hense, Jonas (50%); Hübner, Alexander (25%); Schäfer, Fabian (25%)	Major review from Omega - The International Journal of Management Science on 06.02.2022
3 The revival of retail stores via omnichannel operations: A literature review and research framework	Hense, Jonas (33%); Hübner, Alexander (33%); Dethlefs, Christian (33%)	Accepted for publication in European Journal of Operational Research on 14.12.2021

Remark The versions of Chapter 3 to Chapter 5 may differ slightly from the versions that were published or submitted to the European Journal of Operational Research and Omega - The International Journal of Management Science. This is due to journal-specific guidelines such as formatting or spelling as well as changes that may be undertaken in the course of the peer review process. Yet, relevance and contributions remain unchanged.

2.1 Contribution 1: Assortment optimization in omni-channel retailing

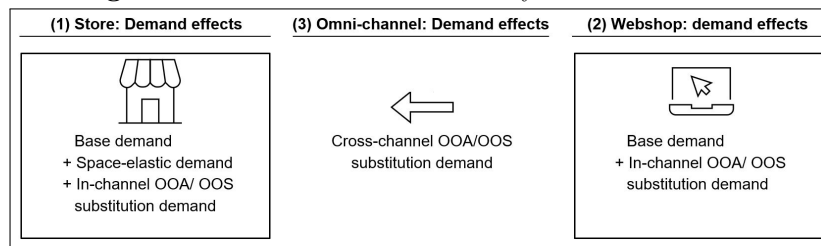
Methodology Based on OR methodology, this article identifies the OC assortment optimization problem. The problem is described qualitatively and formulated as a mathematical model, which depicts the most crucial aspects of the real-world problem. Given that the problem is NP-hard it cannot be solved with commercial solvers. Instead, we develop a problem-specific solution approach, which respects the underlying peculiarities, variables, parameters, objective function, and constraints. Following a detailed description of this approach, various randomly generated test instances are solved. Parameters are called from numerical studies to guarantee practicability. The solution approach is then compared to existing approaches to show the advancement in run time and solution quality. Following that, managerial insights are derived, such as the quantified potential of OC substitution behavior or the relevance and interplay of demand effects in an OC context. The article concludes with a summary of the key findings, practical implications, and future areas of research.

Contribution We first provide an overview on current SC, MC, and OC assortment planning literature. As already noted by Dzyabura and Jagabathula (2018), one of the main publications in this field, a lack of OC models exist. The prevailing contributions by Dzyabura and Jagabathula (2018) and Geunes and Su (2020) are characterized by an absence of crucial OC aspects, most importantly the mutual consideration of in-channel and cross-channel substitution behavior. We therefore develop the integrated OC model for assortment, space, and inventory optimization, which sets itself apart from existing literature in the following ways:

The main contribution is the acknowledgement and consideration of customer substitutions across channels. Both situations, where BOPS substitutions occur as a result of temporary (e.g., sell-offs) or permanent product

unavailability (e.g., product not listed) are considered. Beyond that, we consider stochastic and space-elastic demand and optimize assortments and inventories for each channel as depicted in Figure 2.1. The extension to substitution effects and the consideration of the assortment and inventory decision in each channel is crucial given the interdependence between the channels. For example, delisting a product in one channel may lead to substitutions and a change in demand in another channel.

Figure 2.1: Contribution 1: Summary of demand effects



We also constrain the space in each channel, which makes the problem a NP-hard multi-knapsack problem. A specialized heuristic is therefore applied, which iteratively solves the model, adds substitution demand ex-post, and updates the solutions until there is no more change in solutions. Our numerical tests show that in the general case in-channel substitution creates higher profit advantages than cross-channel substitution. Yet, considering cross-channel substitution in assortment planning offers additional profits, in particular for products with high affinity.

2.2 Contribution 2: An analytical assessment of demand effects in omni-channel assortment planning

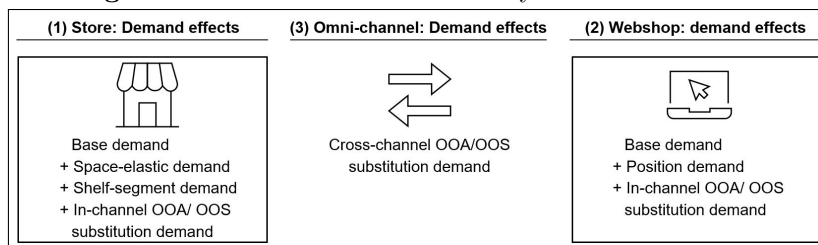
Methodology This article partially resembles the proven approach described in Section 2.1. The OC assortment optimization problem is extended to account for relevant demand effects in the store and in the webshop.

Again, the problem is described qualitatively, formulated as a mathematical model, and solved through a specialized heuristic. The approach is tested, compared, and used to derive managerial insights on the relevance and interplay of demand effects for OC retailers. Lastly, key insights, practical guidelines, and future research topics conclude the article.

Contribution Having analyzed the OC assortment planning problem with a focus on cross-channel substitution demand, we noticed a wide range of demand effects in stores, webshops, and across channels that potentially influence the profit of an OC retailer. Despite that, no existing empirical or optimization literature provides quantitative insights on the relevance and impact of the range of demand effects when determining OC assortments. An empirical measurement would incur high costs, given the large number of demand effects, the dependencies between products and channels, and the mutual reinforcement and compensation of demand effects.

Our major contribution to both the consumer and OR community is therefore an OC model that optimizes the assortment, space and position, and inventory for each channel, and allows an analytical assessment of all relevant demand effects. These include stochastic, space-elastic, shelf-segment, position, in-channel, and cross-channel demand (see Figure 2.2). We therefore differentiate ourselves from existing literature through the analytical approach and the wide range of integrated demand effects.

Figure 2.2: Contribution 2: Summary of demand effects



We build upon the iterative solution approach from the previous article and solve the problem through a tailored, further advanced heuristic. The

analytical assessment of the demand effects shows that space-elastic, position, and shelf-segment demand are particularly important to consider and can cause profit losses of up to 15.5% when being ignored. Ignoring in-channel store substitution can lead to profit losses of up to 1.5% while in-channel webshop, cross-channel store, and cross-channel webshop substitution only have a low impact on the profit. In general, we see that the profit impact of the demand effects is primarily driven by the demand rates itself, the channel package sizes, and the channel size. Our research builds the foundation for further OC assortment models and helps to delineate the scope of future empirical studies on demand effects in OC retailing.

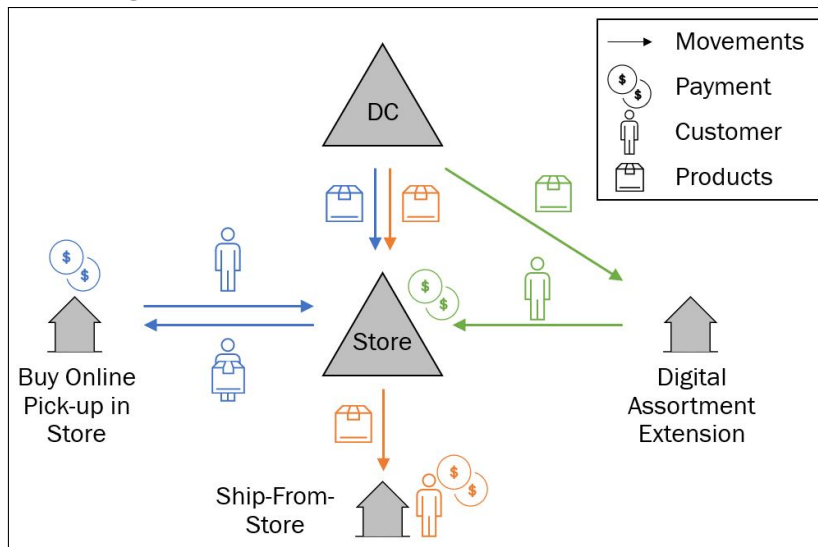
2.3 Contribution 3: The revival of retail stores via omnichannel operations: A literature review and research framework

Methodology This article follows a different methodological approach than the previous two contributions. We identify the increasing relevance of OC store operations and the lack of operational decision models as a research gap. A triangulation approach therefore utilizes multiple research methods to carve out and analyze the most relevant planning problems in this area. The methodologies include a conceptual overview on OC delivery concepts, expert interviews, and a comprehensive literature analysis. Next, a planning framework is derived with relevant planning problems. For each problem we provide a detailed description, depict the underlying challenges in practice, discuss the current coverage of the literature, and lay out areas of future research. The last section summarizes the contributions and overarching avenues of future research.

Contribution We first and foremost contribute to existing literature by acknowledging the central role of the store in novel OC delivery concepts

and the resulting operational challenges in the store (see Figure 2.3). This insight is derived from interviews with industry experts. However, the importance is not mirrored by a holistic coverage in literature, in particular in the form of a state-of-the-art literature review. Consequently, we use the triangulation approach to focus on the revival of bricks-and-mortar stores through OC operations and analyze OC-related planning issues in the store. As a result, we provide practitioners and researchers with the most important store-related planning issues in OC operations, existing decision support systems for such issues, and fruitful research opportunities for each planning issue.

Figure 2.3: Contribution 3: Central role of the store



This helps retailers in their endeavour to introduce and improve OC delivery concepts and tackle challenges in store operations by leveraging quantitative decision support systems from literature. Researchers benefit from a practically relevant planning framework and the layout of promising future directions of research. In a nutshell, we see that some planning issues (i.e. network design, assignment of customer orders, and assortment and inventory planning) are fairly well covered by literature while others (i.e. demand forecasting and inventory replenishment) have received significantly

less coverage and offer great research possibilities in this nascent field of OR.

3 Assortment optimization in omni-channel retailing

Nowadays the majority of retail customers use multiple channels. We investigate the assortment, space and inventory problem for an omni-channel retailer operating with interconnected bricks-and-mortar stores and an online shop. For this problem it becomes essential to consider customers' demand interactions across channels. Current literature mainly focuses on single-channel assortments and ignores cross-channel substitution. We contribute the first integrated omni-channel model that determines assortments for the online and bricks-and-mortar channel with stochastic, space-elastic and out-of-assortment and out-of-stock demand both for in-channel substitution and cross-channel substitution. A specialized heuristic is developed that is based on an iterative solution of a binary problem and demand updates. Our approach achieves near-optimal results for small instances and higher objective values as an alternative heuristic for larger instances. With the full integration of channels, omni-channel retailers can realize a profit increase that mainly depends on the magnitude of substitution rates. We further show numerically that in-channel substitution has a stronger impact on profits than cross-channel substitution when costs are equal across channels.

3.1 Introduction

Retail customers are increasingly utilizing multiple sales channels. Out of 46,000 shoppers surveyed, 73% used multiple channels during their decision process (Sopadjieva et al., 2017). iVend Retail (2019) report that 81% of consumers had already purchased products online, subsequently picking them up in the bricks-and-mortar store. This represents an increase of 30% from 2018 to 2019. Verhoef et al. (2015), Hübner et al. (2016b) and Rooderkerk and Kök (2019) show that many bricks-and-mortar retailers and pure online players are moving from single-channel (SC) via multi-channel (MC) to omni-channel (OC). While SC retailers operate one sales channel, MC retailers use various isolated channels to sell products, such as an online shop and a non-connected bricks-and-mortar store (Beck and Rygl, 2015). In OC, operations are integrated across channels, information is exchanged and assortments are coordinated such that neither the retailer nor the customer distinguishes between the channels anymore. To complement OC assortments, retailers must decide which product to list in which channel. This can only be done by appropriately integrating customer demand transitions between channels and products. For instance, if a product is sold out in the online shop it can be substituted either by another product in the online shop or by the same product from the bricks-and-mortar store. Demand transitions across channels can be enabled for example by buy-online pick-up in store (BOPS) functionalities (also called click and collect) and availability displays in the webshop that redirect customers to the store where the item is stocked in the event of unavailable webshop items (Gallino and Moreno, 2014; Wollenburg et al., 2018a). Thereby, inventories become available to serve customers across channels which resembles virtual inventory pooling.

OC is a very recent phenomenon and new research area that requires advancements in coordinated assortment planning (Melacini et al., 2018; Rooderkerk and Kök, 2019; Hübner et al., 2021). Dzyabura and Jagabathula (2018) conclude that state-of-the art literature solely considers SC or MC

retailers. SC literature for stores or webshops naturally only addresses a single channel and disregards cross-channel substitution (see e.g., K ok et al. (2015)). The most relevant MC models are limited in their practicability for OC by assuming predefined online assortments and only optimizing the bricks-and-mortar store assortment (e.g., Dzyabura and Jagabathula (2018); Lo (2019)). Furthermore, severe limitations are imposed and relevant customer behavior such as substitutions between channels has so far been disregarded (e.g., Wollenburg et al. (2018a); Bianchi-Aguiar et al. (2021)). Hence, the impact of cross-channel substitutions and assortment integration on retail profit and assortment structures has not yet been investigated numerically (Wollenburg et al., 2018a). This paper addresses this research gap and makes the following contributions: It develops the tactical OC assortment, shelf space and inventory problem, where assortments are defined across channels with the objective of maximizing total profit of the retailer and with respect to limited capacities in stores and online warehouses. We account for stochastic, space-elastic and out-of-assortment and out-of-stock demand both for customer substitutions within a channel and substitutions between channels. An OC retailer needs to take the following three decisions: the choice of the products to be listed for each channel, the allocation of space to each listed product and the selection of the stock level. As it is an extension and generalization of the NP-hard multi-knapsack assortment problem, this paper develops a specialized heuristic for the problem. The application to a broad set of test data then provides managerial insights into the impact of coordinating assortments across channels compared to planning for individual channels.

The remainder is organized as follows. Section 3.2 details the setting and outlines related literature. Section 3.3 develops the model and the solution algorithm applied. Section 3.4 completes numerical tests to assess the performance of our approach and derives managerial insights. Lastly, Section 3.5 summarizes the paper and outlines potential areas for future research.

3.2 Planning problem and related literature

Composing assortments across channels is a novel phenomenon. We define the general setting in Section 3.2.1 to gain a common understanding of the related context. This builds the foundation for defining the OC assortment problem in Section 3.2.2. In Section 3.2.3 we review related literature and derive the area for open research.

3.2.1 General setting and category planning

Retailers organize their total assortment within categories. Usually a category contains 60-80 items on average. Products are assigned to categories based on similarities. In most cases, each category is managed separately to overcome the complexity arising from thousands of different products of the retailer. Total available space in stores and warehouses is limited and divided into space for each category. Hence, retailers are required to make a number of decisions related to the tactical management of retail space. For each channel these include category planning, i.e. which categories to offer in which size and depth, and assortment planning, i.e. which products to offer per category, how to allocate each product to the available category space and how to define inventories (see also related frameworks in Hübner et al. (2013); Kök et al. (2015) or Bianchi-Aguiar et al. (2021)). We describe the category decisions hereafter while the subsequent assortment, space and inventory decisions are specified in 3.2.2.

The overarching category planning includes the selection of categories and the definition of each category's role and space (see e.g. Irion et al. (2011); Flamand et al. (2018); Ostermeier et al. (2021)). At this planning stage the perception of the category plays an important role in order to incorporate customers' purchase decision (see e.g., Broniarczyk et al. (1998)). On one side broadening a category generally helps customers to find the desired product, satisfies variety-seeking customers (Broniarczyk and Hoyer, 2008)

and leads to higher consumption benefits (Gijbrecchts et al., 2008). Borle et al. (2005) come to the conclusion that assortment reductions negatively influence online customers' shopping frequency and purchase quantity. Likewise, increasing the number of subcategories provides greater ease of navigation and enhanced customer attitudes towards the webshop. On the other side, beyond a tipping point too many subcategories lead to a more negative attitude (Chang, 2011). In line with that, empirical evidence underlines that categories have become so excessive that reducing variety within a category may increase sales (Iyengar and Lepper, 2000; Dhar et al., 2001; Sloot and Verhoef, 2008). Boatwright and Nunes (2001) found that significant item reductions resulted in a sales increase of 11% across 42 categories examined. Customers may also be faced with higher cognitive loads, trade-off decisions, lower choice accuracy, potentially higher post-purchase regret and therefore expose a greater chance to defer a purchase (Broniarczyk and Hoyer, 2008). Consumer psychology studies identified a wide range of moderating effects related to this ambiguity in customer characteristics (e.g. Chernev (2003); Broniarczyk (2008); Mogilner et al. (2008); Briesch et al. (2009)) and item and category characteristics (e.g. de Clerck et al. (2001); Gourville and Soman (2005); Kalyanam et al. (2007)).

Further elements of OC category planning are showrooming and webrooming. They describes a research behavior where customers collect information in a store or webshop and subsequently purchase the product in another channel or at another retailer. At this stage of the customer decision process, customers collect information in one particular channel. Customers may choose to search a retailer and its channel because of certain expectations regarding the category and the retailers' expertise in this area. In OC it is therefore important to identify the channel complementarity of entire categories and analyze amongst others if the categories can benefit from show- or webrooming. Furthermore, show- and webrooming are primarily used to collect information for a later purchase at any other channel/ retailer. At this stage, a particular preference for a product is not yet defined. We assume in our context, that it becomes necessary to incorporate show- and

webrooming effects when optimizing the role and size of categories. However, other settings and assortment decisions may require the incorporation of web- and showrooming effects in tactical assortment planning (see e.g., Dzyabura and Jagabathula (2018)). In settings, where customers modify evaluations, it can be beneficial to include products in one channel that primarily serves for information collection, followed by purchases in another channel (e.g., instore evaluation and online purchase). In the next step, after defining the category role and size, retailers are required to define the set of products included in each category.

3.2.2 Omni-channel setting and assortment planning

Assortment planning considers the question of which and how many different products to offer within a given category (Fisher, 2009). The main feature of assortment planning is the integration of consumer's willingness to accept a substitute when the desired product is unavailable. At this point, customers already collected information within or across channels and developed a preference for a certain product (e.g., Smith and Agrawal (2000); Hübner and Kuhn (2012); Kök et al. (2015)). If the product is permanently or temporarily unavailable they may settle for a substitute within the channel or across channels. Hence, substitutions become especially relevant when not all conceivable products of a category should or could be listed. It may be beneficial for the retailer to go without some (less profitable) products and thus forcing consumers to switch to substitutes that are more profitable.

The ultimate objective of assortment planning is to maximize the retailers' profit that stems from realized customer demand, which in turn depends on the assortment configuration and availability and accessibility of individual products to the customer. Retailers attempt to offer the assortments and quantities that match customer demand as deviations in either direction are penalized. Offering an item quantity that exceeds demand creates leftover items. Such items entail disposal costs for perishable items, or inventory

holding costs for non-perishable items. An item quantity that falls short of demand results in unsatisfied demand. If customers cannot purchase the desired product or a substitute, sales will be lost and customers will be left unsatisfied. Beyond that, unit costs for the purchase of items have to be considered. Retailers need to take three decisions on a tactical level to maximize profits:

- *Assortment composition* selects the products to list within a category and a channel.
- *Space allocation* assigns space to each listed product in each channel.
- *Inventory planning* determines the stock levels for each product in each channel.

The three decisions are interdependent within and across channels if space capacity is limited. For example, a broader assortment in one channel with more products requires lower inventory levels per product, which increases the risk of out-of-stocks. On the other hand, not carrying some products in the assortment of a channel may end up in demand substitutions of high-affinity products to the other channels or lost demand from switching to competitors. This is a tactical decision as assortments are defined for a mid-term period and before the sales season, e.g., due to supplier negotiations or sourcing contracts with longer lead times. Subsequently, the retailer then has further operational tasks to solve that are related to in-store shelf replenishment, short-term sales incentives (e.g., discounts at the end of a sales period) or personalized assortments in the online shop.

In the following, we specify the setting for the OC context. There are differences between store-, webshop- and OC-retailing in terms of customer reactions and planning issues. Therefore, we will detail the respective decision problems and related demand effects separately for (1) stores and (2) webshops. This builds the foundation for discussing the (3) OC setting. Going forward we will use the term “store” as the equivalent for a bricks-and-mortar sales location and “webshop” as the pendant for the digital sales channel.

(1) Store-related planning issues and demand effects Commonly, retailers select the items of a category (assortment planning) and allocate them to shelves (space and inventory planning). Shelf space is limited in the store and imposes a trade-off between listing broader assortments with less shelf space and shelf inventory per item, and smaller assortments with more space and higher inventory per item. This impacts the availability of each item and consequently the demand for all items. When a desired product is unavailable, customers may decide to replace the desired product with an alternative product within the store. This is called *substitution demand*. Unavailability of items can be the result of two scenarios: an item is either permanently delisted and therefore out-of-assortment (OOA), or an item is temporarily sold out and therefore out-of-stock (OOS). Empirical studies indicate substitution rates of 45% to 84% of the initial demand, where the magnitude depends on attributes of the product, situation and customer (e.g., Gruen et al. (2002); Campo et al. (2004); Aastrup and Kotzab (2009); Tan and Karabati (2013)).

Store inventories are defined by the number of facings and the quantity behind one facing. Facings are the foremost unit of an item on the retail shelf. Increasing the number of facings assigned to a specific item in the store leads to higher item visibility and generates additional item demand called *space-elastic demand* (e.g., Hansen and Heinsbroek (1979), Irion et al. (2012)). Space-elastic demand has been analyzed in multiple studies. Shopper surveys and field experiments conclude that a significant relationship exists between the number of facings and the demand realized. The degree of significance depends on the item type. Brown and Tucker (1961) recognized increasing space effects from the group of unresponsive, inelastic products over general products for everyday purchases to products for impulse purchases. Cox (1964) tests the impact of variations in facings on sales of staples and impulse-purchased items. Frank and Massy (1970) use an experiment to test the influence of facings on sales of grocery products. Curhan (1972) proved that fast-moving products have a higher facing-dependent demand effect than slow-moving items. Drèze et al. (1994) identify the impact on sales through reorganizing shelf configurations.

Chandon et al. (2009) show that the variation of facings is the most significant in-store factor, even stronger than positioning and pricing. Eisend (2014) found by means of a meta-analysis that demand increases by an average of 17% for every facing duplication. In the course of this study, Eisend (2014) also calculates an average cross-space elasticity of -1.6%, which measures the responsiveness in the demand for one item when the space allocated for another item changes. However, Schaal and Hübner (2018) show that low empirical cross-space elasticities either have no or only very little impact on optimal shelf arrangements.

Finally, retailers need to define the vertical and horizontal position of an item within the shelf that determines the *position-dependent demand*. As various studies have found, the position of a product affects the likelihood of it being perceived and purchased. In general, studies show a higher impact of products located on the top- and middle-shelf positions. For example, Underhill (2000) identifies a “reliable zone” roughly ranging from eye to knee level. Products positioned within this zone are likely to be seen; products outside this zone are not. Chandon et al. (2009) found that products positioned on the top-shelf level are more likely to be noticed and chosen than products on the bottom-shelf level. Drèze et al. (1994) found that the vertical position has a much stronger impact than the horizontal position. van Nierop et al. (2008) analyze the interactions between shelf layout and marketing effectiveness and its impact on optimizing positioning. They show, amongst others, that the position of a product within the shelf and the shelf within the aisle impact demand. Valenzuela and Raghurir (2009a), Valenzuela and Raghurir (2009b), Valenzuela et al. (2013), Valenzuela and Raghurir (2015) and Rodway et al. (2012) analyze the center effect. They show that consumers have vertical and horizontal price schemes, translate this into quality perceptions and believe that items in the middle of an array represent the best price/quality trade-off.

To summarize, there are rich consumer studies that identify the impact of substitutions and the effect of space allocation on demand. Space-elastic demand has been unambiguously identified across more than 1,200 studies

(see Eisend (2014)). Demand effects are also attributed to horizontal and vertical positioning effects. The magnitude may be lower (e.g. Chandon et al. (2009)) and mainly imposes demand increases for items positioned in the center of a shelf (e.g. studies of Valenzuela and Raghurir).

(2) Webshop-related planning issues and demand effects Webshop planners have to decide for the tactical and mid-term planning horizon which products to list in the online shop (assortment problem), which space to allocate to items in the webshop (similar to the space problem in stores), and how much inventory to assign to the selected products (inventory problem). Due to the warehouse space constraints, increasing the size of the webshop assortment translates into reducing the inventory of other products or even delisting another product. Determining the assortment, space of products and inventory levels influences substitutions and demand via positioning and salience. The *substitution demand* effects in the online shop are identical to those described for the bricks-and-mortar store. Products can be OOA or OOS and customers may attempt to replace the unavailable product with another product from the webshop (see e.g., Jing and Lewis (2011)). *Positioning demand* emerges through the horizontal and vertical location of products on the online display. For example, Atalay et al. (2012) note that centrally positioned brands positively impact customers' attention, whereas Djamasbi et al. (2010) found that items in the top left corner also receive increased attention. *Salience demand* describes the relative salience of products within an assortment. Products can be visually highlighted by graphically varying the background brightness or color (e.g., Pieters et al. (2010)), or increasing the size of the product displayed. Greater salience results in higher attention, longer fixation of a product and stronger preferences for the same. Usually retailers optimize the salience and product positions on a personalized, operational and short-term horizon to learn from customer behavior (see e.g., Cheung and Simchi-Levi (2017); Bernstein et al. (2019)) in an exploration-exploitation approach or to sell overstocks (see e.g., Golrezaei et al. (2014); Bernstein et al. (2015)).

(3) OC-related planning issues and demand effects OC retailers connect their stores with the webshop. All of the above decisions, demand effects and constraints within the channels therefore also hold true for MC and OC retailers. A major difference however is that OC retailers want to coordinate assortments, space and inventory across channels to capture demand effects between the channels. These demand effects describe a demand shift between channels and are termed *cross-channel substitution*. OC retailers can thereby serve customers with virtually pooled inventories across channels who were left unsatisfied in an SC or MC operation. Just like in-channel substitution, cross-channel substitution is the result of OOA or OOS situations. However, in this case customers replace the unavailable product that is desired by switching to another channel. There, the identical product as well as different products can be purchased as a substitute. The delivery mode for substituted items can be in the form of home deliveries from the online warehouse when substituted from store to webshop. It can also be in the form of BOPS or home deliveries from the store when substituted from webshop to stores. In the latter case of shipments from store, the customer may not even be aware about the shipment location. While substitution within stores has received a significant amount of attention in literature, situations of substitution within webshops or cross-channel substitutions have only recently gained attraction. Gallino and Moreno (2014) and Wollenburg et al. (2018a) show that channel transitions are facilitated by fulfillment options such as BOPS that provide the online shopper with real-time information about inventory availability in the store. Corsten and Gruen (2019) found that 88% of online demand can be substituted, of which 10% switch to a bricks-and-mortar store, 56% opt for a substitute within the webshop, and 22% switch to another webshop. Thus, 66% of the demand potentially stays with the retailer and increases the demand for substitute items. Finally, at this stage customers already have a preferred product (after the collection of information in or across channels). This implies that information seeking and preference formation are independent of the assortment. We follow here a general assumption in well-known exogenous demand (ED) models (see e.g., Smith and Agrawal (2000); K ok and Fisher (2007)). If the preferred product is not available they may settle

for a substitute within the channel or across channels. There is no update of the evaluations. That also means that retailers do not face any demand changes after the information collection behavior via show- and webrooming has been carried out.

To summarize, the goal of this paper is to model and analyze the tactical OC assortment planning problem. When selecting assortments for the mid-term planning horizon, OC retailers need to assign space of store shelves and the webshop to selected items. This goes along with defining inventory levels in the channels and also impacts various demand sources (1) within the store, (2) within the webshop and (3) across channels. (1) Within the store, the most important demand effects are substitutions of unavailable items, space-elastic demand and to some extent positioning. There is a negligible impact of cross-space elasticity. (2) Within the webshops, current empirical research highlights substitution effects and has not yet empirically substantiated the effect of product salience or positioning on customer demand in an online shop. (3) It has been shown that on top of the demand effects within a channel, cross-channel substitution becomes important. To focus on the decision-relevant demand sources in the mid-term context, we do not directly incorporate overarching category effects (e.g., from webrooming) and short-term demand impulses (e.g., from personalized assortment rotation, positioning or salience). The latter is usually applied on the subordinate operational planning stage, e.g., personalization based on cookies.

3.2.3 Related literature

This section reviews related literature based on the setting defined. We apply a similar approach as above by first highlighting related contributions on assortment planning in stores and webshops, followed by a discussion on contributions with multiple channels.

(1) Store-related literature Store assortment planning has gained a great amount of attention in the literature. The works in this area mostly focus on considering some sort of substitution for retailers selling their products through a single store. We refer to recent reviews by Kök et al. (2015) or Shin et al. (2015), and focus within our review on fundamental papers and contributions that are most related to our setting. Smith and Agrawal (2000) develop a demand model for assortment optimization accounting for dynamic OOA substitution while ignoring OOS substitution. Inventory levels are also optimized and shelf constraints respected when optimizing profit with a newsvendor formulation. A specialized heuristic is used to solve the model. Rajaram and Tang (2001) formulate a model that considers OOS substitution and is based on a newsvendor setting. The model optimizes inventories and is solved through a service-rate heuristic. Kök and Fisher (2007) present a model that takes into account OOA. The newsvendor model is solved through an iterative heuristic. Honhon et al. (2010) present a model that focuses on OOS substitution for customer segments based on the sequence of customer preferences. An algorithm based on dynamic programming approaches solves the model. Hübner et al. (2016b) factor OOA and OOS substitution into their model and develop a specialized heuristic to solve large instances. Interestingly, even though assortment planning and space planning are known to be mutually dependent, they are often treated separately in literature (Hübner and Kuhn, 2012; Bianchi-Aguiar et al., 2021). Usually the shelf-space models assume a given assortment and assign facings to space-elastic products under deterministic demand and space constraints (e.g., Hansen and Heinsbroek (1979), Corstjens and Doyle (1981), Bianchi-Aguiar et al. (2015), Geismar et al. (2015), Flamand et al. (2018)). A smaller number of contributions take an integrated assortment and space planning approach. The first stochastic and integrated shelf-space and assortment model by Hübner and Schaal (2017a) includes both OOA and OOS substitution demand and is also based on a newsvendor setting. Another stochastic model by Hübner et al. (2020) optimizes two-dimensional shelves while considering space elasticity and substitution effects.

(2) Webshop-related literature Assortment planning for online channels has gained growing attention in research. The main focus of these contributions is to maximize revenue for dynamically arriving customers and choosing the best assortment of products or online advertising for individual shoppers or customer segments. The majority of contributions deal with dynamic assortment planning and demand learning. Common to all these contributions is the operational problem, where the retailer personalizes the customers' assortment based on available profile information. Some papers apply online optimization techniques and use multinomial logit (MNL) formulations to model unknown customer demand. Such approaches can only be applied to assortments that can be changed frictionlessly. Rusmevichientong et al. (2010) formulate an online policy with unknown MNL purchase probabilities. Sauré and Zeevi (2013) dynamically optimize the assortment for arriving customers. Abeliuk et al. (2016) study the problem of finding an optimal assortment and positioning of products subject to capacity constraints and develop an approach for a small set of products. In a related approach, Agrawal et al. (2016) dynamically optimize the assortment selection, assuming unknown demand parameters using an MNL model. A specific assortment of products is shown to the online shop visitors until they discontinue any further purchasing activity. Cheung and Simchi-Levi (2017) also incorporate uncertainty in their underlying MNL model. They dynamically optimize the assortment, consider resource constraints, and develop an efficient online policy. Bernstein et al. (2019) and Kallus and Udell (2020) study the problem of dynamic assortment customization for customer segments and how these segments can be determined. In another new stream, sales opportunities are investigated when a retailer needs to deal with limited inventory and substitutable products. In such settings, customers arrive sequentially and the retailer decides which subset of products to offer to each customer that arrives, depending on the customer's preferences, inventory levels, and the remaining time in the season. Examples are Bernstein et al. (2015) and Golrezaei et al. (2014).

(3) Literature related to multiple channels With the increasing integration of various sales channels, new contributions emerge that consider retailers with multiple channels. Bhatnagar and Syam (2014) formulate an integer program to allocate items across an MC retailer's stores and a webshop. The model is based on a given demand and not subject to space restrictions. The model does not consider substitution and imposes a strong space limitation on the store by listing only one item. In a further contribution, Dzyabura and Jagabathula (2018) depict a retailer with a given set of products in the online channel. The retailer decides on the subset of products to offer in the store, without specifying inventories, to maximize net returns on sales. Research-offline, purchase-online behavior (showrooming) as well as endogenous product returns are integrated by means of an MNL-model. Demand is modeled using a utility-based model, where the customer's physical evaluation of the store assortment may change the customer's product utilities of the store and online assortment and result in purchasing a different item to the one originally preferred. The impact of the store assortment on online demand is thereby factored in. Lo (2019) follows Dzyabura and Jagabathula (2018) in selecting a subset of the online shop for the bricks-and-mortar store to maximize aggregated expected revenues. Under the usage of an MNL model, customer preferences are updated based on a feature tree. However, in both contributions the assortment composition is limited to the store, as the webshop assortment is assumed to be exogenously given. Geunes and Su (2020) is the first model where a retailer selects products and stocking levels for a store and the online channel. Cost differences for different fulfillment options are considered. Demand is modeled as MNL and considers cross-channel substitutions, but not in-channel substitutions and space-elasticity.

Summary of related literature Table 3.1 summarizes related assortment optimization approaches for stores, webshops and across channels.

The major share of existing contributions related to our problem focuses on stores. These store-related contributions usually apply a newsvendor

Table 3.1: Related literature

Related contribution	Decision ^{1a}	Demand model					Opt. ³	Space constr.	Data ⁴	No of. items ^{5a}
		Stoch.	in-channel		cross-channel					
			OOA	OOS	OOA	OOS				
Store-focused models										
Smith and Agrawal (2000)	A - I	✓	✓				S	✓	SD	5
Rajaram and Tang (2001)	A - I	✓		✓			S		SD	7
Kök and Fisher (2007)	A - I	✓	✓				S	✓	ED/SD	29
Honhon et al. (2010)	A - I	✓	✓	✓			D		SD	8
Hübner and Schaal (2017a)	A - S - I	✓	✓	✓			S	✓	ED/SD	2000
Hübner et al. (2020)	A - S - I	✓	✓	✓			S	✓	ED/SD	100
Webshop-focused models										
Rusmevichientong et al. (2010)	A	✓	✓				D		ED	200
Sauré and Zeevi (2013)	A - S	✓	✓				D		SD	10
Abeliuk et al. (2016)	A - S	✓	✓				S	✓	SD	2
Agrawal et al. (2016)	A	✓	✓				D		SD	— ^{5b}
Cheung and Simchi-Levi (2017)	A	✓	✓				D		SD	25
Bernstein et al. (2019)	A	✓	✓				D		ED	19
Kallus and Udell (2020)	A	✓	✓				D		SD	400
MC and OC models										
Bhatnagar and Syam (2014)	A - I						S		SD	100
Dzyabura and Jagabathula (2018)	A ^{1b}	✓	✓		✓	✓	S		ED	96
Lo (2019)	A ^{1b}	✓			✓		S		SD	32
Geunes and Su (2020)	A - I	✓			✓	✓	S	✓	SD	20
This paper	A - S - I	✓	✓	✓	✓	✓	S	✓	SD	100

^{1a} Included decisions: Assortment (A), space (S), inventory (I); ^{1b} OC model only optimizing store assortment

² Customers' research and information collection behavior: Showrooming (examining products in the store) and webrooming (examining products in the webshop)

³ Optimization: Static (S) or dynamic with sequentially arriving customers (D)

⁴ Data applied: Simulated data (SD) or empirical data (ED)

^{5a} Problem size: Maximum number of items considered; ^{5b} No numerical study

model in combination with exogenous demand models to specify customer choice and factor in limited space. There are recent models that are built on the mutual dependency of assortment and space planning, but do not yet factor in cross-channel substitutions. However, these can serve as starting point for an OC model. In contrast to the amount of store literature, contributions related to specific assortment planning in webshops are only now beginning to emerge. The focus is on online optimization, demand learning and dynamically arriving customers. Such assortment problems are commonly applied to settings where new products can be introduced and removed from assortments in a frictionless manner. Webshop models such as described above follow the objective of gathering information on consumer choice through an exploration-exploitation approach. This setting is hard to transfer to a tactical planning problem where inventories across the channels need to be defined for a mid-term horizon and cannot be exchanged frictionlessly. Furthermore, these papers are limited by not considering space considerations or integrating different demand effects. Just like store papers, literature for webshop assortments also does not

take into account an additional sales channel. To generalize, SC models are limited to one channel and therefore not designed to address switching behavior between channels following an OOS or OOA situation. This issue however is investigated by the small body of literature on assortment planning with multiple channels. As indicated in Table 3.1 none of the models provided so far considers cross-channel demand and in-channel demand jointly, disregarding relevant and related substitution behavior.

This overview accentuates OC assortment optimization as a potential area of valuable research given the absence of an integrated OC model for assortment, space and inventory optimization with stochastic demand and comprehensive substitution effects. This accords with the findings of Melacini et al. (2018) and Rooderkerk and Kök (2019) that already pointed out the shortage of assortment planning with a common objective across channels and the possibility for customers to move seamlessly between channels.

3.3 Model and solution approach

This section develops the formal representation of the problem at hand to maximize the profit of an OC retailer by determining the assortment, space and inventory across channels. We first formalize the decision problem with the objective and constraints in Section 3.3.1. The demand model follows in Section 3.3.2. A solution approach for the NP-hard and nonlinear multi-knapsack problem is presented in Section 3.3.3.

3.3.1 Decision problem and general model

Table 3.2 summarizes the notation.

Table 3.2: Model notation

Indices and sets	
C	Set of channels a retailer operates with $C = \{1, 2, \dots, c, d, \dots, C \}$
K_c	Set of facings a retailer can select for item i in channel c
$I(I^+, I^-)$	Set of (listed, delisted) items with $I = \{1, 2, \dots, i, j, \dots, I \}$
$I_c(I_c^+, I_c^-)$	Set of (listed, delisted) items i within a channel c
Parameters	
α_{ic}	Minimum demand of item i in channel c
β_{ic}	Space-elasticity of item i in channel c
γ_{jc}^{OOA}	Share of demand of item j in channel c which gets substituted by item i in channel c in case item j is out-of-assortment (out-of-stock) in channel c , with $j \neq i$, $i, j \in I$
(γ_{jc}^{OOS})	
δ_{ic}^{OOA}	Share of demand of item i in channel d which gets substituted by item i in channel c in case item i is out-of-assortment (out-of-stock) in channel d , with $d \neq c$, $c, d \in C$
(δ_{ic}^{OOS})	
η_{jd}^{OOA}	Share of demand of item j in channel d which gets substituted by item i in channel c in case item j is out-of-assortment (out-of-stock) in channel d , with $j \neq i$, $i, j \in I$ and $d \neq c$, $c, d \in C$
(η_{jd}^{OOS})	
b_{ic}	Width of one facing of item i in channel c
g_{ic}	Inventory per facing of item i in channel c
$k_{ic}^{max}(k_{ic}^{min})$	Maximum (minimum) number of facings of item i in channel c
S_c	Available shelf (storage) space in channel c
r_{ic}	Revenues for one unit of item i in channel c
u_{ic}	Unit costs for one unit of item i in channel c
s_{ic}	Shortage costs for one unit of item i in channel c
v_{ic}	Salvage value for one unit of item i in channel c
Variables	
k_{ic}	Number of facings assigned to item i in channel c , integer
x_{ic}	Total inventory of item i in channel c , integer (auxiliary variable)

OC retailers must assign a given set of items $i, i \in I$ to a shelf in each channel $c, c \in C$ (i.e., shelves in stores and shelves in warehouses for fulfillment of online orders), where the shelf space for each channel S_c is known. The total set of items I is available for all channels c with $c, d \in C$, i.e., $I_c, I_d, \dots, I_C \subseteq I$, whereas I_c represents the subset of items in a channel c . Because items of the set I_c of a channel c can be delisted, a differentiation between the set of listed items I_c^+ and the set of delisted items I_c^- in each channel c is introduced, with $I_c^+, I_c^- \subseteq I_c$, $I_c^+ \cup I_c^- = I_c$ and $I_c^+ \cap I_c^- = \emptyset$. The set union across all channels represents the set of listed items with $I_c^+ \cup I_d^+ \cup \dots \cup I_C^+ = I^+$ and delisted items with $I_c^- \cup I_d^- \cup \dots \cup I_C^- = I^-$.

To streamline the notation, we use the term “facing” for the store shelves and the online warehouse shelves. It expresses the unit that customers and pickers respectively face when observing the shelf. In both channels the

number of facings are used to compute the inventory. In the store channel, the number of facings also has an impact on demand (i.e., space-elastic demand, see Section 3.3.2). In the webshop, the number of facings visible to the customer can either be zero or one unit (i.e., without any space-elastic demand) and the number of facings for the inventory calculation can be any integer value as this represents the facings in the warehouse. To obtain a compact and general model across channels, we use the uniform term “facings” to calculate the demand and inventory in both channels. The retailer defines the number of integer facings k for each item i in each channel c . Here the retailer can select from a set of integer facings K_{ic} for each item and each channel. The specification of this set for each item and channel allows the incorporation of item- and channel-specific ranges on the number of facings. Channel-specific storage requirements (represented by the space occupied per facing unit b_{ic}) and inventory quantity per facing (represented by g_{ic}) are considered. Hence, the OC retailer optimizes the profit by deciding (1) which products $i, i \in I_c$ to list in each channel c and (2) how many facings to allocate to each listed item $i, i \in I_c^+$ within each channel $c, c \in C$. To express these decisions, the number of facings k_{ic} for each item $i, i \in I$ and channel $c, c \in C$ is employed as a decision variable. Consequently, the number of facings expresses the assortment selection, with $k_{ic} = 0$ representing the case for delisting and $k_{ic} \geq 1$ the case for listing. Furthermore, (3) the number of facings determines the inventory x_{ic} with $x_{ic} = k_{ic} \cdot g_{ic}$. The number of units that are stocked behind one facing are denoted by the parameter inventory per facing g_{ic} , which is determined by the shelf depth in each channel and the item size.

The profit π_{ic} for each item i in each channel c is calculated using Equation (3.1). It depends on the number of facings k_{ic} and inventory x_{ic} of the item i in channel c and, because of demand substitutions across items and channels, also on the number of facings and inventory of all other items $j \in I$ in all channels $d \in C$. The profit equation consists of five parts. Part one represents the unit costs u_{ic} for each item i in each channel c . Unit costs u_{ic} incorporate any kind of purchasing and replenishment costs per channel and item. The random variable for the demand is denoted with y_{ic} .

At the end of a sales period, a mismatch between demand y_{ic} and inventory may lead to overstock ($y_{ic} < x_{ic}$) or unmet demand ($y_{ic} > x_{ic}$). Part two and four calculate the expected revenue for each k_{ic} (and the resulting quantity x_{ic}) given sales price r_{ic} . In part two, for all cases with $y_{ic} < x_{ic}$, the multiplication of y_{ic} with sales price r_{ic} yields the revenues. In part four, for the reverse cases with $y_{ic} > x_{ic}$, only the available stock can be sold and thus x_{ic} is multiplied with the sales price r_{ic} . Part three of the profit function accounts for the expected salvage cost due to items that are left in stock at the end of the period. Leftover stock of item i in channel c is cleared at salvage value v_{ic} , thereby representing a residual value. As $v_{ic} < u_{ic}$, the retailer suffers a loss in profit. The salvage value can also be interpreted as inventory holding costs in the case of non-perishable items (Kök and Fisher, 2007; Hübner et al., 2016b). Part five denotes shortage s_{ic} that are imposed to include a penalty cost for unsatisfied customers.

$$\begin{aligned} \pi_{ic}(\bar{k}, \bar{x}) = & -u_{ic} \cdot x_{ic} + r_{ic} \int_0^{x_{ic}} y_{ic} f_{ic}^* dy + v_{ic} \int_0^{x_{ic}} (x_{ic} - y_{ic}) f_{ic}^* dy \\ & + r_{ic} \int_{x_{ic}}^{+\infty} x_{ic} f_{ic}^* dy - s_{ic} \int_{x_{ic}}^{+\infty} (y_{ic} - x_{ic}) f_{ic}^* dy \end{aligned} \quad (3.1)$$

This generic form of the profit π_{ic} corresponds to the profit calculation in newsvendor problems and can therefore also be found in many other assortment-related decision models (cf. e.g., Smith and Agrawal (2000); Honhon et al. (2010)). The difference always stems from the demand that is taken into account, which is represented through the density function f_{ic}^* . In Equation (3.1) this function accounts for the relevant total demand, which must be quantified in accordance with the assumed customer behavior. We detail the demand function below in Section 3.3.2.

The retailer maximizes total profit Π , which is the sum of item profits per channel π_{ic} of each item $i, i \in I$ in channel $c, c \in C$. The objective function can be described as $\Pi(\bar{k}, \bar{x}) = \sum_{i \in I} \sum_{c \in C} \pi_{ic}(\bar{k}, \bar{x})$. This is subject to the limited shelf space in each channel S_c , i.e. $\sum_{i \in I_c} b_{ic} \cdot k_{ic} \leq S_c$, for all $c \in C$,

with b_{ic} denoting the shelf space occupied by one unit in each channel. The shelf space S_c is the one-dimensional shelf length in each channel, e.g., measured in meters (cf. Kök and Fisher (2007); Irion et al. (2012); Düsterhöft et al. (2020)). The reason for this is twofold: first, the stock per unit (g_{ic}) is fixed and given by the item dimensions and shelf depth. Second, because customers frontally observe store shelves, two different items can only be placed side-by-side, not behind one another. The same holds true for storage in the warehouse. In practice, it is common for retailers to limit the set of facings K_{ic} . In the bricks-and-mortar store this is driven by sales initiatives and marketing contracts regulating the share of facings. In the webshop it is the result of space constraints in warehouses and the online display. As such, we can limit the number of facings with $k_{ic}^{min} \leq k_{ic} \leq k_{ic}^{max}$, for all $i \in I$ and $c \in C$.

3.3.2 Demand model

The most popular approaches for integrating demand substitution in assortment planning are multinomial logit models (MNL) and exogenous demand (ED) models. In this paper, we focus on ED models since MNL models usually neglect limited shelf capacities and ED models are mostly used when inventory levels become relevant. ED models directly specify the demand for each product by depicting consumers that choose from a set of items. If the preferred item is not available, an individual consumer might accept another item as a substitute according to a defined substitution probability. It allows to incorporate exogenous demand factors that are empirically motivated. A further advantage of ED models is the ability to differentiate between the initial choice (i.e. the base demand) and substitution. Each element of the ED model is directly and independently specified. The distribution of each demand element for each product is therefore assumed to be independent. That means, for example, the base demand of product i (specified with α_i) is independent from the base demand of product j , with $i \neq j$. This allows the usage of the convolution concept that also

supports to obtain more tractable integer programs. We focus our analysis on the switching between channels and do not further differentiate demand flows by delivery mode. We model the most popular concept related to the delivery modes, namely BOPS (see e.g., Gallino and Moreno (2014) or Wollenburg et al. (2018a)), where webshop customers are getting access to the store inventory. The webshop is able to access its own inventory but also the store inventory. This enables the webshop to show customers alternatives from the store when OOA or OOS situations occur. The core of our investigation is the planning of assortments across channels. To focus on assortment effects, to streamline the analyzes and obtain a tractable model, we do not integrate further channel-specific demand effects like positioning and salience at the webpage. Furthermore, we assume that research behavior of customers through e.g., web- and showroom has been already taken place. Changing preferences through web- and showroaming of certain products are not considered.

The total expected demand \hat{D}_{ic} of an item i in channel c consists of seven elements (cf. Equation (3.2)). The first element is the space-elastic demand D_{ic}^{SP} . The subsequent three elements are the OOA demand sources. We need to differentiate between substitutions within the same item or across different items and within or across the channel. This gives three potential combinations for substitution flows: different items within the same channel (denoted $OOA^{(1)}$), identical items across different channels (denoted $OOA^{(2)}$), and different items across different channels (denoted $OOA^{(3)}$). The remaining three elements denote the OOS demand, which contains substitutions across different items within the same channel ($D_{ic}^{OOS^{(1)}}$), identical items across different channels ($D_{ic}^{OOS^{(2)}}$) and different items across different channels ($D_{ic}^{OOS^{(3)}}$). We elaborate on the components below.

$$\hat{D}_{ic} = D_{ic}^{SP} + D_{ic}^{OOA^{(1)}} + D_{ic}^{OOA^{(2)}} + D_{ic}^{OOA^{(3)}} + D_{ic}^{OOS^{(1)}} + D_{ic}^{OOS^{(2)}} + D_{ic}^{OOS^{(3)}} \quad (3.2)$$

(I) Space-elastic demand Space-elastic demand D_{ic}^{SP} of item i in channel c describes the effect of an increased demand triggered by an increasing number of facings k_{ic} of item i in channel c (e.g., Hansen and Heinsbroek (1979), Corstjens and Doyle (1981)). The base for the space-elastic demand constitutes the minimum demand α_{ic} , which represents the retailer's forecast for an item that is independent of the facing and assortment configurations, i.e. when an item is not listed and has zero facings. Minimum demand therefore exists regardless of the facing decision for item i in channel c (e.g., Irion et al. (2012); Hübner and Schaal (2017b)). Increasing the number of facings k_{ic} increases the visibility of an item and thereby its demand (cf. e.g., Eisend (2014)). In accordance with prior research (cf. e.g., Hansen and Heinsbroek (1979)), the facing-dependent demand rate is a polynomial function of the number of facings and space-elasticity β_{ic} (with $0 \leq \beta_{ic} \leq 1$). The minimum demand α_{ic} is multiplied by the number of facings k_{ic} to the power of β_{ic} . Thus, if no space-elasticity effect exists and $\beta_{ic} = 0$, this translates into $D_{ic}^{SP} = \alpha_{ic}$. The same holds true if the number of facings is at maximum one as is usually the case for webshops. Hence, space-elastic demand D_{ic}^{SP} is calculated by Equation (3.3) and the corresponding density is denoted by $f_{D_{ic}^{SP}}^*$.

$$D_{ic}^{SP} = \alpha_{ic} \cdot k_{ic}^{\beta_{ic}} \quad \forall c \in C, i \in I \quad (3.3)$$

(II) Out-of-assortment demand OOA demand describes the demand transfer of unsatisfied demand for delisted items to a listed item. This occurs if another item j is delisted ($j \in I^-$) and customers substitute this item j with item $i, i \in I^+$. We assume that if item j is delisted, customers will substitute a certain share of the minimum demand α_j of item j with item i , while some customers will still want to buy item j , even if it is not available. The maximum quantity that can be substituted cannot be higher than the minimum demand α_j as first, the minimum demand exists regardless of the assortment, space and inventory decision for item i , and second, because we follow the usual assumption that substitution takes place

over one round only (cf. e.g., Smith and Agrawal (2000); K ok and Fisher (2007); H ubner and Schaal (2017a)). This implies that if a substitute is also not available, demand is lost. It has been shown that this assumption is not too restrictive (cf. Smith and Agrawal (2000)). In the event of multiple channels, the OOA demand needs to be differentiated between (II.1) different items within the same channel, (II.2) identical items across different channels and (II.3) different items across different channels.

(II.1) The *OOA demand for different items within a channel* $D_{ic}^{OOA(1)}$ for a listed item $i, i \in I_c^+$ in channel c emerges when a customer demands an OOA item j in channel c ($j \neq i, j \in I_c^-$) and chooses to purchase an alternative item i in the same channel c . The substitution rate γ_{jcic}^{OOA} expresses the share to be substituted in this case. Equation (3.4) summarizes the OOA demand within a channel and the corresponding density function is denoted by $f_{D_{ic}^{OOA(1)}}^*$.

$$D_{ic}^{OOA(1)} = \sum_{j \in I_c^- / \{i\}} \alpha_{jc} \cdot \gamma_{jcic}^{OOA} \quad \forall \{c, i\} : c \in C, i \in I \quad (3.4)$$

(II.2) The *OOA demand for identical items across channels* $D_{ic}^{OOA(2)}$ materializes when a customer intends to buy an item i in channel $d, d \in C$ while the item is not listed in channel d with $i \in I_d^-, d \in C$. Given the unavailability, the customer may opt (for reasons of affinity) to switch channels in order to purchase the identical item i in another channel c . The rate δ_{idic}^{OOA} quantifies the share that is substituted. Equation (3.5) summarizes this OOA demand substitution, and the corresponding density function is denoted by $f_{D_{ic}^{OOA(2)}}^*$.

$$D_{ic}^{OOA(2)} = \sum_{d \in C / \{c\} | i \in I_d^-} \alpha_{id} \cdot \delta_{idic}^{OOA} \quad \forall c \in C, i \in I \quad (3.5)$$

(II.3) The *OOA demand for different items across channels* $D_{ic}^{OOA(3)}$ materializes when a customer intends to buy an item j in channel $d, d \in C$ while the item is neither listed in channel d ($j \in I_d^-, d \in C$) nor in channel c ($j \in I_c^-, c \in C$) and therefore opts to switch to a different item $i, i \neq j$ in a different channel $c, c \neq d$. The rate $\eta_{j_d i_c}^{OOA}$ quantifies the share that is substituted. Equation (3.6) summarizes the OOA demand substitution for this relation and the corresponding density function is denoted by $f_{D_{ic}^{OOA(3)}}^*$.

$$D_{ic}^{OOA(3)} = \sum_{d \in C / \{c\} | j \in I_d^- / \{i\}} \alpha_{jd} \cdot \eta_{j_d i_c}^{OOA} \quad \forall c \in C, i \in I \quad (3.6)$$

(III) Out-of-stock demand OOS substitution demand represents unsatisfied demand due to insufficient stock of listed items. In this case, the demand D_j of an item j exceeds the available inventory x_j . D_j is assumed to be equal to space-elastic demand D_j^{SP} as the item is listed and the shelf and webshop representation is still visible to the customer (e.g., via price tags). One round of substitution is assumed as for the OOA. As for the OOA demand, OOS demand needs to be differentiated between (III.1) different items within the same channel, (III.2) identical items across different channels and (III.3) different items across different channels.

(III.1) *OOS demand for different items within a channel* $D_{ic}^{OOS(1)}$ for an item i in channel c ($i \in I_c^+$) occurs when a customer's demand for listed item j ($j \neq i, j \in I_c^+$) exceeds the available quantity x_{jc} of item j in channel c . The customer may then decide to substitute the shortage quantity with item i within the same channel c with the rate $\gamma_{j_c i_c}^{OOS}$. Equation (3.7) calculates this demand type and the corresponding density function is denoted by $f_{D_{ic}^{OOS(1)}}^*$.

$$D_{ic}^{OOS^{(1)}} = \sum_{j \in I_c^+ / \{i\}} [(D_{jc}^{SP} - x_{jc}) | D_{jc}^{SP} > x_{jc}] \cdot \gamma_{jc}^{OOS} \quad \forall c \in C, i \in I \quad (3.7)$$

(III.2) The *OOS demand for identical items across channels* $D_{ic}^{OOS^{(2)}}$ for a listed item i of channel c ($i \in I_c^+$) appears when demand for a listed item i in channel d ($c \neq d, i \in I_d^+$) exceeds the item's inventory x_{id} . Thus, the unsatisfied demand in channel d may be substituted by switching to channel c and purchasing the identical item i . This share is quantified with the rate δ_{id}^{OOS} . Equation (3.8) summarizes this and the corresponding density function is denoted by $f_{D_{ic}^{OOS^{(2)}}}^*$.

$$D_{ic}^{OOS^{(2)}} = \sum_{d \in C / \{c\} | i \in I_d^+} [(D_{id}^{SP} - x_{id}) | D_{id}^{SP} > x_{id}] \cdot \delta_{id}^{OOS} \quad \forall c \in C, i \in I \quad (3.8)$$

(III.3) The *OOS demand for different items across channels* $D_{ic}^{OOS^{(3)}}$ materializes when a customer intends to buy an item $j, j \in I$ in channel $d, d \in C$ but the item is sold out both in channel d ($j \in I_d^+, d \in C$) and in channel c ($j \in I_c^+, c \in C$). The customer then opts to switch to a different item $i, i \neq j$ in a different channel $c, c \neq d$. The rate η_{jd}^{OOS} quantifies the share that is substituted. Equation (3.9) summarizes this and the corresponding density function is denoted by $f_{D_{ic}^{OOS^{(3)}}}^*$.

$$D_{ic}^{OOS^{(3)}} = \sum_{d \in C / \{c\} | j \in I_d^+ / \{i\}} [(D_{jd}^{SP} - x_{jd}) | D_{jd}^{SP} > x_{jd}] \cdot \eta_{jd}^{OOS} \quad \forall c \in C, i \in I \quad (3.9)$$

Calculating the convolution The algorithm uses the convolution concept to generate the distribution of the product's demand, including substitution effects. Since the demand distributions of the products i and j are independent for $i \neq j$, the convolution – represented by the operator \otimes – of the related demand distribution functions results in the distribution of the sum of the demands of products i and j (Hübner and Schaal, 2017a). Furthermore only non-negative demand is allowed, so all following distributions are restricted to \mathbb{R}_0^+ . Assuming the distributions are standardized to the feasible interval, the convolution of the additional demand distributions accounting for OOA and OOS for the item sets I_c^- and I_c^+ , respectively, are given as denoted below. As noted above, the convolution requires independent demand distributions. With the ED model and the given demand components, each element can be specified independently and exogenously. Convolutions are not possible when the independence of the distribution is not given. This is the case for complementary effects across products or across channels. Such cases require coupling techniques and other approximation techniques (see e.g., Netessine and Zhang (2005) who included cross-selling effects).

In our setting, the corresponding density function for $D_{ic}^{OOA(1)}$ is calculated by Equation (3.10). It convolutes the (minimum) demand distribution functions of all OOA items and therefore accounts for the fact that the OOA substitution demand for item i ($i \in I_c^+$) in channel c depends on all OOA items $j \in I_c^-$ of the channel c . Since the substitution parameters γ , δ and η only represent a factor, they will be omitted in the formulas to simplify the notation. Equation (3.11) computes the respective density function for $D_{ic}^{OOA(2)}$. In this case the demand for a listed item i ($i \in I_c^+$) in channel c depends on the convolution \otimes of the demand distribution of all delisted, identical items i in all other channels d ($i \in I_d^-, i \in I_c^+$). Equation (3.12) computes the respective density function for different items across channels $D_{ic}^{OOA(3)}$.

$$\otimes_{j \in I_c^-} f_{\alpha_{jc}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, j \in I_c^-} f_{\alpha_{jc}}^* d\tau \dots dv \quad (3.10)$$

$$\otimes_{i \in I_d^-} f_{\alpha_{id}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, i \in I_d^-} f_{\alpha_{id}}^* d\tau \dots dv \quad (3.11)$$

$$\otimes_{j \in I_d^-} f_{\alpha_{jd}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, j \in I_d^-} f_{\alpha_{jd}}^* d\tau \dots dv \quad (3.12)$$

Equation (3.13) depicts the density function for OOS demand for item i in channel c . As above, we use the convolution function to account for the fact that OOS demand for an item i depends on the expected shortage of all temporarily unavailable items in channel c other than item i . Similarly, Equation (3.14) is used to compute the density function for the OOS demand of identical items across channels. Equation (3.15) computes the density for different items across channels.

$$\otimes_{j \in I_c^+} f_{D_{jc}^{SP}}^* = \int \cdots \int_{x_{jc}, j \in I_c^+}^{\infty} f_{D_{jc}^{SP}}^* d\tau \dots dv \quad (3.13)$$

$$\otimes_{i \in I_d^+} f_{D_{id}^{SP}}^* = \int \cdots \int_{x_{id}, i \in I_d^+}^{\infty} f_{D_{id}^{SP}}^* d\tau \dots dv \quad (3.14)$$

$$\otimes_{j \in I_d^+} f_{D_{jd}^{SP}}^* = \int \cdots \int_{x_{jd}, j \in I_d^+}^{\infty} f_{D_{jd}^{SP}}^* d\tau \dots dv \quad (3.15)$$

To calculate the total demand for item i , Equation (3.16) convolutes the demand density functions of D_{ic}^{SP} , $D_{ic}^{OOA(1)}$, $D_{ic}^{OOA(2)}$, $D_{ic}^{OOA(3)}$, $D_{ic}^{OOS(1)}$, $D_{ic}^{OOS(2)}$ and $D_{ic}^{OOS(3)}$.

$$\begin{aligned}
f_{ic}^* = & f_{D_{ic}^{SP}}^* \cdot \left(\bigotimes_{j_c \in I^-, j \neq i} f_{\alpha_{jc}}^* \gamma_{jcic}^{OOA} \right) \Big|_{\gamma_{jcic}^{OOA} \neq 0} \cdot \left(\bigotimes_{i_d \in I^-, i = i} f_{\alpha_{id}}^* \delta_{idic}^{OOA} \right) \Big|_{\delta_{idic}^{OOA} \neq 0} \\
& \cdot \left(\bigotimes_{j_d \in I^-, j \neq i} f_{\alpha_{jd}}^* \eta_{jdic}^{OOA} \right) \Big|_{\eta_{jdic}^{OOA} \neq 0} \cdot \left(\bigotimes_{j_c \in I^+, j \neq i} f_{D_{jc}^{SP}}^* \gamma_{jcic}^{OOS} \right) \Big|_{\gamma_{jcic}^{OOS} \neq 0} \\
& \cdot \left(\bigotimes_{i_d \in I^+, i = i} f_{D_{id}^{SP}}^* \delta_{idic}^{OOS} \right) \Big|_{\delta_{idic}^{OOS} \neq 0} \cdot \left(\bigotimes_{j_d \in I^+, j \neq i} f_{D_{jd}^{SP}}^* \eta_{jdic}^{OOS} \right) \Big|_{\eta_{jdic}^{OOS} \neq 0}
\end{aligned} \tag{3.16}$$

Complexity of the problem Each item i in each channel c can be assigned a value (π_{ic}) and a weight ($k_{ic} \cdot b_{ic}$) that needs to be assigned to multiple knapsacks with limited space (S_c) in each channel. A knapsack problem assuming a linear objective function and linear constraints is already known to be NP-hard (Kellerer et al., 2010). Our model is a NP-hard multi-knapsack problem with a non-linear and non-separable (quadratic) objective function. Despite the efficient solution of knapsack problems using MIP solvers, the items' mutual dependencies due to the substitution of demand make this problem combinatorial hard. An increasing number of items and available space aggravates this complexity. According to Equation (3.17), there are 2,025 possibilities to fully allocate the assortment in an OC setup with $C=2$, $I_c=3$ and $S_c=8$ for all $c \in C$. An instance with $C=2$, $I_c=10$ and $S_c=30$ for all $c \in C$ already results in $4.4908023 \cdot 10^{16}$ possible solutions.

$$P(I_c, S_c, C) = \binom{I_c + S_c - 1}{S_c}^C = \left(\frac{(I_c + S_c - 1)!}{S_c!(I_c - 1)!} \right)^C \tag{3.17}$$

Each of the solution possibilities implies a varying substitution demand given the mutual dependencies between the items. Thus, to obtain the optimal solution, the demand calculation for each possible solution has to be included in the pre-calculation. With current computing power, however, it is neither reasonable nor feasible to make such a large number of pre-calculations. A Binary Integer Program (BIP) embedded in an iterative heuristic is therefore developed in the following. The BIP ensures

all constraints, but neglects the demand substitution. Substitutions are added ex-post. This degenerates the original quadratic problem into a bounded 0/1 multi-choice multi-knapsack problem given a set of item-facing combinations ($x_{ic} \in \{0, 1\}$). Each combination is associated with size b_{ic} , facing-dependent profit π_{ic} , $i \in \mathbb{N}$ and knapsacks with capacity S_c . The algorithm will be described in the next section.

3.3.3 Solution approach

This section develops a specialized heuristic. It is denoted as **Joint Omni-Channel Inventory and Assortment Optimization (JOCIAO)**. The basic idea is to iteratively solve a BIP model (which is a reformulation of the MIP introduced above) and update the demand distributions until there is no more change in solutions. We first provide an overview and general idea of JOCIAO before detailing the computation process.

Overview We take advantage of the fact that retailers limit the number of facings and that the facings need to be integer. The number of facings for each item i in channel c is accordingly restricted through $k_{ic} = [k_{ic}^{min}, k_{ic}^{max}]$. By limiting k_{ic} , the profit π_{ick} for each item i , each channel c and each facing k can be determined with the help of Equation (3.1). This is subsequently used as an input for the Objective Function (3.18), which computes the overall profit Π . The binary variable z_{ick} thereby states whether an item i in channel c receives k facings ($z_{ick} = 1$) or not ($z_{ick} = 0$). The variable z_{ick} can be split up into the auxiliary variables k_{ic} and x_{ic} as the number of facings can be computed with $k_{ic} = k \cdot z_{ick}$ and the inventory with $x_{ic} = g_{ic} \cdot k_{ic}$. The quantity in each channel c is restricted by a physical space limit S_c . The space consumption issues from the number of facings k_c in channel c and the space b_{ic} occupied by one facing of item i in channel c (cf. Equation (3.19)). Equation (3.20) states that each item i in each channel c receives exactly one facing, and Equation (3.21) defines z_{ick} as a binary variable.

$$\max \Pi(\bar{z}) = \sum_{i \in I} \sum_{c \in C} \sum_{k \in K} \pi_{ick} \cdot z_{ick} \quad (3.18)$$

subject to

$$\sum_{i \in I} \sum_{k \in K} k_c \cdot b_{ic} \cdot z_{ick} \leq S_c \quad \forall c \in C \quad (3.19)$$

$$\sum_{k \in K} z_{ick} = 1 \quad \forall c \in C, i \in I \quad (3.20)$$

$$z_{ick} \in \{0; 1\} \quad \forall c \in C, i \in I, k \in K \quad (3.21)$$

However, this BIP is nonlinear as the item profit and demand depend on the number of facings of the item itself and, given the substitutions, on other items, too. We develop a two-stage solution heuristic to overcome the non-linearity. In the initialization (Stage 1), the item profit π_{ick} for all possible $i \in I$, $c \in C$ and $k \in K$ is precalculated taking into account space-elastic demand excluding substitution demand. This is fed into the BIP to maximize overall profit for an initial solution. This initial solution of the BIP makes it possible to calculate the substitution demand and update the total demand given the assortment and number of facings (k) as well as inventory (x) of all items obtained from the BIP. In the iteration phase (Stage 2), the BIP is solved again as demand changes. Following that, substitution demand and total demand are again updated based on the new solution. This process is repeated until a stop criterion is met (e.g., no more change in facings from one solution to the next). We are applying a related approach of Hübner and Schaal (2017a), which has been proven to deliver efficient results for SC assortment optimization.

Iterative Heuristic After providing the overview above, we will detail the implementation and computation process of the two-stage solution approach in the following.

Stage 1 – Initialization The first stage (see Figure 3.1) helps to overcome the non-linearity by pre-calculating demand and profit for a given set of integer facings. In Step 1.1, the iteration index ℓ is set to zero. In Step 1.2, Equation (3.1) is used to compute the profit π_{ick}^ℓ of iteration ℓ for every item i in channel c and every k in the range from $k_{ic} = [k_{ic}^{min}, k_{ic}^{max}]$. Substitution effects are excluded at this stage and the demand density function only includes space-elastic demand ($f_{ic}^{*,\ell} = f_{D_{ic}^{SP}}^{*,\ell}$) as denoted in Equation (3.3). As this contains only invariants, this demand density function can be calculated for any given facing without invoking the decisions of other items. In Step 1.3, π_{ick}^ℓ serves as input to solve the BIP model and obtain z_{ick}^ℓ . This determines k_{ic}^ℓ and x_{ic}^ℓ in the sets I and C considered. Given this assortment, number of facings and inventory levels, the initial demand density function $f_{ic}^{*,\ell}$ is updated in Step 1.4 using Equation (3.16). Thus, an initial solution up to Step 1.3 excludes substitution demand while the demand density functions f_{ic}^* obtained from Step 1.4 onwards include substitution demand.

Stage 2 – Iterations In the optimization stage, the initial solution is optimized by taking into account substitution effects. Step 2.1 constitutes a loop and will be explained in the course of Step 2.6. In Step 2.2, the iteration index is updated from ℓ to $\ell + 1$. Step 2.3.1 sets the demand distribution being used in the current iteration ($f_{ic}^{*,\ell}$) equal to the one obtained in the previous iteration, which includes OOA and OOS substitution demand for all iterations $\ell \geq 1$. In the next step (Step 2.3.2), π_{ick}^ℓ is pre-calculated and updated for every item, channel and facing. In Step 2.4 we use the updated item profits π_{ick}^ℓ to solve the BIP with substitution effects, which provides us with the corresponding z_{ick}^ℓ . Step 2.5 updates the demand density function $f_{ic}^{*,\ell}$ for each item by means of the convolution of the relevant demand density functions, taking into account k_{ic}^ℓ and x_{ic}^ℓ for all items and channels. Step 2.6 mandates the algorithm to restart at Step 2.1 until z_{ick}^ℓ of two subsequent iterations remains unchanged, which also implies no more change in profit (cf. Equation (3.22))

$$\epsilon = z_{ick}^\ell - z_{ick}^{\ell-1} = 0 \quad \forall i \in I, c \in C, k \in K \quad (3.22)$$

As described, substitution demand can only be calculated for a given assortment, facing and inventory decision for each item in each channel. The heuristic addresses this issue by updating the assortments, facings and inventories in iteration ℓ with the solution of the previous iteration $\ell - 1$, i.e., a one-iteration lag with respect to the demand. This implies potentially non-optimal overall results even though the heuristic itself solves the problem optimally for every single iteration. This potential concern will be analyzed in the numerical analysis.

3.4 Numerical analysis and managerial insights

3.4.1 Overview of tests and data

The following section analyzes the computational performance of our approach and develops managerial insights into substitutions with OC assortments. Table 3.3 summarizes the tests.

Each data set is composed of 50 randomly generated instances. This means all results presented show the average of the corresponding 50 solutions. Unless stated otherwise, we apply identically populated parameters across channels to avoid mixing different effects. This is particularly relevant as we deal with different demand sources, revenues, costs and channel setups. When including too many factors at the same time one obtains mixed results and cannot derive causal insights. In order to simulate close-to-reality conditions, ranges have been defined for all parameters in use. The revenue and cost parameters are defined as $r_{ic} \in [20, 50]$, $u_{ic} \in [15, 30]$, $v_{ic} \in [4, 20]$ and $s_{ic} \in [1, 5]$. In addition to that, the values for each item in each channel have to comply with the inequality $r_{ic} \geq c_{ic} \geq v_{ic} \geq s_{ic}$. Compared to all other revenues and costs, shortage costs are only steering costs to factor in unsatisfied customers. If not specified further, we assume

Table 3.3: Overview of numerical experiments

Section	Rationale	Analysis	Main criteria	# instances
3.4.2	Efficiency of algorithm	Comparison with full enumeration and greedy heuristic; Large instances	Objective values, run time	760
3.4.3	Relative impact of substitutions within and across channels	Varying substitution rates within and across channels; Varying costs across channels	Objective values	550
3.4.4	Impact of cross-channel substitutions for identical products	Varying substitution rates across channels	Objective values, assortment size, facings	250
3.4.5	Impact of connecting assortments across channels (OC vs. MC assortments)	Effect of cross-channel substitution; Value of information in OC retailing	Objective values	2,000

shortage costs of $s_{ic} = 0$ without loss of generality to streamline the analysis. This setting represents customers who encounter unavailable items but do not cause dissatisfaction costs as a convenient substitution is offered. A normal demand distribution is assumed with $\mu_{ic} \in [7, 25]$ and the coefficient of variation $CV_{ic} \in [1\%, 50\%]$. This is based on the premise that a continuous demand distribution closely resembles a discrete demand distribution (Hübner et al., 2016b). To avoid the appearance of negative demand, we compute the demand dispersion using CV_{ic} . We assume conformity of OOA and OOS substitutions (cf. e.g., Campo et al. (2004)) with $\gamma_{jcic}^{OOA} = \gamma_{jcic}^{OOS}$ and $\delta_{idic}^{OOA} = \delta_{idic}^{OOS}$. The case of substitution demand for different items across channels is not considered as this is assumed to pose a small magnitude, hence $D_{ic}^{OOA(3)} = D_{ic}^{OOS(3)} = 0$. We define an aggregated in-channel substitution rate for OOA with $\Gamma_{ic}^{OOA} = \sum_{j \in I} \gamma_{jcic}^{OOA}$ and aggregated cross-channel substitution rate with $\Delta_{ic}^{OOA} = \sum_{d \in C} \delta_{idic}^{OOA}$. The total OOA substitution rate of an item i in channel c is denoted as $\theta_{ic}^{OOA} = \Gamma_{ic}^{OOA} + \Delta_{ic}^{OOA}$. It is analogously defined for OOS with Γ_{ic}^{OOS} , Δ_{ic}^{OOS}

and θ_{ic}^{OOS} . Both aggregated rates θ_{ic}^{OOA} and θ_{ic}^{OOS} need to be below 100%. We assume that $\gamma_{jci_c}^{OOA} = \frac{\Gamma_{ic}^{OOA}}{I-1}$ and $\delta_{iadi_c}^{OOA} = \frac{\Delta_{ic}^{OOA}}{C-1}$. We exclusively consider the OC case of BOPS to once again streamline the analysis and not to mix too many effects. Substitution from the store channel d to the online channel c is therefore $\delta_{iadi_c}^{OOA} = \delta_{iadi_c}^{OOS} = 0$. This makes $\theta_{ic}^{OOA} = \Gamma_{ic}^{OOA}$ for the store channel as $\Delta_{ic}^{OOA} = 0$. Space-elasticity $\beta_{ic} \in [0\%, 35\%]$, for the store channels and $\beta_{ic} = 0$ for all online channels is in line with Eisend (2014). To foreclose any noise in the results, all items are assigned a width $b_{ic} = 1$, a quantity per facing $g_{ic} \in [3, 6]$ for stores and a quantity per package unit $g_{ic} = 10$ for online channels. Available space S_c is limited. It fulfills $\sim 80\%$ of the sum of the basic store demand of all items and $\sim 90\%$ of the sum of the basic online demand of all items, and thus does not completely satisfy the basic demand α_{ic} for all items i in the channel c .

A machine running on Windows 10 64-bit with an Intel Core i7-7600U CPU 2.80GHZ and 16 GB of installed memory was used for numerical tests. The model and algorithm are implemented in Python 3.6 using PyCharm and solved with Gurobi Optimizer.

3.4.2 Efficiency of the algorithm

In this section we execute a range of numerical tests to assess the heuristic's performance. First, we provide the performance results of our algorithm in comparison to an exact approach. Thereafter we test the run time for large instances, for which optimal solutions cannot be computed anymore. A comparison with an alternative heuristic approach follows in the final tests of this section.

Comparison with full enumeration A set of small instances was solved with JOCIAO and a full enumeration to obtain exact results as a benchmark. The test was carried out for small problem sizes. Table 3.4 indicates the exponentially growing run time with increasing instance size of the exact

solution approach. Compared to the exact approach our solution approach provides significantly better run times while delivering near optimal results. On average, the model accomplishes a solution quality of 99.42% to 99.98%.

Table 3.4: Comparison of JOCIAO to full enumeration: run time and solution quality

Number of items I_c per channel	2	2	3	3	4
Space S_c in store, online warehouse	4, 2	6, 3	6, 3	8, 4	12, 6
Number of instances	30	30	30	10	10
Avg. run time JOCIAO [seconds]	2.2	2.4	5.3	7.4	10.3
Avg. run time full enumeration [seconds]	260.5	443.7	3844.0	14,305.2	> 18,000
Avg. solution quality ¹	99.42%	99.49%	99.89%	99.98%	n/a

¹ Total profit obtained by JOCIAO / Total profit obtained by full enumeration

Run time tests with large instances Retailers usually run assortment optimization for categories with 50 to 100 items. Assortment, space and inventory selection is commonly carried out periodically in the course of a cyclical assortment review (e.g., for fashion retailers with seasonal items or for food retailers with an annual review of permanent assortments). In order to allow for the computation of different scenarios, a reasonable processing time even for large assortments is required. To test this prerequisite, we generated a set of large instances and solved it with our model. Table 3.5 illustrates the results that exhibit reasonable solution times for the tactical problem.

Table 3.5: Application of JOCIAO to large instances: run time

Number of items I_c per channel	20	40	60	80	100
Space S_c in store, online warehouse	60, 30	120, 60	180, 90	240, 120	300, 150
Avg. run time [minutes]	4.3	13.1	40.3	74.0	117.1

Comparison with a greedy heuristic This section performs the comparison of JOCIAO with a greedy heuristic. The greedy heuristic strikes through its short run time while providing solid results for assortment and knapsack problems. The item profit is calculated for each potential facing

value in the store and in the online warehouse. Equation (3.3) is called to base the profit on space-elastic demand D_{ic}^{sp} . All items I_c for each channel are then ranked according to their item profit taking into consideration the space requirements b_{ic} and g_{ic} and allocated to space S_c in decreasing order. Once space S_c is used up, the decision and auxiliary variables k_{ic} and x_{ic} are obtained to compute $D_{ic}^{OOA^{(1)}}$, $D_{ic}^{OOS^{(1)}}$, $D_{ic}^{OOA^{(2)}}$ and $D_{ic}^{OOS^{(2)}}$ using Formulas (3.4), (3.5), (3.7) and (3.8). Lastly, the resulting profit is calculated ex-post with Equation (3.1). The greedy heuristic has the disadvantage of not directly incorporating the substitution effects. As such we first use randomly generated substitution rates, and thereafter substitution rates within certain ranges.

Table 3.6: Comparison of JOCIAO with greedy heuristic: run time and profit change

Items I_c per channel	5	10	20	30	20	20	20	20	20
Subst.rate ¹ Γ_{ic}^{OOA} , Γ_{ic}^{OOS}	rand.	rand.	rand.	rand.	rand.	rand.	rand.	rand.	rand.
Subst.rate ² Δ_{ic}^{OOA} , Δ_{ic}^{OOS}	rand.	rand.	rand.	rand.	0%	10%	20%	30%	40%
Space S in store, online	15, 7	30, 15	60, 30	90, 45	60, 30	60, 30	60, 30	60, 30	60, 30
Avg. run time [sec.]	16.0	56.5	259.9	581.9	179.5	201.4	181.3	310.1	445.1
Avg. profit change ³	1.8%	1.1%	0.8%	0.9%	0.6%	0.6%	0.7%	0.9%	1.0%

¹ In-channel

² Cross-channel from webshop d to store c

³ (Total profit obtained by JOCIAO / Total profit obtained by greedy heuristic) - 1

Table 3.6 summarizes the results for different assortment sizes and substitution rates. Generally, JOCIAO provides a clear profit advantage at the cost of longer run times. It yields an average profit advantage of up to 1.8% over the greedy heuristic. The profit advantage decreases with increasing assortment sizes as additional iterations and thereby alterations of k_{ic} have, in relative terms, a lower impact on profit for large instances. Furthermore, the profit advantage increases with higher cross-channel substitution rates.

3.4.3 Managerial insights on the effect of in-channel vs. cross-channel substitution

In our first analysis to obtain managerial insights into substitution effects and their impact on OC assortments and total retail profitability, we investigate different substitution patterns within and across channels with a data set with $I_c = 20$. We apply a benchmark with an aggregated in-channel substitution rate of 20% for all items within the online channel, 20% within the store, and additionally an aggregated cross-channel substitution rate of 20%. A total of 60% of the demand for a certain product may be substituted as a result. As a comparison, we created five further data sets without any in-channel substitution $\Gamma_{ic}^{OOA} = \Gamma_{ic}^{OOS} = 0\%$, but aggregated cross-channel substitutions of $\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS}$ in the range of 40% to 80%. Only in the case of 60% do we have the identical total substitution rate $\theta_{ic} + \theta_{id}$ (within and across channels), whereas the other data sets go along with lower or higher substitution potential.

Table 3.7: Impact of in-channel and cross-channel substitution rates, with identical costs across channels: profit change

	Γ_{ic} webshop	Γ_{id} store	Δ_{ic} webshop to store	Sum	Avg. profit change ¹
Benchmark	20%	20%	20%	60%	
Data set 1	0%	0%	40%	40%	-3.4%
Data set 2	0%	0%	50%	50%	-3.3%
Data set 3	0%	0%	60%	60%	-3.1%
Data set 4	0%	0%	70%	70%	-2.9%
Data set 5	0%	0%	80%	80%	-2.7%

¹ (Total profit of data set / Total profit of benchmark) - 1

Table 3.7 shows that the benchmark clearly results in higher profits. It can thus be concluded that in-channel substitution is more beneficial than equally high webshop-to-store substitution. In fact, not even a higher aggregated substitution rate of 70% or 80% leads to a profit advantage. The main driver is additional in-store demand that fuels the profit advantage. Substitutions to items with higher profits are listed within the channel,

instead of the identical items that are listed in the other channel, but with the same profitability as the unavailable item.

Next, we analyze the impact of cross-channel substitutions in combination with cost differences across channels. This serves to understand the cost differences required between channels to obtain break-even. Unit costs in the store, as the demand-receiving channel, were therefore gradually reduced while all other parameters remain constant. The above-described benchmark and data set 3 are applied to obtain identical substitution levels in both cases.

Table 3.8: Sensitivity analysis of varying store unit costs for data set 3: profit change

	Decrease in store unit cost u_{ic} ¹					
	-10%	-8%	-6%	-4%	-2%	0%
Avg. profit change ²	3.3%	2.0%	0.7%	-0.5%	-1.8%	-3.1%

Note: Based on 50 randomly generated instances for $I_c = 20$; no shortage costs

¹ Decrease in store unit costs for data set 3 compared to store unit costs for benchmark

² (Data set 3 profit / Benchmark profit) - 1

Table 3.8 summarizes the impact on total profit. As already stated above, with identical unit costs across channels and no in-channel substitution, we obtain a 3.1% lower profit on average than with in-channel substitution. However, the profit disadvantage turns into a profit advantage once a unit cost decrease of around 4-6% can be realized in the store. This is the break-even point, where lower unit costs counterbalance the unfavorable substitutions across channels instead of within channels. Given that a cost reduction of 4-6% is a challenging objective for low-margin retailers, we can therefore summarize that it is clearly worth supporting and incentivizing in-channel substitution.

To sum up, substitution across channels gives the retailer more opportunities to offer the desired products, but as the analysis shows, substitution within a channel is more beneficial for retailers when profits for each item are identical across channels. Missing substitutions within a channel cannot be compensated by substitutions across channels unless there are significant cost differences across channels. Thus, when optimizing assortments, a

primary focus has to be given to in-channel substitution for profitable items. This does not just present higher profit potential: it is also easier and less costly to execute, using aids such as technology-enabled mechanisms.

3.4.4 Managerial insights on the effect of substituting identical products across channels

Customers exhibit a notably high affinity for certain items or brands in industries such as fashion or electronics, which poses a great opportunity for OC compared to SC or MC retailers. In order to obtain the item they want, customers are very willing to switch channels, especially if the transition is convenient, and do not consider other items an alternative. This special case is examined by setting the cross-channel substitution rate with $\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS}$ to up to 80% for all items. To allow for such a high product affinity, in-channel substitution rate across different items is $\Gamma_{ic}^{OOA} = \Gamma_{ic}^{OOS} = 0\%$. As we want to assess the potential of substitution across channels, we apply no cross-channel substitution as the reference case ($\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS} = 0\%$). The reference setting resembles an MC retailer that operates two separate channels, but without any links between them. This benchmark analysis indicates the overall potential for integrating the channels and allowing easy cross-product substitutions when moving from MC to OC for categories with high product affinity. As we focus on substitutions from online to store, only the solution structures within the store are affected and reported in the following.

Table 3.9 shows that for OC assortments with high-affinity items and equal costs across channels, the average profit advantage can reach up to 1.42% compared to an MC setup. By motivating customers to switch across channels for high-affinity items, profits increase in all cases. The increase in total profits is solely driven by an increase in store profits as the store receives additional cross-channel substitutions from unsatisfied demand of the webshop. Less profitable items are thereafter delisted or decreased in

Table 3.9: Impact of omni-channel substitution on high-affinity products: profit change and assortment structure

		Cross-channel subst. rate Δ_{ic}^{OOA} , Δ_{ic}^{OOS}			
		20%	40%	60%	80%
Avg. profit change ¹	Total	0.33%	0.66%	1.03%	1.42%
Avg. change of assortment size ²	Store	-1.6%	-2.3%	-2.8%	-4.2%
Avg. no of items with facing changes ³	Store	6.0%	10.6%	15.9%	19.9%

Note: Based on 50 randomly generated instances for $I_c = 20$; no shortage costs

¹ (Total profit with substitutions / Total profit without substitutions) - 1

² (No of. items i_c in OC assortment / No of. items i_c in MC assortment) - 1

³ Share of items in OC setting with different facing number than in MC setting

stock while more profitable items with additional cross-channel substitution demand are listed or increased in stock. This redistribution of store space is also mirrored by decreasing assortment size and increasing facing changes when substitution rates grow.

3.4.5 Managerial insights on the effect of omni-channel vs. multi-channel retailing

Impact of omni-channel substitution In the following analysis we further investigate the effects of OC and MC assortments in a more general setting by comparing an OC retailer with an MC retailer. To compute the profit impact, the assortment optimization for the MC retailer is performed without cross-channel substitution ($\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS} = 0$). Table 3.10 shows the results of the analysis and demonstrates that without shortage costs, cross-channel substitution leads to a profit increase of up to 1.06% on average. Of the 800 instances tested in Table 3.10, over 98.6% of the items achieved a profit advantage from applying cross-channel effects compared to the MC setup. As expected, higher substitution rates lead to growing profit advantages over MC retailing. Quadrupling the cross-channel substitution rate leads to at least a quadruplication of the profit advantage.

Therefore, achieving an increase of cross-channel substitution rates, for example through sales incentives or supportive technology, depicts a great opportunity for retailers to mark up their total profit. This is despite the higher impact of in-channel substitutions (see Section 3.4.3).

Table 3.10: Impact of omni-channel substitution with $s_{ic} = 0$: profit impact of OC over MC

	I_c	Cross-channel subst. rate $\Delta_{ic}^{OOA}, \Delta_{ic}^{OOS}$				Growth factor from 10% to 40%
		10%	20%	30%	40%	
Avg. profit change ¹	5	0.25%	0.50%	0.80%	1.06%	4.3
	10	0.19%	0.40%	0.61%	0.82%	4.3
	20	0.17%	0.35%	0.55%	0.74%	4.4
	30	0.18%	0.38%	0.58%	0.78%	4.4
Max. profit change ¹	5	1.05%	2.21%	3.22%	4.06%	na
	10	0.46%	1.46%	1.94%	2.39%	na
	20	0.38%	0.76%	1.12%	1.42%	na
	30	0.35%	0.78%	1.16%	1.60%	na

¹ (OC profit / MC profit) - 1

In Table 3.11 we detail the solution structure of one representative data set from Table 3.10 with $N = 20$ and $\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS} = 20\%$. For this data set, 55.5% of the profit is contributed by the store and 44.5% by the webshop. All of the products listed in the store are also listed in the webshop as the OC retailer focuses on the most profitable items, which are identical across channels. The store receives more demand given the space elastic demand and cross-channel substitutions from the webshop to the store. Additional demand can be leveraged by the store by delisting less profitable items and vacating space for the (already listed) more profitable items. These effects in combination with slightly more total space in warehouses result in 48% larger assortments in the webshop than the store. This constitutes a realistic setting as retailers tend to list more items online, particularly items with low demand (i.e. the long tail).

Table 3.11: Representative solution in detail, N=20

	Profit	Assortment size		Space-el. demand	Substitutions
	% of total	no. products	% of total	% of total	% of total
Store	55.5%	12.5	62.6%	51.5%	5.0%
Webshop	44.5%	18.5	92.5%	41.8%	1.4%

Impact of position-dependent demand We further analyse the impact of OC and MC assortments via an extension of the demand model with position-dependent demand in the store. We assume that the positioning of items on shelf segments has an impact on demand (see also Hwang et al. (2005); Hübner and Schaal (2017b); Bianchi-Aguiar et al. (2021)). Total space is equally split up into a number of shelf segments $t, t \in T$, which are thereby also space constrained. The shelf segments can be differentiated vertically (e.g., eye-level vs. knee-level) and horizontally (e.g., beginning vs. center of aisle). The variables z_{ickt} and k_{ict} are extended with the shelf segments t to also include the assignment of facings to shelf segments. Different customer visibility for each segment may result in a further demand effect that is modeled with the parameter λ_{ct} . It denotes the attractiveness of each shelf segment $t \in T$ in each channel $c \in C$. In the general case, an item can be assigned to multiple shelf segments t at the same time. We therefore calculate an average attractiveness parameter λ_{ickt} with $\lambda_{ickt} = [\sum_{t \in T} k_{ict} \cdot \lambda_{ct}] / k_{ic}$ which depends on the facings distribution across the segments. The parameter λ_{ickt} can be obtained in precalculations. Consequently, we incorporate the position-dependent demand in the total demand calculation of Equation (3.2). This requires to replace space-dependent demand D_{ic}^{SP} with the space- and position-dependent demand $D_{ickt}^{SP,Pos}$ that is calculated by:

$$D_{ickt}^{SP,Pos} = \alpha_{ic} \cdot \lambda_{ickt} \cdot k_{ic}^{\beta_{ic}} \quad \forall c \in C, i \in I, k \in K, t \in T \quad (3.23)$$

By means of the replacement, $D_{ickt}^{SP,Pos}$ is also integrated into the calculation of OOS demand and into the algorithm. In the initialization in Stage 1, Step 1.2 needs to be extended for all shelf segments $t \in T$ to calculate the profit π_{ickt}^{ℓ} with $f_{ickt}^{*,\ell} = f_{D_{ickt}^{SP,Pos}}^{*,\ell}$. This is then applied to the BIP to obtain z_{ickt}^{ℓ} and an initial solution. The updated demand is applied in Step 2.3, where demand $f_{ickt}^{*,\ell}$ and profit π_{ickt}^{ℓ} are again calculated additionally for all shelf segments $t \in T$. In Step 2.4 we obtain z_{ickt}^{ℓ} and consequently the facings for all shelf segments to update $f_{ickt}^{*,\ell}$ in Step 2.5. This is iterated

until the stop criteria in Step 2.6 is met. We extended the stop criteria to also hold true for all shelf levels $t, t \in T$.

We want to identify the impact of position-dependent demand on OC assortment planning. For this purpose we focus on a representative data set with $N = 10$ and the same parameter settings as in Table 3.10. This is extended by dividing the total space into three segments ($T = 3$). We streamline the analysis and apply position-dependent demand only in the store and not to the webshop. That means λ_{ct} is zero for the webshop and for the store $\lambda_{ct} = \{1.0, 1.2, 1.4\}$ for $t = \{1, 2, 3\}$. We differentiate the segments into vertical levels where the top level is characterized by higher visibility (e.g., eye-level position) and thereby higher demand for this segment (see e.g., Drèze et al. (1994)).

Table 3.12 shows that in a setting with position-dependent demand the profit advantage of OC over MC is 0.43% for a cross-channel substitution rate of 40%. This is approx. 50% lower than the profit advantage of OC over MC without position-dependent demand (0.82%, see Table 3.10). The difference can be attributed to the higher demand in the retail store through the added position-dependent demand, which reduces the relative profit advantage of OC over MC. Assortment size changes have a similar magnitude as without position-dependent demand. Overall, it can be constituted that the position-dependent demand has an impact on the profitability, but when assessing OC vs. MC assessments it is relatively small.

Table 3.12: Impact of omni-channel substitution in a demand model with position-dependent demand

		Cross-channel subst. rate	
		$\Delta_{ic}^{OOA}, 20\%$	$\Delta_{ic}^{OOS}, 40\%$
Avg. profit change ¹	Total	0.21%	0.43%
	Store	0.36%	0.73%
	Webshop	0.00%	0.00%

Note: Based on 50 randomly generated instances for $I_c = 10$; no shortage costs

¹ (Total profit with substitutions / Total profit without substitutions) - 1

Impact of shortage costs A further analysis showcases the impact of the shortage costs that act as steering costs and additionally penalize non-available items. Hence we now assume $s_{ic} > 0$. Table 3.13 highlights that cross-channel substitution leads to a profit increase of up to 1.84%, but only up to 0.23% on average. The relatively small magnitude is attributed to shortage costs as we assumed that on average only 80 to 90% of total basic demand can be provided with the limited shelf and warehouse space. Assortments therefore tend to have too little space for too much demand. Since demand is added via cross-channel substitution to stores, a higher share of customer demand is unfulfilled and total shortage costs rise. In test cases where the sum of the basic demand exceeds the total space by 25%, the magnitude may even become negative. Only items with relatively high margins are able to leverage their favorable demand-to-space ratios, exploit the additional demand and avoid customer dissatisfaction.

Table 3.13: Impact of omni-channel substitution with $s_{ic} > 0$: profit impact of OC over MC

	I_c	Cross-channel subst. rate $\Delta_{ic}^{OOA}, \Delta_{ic}^{OOS}$			
		10%	20%	30%	40%
Min. profit change ¹	10	-0.29%	-0.53%	-0.76%	-1.01%
	20	-0.19%	-0.33%	-0.48%	-0.63%
	30	-0.20%	-0.40%	-0.54%	-0.68%
Avg. profit change ¹	10	0.05%	0.10%	0.15%	0.22%
	20	0.04%	0.07%	0.12%	0.20%
	30	0.05%	0.10%	0.16%	0.23%
Max. profit change ¹	10	0.75%	1.03%	1.37%	1.84%
	20	0.24%	0.45%	0.62%	0.88%
	30	0.30%	0.53%	0.77%	1.16%

¹ (OC profit / MC profit) - 1

Value of information Finally, we assess the effect of ignoring cross-channel substitution when taking the assortment decision, which can also be interpreted as value of information. This situation may arise when online customers face OOA or OOS and transition to the store, even though the MC retailer does not offer OC services, does not advertise the store in the online channel or is simply not aware of such customer behavior. We look at

an MC retailer that optimizes assortments assuming ($\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS} = 0\%$) while in reality costumers transition from online to retail and substitute items with ($\Delta_{ic}^{OOA}, \Delta_{ic}^{OOS} > 0\%$). The MC solution structure obtained without substitutions is then applied with the actual substitution rates ex-post and compared to the solution with direct integration of substitution effects as in the OC model. We denote this scenario as “Ignoring”. The benchmark is the scenario as described in this paper that directly integrates cross-channel substitution. This is denoted as “Integrating”. Table 3.14 reports the profit increase for the two different scenarios with different substitution rates. The profit increases of up to 1.42% when channel switching is directly integrated the channels (in “Integrating”), whereas the profit with an ex-post evaluation of substitution effects results in an average profit increase of up to 1.12%. The results reveal that retailers suffer up to 0.30% on average and up to 0.81% profit loss (for $\Delta_{ic}^{OOA} = \Delta_{ic}^{OOS} = 80\%$) if they disregard OC substitution in assortment planning. It is therefore recommended that retailers consider OC substitution in their assortment composition to yield additional profits.

Table 3.14: Impact of disregarding omni-channel substitution: profit change

		Cross-channel subst. rate $\Delta_{ic}^{OOA}, \Delta_{ic}^{OOS}$			
		20%	40%	60%	80%
Profit change ¹	“Integrating” Scenario	0.33%	0.66%	1.03%	1.42%
	“Ignoring” Scenario	0.31%	0.59%	0.85%	1.12%
Value of information ²	Avg.	0.02%	0.08%	0.17%	0.30%
	Max.	0.09%	0.31%	0.59%	0.81%

¹ (Retailer profit / MC profit) - 1

² Difference between both scenarios

3.5 Conclusion and future research

In order to account for cross-channel shopping behavior, we described and defined the novel problem of integrated assortment, space and inventory planning in OC retailing. The major difference vs. existing contributions is the acknowledgment and integration of cross-channel substitution behavior in the case of OOA or OOS. We developed the corresponding model to

maximize the retailer's total profit taking into account stochastic demand and substitutable products. The model incorporates space-elastic demand, OOA and OOS substitution demand within a channel as well as OOA and OOS substitution demand across channels. The NP-hard multiple-knapsack problem exhibits increasing run times with the number of items and space constraints. A BIP was therefore developed and employed in the specialized heuristic JOCIAO to efficiently solve the novel problem. In the numerical results and managerial insights we particularly show (counter intuitively) that in-channel substitution matters. This gives retailers the opportunity to transfer demand to more profitable items. Retailers make further gains with cross-channel substitution in cases with high product affinity. A further highlight is the finding that the increase of cross-channel substitution rates increases profits at a similar rate. Being able to limit or control customer dissatisfaction, OC retailing may generate more beneficial profit advantages. Increasing the substitution rate and a seamless transition from online to retail should be a major objective for OC retailers to boost profits. Beyond that, considering cross-channel substitution in the assortment optimization offers extra profits.

Given the novelty of OC assortment planning there is a wide range of future research opportunities. The current model could be extended by including further variables. The first opportunity is to include elements of category planning and expand the scope towards defining category roles and sizes. This includes taking account of complementary effects across categories and channels, such as show- and webrooming effects. Equivalent to space-elasticity in the bricks-and-mortar store, one could include the effects of positioning or highlighting products on a webpage. Depending on the position or the highlight, visibility is improved and therefore demand increases. Likewise, it may be insightful to investigate the effect of assortment sizes and customers' variety perceptions. The extension of the demand model with various effects goes along with the research question on which demand effect matters. A modeling and optimization approach can serve here as input to specify empirical research efforts as well. Another demand effect is cross-selling when customers pick-up a product in

the store and decide to buy another product on top of that. Given the OC customers' superior value (Song et al., 2020), this is often a rationale for implementing BOPS and could be included in the model. As Rooderkerk and Kök (2019) state, retailers also support customers in their customer journey from offline to online, for example by providing terminals or tablets to access the webshop while shopping in the bricks-and-mortar store, laying the groundwork for another extension of the current model. Within our model, webshop customers have access via BOPS to alternatives in the store. Our model can be extended by adding another dimension to the substitution matrix (by product, by channel and by delivery mode) and by differentiating the unit costs by delivery mode to incorporate the differences between BOPS and ship-from-store. While we investigate the assortment planning problem from a tactical perspective without reorder options from suppliers, we also deem an operational or multi-period model valuable research. This requires incorporating reorder options from suppliers, customer fulfillment with backlogging or customer returns. Furthermore, current assumptions of the model provide opportunities for relaxation. For example, future models could assume several rounds of substitution in the event that products are unavailable, potentially with decreasing substitution rates. In addition to that it would be worth focusing on the economics behind different channels by adjusting unit costs and investigating the potential of steering customers into certain channels. Interesting insights could also be generated by examining the relevant demand effects empirically. While OC demand forecasting has already received some attention, (e.g., Cao et al. (2016)), it would be interesting to compare this with empirical results. Given the diverse nature of OC retailing in terms of categories (e.g., fashion, electronics, groceries), store formats (e.g., hypermarkets or convenience stores) and business models (e.g., showrooming or webrooming), it is also worth determining the differing impact OC has on certain combinations. This would also make it possible to derive the most beneficial conditions for specific OC business models, such as categories or store formats, as well as substitution rates and/or assortment structures. This paper and the model and insights derived can serve as starting point for such future research.

Figure 3.1: Pseudo code of JOCIAO

Stage 1 – Initialization: Solve BIP without substitution effects
Input: Set of channels C , set of items in channel I_c and set of possible facings K_{ic}

Step 1.1 Set $\ell = 0$

Step 1.2 For all channels $c \in C$:
 For all items $i \in I_c$:
 For all facings $k \in K_{ic}$:
 Calculate π_{ick}^ℓ with $f_{ic}^{*,\ell} = f_{D_{ic}^{SP}}^{*,\ell}$
 End for
 End for
End for

Step 1.3 Solve BIP using Equation (3.18) to (3.21) to obtain z_{ick}^ℓ and consequently all k_{ic}^ℓ and x_{ic}^ℓ and Π^ℓ

Step 1.4 For all channels $c \in C$:
 For all items $i \in I_c$:
 Deploy k_{ic}^ℓ and x_{ic}^ℓ in Equation (3.16) to update $f_{ic}^{*,\ell}$
 End for
End for

Return: z_{ikl}^ℓ , $f_{ic}^{*,\ell}$ and Π^ℓ

Stage 2 – Iterations: Solve BIP with substitution effects
Input: Set of channels C , set of items in channel I_c , set of possible facings K_{ic} and z_{ikl}^ℓ , $f_{ic}^{*,\ell}$ and Π^ℓ

Step 2.1 Repeat

Step 2.2 Set $\ell = \ell + 1$

Step 2.3 For all channels $c \in C$:
 Step 2.3.1 For all items $i \in I_c$:
 Set $f_{ic}^{*,\ell} = f_{ic}^{*,\ell-1}$
 Step 2.3.2 For all facings $k \in K_{ic}$:
 Calculate π_{ick}^ℓ with $f_{ic}^{*,\ell}$
 End for
End for

Step 2.4 Solve BIP using Equation (3.18) to (3.21) to obtain z_{ick}^ℓ and consequently all k_{ic}^ℓ and x_{ic}^ℓ and Π^ℓ

Step 2.5 For all channels $c \in C$:
 For items $i \in I_c$:
 Deploy k_{ic}^ℓ and x_{ic}^ℓ in Equation (3.16) to update $f_{ic}^{*,\ell}$
 End for
End for

Step 2.6 Until Stop Criteria is met and Equation (3.22) holds true, otherwise continue with Step 2.1

Return: z_{ikl}^ℓ and Π^ℓ

4 An analytical assessment of demand effects in omni-channel assortment planning

The advent of omni-channel (OC) retailing makes assortments seamlessly available across channels and affects customer behavior. Whereas the demand effects within channels are well known, the effects across channels are less clear. A vast variety of assortment-related demand effects in bricks-and-mortar stores, webshops, and across channels has a potential impact on retailers' profitability. The multitude makes it costly to measure each effect empirically, which is further aggravated by interdependencies between products, channels and effects, and the resulting numerical complexity. Despite most retailers adopting OC assortments, the relevance and interplay of these demand effects is neither fully clear from an empirical nor an optimization perspective. It becomes necessary to better understand how assortment-related decisions impact customer behavior and optimal assortments.

We approach this research gap by identifying and integrating demand effects in a novel model for OC assortment optimization. Our results show that in-channel effects matter more than cross-channel transitions. This also holds true when demand effects exceed the empirically measured values in a single-channel context. Generalizing, the impact depends on demand rates, channel package sizes, and channel size. These findings are relevant for the OR community, empirical researchers, and retailers and help streamlining further research in two ways. First, further advances of OC assortment models with cross-channel effects should be based on our findings. Second, given that the empirical tests for these effects are very voluminous and costly, our findings serve as "guardrails" to define the scope of such empirical investigations.

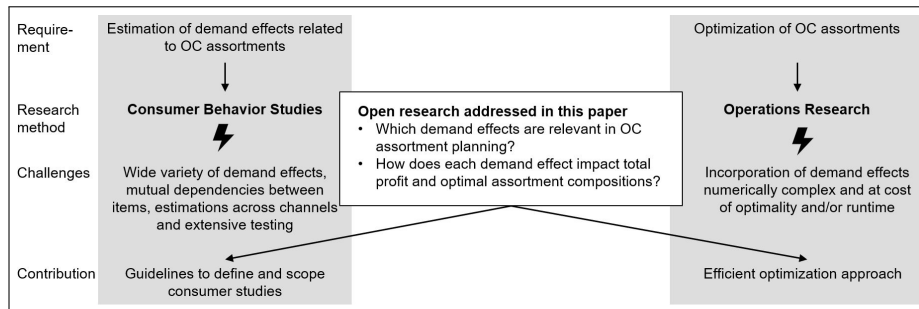
4.1 Introduction

The growth of e-commerce and the introduction of online channels has been the largest change in retail over recent decades. Seamless shopping options across digital and bricks-and-mortar channels are enabled along with enhanced operations and additional customer interfaces to allow frictionless OC retailing (Beck and Rygl, 2015; Hübner et al., 2016b; Rooderkerk and Kök, 2019). Examples are buy-online pick-up in store (BOPS), where customers can pick-up pre-ordered products in stores, or digital assortment extensions (DAE), where customers can access online assortments in stores via digital point of sales such as terminals or tablets (e.g., Hübner et al. (2021)). These options make assortments seamlessly available for customers across channels (e.g., Gallino and Moreno (2014); Ishfaq et al. (2016)), which calls for the incorporation of manifold customer behavior. Customer demand at the point of sale is influenced by the visibility of product on the stores shelves (e.g., Chandon et al. (2009)) and on the webpage of the online shop (e.g., Atalay et al. (2012)). Furthermore, if preferred products are unavailable, customers may settle for a substitute (Kök et al., 2015). With OC assortments, this is not just possible within the channel, but also across channels (Dzyabura and Jagabathula, 2018). For example, if a preferred product is not available at the store, the customer may opt to purchase a different product from the store or the identical product from the online channel.

Insights into demand effects within a store are rich (e.g., Drèze et al. (1994), Eisend (2014)), whereas influencing factors in the webshop receive less coverage (e.g., Djamasbi et al. (2010)), and cross-channel substitution is barely covered (e.g., Corsten and Gruen (2019)). The empirical studies usually focus on a single demand effect within one channel as data collection is costly, extensive and time consuming. The impact of all demand effects such as the quantity and position of products within store shelves, the position of products on the webpage, substitutions within a channel and transitions across channels would need to be estimated simultaneously. Furthermore, as

some effects reinforce each other (e.g., high quantity in a good position) and some compensate for suboptimal decisions (e.g., substitutions), it becomes necessary to understand how these effects interact and impact optimal assortment compositions. From an optimization perspective, first contributions have appeared that cover OC assortment planning (e.g., Dzyabura and Jagabathula (2018), Hense and Hübner (2021)). However, these either do not cover the entire width of demand effects or lack a comprehensive analytical assessment of the various demand sources. This is mainly caused by the complexity of the decision problem. It is a quadratic, nonlinear, and \mathcal{NP} -hard problem due to the reciprocal demand dependence of products (Hübner et al., 2016a).

Despite most retailers adopting OC services, the relevance and interplay of various demand effects is neither fully clear from an empirical point nor an optimization perspective. It becomes necessary to better understand how assortment-related decisions impact customer behavior and assortment optimization. Figure 4.1 visualizes the intersection of the problem. As the empirical research of multiple demand effects across channels is scarce, it is important to develop insights into which demand effects matter when determining assortments. Given that such consumer studies will need to be very extensive, it is necessary to limit the scope of future studies by understanding which demand sources under which circumstances have a high impact on optimal assortments. For these demand effects, consumer studies need to be designed that generate precise estimations. However, this also means that Operations Research (OR) needs to connect the demand effects within a decision model and develop tractable models and efficient solution approaches. We therefore contribute to both consumer research and OR by developing an optimization approach for OC assortments with relevant demand effects. The model enables analytical demand assessment and is crucial for substantiating the relevance of each demand effect, and determining whether and when it becomes necessary to integrate these into OC assortment planning.

Figure 4.1: Research scope and link between consumer studies and Operations Research

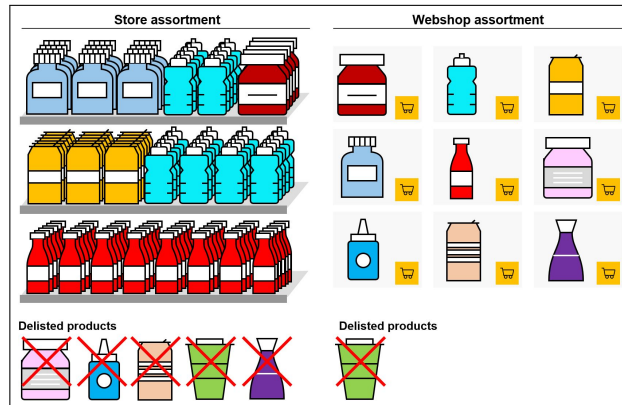
The remainder is as follows. We first introduce the decision problem and identify its related demand effects (Section 4.2) and review associated literature (Section 4.3) before then developing a comprehensive demand and decision model and solution approach (Section 4.4). The heart of the paper consists of numerical analysis. We develop managerial insights by means of computational studies (Section 4.5). Lastly, we synthesize and highlight potential areas for future research (Section 4.6).

4.2 Decision problem and related demand effects

Figure 4.2 gives an overview of the three decisions an OC retailer has to undertake for a set of products. First, the items to be listed have to be defined for both the store and the webshop. The decision problem is called *assortment composition*. Second, in the store, the number of customer-facing units for each listed item need to be determined and placed on vertical and horizontal segments of a shelf. This is called *space and position allocation*. The number of customer-facing units also defines the available quantity, which constitutes the third planning issue *inventory*

management. Correspondingly, in the webshop, for listed items retailers need to decide the position on the webpage as well as the inventory in the online warehouse. As space is limited, the three decisions are interdependent within and across channels. For instance, the listing of additional products in the store requires lower inventory levels of other listed products, which increases the risk of out-of-stocks and may result in and may result in customers searching for substitutions in the webshop. In the following we will detail the decision problems and demand effects. These differ between channels and we will analyze these separately for (1) stores, (2) webshops, and, synthesized, for the (3) OC setting. Going forward we will use the term “store” as the equivalent for a bricks-and-mortar sales location and “webshop” as the pendant for a digital sales channel.

Figure 4.2: Illustration of the assortment-related decision problems of an OC retailer



(1) Store-related planning issues and demand effects Typically, retailers decide which items of a category to list (assortment composition) and assign them to shelf segments in the store (space, position, and inventory planning). Total shelf space is limited, leading to a trade-off between wider assortments with more products that each occupy less shelf space and inventory, and smaller assortments with more space and inventory per item. The underlying decisions then interact with three demand effects: space-elastic, shelf-segment, and substitution demand.

The inventories of an item are denoted by the number of facings (i.e., the foremost unit of an item on the retail shelf) assigned to an item and the number of units behind each facing. The more facings an item receives, the higher the item's visibility on the shelf. For example, the blue bottle in Figure 4.2 currently has 3 facings. Increasing it to 4 facings would increase its visibility. This effect is known as *space-elastic demand*. Shopper surveys and field experiments find a positive correlation between the number of facings and the demand. For example, the influence of facings on demand has been tested for impulse-purchased items and staples (Brown and Tucker, 1961; Cox, 1964), grocery products (Frank and Massy, 1970), and fast-moving products (Curhan, 1972). Chandon et al. (2009) show that out of several in-store demand factors such as pricing or positioning, the variation of facings is the most influential one. Eisend (2014) carry out a meta-analysis covering over 1,200 consumer studies showing an average rise in demand of 17% every time the number of allocated facings is doubled. In the course of this study, it is also shown that the demand for one items changes by -1.6% when the space assigned to another item is varied. This effect called cross-space elasticity is investigated by Schaal and Hübner (2018), who state that they have found no significant impact of this effect on shelf optimization exists.

The likelihood of an item being perceived and purchased also changes depending on the vertical and horizontal position of an item on the shelf. For example, moving the blue bottle from top level to the mid-level may increase the visibility as it comes to the eye-level. This is called *shelf-segment demand*, which has been confirmed in various studies. For example, Underhill (2000) defines a "reliable zone", roughly ranging from eye to knee level, where products are much more likely to be seen. Adding to that, Chandon et al. (2009) notices the same effect for items placed on the top-shelf level. Drèze et al. (1994) also analyses position demand concerning horizontal positioning and states that horizontal positions have a much weaker impact than vertical positions. van Nierop et al. (2008) analyze that, amongst others, the number of facings and the item positioning on shelf levels impact demand. Valenzuela and Raghuram (2009a), Rodway

et al. (2012), and Valenzuela and Raghurir (2015) analyze the center effect. They show that consumers perceive items in the middle of an array as the best price/quality trade-off.

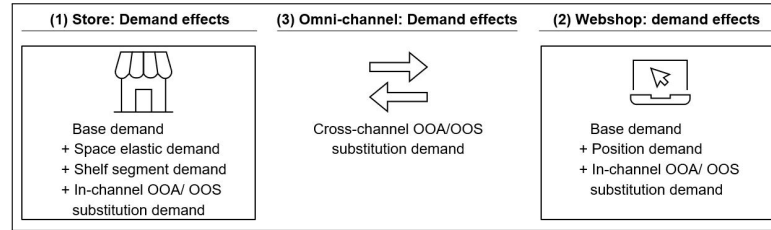
Finally, the demand for an item is impacted by the availability. The so-called *substitution demand* takes place when a desired product in the store is unavailable (in Figure 4.2 all the products with cross through them) and the customer substitutes the desired product for an alternative product within the store. Unavailability of items can be caused by out-of-assortment (OOA) or out-of-stock (OOS) situations. An item is OOA when it is permanently delisted, while OOS describes a temporary sellout. According to empirical studies, 45% to 84% of the initial demand can be substituted, where the magnitude depends on product-, situation-, and customer-attributes (e.g., Campo et al. (2004); Aastrup and Kotzab (2009); Tan and Karabati (2013)).

(2) Webshop-related planning issues and demand effects Retailers also need to decide which products to list for the webshop (assortment composition), where to position them on the webpage (space and position allocation), and how much inventory to assign to the selected products in the warehouses (inventory planning). Given that warehouse space is also constrained, the webshop faces the same trade-off as the store: an increase in the size of the webshop assortment requires a reduction of the inventory or even delisting of other products. Determining the assortment, space, and position of products and inventory levels has an interrelation with the following two demand effects: position and substitution demand.

A webpage can be described by an array of rows and columns, where the horizontal and vertical location of products leads to increased customer fixation and demand (e.g., Faraday (2000)). *Position demand* usually increases for items positioned in the center or the top-left corner of a page. For example, Atalay et al. (2012) note that centrally positioned brands positively impact customers' attention. Still (2017) states that spatial

location predicts user fixation. Djamasbi et al. (2010) found that items in the top left corner are also often attended to. Position effects are also omnipresent in similar areas such as search engines, where top-ranked results receive more clicks than lower-ranked results (Craswell et al., 2008). A further variant of the position demand is the salience of an item compared to the remaining assortment. This effect may come in the form of visual highlights by graphically varying the background brightness or color, or increasing the size of the product displayed (e.g., Faraday (2000); Grier et al. (2007)). Greater salience results in higher attention, longer fixation on a product, and stronger preferences for the same. *Substitution demand* in the webshop is identical to this demand effect in the bricks-and-mortar store. Products can be OOA or OOS and customers may substitute the unavailable product with an available product from the webshop (see e.g., Jing and Lewis (2011)). Substitution within the webshop has little empirical coverage in the literature.

(3) OC-related planning issues and demand effects All of the above decisions, demand effects, and constraints within the channels also apply to OC retail. Additionally, OC retailers need to take into account *cross-channel substitution demand*. When customers face products that are OOA or OOS, instead of staying within the channel, they replace the desired, unavailable product with an identical or different product in another channel. Substitutions can take place from the store to the webshop and vice versa. Compared to substitution within stores, cross-channel substitution has only recently gained attraction in literature. Gallino and Moreno (2014) and Wollenburg et al. (2018a) show that channel transitions are facilitated by fulfillment options such as BOPS that provide the online shopper with real-time information about inventory availability in the store. Corsten and Gruen (2019) investigate customer behavior when faced with unavailable items in the online channel. 88% of online demand can be substituted, of which 10% switch to a bricks-and-mortar store, 56% opt for a substitute within the webshop, and 22% switch to another webshop.

Figure 4.3: Summary of omni-channel demand effects

As summarized in Figure 4.3, we note a bandwidth of demand effects and their potential interplay in an OC context. While some demand effects are already difficult to assess empirically in isolation, the measurement of such interplay and customer behavior is particularly challenging. Relevant demand effects that require consideration (1) in the store are space-elastic, shelf-segment, and substitution demand. There is a negligible impact of cross-space elasticity and horizontal positioning. (2) Position and substitution effects in the webshop have been substantiated by current research but still lack further empirical assessment. (3) In an OC context, it has been shown that cross-channel substitution is an important area to study.

4.3 Related literature and contribution

This section analyzes pertinent assortment planning literature. We first introduce fundamental SC literature, and secondly we review relevant OC contributions. *Store*-related contributions primarily focus on various forms of in-channel substitution demand. Smith and Agrawal (2000) and Kök and Fisher (2007) optimize a retailer's profit through a newsvendor formulation that takes into account OOA substitution but ignores OOS substitution. Honhon et al. (2010) additionally considers OOS substitutions for customer segments based on sequential customer preferences. Hübner and Schaal

(2017a) are the first to add space-elasticity to substitution demand in their formulation. This is extended by Hübner et al. (2020) to two-dimensional shelves. Operational assortment optimization for *online channels* mainly utilizes dynamic approaches and demand learning to personalize assortments based on available customer data for customers who arrive sequentially. This short-term problem requires assortments that can be changed frequently and without friction. Rusmevichientong et al. (2010) formulate an online policy with unknown purchase probabilities. Abeliuk et al. (2016) generalize Rusmevichientong et al. (2010) as they study the problem of finding an optimal assortment and positioning of products subject to capacity constraints. Chen et al. (2016) carry out a space optimization, where inventory and the placement of products is optimized. Kallus and Udell (2020) study dynamically personalizing assortments for customer segments and how to define such segments. However, the scope of these papers is different from our setting. These papers deal with a given assortment and decide about the placement of products for customers or segments.

As store and webshop models are restricted to a single channel we henceforth review models with *omni-channels*. The first contribution stems from Dzyabura and Jagabathula (2018). They depict a retailer with a webshop and a store. Sales are maximized by deciding on the subset of products to offer in the store. In this scenario only the store assortment is optimized, without optimizing inventories. Demand is modeled using a utility-based model, where the customer's physical evaluation of the store assortment may change the customer's product utilities of the store and online assortment and result in purchasing a different item than the one originally preferred. In-channel and DAE OOA substitution are factored in but no OOS substitution. Geunes and Su (2020) develop the first model where a retailer optimizes assortments and inventory for a store with limited shelf space and a webshop with limited distribution center capacity. The utility-based model includes store, online, and hybrid online customer segments. Webshop-to-store OOS substitution is modeled through hybrid online customers that choose to substitute OOS webshop products for identical products via drop-shipment or ship-from-store (SFS). Regardless, the authors do not consider in-channel

substitution or cross-channel substitution for different products. Most recently, Hense and Hübner (2021) investigated the assortment, space, and inventory problem for an OC retailer that offers BOPS through a webshop to store substitution. The model includes OOA and OOS substitutions for different products in the same channel or a different product in the other channel. Space-elastic demand for the store and limited space for both channels is also considered. However, relevant effects within the channels, such as shelf-segment and position demand, and demand flows from the store to webshop via DAE substitution are not taken into account. Furthermore, the solution approach requires considerably higher computation times the more demand effects are incorporated.

Summary of related literature and contribution Table 4.1 summarizes related literature.

Table 4.1: Related literature

Related contribution	Decision ^{1a}			Demand model							Space constr.	No of items	
	A	I	S	In-channel		Omni-channel ²		Other					
				Space	Shelf. Pos.	OOA	OOS	OOA	OOS	Stoch.	Arrival ³		
Store													
Smith and Agrawal (2000)	✓	✓				✓				✓	S	✓	5
Kök and Fisher (2007)	✓	✓				✓				✓	D	✓	29
Honhon et al. (2010)	✓	✓				✓	✓			✓	D		8
Hübner and Schaal (2017b)	✓	✓	✓	✓	✓					✓	S	✓	200
Hübner et al. (2020)	✓	✓	✓	✓	✓		✓			✓	S	✓	100
Webshop													
Rusmevichientong et al. (2010)	(✓) ^{1b}					✓				✓	D		200
Abeluk et al. (2016)	(✓) ^{1b}	✓			✓	✓				✓	S	✓	2
Chen et al. (2016)		✓	✓		✓						S		12
Kallus and Udell (2020)	(✓) ^{1b}					✓				✓	D		400
Omni-channel													
Dzyabura et al. [2018]	(✓) ^{1c}					✓		DAE		✓	S		96
Geunes and Su (2020)	✓	✓						SFS	SFS	✓	S	✓	20
Hense and Hübner (2021)	✓	✓	✓	✓		✓	✓	BOPS	BOPS	✓	S	✓	100
This paper	✓	✓	✓	✓	✓	✓	✓	BOPS, DAE	BOPS, DAE	✓	S	✓	100

^{1a} Decisions included: Assortment (A), space (S), inventory (I); ^{1b} Personalizing assortment display to customers with given total assortment; ^{1c}

OC model only optimizing store assortment

² Cross-channel substitution demand: ship-from-store (SFS), buy-online pick-up in store (BOPS), or digital assortment extensions (DAE)

³ Optimization: Static (S) or dynamic with sequentially arriving customers (D)

The store-focused literature often uses exogenous demand models, constrained space, and some sort of in-channel substitution. Recent models also integrate space-elasticity but ignore certain demand factors such as shelf-segment or position demand. Webshop-focused literature usually utilizes demand learning for dynamically arriving customers. Given the setting, such models are not applicable to tactical assortment planning where inventories across the channels need to be defined for a mid-term horizon and cannot be altered frictionlessly. Above that, no space constraints,

single-channel demand effects, or additional sales channels are considered. Finally, both store- and webshop-models are limited to one channel and do not cover cross-channel substitutions. This problem is examined by the small body of OC-related assortment literature. As indicated in Table 4.1, none of the models provided so far considers the whole spectrum of SC demand effects as well as in- and cross-channel substitution. Most strikingly, analytical perspectives on the integrated demand effects are either fully absent (e.g., Dzyabura and Jagabathula (2018); Geunes and Su (2020)) or focus on BOPS (e.g., Hense and Hübner (2021)). The overview shows the need for the development of a model for assortment optimization across channels taking into consideration the relevant demand effects in each channel. Resulting insights will extend existing models and respond to the findings of Wollenburg et al. (2018a), Rooderkerk and Kök (2019) and Hense and Hübner (2021), who have already pointed out the shortage of analytical insights on assortment planning with a common objective across channels and the possibility for customers to move seamlessly between channels.

4.4 Model and solution approach

In this section we first develop a profit function and binary-integer program (BIP) that maximizes the profit of an OC retailer (Section 4.4.1). This involves determining the optimal assortment, facing, position, and inventory level for all items across all channels. The second stage is to derive the demand for each item across all channels (Section 4.4.2). Finally, an advanced specialized heuristic is developed to solve real-world applications (Section 4.4.3).

4.4.1 Decision problem and optimization model

Table 4.2 summarizes the notation.

Table 4.2: Model notation

Indices and sets	
C	Set of channels a retailer operates with $C = \{1, 2, \dots, c, d, \dots, C \}$
K_{ic}	Set of facings k a retailer can select for item i in channel c
I (I^+ , I^-)	Set of (listed, delisted) items with $I = \{1, 2, \dots, i, j, \dots, I \}$
I_c (I_c^+ , I_c^-)	Set of (listed, delisted) items i within a channel c
T_c	Set of shelf-segments t in channel c
M_c	Set of webpage rows m in channel c
N_c	Set of webpage columns n in channel c
Parameters	
α_{ic}	Base demand of item i in channel c
β_{ic}	Space-elasticity of item i in channel c
λ_{ct}	Attractiveness factor of shelf-segment t in channel c
ψ_{imn}	Position-elasticity of item i on webpage position in row m and column n
γ_{jc}^{OOA}	Share of demand of item j in channel c that gets substituted by item i in channel c
(γ_{jc}^{OOS})	in case item j is out-of-assortment (out-of-stock) in channel c , with $j \neq i$, $i, j \in I$
δ_{id}^{OOA}	Share of demand of item i in channel d that gets substituted by item i in channel c
(δ_{id}^{OOS})	in case item i is out-of-assortment (out-of-stock) in channel d , with $d \neq c$, $c, d \in C$
η_{jd}^{OOA}	Share of demand of item j in channel d that gets substituted by item i in channel c
(η_{jd}^{OOS})	in case item j is out-of-assortment (out-of-stock) in channel d , with $j \neq i$, $i, j \in I$ and $d \neq c$, $c, d \in C$
b_{ic}	Width of one facing of item i in channel c
g_{ic}	Inventory per facing of item i in channel c
$k_{ic}^{max} (k_{ic}^{min})$	Maximum (minimum) number of facings of item i in channel c
$S_c(S_{ct})$	Available shelf (storage) (in shelf-segment t) in channel c
r_{ic}	Revenues for one unit of item i in channel c
u_{ic}	Unit costs for one unit of item i in channel c
s_{ic}	Shortage costs for one unit of item i in channel c
v_{ic}	Salvage value for one unit of item i in channel c
Variables	
$k_{ic}(k_{ict})$	Number of facings assigned to item i (in shelf-segment t) in channel c , integer
p_{icmn}	Binary variable indicating whether item i in channel c was placed in position mn
x_{ick}	Total inventory of item i in channel c for facing k , integer (auxiliary variable)

The OC retailer assigns items from a given set of items $i, i \in I$ to one or more channels $c, c \in C$. The total set of items I can be sold in all channels c with $c, d \in C$, i.e., $I_c, I_d, \dots, I_C \subseteq I$. The subset of items in channel c is given by I_c . Given that items in this subset can either be listed or delisted, we differentiate between the set of listed items I_c^+ and the set of delisted items I_c^- in each channel c , with $I_c^+, I_c^- \subseteq I_c$, $I_c^+ \cup I_c^- = I_c$ and $I_c^+ \cap I_c^- = \emptyset$.

The set union across all channels represents the set of listed items with $I_c^+ \cup I_d^+ \cup \dots \cup I_C^+ = I^+$ and delisted items with $I_c^- \cup I_d^- \cup \dots \cup I_C^- = I^-$. Channels can be configured both as store or webshop. From a customer perspective, items in stores are assigned to shelf-segments $t \in T$ (i.e., edge vs. center of the aisle and eye-level vs. knee-level), and in webshops they are assigned to specific positions (i.e., top vs. bottom and left vs. right). In stores, shelf space is dedicated to each shelf-segment S_{ct} , while in webshops it refers to the entire warehouse capacity S_c . The shelf space $S_c(S_{ct})$ can be considered as a one-dimensional shelf length in each channel, e.g., measured in meters (cf. Kök and Fisher (2007); Irion et al. (2012); Düsterhöft et al. (2020)). This one-dimensionality is dictated by the fixed stock per unit (g_{ic}) (i.e., denoted by the item dimensions and shelf depth in the channel) and the circumstance that two different items can only be placed side by side, not behind one another when customers and pickers frontally observe the store or warehouse shelf. To provide a general and lean model across channels, the uniform term “facing” is used both for the store shelves and the online warehouse shelves. It represents the unit that customers face when observing the store shelf or pickers when facing the warehouse shelf. Facings can be placed across shelf-segments in the store. Facing-dependent store inventory x_{ick} is then defined using the integer number of facings across all shelf-segments multiplied with the stock per facing ($x_{ick} = k \cdot g_{ic}$). In the webshop customers do not face different shelf-segments. The webshop therefore does not consist of any shelf-segment $T_c = \emptyset$. Instead, the customers observe items on the webshop’s homepage. Items are placed on an array that is defined by rows $m, m \in M$ and columns $n, n \in N$. An item’s position on this array is uniquely defined by binary position variable p_{icmn} . Items are either not listed or are placed on one position on the webpage. As no such thing exists in the store, the binary variable p_{icmn} is not applied to the store channel (i.e., $p_{icmn} = 0$ and $N, M \in \emptyset$). While space on the homepage is sufficient to list all items, warehouse space for online fulfillment is constrained. Warehouse inventory is then calculated via multiplication of the integer number of facings k with the stock per facing g_{ic} in the warehouse ($x_{ick} = k \cdot g_{ic}$).

To optimize profits the OC retailer needs (1) to decide which products $i, i \in I_c$ in each channel c to list (I_c^+) and to delist (I_c^-), (2) on which shelf-segment (store) t or which position (webshop) p_{icmn} to place item $i, i \in I_c^+$ and how many facings k to allocate to listed item $i, i \in I_c^+$ in each channel $c, c \in C$. (3) The amount of inventory x_{ick} is defined based on the number of facings. These decisions are depicted via decision variables k_{ic} (k_{ict}), and p_{icmn} . k_{ic} defines the number of facings for each item $i, i \in I$ and channel $c, c \in C$. In the case of the store channel, the total number of facings is calculated using $k_{ic} = \sum_{t \in T} k_{ict}$. In general it is valid that $k_{ic} = 0$ represents the case for delisting and $k_{ic} \geq 1$ the case for listing. In practice, it is common for retailers to limit the set of facings K_{ic} . In the store this is driven by sales initiatives and marketing contracts regulating the share of facings. In the webshop it is the result of space constraints in warehouses. Hence, the retailer selects k_{ic} from a set of integer facings $K_{ic} = [k_{ic}^{min}, k_{ic}^{max}]$ where $k_{ic} \geq k_{ic}^{min}$ and $k_{ic} \leq k_{ic}^{max}$. Introducing item- and channel-specific ranges for the number of facings enables the consideration of channel-specific storage requirements (represented by the space occupied per facing unit b_{ic}) or the fixed stock of units behind each facing (represented by inventory per facing g_{ic}). Binary variable p_{icmn} complies with the listing decision for the webshop denoted by k_{ic} . Any item i can only be allocated to a single position mn on the webpage. Thus $p_{icmn} = 1$ if product i is displayed at position mn and $p_{icmn} = 0$ if otherwise.

Objective function Equation (4.1) is used to compute the profit π_{ick} for each item i in each channel c and number of facings k . The profit depends on the inventory x_{ick} , the demand realized and the associated revenues and costs. The profit equation consists of five parts and follows the profit calculation for newsvendor-like problems (for application in assortment-related decision models see e.g., Smith and Agrawal (2000); Honhon et al. (2010)). Part one represents the unit costs u_{ic} . Unit costs u_{ic} include any type of costs involved per channel and item, such as purchasing, replenishment and fulfillment costs. To account for the possibility that demand does not match inventory at the end of a sales period (i.e., overstock

or shortage) we introduce the demand variable y_{ick} . Overstock occurs when $y_{ick} < x_{ick}$, while $y_{ick} > x_{ick}$ describes the shortage case. Parts two and three represent overstock situations. Part two calculates the expected revenue for inventory quantity x_{ick} by multiplying y_{ick} by sales price r_{ic} . Part three of the profit function accounts for the expected salvage cost due to items that are left in stock at the end of the period. Leftover stock of item i in channel c is cleared at salvage value v_{ic} , thereby representing a residual value. As $v_{ic} < u_{ic}$, the retailer suffers a loss in profits. The salvage value can also be interpreted as inventory holding costs in the case of non-perishable items (Kök and Fisher, 2007; Hübner et al., 2016b). Parts four and five cover shortage situations. Part four calculates the expected revenue for each inventory quantity x_{ick} . However, the retailer can only sell the available stock in this case. x_{ick} is therefore multiplied by the sales price r_{ic} . Finally, part five introduces shortage costs s_{ic} that impose a penalty cost on the retailer for unsatisfied demand. Please note that the demand is determined by the assortment and facing selection and position on the store shelf and webpage. This is specified by the density function f_{ick}^* that is developed in Section 4.4.2. The sum of all single item profits π_{ick} constitutes the overall profit Π of the retailer.

$$\begin{aligned}
\pi_{ick}(x_{ick} \mid x_{ick}=k \cdot g_{ic}) &= -u_{ic} \cdot x_{ick} + r_{ic} \int_0^{x_{ick}} y_{ick} f_{ick}^* dy \\
&+ v_{ic} \int_0^{x_{ick}} (x_{ick} - y_{ick}) f_{ick}^* dy \\
&+ r_{ic} \int_{x_{ick}}^{+\infty} x_{ick} f_{ick}^* dy \\
&- s_{ic} \int_{x_{ick}}^{+\infty} (y_{ick} - x_{ick}) f_{ick}^* dy
\end{aligned} \tag{4.1}$$

Decision model In the following we embed Equation (4.1) into a BIP and introduce a single binary variable z_{ick} , where z_{ick} expresses the listing decision for item i in channel c with the quantity of facings k , i.e., whether it receives k facings ($z_{ick} = 1$) or not ($z_{ick} = 0$). The objective function

(Equation (4.2)) applies Equation (4.1) to calculate the item profit π_{ick} and sum it up for all selected facings ($z_{ick} = 1$). Equation (4.3) (Equation (4.4)) ensures that the limited available shelf-space in the store (webshop) is not exceeded. In webshops, items must be assigned to positions. Equation (4.5) makes sure that items i that are listed (i.e., $z_{ick}=1$) are also allocated to a position (i.e., $p_{icmn} = 1$). In the even that an item with 0 facings is selected $z_{ic0} = 1$, Equation (4.6) prevents a position assignment. Since in stores items can be assigned to different shelf-segments t , Equation (4.7) is required to sum up the facings across the different segments z_{ickt} . Equation (4.8) expresses that each item i in each channel c receives exactly one facing value. Equation (4.9) defines the binary variables applied.

$$\max \Pi(\bar{x}) = \sum_{i \in I} \sum_{c \in C} \sum_{k \in K_{ic}} \pi_{ick} \cdot z_{ick} \quad (4.2)$$

subject to

$$\sum_{i \in I} \sum_{k=1}^{k_{ic}^{max}} k \cdot b_{ic} \cdot z_{ickt} \leq S_{ct} \quad \forall c \in C \mid T_c \neq \emptyset, t \in T \quad (4.3)$$

$$\sum_{i \in I} \sum_{k \in K_{ic}} k \cdot b_{ic} \cdot z_{ick} \leq S_c \quad \forall c \in C \mid M_c \neq \emptyset, t \in T \quad (4.4)$$

$$\sum_{m \in M} \sum_{n \in N} p_{icmn} = \sum_{k \in K_{ic}} z_{ick} \quad \forall c \in C \mid M_c \neq \emptyset, i \in I \quad (4.5)$$

$$\sum_{m \in M} \sum_{n \in N} p_{icmn} = 0 \cdot z_{ic0} \quad \forall c \in C \mid M_c \neq \emptyset, i \in I \quad (4.6)$$

$$\sum_{l=1}^{l_{ic}^{max}} \sum_{t \in T_c} l \cdot z_{ickt} = k \cdot z_{ick} \quad \forall c \in C \mid T_c \neq \emptyset, i \in I, k \in K_{ic}, t \in T_c \quad (4.7)$$

$$\sum_{k \in K_{ic}} z_{ick} = 1 \quad \forall c \in C, i \in I \quad (4.8)$$

$$z_{ick}, z_{ickt}, p_{icmn} \in \{0; 1\} \quad \forall c \in C, i \in I, k \in K, m \in M, n \in N \quad (4.9)$$

Based on Equation (4.1), the BIP also takes into consideration density function f_{ick}^* , and thereby accounts for the total demand of item i in

channel c with facing k . f_{ick}^* quantifies assumed customer behavior and takes into consideration all the demand peculiarities of the underlying problem.

4.4.2 Demand model

We apply an ED model that directly specifies consumer behavior and demand. ED models are appropriate for our research question as they model each effect separately, but also aggregate the single demand sources in a total demand function. This allows analysis of the various effects on assortment compositions and arrangements of products within the channels. The ED model represents customers that choose their favored variant from a set of items. In the event that an item is unavailable, the customer substitutes their second favorite item with a defined probability. Demand distributions are assumed to be independent as all elements of the ED models are set independently. The base demand of product i and product j , with $i \neq j$ are independent of each other, for instance. We can use convolutions to aggregate demand. Our demand model focuses on substitution behavior across channels. To enable demand substitutions across channels, we consider BOPS and DAE. In BOPS, the webshop provides online customers with access to the webshop as well as store inventory when substitution (i.e., OOA or OOS) situations occur. Demand is transferred from the webshop to the store. DAE on the other hand provides store customers with access to store and webshop inventory when substitution situations occur. Demand transfers from the store to the webshop are facilitated.

The total expected demand \hat{D}_{ick} is composed of three elements (Equation (4.10)). The first element $D_{ick}^{SP,SH,PO}$ combines space-elastic D_{ick}^{SP} , shelf-segment D_{ick}^{SH} , and position demand D_{ic}^{PO} . OOA (OOS) substitution demand is stated as the second (third) element $D_{ic}^{OOA(1,2,3)}$ ($D_{ick}^{OOS(1,2,3)}$). It encompasses substitutions across different items within the same channel $D_{ic}^{OOA(1)}$

($D_{ic}^{OOS^{(1)}}$), identical items across different channels $D_{ic}^{OOS^{(2)}}$ ($D_{ic}^{OOS^{(2)}}$), and different items across different channels $D_{ic}^{OOA^{(3)}}$ ($D_{ick}^{OOS^{(3)}}$).

$$\hat{D}_{ick} = D_{ick}^{SP,SH,PO} + D_{ic}^{OOA^{(1,2,3)}} + D_{ic}^{OOS^{(1,2,3)}} \quad (4.10)$$

Regardless of the three components of the total expected demand \hat{D}_{ick} , each item i in channel c has a base demand α_{ic} that corresponds to demand for the item when $k = 0$. This specifies the forecast demand for an item and is independent of the decision for item i in channel c and the total assortment composition. All demand effects are summarized in Table 4.3.

Table 4.3: Overview of demand effects

Demand effect	Channel ¹	Description	Parameter(s)
α_{ic}	S, W	Base demand regardless of assortment, segment and position	α_{ic}
D_{ick}^{SP}	S	Space-elastic demand from an increased number of facings	β_{ic}
D_{ick}^{SH}	S	Shelf-segment demand from placing facings on certain store shelf-segments	λ_{ct}
D_{ic}^{PO}	W	Position demand from placing items on certain webpage positions	ψ_{imn}
$D_{ic}^{OOA^{(1)}}$, $D_{ic}^{OOS^{(1)}}$	S, W	OOA/OOS substitution demand for different items within channel	$\gamma_{j_c i_c}^{OOA}$, $\gamma_{j_c i_c}^{OOS}$
$D_{ic}^{OOA^{(2)}}$, $D_{ic}^{OOS^{(2)}}$	S, W	OOA/OOS substitution demand for identical items across channels	$\delta_{i_d i_c}^{OOA}$, $\delta_{i_d i_c}^{OOS}$
$D_{ic}^{OOA^{(3)}}$, $D_{ic}^{OOS^{(3)}}$	S, W	OOA/OOS substitution demand for different items across channels	$\eta_{j_d i_c}^{OOA}$, $\eta_{j_d i_c}^{OOS}$

⁽¹⁾ Application in channel: Store (S), Webshop (W)

(I) Space-elastic demand Space-elastic demand D_{ick}^{SP} describes the effect that the visibility of and therefore customer demand for an item increases when the number of assigned facings k of item i in channel c is increased (e.g., Hansen and Heinsbroek (1979), Corstjens and Doyle (1981), Eisend (2014)). We define space-elastic demand (4.11) through a polynomial function (cf. Hansen and Heinsbroek (1979)). Base demand α_{ic} is multiplied by the number of facings k and raised to the power of β_{ic} (with $0 \leq \beta_{ic} \leq 1$). Hence, when $\beta_{ic} = 0$ (i.e., an item has no space-elasticity), $D_{ick}^{SP} = \alpha_{ic}$.

$$D_{ick}^{SP} = \sum_{t \in T} \alpha_{ic} \cdot k^{\beta_{ic}} \quad (4.11)$$

(II) Shelf-segment demand Shelf-segment demand D_{ick}^{SH} (Equation (4.12)) of item i in channel c stems from a varying attractiveness of vertical and horizontal shelf-segments given differing visibility (cf. e.g., Drèze et al. (1994); Hwang et al. (2005)). The demand effect is modeled with the attractiveness parameter λ_{ct} that is assigned to each shelf-segment $t \in T$ in each channel $c \in C$. It describes the magnitude with which demand increases if the item is positioned on shelf-segment t , where higher values of λ_{ct} correspond to higher visibility. Commonly, an item can be assigned to multiple shelf-segments t at the same time. To account for this case, we calculate an average attractiveness parameter λ_{ick} with $\lambda_{ick} = [\sum_{l=1}^k \sum_{t \in T} l \cdot \lambda_{ct}] / [k \cdot z_{ick}]$ which depends on the assigned facings across all segments.

$$D_{ick}^{SH} = \alpha_{ic} \cdot \lambda_{ick} \quad (4.12)$$

(III) Position demand Position demand D_{ic}^{PO} (Equation (4.13)) for item i in the webshop describes an elastic value that depends on the position of an item on a webpage. Faraday (2000) and Djamasbi et al. (2010) found that the closer an item is placed to the top left corner on a webpage, the higher the visibility to the customer. The position effect ψ_{imn} for every item i in every position, defined by rows M and columns N , is stated in an array. Following previous research (cf. e.g., Chen et al. (2016)), the position demand rate is also a polynomial function. It multiplies the base demand α_{ic} by binary position variable p_{icmn} to the power of ψ_{imn} . An item i in the webshop channel is either delisted (i.e., $\sum_{m \in M} \sum_{n \in N} p_{icmn} = 0$) or listed and therefore placed on one position on the array (i.e., $\sum_{m \in M} \sum_{n \in N} p_{icmn} = 1$). For the store, $M, N = \emptyset$ because no positioning in this sense exists.

$$D_{ic}^{PO} = \sum_{m \in M} \sum_{n \in N} (\alpha_{ic} \cdot p_{icmn})^{\psi_{imn}} \quad (4.13)$$

We are able to condense D_{ick}^{SP} , D_{ick}^{SH} and D_{ic}^{PO} into a single demand component $D_{ick}^{SP,SH,PO}$ (Equation (4.14)). The corresponding density function is denoted by $f_{D_{ick}^{SP,SH,PO}}^*$.

$$D_{ick}^{SP,SH,PO} = \left(\sum_{m \in M} \sum_{n \in N} \alpha_{ic} \cdot p_{icmn} \right)^{\psi_{imn}} \cdot k^{i\beta_{ic}} \cdot \lambda_{ick} \quad (4.14)$$

(IV) Out-of-assortment substitution demand OOA substitution demand occurs when a customer demands a delisted item ($j \in I^-$) and instead substitutes a listed item ($i, i \in I^+$). We assume that if item j is delisted, customers will substitute a certain share of the base demand α_{jc} of item j with item i . Some customers will maintain their wish to purchase item j , even if it is not available. Base demand α_{jc} constitutes the maximum quantity that can be substituted. This is due to the independence of the base demand from any decision for item i and the usual assumption that substitution takes place over only one round (cf. e.g., K ok and Fisher (2007); H ubner and Schaal (2017a)). This implies that if a consumer's substitute is also unavailable, demand is lost. It has been shown that this assumption is not too restrictive (cf. Smith and Agrawal (2000)). In the event of multiple channels, the OOA demand needs to be differentiated between (IV.1) different items within the same channel, (IV.2) identical items across different channels and (IV.3) different items across different channels.

(IV.1) The OOA demand for different items within a channel $D_{ic}^{OOA(1)}$ (Equation (4.15)) for a listed item $i, i \in I_c^+$ in channel c takes place when a customer intends to buy a delisted item j in channel c ($j \neq i, j \in I_c^-$) and substitutes it with an alternative item i in the same channel c . The

substitution rate γ_{jcic}^{OOA} quantifies the share that is substituted. The density function is denoted by $f_{D_{ic}^{OOA(1)}}^*$.

$$D_{ic}^{OOA(1)} = \sum_{j \in I_c^- / \{i\}} \alpha_{jc} \cdot \gamma_{jcic}^{OOA} \quad (4.15)$$

(IV.2) The *OOA demand for identical items across channels* $D_{ic}^{OOA(2)}$ (Equation (4.16)) occurs when a customer demands an OOA item i in channel $d, d \in C$ with $c \neq d, i \in I_d^-$. The related density function is defined by $f_{D_{ic}^{OOA(2)}}^*$. Given the OOA situation, the customer may decide (for reasons of affinity) to switch channels to buy the identical item i in another channel c . The share to be substituted is denoted by δ_{idic}^{OOA} .

$$D_{ic}^{OOA(2)} = \sum_{d \in C / \{c\} | i \in I_d^-} \alpha_{id} \cdot \delta_{idic}^{OOA} \quad (4.16)$$

(IV.3) The *OOA demand for different items across channels* $D_{ic}^{OOA(3)}$ (Equation (4.17)) results from customers with an intention to purchase item j in channel $d, d \in C$ even though the item is delisted in both channels d ($j \in I_d^-, d \in C$) and c ($j \in I_c^-, c \in C$). The corresponding density function is denoted by $f_{D_{ic}^{OOA(3)}}^*$. Customers subsequently decide to purchase the alternative item $i, i \neq j$ in a different channel $c, c \neq d$.

$$D_{ic}^{OOA(3)} = \sum_{d \in C / \{c\} | j \in I_d^- / \{i\}} \alpha_{jd} \cdot \eta_{jdic}^{OOA} \quad (4.17)$$

(V) Out-of-stock substitution demand OOS substitution demand emerges in the event of unsatisfied demand for a listed item j in channel c due to temporary unavailability due to stock-outs, i.e., demand $D_{jck}^{SP,SH,PO}$ is greater than the available inventory x_{jck} . The underlying assumption is

that listed items and their representation on the shelf and in the webshop is firstly still visible to the customer when OOS occurs (e.g., via price tags or product pages), and secondly because customers who purchased the product when it was available were still under the influence of these demand effects. As for OOA demand, we assume one round of substitution and draw a distinction between different/identical items within the same/across channels as for OOA.

(V.1) *OOS demand for different items within a channel* $D_{ic}^{OOS(1)}$ (Equation (4.18)) for an item i in channel c ($i \in I_c^+$) emerges when a customer intends to buy listed item j ($j \neq i, j \in I_c^+$), while the available quantity x_{jck} of item j in channel c is insufficient to fulfill the demand. Customers therefore potentially decide to purchase item i within the same channel c at the rate of $\gamma_{jci_c}^{OOS}$. The corresponding density function equals $f_{D_{ic}^{OOS(1)}}^*$.

$$D_{ic}^{OOS(1)} = \sum_{j \in I_c^+ / \{i\}} [(D_{jck}^{SP,SH,PO} - x_{jck}) | D_{jck}^{SP,SH,PO} > x_{jck}] \cdot \gamma_{jci_c}^{OOS} \quad (4.18)$$

(V.2) The *OOS demand for identical items across channels* $D_{ic}^{OOS(2)}$ (Equation (4.19)) for a listed item i in channel c ($i \in I_c^+$) appears when the available stock x_{id} for the identical listed item i in channel d ($c \neq d, i \in I_d^+$) is insufficient to satisfy the demand. Customers may then substitute by switching to channel c and buying the identical item i there. The substitution rate is specified via $\delta_{idi_c}^{OOS}$ and the density function by $f_{D_{ic}^{OOS(2)}}^*$.

$$D_{ic}^{OOS(2)} = \sum_{d \in C / \{c\} | i \in I_d^+} [(D_{idk}^{SP,SH,PO} - x_{idk}) | D_{idk}^{SP,SH,PO} > x_{idk}] \cdot \delta_{idi_c}^{OOS} \quad (4.19)$$

(V.3) The *OOS demand for different items across channels* $D_{ic}^{OOS^{(3)}}$ (Equation (4.20)) comes into play when item $j, j \in I$ is temporarily unavailable in both channels $d (j \in I_d^+, d \in C)$ and $c (j \in I_c^+, c \in C)$. Customers substitute it by item $i, i \neq j$ in a different channel $c, c \neq d$. The substitution share is quantified by rate η_{jaic}^{OOS} and the density function by $f_{D_{ic}^{OOS^{(3)}}}^*$.

$$D_{ic}^{OOS^{(3)}} = \sum_{d \in C / \{c\} | j \in I_d^+ / \{i\}} [(D_{jdk}^{SP,SH,PO} - x_{jdk}) | D_{jdk}^{SP,SH,PO} > x_{jdk}] \cdot \eta_{jaic}^{OOS} \quad (4.20)$$

Calculating the convolution Given the peculiarities and the demand components of our ED model, we can specify each element independently and exogenously. Consequently, demand distributions of the products i and j are also independent for $i \neq j$. That opens up the possibility of calling on the convolution concept to generate the distribution of the items's demand along with all three demand elements. The convolution – represented by the operator \circledast – of the related demand distribution functions can be used to compute the distribution of the sum of the demands for items i and j . We limit the subsequent distributions to \mathbb{R}_0^+ in order to exclude negative demand. Following the assumption that distributions are standardized to the feasible interval, we convolute the additional demand distributions accounting for OOA and OOS for the item sets I_c^- and I_c^+ . The density function for $D_{ic}^{OOA^{(1)}}$ (Equation (4.21)) computes the convolution of the (base) demand distributions of all OOA items. This takes into account the dependence of the OOA substitution demand for listed item $i (i \in I_c^+)$ on the convolution \circledast of all delisted (OOA) items $j \in I_c^-$ in channel c . Correspondingly, the density function for $D_{ic}^{OOA^{(2)}}$ (Equation (4.22)) considers the dependence on the demand distribution of all delisted, identical items i in all other channels $d (c \neq d, i \in I_d^-)$ and the density function for $D_{ic}^{OOA^{(3)}}$ (Equation (4.23)) incorporates the dependence on all different delisted items j in all other channels d . Since the substitution probabilities γ , δ and η only represent a factor, they will be omitted in the equations to simplify the notation.

$$\otimes_{j \in I_c^-} f_{\alpha_{jc}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, j \in I_c^-} f_{\alpha_{jc}}^* d\tau \dots d\nu \quad (4.21)$$

$$\otimes_{i \in I_d^-} f_{\alpha_{id}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, i \in I_d^-} f_{\alpha_{id}}^* d\tau \dots d\nu \quad (4.22)$$

$$\otimes_{j \in I_d^-} f_{\alpha_{jd}}^* = \int \cdots \int_{\mathbb{R}_0^{+,n}, j \in I_d^-} f_{\alpha_{jd}}^* d\tau \dots d\nu \quad (4.23)$$

As for OOA demand, we use the convolution concept to calculate OOS demand. Equation (4.24) computes the density function and provides for the fact that OOS demand for item i in channel c is dependent on the expected shortage of all other OOS items j in channel c . Resembling that, Equation (4.25) and Equation (4.26) calculate density functions that account for the OOS demand for identical items across channels and different items across channels, respectively.

$$\otimes_{j \in I_c^+} f_{D_{jck}^{SP,SH,PO}}^* = \int \cdots \int_{x_{jck}, j \in I_c^+}^{\infty} f_{D_{jck}^{SP,SH,PO}}^* d\tau \dots d\nu \quad (4.24)$$

$$\otimes_{i \in I_d^+} f_{D_{idk}^{SP,SH,PO}}^* = \int \cdots \int_{x_{idk}, i \in I_d^+}^{\infty} f_{D_{idk}^{SP,SH,PO}}^* d\tau \dots d\nu \quad (4.25)$$

$$\otimes_{j \in I_d^+} f_{D_{jdk}^{SP,SH,PO}}^* = \int \cdots \int_{x_{jdk}, j \in I_d^+}^{\infty} f_{D_{jdk}^{SP,SH,PO}}^* d\tau \dots d\nu \quad (4.26)$$

To calculate the total demand for item i , Equation (4.27) convolutes the demand density functions of $D_{ick}^{SP,SH,PO}$, $D_{ic}^{OOA(1)}$, $D_{ic}^{OOA(2)}$, $D_{ic}^{OOA(3)}$, $D_{ic}^{OOS(1)}$, $D_{ic}^{OOS(2)}$ and $D_{ic}^{OOS(3)}$.

$$\begin{aligned}
f_{ick}^* = & f_{D_{ick}^{SP,SH,PO}}^* \cdot \left(\otimes_{j_c \in I^-, j \neq i} f_{\alpha_{jc}}^* \gamma_{jc i_c}^{OOA} \right) \Big|_{\gamma_{jc i_c}^{OOA} \neq 0} \\
& \cdot \left(\otimes_{i_d \in I^-, i = i} f_{\alpha_{id}}^* \delta_{i_d i_c}^{OOA} \right) \Big|_{\delta_{i_d i_c}^{OOA} \neq 0} \\
& \cdot \left(\otimes_{j_d \in I^-, j \neq i} f_{\alpha_{jd}}^* \eta_{j_d i_c}^{OOA} \right) \Big|_{\eta_{j_d i_c}^{OOA} \neq 0} \\
& \cdot \left(\otimes_{j_c \in I^+, j \neq i} f_{D_{jck}^{SP,SH,PO}}^* \gamma_{jc i_c}^{OOS} \right) \Big|_{\gamma_{jc i_c}^{OOS} \neq 0} \\
& \cdot \left(\otimes_{i_d \in I^+, i = i} f_{D_{idk}^{SP,SH,PO}}^* \delta_{i_d i_c}^{OOS} \right) \Big|_{\delta_{i_d i_c}^{OOS} \neq 0} \\
& \cdot \left(\otimes_{j_d \in I^+, j \neq i} f_{D_{jdk}^{SP,SH,PO}}^* \eta_{j_d i_c}^{OOS} \right) \Big|_{\eta_{j_d i_c}^{OOS} \neq 0}
\end{aligned} \tag{4.27}$$

According to Kellerer et al. (2010) a knapsack problem with a linear objective function and linear constraints is already known to be \mathcal{NP} -hard. As our model represents an \mathcal{NP} -hard problem with multiple knapsacks, a non-linear and non-separable (quadratic) objective function, and mutual dependencies between the items we embed the BIP (Section 4.4.1) in an iterative heuristic (Section 4.4.3). While the BIP respects all constraints, demand substitutions are only added ex-post.

4.4.3 Solution approach

This section develops the specialized heuristic for the optimization of **O**mnichannel **C**hannel **A**ssortment, **S**pace, **P**osition and **I**nventory (OC-ASPI). It is an advance on the iterative heuristic for the Joint Omnichannel Inventory and Assortment Optimization (JOCIAO) of Hense and Hübner (2021). We apply the same algorithmic principles as JOCIAO. It constitutes an appropriate starting point for the underlying problem as it considers some of the demand components described and achieves near-optimal solutions. However, shelf-segment, position, and DAE cross-channel demand are not part of JOCIAO. By integrating these demand effects into our decision problem, the solution possibilities and numerical complexity increase dramatically. This calls for a refinement, which we achieve by effectively guiding the iteration process.

In the following, we first give an overview and general idea of the heuristic and thereafter detail the computation process.

Overview The BIP formulated in Section 4.4.1 is non-linear as the item demand and subsequently the profit depend on the number of facings on shelf-segments, the position of the item itself and, given the substitutions, on other items. We circumvent the non-linearity through a two-stage solution heuristic. In the initialization (Stage 1), we pre-calculate item profit π_{ick} and thereby extend JOCIAO (which only considers space-elastic demand) by shelf-segment and position demand. Any form of substitution demand is thereby excluded. The pre-calculated profits are provided to the BIP to maximize overall profit for the initial solution. Based on the initial solution of the BIP we compute the substitution demand and update the total demand given the assortment and number of facings (k) as well as position (p_{icmn}) and inventory (x_{ick}) of all items obtained (z_{ick}) from the BIP. As demand input has now changed, the BIP is solved again in the iteration phase (Stage 2). Yet this time the unique proposition of OC-ASPI comes into play as we preempt potentially inefficient solutions. The item profit π_{ick} is not calculated for all possible $k \in K_{ic}$ as in JOCIAO. Instead $k \in K_{ic}$ is limited to deviate from the previous iteration's solution $k^{\ell-1}$ to a certain extent e in each channel c .

To analyze the magnitude by which the facings of a single item i change from one iteration $\ell - 1$ to the next iteration ℓ , we solved 50 instances of different assortment sizes up to $N = 30$. For the store, we found that the facings of an item change with a magnitude of > 2 in only about 1% of the cases. In the webshop, facing changes are even smaller and only happen with a magnitude of 1 (cf. Table 4.4). We therefore set the extent for each facing iteration to $e_c = \{2, 1\}$ for $c = \{0, 1\}$, with $\sum_{k \in K_{ic}} k \cdot z_{ick}^{\ell-1} - e \leq k \leq \sum_{k \in K_{ic}} k \cdot z_{ick}^{\ell-1} + e$. Our contribution is to exclude a major proportion of inefficient solutions that would have been most likely outside the next iteration solution, thereby reducing the number of calculations and eventually runtimes. The new solution within e_c is drawn upon to update substitution demand and total

demand. This process is repeated until a stop criterion is met (e.g., no more change in profit from one iteration to the next).

Table 4.4: Magnitude of facing changes from one iteration $\ell - 1$ to the next iteration ℓ , average of 50 instances

Channel	Items	0	1	2	>2
Store	10	90.3%	8.0%	0.8%	0.8%
	20	91.7%	6.8%	0.4%	1.2%
	30	92.4%	6.2%	0.4%	0.9%
Webshop	10	96.2%	3.8%	0.0%	0.0%
	20	94.8%	5.2%	0.0%	0.0%
	30	95.1%	4.9%	0.0%	0.0%

Iterative Heuristic Building upon the overview, we further specify the implementation and computation process of the two-stage solution approach along the pseudo code displayed in Figure 4.4.

Stage 1 – Initialization The non-linearity is bypassed in Stage 1. It pre-calculates both demand and profit for a given set of integer facings and the webpage-position. In Step 1.1, we set the iteration index ℓ to zero. In Step 1.2, the profit π_{ick}^ℓ of iteration ℓ is calculated for every item i in every shelf-segment t in channel c , every k in the range from $\sum_{k \in K} k_{ic} = [k_{ic}^{min}, k_{ic}^{max}]$ and every position allocation p_{icmn} (Equation (4.1)). The demand density function at this point includes space-elastic, shelf-segment, and position demand ($f_{ick}^{*,\ell} = f_{D_{ick}^{SP,SH,PO}}^{*,\ell}$), as denoted in Equation (4.14). Because substitutions are still excluded, the demand density function only includes invariants and allows the computation for any given facing and any given position without relying on the decisions of other items. In Step 1.3, π_{ick}^ℓ solves the BIP model and issues z_{ick}^ℓ , z_{ickt}^ℓ and p_{icmn}^ℓ . In Step 1.4 the initial demand density function $f_{ick}^{*,\ell}$ is updated based on this assortment, number of facings per shelf-segment, positions and inventory levels (cf. Equation (4.27)). This updated version therefore accounts for substitution demand while the initial solution (up to Step 1.3) omits substitution.

Stage 2 – Iterations Stage 2 accounts for substitution demand and thereby optimizes the initial solution from Stage 1. In Step 2.1, a loop is implemented, which will be introduced in Step 2.6. In Step 2.2 we register the current iteration by updating ℓ to $\ell + 1$. In Step 2.3.1 we include OOA and OOS substitution in the demand distributions of all following iterations $\ell \geq 1$ by setting $(f_{ick}^{*,\ell})$ equal to the demand distribution from the previous iteration. In Step 2.3.2, π_{ick}^ℓ is pre-calculated and updated for every item, channel, shelf-segment, row and column. Facings are constrained with $\sum_{k \in K_{ic}} k \cdot z_{ick}^{\ell-1} - e \leq \sum_{k \in K_{ic}} k \cdot z_{ick}^{\ell-1} + e$. In Step 2.4., we are able to solve the BIP with substitution effects based on the updated item profits π_{ick}^ℓ and obtain z_{ick}^ℓ , z_{ickt}^ℓ and p_{icmn}^ℓ . In Step 2.5 the demand density function $f_{ick}^{*,\ell}$ is updated for each item. The convolution of the relevant demand density functions and z_{ict}^ℓ , z_{ickt}^ℓ and p_{icmn}^ℓ are put to use to perform this. In Step 2.6, a repetition of the algorithm from Step 2.1 is demanded until the Π^ℓ of two subsequent iterations remains unchanged (cf. Equation (4.28)).

$$\epsilon = \Pi^\ell - \Pi^{\ell-1} = 0 \quad (4.28)$$

As described, to circumvent the non-linearity we are constrained to calculate the substitution based on a given z_{ickt} and p_{icmn} for each item in each channel. This necessity is satisfied as the update of the assortments, facings, positions and inventories in iteration ℓ is carried out by solving the previous iteration $\ell - 1$. While the solution is always based on a one-iteration lagged demand, which potentially leads to non-optimal results, the problem is solved optimally for every single iteration. We address this potential concern in the numerical analysis.

4.5 Numerical results and managerial insights

This section gives an overview of the test setting (Section 4.5.1), analyzes the computational performance of the heuristics (Section 4.5.2) and provides insights on the relevance of demand effects (Section 4.5.3).

4.5.1 Overview of the test setting and applied data

We want to provide generally valid insights into demand effects with different randomly generated data sets in line with empirical studies from literature. As no empirical OC studies are available, we refer to insights from SC studies. All data sets are available at <https://github.com/JonasHen90/OCAssortments>, with each data set consisting of 30 instances. To derive causal insights and avoid mixing effects, parameters are populated identically across channels if not specified further. To simulate close-to-reality conditions, the values of all parameters are confined by a defined range. Available space $\sum_{t \in T} S_{ct}$ is limited and serves $\sim 80\%$ of the aggregated base demand of all store items and $\sim 90\%$ of the aggregated base demand of all webshop items. The revenue and cost parameters are set as $r_{ic} \in [20, 50]$, $u_{ic} \in [15, 30]$ and $v_{ic} \in [4, 20]$, with $r_{ic} \geq c_{ic} \geq v_{ic} \geq s_{ic} \forall i, c$. Shortage costs primarily represent the dissatisfaction of customers who cannot satisfy their demand. In our setting, convenient substitutions are provided within the channels as well as via BOPS and DAE. We therefore set $s_{ic} = 0$ without loss of generality. We assume that demand is normally distributed with $\mu_{ic} \in [7, 25]$ and the coefficient of variation $CV_{ic} \in [1\%, 50\%]$. Campo et al. (2004) find conforming OOA and OOS substitution rates. We follow accordingly with $\gamma_{jci_c}^{OOA} = \gamma_{jci_c}^{OOS}$ and $\delta_{idi_c}^{OOA} = \delta_{idi_c}^{OOS}$. Substitutions for different items across channels are disregarded (i.e., $D_{ic}^{OOA(3)} = D_{ic}^{OOS(3)} = 0$). We denote the aggregated OOA substitution rates for in-channel substitution ($\Gamma_{ic}^{OOA} = \sum_{j \in I} \gamma_{jci_c}^{OOA}$) and cross-channel substitution ($\Delta_{ic}^{OOA} = \sum_{d \in C} \delta_{idi_c}^{OOA}$), which add up to the total OOA substitution rate of item i in channel c , i.e., $\theta_{ic}^{OOA} = \Gamma_{ic}^{OOA} + \Delta_{ic}^{OOA}$.

OOS substitution rates are defined in the same manner using Γ_{ic}^{OOS} , Δ_{ic}^{OOS} and θ_{ic}^{OOS} . The aggregated OOA and OOS rates θ_{ic}^{OOA} and θ_{ic}^{OOS} cannot exceed 100%. We assume that $\gamma_{jcic}^{OOA} = \frac{\Gamma_{ic}^{OOA}}{I-1}$ and $\delta_{iaic}^{OOA} = \frac{\Delta_{ic}^{OOA}}{C-1}$. We define space-elasticity $\beta_{ic} \in [0\%, 35\%]$ for the store and $\beta_{ic} = 0$ for the webshop (Eisend, 2014). The attractiveness factor λ_{ct} is differentiated for each vertical shelf-segment t with the top level attracting the highest additional demand (see e.g., Drèze et al. (1994)). For the store, we define three vertical segments ($T = 3$) with limited space S_{ct} . Following Drèze et al. (1994) we set $\lambda_{ct} = \{1.0, 1.2, 1.4\}$ for $t = \{1, 2, 3\}$. Meanwhile, as there is no shelf-segment demand in the webshop we assume one shelf-segment ($T = 1$) and shelf-segment attractiveness $\lambda_{ct} = 1.0$ for the webshop. Accordingly, for the store $M = N = 1$ and $\psi_{imn} = 1.0$ as no webpage positioning is carried out for the store. The webshop page, represented by $|M| \times |N|$, is always assumed to be sufficient for placing all listed items. However, space is limited for the webshop in the warehouse, resulting in the situation that some webshop positions are discarded if products cannot be listed due to warehouse space constraints. According to Djasasbi et al. (2010) and Faraday (2000), position elasticity ψ_{imn} depends on item i and the location mn on the webpage and is higher for positions on the top left of the page. We therefore assume $\psi_{imn} \in [0\%, 16\%]$ for the webshop (cf. Chen et al. (2016)). To prevent any noise in the results, all items are assigned a width $b_{ic} = 1$, a quantity per facing $g_{ic} \in [3, 6]$ for stores, and a quantity per package unit $g_{ic} = 10$ for online channels.

A machine running on Windows 10 64-bit with an Intel Core i7-8665U CPU 1.90GHZ and 16 GB of installed memory was used for the numerical tests. The model and algorithm are implemented in Python 3.6 and solved with Gurobi Optimizer 8.0.

4.5.2 Computational efficiency of the OC-ASPI algorithm

Runtime tests Our decision problem at hand typically arises in the course of a periodical assortment planning cycle (e.g., for fashion retailers with seasonal items or food retailers with stable assortments). A reasonable computation time for the tactical problem is required to allow the retailers to calculate and assess different scenarios, even for large assortments. Categories often include as many as 60 to 80 items. Our model solved a set of large instances with up to 100 items to assess the runtimes. That means we have a total of 200 items across the channels. Table 4.5 illustrates the results, demonstrating reasonable solution times for the tactical problem.

Table 4.5: OC-ASPI vs. benchmark: run time and profit change, *average of 30 instances*

Number of items I_c per channel	10	20	30	40	60	80	100
Space S_c in store, online warehouse	30, 15	60, 30	90, 45	120, 60	180, 90	240, 120	300, 150
Avg. run time OC-ASPI [minutes]	1.52	6.97	17.35	40.0	101.1	219.1	– ²
Avg. run time saving vs. benchmark	87.1%	86.7%	87.2%	– ³	– ³	– ³	– ³
Avg. profit change ¹	-0.04%	-0.09%	-0.12%	– ³	– ³	– ³	– ³
Median profit change	0.00%	-0.02%	-0.06%	– ³	– ³	– ³	– ³

¹ (Total profit obtained by OC-ASPI / Total profit obtained by benchmark of Hense and Hübner (2021)) - 1

² Could not be solved due to memory limitations

³ No solution obtained with benchmark within 300 minutes

Comparison with benchmark We chose to compare OC-ASPI with an alternative heuristic. The heuristic provided by Hense and Hübner (2021) is the only available approach that solves OC assortments for store and online channels, but without shelf-segment- and position demand. As such, the benchmark heuristic solves a special case. Our contribution results from the extension of demand effects, a modified stop criterion, and a streamlined iteration procedure. While the benchmark calculates in each iteration ℓ the demand and profit for all possible facings k for each item i in each channel c , OC-ASPI only iterates across all facings k , with $\sum_{k \in K_{ic}} k \cdot z_{ick}^{l-1} \pm 2$ for the store and $\sum_{k \in K_{ic}} k \cdot z_{ick}^{l-1} \pm 1$ for the webshop. We assessed the run times for different assortment sizes in Table 4.5. OC-ASPI provides enormous run time savings of around 87% at the cost of marginal profit losses of less than 0.12%, both driven by the limitation of iterations.

4.5.3 Managerial insights into related demand effects

We analytically assess the magnitude of each demand effect on profit and assortments and develop propositions based on the insights obtained. This allows us to identify decisive demand effects. We assess the consequences of ignoring one of the demand effects when optimizing OC assortments in order to understand the relevance of each demand source. This describes the situation where consumers exhibit the demand effect examined but the retailer assumes its irrelevance. We term this “value of information” as it describes the possession of information about the existence and magnitude of a particular demand effect and consequently the value of composing assortments accordingly. “Value of information” is the difference between the following two scenarios:

- “*Ignoring*” describes a retailer who disregards the particular demand effect (e.g. $\beta_{ic} = 0\%$), while in reality the demand effect is taking place (e.g., $\beta_{ic} = 35\%$). The solution is then evaluated ex-post for varying degrees of the demand effect (e.g., $\beta_{ic} = 5\%, 15\%, 25\%, 35\%$).
- “*Integrating*” is the scenario as described in this paper with full information about demand effects and direct integration into the optimization approach.

The analyses are structured as follows. First, we assess each demand effect in relation to profit (cf. Table 4.6 and propositions (1) to (4)). Next, the composition of assortments is analyzed in detail (cf. Table 4.7 and propositions (5) to (7)). Third, we assess the impact of channel sizes and cross-channel substitution on profits (cf. Table 4.8 and proposition (8)).

(1)-(4) Impact of demand effects on profit We use the “Integrating” scenario and “value of information” as measurements to analyze the impact of increasing demand rates on profit.

(1) Demand effects within the channel are stronger than across channels. The magnitude of each effect depends of course on the expected customer behavior. The values we apply are informed by related (single-channel) empirical studies. Table 4.6 highlights the importance of integrating space-elastic demand in OC assortment planning, followed by position demand, shelf-segment demand, in-channel substitution, and cross-channel substitution. The results demonstrate a potential profit loss of up to 15.5% on average (for $\beta_{ic} = 35\%$) if space-elastic demand in the store is ignored in OC assortment planning, a profit loss of up to 4.9% (for $\psi_{imn} = 16\%$) when ignoring position-elasticity in the webshop, and a profit loss of up to 4.7% on average (for $\lambda_{ct} = 35\%$) by not integrating shelf-segment demand. In-channel substitution marks a significantly smaller profit loss of up to 1.5%. Ignoring cross-channel substitution causes retailers to potentially forego profit losses of up to 0.4% on average. The order is largely driven by the rate of the demand effect and the size of the demand it applies to. Space elasticity causes higher profit losses when being ignored as the demand rate is particularly high (i.e., for “strong” = 35%), and applies to the full base demand. The demand rate for position elasticity is lower (i.e., for “strong” = 16%), and decreases the more a product is placed in a less visible position. Similar to that, the demand rate for shelf-segment attractiveness (e.g., for “strong” = 35%) only applies to the most visible shelf-segment and is reduced for lower shelf-segments. Lastly, even though both in-channel and cross-channel substitution have a high demand rate (i.e., for “strong” = 40%), this only applies to OOS and OOA demand, and is constrained to substitutions within or across channels. This highlights the need for retailers to put a clear focus on integrating in-channel effects over cross-channel effects when defining OC assortments.

(2) Flexibility in stocking quantities determines substitution potential within the channel. Ignoring in-channel store substitution causes greater profit losses than ignoring in-channel webshop substitution. Despite identical margins, base demand, and substitution rates, the profit loss from ignoring customer substitution within the store (up to 1.5%) is higher compared to disregarding substitution within the webshop (up to

0.2%). This stems from the case that larger package sizes are stored for the webshop (quantity per package unit $g_{ic} = 10$), which allows less flexibility in changing the assortment and requires more additional demand to make the change worthwhile, compared to the store (with quantity per package of $g_{ic} \in [3, 6]$). This means inventories in the webshop are less flexible in terms of quantity adjustments and react less sensitively to additional demand. Also, the store creates more substitution demand given the smaller share of fulfilled base demand, which consequently creates more opportunities to change assortments. Looking at the “Integrating” scenario, we verify a differing trend. Higher demand rates for in-channel webshop substitution lead to higher profit advantages of up to 3.2% over a 0% scenario. Retailers should therefore prioritize integrating in-channel store substitution (with units that usually have smaller stocks) over in-channel webshop substitution (with units that usually have larger stocks).

(3) Digital Assortment Extension has a higher profit potential than buy-online pick-up in store. Ignoring cross-channel substitution to the webshop (i.e., DAE) results in higher profit losses than ignoring it towards the store (i.e., BOPS). This is mainly driven by more unfulfilled demand in the store and therefore higher substitution demand for the webshop. The additional demand primarily helps to sell units of inventory that would have otherwise be unsold. When integrating these demand effects, it is more important for retailers to concentrate on cross-channel substitutions from stores to webshops. This results in higher profit advantages when demand effects increase.

(4) In-channel store substitution is the most important substitution option out of all substitution possibilities. Ignoring in-channel store substitution can lead to profit losses of up to 1.5% and is far more severe than disregarding any other substitution possibility. First of all, the store generates more substitution demand than the webshop given the smaller share of fulfilled base demand and more demand effects. In addition to that, the store is more flexible in changing assortments given the smaller stocking quantities. Lastly, in-channel substitutions spread

the additional demand to more items while cross-channel substitution only allows substitutions of identical items. Therefore, in-channel substitution potentially adds demand to more profitable items than the one originally demanded, which makes an assortment change worthwhile.

Table 4.6: Impact of disregarding demand effects: profit change

Demand effect	Analysis, profit change	Demand impact ³			
		Weak	Med.-Weak	Med.-Strong	Strong
Space-elasticity β_{ic}	Integrating ¹	2.58%	7.71%	13.07%	18.21%
	Value of information ²	1.07%	5.03%	10.2%	15.48%
Shelf-segment attractiveness λ_{ct}	Integrating ¹	2.32%	4.22%	5.9%	7.42%
	Value of information ²	1.14%	2.31%	3.52%	4.74%
Position elasticity ψ_{imn}	Integrating ¹	2.81%	5.59%	8.38%	11.37%
	Value of information ²	1.14%	2.32%	3.49%	4.87%
In-channel store sub. $\gamma_{j_{cic}}^{OOA}, \gamma_{j_{cic}}^{OOS}$	“Integrating” ¹	0.83%	1.47%	2.15%	2.84%
	Value of information ²	0.25%	0.52%	0.97%	1.53%
In-channel webshop sub. $\gamma_{j_{cic}}^{OOA}, \gamma_{j_{cic}}^{OOS}$	Integrating ¹	0.79%	1.59%	2.39%	3.2%
	Value of information ²	0%	0.04%	0.11%	0.22%
Cross-channel store sub. ⁴ $\delta_{i_{dic}}^{OOA}, \delta_{i_{dic}}^{OOS}$	Integrating ¹	0.46%	0.84%	1.16%	1.52%
	Value of information ²	0%	0.05%	0.14%	0.36%
Cross-channel webshop sub. ⁵ $\delta_{i_{dic}}^{OOA}, \delta_{i_{dic}}^{OOS}$	Integrating ¹	0.16%	0.23%	0.32%	0.41%
	Value of information ²	0.1%	0.12%	0.16%	0.19%

¹ (Retailer profit / Retailer profit with demand effect = 0%) - 1

² Profit loss as difference between Integrating scenario and Ignoring scenario

³ For $\beta_{ic} = 5\%, 15\%, 25\%, 35\%$, $\lambda_{ct} = 5\%, 15\%, 25\%, 35\%$, $\psi_{imn} = 4\%, 8\%, 12\%, 16\%$, $\gamma_{j_{cic}}^{OOA}, \gamma_{j_{cic}}^{OOS} = 10\%, 20\%, 30\%, 40\%$, $\delta_{i_{dic}}^{OOA}, \delta_{i_{dic}}^{OOS} = 10\%, 20\%, 30\%, 40\%$

⁴ DAE; ⁵ BOPS

(5) - (7) Impact of demand effects on assortment composition After the analysis of the demand effects on total profit, we will analyze the impact on the solution structure. In Table 4.7 we introduce additional measurements to assess the assortment composition. *Identical solution* measures the share of identical solutions between the “Integrating” and “Ignoring” scenario in terms of listed and delisted products as well as the number of facings (and therefore inventory) assigned. *Product overlap* reports the share of identical solutions with respect to products only (i.e., in both listed or delisted scenarios). *Average facing change* describes the average number of facings by which an item differs between the scenarios. *Channel congruence* assesses the alignment of the store and webshop assortment

for each of the two scenarios. Positive numbers indicate that assortments between the channels become more aligned when integrating the demand effect.

(5) Assortments diverge from optimal configurations the more inaccurate demand is considered. As expected, the optimal solutions increasingly diverge from the assortments that ignore a demand effect with increasing demand rates. The stronger the demand effect, the more products and facings are changed in the “Integrating” scenario. The change often happens primarily in one of the two channels, as the demand effects also mainly focus on one channel. This leads to additional demand in one channel and enables the channel to focus on the more profitable items. For example, cross-channel store substitution causes additional demand in the webshop. This, in turn, makes a change of the webshop assortment reasonable and leads to a share of non-identical webshop assortments in comparison with the inaccurate “Ignoring” scenario (for $\delta_{i_{dic}}^{OOA} = \delta_{i_{dic}}^{OOS} = 40\%$). In this context, the change in the share of identical solutions is also strongly connected to the change in profit, i.e., the higher the change in profits, the stronger the assortments diverge. To sum up, this implies that the greater the demand effect, the higher the chances that OC retailers will miss out on listing the right quantities of their profit-maximizing products.

(6) Getting the facings and inventories right is a more pressing issue compared to getting the products right. When erroneously ignoring demand effects, more facings are chosen suboptimally than are assortment configurations. Differences between the “Integrating” and “Ignoring” assortments are often driven by varying facings rather than product selection. In many cases the assortment configuration only differs slightly, but facings and inventories drive the difference. When varying space elasticity, for instance, this results in no identical solution at all, but 71% of the store products and 97% of the webshop products still overlap for $\beta_{ic} = 35\%$. The change is then driven by a change in the number of facings, which can be as small as 0.1 facings for the webshop. For the strong magnitude of the

demand effects, an overlap of 71% of the store assortment also constitutes the smallest overlap across all channels and demand effects.

(7) Increasing store-related demand effects decrease the similarity of assortments across channels, while increasing webshop-related demand effects have the opposite effect. Demand effects that mainly impact optimal solutions within the store (i.e., space elasticity, shelf-segment attractiveness, in-channel store substitution, and cross-channel webshop substitution) lead to compositions where the store and webshop assortment increasingly diverge when integrating the demand effect. The underlying reason is the number of listed products per channel. In the store, fewer products are listed than in the webshop. With increasing demand effects the store focuses even more on the most profitable items, cuts down the assortment size, and therefore distances itself further from the larger webshop assortment. The channel congruence for shelf-segment attractiveness $\lambda_{ct} = 35\%$ decreases by -9% , for example. If an “Ignoring” store assortment had 15 listed products, this would indicate that an average of 1.35 fewer products (as $9\% \cdot 15 = 1.35$) are now listed in the “Integrating” store assortment given that the webshop assortment remains unchanged. The opposite is true for demand effects focused on webshop demand (i.e., position elasticity, in-channel webshop substitution, and cross-channel store substitution). Webshop assortments approximate the store assortment. More products are listed in the webshop, but with increasing demand effects the webshop focuses on the most profitable items, cuts down assortment sizes, and therefore increasingly assimilates the store assortment.

Table 4.7: Impact of disregarding demand effects: Assortment composition

Demand effect	Analysis	Channel	Demand impact ⁵			
			Weak	Med.-Weak	Med.-Strong	Strong
Space-elasticity β_{ic}	Identical solutions ¹	Store	0%	0%	0%	0%
		Webshop	57%	30%	7%	0%
	Product overlap ²	Store	96%	86%	77%	71%
		Webshop	100%	99%	98%	97%
	Avg. facing change ³	Store	0.4	1.0	1.6	2.2
Webshop		0	0.1	0.1	0.1	
Channel congruence ⁴	Total	-4%	-15%	-25%	-32%	
Shelf-segment attractiveness λ_{ct}	Identical solutions ¹	Store	3%	0%	0%	0%
		Webshop	70%	53%	53%	43%
	Product overlap ²	Store	97%	96%	94%	92%
		Webshop	99%	99%	99%	99%
	Avg. facing change ³	Store	0.2	0.4	0.6	0.8
Webshop		0	0	0	0	
Channel congruence ⁴	Total	-2%	-4%	-6%	-9%	
Position elasticity ψ_{imn}	Identical solutions ¹	Store	70%	50%	47%	43%
		Webshop	10%	0%	0%	0%
	Product overlap ²	Store	100%	99%	99%	99%
		Webshop	98%	95%	92%	90%
	Avg. facing change ³	Store	0	0.1	0.1	0.1
Webshop		0.1	0.2	0.3	0.4	
Channel congruence ⁴	Total	4%	8%	12%	16%	
In-channel store sub. $\gamma_{jctc}^{OOA}, \gamma_{jctc}^{OOS}$	Identical solutions ¹	Store	13%	0%	0%	0%
		Webshop	97%	97%	77%	70%
	Product overlap ²	Store	99%	97%	96%	94%
		Webshop	100%	100%	100%	100%
	Avg. facing change ³	Store	0.1	0.2	0.4	0.5
Webshop		0	0	0	0	
Channel congruence ⁴	Total	-2%	-4%	-6%	-8%	
In-channel webshop sub. $\gamma_{jctc}^{OOA}, \gamma_{jctc}^{OOS}$	Identical solutions ¹	Store	93%	90%	87%	83%
		Webshop	53%	37%	23%	17%
	Product overlap ²	Store	100%	100%	100%	100%
		Webshop	99%	98%	98%	97%
	Avg. facing change ³	Store	0	0	0	0
Webshop		0	0	0.1	0.1	
Channel congruence ⁴	Total	1%	1%	2%	3%	
Cross-channel store sub. $\delta_{idtc}^{OOA}, \delta_{idtc}^{OOS}$	Identical solutions ¹	Store	80%	70%	73%	53%
		Webshop	47%	13%	10%	0%
	Product overlap ²	Store	100%	99%	99%	99%
		Webshop	99%	98%	98%	97%
	Avg. facing change ³	Store	0	0	0	0.1
Webshop		0	0.1	0.1	0.2	
Channel congruence ⁴	Total	0%	0%	0%	0%	
Cross-channel webshop sub. $\delta_{idtc}^{OOA}, \delta_{idtc}^{OOS}$	Identical solutions ¹	Store	63%	40%	27%	20%
		Webshop	100%	100%	93%	93%
	Product overlap ²	Store	100%	100%	99%	99%
		Webshop	100%	100%	100%	100%
	Avg. facing change ³	Store	0	0.1	0.1	0.1
Webshop		0	0	0	0	
Channel congruence ⁴	Total	0%	-1%	-1%	-2%	

¹ Share of identical items and inventory between “Integrating” and “Ignoring” scenario² Share of identical items between “Integrating” and “Ignoring” scenario³ Average magnitude by which the facings of an item differ between “Integrating” and “Ignoring” scenario⁴ Difference in assortment similarity across channels, i.e., positive numbers indicate that assortments become more identical across channels for the “Integrating” vs “Ignoring” scenario, and vice versa⁵ For $\beta_{ic} = 5\%, 15\%, 25\%, 35\%$, $\lambda_{ct} = 5\%, 15\%, 25\%, 35\%$, $\psi_{imn} = 4\%, 8\%, 12\%, 16\%$, $\gamma_{jctc}^{OOA}, \gamma_{jctc}^{OOS} = 10\%, 20\%, 30\%, 40\%$, $\delta_{idtc}^{OOA}, \delta_{idtc}^{OOS} = 10\%, 20\%, 30\%, 40\%$ ⁶ DAE; ⁷ BOPS

(8) Impact of channel sizes To understand the impact that cross-channel substitutions can have when retailers operate with differently sized stores or warehouses, we carried out a sensitivity analysis across the magnitude of demand effects and differing channel sizes. For cross-channel store substitution we assumed a store size with shelves for 90 facings, which

fulfills $\sim 80\%$ of the total base store demand. Next, we assumed stores at 50% (45 facings) and 25% of the initial size (23 facings). The warehouse size remains unchanged. Similar to that, a warehouse with a shelf space of 45 facings was assumed for cross-channel webshop substitution. This satisfies $\sim 90\%$ of the total base webshop demand. Subsequently, we cut the warehouse to 50% (23 units) and 25% (11 units) of the initial space while leaving the store shelf at 90 facings.

(8) The impact of cross-channel substitutions depends on channel size. The smaller a channel, the less inventory can be stored and the more substitutions occur to the other channel. In the case of the stores, this means that a smaller store and stronger cross-channel substitution from the stores to the webshop lead to increasing profit losses when wrongly ignoring this effect. The underlying cause is the additional, unfulfilled demand in the store. Customers consequently switch more intensively to substitute products from the webshop, which results in changing demand and the rearrangement of assortments in the webshop. If the demand effect is ignored, the additional demand cannot be leveraged by changing the assortment, which explains the increasing “value of information”, which goes up to 1.46%. When integrating this demand effect, decreasing store sizes and increasing demand effects can also make a great difference and generate up to 11.07% more profit. The same holds true for webshops: it is just that channels change. Additional demand in the webshop that is unfulfilled causes higher substitutions to the store, and can be used to optimize the assortment in the store.

Table 4.8: Impact of disregarding cross-channel substitution: profit change

Channel size	Analysis	Demand impact ³			
		Weak	Med.-Weak	Med.-Strong	Strong
Cross-channel substitution to webshop					
90, 45	“Integrating” Scenario, profit change ¹	0.46%	0.84%	1.16%	1.52%
	Value of information, profit loss ²	0%	0.05%	0.14%	0.36%
45, 45	“Integrating” Scenario, profit change ¹	1.68%	3.11%	4.51%	5.64%
	Value of information, profit loss ²	0.07%	0.15%	0.49%	0.89%
23, 45	“Integrating” Scenario, profit change ¹	3.25%	6.14%	8.81%	11.07%
	Value of information, profit loss ²	0.09%	0.35%	0.88%	1.46%
Cross-channel substitution to store					
90, 45	“Integrating” Scenario, profit change ¹	0.16%	0.23%	0.32%	0.41%
	Value of information, profit loss ²	0.10%	0.12%	0.16%	0.19%
90, 23	“Integrating” Scenario, profit change ¹	0.26%	0.5%	0.79%	1.10%
	Value of information, profit loss ²	0.08%	0.18%	0.34%	0.52%
90, 11	“Integrating” Scenario, profit change ¹	0.65%	1.25%	1.81%	2.39%
	Value of information, profit loss ²	0.19%	0.51%	0.89%	1.31%

¹ (Retailer profit / Retailer profit with $\delta_{i_d^{ic}}^{OOA}, \delta_{i_d^{ic}}^{OOS} = 0\%$) - 1

² Difference between “Integrating” Scenario and “Ignoring” Scenario

³ For $\delta_{i_d^{ic}}^{OOA}, \delta_{i_d^{ic}}^{OOS} = 10\%, 20\%, 30\%, 40\%$

4.5.4 Summary of insights and implications for retailers and researchers

Summary of managerial insights We analyzed the impact of different demand effects on profits and solution structures. To do so, a comparison between the “Integrating” and the “Ignoring” scenarios revealed several general insights. We note that demand effects within the channels are greater in terms of profit impact than across channels. One driving factor is the storage quantity and handling unit, which is commonly larger in online warehouses than in stores. This makes the webshop less flexible in terms of quantity adjustments, and it reacts less sensitively to additional demand in that channel. Solutions also increasingly change with increasing demand rates. The main driver in this context is not the assortment configuration, as they even resemble each other for highly differing demand rates, but the changes in facings. Beyond this, demand effects that are focused on increasing demand in the store, cause the similarity of assortments

across channels to decrease, while increasing webshop-related demand effects have the opposite effect. Lastly, we identified that profits caused by increasing cross-channel substitutions are highly dependent on the size of the channel.

Guidelines for retailers Retailers benefit from our insights as they help to prioritize the integration of various demand effects in retailers' OC assortment planning. The integration of space-elasticity, shelf-segment demand, position demand, and in-channel store substitution are especially highly recommended. While in-channel and cross-channel webshop substitution is less impactful. We have also outlined the underlying reasons and highlighted dependence on the magnitude of demand rates, channel package sizes, and channel size. In combination, these factors define the profit impact for retailers and options for steering the assortment compositions and determine whether it is worth integrating certain demand effects. Leveraging this knowledge and monitoring the factors helps to optimize profits.

Discussion in light of the literature Our findings have implications for empirical research, given the complexity of measuring various demand effects, especially for cross-channel substitution. Only few empirical studies have analyzed this phenomenon so far. Corsten and Gruen (2019) found that 66% of the demand potentially remains with the retailer and increases the demand for substitute items. Using these insights, we assumed cross-channel substitutions in the magnitude of the empirically measured values and found that these effects have a minor impact on profits and solution structures. This shows that the significant effort required for empirical measurement needs to be carefully evaluated.

Our findings also further reveal insights for modeling. A very limited number of optimization models account for cross-channel effects. Most of the models are constrained in their applicability to instances of practice-relevant size, runtime or solution quality due to the combinatorial complexity arising

from adding channels. Our findings show that cross-channel substitutions have an effect, but at a moderate scale. This means that complex solution approaches suffer from either runtime inefficiency or low solution quality problems and need to be carefully considered. Research should particularly focus on optimization models that account for in-channel effects.

4.6 Conclusion and future research

In this paper we detail the demand effects related to OC assortment planning and the complexity involved in their empirical assessment. To allow for a numerical assessment, we developed a BIP model for OC retailers to optimize the assortment, space and position, and inventory decision for each channel. We incorporated stochastic, space-elastic, shelf-segment, position, OOA and OOS in-channel, and cross-channel demand taking into account space constraints. In comparison with existing contributions, we provide a model that is the first to integrate such a range of decisions and demand effects simultaneously. Given the NP-hard multiple-knapsack problem, we formulated the specialized heuristic OC-ASPI. We are able to apply the program to large-scale, practical problem instances. Our primary contribution is the analytical assessment of all demand effects. We found evidence that it is of utmost importance to include space-elastic, shelf-segment and position demand in OC assortment planning. Ignoring these effects in assortment planning despite their existence can lead to dramatic profit losses of up to 15.5% on average. The consideration of OOA and OOS in-channel store substitution is a little less important. Yet this demand effect can still cause significant profit losses of up to 1.5% when ignored. The remaining three substitution effects, i.e., OOA and OOS in-channel webshop, cross-channel store, and cross-channel webshop demand generally have only a small impact on profits. However, this greatly depends on channel size.

Our practical contribution and the underlying model and heuristic can build the basis for future research. As our analytical assessment is mainly built upon existing empirical insights for SC retailers, we deem an empirical confirmation of assumed cross-channel substitution rates meaningful. Along these lines, price policies for services (e.g., BOPS) and products across channels are further potentially crucial drivers of demand that could be worth investigating as well. Another driver of demand is cross-selling when customers pick-up a product in the store and decide to buy another product on top of that. In addition to that, it is worth focusing on the economics behind different channels and fulfillment models like BOPS or DAE, incorporating adequate costs and thereby examining opportunities for active customer steering within and across channels. Moreover, the problem could be related to operational planning to salvage overstocks. In this context, a tactical perspective could also include reorder options from suppliers, customer fulfillment with backlogging or customer returns. Lastly, we examined a general case, but special cases such as showrooms, special product categories or characteristics (such as impulse purchases or products with high affinity) may cause the results to vary and therefore justify further investigations.

Figure 4.4: Pseudo code of the specialized heuristic

Stage 1 – Initialization: Solve BIP without substitution effects
Input: Set of channels C , set of shelf-segments in channel T_c , set of items in channel I_c , set of possible facings K_{ic} , set of rows M and set of columns N

Step 1.1 Set $\ell = 0$
Step 1.2 For all channels $c \in C$:
 For all items $i \in I_c$:
 For all facings $k \in K_{ic}$:
 For all shelf-segments $t \in T_c$:
 For all positions $p \in M \cdot N$:
 Calculate π_{ick}^ℓ with $f_{ick}^{*,\ell} = f_{D_{ick}^{SP,SH,PO}}^{*,\ell}$
 End for
 End for
 End for
 End for
Step 1.3 Solve BIP using Equation (4.2) to (4.9) to obtain $z_{ick}^\ell, z_{ickt}^\ell, p_{icmn}^\ell$ and Π^ℓ
Step 1.4 For all channels $c \in C$:
 For all items $i \in I_c$:
 Deploy $z_{ick}^\ell, z_{ickt}^\ell$ and p_{icmn}^ℓ in Equation (4.27) to update $f_{ick}^{*,\ell}$
 End for
Return: $z_{ick}^\ell, z_{ickt}^\ell, p_{icmn}^\ell, f_{ick}^{*,\ell}$ and Π^ℓ

Stage 2 – Iterations: Solve BIP with substitution effects
Input: Set of channels C , set of shelf-segments in channel T_c , set of items in channel I_c , set of possible facings K_{ic} , set of rows M , set of columns N and $z_{ick}^\ell, z_{ickt}^\ell, p_{icmn}^\ell, f_{ick}^{*,\ell}$ and Π^ℓ

Step 2.1 Repeat
Step 2.2 Set $\ell = \ell + 1$
Step 2.3 For all channels $c \in C$:
Step 2.3.1 For all items $i \in I_c$:
 Set $f_{ick}^{*,\ell} = f_{ick}^{*,\ell-1}$
Step 2.3.2 For all facings $k, e \in K_{ic}$ with $k_{ic}^{\ell-1} - e \leq k \leq k_{ic}^{\ell-1} + e$:
 For all shelf-segments $t \in T_c$:
 For all positions $p \in M \times N$:
 Calculate π_{ick}^ℓ with $f_{ick}^{*,\ell}$
 End for
 End for
 End for
Step 2.4 Solve BIP using Equation (4.2) to (4.9) to obtain z_{ick}^ℓ and all $z_{ickt}^\ell, p_{icmn}^\ell$ and Π^ℓ
Step 2.5 For all channels $c \in C$:
 For items $i \in I_c$:
 Deploy z_{ick}^ℓ and all z_{ickt}^ℓ and p_{icmn}^ℓ in Equation (4.27) to update $f_{ick}^{*,\ell}$
 End for
 End for
Step 2.6 Until Stop Criteria is met and Equation (4.28) holds true, otherwise continue with Step 2.1
Return: $z_{ick}^\ell, z_{ickt}^\ell, p_{icmn}^\ell$ and Π^ℓ

5 The revival of retail stores via omnichannel operations: A literature review and research framework

The increasing importance of integrated fulfillment concepts revitalizes bricks-and-mortar stores and puts them at the center of retail operations. So-called omnichannel (OC) concepts leverage stores to offer seamless and enhanced operations for offline and online shoppers. Stores are used to fulfill online orders, offer shorter lead times to customers, and extend the assortment across channels. The role of the store and the underlying store operations are thus impacted by profound changes. This transformation has not yet been assessed comprehensively from a practical or an Operations Research (OR) lens.

This paper identifies cross-cutting store-related planning issues and develops a planning framework for OC operations. We apply industry interviews and a systematic literature analysis to derive five planning issues. Research gaps are revealed by matching the pertinent OR literature with managerial needs. The planning issues network design of fulfillment locations, assignment of customer orders, and assortment and inventory planning have been discussed in several store-related OC publications. Demand forecasting and inventory replenishment and returns have received less coverage, and offer significant research opportunities.

5.1 Introduction

Retailing is experiencing a continuous growth of online sales at the expense of bricks-and-mortar stores sales. Customers no longer differentiate between channels, but increasingly expect an integrated offering and seamless channel switching (Agatz et al., 2008; Verhoef et al., 2015). Bricks-and-mortar retailers are reacting to these developments by expanding their online services while trying to leverage their store network (see e.g., Gallino and Moreno (2014); Hübner et al. (2016b); Ishfaq et al. (2016); Wollenburg et al. (2018b); Caro et al. (2020); Shao (2021)). In the past, the online shop mainly supplemented stores. Now, stores not only supplement online sales but are also becoming a central piece of the customer journey across seamlessly integrated channels. This has hugely affected the role of the store. Additional services have been offered, such as delivering products from the stores to the customer's home, enabling the collection of pre-picked shopping carts in stores, or accessing extended assortments via different channels (e.g., Digital Commerce 360 (2018)). The COVID-19 pandemic has accelerated the expansion of such offerings and reinforced the store's role change from being purely a sales area to also acting as a logistics fulfillment hub (see e.g., Caro et al. (2020)). The integration of stores and digital channels is called omnichannel (OC). The store now serves as the epicenter for OC retail operations, functioning as an additional warehouse for online orders besides the traditional purpose as a customer shopping area (see e.g., Brynjolfsson et al. (2013); Bell et al. (2014); Gallino et al. (2017); Gallino and Moreno (2019); Janjevic et al. (2020)). Bell et al. (2018b) describe this development as: "Offline is dead and dying, yet it is also alive and thriving".

The revitalization of stores requires efficient operations planning now more than ever. It needs agile, connected, and responsive retail operations. As stores may now be used as alternative picking locations, retailers have to decide which stores to include in their fulfillment network and from which specific location to pick each online order. In addition, assortments and

inventory levels for each point of sales need to be determined, and how they are to be shared across locations. This requires quantifying, modeling and solving the trade-off between costs and operational advantages. Picking costs in stores are usually higher than in warehouses, for example (see e.g., Boysen et al. (2021); Difrancesco et al. (2021)). However, the access to inventories across stores generates inventory pooling effects that result in lower overstocks at the same time as higher service levels (Alptekinoglu and Tang, 2005). Effectively mastering the renaissance of the store in OC fulfillment requires the development and application of advanced models and OR methods.

The number of publications dealing with such planning models has grown significantly and amounts to over 40 papers. Most of them have been published after 2016 and the *European Journal of Operational Research* has been the major outlet for these papers. The growing importance of OC operations is also reflected in an increasing number of related literature reviews. Agatz et al. (2008) review literature on operational challenges in single and multichannel fulfillment. Operational issues and relevant literature is structured along purchasing, warehousing, delivery, and sales. Montreuil (2016) conceptualize the warehouse and transportation network based on Physical Internet concepts for OC fulfillment. Galipoglu et al. (2018) identify the research front through a content analysis and a citation and co-citation analysis. Kembro et al. (2018) carry out a literature review related to OC warehousing. Melacini et al. (2018) investigate the move towards OC and map emerging issues along distribution network design, inventory and capacity management, and delivery planning and execution. Bijmolt et al. (2021) discuss demand- and supply-side challenges in OC assortment and inventory, distribution and delivery, and returns. Another review is provided by Mou et al. (2018), but they focus on operations within bricks-and-mortar stores only. Cai and Lo (2020) and Raza and Govindaluri (2021) conducted a systematic literature review on omnichannel management. Both descriptive analysis identified in particular supply chain and distribution topics as future research areas. Caro et al. (2020) identified

the distribution approach through e-commerce and omnichannel as one of the future research topics.

The overview shows that current reviews lack topicality (e.g., Swaminathan and Tayur (2003); Agatz et al. (2008)), an OR perspective on the store (e.g., Kembro et al. (2018); Caro et al. (2020); Cai and Lo (2020); Bijmolt et al. (2021)), or an analysis of latest developments in practice (e.g., Galipoglu et al. (2018); Melacini et al. (2018); Cai and Lo (2020); Raza and Govindaluri (2021)). This is aggravated by cross-cutting OC operations topics, which are not fully discussed and evaluated by the literature reviews. Examples include an integrated assortment for buy online pick-up in store purchases (see e.g., Rooderkerk and Kök (2019); Hense and Hübner (2021)), the fulfillment of online orders through stores (see e.g., Ishfaq and Bajwa (2019); Arslan et al. (2020); Bayram and Cesaret (2021)), or the consideration of a single virtual stock for all fulfillment locations (see e.g., Aflaki and Swinney (2021)).

A comprehensive, up-to-date review on operational issues and the central role of the store in OC operations constitutes a research gap. Hence, this review particularly targets contributions concerned with OR applications, quantitative model-based analyses, and decision support systems for stores within OC operations. Obtaining a state-of-the-art overview requires blending knowledge from retail practices and academia. Industry insights are core for this research area as retailers have been forerunners in quickly responding to fundamentally changing market requirements, such as the growth of online businesses. Resulting retail practices materialized in store innovations and heavily drove the implementation of OC operations.

To fill these gaps we investigate the revival of the store through OC retailing and more specifically the store-related planning issues in OC operations. This can be traced back to three questions displayed in Figure 5.1. First, we structurally identify the most relevant store-related planning issues derived from interviews with industry experts (RQ1). Second, we match this overview with an analysis of existing literature and evaluate whether and to what extent these planning issues are covered in pertinent analytical

and modeling literature (RQ2). We use the combination of RQ1 and RQ2 to ensure a structured literature analysis that responds to retailers' challenges and reveals answers in the literature. Third, the combination of both questions makes it possible to state which planning issues are not yet sufficiently covered and offer further research potential (RQ3).

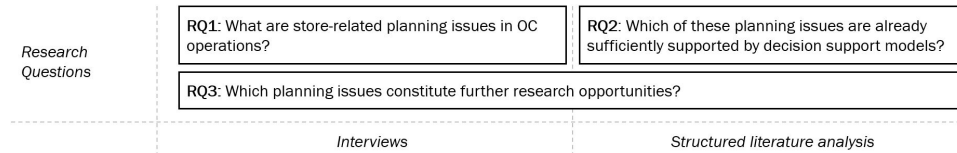


Figure 5.1: Overview of research questions

As this is a new and under-explored research area, we apply multiple research methods to shed light from different angles. Section 5.2 details the research approach taken. In Section 5.3, we utilize the different research methodologies to systematically identify and analyze planning issues, discuss related literature, and outline further research opportunities. Section 5.4 concludes the paper.

5.2 Research Methodology

We aspire to generate a holistic understanding for the nascent topic of integrating stores into omnichannel operations. We particularly aim to understand the planning aspects involved in the OC setting, while we need to cope with the scarcity of existing contributions as the topic has evolved only recently. Multi-method approaches are an imperative in such cases (see Boyer and Swink (2008); DeHoratius and Rabinovich (2011) for examples). We first develop a conceptual overview that is appropriate to build a theoretical foundation for emerging research topics (Webster and Watson, 2002). In doing so, we define the scope and create an overview of relevant OC concepts. We then enrich the concepts with insights from practice by conducting semi-structured expert interviews. We additionally perform a systematic literature review to incorporate the latest research.

Figure 5.2 summarizes our methodological approach, which places the triangulation of multiple sources in the center. The sources are detailed in the following.

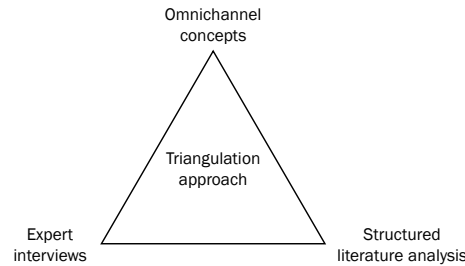


Figure 5.2: Applied research methodology using a triangulation approach

Omnichannel concepts OC operations imply fully integrated channels where consumers can shop without noticing the different channels operating in the background (e.g., which picking location was used). It extends the multichannel (MC) concept, which characterized retailers with various isolated sales channels, such as an online shop (henceforth referred to as “webshop”) and a bricks-and-mortar store (henceforth referred to as “store”) (Brynjolfsson et al., 2009; Bell et al., 2014; Verhoef et al., 2015; Beck and Rygl, 2015; Hübner et al., 2016b). The predecessor of MC is termed single channel (SC) and describes retailers that operate a single sales channel. This concept is becoming less relevant (e.g., Hübner et al. (2016b); Rooderkerk and Kök (2019)). The novelties of OC operations surface at the store and its interactions with different fulfillment concepts. These can be differentiated by the point of receipt of goods (at store vs. at home) and the point where the customer orders and pays (at store vs. at home). Please note that “at home” symbolizes here any location to order online outside the store (e.g., home, during travel or at office) and to receipt goods outside the store (e.g., home, parcel locker or pickup station). Figure 5.3 summarizes these options. The resulting OC-enabled fulfillment concepts (henceforth referred to as “OC concepts”) are ship-from-store (*SFS*), buy online pick-up in store (*BOPS*), and digital assortment extension (*DAE*). As “ordering at store” and “receiving at store” when taken together constitutes conventional in-store shopping and not an OC concept, we will not further study this

aspect. Figure 5.4 further illustrates the product flows for the three OC concepts, which are at the center of our research on operations. Although we focus on the role of the store we have added a distribution center (DC) to complement the product flows and our definition of the concepts.

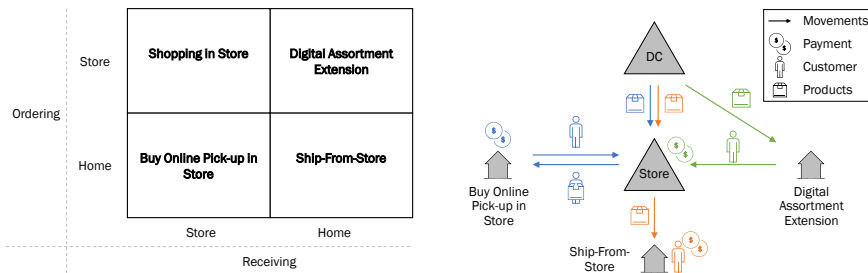


Figure 5.3: Relation between ordering and receiving

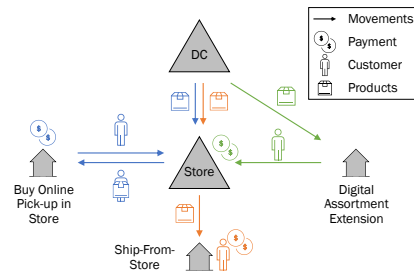


Figure 5.4: Product flows in OC retailing

Buy Online Pick-up in Store (*BOPS*, also called click and collect) enables demand transfer from the webshop to the store. Customers can order online to pick up products in the store, possibly while observing store inventory (see e.g., Hübner et al. (2016a); Bayram and Cesaret (2021)). Products originate from a DC and are shipped to the store for pick-up either prior to a specific order (i.e., use of store inventory) or after the order is placed (i.e., specific online inventory). This practice helps to shorten lead times or to substitute unavailable webshop items by redirecting customers to the store where the item is available. A variation depicts reserve online, pick-up and pay in store (*ROPS*), where items are reserved online but payment is only carried out in the store upon pick-up. *BOPS* is particularly adequate for low-value products (e.g. discounter products) to avoid that shipping costs outweigh the merchandise value and discourage purchases. Also, products with high return rates (e.g. fashion wear) are suitable as the pick-up poses the opportunity to try and potentially replace the ordered product immediately so that return costs can be saved.

Ship-from-store (*SFS*) describes the process of accessing store inventory for online orders and using stores as an alternative fulfillment location to DCs (see e.g., Hübner et al. (2016a); Difrancesco et al. (2021)). Orders are picked in the store and shipped directly to the customer. Benefits arise

from shorter transportation distances to customers compared to remote DCs and from inventory pooling across different locations. In this concept, products originate from a DC and are shipped to stores prior to the customer order as regular DC-to-store deliveries. Industries with high customer expectations regarding delivery time (e.g., consumer goods) and areas that are characterized by heavy mark-downs (e.g., seasonal fashion products or technology-heavy electronics articles) greatly benefit from this OC concept. It helps satisfying customers and selling off otherwise marked-down inventory.

A **Digital assortment extension** (*DAE*) provides an “endless aisle” to the store via demand transfer from the store to the webshop. Alternative terms are digital shelf extension or virtual shelf expansion. Digital devices in the store enable access to the online assortment and expand the store’s offering. In an extreme variant of *DAE*, the store has no sales inventory and only serves as a showroom with digital order options (see e.g., Dzyabura and Jagabathula (2018); Gao and Su (2017a); Park et al. (2020)). The role of the DC in this concept differs from the previous two. Here, products are typically shipped to customers directly from a DC after an order is placed in the store via *DAE*. The concept is relevant for retailers with large assortments and small stores, large products (e.g. furniture or bikes), and products with a wide range of designs or sizes (e.g. fashion). *DAE* helps in such cases to offer customers the full assortment through a digital shelf extension while using shelf space for the most adequate products.

Enabling *BOPS*, *SFS*, and *DAE* revolutionizes the role of the store, the underlying retail operations, and the interface to the customers. It requires the integration and coordination of operations, resources, and information systems and entails novel structures and planning systems. This creates the need to identify and analyze the planning problems that are affected the most by the transition towards OC concepts. In particular, enhanced approaches across channels are required for the demand forecast, the selection of fulfillment locations of online orders and the assortment and inventory definition. We aim to distill the essentials of the new role of the store in

OC operations. We therefore analyze OC retail settings with integrated channels. This leads us to focus on tactical and operational planning problems, which excludes strategic aspects like moving from SC or MC to OC. Furthermore, we focus on the impact of operations and stores. We therefore do not mix our analysis with special cases of the OC concept that do not involve store operations. One example is drop-shipping, where the retailer redirects the order to a third party, which directly ships the products to the customer. Finally, we focus on the distribution of physical products in B2C retail. Altogether, this builds the basis for our analysis and is also applied to the data collection from practice and research, which is detailed in the following.

Interviews with experts from industry Input of industry experts helps to ensure external validity, enhances the practical relevance, especially for emerging topics, and uncovers new facets of the research topic (Eisenhardt, 1989). From June to August 2020, we interviewed managers of OC retailers and retail and supply chain consultants to gain a holistic perspective on the topic. All participants currently work or worked for European head-quartered, multi-national retailers in fashion, DIY, electronics, furniture, books, home-ware, or food. The retailers' diversity allowed us to view their challenges from various angles. We set out to target retailers with an online channel and proprietary bricks-and-mortar stores that offer company-wide OC fulfillment concepts. Beyond that, the OC retailers need complete control and responsibility for their distribution processes. Interview partners at the companies were either self-selected by the company or directly approached by us. The interviewees possess fundamental knowledge of OC operations. All interview partners and details of the interviewing approach are summarized in the Appendix.

Structured literature analysis We deployed a systematic literature review consisting of five steps to identify the most relevant articles published up to March 2021. First, we carried out a keyword-based search on Scopus

and EBSCO. The search was limited to articles that were published after 1999 in peer-reviewed, international journals in the areas of operations research, logistics, marketing-operations, and general management studies. For further details we refer to *Literature search approach* in the Appendix. Second, all 132 articles obtained were read in their entirety by at least two authors, re-assessed, and subsequently either included on or excluded from our research paper. 33 articles classified as relevant and were mapped to the planning issues derived from the interviews. Third, the bibliography of selected articles was scanned to identify further relevant contributions. 16 additional articles were found that the authors read in their entirety, out of which three were categorized as relevant. Fourth, Google Scholar was utilized to screen literature that cited articles that had been selected in the previous two steps. Here, 46 papers were reviewed in full and three were eventually added to our paper. Steps three and four implied the review of additional articles but also revealed further matching publications. Articles identified in steps three and four underwent the same thorough process described in step two. Fifth, a manual search of the leading OR journals was carried out to verify our approach via a small, highly relevant sample. This last step identified and added eight additional publications. Altogether, we read 211 articles in their entirety and identified a total of 47 articles that were to be mapped to the categories.

Comprehensive analysis with a triangulation approach We use the different methodologies to systematically identify and analyze the relevant planning problems. At regular meetings, all team members discussed the codes, categories, and findings from interviews to set aside subjective impressions from single researchers and come to an objective interpretation of interviewee perceptions. This ensures repeatability of our findings (Lincoln and Guba, 1985). The meetings also served to review and map relevant literature. The empirical findings in Section 5.3 are structured along five elements (demand forecasts, network design of fulfillment locations, assignment of customer orders, assortment and inventory management, and inventory replenishment and returns) that emerged from our conceptual

overview, expert interviews and literature analysis. Each element depicts one planning issue. The elements are structured along planning horizons and their relation to each other.

To specify the planning issues in the following sections, we first define and outline the problems through our findings from the interviews and literature and subsequently relate these to the various OC concepts. The specification of each issue is enriched by current challenges and systems applied in practice. Literature is then related to each issue. Literature, interviews, and the OC concepts are jointly and iteratively applied, e.g., findings from literature are leveraged to sharpen the scope of the planning issues identified from interviews. Applying such a holistic view is intended to develop new insights for academics and to guide practitioners.

5.3 Store-related planning issues in omnichannel operations

This section develops the planning framework for store-related issues in OC operations. We will first provide an overview of the planning tasks and their interdependencies in Section 5.3.1. Afterwards we will analyze each task separately in Sections 5.3.2 to 5.3.4.

5.3.1 Overview of the planning framework

OC merges operations across channels. This brings forth planning questions on how to match supply and demand across channels and puts the store at the center of those operations. In particular, it requires answers as to what and how much to offer and where to process an order across integrated channels. However, integral planning implies more complex approaches with multi-dimensional interdependencies. Efficient planning of the entire OC

operations is neither possible in the form of a monolithic system that plans all tasks simultaneously nor by simply performing the various planning steps successively. Varying decision owners, time horizons, planning frequencies, and levels of aggregation necessitate a decomposition of the entire planning problem (see also Schneeweiss (2003)). The compromise is hierarchical planning that balances practicability, the integration of interdependencies, and the breakdown of the overall problem into partial planning modules. It enables coordination among the planning issues (Miller, 2001; Schneeweiss, 2003; Stadtler and Kilger, 2008). Using our multi-method approach allowed us to derive such a planning framework that aligns the novel OC planning issues (see Figure 5.5).

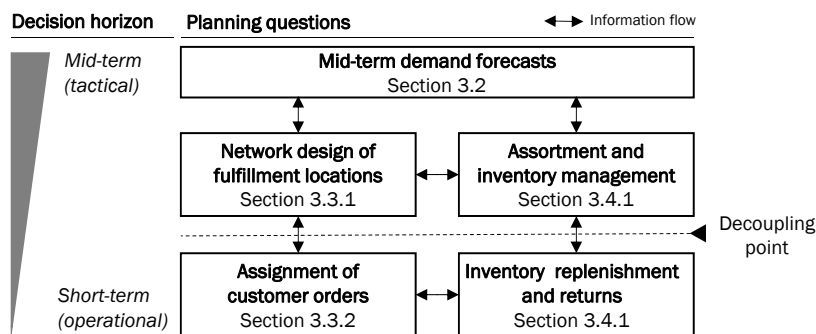


Figure 5.5: Overview of store-related planning questions in OC operations

Dimensions of the planning framework The planning problems can be differentiated into two dimensions. The first dimension is the *planning and decision horizon*, which differentiates planning problems with a mid-term tactical scope and those with a short-term operational scope. The mid-term planning problems usually have a horizon of multiple months and are based on forecasts, whereas the short-term issues require handling on a daily level and are based on actual customer orders. This also defines the decoupling point that divides planning tasks into forecast-driven and order-driven (Hübner et al., 2013). Retailers have to anticipate consumer demand down the supply chain until the customer interaction. The decoupling point in retailing is located at the store shelf and webshop respectively, at the very end of the supply chain when an actual customer order is placed. The second dimension can be differentiated along the *planning questions* from planning

of demand through fulfillment locations to assortments and inventories. This is in line with Fisher and Raman (2010), who noted that retailers have three tactics at their disposal for matching supply with demand: accurate forecasting, supply flexibility and inventory stockpiling.

Planning questions, interdependencies and information flow At the mid-term level and with the availability of **demand forecasts**, OC retailers are required to decide which locations in their network to make available for fulfillment (**network design of fulfillment locations**). This entails the set up of picking and shipping locations of online orders as well as in-store OC offerings. Besides that, **assortment and inventory management** needs to be carried out for each channel and location. This results in defining which products to list in which channel and in which quantity. Planning of locations, assortments and inventories are interdependent and need to be aligned. For example, selecting assortments for *SFS* or *DAE* depends on the availability of singular locations in the network and the available assortments in these locations. On the operational level and with the availability of actual customer orders, retailers need to take care of the **assignment of customer orders** to fulfillment locations. These locations are responsible for picking, packing and delivering the orders. **Inventory replenishment and returns** complements this decision by defining how much and when to replenish across channels. Both operational planning issues are also interdependent as order assignment depends on the available inventory of a location.

The decisions are also characterized by a hierarchical dependency: forecasts are input to network, assortment and inventory selections. But networks, assortments and inventories also impact forecasts, for example, if substitutions between products or locations are integrated into the forecasts. Furthermore, order assignment depends on the locations previously selected. The same hierarchical dependency exists for inventory-related questions. For example, inventory needs to be replenished up to levels that have been determined in the mid-term module. The target levels depend in turn

on the feasible replenishment frequency. The OC retailer which operates an own set of stores and DCs is the owner of the planning and decision process. Within the retailer, different business units may be responsible for the planning process. All planning purposes require an involvement of marketing, sales and operations related departments.

Structure of following analysis In the following we will analyze each planning decision separately to focus on the core contents. We first delineate the requirements for demand forecasting across channels in Section 5.3.2, as forecasts are required inputs to the mid-term planning problems. This is followed by the topics related to fulfillment locations in Section 5.3.3 and assortment and inventory topics in Section 5.3.4. Each section firstly describes each planning problem and its scope (paragraph: scope of the planning problem), and how the planning problem impacts OC retailers in practice (paragraph: challenges in practice). This answers RQ1. Second, our literature review unveils relevant publications and details the decision problem that is investigated in the respective contribution (RQ2; paragraph: current state of literature). We focus on the decision models and structures of the novel problems to accomplish our ultimate objective of creating a planning architecture. Lastly, our outlook for future research brings together the gaps identified in the literature view and the combined view of literature and industry. This serves to develop areas of future research (RQ3; paragraph: future areas of research).

5.3.2 Mid-term demand forecasts

Scope of the planning problem The customers' role and its' direct impact on retail operations is growing through OC retailing. In contrast to that, traditional logistics to supply bricks-and-mortar stores usually end at the store without direct customer contact. Hence, a more comprehensive understanding of consumer behavior is required to understand the demand effect of cross-channel switching. Such an understanding translates into the

required generation of mid-term demand forecasts for OC retailers for each channel, each store and each DC, and for each product to define fulfillment locations, assortment sizes, and inventory levels.

Challenges in practice OC retailers are confronted with issues related to accurate demand forecasts that can be clustered into three areas. First, a sufficient historical OC database and OC experiences are often unavailable due to very recent OC service enhancements and significant developments of OC shopping. Many interviewees state that processes, hardware, and software to collect and store necessary data into a single, aggregated system and to make this data available are not implemented. Even if available, only limited sales figures are directly retrievable. One interviewee (VAR01) described a case of an electronics retailer where *“cash register systems are not connected real-time to the retailer’s merchandise management system. Also, cross-channel returns are not updated in the same system and not in real-time.”* Hence, sales figures are updated with a time lag and do not feed correctly into forecasts. Returns might not be considered at all. Second, demand models across channels are required to capture additional customer attributes. A sales manager from a fashion retailer (FAS01) confirmed that new OC customer segments emerge and existing segments change their purchasing behavior. It becomes more challenging to understand customer needs and how customers use channels to make a purchasing decision, such as, if one channel is used for the information search and another for product purchases. A manager from a fashion retailer (FAS02) emphasized that the SC models do not consider these different OC-induced customer segments and their individual demand patterns. As an example, *BOPS* customers, who are mobile, prefer to see the products at pick-up or like to add additional products on-site. Forecasts must consider these customer groups and their respective, varying preferences, and shopping cart compositions. Furthermore, existing SC or MC models used by retailers do not sufficiently capture OC demand flows across channels, e.g., substitutions from store to online. The positive impact of webshop promotions on online demand, for instance, may have a negative impact on store demand. To

increase OC forecasting accuracy, one needs to identify relevant drivers of demand. Such drivers may include external factors such as weather, season, or the activities of a competitor and internal factors such as advertisements across or within a channel. Third, *DAE* demand is greatly influenced by in-store inventory as customers often use *DAE* orders to buy temporarily sold out, out-of-stock (OOS) items. However, in-store inventory information is often not reliable or available in real-time, making predictions more difficult. This may result in censored demand observations since sales are limited by the available supply information.

None of the companies interviewed currently applies advanced predictive models for a channel integrated OC demand forecast regardless of the OC concept used. Instead, rather rudimentary methods are used (e.g., that do not consider cannibalization or reinforcing effects via OC). For example, when considering *SFS*, two interviewees (VAR02, DIY01) described that they consider only the top x% of products and calculate the expected offline demand using a general flat adjustment factor. Three interviewees (DIY01, FAS01, VAR05) reported attempts to break down total forecasts to certain regions and ZIP codes but lack the required data to connect all OC-sales.

Current state of the literature A limited number of contributions currently addresses demand forecasting in an OC context. Table 5.1 summarizes the related literature.

Table 5.1: Related literature on demand forecasting in OC retailing

Related contribution	OC concepts			Method ¹	Per	Problem size ²			
	<i>BOPS</i>	<i>SFS</i>	<i>DAE</i>			Prod	Stores	DCs	Data ⁴
Cao et al. (2016)	✓			AM	S	1	1	1	SD
Lee (2017)	✓			ARM, AA	S	10	- ³	5	SD
Pereira and Frazzon (2020)	✓	✓		Cl, NN	S	10	16	4	ED
Klibi et al. (2021)	✓	✓		GT, AR	S	24.029	1	1	ED

¹ Formulation of the underlying problem: Analytical model (AM), Association rule mining (ARM), Apriori algorithm (AA), Clustering (Cl), Neural network (NN), Graph theory (GT), Arimax regression model (AR)

² Problem size: Multiple (M) or single (S) periods (Per), No. of product variants (Prod), No. of stores, No. of DCs

³ Instead of no. of stores, 500 transactions per DC cluster are considered

⁴ Data applied: Simulated data (SD), Empirical data (ED)

Cao et al. (2016) develop an analytical model to analyze the introduction of *BOPS* and the resulting allocation of customer demand across a store

channel, an online channel, and the new *BOPS* channel. Demand for one product is allocated across channels based on the utilities of heterogeneous customers. Different price scenarios are considered, where the *BOPS* product can or cannot be purchased in the retail store. In most scenarios, a fraction of the existing store and webshop customers shift to the *BOPS* channel. Lee (2017) takes a more granular forecasting approach by modeling demand points (e.g., stores with *BOPS*) aggregated in a cluster. However, in their numerical experiment demand points are simplified into the total number of transactions. Demand patterns among purchased items are derived using association rule mining and an apriori algorithm. Resulting forecasts are expressed through if-then rules. Pereira and Frazzon (2020) consider *BOPS* and *SFS*. They also take a step-wise approach to OC forecasting on a store-level. Just as in Lee (2017), consumption patterns are clustered based on sales data. In contrast, demand forecasts are created for every single time series. A neural network as part of a machine learning approach is used to improve forecasting accuracy. Pereira and Frazzon (2020) then test the solution with a simulation-based optimization to optimize financial and material flows in an OC supply chain. Lastly, Klibi et al. (2021) analyse a similar setting of an OC retailer with *BOPS* and *SFS*. Also, a two-step approach to OC demand forecasting is applied. The authors firstly use graph theory to create a network of relationships between products based on customer shopping baskets and specific historic time horizons. The network is then used with an ARIMAX model, which provides a demand forecast for each product in each channel.

Future areas of research Future research can contribute in four areas, namely coverage of OC concepts, data issues, OC-specific attributes, and methodological extensions. First of all, while some first research on forecasting *BOPS* demand exists, there is little research covering *SFS* and *DAE*. Forecasts for *SFS* need to predict location- and time-specific demand for stores and the webshop to enable retailers to plan their store assortments and inventories. *DAE* depends on factors such as *DAE*-devices in stores, accessibility, and store staff to support customers in changing their shop-

ping channel. Most important however is the consideration of customer volumes in the store and their propensity to change channels when offered the possibility to shop online.

Second, precise information about demand and inventory is vital for channel interactions but not available due to infrastructural and setup issues. Methods for estimating demand and inventory with missing data and time lags are therefore required. This particularly involves approaches to replace missing data points (e.g., current inventory, current forecast in the store) with forecasts.

Third, there is room for methodological extensions. Existing OC forecasting models lack the completeness of internal and external variables within and across channels. These include variables representing general demand patterns, such as seasonality or weekdays, uncontrollable, external factors, such as the weather, socio-economic conditions, or competitor activities, and unknown factors such as roadworks in front of a store or web-page breakdowns. It would also be worth investigating further attributes such as in-store displays, display restrictions, store and product attributes, or assortment depth and diversity. Particular attention must be paid to factors that help to estimate the interactions between channels: cross-channel promotions, the type, number, and position of *DAE* devices in stores, or the level of store staff involvement, for instance. In general, it is also worth taking into account effects like sales cannibalization, halo effects, and inter- and intra-category effects. Also, given the magnitude of cross-channel returns, these should be estimated for calculating available inventory. Customer segment- and channel-specific demand factors will improve forecasting accuracy. Another crucial dimension to consider is censored demand, and accounting for out-of-stock (OOS) items that have or have not been balanced by cross-channel substitutions. Finally, another opportunity for enhancing OC forecasting models is the combination of different methods. While the above models have a quantitative nature, one could integrate qualitative forecasting to compensate for missing data points in the new OC services. Alongside this, models could include a blend of quantitative

methods, such as time series, linear regressions, or causal modeling, to select the method most appropriate for a certain situation. As already shown by Pereira and Frazzon (2020), such models could be further enhanced via machine learning algorithms to refine forecasts. Machine learning models deem particularly useful to assess above mentioned internally generated and externally available data and derive potentially underlying demand patterns. Given their capability of handling enormous amount of data, the more data is fed, the more patterns can be uncovered, the more granular forecasts turn (e.g., aggregation on item-, consumer-, single-location, or hour-level) and the more adaptable models become to changes like seasonality or market trends. Next to that, one should investigate whether available store or e-commerce demand forecasting models can be transferred and enriched to an OC setting with demand flows across channels. The forecasts serve as input to the mid-term location and assortment planning. These will be delineated in the subsequent sections.

5.3.3 Fulfillment locations

The fulfillment location selection takes place on two planning horizons. As such, we first outline the tactical network decision and complement it with the operational order assignment problem.

Network design of fulfillment locations

Scope of the planning problem At this planning stage, an OC retailer defines the subset of possible fulfillment locations among the entire network of stores and DCs. In other words, the retailer needs to define which of the depots (e.g., central DC, regional DC, stores) to set up for picking and shipping of online orders or in-store OC offerings. This is relevant for *SFS*, where retailers select stores for home delivery, but also applies to *BOPS* when locations other than the pick-up location supply the orders

(see Figure 5.4). For example, a fashion retailer (FAS03) explained that *BOPS* orders are often shipped from DCs to stores together with regular store replenishment. This is common practice for low-cost retailers, that can afford asking customers to wait for several days to pick up the order, to reduce logistics costs. For the *DAE*, it means selecting which depots are included in the offerings. Available storing, picking and processing capacities as well as assortments and inventories are prerequisites for including a depot in the set of potential fulfillment locations. Ultimately, expected inventory holding and picking costs at the fulfillment location and transportation costs to stores and to the customers determine the selection of depots. In contrast to the assignment of a specific order to one location (see Section 5.3.3), this is based on expected demand and goes along with setting up information systems between inventory locations as well as processes.

Challenges in practice OC retailers pointed out challenges with selecting the appropriate fulfillment locations that can be clustered around three topics: development paths, operational efficiency, and heterogeneous depots. First, several interviewees (VAR01, DIY01, FAS03, FAS04) explained how retailers started their OC fulfillment in stores with an *SFS* concept as it was easy, fast, and cost-efficient to implement compared to opening new facilities. The enduring business problem here is to define the optimal volume processed in stores. Processing in stores has benefits compared to DCs if stores are in customer proximity. One interviewee (VAR01) predicts that *SFS* will be a critical retail element of the future as integrated delivery through city locations can satisfy short-term delivery (e.g., as same-day delivery). Second, the share of online sales processed via stores has operational and economic limitations despite the benefits due to lower delivery times (FAS01). For example, picking, and packing orders needs space that often does not exist (DIY01). One interviewee (VAR02) commented on the furniture sector: “*We have seen store managers who did not want to assign store space for services of SFS and BOPS channels and refused to assign packing spaces in their stores*”. Some experts (VAR01, VAR05, DIY03) discuss the correlation of structural factors such as typical customer order

sizes and location decisions. For retailers with small order sizes, *SFS* can be beneficial. For those experiencing large order sizes, the use of stores is less cost-efficient than processing in DCs. Some retailers indicated that not more than 10% of total store sales could be allocated to *SFS* and *BOPS* as it would otherwise disturb store and customer processes too much. Finally, the location selection becomes more relevant but also more complex when retailers hold a number of heterogeneous depots, e.g., DCs and stores with different assortments and fulfillment costs. The picking efficiency is different across stores (as different walking distances occur) and between stores and DCs (due to different automation levels). A further related organizational issue mentioned is the legal separation of stores and DCs (in franchise systems or at retailers with later added online offerings, for instance). In these cases, a fully integrated concept may become impossible, particularly if independent store managers refuse *SFS* (VAR02). In fact, inventories in one location may belong to different entities, for example because a store accepts returns of online inventory. In this case, certain shares of store inventory cannot directly be used for fulfillment of online orders. In this case only parts of the inventory (e.g., sourced from DCs) or services (e.g., only *SFS*, no returns) may be offered. Such organizational boundaries and limited availability of products and services restrict the network.

In the course of recent introductions of OC concepts, most retailers decided on fulfillment locations based on speed and ease of implementation and only partially based on induced costs. Because many retailers have introduced OC fulfillment in recent years, most of them are still in an improvement phase. For example, in some cases stores were mostly used for online order fulfillment due to low set-up efforts. In other cases, retailers tried to leverage existing systems (e.g., transportation between DC and store) to supply *BOPS*-orders. These considerations, however, are unsustainable solutions as actual fulfillment costs are not fully taken into account.

Current state of the literature Table 5.2 summarizes papers that deal with the mid-term network design decision. The problems are typically

formulated as an inventory problem where retailers define the stocking volume in possible fulfillment locations and assign them a share of the online demand. Through this inventory definition and demand assignment, the papers derive the most beneficial depots out of a set of all potential fulfillment locations (picking and delivery from stores or DCs, or dropshipping).

Table 5.2: Related literature on OC network design of fulfillment locations

Related contribution	OC concepts			OR characteristics						Costs ⁷			Problem size ⁸			
	<i>BOPS</i>	<i>SFS</i>	<i>DAE</i>	Mod ¹	SolA ²	OF ³	DecV ⁴	Cons ⁵	Dem ⁶	Inv	Pi	Tra	Per	Prod	Location	Data
Bendoly (2004)		✓		Sim	Sim	I	O	BL	sto, st	(✓)	(✓)		M	1	St	SD
Alptekinoglu/Tang [2005]		✓		MIP	Dec	C	LA	D, C	sto, st	(✓)		I	M	1	St	SD
Bendoly et al. (2007)		✓		MIP	Enum	C	O, LA	BL	sto, st	(✓)	(✓)	I, O	M	1	St/DC	SD
Hovelaque et al. (2007)		✓		Sim	Sim	P	O	-	sto, st	(✓)	(✓)		1	1	St/DC/Dr	SD
Chen et al. (2011)		✓		DP	AM	R	O, LA	-	sto, st		(✓)	O	1	1	St/Dr	SD
Mahar et al. (2012)	✓			Sim	Sim	C	LA	-	sto, st	(✓)			1	1	St/Dr	SD, ED
Mahar et al. (2014)	✓			MIP	Opt	C	LA	D, CR	sto, st	(✓)		I	M	1	St/DC	SD, ED
Ma et al. (2017)		✓		AM	AM	P	O	-	sto, st	(✓)	(✓)		1	1	St/Dr	SD
Mahar and Wright (2017)	✓			NLO	Opt	C	LA	Dis	sto, st	(✓)	(✓)	I	M	1	St/DC	SD, ED
Ishfaq and Raja (2018)		✓		MIP	Opt	C	LA	D, I	det, st	(✓)	(✓)	I, O	M	M	St/DC/Dr	SD, ED
Ishfaq and Bajwa (2019)		✓		NLO	Dec	P	LA	D, I	det, st	(✓)	(✓)	I, O	M	M	St/DC/Dr	SD, ED
Arslan et al. (2020)		✓		SDP	Opt	P	LA	C, I, LN	sto, st	(✓)	(✓)	I, O	M	1	St/DC	SD, ED
Bayram/Cesaret [2021]		✓		SDP	Heu	P	LA	I	sto, st		(✓)	O	M	1	St/DC	SD
Prabhuram et al. (2020)		✓		MCDM	MCDM	V	-	-	-	(✓)			1	1	St/DC	SD

¹ Formulation of the underlying problem: Simulation (Sim), Mixed-integer program (MIP), Dynamic program (DP), Analytical model (AM), Non-linear optimization (NLO), Stochastic dynamic program (SDP), Multi-criteria decision making (MCDM)
² Solution approach: Simulation (Sim), Decomposition scheme (Dec), Enumeration (Enum), Analytical modeling (AM), Optimal (Opt), Heuristic (Heu), Multi-criteria decision making (MCDM)
³ Objective function: Inventory (I), Costs (C), Profit (P), Revenue (R), Value (V)
⁴ Decision variables: Order-up-to-level (O), Location assignment (LA) or no decision variables specified (-)
⁵ Constraints: Backorder level (BL), Demand satisfaction (D), Capacity (C), Customer returns (CR), Distance (Dis), Inventory (I), Location number (LN) or no constraints specified (-)
⁶ Demand model: Stochastic (sto), Deterministic (det), Seasonal (sea), Stationary (st) or no demand model specified (-)
⁷ Inventory costs (Inv) either fully including inventory holding costs, shortage/backorder costs and salvage value ✓, or only partially (✓), Picking costs (Pi) as depot-specific costs for fulfillment either fully including actual picking, packing, and handling costs ✓, or only partially or in abstract terms (✓), Transportation costs (Tra) as inbound costs to DCs and stores (I) or outbound costs to customers (O)
⁸ Problem size: Multiple (M) or single (S) periods (Per), single (1) or multiple (M) no. of products (Prod), No. of location variants (Stores (St), DCs (DC) and drop-shipping (Dr)), simulated (SD) or empirical data (ED) applied

Bendoly (2004) is the first contribution that constitutes a related OC network problem. It applies multiple stores and a webshop where the entire store and online demand is fulfilled by the stores. The retailer defines the fulfillment shares across stores for *SFS*. The paper develops a combined simulation and optimization model where the store customers are served first and the online customer second. Products that are not available for online sales in one store can be substituted by a neighboring store and shipped to the customer instead. This imposes additional shipment costs, but may reduce total inventory. To solve this trade-off, stores reserve a fraction of their inventory for online sales. Steering the substitution process across stores in this way allows inventory pooling effects to be obtained. The model optimizes for lowest inventory costs and determines order-up-to-levels for the locations in the network. Inventory holding costs are in focus without considering backorder and salvage costs. Fulfillment costs are only considered as an aggregated cost factor for penalizing substitution. Bendoly et al. (2007) further specify the related costs and additionally model an online DC as a potential location. They formulate an optimization approach

to minimize total costs while maintaining a service level with an order-up-to level policy and backorder option. For this, the authors solve the MIP and determine the impact of a decision variable (fraction of online orders served by online depot) at a fixed value on total costs. Doing this, they determine how much of the online demand should be fulfilled with the inventory of the stores and at which costs. There are two competing pooling effects with pooling online demand in the central online DC and pooling store demand with online demand in the stores. Costs considered include the fixed costs of setting up operations at the online DC and the stores, the inventory holding cost in the entire system, as well as the variable shipping and handling costs of supplying the online demand to DC, stores (inbound), and to customers (outbound). Transportation costs are modeled without order bundling (inbound) or routing (outbound) effects. All cost elements are simplified with linear costs. In the related paper of Alptekinoglu and Tang (2005) the retailer can also fulfill online orders from stores and DCs. The authors propose a decomposition scheme that first transforms the multi-depot problem into several single-depot problems with ordering and allocation decisions. In the second step, demand is assigned through a convex nonlinear program. They minimize the total expected costs subject to depot capacities and according to stochastic demand. The total costs comprise linear transportation, inventory and backorder costs. Transportation costs cover inbound processes from supplier to DCs as well as to fulfillment locations but without consideration of bundling effects and order consolidation. A case application also models the customer-dependent shipment costs from fulfillment locations to customer zones. Hovelaque et al. (2007) add drop-shipping as another fulfillment option besides store and DC fulfillment, and compare this within a simulation. They apply a Newsvendor formulation to determine the optimal stocking level for each fulfillment type. Similarly, Ma et al. (2017) formulate a Newsvendor model to define the optimal mix of the fulfillment of store and *SFS* orders with a store and a drop-shipping inventory. They further include return rates. Sales prices, purchasing costs, salvage and shortage costs are considered in both papers to display overall retailer profits. Chen et al. (2011) develop a model with two stores. These stores need to fulfill in-store demand and

can decide whether to accept an incoming online order or decline it. This way, they act in the form of a drop-shipper serving online customers from their inventory. In contrast to previous contributions, dynamic fulfillment is applied such that the second store can only fulfill orders when the first store refuses. They determine inventories for both stores and develop admission policies that help to decide when to accept or decline an online order for each location. The authors do not consider inventory costs for the dynamic programming model. Instead, they include location-depending costs such as handling, packing, and shipping costs for orders to compare expected revenues between the store fulfillment options. Shipping costs are modeled as direct per-customer costs without routing effects.

Mahar et al. (2012) extend the problem to offer stores for *BOPS* in a dynamic fashion. Each time an online customer reaches checkout, the model uses information on current inventory levels and expected demands to specify which of the stores should be presented as available pick-up locations. The policy attempts to discard stores with low inventory levels. The authors formulate a decision problem to minimize the total costs of inventory holding and backorder as well as lost sales and redirecting customers (so called customer goodwill costs). The latter two express additional refill costs of stores and penalty costs when customers may not select the preferred channel and switch to competitors or change to direct-to-customer shipments with additional fees. Direct-to-customer shipments are assumed as an alternative option for online orders directly from the supplier (which does not impact the *BOPS* decision). In an extension, Mahar et al. (2014) determine how many and which stores to be set up to handle *BOPS* and in-store return. They thereby extend the cost function with fixed setup costs per period and transportation costs between the DC, stores and customers, which are modeled as direct distance costs. Fixed setup and inventory holding costs favor central fulfillment from the DC, whereas transportation and goodwill costs favor providing pick-up/return at all stores to essentially pass “free shipping” costs to the customer and avoid any lost sales. While the decision problem of their first paper was solved as a simulation, this contribution formulates a MIP that is solved to

optimality. In another extension, Mahar and Wright (2017) develop a NLO model to assess costs and customer value for *BOPS* and in-store returns. More specifically, their model determines the set of stores that should handle pick-ups and returns within their entire store network. Their model optimizes total costs including fixed operating, inventory, lost-sale, and shipping costs as well as initial setup costs and variable costs for providing both pick-up and return services. Costs for *BOPS* are compared against delivering online orders directly from the DC.

Ishfaq and Raja (2018) develop a framework with a retailer that can fulfill deterministic, multi-periodic online demand from an integrated DC for online customers and stores, a dedicated DC for online orders only, and a vendor for direct vendor-shipments. They formulate and solve a MIP with the objective to minimize total costs. They include location-specific operational costs due to different labor wages, facilities, and operations, inventory costs that vary across echelons, and transportation costs to the inventory locations and from them to customer markets. The model selects the best order fulfillment location for a number of orders in a customer market. This means orders are still unknown and modeled on an aggregate level. Outbound transportation costs are direct-shipment costs to markets and not customer-specific or based on routes. In a related extension, Ishfaq and Bajwa (2019) optimize the webshop prices and include a price-dependent demand function. Their non-linear optimization model aims to maximize profits subject to customer demand and limited available product supply. Arslan et al. (2020) consider *SFS* in a two-stage stochastic model to compare online order fulfillment from DCs, urban fulfillment platforms, and stores. The authors consider stochastic in-store and online demand as well as available store capacities for their model. Profits are maximized while being limited by available locations, product supply, and fulfillment and transportation capacity. Decisions in the model are twofold: in a first stage, it determines which of the three fulfillment locations are to be used on a tactical level. For the decision, the paper considers a number of cost elements ranging from inventory holding and replenishment costs and fulfillment costs per location through to customer-specific transportation

costs. In the second stage (which is an operational decision as described in Section 5.3.3), it assigns actual orders to locations. The model assumes that online orders arrive with physical store customers in parallel. In contrast to other authors, Arslan et al. (2020) model a profit function and also incorporate revenues generated from in-store sales. They then develop an integrated view on both offline and online operations and find that *SFS* can increase profitability, mainly due to more online orders being satisfied. Bayram and Cesaret (2021) formulate a Markov decision process and apply a hybrid heuristic solution approach as integrative cross-channel fulfillment policy. The authors evaluate whether *SFS* is a preferable alternative to DC shipping. They compare different policies for dynamic order fulfillment based on stochastic demand predictions. Outbound transportation costs are included as well as location-dependent costs for picking, handling, and packaging online orders in stores. A multi-criteria problem is formulated by Prabhuram et al. (2020). They compare different network configurations that distinguish the use of DCs (either separate for online and store or combined), stores (either for store only or combined), or the combination of all options. A multi-criteria objective includes service factors for response time, product variety, availability, order visibility and returnability as well as location-specific costs.

Future areas of research Current literature on network design has originated from inventory allocation problems and developed since then to more comprehensive cost functions, demand models and decision criteria. However, matching challenges from practice with literature highlights some research gaps. These would range from providing models and solutions for the expanding services, using inventory pooling across stores, more deeply grounding the relevant costs empirically, extending models to include dynamic and stochastic demand effects, through to developing managerial insights and assessing the impact of new technologies related to store operations.

The first topic is related to OC concepts. Table 5.2 shows that *SFS* concepts are mainly considered. *BOPS* has particularly gained traction in recent years (see e.g., Gallino and Moreno (2014), and further research has elaborated how *BOPS* can be integrated in fulfillment locations. The question arises as to whether orders for customer pick-up in stores should directly be picked in the store (with potentially higher picking costs and lower availability), in DCs, or in other stores for delivery to the pick-up store. Furthermore, retailers will benefit from *BOPS* when this can be used for cross-selling opportunities in the store. This entails the research questions on how cross-selling can be materialized. A further opportunity in this context are processes and decision rules when redirecting customers to *SFS* or *DAE* is an option. It could be evaluated whether extended assortments that are not present in customer-facing shelves could be stored in the backroom (i.e., can be picked immediately for customer take-away) and how this impacts operations and sales. Also, correlations of order size and location availability can be assessed further to determine, if certain location types are specifically suitable for certain orders.

A second area of research opportunities deals with related costs. Although recent literature considers a broader set of costs (e.g., Ishfaq and Bajwa (2019)), a cost-holistic comparison of different fulfillment locations is still limited. As shown in Table 5.2, many contributions are limited to few cost elements while not even the basic costs such as inventory, picking, and transportation costs are location- and context-specific. In particular, the transportation costs to customers are only approximated (in most cases with one-way tours or as route length estimates (RLE) (see e.g., Janjevic et al. (2020)), and the actual routing costs are not taken into account. This is a strong simplification on behalf of numerical complexity, but does not represent the real costs of transportation. Most of the contributions are based on synthetic data. An application with empirically grounded decision-relevant costs will provide more managerial insights. The focus is on economic values. Another view is the impact on sustainability factors (e.g., carbon footprint of location types or the reduction of overstocks) and service factors (e.g., lead time, availability).

The third area is related to modeling approaches. In many cases independent demand models are applied that do not factor in cross-channel demand when strictly prioritizing online (e.g., Bendoly (2004); Chen et al. (2011)) or offline (e.g., Hovelaque et al. (2007)) demand, for example. This allows the development of first managerial insights, but these are limited to the assumptions mentioned. Table 5.2 indicates that early contributions are based on MIP models. However, the location selection problem may be a dynamic problem with opening and closures (e.g., because of seasonal demand) and therefore requires to extend the cost functions by opening/closing costs or in-season replenishment costs. Furthermore, the demand is subject to variations which then need to be built into stochastic demand models. Extending the more comprehensive cost formulations (e.g., by Ishfaq and Raja (2018) and Ishfaq and Bajwa (2019)) by the dynamic and stochastic components and incorporating further variables (e.g., dynamic adjustments) requires advanced solution approaches. This also includes innovative approaches to take into account actual transportation costs.

Further extensions can be identified that factor in the findings from the expert interviews. Managerial insights are required to define the optimal ratios between store and DC fulfillment and to define the most economically rational level of store fulfillment. Here it will be important to identify the drivers of the relative shares, e.g., rural vs. urban deliveries, and low- vs. high-traffic stores. Thresholds may be used to provide retailers with practical rules of thumb. As the topic seems one of great concern for retailers (VAR02), one could question whether simplistic assumptions sufficiently solve the problem of identifying efficient shares between fulfillment locations. Retailers could identify the maximum percentage of online orders that should be handled by stores by modeling all decision-relevant costs.

Finally, there are further research opportunities with the use of new technologies. Efficiency gains via automation might come into play when process automation leads to lower fulfillment costs in certain locations. At the same time, models can be extended by new shipping modes such as robots, drones, using parcel lockers for order pick up, or crowd concepts. One should also

look into innovative fulfillment concepts to acknowledge concerns store managers have with the increasing use of regular store space for online fulfillment. One recently developed concept is the micro-fulfillment center. This features small-space fulfillment centers that can be placed within other buildings. This raises the general question of what space allocation in multi-used locations is desired. In stores, this would be how much space should be assigned to online order fulfillment (e.g., additional picking/packing areas, storage space), and how employees should be assigned to store operations or online order tasks – concerns mentioned by several experts (e.g., VAR02, DIY01).

Assignment of customer orders to fulfillment locations

Scope of the planning problem Whereas the network problem determines the set of available fulfillment locations, the related and subsequent operational problem is the actual assignment of online orders to the locations selected mid-term. At this stage, OC retailers specifically consider the currently available inventories and picking capacities at each location as well as lead times and requested time windows. This may further reduce the set of locations. The assignment of orders aims to minimize the total costs consisting of inventory (i.e., costs for inventory holding and over- and underage), picking (i.e., processing an order at fulfillment location), and transportation costs (i.e., from inventory location to customers). Each of these elements are location-specific. While this decision problem seems obvious for *SFS*, it is also relevant for *BOPS* and *DAE* in cases where orders are supplied from other locations than the order location (i.e., the structure outlined in Figure 5.4 is extended to multiple store locations).

Challenges in practice The challenges related to order assignment build upon the efficiency, heterogeneity, and capacity issues for the selection of locations discussed above. Further questions on the operational level arise from the necessity to include inventories available in real-time at

each location. In a fully integrated *SFS* concept, customers can select all products offered in stores at the online checkout. As a consequence, it needs to be based on real-time product availability in each location. However, real-time availability about store inventory is not only an issue of IT systems but mainly restricted by the time lag between in-store customers putting items into the basket and check out (VAR01). Moreover, inventory shortages may result in conflicts between in-store pickers and customers. Fighting for products that would be used for online customers gives a negative signal to the store customers (FAS02). In a store with order picking and customer presence, employees who perform picking usually cannot consult customers or replenish shelves during this time. Hence, either additional staff are required or tasks need to be reassigned among existing staff. This requires procedures to integrate in-store picking with little customer disturbance. Using inventories in stores for the fulfillment of online orders decreases the inventory available for in-store revenues and may require more store space if inventories are increased, but may decrease fulfillment costs and lead time for online orders. Higher inventory levels may also compensate for inventory record inaccuracy (FAS03). Offering *BOPS* leads to situations where retailers have to prioritize orders, e.g., if an in-store customer wants to purchase an item that is already reserved for a *BOPS* order (DIY02). Further, unclaimed *BOPS* orders take-up valuable space in the store (FAS05). Moreover, the in-store customer would buy certainly the product, whereas there is also a risk that the customer who reserved the product online may not show up and buy it. OC retailers therefore require order acceptance rules and avoid accepting all orders on a first-come-first-served principle. Such support systems and above mentioned order assignment processes that are updated regularly are required to ensure efficient OC operations (VAR02). Sourcing from distributed locations can yield economic advantages if idle capacities and overstocks in stores are utilized. However, assortment availability or stockouts can lead to split orders or locations dropping out for one order. As one DIY retailer (DIY01) explained, order splitting requires additional processes and operations regarding downstream order consolidation before delivery or substitutions between products. To mitigate these challenges, retailers think about

steering customers into other channels (e.g., *SFS* instead of *BOPS*) in the event of stock-outs, giving retailers flexibility in accessing different inventories.

Current state of the literature Table 5.3 summarizes the related literature, which is so far only based on *SFS*.

Table 5.3: Related literature on the assignment of customer orders in OC retailing

Related contribution	OC concepts			OR characteristics						Costs ⁷			Problem size ¹⁰				
	<i>BOPS</i>	<i>SFS</i>	<i>DAE</i>	Mod ¹	SolA ²	OF ³	DecV ⁴	Cons ⁵	Dem ⁶	Inv	Pi	Tra	Route ⁸	Inv ⁹	Prod	Store	Data
Aksen/Altinkemer [2008]	✓			MIP	LR	C	OA	D,VC	det,st			✓	R	nc	1	M	SD
Mahar et al. (2009)	✓			MIP	Sim	C	O,OA	S	sto,st	✓		(✓)	DD	S	1	M	SD
Mahar and Wright (2009)	✓			MIP	Sim	C	OA	S	sto,st	✓		(✓)	DD	S	1	M	SD,ED
Xiao et al. (2009)	✓			DP (NV)	AM	R/P	O,OA	-	sto,st		(✓)	(✓)	DD	S	1	2	SD
Bretthauer et al. (2010)	✓			MIP	Opt	C	O,OA	C	sto,st	✓		(✓)	DD	S	1	M	SD
Andrews et al. (2019)	✓			MCDM	Dec	V	OA	D,I	sto,st			(✓)	DD	S	M	M	ED
Ni et al. (2019)	✓			DP	Heu	C	OA,DT	LC,I,TL	sto,st			(✓)	DD	S	1	M	SD,ED
Arslan et al. (2020)	✓			SDP	Opt	P	OA	C,I,LN	sto,st	✓		(✓)	DD	S	M	M	SD,ED
Dethlefs et al. (2021)	✓			MIP	CFRS	C	OA,DT,R	LC,MO,TL,VC	det,st		✓	✓	R	S	M	M	SD,ED
Difrancesco et al. (2021)	✓			MIP	Sim	C	CW,P,T	C,DT,SL	sto,st		(✓)	✓	R	S	M	1	SD,ED

¹ Formulation of the underlying problem: Mixed-integer program (MIP), Dynamic program (DP), Multi-criteria-decision making (MCDM), Stochastic dynamic program (SDP), News vendor (NV)

² Solution approach: Lagrangian relaxation (LR), Simulation (Sim), Analytical modeling (AM), Optimal (Opt), Decomposition scheme (Dec), Heuristic (Heu), Cluster-first-route-second heuristic (CFRS)

³ Objective function: Costs (C), Revenue (R), Profit (P), Value (V)

⁴ Decision variables: Order assignment (OA), Order-up-to-level (O), Delivery type (DT), Delivery Cut-off Window (CW), Pickers and Packers (P), Timing (T), Routing (R)

⁵ Constraints: Capacity (C), Delivery time (DT), Demand satisfaction (D), Inventory (I), Location capacity (LC), Location number (LN), Minimum orders per location (MO), Service level (SL), Tour length (TL), Vehicle capacity (VC) or no constraints specified (-)

⁶ Demand model: Demand model: Stochastic (sto), Deterministic (det), Seasonal (sea), Stationary (st) or no demand model specified (-)

⁷ Specific costs considered: Over- and underage costs (Inv); Picking costs (Pi) calculated as fulfillment costs including picking, packing, and handling orders per product and location ℓ , or approximated per customer or modeled in abstract terms (ℓ); Transportation costs (Tra) to customers calculated based on vehicle routing and tours ℓ , or approximated with distance-based measures (no tour building) (ℓ)

⁸ Route: Solution of the order shipment with actual vehicle routing (R), or route length estimates approach (RLE), or direct distance estimates (DD)

⁹ Inventory: dedicated to one channel (D), or shared across channels (S), or no inventory considered (nc)

¹⁰ Problem size: single (1) or multiple (M) no. of products, two (2) or multiple (M) no. of stores, simulated (SD) or empirical data (ED) applied

Aksen and Altinkemer (2008) develop an order assignment problem as a combination of location-allocation and multi-depot vehicle routing problems. They formulate a MIP and use the Lagrangian relaxation method to generate solutions with minimum costs. Online orders are exclusively fulfilled from a set of stores. Besides fixed location costs, their single-period model with known demand considers transportation costs from DCs to stores and from stores to customers. Compared to many subsequent publications on this topic, it also determines the vehicle routes connecting all stores and customers with each other. Hereby, vehicle capacities are considered as model constraints. Bretthauer et al. (2010) are the first to apply a two-echelon fulfillment system with a DC and multiple stores. Incoming goods are received at the DC, which serves as a break bulk facility. Goods are then shipped to the stores in equal quantities. After that, each store incurs independent store and online demand. The online demand originates from the surrounding region. They apply a static order assignment policy so that each online customer is served from the closest store. For this, the authors apply a branch-and-bound algorithm and associated tight bounds to solve an MIP that aims to minimize total cost. This is further relaxed

by Mahar et al. (2009) with a dynamic assignment policy of online orders that accounts for current inventory positions at the stores. This enforces the assignment of online customers not only to the next store, but also to the store with higher availability. In both papers, at the end of the period, inventory holding, backorder, fixed operational, and transportation costs are assessed. Bretthauer et al. (2010) also consider handling costs per product and customer in each location. In a further extension, Mahar and Wright (2009) introduce an assignment policy where the online orders are accumulated over time and allocations are made after a time interval. This further reduces costs. However, common across these models is that they are limited to one product and the transportation costs to customers are only modeled with direct shipments. They thus use distance metrics even when customers are served together within a tour. Both papers by Mahar et al. (2009) and Mahar and Wright (2009) use a simulation to generate cost-minimizing results for their decision problems. Xiao et al. (2009) consider with a DP a retailer that assigns inventory to two stores before the sales season and dynamically assigns any online order that arrives at the stores or a drop-shipper. For this, the authors combine a newsvendor problem with an admission model. To reduce transportation costs (modeled as direct distance costs), the online orders are allocated to the next store. Drop-shipment is only applied as a backup for stores with insufficient availability. Andrews et al. (2019) is the first contribution with multiple products. They apply an online optimization approach and solve a multi-criteria decision problem with an algorithm based on a primal-dual schema. Their model assigns orders to stores and a DC based on available inventory, transportation costs, lead time and picking capabilities. Transportation costs are also based on direct distances, but additional costs for split orders are integrated. Differences in picking costs across locations are not considered but sufficient inventory in the DC is assumed to supply all orders. Ni et al. (2019) extend the assignment problem to multiple periods and with transportation aspects. They assign orders to stores over multiple periods to minimize transportation and penalty costs for late deliveries. Unfulfilled and newly incoming orders are consolidated until the end of each time period and then assigned to specific stores and specific

delivery modes. These delivery modes include crowdshipping and truck delivery. They approximate transportation costs to customers by modeling the shipping costs to a delivery zone. Difrancesco et al. (2021) evaluate in-store fulfillment processes (including picking and packing) for a retailer with one store and both in-store and online sales. Through a simulation, the authors analyze the impact of employee numbers and timing decisions on order fulfillment costs. Costs include in-store picking and packing as well as outbound transportation costs based on transportation time per order. As only one store is considered, the assignment problem should be seen as internal decision with regards to picking time and picker/packer allocation. Dethlefs et al. (2021) develop and apply a cluster-first-route-second algorithm that assigns orders to a heterogeneous set of picking locations. They are the first to consider location- and product-specific picking costs, vehicle routing costs, and to also assign orders to tours. They show that integrated fulfillment from both stores and DCs can be beneficial, especially under time restrictions when orders must be fulfilled rapidly.

There are further related contributions like Abdulkader et al. (2018), Paul et al. (2019), or Bayliss et al. (2020) that apply joint delivery to stores and online customers. However, their focus is on solving the VRP in OC settings by using consolidation benefits, e.g., in vehicle capacity. Orders are either pre-assigned or are shipped from DCs, independently from the store network. These contributions are a valuable source to enhance order assignment problems with VRPs.

Future areas of research There are opportunities for further research in extending the assignment problem to *BOPS* and *DAE* concepts, enhancing models with further decision-relevant costs and criteria and last but not least, investigating lead-time aspects. The first area is related to the OC concepts. Table 5.3 shows that *BOPS* and *DAE* have not been studied for order assignment. However, these could also leverage the advantages of location assignment. While customers choose a specific store for order pick-up (in *BOPS*), the picking process can be handled in other locations

with the aim of minimizing costs or using spare capacities and stocks, e.g., in particular at the end of sales season. Similarly, *DAE* has not been discussed, but may become relevant if online customers have digital access to inventories across stores.

The second topic area is related to the fulfillment costs considered and other factors. A holistic assignment decision consists of two cost categories: depot-specific processing costs (incl. inventory, picking) and transportation costs (incl. last-mile delivery to customers). The assignment decision depends on inventory positions, picking systems at a depot, as well as transportation means and options for routing. Table 5.3 reveals that in most applications so far only direct distance-based costs have been modeled. The building of customer tours needs to be extended either to RLE or full VRP solutions to obtain an efficient representation of the actual transportation costs. This also includes an extension to split orders or the consolidation of orders across fulfillment locations. The effect of alternative transportation modes (e.g., with cargo bikes, autonomous robots) and alternative pick-up locations may be included (e.g., parcel lockers or solo pick-up stations).

One needs to further analyze the impact of order assignment on customers in stores based on current practical challenges and the primary concern of many store managers. How does a certain level of customer presence in stores impact assignment decisions? For example, stores should be avoided during peak times. Assignment decisions may therefore be impacted not just by the available inventory in addition to costs but also by store customer presence and picking capacity. Sustainability issues may have additional impact, e.g., the desire to avoid overstock. Emission costs may also be integrated into holistic cost evaluations in the future. Finally, an increasing demand for shorter delivery times (i.e., same hour, same day) and the use of tight time windows raise questions regarding how these factors impact assignment decisions. Can each customer on a tour be reached from each picking location within the desired time window?

5.3.4 Assortment and inventory

Assortment and inventory planning is concerned with the selection of products and stocking levels across channels, DCs and stores (see Section 5.3.4). This is a tactical decision that needs to be accomplished with the operational inventory replenishment (see Section 5.3.4).

Assortment and inventory planning

Scope of planning problem To optimize retailers' profit, assortment planners must decide, out of all potentially available products, which set of products to list in which channel and inventory location (DCs and stores), which products to make available for *BOPS*, *SFS*, and *DAE*, and in which quantities. As shown in Figure 5.4, assortment and inventory planning builds upon the DC and store as fulfillment locations. The assortment is defined on a mid-term level as it is usually related to periodical supplier negotiations (e.g., in grocery) and seasonal cycles (e.g., in fashion), and imposes changes on store shelf arrangement as well as webshop configurations. Assortment composition and inventory target levels must be defined jointly as these decisions are interdependent when space in the store or the online DC is limited. For example, adding one product requires the reduction of inventory of another listed product or the delisting of another product to provide space. A meaningful allocation of products and inventories to the webshop and stores can be achieved to serve customers across channels by disentangling and quantifying demand transitions between channels and products. In OC retailing, not only in-channel substitutions but also demand transitions between the webshop and stores and vice versa are very common (Rooderkerk and K ok, 2019). Next to OC-specific demand, OC assortment and inventory planning must consider product margins, distribution and replenishment costs as well as inventory holding costs for each inventory location. To quantify the demand fulfillment, it becomes additionally necessary to consider over- or underage costs.

Challenges in practice Enabled through OC information systems, customers use multiple channels along their purchasing journey and frequently change channels to gather pre-purchase product information (i.e., research-online, purchase-offline behavior, and vice versa), change preferences, and substitute products (VAR01). This requires demand models that integrate such cross-channel customer effects as well as inventory location and product-specific economics. Assessing these effects and costs of varying assortments across stores and the webshop on customer demand is a challenge.

A new aspect is the product and channel complementarity. For instance, being able to offer a broader assortment in one category might convey the impression of elevated competencies in this category and lead to additional sales. On the same line, offering a complementary channel to existing channels provides both a source of supplementary information and additional substitution possibilities for customers. Transferable substitution demand, walk-rates, and cannibalization become even more crucial as a result, but also harder to estimate. As an example, one manager from a DIY retailer (DIY01) referred to the difficulty of assessing the potential of *“preventing customers from buying online and steering them into the store to utilize up- or cross-selling opportunities”*. Moreover, OC assortment planning requires the incorporation of different revenue and cost structures of products and fulfillment locations via the introduction of OC concepts. For example, purchases following cross-channel research or products that are sold using *SFS* make it more difficult to account for the actual fulfillment cost of items.

Despite using some kind of software-based decision support, planning approaches currently applied in practice usually take channel-isolated views and do not account for OC channel demand interactions and cost implications. Two fashion managers who we interviewed, (FAS01, FAS02) explained that retailers often do not take a differentiated item- and data-based decision but mainly aim to list the entire assortment in the online channel to make the webshop a virtual shelf extension. Exceptions are original bricks-and-mortar retailers with small online volumes that also exclude slow-moving

products from their online DC. Another fashion manager (FAS03) stated that a minimum required margin is the only deciding factor when defining online assortments. In the stores, retailers usually list a reduced assortment, often physically constrained by the store's shelf space. Other retailers only list products in the store that are characterized by comparatively low values and high shipping costs (FAS06, DIY02). This is however often done without profound insights into channel buying and switching behavior and reduces the potential of revenue-driving OC concepts. The decision on what and how much to offer via *BOPS*, *SFS*, and *DAE* is often either somewhat disregarded or not based on channel-specific cost, revenue and demand implications. A DIY manager (DIY02) revealed that competitors simply listed every possible item for *BOPS*, whereas they used a more advanced approach for *SFS* and decided between DC shipment and *SFS* via a set of performance indicators such as the number of products in the store, delivery method, or inventory turnover. One OC expert summarized that "*there is no winning philosophy and much depends on industry and company characteristics*" (VAR04).

Current research The OC assortment and inventory planning literature is summarized in Table 5.4. Notably, all these papers were published after 2016. Related research can be divided into contributions that only consider inventory decisions, only assortment decisions, or both jointly. We start our review with inventory models in the OC context, and subsequently expand this to assortment models and integrated models.

The first stream of related papers deals with defining inventory target levels across channels. Gao and Su (2017b) developed the first analytical model that defines inventory levels for a single product in an OC setting. The inventory in the store channel is optimized with closed form expressions taking into account stochastic demand where customers can substitute from the store channel to the online channel via a *DAE* in the event of stockouts. In an extension, inventory in the dedicated online DC is defined endogenously, i.e., by the model. The authors analyze the profit impact

Table 5.4: Related literature on OC assortment and inventory planning

Related contribution	OC concepts		Modelling approach				Decision ⁶				Substitution ⁷			Problem size ⁹		Data ¹⁰			
	<i>BOPS</i>	<i>SFS</i>	<i>DAE</i>	Model ¹	SolA ²	OF ³	Specifics ⁴	Dem ⁵	I-st	L-ws	A-st	A-ws	in	ws-st	st-ws		Per	Prod	Sto
Gao and Su (2017b)			✓	AM(NV)	CFS	P	CS,SEQ	sto, st	✓	✓					S	1	1	1	SD
Shi et al. (2018)	✓			AM(NV)	CFS	P	CS	sto, sea	✓							2	1	1	SD
Zhang et al. (2018)	✓			AM(NV)	CFS	P	CR	sto, st	✓	✓						1	1	1	SD
Lu et al. (2020)	✓			AM(NV)	CFS	P	C,SL,CR	sto, st	✓							1	1	1	SD
Saha et al. [2020]	✓			SDP	Opt	C	CS,SEQ	sto, st	✓							1	1	1	SD
Gu and Tayi (2017)			✓	AM	CFS	P	CS	sto, st			✓				A	2	2	1	SD
Dzyabura et al. [2018]			✓	NLP	GH	NS	C	sto, st			✓		A ⁸		A ⁸	1	96	1	ED
Li et al. (2020)			✓	AM	CFS	P	CS	sto, st			✓				A	1	2	1	SD
Park et al. (2020)			✓	MIP	Opt	U	C	sto, st			✓				A	1	3	17	ED
Mandal et al. (2021)			✓	AM	CFS	P		sto, st			✓					1	1	1	SD
Gao and Su (2017a)	✓			AM(NV)	CFS	P		sto, st	✓	✓	✓				S	1	1	1	SD
Geimes and Su (2020)		✓		SP	SH	P	C,SL,CS	sto, sea	✓	✓	✓			S		1	20	1	SD
Hense/Hübner [2021]	✓			BIP(NV)	SH	P	C	sto, st	✓	✓	✓	✓	A/S	A/S		1	100	1	SD

¹ Formulation of the underlying problem: Analytical model (AM), Stochastic dynamic program (SDP), Binary integer program (BIP), Mixed-integer program (MIP), Non-linear program(NLP), Stochastic program (SP), Markov model (MM), Newsvendor (NV)

² Solution approach: Closed form solutions (CFS), Optimal (Opt), Greedy heuristic (GH), Specialized heuristic (SH)

³ Objective function: Profit (P), Costs (C), Sales net returns (NS), Utility of showcase (U)

⁴ Specifics: Capacity constraints (C), Service level constraints (SL), >1 Customer segment (CS), Sequential customer arrival (SEQ), Cross-selling (CR)

⁵ Demand model: Stochastic (sto), Deterministic (det), Seasonal (sea), Stationary (st)

⁶ Decision taken: Inventory (I), Assortment (A), Store (st), Webshop (ws)

⁷ Substitutions considered: Within channel (in), webshop to store (ws-st), store to webshop (st-ws); options include out-of-stock (S) and out-of-assortment (A)

⁸ Substitution also possible if utilities of available products change

⁹ Problem size: Periods (Per), No. of product variants (Prod), No. of stores (Store)

¹⁰ Data applied: Simulated data (SD), Empirical data (ED)

of *DAE* using physical showrooms in the store. This enables customers with product value uncertainty and availability risk to realize their product valuation even when the product is OOS in the store. The model only considers the store channel as a whole and does not define inventory levels for each store separately. The demand model only specifies a transition from the store to the webshop, but not vice versa or to other products in the same channel. In a related setting with product value uncertainty, Shi et al. (2018) optimize with a closed form expression the seasonal store inventory level and price for a single product when *BOPS* is possible. The retailer optimizes a pre-order discount and uses pre-order information to define the inventory in the store channel. In a pre-sales period, informed customers can order discounted products online for pick-up in store. In the sales period, those customers pick up their orders, realize their product valuation, and potentially return their resalable *BOPS* order direct to the store. Online inventory is exogenous and unlimited. Substitution between channels is not considered at all. In a similar application, Zhang et al. (2018) study with an analytical model an online retailer with *BOPS* that optimizes one single retail price as well as one aggregated inventory across

channels for a single product. Online customers can decide between home delivery and a potential fee-based return, or choose *BOPS*, visit the store at a particular traveling cost, evaluate the product, and either directly cancel the order or purchase the product. Lu et al. (2020) analyze with an analytical model an OC retailer with growing *BOPS* and dropping in-store demand for one product. The retailer generates additional cross-selling revenue via *BOPS* sales while being required to fulfill a minimum share of in-store demand. The optimal inventory target levels are explored for the store under scenarios where inventory for *BOPS* and in-store demand is shared and jointly ordered. While the previous inventory contributions formulate MIPs, Saha and Bhattacharya (2020) add to the existing single-item inventory models through a Markov decision process. They consider a retailer with *BOPS*. Store inventory is optimized taking into account independent store and *BOPS* demand while the online DC has unlimited capacity. Whereas store demand is fulfilled immediately, *BOPS* orders are reserved until pick-up, taking up inventory space and delaying replenishment requests. No substitution within or between channels is considered.

The models above provide some first approaches to derive analytically inventory target levels in operations across channels. The OC concepts applied are limited to one product and disregard assortment compositions. A main limitation for the application to practice is the consideration of channels without specifying the inventory for concrete stores. Furthermore, the models are not comprehensive in terms of substitutions between and within sales locations as well as considering actual fulfillment costs.

The second related stream comprises assortment planning. Gu and Tayi (2017) use an analytical model to analyze research-offline, purchase-online behavior for a retailer with a webshop and a store with a connected *DAE*. The retailer decides on products, what price to sell them at, and whether to sell the products via both channels or only the webshop. In the store, customers receive information about the available product(s) and thereby fully resolve their product uncertainty on available products or reduce their uncertainty about unavailable products. Subsequent online purchases are

enabled via *DAE* for products not available in the store. Customers can return products after evaluating the product received. Inventory levels are not defined, substitution rates are not considered, and costs are limited to handling returned products. Dzyabura and Jagabathula (2018) also formulate a NLP and propose a greedy heuristic to solve the problem. They consider showrooming as a special case of *DAE* where the store assortment is optimized without optimizing inventories. Out of a given set of products in the online channel, the retailer decides on the subset of products to offer in the store. Net returns on sales are maximized by the retailer, thereby ignoring any costs. Demand is modeled using a utility-based model. The customer's physical evaluation of the store assortment may change the customer's product utilities of the store and online assortment and result in purchasing a different item to the one originally preferred. The impact of the store assortment on online demand is factored into these considerations. Li et al. (2020) also analyze with closed-form expressions an online retailer with *DAE* in the form of a physical showroom. Two scenarios are considered in which either both or only one of two products is listed in the showroom. Within each scenario, prices and the level of information provided in the showroom are defined. The authors resemble Gu and Tayi (2017) in their substitution approach when customers visit the showroom to reduce their evaluation uncertainty. Again, customers can return their products and the inventory decision is disregarded. Park et al. (2020) develop a MIP to optimize a store's showroom assortment of a retailer with *DAE*. Space and budget constraints are considered. The showcased products can be varied along various feature categories, where each feature is assigned a customer utility. In a special case of automobile retailing, this contribution does not consider inventory and thus no OOS substitution. Just as Li et al. (2020), Mandal et al. (2021) utilize an analytical model to study the setting of an online retailer with one product that varies along standardization and product value. The authors consider a scenario with a showroom and another scenario with a store that accepts online returns. Similar to the previous assortment papers, no inventory is considered and customers have unknown product valuations, which can be resolved by visiting the showroom or the store. Yet no OOA or OOS substitution is considered.

The third stream blends inventory and assortment decisions. Gao and Su (2017a) provide the first combined model with *BOPS*. Through an analytical approach, they depict an OC retailer that provides store inventory availability to online customers and reduces customers' hassle cost of store shopping. They may also exclude *BOPS* products in the webshop that are below a specific inventory level (i.e., select assortment for *BOPS*). The demand model considers OOS situations, where customers switch from the store to the webshop, cross-selling opportunities, and exogenous online demand. Geunes and Su (2020) develop a stochastic program which is solved through a specialized heuristics. They apply *SFS* and model inventory and assortment optimization with OOS situations in the online DC. This is the first contribution with more detailed fulfillment costs. The retailer selects the assortment and inventory target levels for a store with limited shelf space and for the online channel with limited DC capacity. Different costs are considered for each fulfillment option. Demand is modeled with store, online, and hybrid online customer segments. When online products become OOS, hybrid online customers can choose to substitute for the identical product via drop-shipment or *SFS*. Drop-shipment is carried out at the full price through an external supplier with unlimited inventory. *SFS* uses the inventory of the retailer's own stores and charges the customer a discounted price as products are only shipped at the end of the sales season. Regardless, customers can neither substitute different products within the same channel nor unavailable products in the store. Hense and Hübner (2021) formulate a binary integer program and propose a specialized heuristics to solve larger instances. They investigate the assortment, space, and inventory problem for an OC retailer that offers *BOPS*. Customers can substitute OOA and OOS products for different products in the same channel or another product in the other channel. The authors also include space-elastic demand for the store and limited space for both channels. Interdependence between the assortment size, space assignment, and demand implications are investigated.

Future areas of research Current literature provides structural insights into assortment compositions and inventory levels across channels. However, many models lack the integration of comprehensive demand models with substitutions across and within channels, incomplete fulfillment costs and overly simplistic assumptions such as pre-defined online assortments and inventories. Table 5.4 also reveals that many models are restricted to one or two channels (without considering multiple stores), and one or two products. To advance the OC operations literature, research opportunities lie in a more thorough consideration of demand effects across channels, stores as showrooms, cost and revenue parameters, safety stocks, and more solution approaches applicable for practice.

First of all, relevant customer search behavior across channels, as well as substitution, complementary, and cross-selling effects, must be taken into account when configuring OC assortments. At the same time, only Dzyabura and Jagabathula (2018) and Hense and Hübner (2021) consider in-channel substitution demand and substitution demand into two directions within one model. To reflect actual OC demand, it is beneficial to complement the models mentioned and account for all relevant OC research, i.e., substitutions within a channel (see e.g. Hense and Hübner (2021)), substitutions from the webshop to the store (see e.g. Geunes and Su (2020)) and substitutions from the store to the webshop (see e.g. Dzyabura and Jagabathula (2018)). Alongside that, product and channel complementarity as well as cross- and upselling opportunities are worth integrating. It is also crucial to respect relevant single- and multichannel demand effects such as space-elasticity and cross-space elasticity in the bricks-and-mortar store (see e.g., Hansen and Heinsbroek (1979), Chandon et al. (2009), Eisend (2014), or Schaal and Hübner (2018)), or positioning- and salience-effects on a web page (see e.g., Djamasbi et al. (2010), Pieters et al. (2010), or Atalay et al. (2012)). The broad range of demand effects that potentially influence OC assortments calls for empirical investigation and modeling of the relevance and magnitude of the demand effects mentioned.

Secondly, assortment and inventory related research is necessary for showrooms. Showrooms depict a specific case as the stores display products but only sell them through *DAE*. Stores are only required to have one unit in stock as a result. Fulfillment is carried out at scale via the DC. Despite benefiting from inventory pooling, assortment decisions gain even greater importance in this context. To define the showroom assortment it is particularly valuable to empirically and analytically understand which products need to be explained or experienced by customers and which characteristics help convince in-store customers to carry out their purchase via *DAE*.

Thirdly, OC retailers must look beyond pure unit costs and revenues and take total profitability and feasibility into account. OC retailers need to identify all decision-relevant costs, including unit and purchasing costs but also fulfillment costs, safety stock cost, salvage values, shortage costs, and hidden OC costs. This concerns products bought through cross-channel research, *BOPS*, *SFS*, or *DAE*, causing information provision or fulfillment costs across different channels. Feasibility, on the other hand, mainly describes whether it is operationally possible to offer OC concepts for certain products. For example, a paint bucket might provide highly beneficial OC margins, yet it is impossible to guarantee undamaged *SFS* for the product. Other dimensions to consider in this context are the weight and size of products. A further factor that has not yet been considered in inventory management in OC is the determination of safety stocks for each sales and inventory location. As one inventory location can back up others and provide substitutions across channels, this also constitutes a novel problem caused by OC operations. Finally, we have seen that in practice, many assortment and inventory models fall short of accounting in a practical manner where OC demand effects are concerned. Given the magnitude such effects can have on profits, retailers require related decision support that is not only easily applicable but also delivers efficient solutions without extensive OC data that is potentially unavailable. In these cases, algorithms to estimate demand probabilities are required to obtain inputs for more detailed demand models (see also Section 5.3.2).

Inventory replenishment and returns

Scope of planning problem Inventory replenishment and returns is the subsequent planning step after the mid-term assortment and target inventory level definition. It deals with when and how much to refill and how to integrate returned inventory. Retailers need to define the replenishment frequency (when) and quantity (how much) for each inventory location (stores and DCs) after actual customer orders have been fulfilled to meet the specified inventory target levels. Retailers usually apply a periodic review policy with a constant order cycle and variable order size (Holzapfel et al., 2016). The retailers need to check available inventory positions against expected demand (within and across channels) and take into account service levels, lead times, target inventory levels, and inventory that is currently available in order to define the order-up-to-level. The available inventory in the store is then a composition of physical inventory plus scheduled receipts from DCs or suppliers, plus expected receipts from customer returns, minus expected store sales, reservations, backlog and orders from the webshop and the safety stock. The available store inventory is particularly relevant for OC concepts as this is displayed in the webshop and serves as a basis on which customers can decide whether to go to the store or use *BOPS* (see Figure 5.4 and Hübner et al. (2016b)).

Challenges in practice OC retailers face difficulties achieving real-time inventory accuracy, managing cross-channel product returns, and orchestrating OC replenishment rules that integrate dependent decision systems. First, obtaining real-time inventory transparency and accuracy, i.e., “*which SKU is available in which moment in which location*” constitutes the most critical challenge in this context (VAR03). One manager (VAR04) also emphasized the importance of an integrated, real-time inventory and point-of-sale (POS) data management system. This not only enables synchronized OC availability information but also prevents inventory record inaccuracy (IRI). IRI has severe consequences for *BOPS* or *SFS*, such as overselling stock, using unnecessarily high safety stock, or achieving unsatisfactory

service levels and fill rates (VAR02). Accordingly, customer satisfaction as well as logistics costs for inventory holding or replenishment are negatively affected. The causes of this problem are manifold. A large number of OC channels and different fulfillment concepts complicate the management and recording of accurate data. Manual errors such as late checkouts of *SFS* products further impair the IRI (VAR02). Moreover, a lack of integrated POS inventory control systems across different channels and along different stages of the retail supply chain, as well as low data update frequencies, prevent real-time inventory visibility. A further complication is the option to return online orders in the store. This burdens store operations through potentially flawed processes, for example through the assignment of cross-channel returns to wrong locations, and increases store inventories. The anticipation and management of such returns are difficult, yet necessary (FAS06). Rules also need to incorporate cross-channel returns as these quantities can potentially complement or even replace restockings (e.g., FAS06, VAR05). Processes are required that decide how and where incoming returns are fed back into the store inventory (VAR05). Finally, managers are urged to optimize inventory control to avoid unnecessary stockouts and rush orders, or overstocks, respectively (e.g., FAS01, VAR05). The definition of reorder periods and quantities requires that the retailers consider a vast array of input factors. OC demand forecasting, assortment and inventory planning, replenishment costs, and inventory accuracy serve as inputs, which are often incomplete or faulty. Such deficiencies require higher cycle inventories or safety stocks (Gallino et al., 2017). Thus, as summarized by a fashion retailer (FAS03), replenishment can only offset preceding flaws at the expense of unnecessary higher costs.

Today, only few OC retailers operate fully integrated, real-time inventory control systems. Most retailers use basic tools like spreadsheet-based, channel- or location-isolated control systems, or integrated control systems that update inventories only once a day, usually overnight. Barely any retailer records accurate inventories (DIY02) due to processes that are prone to manual errors, such as having to check out *BOPS* goods through the cash desk. One fashion manager (FAS03) even pointed out that this

impacts OC retailers to the point where they no longer trust their in-store inventory and instead ship *BOPS* orders from a DC to the store for pick-up. Furthermore, cross-channel returns receive a comparatively lower degree of attention. Many consider this a downstream topic that is not worth considering when setting optimal order-up-to levels (VAR05).

Current research Starting from 2017, the planning issue in OC replenishment is addressed by a small body of multi- and single-period inventory literature. Table 5.5 summarizes related literature.

Table 5.5: Related literature on OC inventory replenishment and returns

Related contribution	OC concepts			OC return	Modeling approach						Costs ⁷			Problem size ⁸				
	<i>BOPS</i>	<i>SFS</i>	<i>DAE</i>		Mod ¹	SolA ²	OF ³	DecV ⁴	Specs ⁵	Demand ⁶	Ov/Sh	Inv	Pi	Per	Prod	Sto	DCs	Data ⁹
Xu et al. (2017)	✓				MIP	DP	C	q	CS	det,sea,cr	✓	✓		M	1	1	1	ED
Xu and Cao (2019)	✓				SDP	Sim	P	q	SL	sto,st	✓	✓		M	1	1	1	SD
Govindarajan et al. (2020)		✓			SDP	SH	C	q		sto,st	✓	✓	✓	M	1	150	10	SD
Li (2020)		✓			AM	DP	P	q	TS	sto,st	✓	✓		M	1	2	1	SD
Muir et al. (2019)				✓	Sim	TA	T, IA	q, r		det, sea				M	1	45	1	ED
Radhi and Zhang (2019)				✓	AM	CFS	P	q		sto,st	✓			S	1	1	1	SD
He et al. (2020)				✓	AM	CFS	P	q, p		sto,st				S	1	1	1	SD

¹ Formulation of the underlying problem: Mixed-integer program (MIP), Stochastic dynamic problem (SDP), Simulation (Sim), Analytical model (AM)

² Solution approach: Dynamic programming (DP), Simulation (Sim), Specialized heuristic (SH), Thinning algorithm (TA), Closed form solution (CFS)

³ Objective function: Costs (C), Profit (P), Timeliness (T), Inventory availability (IA)

⁴ Decision variable: Quantity (Q), Reorder point (R), Price (P)

⁵ Specifics: >1 Customer segment (CS), Service level (SL), Transshipments between stores (TS)

⁶ Demand model: Stochastic (sto), Deterministic (det), Seasonal (sea), Stationary (st), Cross-channel demand, where channels demand depend on each other (cr)

⁷ Specific costs on top of unit costs: Overage and shortage/backlogging costs (Ov/Sh), Inventory holding costs (Inv), Picking costs (Pi)

⁸ Problem size: Multiple (M) or single (S) periods (Per), No. of product variants (Prod), No. of stores, No. of DCs

⁹ Data applied: Simulated data (SD), Empirical data (ED)

Xu et al. (2017) propose a multi-period inventory replenishment policy for a single product as a non-capacitated lot-sizing problem with known demand. They derive structural properties to apply them within dynamic program for solving the model. An OC retailer satisfies periodic online, *BOPS*, and store demand via one centralized inventory. With the objective of minimizing total operational costs, the OC retailer uses advanced demand information to define how much to replenish in each review period. The OC retailer can delay online orders at the cost of a penalty. Demand transitions across channels are modeled via online customers choosing between home delivery and *BOPS*. No location-specific order-up-to levels are defined. Xu and Cao (2019) formulate a Markov decision process to depict an OC retailer's particular case with *BOPS*. The retailer has to satisfy multi-periodic and stochastic demand for a single product from the store and *BOPS* of a franchisor. A periodic inventory review is applied that optimizes the order-up-to level for one store and the fraction of store inventory to reserve for *BOPS* orders. Once the inventory is reserved, it can only be

reassigned as leftovers at the end of a period. Revenues are considered, as well as backlogging, shortage, inventory holding, and purchasing costs. The demand model is limited to the store and no OC interaction across channels is considered. Govindarajan et al. (2020) extend the problem to multiple inventory locations (i.e., multiple stores and DCs), but again only for a single product. Optimal order-up-to levels are defined within a stochastic dynamic problem for each location and each review period. While store demand is fulfilled as it arrives, incoming online orders are fulfilled by shipping from DCs or *SFS*, depending on a threshold fulfillment policy. The model accounts for shortage, overage, inventory holding, and distance-dependent unit costs (i.e., picking, packing, labeling, and shipping), but disregards demand interactions between the channels or stores. A specialized heuristic for integrated inventory planning and fulfillment is applied. Li (2020) add to the existing literature on multi-period, multi-store and single-item inventory replenishment. Their contribution stems from the consideration of transshipments between stores, which fulfill both online and offline demand from their region. For each period, the retailer needs to decide for each store how much to replenish from other stores and from the DC. This results in the total quantity ordered from the supplier and shipped to the DC. The model takes into account purchasing, shortage, overage, inventory holding and shipping costs from the DC to the stores and between the stores. Yet, neither picking costs nor cross-channel demand substitutions are considered. A dynamic program solves the problem optimally.

The effect of returns on inventories is picked up for single products in an additional stream. By the means of a simulation and a thinning algorithm, Muir et al. (2019) take resalable OC returns into account when deciding on order quantity and reorder point of a retailer's stores and online DC. However, OC distribution concepts are disregarded. Online orders are exclusively fulfilled by the DC but can be returned in the stores. Online and in-store customer demand is periodic and deterministic while returns are stochastic. Inventory policies for all locations are derived to optimize inventory availability and lead times via a discrete-event simulation. In contrast to the multi-period contributions described so far, Radhi and

Zhang (2019) and He et al. (2020) focus on single-period inventory control, taking OC returns into account. Radhi and Zhang (2019) study analytically a retailer with resalable cross-channel returns for online orders. The authors consider overage, shortage, and shipping costs as well as refurbishment costs for the resalable returns. He et al. (2020) uses a game theoretic approach to investigate a similar setting, where returned products can be resold at a discount.

Future areas of research The papers analyzed above help to determine replenishment quantities in varying OC settings. However, all of the models suffer from constrained OC concepts, costs, and demand as well as problem sizes. Practically relevant future research can be obtained from models that represent real-life settings, the consideration of OC returns, the incorporation of inventory accuracy, and advances in short-term revenue management. First, it is necessary to extend the scope in terms of products and inventory locations and OC concepts. Common across the related papers is that they optimize the refill quantity for a single product only without considering demand interactions across channels (e.g., for OOS substitutions), across products, and for the various OC concepts of *BOPS*, *SFS*, and *DAE*. The different OC concepts are only partially covered. The contributions are limited to either channels or only a limited number of stores. This means the refill quantities are not specified for all possible inventory locations and fulfillment from other locations in OOS situations is not yet considered. Second, the existing literature does not sufficiently close the gap between OC returns and inventory control. Most evidently, models considering the reintegration of returns to the stores or returns via shipment to the online DC while also looking at OC concepts are lacking. Also, the actual replenishment and return costs (e.g., picking, packing, labeling and transportation costs between DCs and stores) are so far only partially factored in by Govindarajan et al. (2020). Third, a problem that did not seem to receive any attention in the current OC literature is the management of IRI, despite being the most critical issue in OC inventory control according to practitioners (see also Hauser et al. (2020)). The

available research of IRI for stores should be extended to OC operations. On the one hand, factors that mitigate IRI in OC can be researched, such as efficient control-mechanism-like rules for efficient inventory auditing. Retailers will also benefit from understanding factors that exacerbate IRI in OC operations, such as slow or non-integrated inventory systems. Another factor is that OC replenishment models are necessary that account for unavoidable IRI, e.g., by incorporating probabilistic inventory levels. Lastly, another opportunity is the alignment of replenishment models with revenue management. A synchronization of replenishment and customer steering would address excess and shortage stock.

5.4 Conclusion

Summary and contribution Digitization and OC retailing result in considerable challenges for the management and optimization of retail operations. Retail practice itself has transformed during the last decade from offline only or online only to seamlessly integrated channels. This tremendous progress has been largely driven by retail itself to react quickly to the growing and cross-cutting online market. The dynamic environment has uncovered new topics that have been predominantly discussed in practitioner-oriented or empirical journals to describe and identify the new phenomena. The continued absence of quantitative insights, their practical need, and the growing availability of data motivates an increasing number of scientists to intensify OR-based research on OC operations. This review provides an overview and structure for this nascent and growing research field of operational research in OC retailing.

The OC concepts *BOPS*, *SFS*, and *DAE* make the store the epicenter of OC operations. This makes it necessary to investigate emerging challenges and planning questions imposed on store operations. We focus on the discovery of tactical and operational, store-related planning issues, the discussion of such in the light of existing academic literature, and the

derivation of future directions of research. This review is distinct from current (empirical) literature reviews as we apply an OC lens, center on the role of the store, identify practically relevant planning issues by means of interviews, and investigate applications of quantitative decision support. The underlying challenges of these planning issues are recognized and faced by the large share of interviewees. This allows us to contribute to literature by delineating a modeling framework for OC operations with a store focus and five specific planning issues. We relate and analyze quantitative literature for each planning issue and derive future areas of research by merging insights from retail practice and state-of-the-art literature. We found stark contrasts in the literature coverage of the planning issues under investigation. There is a growing body of literature around network design, assignment of customer orders, and assortment and inventory planning. We have noticed a peculiar focus on *BOPS* and *DAE* for assortment and inventory planning and *SFS* for network design and assignment of customer orders. In stark contrast to that, demand forecasting and inventory replenishment are significantly under-explored.

Overarching avenues of future research In addition to the future research for each planning issue, our analysis revealed some prominent topics that are present across all planning issues. Advanced modeling and solution approaches have the potential to enhance decision making and provide new insights into novel OC problems. First of all, we noticed a common shortcoming of OC-specific variables and parameters in demand modeling. In particular, there is limited integration of novel cross-channel customer interactions and their implications for demand modeling in each channel. One can also drill down into revenue upsides and specific attribute-based consumer utilities. This also calls for further multidisciplinary approaches to integrate insights and techniques from marketing, consumer psychology, and revenue management. Predictive and prescriptive analytics (based on big data or machine learning techniques, for example) could be applied. Many models centre on a stationary demand. This should be further relaxed, as retail sales are usually stochastic, seasonal, and non-stationary. This

requires approaches of stochastic dynamic programming. Furthermore, the majority of current approaches for planning fulfillment locations and assortments are based on optimization approaches. When considering dynamic and non-stationary problems (e.g., with growth of online demand) it will be beneficial to combine the optimization approaches with simulations and develop simulation-optimization tools.

Secondly, more comprehensive modeling of decision-relevant costs across products and inventory locations is required. Another topic that extends through all planning issues is the insufficient consideration of real-time, accurate, and transparent data systems. More research on how to collect, aggregate, analyze, and share data and potentially compensate for data flaws will be of use in all planning issues. Aspirations relating to service levels or delivery times in an OC setting should be also included in the models.

Thirdly, the literature review has shown that early OC literature mainly focused on developing analytical models to derive structural properties and insights into OC operations. These approaches cannot be transferred easily to ongoing planning purposes in practice. This calls for advanced models that can be solved numerically. Moreover, there is not yet a comparison or benchmark study available that tests the effectiveness of various solution approaches for each problem. Further investigating heuristic solution approaches, both in terms of solution quality and run-time as well as more generalized models, would be a beneficial area of research. Efficient solution approaches for the related general OR problems from single channels could be a valid starting point. Available models and OR approaches from SC and MC planning problems should be assessed with respect to their ability to transfer them to OC operations. Further enhanced models may bear the potential to provide meaningful solutions for store-related OC operations issues. Most computational experiments have used instances with a very limited number of products and stores. Hence, it is crucial to gather a set of benchmark instances that can be used to compare solution approaches covering realistic features and sizes.

Fourthly, our analysis has revealed the relevance and impact of uncertainties for the different planning issues. Most retailers confirmed that they need to deal with uncertain demand. Accounting for that, scientific contributions model stochastic demand. While customer demand represents one uncertainty factor, the supply side entails additional uncertainties that have not been discussed. The reliability of suppliers and lead time determine the inventory availability at the point of sale. Recent events such as the COVID-19 pandemic or the Suez Canal obstruction have shown how fragile inbound logistics can be and how much this can impact inventory planning for retailers. A further analysis can entail questions on multi-stakeholder perspectives. This paper details the planning framework for an OC retailer with own stores. However, there are varying business models with suppliers that operate the online channels and compete with a retailer that only operates a store.

Last but not least, while we have outlined connections between the planning issues identified, we have not assessed the dependencies in detail. A model that quantitatively assesses the horizontal and vertical interdependencies of planning steps is still necessary. Here, one can investigate for example how designing the network of fulfillment locations can alter assortment availability. Similarly, defining assortments across channels can be analyzed by considering options for replenishment practices and returns in the channels. Moreover, there is most certainly a wide array of further dependencies with planning aspects outside our scope. Hence, we encourage embedding the store-related planning issues within a greater framework that also covers topics such as purchasing and warehousing or areas related to marketing operations such as product or delivery pricing and promotions. Setting prices for both products and delivery services heavily impacts demand, which feeds subsequent decisions. Moreover, pricing also entails the question how to handle returns in an OC setting. Questions regarding pricing should therefore be addressed in further research in the context of store operations. Future generalizations could also build upon and extend our results to differentiate between food and non-food applications.

Our insights from practice, the literature review, and the framework developed build the foundation for this ongoing research and will foster the creation of further advanced models and solution approaches for OC operations and store management.

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Appendix

Interviewing approach

Table 5.6: Overview of interview partners

ID	Role	Sector	Annual sales in €	No. of stores
VAR01	Industry Expert	Electronics, fashion, food	n/a	n/a
VAR02	Industry Expert	DIY, furniture, electronics	n/a	n/a
DIY01	Head of Supply Chain Processes	DIY	1bn-inf	300-inf
FAS01	Senior Director Sales	Fashion	1bn-inf	300-inf
FAS02	Global Lead Digital Solutions	Fashion	300m-1bn	0-100
VAR03	Industry Expert	DIY, fashion, furniture, food	n/a	n/a
FAS03	Director Strategy & PMO	Fashion	300m-1bn	300-inf
FAS04	Senior Store Manager	Fashion	1bn-inf	300-inf
FAS05	Head Corporate Development	Fashion	1bn-inf	0-100
OPT01	Manager Business Development	Optical	0-300m	0-100
VAR04	Industry Expert	Fashion, homeware	n/a	n/a
FAS06	Director Marketing	Fashion	300m-1bn	0-100
FAS07	Managing Director	Fashion	0-300m	0-100
VAR05	Industry Expert	Electronics, fashion, food	n/a	n/a
DIY02	Director Multi-/Omnichannel	DIY	1bn-inf	100-300
DIY03	Chief Operations Officer	DIY	1bn-inf	100-300
VAR06	Director Multi-/Omnichannel	Books, electronics, homeware	1bn-inf	300-inf

A semi-structured interview approach with open-ended questions was applied to retrieve relevant information and gain sufficient flexibility, which is appropriate when exploring a relatively little-known area of research (Edmondson and Mcmanus, 2007; Creswell, 2009). The questions aimed to understand which store-related decisions have to be taken by a retail manager in an OC context. We also probed for underlying challenges and issues that prompted the necessity for management attention. Questions were guided along the main customer-facing activities of the OC supply chain (Melacini et al., 2018; Bijmolt et al., 2021). Two interviewers conducted the interviews to ensure objectivity. The interviews lasted 45min on average. After the interviews, both interviewers immediately wrote down, compared, and transcribed their field notes. These notes were subsequently coded with the help of the qualitative data analysis software MAXQDA 11. Utilizing the software, we structured and categorized the interview insights into planning tasks. A natural information saturation was concluded after the marginal value of interviews 16 and 17 was found to be close to zero, only confirming existing insights, and coding did not change anymore. The number of interviews conducted (17) is in line with recommendations to

ensure academic rigor and generalizability (see Eisenhardt (1989), Ellram (1996) or Guest et al. (2006) for examples) The interviews were analyzed in two layers. First, an objective content analysis was conducted focusing on the identification of the planning decisions and underlying challenges. In a second layer, we concentrated on how the planning tasks relate to each other and how they can be integrated into a comprehensive planning framework.

Literature search approach

Table 5.7: Overview of keywords used in EBSCO and SCOPUS search

Area 1: Omni- and Multi-Channel	<i>OR</i>	Area 2: Two Single-Channels
omnichannel	click & reserve	online
omni-channel	click-and-reserve	e-com*
multichannel	bricks-and-clicks	ecom*
multi-channel	bricks and clicks	<i>AND</i>
multistore	online-to-offline	offline
multi-store	online to offline	store
crosschannel	O2O	stationary
cross-channel	offline-to-online	
pickup instore	offline to online	
pick-up in-store	ship-from-store	
pick-up-in-store	ship from store	
BOPS	SFS	
click and collect	transport-from-store	
click & collect	transport from store	
click-and-collect	send-from-store	
click and reserve	send from store	

Any keyword in Area 1 or a combination of the keywords in Area 2 in abstract, title or keywords qualified for a hit. Altogether, we screened 1732 papers from EBSCO and Scopus. Additional manual searches covered the following journals: Management Science, Production and Operations Management, Journal of Operations Management, Manufacturing and Service Operations Management, European Journal of Operational Research.

Search queries consisted of all MC and OC variations or variations of two single channels, such as e-commerce and stores (see Table 5.7). We also included plural forms, delimiters, prefixes, and suffices in our search strings. Acknowledging the importance of distinct borders for the literature under review and following our scope outlined above, we excluded literature from

areas such as pure marketing or general service management. Pure SC contributions such as online or offline retailing are also out of scope due to the distinct differences between SC and OC retailing (e.g., Agatz et al. (2008); Brynjolfsson et al. (2013); Bell et al. (2014)). MC retailing, on the other hand, is included in this first search, given the possibility of authors falsely describing OC concepts as MC fulfillment. Moreover, articles have to address a store-related operations problem.

The resulting 1,732 publications underwent an initial screening based on title, abstract, and keywords. All authors analyzed every article concerning whether the article addresses any of the planning issues in OC. In the rare event that only one of the authors classified an examined publication as relevant, it is nevertheless included in the further process to avoid any false negatives. Out of 1,732 articles, 132 articles qualified for step two.

6 Conclusion and outlook

This doctoral thesis deals with OC retailing. It equally supports practitioners and researchers in planning and optimizing assortments and store operations. Chapter 3 to Chapter 5 each state the respective key findings, methodological advancements, and areas of further research. This section is therefore used to aggregate the findings and research opportunities.

Conclusion

Chapter 3 to Chapter 5 define and formulate the novel problem of OC assortment planning, provide efficient solution approaches for different variations of the problem, quantitatively assess demand effects with a particular focus on OOA and OOS cross-channel shopping behavior, derive the most crucial challenges for OC store operations, list and analyse existing solution approaches in literature, and propose research opportunities in this field of operations research.

Omni-channel assortment planning Chapter 3 and Chapter 4 contain a detailed description of the problem of integratively optimizing assortments, space and position, and inventory across channels. A wide array of demand effects is accounted for: stochastic, space-elastic, shelf-segment, position, OOA and OOS in-channel substitution, and OOA and OOS cross-channel substitution demand under the consideration of space constraints in the

store and the online warehouse. The objective is always the maximization of the retailer's profit. Through the development of a BIP and a specialized heuristic, the interdependence of the assortment composition, space and position allocation, and inventory management is respected, the decisions are taken in an integrative way, and the NP-hard multiple-knapsack problem can be solved heuristically.

An application to various problem instances shows that the impact of demand effects on profits strongly depends on channel sizes, demand rates, and channel package sizes. Across all problem settings we see that omni-channel shopping behavior matters and that the consideration in assortment planning can generate valuable profits in traditionally low-margin retail businesses. In the general case, taking cross-channel substitution into account leads to profit gains of 1.5%, but in special case (e.g., reduced space in the store, or products with a particularly high affinity) it can create profit gains of up to 11.1%. Elevating substitution rates, for example by facilitating pick-ups in stores or providing easy access to in-store devices that exhibit online assortments, helps to grow profits through cross-channel substitution at a similar rate. On the other hand, ignoring cross-channel OOA and OOS substitution can lead to profit losses of up to 0.4%. Yet, other analyzed demand effects cause a significantly larger impact on profits when being ignored. In particular, space-elastic, position, and shelf-segment demand are crucial when defining assortments across channels. Ignoring these demand effects leads to profit losses of up to 15.5%, 4.9%, and 4.7% on average respectively, while in-channel OOA and OOS substitution demand potentially causes profit losses of up to 1.5% on average.

The combination of assortment decisions and demand effects has not been assessed by previous literature. Researchers thus benefit from this work as it provides a proven approach to OC assortment planning and can be leveraged when further developing approaches for this and neighbouring problems. It also defines a prioritized list of demand effects that are imperative for OC assortment planning and can be assessed through a numerical lens. Practitioners will capitalize on an enhanced understanding of customer

behavior, the acknowledgement of OOA and OOS cross-channel substitution behavior, and insights on how to improve their assortments, space and position, and inventories across channels to boost profits.

Omni-channel store operations Chapter 5 accommodates the latest trends in OC retailing and digitization, which enable OC delivery concepts (i.e. *BOPS*, *SFS*, and *DAE*) that greatly influence the way store operations need to be set-up and organized. Through a triangulation approach the underlying article uncovers demand forecasting, network design, assignment of customer orders, assortment and inventory planning, and inventory replenishment as crucial planning problems in this area.

In demand forecasting, a prevalent challenge for many retailers constitutes the lack of historical data but also the aggregation of processes, hardware, and software to make data accessible through a single, aggregated system. This planning problem is relatively under-researched with few contributions and would benefit from extensions to applicable OC concepts, workarounds for data issues, and alternative methodologies. When designing the network of fulfillment location retailers often deal with questions around which volume to assign to OC stores and how to guarantee efficient operations (e.g., picking and packing). A large share of articles approaches this topic, with a particular focus on *SFS*. Future research should include other OC delivery concepts, also taking into account relevant costs and dynamic and stochastic demand effects. The assignment of customer orders to fulfillment locations is, in practice, hampered by inefficient operations, questions on when to use stores to utilize idle capacities and inventories, and the lack of location-specific, real-time inventories. A range of articles covers this topic from a *SFS* view but extending it to *BOPS* and *DAE* concepts and, very importantly, including lead-time aspects represent valuable research opportunities. Assortment and inventory optimization is challenged through novel cross-channel customer behavior and location and product-specific attributes like costs. A peculiar focus of existing literature lies on *BOPS* and *DAE* but contributions often ignore multi-product OC environments, safety

stocks, showrooms, and relevant OC demand effects. Lastly, inventory replenishment in practice is characterized by issues regarding the achievement of real-time inventory accuracy and the management of OC returns. Existing literature is rather scarce and particularly does not cover OC returns in combination with inventory replenishment as well as workarounds to compensate for the lack of inventory accuracy.

Through the applied approach we uncover a wide range of practically relevant challenges and solution approaches in OC store operations. While it provides a detailed and well defined research agenda for researchers interested in this area, practitioners are supported in their practical need of quantitative decision models for OC store operations.

Future areas of research

Omni-channel assortment planning As OC assortment planning is constantly influenced by upcoming trends in customer behavior and latest advancements in customer technology and digitization, numerous opportunities for future research are identified. The developed models could be extended to neighboring, preceding, or succeeding decisions such as category planning (i.e. deciding on the size and depth of categories), pricing (i.e. deciding on the retail price), or inventory replenishment (i.e. deciding when and how much to replenish). In the latter case, a multi-period model seems highly suitable. Such a model should also integrate the decisive issue of inventory accuracy, for example by considering safety stocks, to avoid customer dissatisfaction when being unavailable to fulfill BOPS or SFS orders. Moreover, additional demand effects can further enhance the existing models. First and foremost, cross-selling could be included as it often poses a major motivation for retailers to construct assortments in a certain way. Showrooming and webrooming behavior of customers may also be incorporated. These effects help develop a more profound understanding of customer decisions and purchasing journeys in practice and

potentially increase the validity of assortment planning but also category planning. Next to that, the models could consider the impact of assortment variety on customer demand or several rounds of substitution, which is a particularly representative customer behavior for alike items. Besides that, existing models should be leveraged to compare the different OC delivery concepts (i.e., SFS, BOPS, DAE, click and reserve). Cost structures and specifics would require adaptations of the respective model, such as 100% substitution rates for SFS, higher picking costs for BOPS, or the integration of no-show rates for click and reserve. Furthermore, the characteristics of novel store formats such as dark stores (e.g., from uprising fast-delivery grocery retailers like Gorillas or Instacart) could be included just as trends in personalization or localization of assortments. An enabler for the latter is the collection and utilization of data. Being able to include advanced analytics, artificial intelligence, and automation in retailers' planning processes offers more than just a commercial optimization. It gives insights into emerging trends, which can be broken down into locations, store formats, and customer segments. Lastly, omni-channel assortment planning also offers methodological research opportunities. Given the continuous increase in computational power, large instances of our heuristics may soon be solved by exact methods. This progress allows a comparison of existing heuristics to exact methods but also the chance to tune existing heuristics and reduce computation times.

Omni-channel store operations Single research opportunities for every planning problem in OC store operations are already outlined in Chapter 6. Above that, further areas for future research are identified across all planning issues. In general, we notice a shortcoming of OC-specific variables and parameters such as cross-channel substitutions or research behavior. In this context, existing models would benefit from bringing together alternative modeling approaches such as simulations but also insights from varying disciplines such as marketing or consumer psychology. The integration of specific costs for the OC delivery concepts also applies to the large share of planning issues. On top of that, existing models need to be improved in

relation to their general applicability, run-time performance, and optimality gaps. This will also allow the calculation of larger problem sizes such as a higher number of channels, stores, or products. Existing models can build the starting point for such enhancements but need to be assessed and advanced carefully. Moreover, we believe that the dependencies between each of the planning problems can yield valuable insights. Above all, it will help in the conceptualization, formulation, and modeling of an integrative system covering all relevant areas in store operations. Such a system could cover the exchange, aggregation, and common utilization of product and information flows.

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