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# THE INTERACTION BETWEEN DESIGN AND QUALITY CONTROL OF CONCRETE STRUCTURES

by
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Gilbert W. Winslow Career Development Chair
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# THE INTERACTION BETWEEN DESIGN AND QUALITY CONTROL OF CONCRETE STRUCTURES

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## 1. PRELIMINARY REMARKS

Within the probabilistic concept of structural safety, the outcome of any production process is considered to be random. So, too, are the dimensions to be realized, the configuration of loads considered as possible actions on the structure as well as their maximum intensities during the anticipated lifetime. Structural safety is usually measured in terms of mathematical reliability or its complement "probability of failure." Conceptionally it is then assumed that the mathematical probability of failure is related in some form to its statistical equivalence, the failure rate. Once a measure of safety is defined - there are other simpler but less informative measures of safety than that mentioned - optimization of costs, weight, or other important parameters versus reliability is possible, which no doubt reveals numerous ways to invest available resources more efficiently.

Design and construction of buildings has been codified from the beginning of urbanized societies.This is not only to set commonly agreed rules and responsibilities within the profession but also because society is very sensitive to fatal structural accidents and the impact of design rules on the economy of several human activities is considerable. Therefore, optimization as defined here, means optimization of building codes, with the tacit assumption that a socio-economic mechanism exists which enforces the realization of more or less "optimal" structures throughout.

Considerable efforts have recently been made in the formulation of probability-based formats for design codes, where the input information on the physical parameters is no longer deterministic but stochastic. It appears, indeed, that the tools necessary for code optimization are now ready for immediate use. Unfortunately in some areas there still exists a problem of processing not so much data as the entire information about uncertain phenomena into basic code parameters. The knowledge about the stochastic behavior and appropriate models of actions is still insufficient but is improving rapidly. However, little attention has been paid to the production and construction processes which necessarily must follow any design when realizing a structure. The processes of production need guidance by adequate rules of control; their outcome should be judged for acceptance which then ensures that the designer's assumptions on the structural parameters are realized <u>in situ</u>. Being based on statistical tests, the decisions made during production and for compliance involve uncertainties which have to be quantified and considered in the overall evaluation of risks.

Different types of uncertainties can be distinguished. At first, there is not certainty about the appropriate stochastic model. However, concerning concrete strength the assumption of normally distributed strength values can usually not be rejected. In addition, this question is not a crucial one as long as we treat probabilities as operative measures of uncertainty /1/. Secondly, there always is, due to the limited sample size, considerable ordinary statistical uncertainty and thirdly, the distribution parameters to be tested are themselves random-like variables. The designer has to face all these uncertainties.

As of yet, statistical observations on the variability of the properties of materials have been processed into basic probabilistic input information for codes in a rather arbitrary way. Typically, the extent of control is used for classification of variabilities. But, the relation between variability and control does not appear to be fully understood, nor is it known how to build up optimal control mechanisms, taking into account the technical standard of concrete production and that design, production and acceptance, due to the distribution of responsibilities, remain separate functions when realizing a structure.

The following study aims at a better understanding of the interaction of uncertainties and control.



## 2. GENERAL REMARKS, PROBLEM DEFINITION

When studying the problems of interaction among risks during design, production and acceptance, it is reasonable to assume the cost of control to be in general negligibly small as compared with the total building cost. If this holds, optimization between total costs and risks of the structure can be left entirely to the designer (or a code committee), who then just must know (specify) the regimes for production and control in order to assess in advance appropriate values to concrete variabilities. The designer will come up with an individual job specification which includes the targets for production and the rules for production and acceptance control.

The specification for concrete with reference to its strength, measured on standard test specimens, is usually given as one number representing a strength value below which a certain fraction, $p_c$, of all possible strength values is expected to fall. That strength is often called characteristic strength, $f_c$ /2/. Such a definition is necessary. It represents a limiting quality for the ideal case where the infinity of strength values would be measured. The general assumption thereby is that this limiting quality would lead to equal probabilities of failure irrespective of the concrete variability - which is theoretically incorrect but apparently a good practical concept.

Besides a strength specification, the system of job specifications normally contains:

- a design rule for the target mix.

  According to most of the rules presently in use, the mix should be designed at least for a target mean equal to $k_M \cdot \sigma$ above a nominal strength $f_c$ where $\sigma$ is the expected standard deviation and $k_M$ a mix design factor of the order of 1.5 to 2.5. It should be noted that $k_M$ generally does not comply with the standardized normal variate $u = k_c$ belonging to the characteristic strength $f_c$ for the specified limiting fraction of defectives.

- an enumeration of characteristic components of the process of production to be checked or even inspected by measurements together with acceptable limits for these variables.

- a compliance criterion.

  It contains the type and frequency of sampling, the testing procedure, one or more acceptance functions as functions of the test results and the corresponding set of acceptance limits with which the acceptance functions are to be compared. The acceptance limits generally are linear functions of the characteristic or nominal strength.
  A variety of acceptance functions are applied at present. Most of them are linked with the concept of an underlying normal distribution and contain estimators for the mean and sometimes the standard deviation. Such functions are used as independent or running sampling functions and are very frequently amended by one or more extreme value or any other order statistic condition.

- a description of sanctions in the case of negative acceptance decisions.

  Generally, it is stated how and to what extent in-situ strength testing is required and/or proof load tests must be carried out in order to reach the final decision. It is a matter of the contract between producer and consumer (contractor and owner), if not the civil right, of who pays for the additional cost of testing, eventually the cost of delay of construction or even cost for removal and reconstruction. Occasionally, a guide is given as to how to carry out additional investigations of structural safety. Also, some examples are known where monetary penalties are defined for poor quality.



Given that format, it must be underlined that there cannot be any direct feedback from production to the designer which would enable some form of updating previous assumptions. Production takes place long after the design process has been finished. Normal standard tests usually allow decisions even not earlier than 28 days after the concrete has been cast. If compliance tests fail to comply there is no simple course of actions. The cost of delay of construction, of destruction, removal and replacement of concrete that has been identified to fail may be immense and not justified in the face of the statistical uncertainty of the regular compliance decision. Also, testing of the strength of in-situ concrete may turn out to be costly and may involve major difficulties when converting such results into standard strength test results.

In summary, the statistical problem of the designer is a problem of prediction, given some diffuse prior statistical information on the outcome of production processes, the existence of a set of job specifications, which include the specification of quality, a mix design formula, some rules of statistical production control and lenient compliance criteria which all together work as a guiding mechanism only because there would be a penalizing action on the very end of the process, if and only if sufficient evidence of poor quality has been found through tests or otherwise. Only in the last step of decision-making is estimation of concrete strength in the strict sense required.

The flow-diagramm, displayed in Fig. 1, illustrates without major sophistication the course of actions during the whole procedure. Design, production and acceptance have been distinguished as separate consecutive blocks. In fact, production, compliance and acceptance testing form some kind of sequential analysis. However, due to the different types of measurements taken in each step and due to the separation of responsibilities for sampling and decision-making direct processing of data through the sequence of actions is limited.

In the following we shall look first at the information on concrete strength available to the designer. Then, the fundamental relationships between design, production, compliance and eventual final acceptance testing will be studied. For illustration some numerical results are given.

## 3. THE PRIOR DISTRIBUTION OF STRENGTH

In probability-based design, the designer has to predict the probability distribution of concrete strength at each design point in the structure. An example of the outcome of present production /3/ is shown in Fig. 2, where a certain system of specification, control and acceptance has been in use. Each point represents the mean and standard deviation of a particular concrete grade produced at jobs under apparently constant conditions. The sample size for determination of each point is in any case larger than 40, which is taken to be sufficient for setting population parameter ≈ sample parameter. The assumption of a normal distribution of strength values has been verified at a significance level of 95%.

The individual mean, m, varies about an overall mean, $m_M$, with standard deviation $\sigma_M \approx 5 N/mm^2$ and the individual standard deviation about an overall standard deviation $m_\sigma \approx 5 N/mm^2$ with approximately $\sigma_\sigma \approx 1 N/mm^2$. Obviously some producers have taken into account the special production conditions when designing the individual target mean since a positive correlation of $\rho_{M,\sigma} \approx 0.45$ is observed. The variability of m obviously expresses the varying risk-willingness of the producers. It may depend on the economic structure of the building industry and/or the professional morals of the producers in that country but apparently is not affected



by the form of the compliance criterion. On the other hand, the overall mean $m_M$ seems to be affected by an eventually existing design rule for the mix but mainly by the severity of the compliance criterion and the sanctions for negative acceptance decisions.

If there were no further information, the designer should use the following predictive distribution of concrete strength X /4/

$$f(x) = \int_\sigma \int_m f(x;m,\sigma) \cdot f(m,\sigma)\, dm\, d\sigma \qquad (1)$$

which has a considerably larger variability of strength than any average population. Since the variability of σ is small in comparison with that of m, $\sigma_\sigma$ may be neglected. Under the assumption of normally distributed mean values, the predictive (Bayesian) distribution of X is then normally distributed with mean $m_M$ and variance $\sigma_X^2 = m_\sigma^2 + \sigma_M^2$. For the results given in Fig. 2 one obtains $\sigma_X \approx 7 N/mm^2$ and a percentage of strength values falling below the nominal strength of approximately 12%, which is in good agreement with observations for other concrete grades and other similar compliance criteria.

Next we study the interaction of safety, economy of production and compliance control.

## 4. THE FUNDAMENTAL CONCEPT

### 4.1 The safety criterion

Assume that there is a design situation which is representative for the type of structure under consideration. Assume further that there is a system of safety elements in the design procedure (partial safety factors or load and strengthreducing factors, additive elements, nominal or characteristic values) which leads to a given probability of failure if the stochastic properties of the variables were known.

To meet the designer's overall evaluation of socially admissible and economically optimal risks it is required that the failure probability, $P_f$, assigned to any design should be less than or equal to some optimal value OPT $P_f$.

$$P_f \leq \text{OPT } P_f \tag{2}$$

However, the actual concrete quality is uncertain. The failure probability can then be considered as a compound of three components /5/

i. the failure probability $F_f(m,\sigma)$ of a given quality of concrete, expressed as a point function of m and σ for the reference design situation,

ii. the probability $o(m,\sigma)$ with which that quality is offered and

iii. the point probability $A(m,\sigma)$ of accepting this quality during the compliance procedure.

In other words, the total probability of failure is the joint probability of these three events falling into the failure region and integrated over all offered qualities, given that they have been accepted. Note that the meaning of (2) must necessarily be modified such that $P_f$ represents an average value and thus, allowance must be given for particular designs to deviate from this value.

$F_f(m,\sigma)$ is the volume integral in the failure domain of the joint probability density function of all contributing variables such as loads, dimensions, the material strength, and other variables.

$$F_f(m,\sigma) = \int_{\{g(x,\xi)\leq 0\}} \int f(x,\xi;m,\sigma)\, dx\, d\xi \tag{3}$$



where $g(x,\xi)$ represents the chosen limit state criterion, X the material strength with parameters m,σ, and ξ the stochastic vector of all other variables.

$A(m,\sigma)$ is equivalent to the operation characteristic, $L(m,\sigma)$, of the compliance criterion in use. It is a function of the number of test results, the type of the compliance function and again the underlying distribution of strength.

Finally, the distribution of offered qualities $o(m,\sigma)$ is, of course, a function of the technical standard of production, but mainly the risk-willingness of the producer, and thus also a function of the operating characteristic of the compliance criterion and the sanctions of non-acceptance.

Hence, the three components of the probability of failure are multiply interrelated. Major simplifications are obtained when one assumes for the moment no effect of production control on the type and parameters of the distribution of strength which is considered as normal. For simplicity, assume further that the standard deviation σ of X is a constant. Then, the point probability of failure is independent of the two other components. Introducing a function H(m) which is the joint probability of offering and accepting a given quality

$$H(m) = P(\text{offer of } m \cap \text{accepting } m) \tag{4}$$

the basic concept (1) can be written as follows:

$$P_f = P\{\text{Failure}\,|\,H(m)\} \leq \text{OPT } P_f \tag{5}$$

or

$$\frac{1}{\text{OPT } P_f} \; \frac{\int_m F_f(m)\cdot H(m)\, dm}{\int_m H(m)\, dm} \leq 1 \tag{6}$$

## 4.2 The Operation Characteristic

We shall decide on the form and the parameters of the compliance criterion. Taking into account statistical efficiency and the ease in practical use as well the Joint Committee on Statistical Control of Concrete Quality has recommended a criterion of the following form /6/

$$\text{Accept if} \quad f_c \leq z = \bar{x}_{(n)} - \lambda \cdot \sigma \qquad (7)$$

otherwise: reject and impose a penalty

in which $f_c$ is the acceptance limit and, for convenience, equal to the nominal characteristic strength, $\bar{x}_{(n)}$ the independent or running mean of a sample of size n, and $\lambda$ an acceptance factor. If the standard deviation is not known, $\sigma$ may be estimated by the sample standard deviation $s_{(n)}$. For independent means, known standard deviation and an underlying normal distribution of strength values the operation characteristic as a function of $f_c$, $\lambda$, and n is then given by /7/

$$L(m;\lambda,n) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-t^2/2}\, dt = 1 - \Phi(u) \qquad (8)$$

where $u = \{ \frac{f_c - m}{\sigma} + \lambda \} \cdot \sqrt{n}$

and $\Phi(u)$ the normal integral. In principle, any other criterion, more appropriate for the particular situation could be chosen.

It is useful to recall briefly some properties of operation characteristics. A graphical representation is given in Fig. 3. $L(m,\lambda,n)$ is monotonically increasing



with m and equal to the probability of acceptance (strictly spoken the probability of non-rejection). The term $G(m;\lambda,n) = 1 - L(m;\lambda,n)$ may be called probability of rejection. Given a quality m which is sufficient by definition, $G(m;\lambda,n)$ represents the type-I-error as defined in the test theory /7/ or more commonly, the producer's risk. On the other hand, given an insufficient quality $L(m;\lambda,n)$ might be called type-II-error or the consumer's risk. Note that for any high quality the producer's risk will remain finite. It should also be recalled that decreasing of $\lambda$ shifts $L(m;\lambda,n)$ to the left wheras an increasing sample size results in a higher selectivity of the criterion for qualities in the neighbourhood of the so-called "indifference point" where $L(m;\lambda,n) = G(m;\lambda,n) = 0.5$.

In statistical interpretation the operation characteristic indicates that conditional relative frequency with which positive acceptance decisions would occur for any given quality.

Thus, there obviously exists a strong relationship between the the decideability of the compliance criterion and the distribution of offered qualities.

Therefore, we consider separately the economics of production for various sets of compliance parameters $f_c$, $\lambda$, n and the consequences of negative acceptance decisions.

## 4.3 Economics of Production

Let B be the payment per unit volume of concrete if accepted, C the basic cost of concrete production, a(m+b) the cost of production depending on m, c•n the cost of compliance testing, and $\alpha \cdot B$ a penalty in some form imposed on the producer in case of a non-acceptance decision. Then,

$$Q(m,\lambda,n) = (B - C - a(m+b) - c \cdot n) \cdot L(m,\lambda,n)$$
$$- \alpha \cdot B \cdot (1 - L(m,\lambda,n)) \qquad (9)$$

represents the benefit function of the producer.

B and C are assumed to fixed values but may vary according to the technical standard and the general competitive situation. Under such conditions, the cost of production is approximately a linear function of the cement content which in turn affects m linearly /8/. The cost of sampling and testing are assumed to be linear functions of the number of specimens. In the simplest form of sanctions $\alpha$ is a constant. If $\alpha = 1$, the meaning is that there will be no payment in the case of rejection. More realistically, $\alpha$ should be a function of the "true quality" (see Fig. 13 which is discussed in more detail later on) or other relevant parameters. To give rough figures, the constants become $C \approx 75$, $a \approx 0.05$, $b \approx 150$, $c = 0.005$ to $0.03$, for medium strength levels of concrete when setting $B = 100$.

For any value of m, equation (9) is decreasing with $\lambda$,n and $\alpha$ and possesses just one optimum value max $Q(m,\lambda,n)$ for $m = m_{opt}$ and a given set of values $f_c$, $\alpha$, $\lambda$, n. Fig. 4 demonstrates its general form. The profit is very sensitive for qualities $m \leq m_{opt}$, but decreases also considerably for $m > m_{opt}$. Fig. 5 shows the variation of the profit function with $\lambda$. Astonishingly, there is only a small variation of $Q(m;\lambda,n,\alpha)$ with $\alpha$ and n. (See Fig. 6 and 7). Apparently, it is just necessary that a penalty in the order of the production costs exists. For n we find optimal values in the range of 4 to 10. For information, Fig. 8 is added which definitely suggests that the magnitude of the standard deviation is also a significant factor of concrete production.

As shown, only a small region exists where profitable production is possible. In the long run, production will only be positive if at least

$$\int_{m_u}^{m_o} Q(m,\lambda,n) \cdot o(m)\, dm \geq 0 \qquad (10)$$

where $m_o$ and $m_u$ are extreme qualities the particular producer



might wish to risk and o(m) is "his" frequency function of produced qualities. Again, o(m) and the limits $m_u$, $m_o$ are generally unknown.

However, given a set of constants B, C, a, b, c, $\alpha$ and for any $\lambda$, one can find a sample size for which Q is less than or equal to zero for any value of m, thus a maximum acceptance factor can be found as

$$\lambda_{max} = \lambda_1(n) \text{ for } Q(m,\lambda_{max},n) = 0 \qquad (11)$$

The choice of $\lambda > \lambda_{max}$ would always result in economic losses.

Furtheron, for fixed $\lambda$ and n an optimal target value for the production is obtained by

$$m_{opt} = \max Q^{-1}(\lambda,n) \qquad (12)$$

from which a mix design factor can be derived

$$k_M = \frac{f_c - m_{opt}}{\sigma} \qquad (13)$$

which obviously describes the best production policy for the producer. If the overall mean, $m_M$, of offered qualities is now assumed to equal that optimum target mean, $m_M \approx m_{opt}$, we get together with the standard deviation, $\sigma_M$, estimated statistically as shown in chapter 3, and an appropriate assumption for the type of distribution sufficient information to calculate a minimum acceptance factor $\lambda_{min}$ as a function of n, which results in a family of operation characteristics with parameters $\lambda$, n just fulfilling the equality of the safety criterion (6) where

$$H(m) = o(m;\max Q^{-1}(\lambda,n),\sigma_M) \cdot L(m;\lambda,n) \qquad (14)$$

Actually, we easily find an indicator function

$$\lambda_{min} = \lambda_2(n) \quad (15)$$

representing the set of possible solutions.

The best set of values $\lambda$,n is obtained for

$$\max Q(m,\lambda,n); \qquad \lambda,n \text{ for } \begin{cases} \lambda \leq \lambda_1(n) \\ \lambda = \lambda_2(n) \end{cases} \quad (16)$$

In words, that combination of $\lambda$, n out of $\{\lambda = \lambda_2(n)\}$ is most economical and simultaneously "safe" which gives the maximum of max Q(m,$\lambda$,n) under the restriction that $\lambda_{min} \leq \lambda_{max} = \lambda_1(n)$. Additionally, we might search for combinations $\lambda$, n giving max Q(m,$\lambda$,n) for the whole possible range of combinations, that is

$$\lambda_{opt} = \lambda_3(n) = \max Q^{-1}(m;\lambda,n) \text{ for } \begin{cases} \lambda \leq \lambda_1(n) \\ \lambda \geq \lambda_2(n) \end{cases} \quad (17)$$

Fig. 9 demonstrates the domain of possible values $\lambda$ for n = 6 which gives profit to the producer and meets the reliability criterion as well. It also shows that the profit is best at the crossing of lines $\lambda_2$ and $\lambda_3$. The figure will be discussed in the next chapter in more detail.

## 4.4 The Effect of Production Control

Even for the simplest technical standard of production, some form of production control is usually codified, for example, check on the measuring units of the plant, the gradation of aggregates in an attributive manner, examination of the consistency of the mix by measurements of slump.



Various other techniques of measuring strength-indicating properties are applied at jobs at higher technical levels, for example, measurements of the water content of the mix which give estimates of potential strength in combination with the knowledge of content and grade of cement or even analysis of production by accelerated strength tests. Directive actions usually follow statistical estimates of the present state of the process. More advanced methods, such as control on the basis of the predicted future outcome, may become common for automatically controlled plants.

Any production control will, of course, narrow the "original" variability of the process. It depends on the autocorrelation structure of the process and on the frequency of testing and readjustment of the process. Due to the multivariate nature of control, the partly attributive type of judgement, as well as the "noisy" relationship between measurements and potential strength, the mathematical handling of the effects of control becomes extremely difficult and will be dealt with elsewhere.

However, if there is effective production control by means of strength-indicating tests (for simplicity a functional relationship between measurements and potential strength is now assumed) and if approval of the control procedures is a necessary condition of acceptance, the variability in the qualities offered should be limited to that which is unavoidable during production control. Let production control be carried out by the well-known control chart technique on independent means, where the mean $\bar{x}_{(r)}$ of r results is compared with a two-sided control limit. The control criterion is then

$$\text{Correct if: } \bar{x}_{(r)} < m_M - \kappa \cdot \frac{\sigma}{\sqrt{r}} \text{ and } \bar{x}_{(r)} > m_M + \kappa \cdot \frac{\sigma}{\sqrt{r}} \quad (18)$$

the operation characteristic of which is known to be

$$L(m,\kappa,r) = \phi\left(\frac{m_M - m}{\sigma}\sqrt{r} + \kappa\right) - \phi\left(\frac{m_M - m}{\sigma}\sqrt{r} - \kappa\right) \quad (19)$$

κ may be called the control factor. $L(m,\kappa,r)$ can be interpreted as follows. If there is a population with mean m, the control procedure would detect the deviation of m from $m_M$ with probability $1 - L(m,\kappa,r)$. In other words, a process being in state m would be adjusted to the target mean $m_M$ with this probability. The probability of no action is the operation characteristic itself. The family of processes out of o(m) where no action was necessary, forms that part of the distribution of offered qualities which cannot be identified to depart from the target quality. The remaining part undergoes some readjustment. If the readjustment is done with a precision expressed exactly by the distribution of the first part,

$$o'(m) = N \cdot L(m,\kappa,r) \cdot o(m) \qquad (20)$$

is obtained which represents the corrected (filtered) distribution of offered qualities. N is a normalizing constant which is computed from

$$N = \frac{1}{\int_m L(m,\kappa,r) \cdot o'(m)\, dm} \qquad (21)$$

Its physical meaning is that no fresh concrete will be thrown away and so would go out of the process. No simple solution to find N exists. However, assuming normality for o(m), the mean of o'(m) should still be $m_M$ since $L(m,\kappa,r)$ is symmetrical with respect to $m_M$. Its standard deviation as obtained by evaluating

$$\sigma'_M = \{ \int (m-m_M)^2 \cdot o'(m)dm\}^{1/2} \qquad (22)$$

is shown in Fig. 10 whereby κ = 1.96 which corresponds to



a 5% probability of making type-II-errors in production control. Neglecting the side-effect of production control on the distribution of strength and therefore on $F_f(m)$ in (5), new sets of limiting values $\lambda_1$, $\lambda_2$, $\lambda_3$ and n can be found for any combination of κ and r. The first term in (10) should be amended by d · r and o(m) in (5) must be replaced by (22). d would represent the unit costs of production control tests.

## 5. SOME NUMERICAL RESULTS

Consider a refernce design situation in which the point probability of failure is given by

$$F_f(m) = \frac{1}{2}\,\{1 - \mathrm{erf}\,(\beta(m))\} \qquad (23)$$

where

$$\beta(m) = \frac{\ln(m/\check{s})}{\{(\sigma'_M/m)^2 + V_S^2\}^{1/2}}$$

For a coefficient of variation $V_S = 0.2$ which might represent the average maximum disturbances of the effects of actions upon the family of structures under consideration, choose $\check{s}$ and $f_c$ such that for some average m, β(m) would be in the order of 4 to 6. This magnitude of the safety index β may be attributed to present practice of design and is assumed to be optimal.

In particular, if $f_c = 25N/mm^2$, a calibration procedure gives OPT $P_f \approx 10^{-8}$ (OPT β ≈ 5.7) and $\check{s} \approx 8N/mm^2$ for $\sigma = \sigma_M = 5N/mm^2$. The necessary calculation to find $\lambda_1$, $\lambda_2$ and $\lambda_3$ can easily be carried out numerically.

Returning to Fig. 9, it can be said that there is only a small domain of possible combination of the mix design and the acceptance factor. Mix design factors less than 2 are unsafe and uneconomical which is in good agreement with present experience. The optimum acceptance factor is in the immediate neighbourhood of the minimum mix design factor.

The optimum sample size used for decision has been determined to be near n = 5 - 7. Fig. 11 demonstrates, that $\lambda_{min}$, $\lambda_{max}$ and $\lambda_{opt}$ are significantly changed if the sample size changes. In spite of this, the optimum possible $\lambda$ is observed to be relatively insensitive with sample size. In good approximation it can be set to be $\lambda \approx 1.3$ throughout (line A). The lower half of Fig. 11, where also the corresponding profit is given, indicates clearly, that $\lambda = 1.3$ should be used only in combination with sample sizes around n = 6, (see maximum on line B). It can be shown, that the mix design factor equals approximately 1.2 + $\lambda$, if n = 6, but also different constants $\alpha$, B, C, a, b, c etc.

As expected, these results will change slightly if the effect of production control, that is diminishing of $\sigma_M$ with the number r of control tests, is included. Using the rough estimate as given in Fig. 10, consideration of production control decisions further diminishes the optimal acceptance factor ($\lambda \approx 1.1$) and increases the profit through reducing the mix design factor. The best sample size would lie near r = 4 which increases the nominal profit by approximately 20% as compared with the situation r = 0. Keeping the profit as a constant, it is easily shown, that application of production control equivalent to r = 4 would allow to reduce the margin ($m_{opt} - f_c$) between acceptance limit $f_c$ and target mix $m_{opt}$ by approximately 0.2 · $\sigma$.

Summarizing, given definite values for $\sigma_M$, B, C, a, b, c, d and $\alpha$, a representative design situation including information about the probabilistic behavior of the action-effects and an operative value assigned to OPT $P_f$, which is best evaluated through calibration of the probabilistic model with present practice, optimal parameters of the entire control regime can be derived. Concerning sample sizes, it seems to be more reasonable to choose higher values since the economic optima are rather smooth. Numerical



solutions are still possible but much more time-consuming if the standard deviation $\sigma$ would be introduced as an additional Bayesian random variable.

The appropriate design situation might vary according to the type of structure (e. g. structures which carry loads mainly by normal forces react more sensitively to variations in concrete quality and thus need more rigid rules for specification and control than concrete structures which fail primarily due to insufficient strength or position of the reinforcement). So, it might, indeed, be useful fo classify the types of structures according to their sensitivity against concrete failures.

The constants B, C, a, b, c are characteristic for the particular technical and economic situation of concrete production and may vary among sites, regions and countries. They seem to be rather independent of the type of structure. As mentioned, the same statements are valid for the risk-willingness of the producers, expressed by $\sigma_M$.

A very difficult question is that of the factual cost of consequences of non-acceptance decisions. Those have been measured as a fraction $\alpha$ of the regular payment B. Actually, they depend significantly on the type of the structure, the structural importance of the part of the structure where concrete has failed to comply, the cost of in-situ testing and eventually removal and replacement of the poor concrete but also on possible delay of further construction works and, last but not least, on the psychological and professional attitudes of the persons involved. Hence, it seems to be almost impossible to specify beforehand a set of sanctions which would total exactly an envisaged $\alpha$. Fortunately, the results shown in Fig. 5 suggest, that the effectiveness of the system of control is not significantly weakened as long as there is a set of sanctions which would amount to an $\alpha$ - value of 0.5 - 1.0.

Taking into consideration, that the probability of rejection for an average concrete lot, having a target mean $m_t = m_{opt} = f_c + k_M \cdot \sigma$ and tested against the criterion (7) with $\lambda = 1.1$ and $n = 6$ is $P_R \approx 10^{-3}$ any negative compliance-decision indicates highly the presence of an insufficient quality. Even if $m_t = f_c + k_c \cdot \sigma$, where $k_c$ relates $m_t$ directly to the specified characteristic strength, the probability of rejection still would be $P_R \approx 0.1$ for $k_c = 1.65$. Under such circumstances, it seems to be reasonable, that the producer should be charged for any re-testing of the concrete. Thus, those sanctions may include at least re-testing of the <u>in-situ</u> concrete at the producer's expense, a financial penalty imposed on the producer, eventually depending on the true quality and, of course, the cost of removal and replacement of the concrete concerned which has been proved to be insufficient with respect to structural safety. Clearly, the producer usually will not accept any of these sanctions except the cost of re-testing if the negative compliance decision was due to statistical uncertainties rather than to true insufficiency of the concrete.

In principal, there are two ways how to judge finally on concrete quality.

a). It might be possible to identify all critical cross-sections and test their strength - destructively and/or non-destructively. The strength observed at a particular point might be set to be the "true" strength except for some small random disturbances caused by testing errors. Rechecking of structural safety has to be done in the regular way, but treating concrete strength as "quasi-deterministic" and with due consideration of the random and systematic differences between strength values obtained by standard and <u>in-situ</u> testing. If the usual reliability level of that part of the structure is ensured, the concrete has to be accepted. The producer must receive full payment but might have paid an considerable amount of money for re-testing and re-calculation.



However, a very undesirable situation can arise, if only one or several cross-sections do not fulfill the safety requirements. The producer might argue and remain fully within the concept that due to the unavoidable variabilities in production any other structure would have such weak points which do not require repair. In fact, the probabilistic approach to structural safety then becomes, at least in parts, questionable. In consistency with the probabilistic concept one or more "unsafe" sections should still be allowed as it is admitted for "accepted" structures. The very difficult statistical aspects of this problem will not be studied herein.

b). Such difficulties can be avoided, if random sampling is chosen. The actual characteristic strength can be estimated and be compared with the specification - a procedure which enables clear decisions. Again, conversion of strength values obtained for standard tests and <u>in-situ</u> tests is necessary. It seems to be reasonable to require that the confidence level of the estimate should correspond to the risk of non-detecting insufficient qualities multiplied by the probability of their occurence, which is simply the portion of "unsafe" structures not detectable by the normal compliance procedure. Therefore, we have

$$P(x_{5\%} \geq f_c) = 1 - \alpha_p \qquad (24)$$

where $$\alpha_p = \int_0^{f_c + k_c \cdot \sigma} L(m;\lambda,n,f_c) \cdot o(m;m=f_c+k_M \cdot \sigma,\sigma_M)\, dm$$

Using the criterion (5) with $\lambda = 1.1$, $n = 6$, $k_M = 1.2 + \lambda$ and $\sigma_M = 5 N/mm^2$, one obtains by numerical integration $\alpha_p \approx 0.032$. Still assuming an underlying normal distribution and known standard deviation, the final acceptance criterion yields

$$f_c \leq x_{5\%,1-\alpha_p} = \bar{x}(t) - (\frac{1.85}{\sqrt{t}} + 1.65) \cdot \sigma \qquad (25)$$

in which t is the number of samples taken.

## 7. SUMMARY AND CONCLUSIONS

A rational system of control including recommendations for the design of mixes and sanctions to be specified for the case of non-compliance decisions is constructed. It rests on the assumption that a limiting quality, defined as a fraction to be admitted to fall below the specified strength which is considered to be optimal with respect to overall economy and reliability of the structure can be defined.

The designer must decide upon the feasibility of statistical production control as part of the compliance procedure which highly influences his pre-evaluation of concrete variabilities. If the effectiveness of production control is part of the compliance procedure, the variability of concrete strength, measured in standard deviations, is significantly less (≈ 50%) than otherwise because major parameter uncertainties are taken away.

For both types of the controlling regime different margins between characteristic strength and target mean should be used. Mixes designed according to $m_t = f_c + 2.3(2.5) \cdot \sigma$ are optimal under the following conditions.

a). a compliance function of the type given in (7) with parameters $n = 6$ and $\lambda = 1.1 (r=4)$ and $\lambda = 1.3 (r=0)$ will be used. r denotes an equivalent sample size for production control.

b). a system of sanctions in combination with confidential estimates of the characteristic strength through in-situ testing as given by (25) is defined beforehand.



The numbers given in the above are of illustrative nature although the underlying assumptions on various input-parameters have been chosen to be as realistic as possible.

For design the mean strength of concrete is $m_t = f_c + 2.3(2.5) \cdot \sigma_x$. The standard deviation should be chosen to be $\sigma_x = \{\sigma_M'^2 + \sigma^2\}^{1/2}$ where $\sigma_M' = \sigma_M$ in the regular case and $\sigma_M' \approx 0.5 \cdot \sigma_M$, if functioning of production control is a part of the information needed for acceptance decisions.

Further research is needed in probabilistic modelling of concrete production in order to determine effective control procedures and for estimating in-situ strength. Also, the question of the type of the penalty function needs to be studied furtheron.

Acknowledgements

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| | | |
|---|---|---|
| $\alpha$ | = | penalty/payment |
| $1-\alpha_p$ | = | confidence probability |
| $\beta(.)$ | = | safety index |
| $\xi$ | = | random vector |
| $\rho_{M,\sigma}$ | = | correlation coefficient |
| $\sigma$ | = | population standard deviation |
| $\lambda$ | = | acceptance factor |
| $\kappa$ | = | control factor |

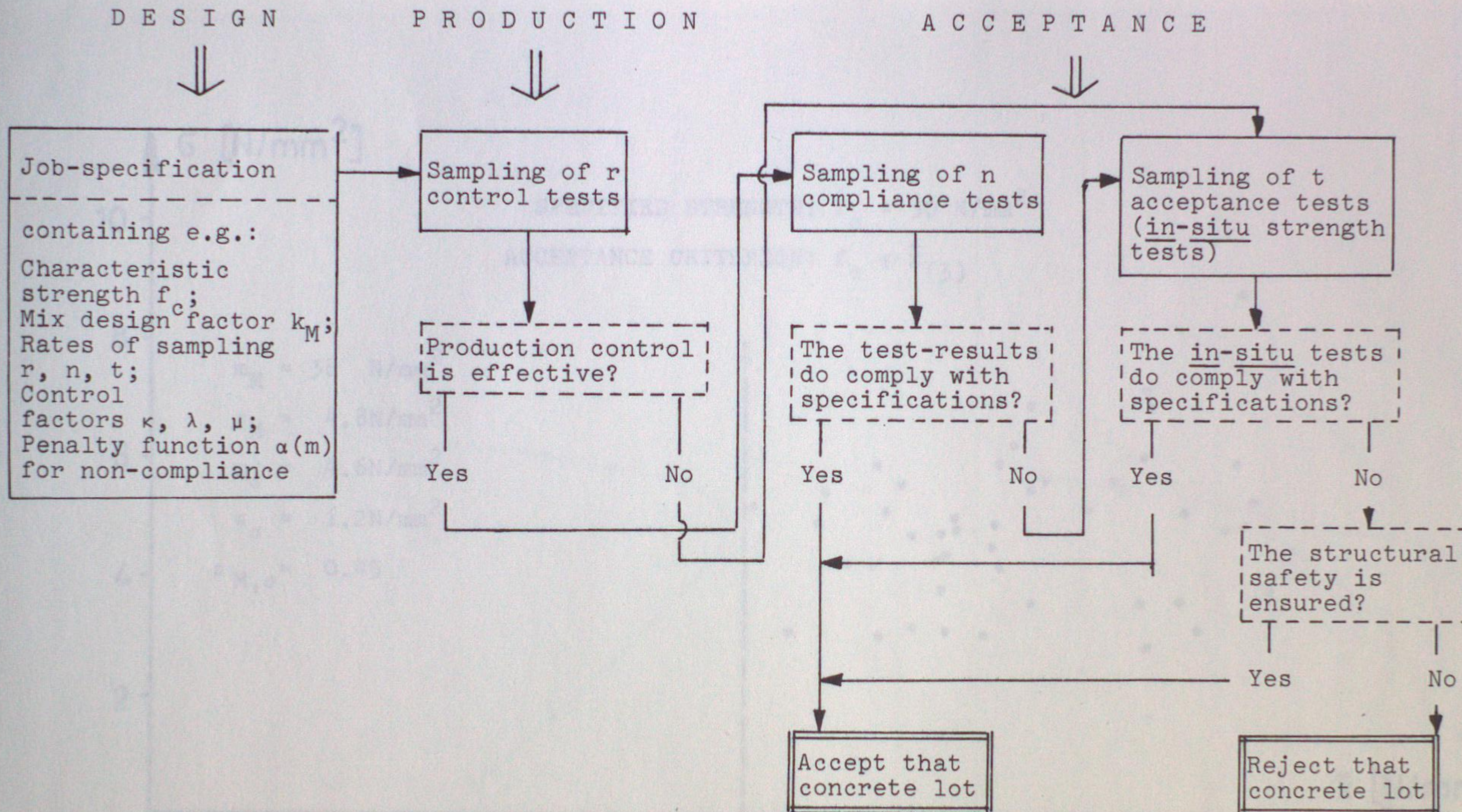


Fig.1: Simplified course of actions during production and acceptance of structural concrete following the job-specification according to optimal design.

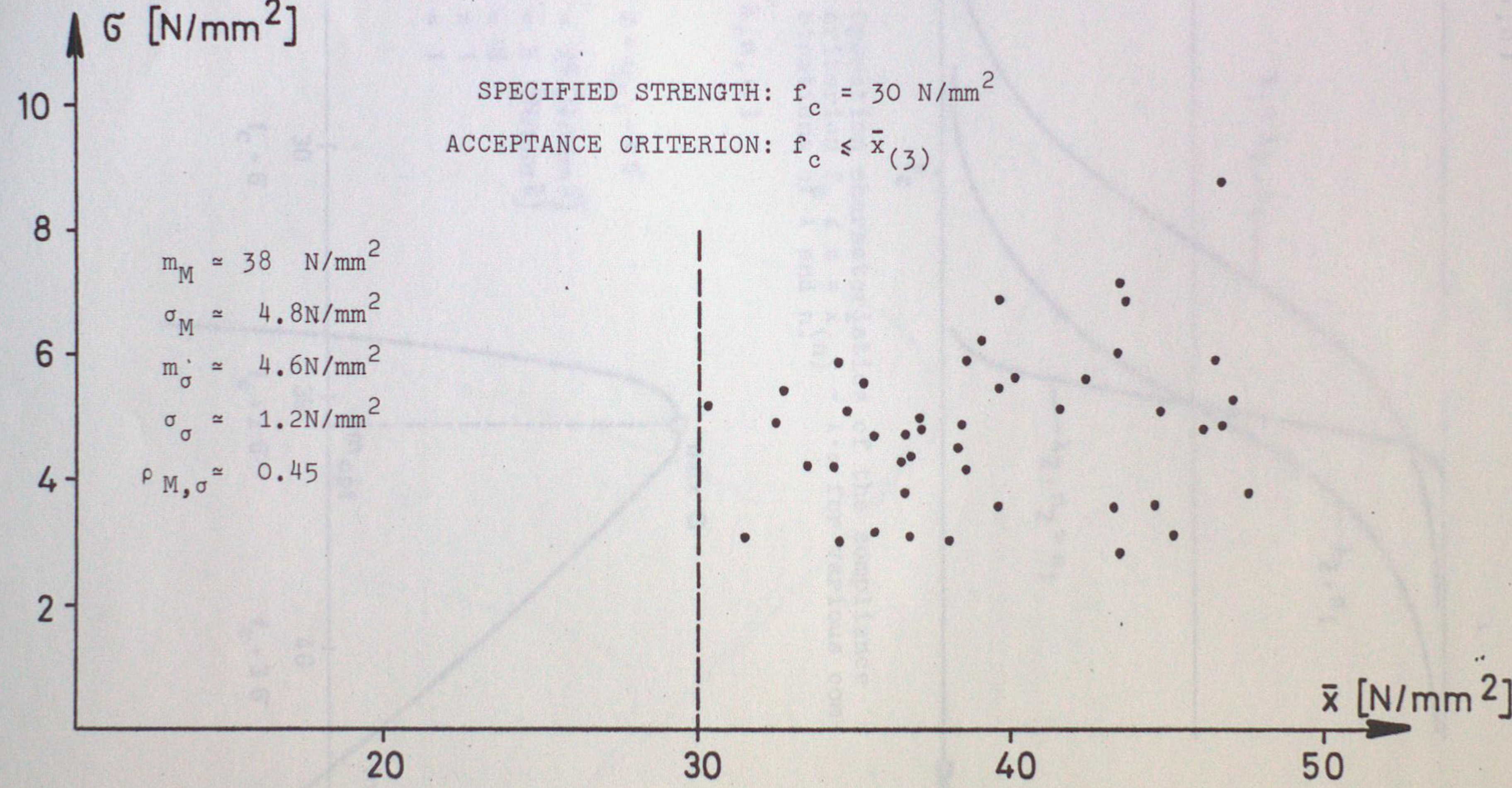


Figure 2: Observed values of mean m and standard deviation σ for a given concrete grade and given acceptance criterion

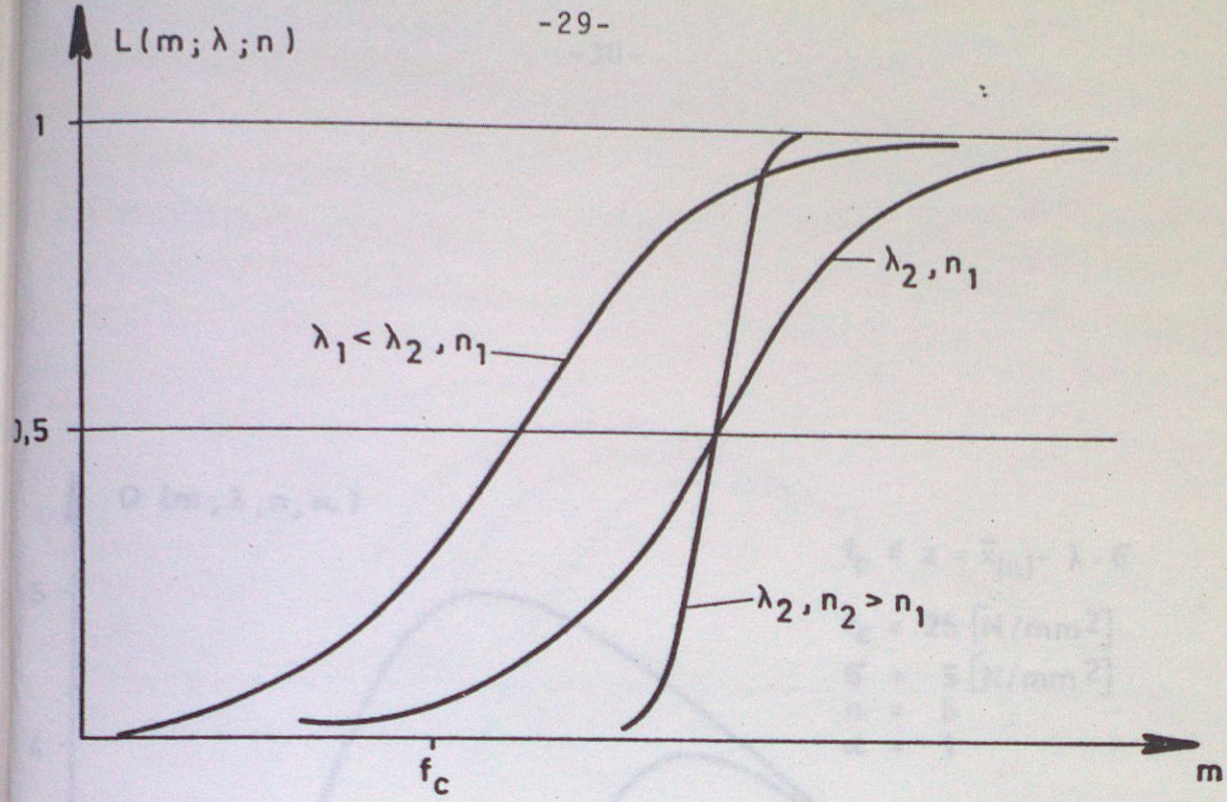


Fig. 3: Operation characteristics of the compliance criterion $f_c \leq z = \bar{x}_{(n)} - \lambda \cdot \sigma$ for various combinations of λ and n.

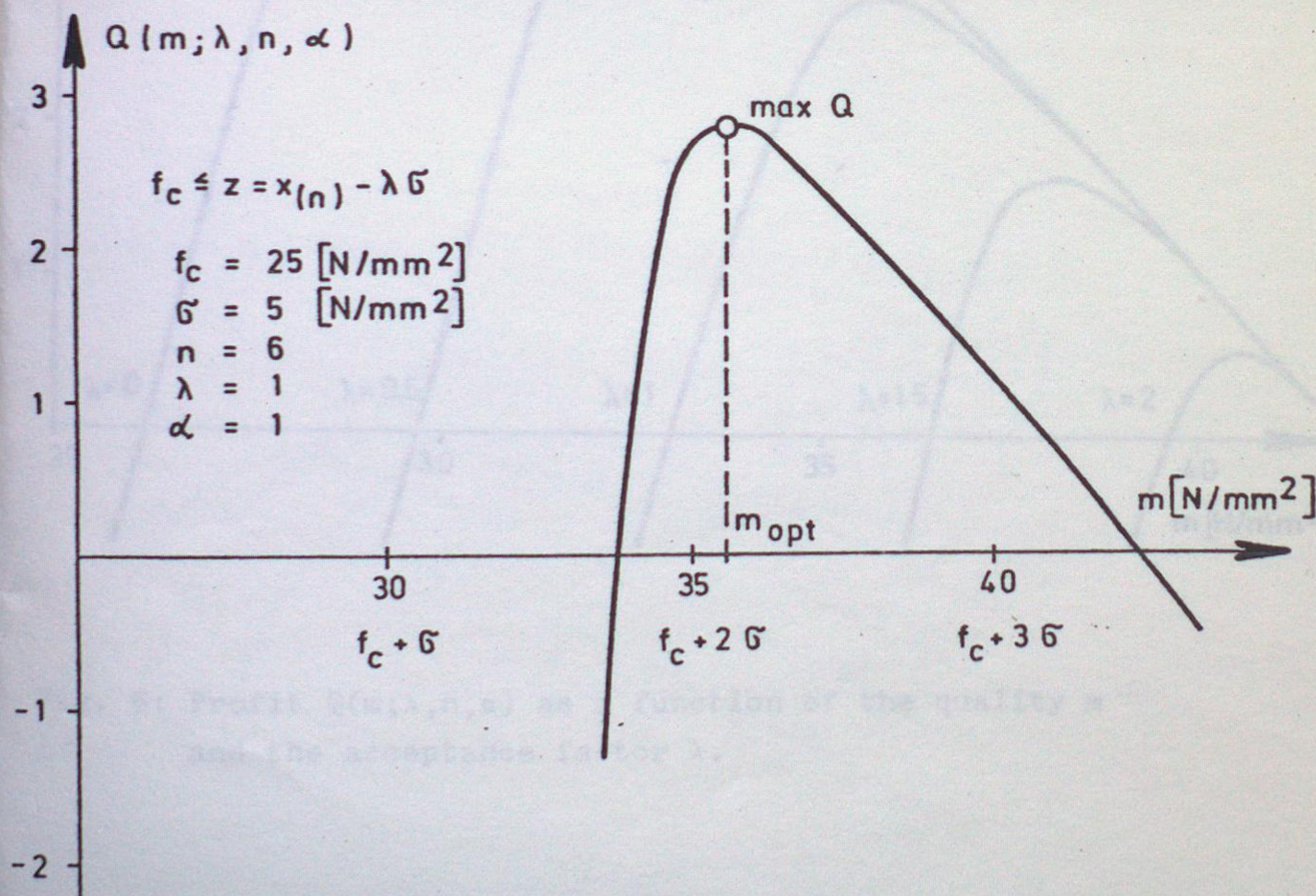


Fig. 4: General form of the profit function

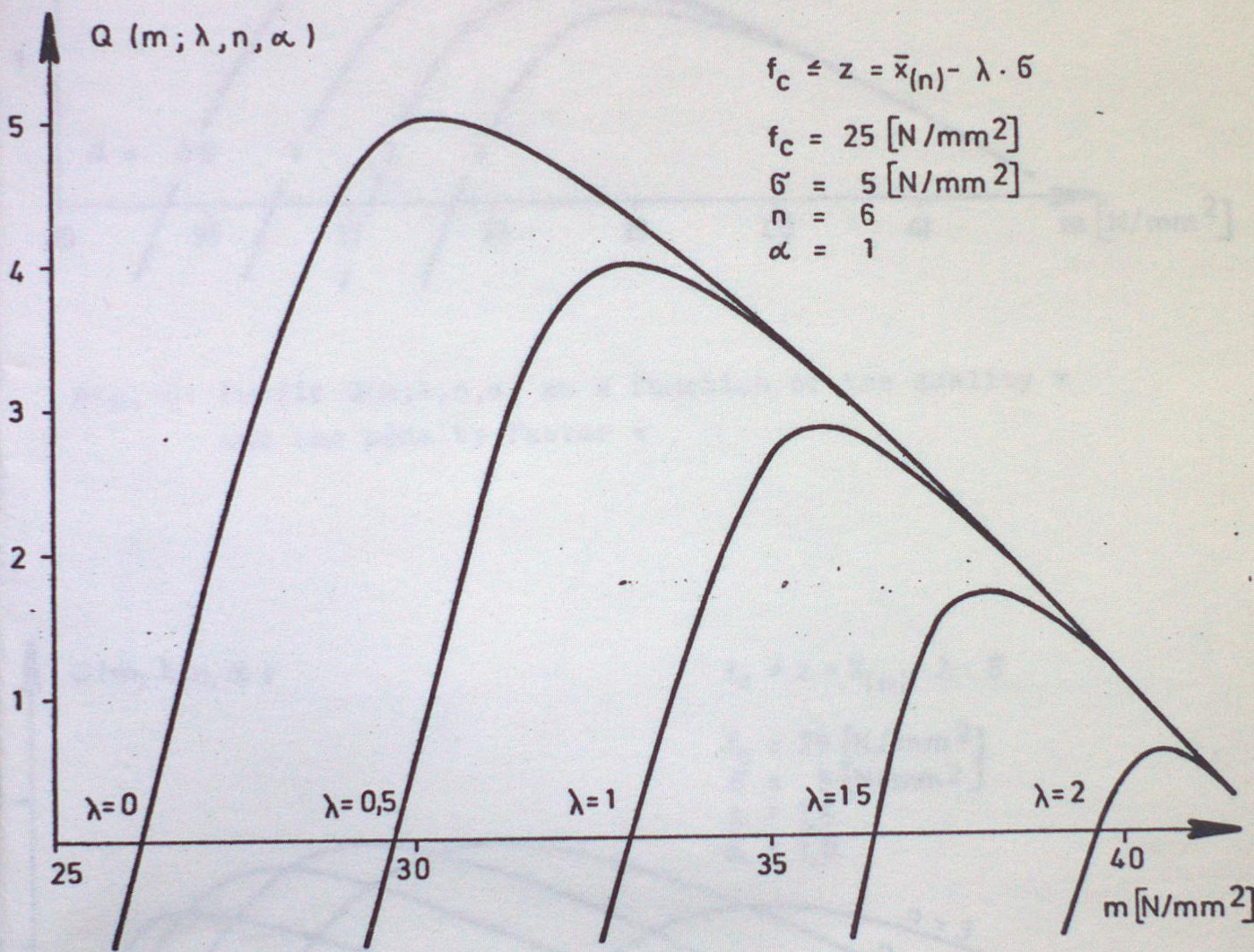


Fig. 5: Profit Q(m;λ,n,α) as a function of the quality m and the acceptance factor λ.

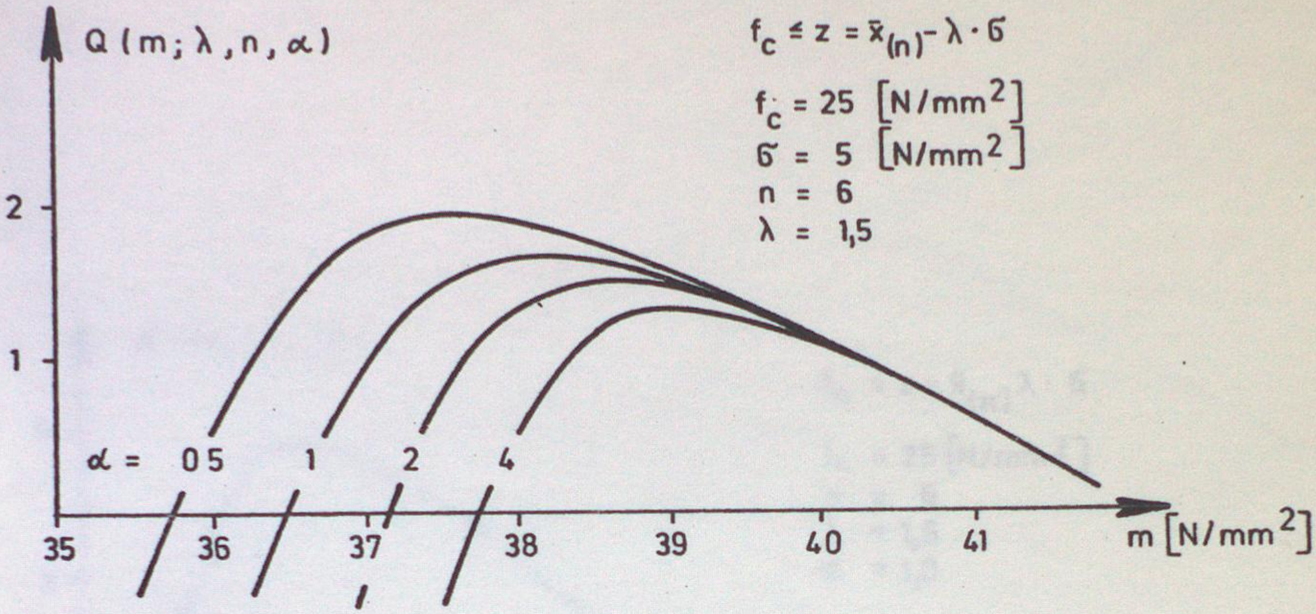


Fig. 6: Profit Q(m;λ,n,α) as a function of the quality m and the penalty factor α

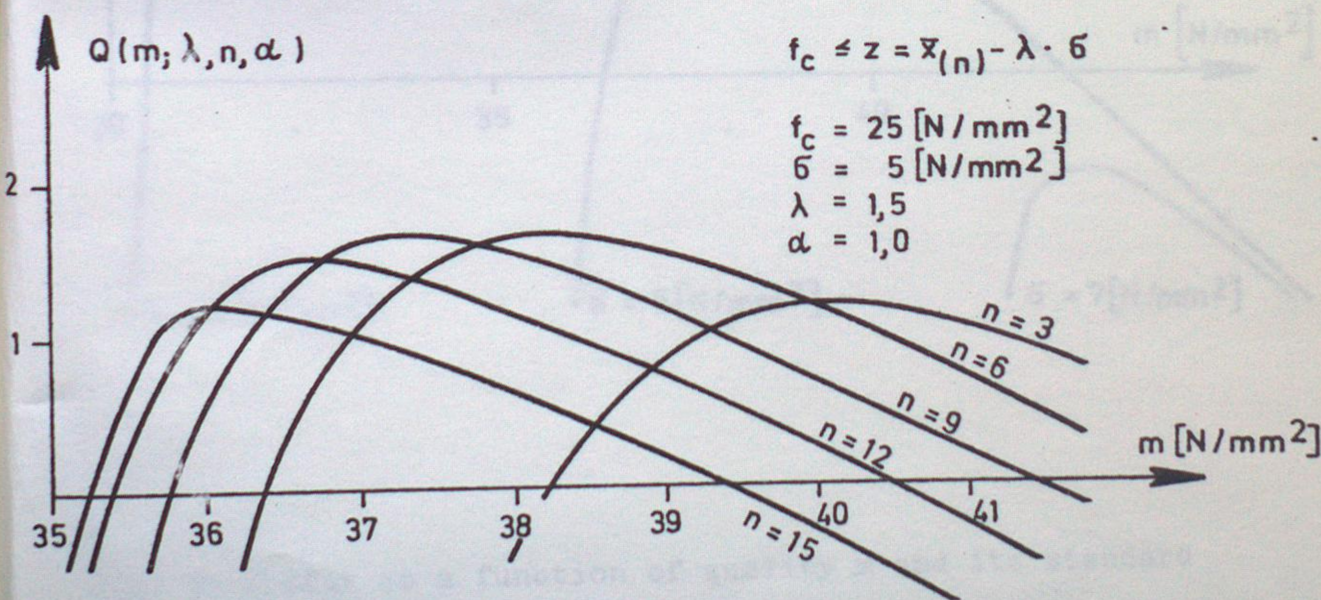


Fig. 7: Profit Q(m;λ,n,α) as a function of the quality m and the sample sizen.

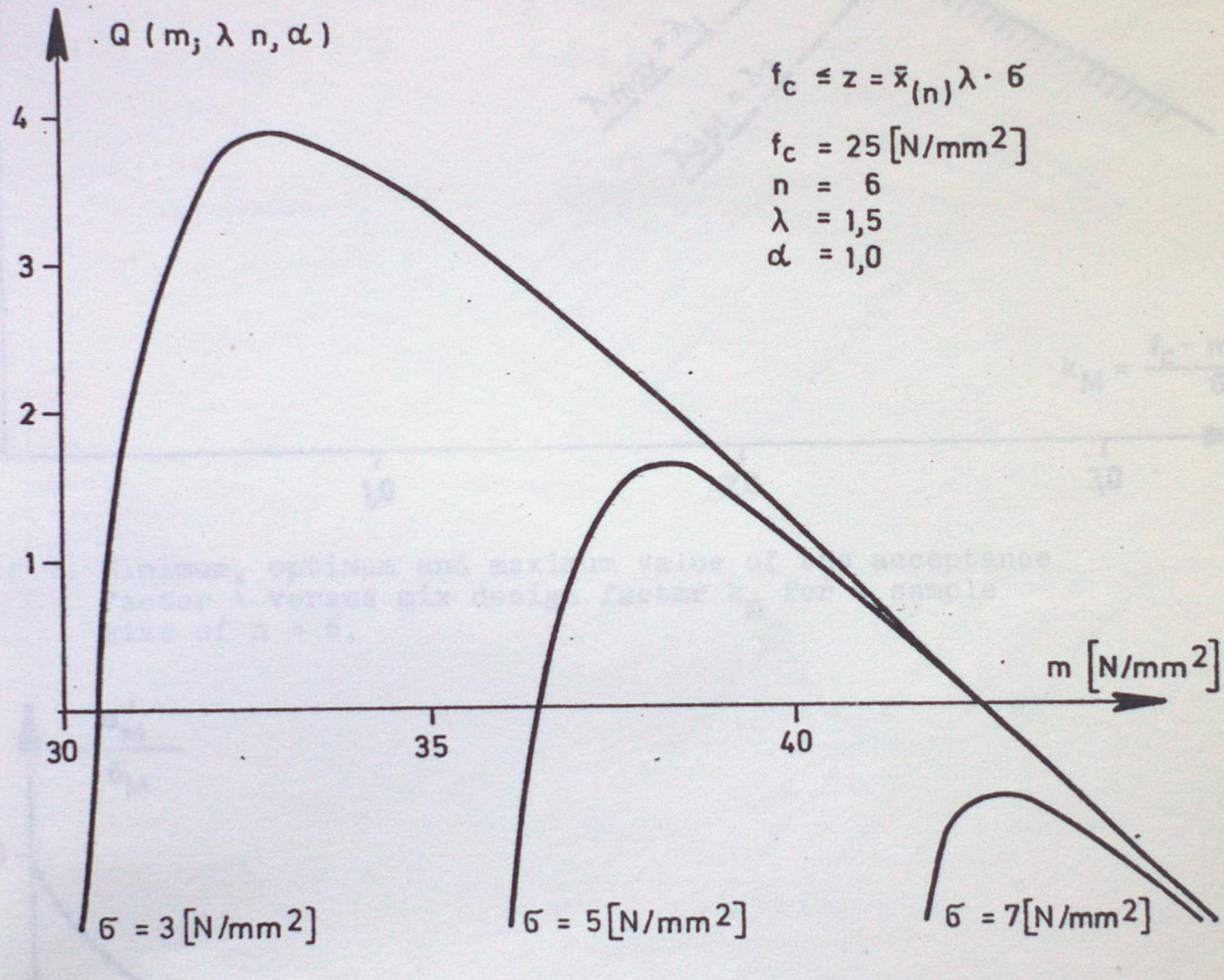


Fig. 8: Profit as a function of quality m and its standard deviation σ.

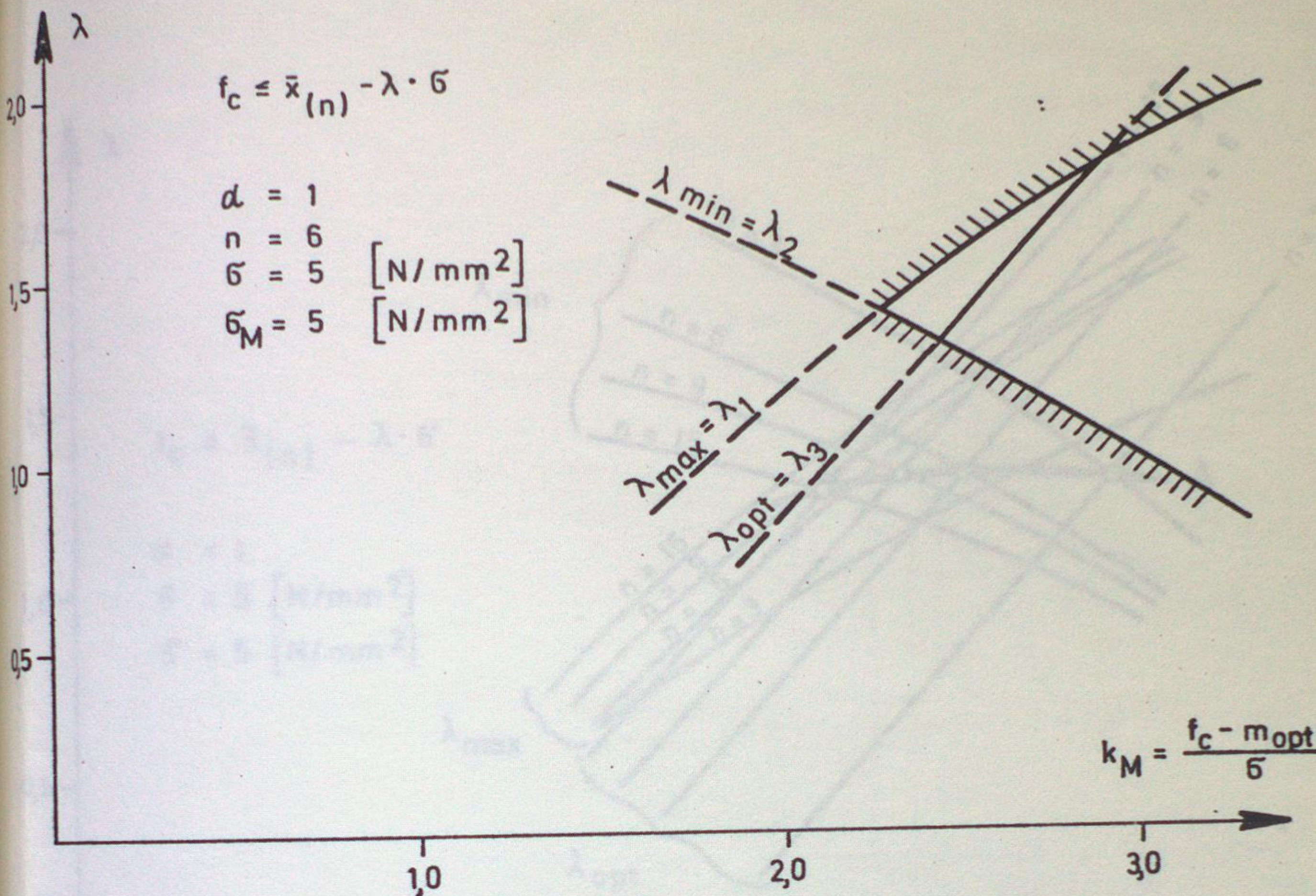


Fig.9: Minimum, optimum and maximum value of the acceptance factor λ versus mix design factor $k_M$ for a sample size of n = 6.

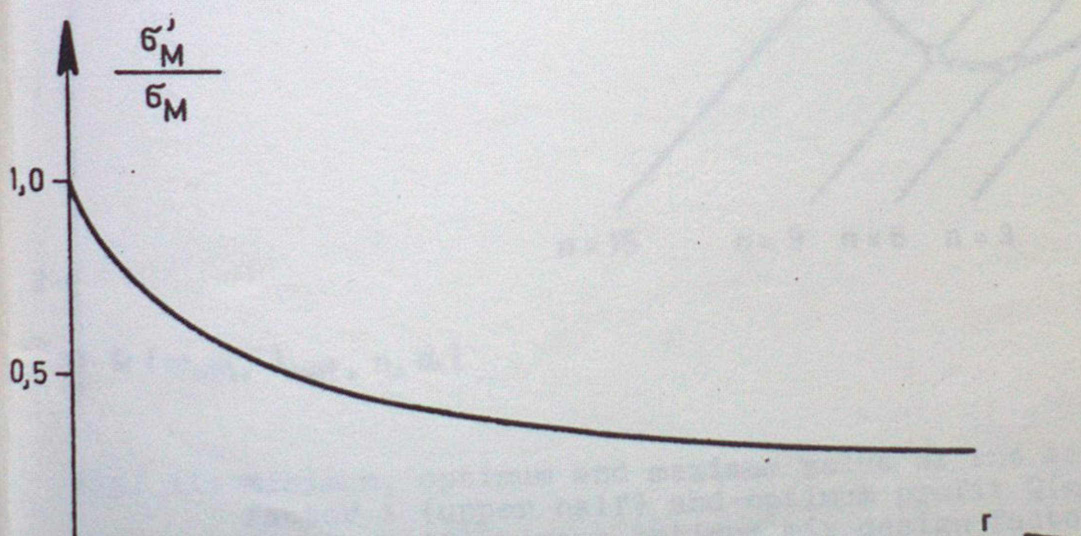


Fig.10: Approximate decrease of the uncertainty of the actual target mean with the number of samples for production control, κ = 1.96.

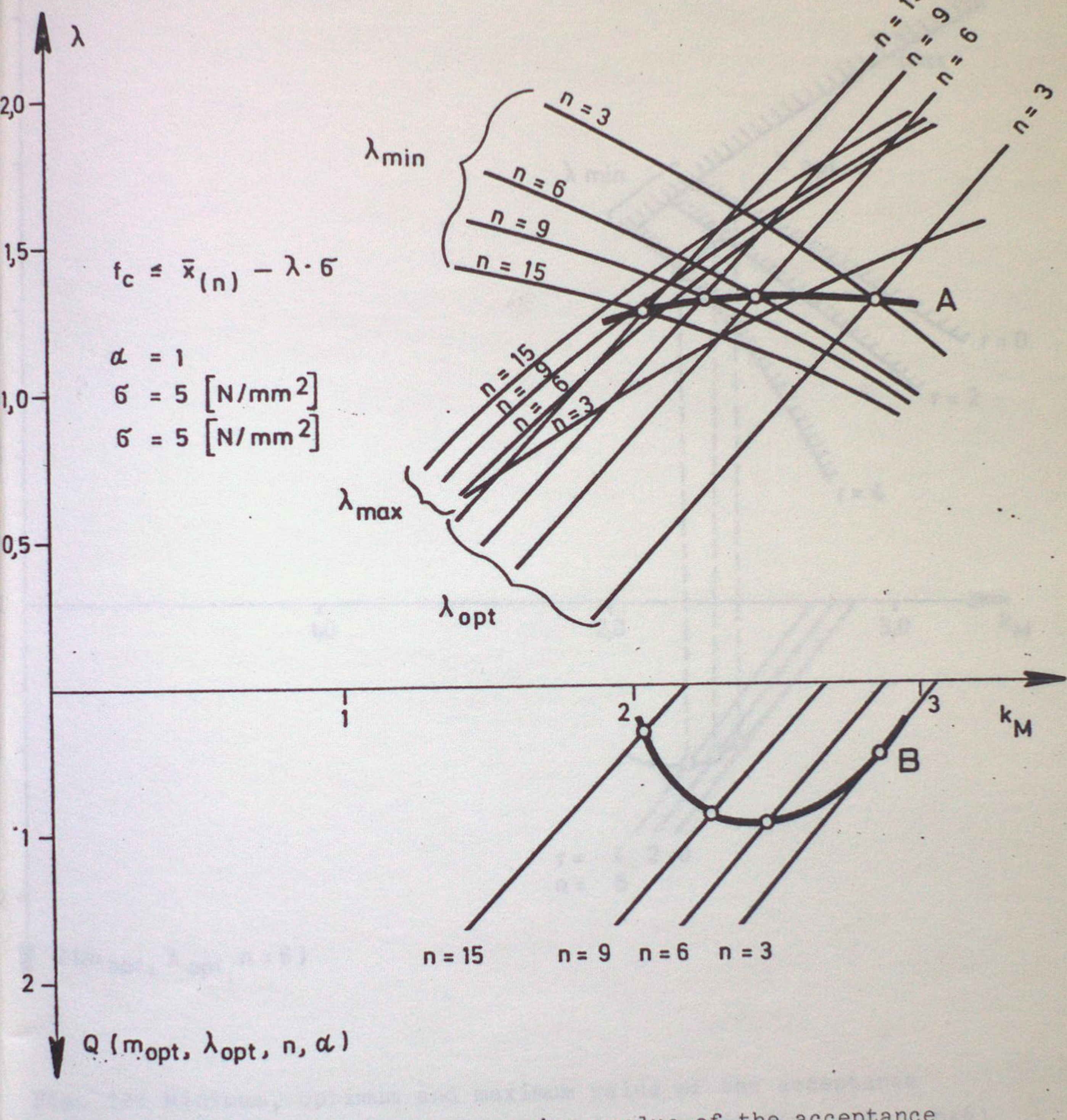


Fig. 11: Minimum, optimum and maximum value of the acceptance factor $\lambda$ (upper half) and optimum profit $Q(m_{opt}, \lambda_{opt}, n, \alpha)$ (lower half) versus optimum mix design factor $k_M$ for different sample sizes n = 3, 6, 9, 15.

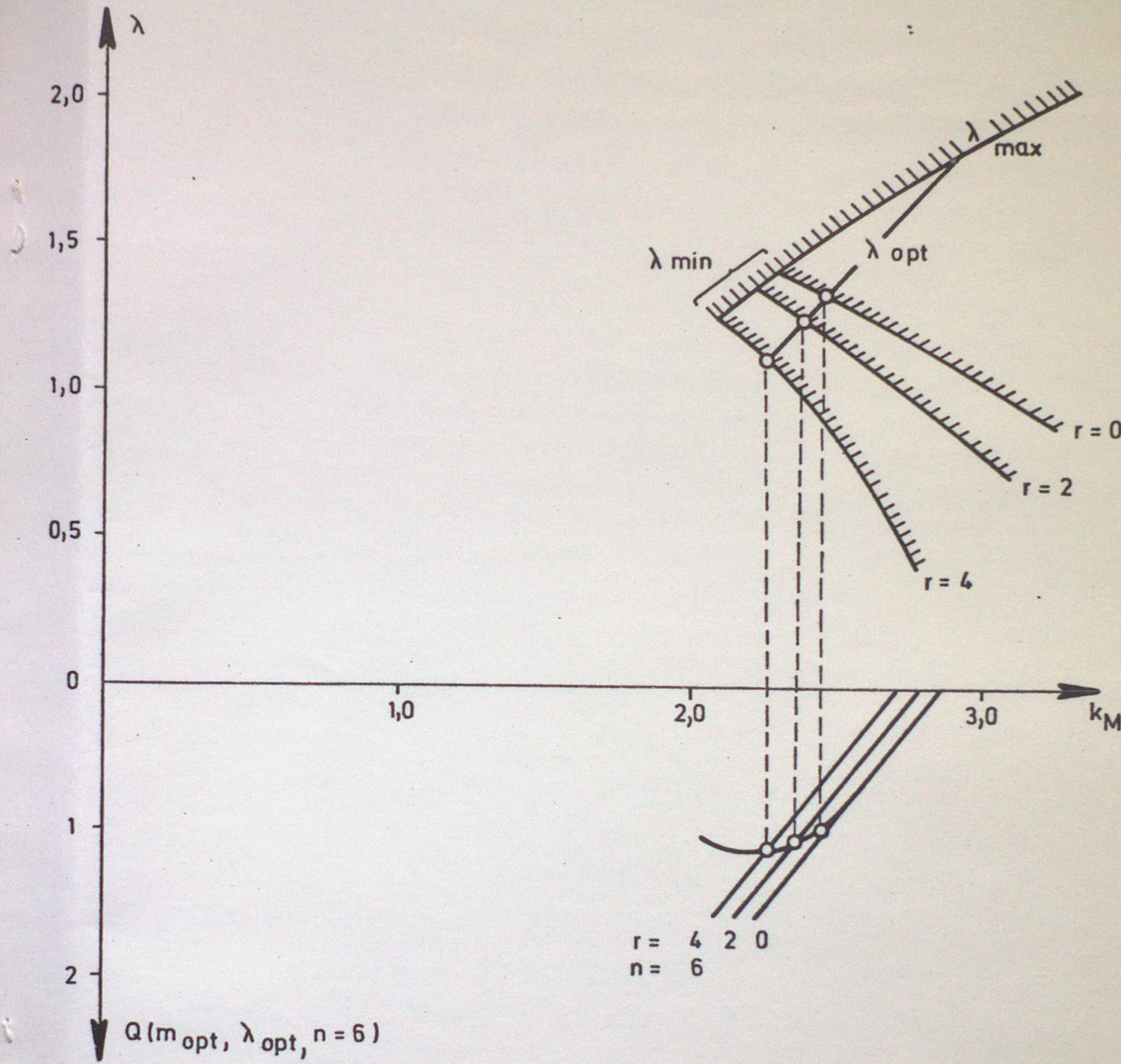


Fig. 12: Minimum, optimum and maximum value of the acceptance factor λ (upper half) and optimum profit $Q(m_{opt}, \lambda_{opt}, n=6)$ versus optimum mix design factor $k_M$ for different efforts in production control (r = 0, 2, 4).