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Estimation of Structural Properties by Testing for Use in Limit State Design

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with contributions by F. Bijlaard, H. Mathieu, G. Sedlacek, J. Stark

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Estimation of Structural Properties by Testing for Use in Limit State Design

General principles for planning, sampling and evaluation for the elementary case

by

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Introduction

This is one of the documents of a series of publications, prepared by individual authors but discussed within the Joint Committee on Structural Safety (JCSS), in particular within its Working Party. The series up to now consists of the following titles:

Proposal for a Code for the Direct Use of Reliability Methods in Structural Design
O. Ditlevsen, H.O. Madsen

Estimation of Structural Properties by Testing for Use in Limit State Design
M. Kersken-Bradley, W. Maier, R. Rackwitz, A. Vrouwenvelder

Structural Performance Criteria
L. Östlund

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G.I. Schuëller

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R. Giannini, P.E. Pinto, R. Rackwitz

Geometrical Variability in Structural Members and Systems
F. Casciati, I. Negri, R. Rackwitz

Bayesian Decision Analysis as a Tool for Structural Engineering Decisions
O. Ditlevsen

The papers are referred to as "Working Documents" since they generally give information on the state of development of certain concepts or subjects, rather than giving approved guidelines.

This paper specifically is concerned with the methodology of testing and the use of test results, where tests are performed to give data on structural properties for limit state design. The objective is to obtain data that are comparable to data derived by acknowledged calculation models in terms of associated uncertainties. The paper focusses on a very model for explaining the basic features; it is intended as a basis for more operational documents, which would include more details on testing procedures relevant for specific testing tasks.

This series of publications is intended to initiate discussions and exchange of comments. Comments may be sent to the Headquarter of IABSE, which will take care of sending these to the respective bodies of the JCSS.

Future papers of the JCSS will appear in appropriate international Engineering Journals. This series published by IABSE is closed.

The above papers are issued in honour of Professor Julio Ferry Borges, former President of the JCSS, expressing our deep appreciation and sincere thanks for successfully guiding the Joint Committee for more than 18 years.

The General Reporter of the JCSS:
M. Kersken-Bradley (until November 1990)
A. Vrouwenvelder (at present)

The President of the JCSS:
J. Schneider

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ESTIMATION OF STRUCTURAL PROPERTIES BY TESTING FOR USE IN LIMIT STATE DESIGN

General principles for planning, sampling and evaluation for the elementary case

This document is intended to serve as a basis for deriving practical guidelines for the planning and evaluation of tests and is motivated by the following considerations:

- Modern design codes have progressed to an extent that, at least conceptually, they follow a probabilistic basis. As concerns principles and rules for verifications by calculation, a consistent format in terms of design values - determined from characteristic values and partial safety factors - is used.

- However, where codes do not give sufficient information on structural properties and tests need to be performed, no rules are available at present to ensure that structural properties determined by testing are associated with the same level of reliability as when determined according to codified calculation models.

- Compatibility between tested and calculated structural properties does not only refer to statistical aspects in the evaluation of test results, but also to the planning of tests including sampling of specimens and documentation of tests, such that tests can be checked comparable to calculations.

- In administrative terms the situation referred to herein, is often covered by agreement or approval procedures. In this sense acknowledged rules for determining structural properties in testing may serve as a technical basis in the context of agreement or approval procedures.

In 1985 a CEB/ECCS Cooperation Group prepared two notes (Nos. 10 and 11) on the topic. Application of the rules suggested in these notes in the following years provided evidence for their consistency, but also showed the need for some developments and improvements.

This document is an attempt to improve the aforementioned notes. Conceptual extensions are based on preliminary drafts and discussions in the JCSS Working Party. Experience in application within the ECCS and on national level (cf. for example, document by the Institut für Bautechnik, Berlin, as a basis for agreement procedures) are tentatively considered.

In order to illustrate the concept, only the most simple testing task -"elementary case" - is treated herein. The extention to more complex tasks is straightforward in conceptional terms, but not necessarily with regard to operational rules, which are still under discussion.

It is emphasized that this document focuses on reliability aspects only.

1. SCOPE AND OBJECTIVE

1.1 General scope

This document gives rules for the experimental assessment of the resistance of a structural member for use in limit state design, in the case that:
- a physically based or empirical calculation model is available;
- only one global model uncertainty coefficient D is involved;

This case is referred to as the "elementary case".

Design parameters may be fixed or may be varied. For the case where design parameters are varied, only some elementary solutions are presented.

The aim of the experiments can be described as to reduce the uncertainty with respect to the coefficient D. The presented concepts in this document should lead to a design having associated the same level of reliability as a design which is exclusively based on calculations.

1.2 Field of application

The rules in this part refer to the resistance of a structural member, which can be any part of a load bearing structure. Application to other than load bearing properties is in general possible but not dealt with in detail.

The rules are intended for situations where experimental evidence is sought for a restricted population of structural members as, for example, in the context of technical approvals (agrément). Assessment of a unique structure or structural member by laboratory testing or proof loading on site is not in the scope of this document.

In principle, the rules in this document may also be applied for the derivation or justification of code formula. However, additional considerations are required to cover problems related to the justification of simplified code formula and the interpretation, validity and acceptance of unsatisfying tests and test documentation.

1.3 Probabilistic aspect

This document does not cover all features relevant for an experimental assessment. Main emphasis is on the probabilistic aspects as planning of tests, sampling procedures and evaluation of test results.

Comment:
Supplementary rules are generally required to account for specific aspects of different types of material, structural members and investigated properties, including rules for the manufacturing of specimens and execution of the tests. This also holds for aspects of model analysis, including e.g. size effects, duration effects and conversion factors.

1.4 Objective

The objective of this document is to describe a procedure which leads to the establishment of a probability distribution (type, mean, standard deviation) for the prior unknown coefficient D, given a number of experimental results. Based on the probability distribution of D, one or more of the subsequent quantities will be derived for application in partial safety factor design methods (level I):
- a design value for the unknown coefficient D
- a characteristic value for the unknown coefficient D
- a design value for the resistance of the structural member
- a characteristic value for the resistance of the structural member
- a partial safety factor for the resistance of the structural member

Comment:
Design values may be obtained directly from the test results or calculated using characteristic values from test results and safety factors given by the code. In both cases, the application in design shall render the same level of reliability as intended for calculation models given in the relevant design code.

2. DEFINITIONS

2.1 Structural member

It is presumed that a structural member is comprehensively specified by a set of specifications.

Comment:
Examples of specifications are: a grade of material, a nominal dimension, and so on.

2.2 Populations and samples

The entity of members produced according to a unique set of specifications is referred to as an elementary population. A population in which specification parameters vary, is referred to as a composite population. The set of test specimen associated to a certain population is referred to as elementary sample or composite sample respectively.

A sampling procedure may be:
- representative
- artificial

with respect to basic variables and (for composite populations) to varying specifications.

Representative refers to an elementary population for which the experimental evidence is intended. For obtaining representative samples or representative realization of basic variables, random sampling is always a correct procedure.

Artificial means that no direct relation exists between the distribution in the sample and in the population.

2.3 The resistance of a structural member

It is presumed that the resistance of a structural member is comprehensively determined by a set of measurable quantities. Quantities, which are deterministic with respect to an elementary population are referred to as specifications $\underline{w}$. Quantities which are random with respect to an elementary population are referred to as basic variables $\underline{X}$.

Comments:
The calculation model usually only contains a subset of all specifications and basic variables. Specifications may be constant within a population (elementary population) or vary in a predictable manner (composite population). Note that for instance the means of some basic variables may have to be considered as (implicit) specifications.

2.4 Load specification

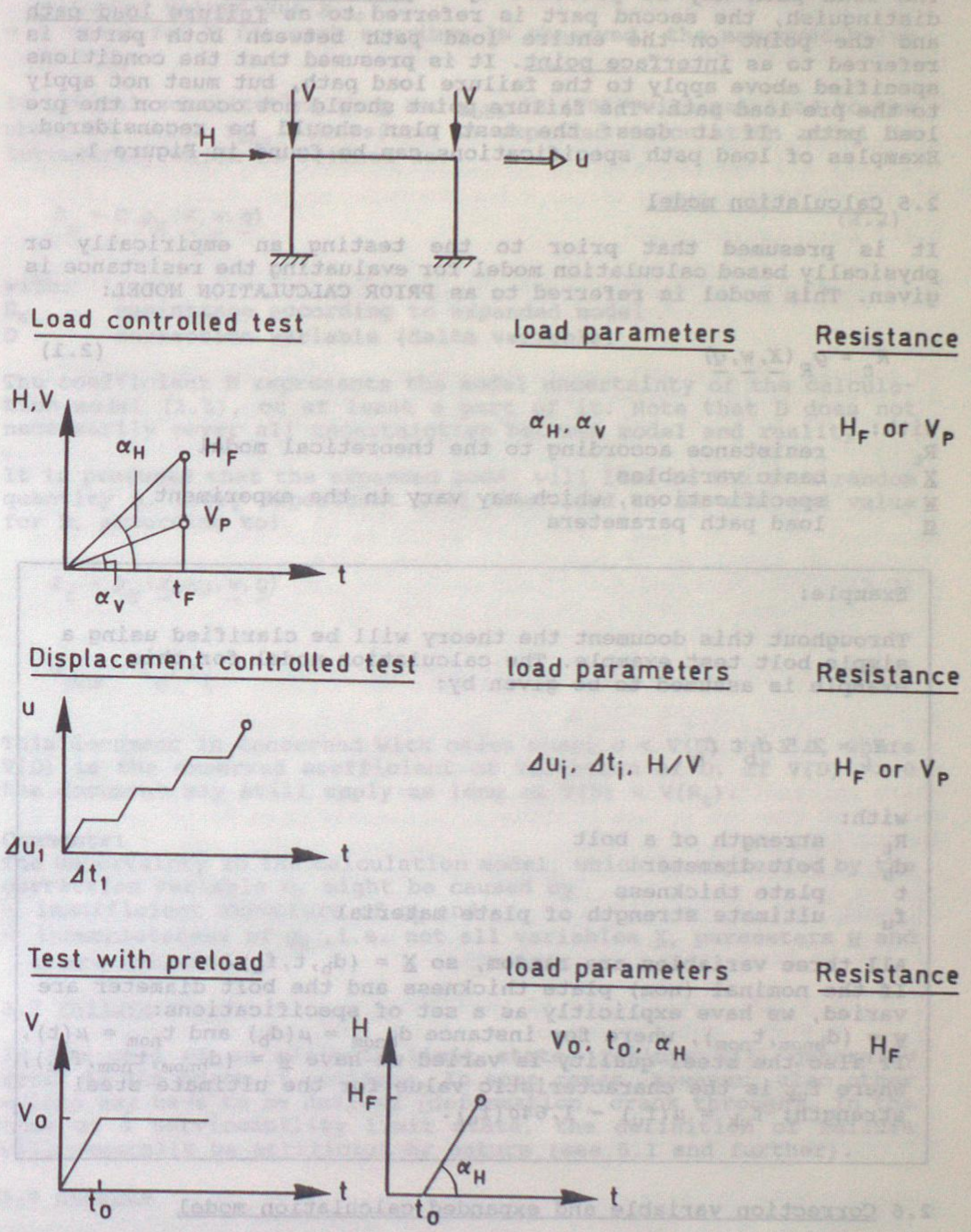
It is presumed, that the load to be applied to a test specimen during the test is comprehensively defined by a set of load path parameters $\underline{q}$, describing shape, location, intensity of the loads and their development in time, if necessary.

Load parameters determine the planned load path (forces and deformations as function of space and time). It is presumed in this document that the loads develop proportional to each other (that is proportional to a load factor) or that they equal zero.

The case, where all specimen are subjected to the same load path (same value for all load path parameters) is referred to as case without variation of load path parameter, otherwise with variation of load path parameter.

If internal forces are used instead of loads to describe the limit state, they can be treated like loads, as long as they are proportional to imposed loads (due to equilibrium) or measured.

Fig. 1: Load path examples for a simple frame structure



Comment:
The load path may be preceded by a pre load path. In order to distinguish, the second part is referred to as failure load path and the point on the entire load path between both parts is referred to as interface point. It is presumed that the conditions specified above apply to the failure load path, but must not apply to the pre load path. The failure point should not occur on the pre load path. If it does: the test plan should be reconsidered. Examples of load path specifications can be found in Figure 1.

2.5 Calculation model

It is presumed that prior to the testing an empirically or physically based calculation model for evaluating the resistance is given. This model is referred to as PRIOR CALCULATION MODEL:

$$R_t = g_R(\underline{X}, \underline{w}, \underline{q}) \tag{2.1}$$

with:
R_t resistance according to the theoretical model
$\underline{X}$ basic variables
$\underline{w}$ specifications, which may vary in the experiment
$\underline{q}$ load path parameters

Example:
Throughout this document the theory will be clarified using a simple bolt test example. The calculation model for this example is assumed to be given by:

$$R_t = 2.5\, d_b\, t\, f_u$$

with:
R_t strength of a bolt
d_b bolt diameter
t plate thickness
f_u ultimate strength of plate material

All three variables are random, so $\underline{X} = (d_b, t, f_u)$.
If the nominal (nom) plate thickness and the bolt diameter are varied, we have explicitly as a set of specifications:
$\underline{w} = (d_{bnom}, t_{nom})$, where for instance $d_{bnom} = \mu(d_b)$ and $t_{nom} = \mu(t)$.
If also the steel quality is varied we have $\underline{w} = (d_{bnom}, t_{nom}, f_{uk})$, where f_{uk} is the characteristic value for the ultimate steel strength: $f_{uk} = \mu(f_u) - 1.64\sigma(f_u)$.

2.6 Correction variable and expanded calculation model

A standard experiment within the context of this document is carried out as follows:
- fixed values are chosen for specifications $\underline{w}$ and load path parameters $\underline{q}$;
- all random variables $\underline{X}$ for each specimen are measured; the measured values are $\underline{x}_{obs}$;
- a value for R for each specimen is observed; the measured value is r_{obs}.

In general substituting $\underline{w}$, $\underline{q}$ and $\underline{x}_{obs}$ in (201) will not lead to the observed value r_{obs}. Therefore an expanded calculation model is introduced, which is defined as:

$$R_e = D\, g_R(\underline{X}, \underline{w}, \underline{q}) \tag{2.2}$$

with:
R_e resistance according to expanded model
D correction variable (delta variable)

The coefficient D represents the model uncertainty of the calculation model (2.1), or at least a part of it. Note that D does not necessarily cover all uncertainties between model and reality.

It is presumed that the expanded model will lead to the true random quantity R. Every experiment will then lead to an observed value for D, according to:

$$r_t = g_R(\underline{x}_{obs}, \underline{w}, \underline{q}) \tag{2.3}$$

$$d_{obs} = r_e / r_t \tag{2.4}$$

This document is concerned with cases where $0 < V(D) << 1.0$, where V(D) is the observed coefficient of variation of D. If $V(D) >> 0$ the document may still apply as long as $V(D) < V(R_t)$.

Comments:
The uncertainty in the calculation model, which is expressed by the correction variable D, might be caused by
- insufficient structure of g_R and
- incompleteness of g_R, i.e. not all variables $\underline{X}$, parameters $\underline{w}$ and $\underline{q}$ are included.

2.7 Failure

In the case of an ultimate limit state, failure will generally refer to the event of collapse. In some cases, however, also other events may have to be defined (deformation, crack through). In the case of a serviceability limit state, the definition of failure will generally be artificial by nature (see 5.1 and further).

2.8 Subsets

If design parameters are varied during the test, the complete set of experiments can be subdivided into a number of subsets. Within a subset the variation of the design parameter (or at least the effect of it) should be negligible (see 5.1 and further).

3. PRIOR INFORMATION, SAMPLING, LOAD PATH SELECTION AND MEASUREMENTS

3.1 Prior information

3.1.1 Principal of prior information

It is assumed that planning of tests and evaluation of test results utilizes all available prior information regarding the properties of concern.

3.1.2 Calculation model

A calculation model is assumed to be known for the failure mode that dominates low strength failures.

Comment:
In general a structural member may posses a number of fundamentally different failure modes, e.g. a girder may fail by bending at midspan or shear at the supports. It is possible that the average strength region is governed by other modes than the low strength region. As the low strength region (say mean value minus two to three standard deviations) is most important in reliability analysis, the modelling of the member should focus on the corresponding mode. In item 3.2.2 this point is discussed further with respect to sampling of basic variables. Item 4.2.2 deals with a check whether presumed and actual mechanism comply with each other. In cases where relevant failure modes are unknown, preliminary tests should be performed.

3.1.3 Basic variables included in the calculation model

For basic variables $\underline{X}$, which are included in the calculation model, statistical parameters have to be known. If those values are unknown, it is recommended to supply them by preliminary tests. Where information is available only from a limited population (e.g. only from one lot or one producer), variances need to be increased accordingly.

Bolt Test Example (continued)		
Let be known from pre-knowledge:		
V_{db}	= 0.005	coefficient of variation of the bolt diameter
V_t	= 0.05	coefficient of variation of the plate thickness
V_{fu}	= 0.07	coefficient of variation of the ultimate strength of the plate material

3.1.4 Basic variables not included in the calculation model

For basic variables not included in the calculation model it has to be made sure, that they have only small influence or that the actual values are representative (see item 3.2.3).

3.1.5 Composite populations

Specifications which are varied in the experiments, should be included in the model. The representative range of application and production statistics for these variables have to be known.

3.1.6 Prior distribution for statistical parameters of D

If no specific information is available, the standard noninformative prior for the mean μ and standard deviation σ of ln D should be used, that is $f_\mu(m)$ is constant and $f_\sigma(s)$ is proportional to 1/s.

Comment:
This is a mathematical convention which results in a normal distribution for μ and a chi-square distribution for σ for one or several sets of observations (posterior distributions).

3.2 Sampling of specimen

3.2.1 Sources of specimens

Specimens may be sampled from production or manufactured for the purpose of testing.

3.2.2 Basic variables included in the model

For basic variables which are included in the model sampling might be performed in such a way that they attain values in the vicinity of the estimated design point ("design point orientated" sample). The design point should be inferred from the selected failure mode (see 3.1.2). If no other information is available, design point values should be chosen corresponding to 0.8β for a dominating variable and to 0.32β for others, β being the target reliability index. This type of sampling is recommended if samples are small and/or when the failure mode may change with variation of the basic variable.

Comment:
In general the above procedure is strongly recommended for geometrical imperfection. For strength parameters this concept needs to be assessed with care. For instance there might be a difference between a bad sample of concrete grade 30 and an average sample of concrete grade 20, even if both have the same cubic strength.

3.2.3 Basic variables not included in the model

Basic variables which are not included in the model should be sampled representatively. It may be necessary to account for populations from different producers.

3.2.4 Specifications

For specifications sampling will often be such that the whole range is covered uniformly, which may differ strongly from production statistics (artificial sampling).

3.3 Determination of loading

3.3.1 General requirement

Depending on the anticipated scope of application of the structural member population, all relevant action effects shall be considered by applying corresponding loads, forces, imposed deformations and other relevant influences (e.g. temperature) to the specimens.

3.3.2 Loading path

Loading paths shall be selected such that they are representative for the anticipated scope of application of the structural member, account for possible unfavorable paths and/or account for those paths, which are considered in calculations in comparable cases.

3.3.3 Special action effects

Where structural properties are conditioned by one or several effects of actions which are not varied systematically, then these effects should be specified by their design values. Where they are independent of the other parameters of the loading path, design values related to estimated combination values may be adopted.

3.4 Measurements

3.4.1 General requirements

The rules and formulas for evaluating test results form only a part of the information the tests make available. It is highly recommended to make as many quantitative and qualitative observations as possible in order to check the presumptions made implicitly or explicitly to allow for alternative evaluations afterwards.

3.4.2 Basic variables included in the calculation model

In principle the actual values X_{obs} of the basic variables included in the calculation model have to be determined for each specimen for each experiment, by direct or (if otherwise impossible) by indirect measurement. In the case of indirect measurement, all relevant conversions should be taken into account. Additional uncertainties have to be accounted for by increasing the variance (see 4.2.4).

3.4.3 Load path

The actual load path parameters and loads have to be determined for each test and during the test - especially at the failure point (limit state point) and if needed at the interface point (see item 2.4). The actual values may be used to check whether the intended load path was met sufficiently or not.

3.4.4 Resistance

In a test the resistance R of a structural member will in general be identified with the load intensity at which the limit state under consideration is reached.

The actual value of the limit load should be measured with extreme care for every experiment.

4. EVALUATION OF TEST RESULTS

4.1 General

4.1.1 Statistical uncertainty

The test results shall be evaluated accounting for statistical uncertainties by determining the predictive distributions of the correction variable D. Generally a model shall be assumed which renders a central t-distribution.

Comment:
The predictive distribution "estimates" the distribution of a variable, accounting for the prior information on its statistical parameters and the relevant observations; the specifications of 3.1.6 automatically result in a central t-distribution for D.

4.1.2 Additional model uncertainties

Additional model uncertainty parameters may be required to deal with the discrepancy between experimental model and reality.

Where codes include implicit safety provisions related to comparable members and properties, these provisions shall not be ruled out by testing and may give rise to an additional safety elements in the formulas.

Comment:
No provisions are required for those model uncertainties which are identified by testing. However, uncertainties in predicting the performance of members in actual service conditions on the basis of test results may need to be considered, depending on the degree to which the experimental model is conservative.

4.1.3 Contradiction to experience

Where a statistical evaluation of tests gives results which are incompatible with experience, the reasons for deviating from the statistical results shall be investigated and recorded.

4.1.4 Extrapolation

The result of a test evaluation is valid for the specifications and load characteristics considered. Extrapolation to cover other design parameters and loadings requires additional information, e.g. from previous tests.

4.2 Predictive distribution for the correction variable D

4.2.1 Observed values for the correction variable

The values for the correction variable d_i for each specimen i, (i = 1,2 ... n) has to be calculated from:

$$r_{t_i} = g_R(\underline{x}_i, \underline{w}_i, \underline{q}) \tag{4.1}$$

$$d_i = r_{e_i} / r_{t_i} \tag{4.2}$$

with:

$\underline{x}_i$ observed basic variable for specimen i

$\underline{w}_i$ specification parameters for specimen i

$\underline{q}$ load path parameters

r_{e_i} observed resistance for specimen i

4.2.2 Pre-analysis

The observed resistance values r_e shall be plotted versus:

- the calculated resistance r_t according equation (4.1);
- each of the observed basic variables x_{obs} and varied specifications;

This plotting procedure is intended respectively (see also chapter 5):

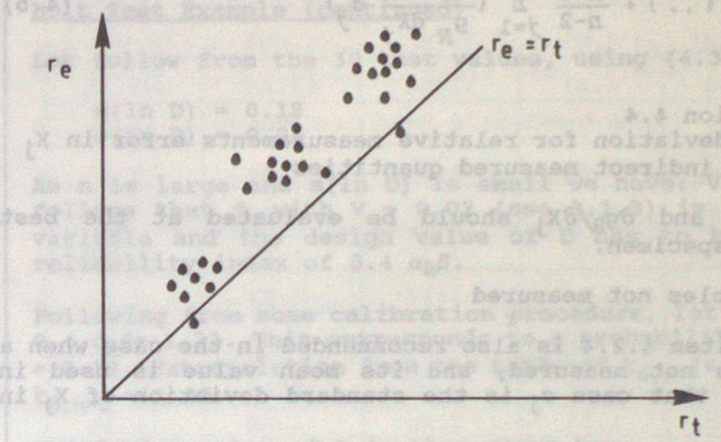
- to establish corresponding relations or classifications;
- to check whether calculation models adequately account for the respective variables.

For all tests the responsible mechanism should be indicated. Where engineering judgement supports the predefined calculation model, the analysis may proceed according to the lines of 4.2.3 and further. Where appropriate, engineering judgement should be supported by statistical hypotheses testing.

If more than one failure mode is observed in the test results, it is recommended to repeat the tests in a number of series. In every series all modes but one should be excluded.

> Bolt Test Example (continued)
>
> Suppose that a number of 30 tests are performed. The specification parameters d_{bnom} (nominal bolt diameter) and t_{nom} (nominal plate thickness) are varied in a representative way. The basic variables d, t and f_u are measured for every test. The resulting r_e - r_t diagram is given by Figure 2.

Fig. 2: Example of r_e-r_t-diagram



4.2.3 Standard case

The standard case is defined by the following requirements:

- Load path parameters are presumed to be equal for all tests;
- Specifications are presumed to be equal for all test specimen or to be sampled in a representative manner. (Comment: For arbitrary sampling of the specifications, see 5.1 and further).
- The resistance R_t and the correction variable D are assumed to follow a log-normal distribution and therefore also the observed resistance R_e.

As R_t, R_e and D follow a lognormal distribution, it follows that $\ln(R_t)$, $\ln(R_e)$ and $\ln(D)$ follow a normal distribution. This means that estimators for mean $\mu(\ln D)$ and standard deviation $\sigma(\ln D)$ can be derived from:

$$m(\ln D) = \frac{1}{n} \sum_{i=1}^{n} \ln d_i \tag{4.3}$$

$$s(\ln D^2) = \frac{1}{n-1} \{ \sum_{i=1}^{n} (\ln d_i)^2 - n.m(\ln D)^2 \} \tag{4.4}$$

Based on a prior distribution for the parameters of lnD according to item (3.1.6), the posterior distribution of lnD, including the statistical uncertainties, is a central t-distribution with the parameters $\{ m(\ln D),\ s(\ln D)\sqrt{1 + 1/n},\ n-1 \}$.

4.2.4 Indirect measurement

When for some of the basic variables indirect measurements are performed the procedure has to be adjusted; the estimation of $\sigma(\ln D)$ should conservatively be increased according to:

$$s(\ln D^2) = \frac{1}{n-1}(\,..\,) + \frac{n-1}{n-2}\sum_{j=1}^{J}\left(\frac{1}{g_R}\frac{\partial g_R}{\partial X_j}\sigma_j\right)^2 \quad (4.5)$$

with:

- $(.)$ see equation 4.4
- σ_j standard deviation for relative measurements error in X_j
- J number of indirect measured quantities

The functions g_R and $\partial g_R/\partial X_j$ should be evaluated at the best estimate for the specimen.

4.2.5 Basic variables not measured

The procedure of item 4.2.4 is also recommended in the case when a basic variable is not measured, and its mean value is used in element 4.5.1. In that case σ_j is the standard deviation of X_j in the sample.

In this case, however, it is not necessary to consider X_j as random, when evaluating the design value for the resistance r_d according to 4.5.1, as the procedure of 4.2.4 takes full account for the variability of X_j. In the procedure of 4.5.1 one can simply replace X_j by $\mu(X_j)$.

Comment:
If one can make sure that the sample is representative with respect to X_j, one might consider to replace X_j by its mean value from the beginning. Following the standard procedures (4.3.1) the variability of X_j is then incorporated in the measured variability of D. It should be stressed however that this procedure is not recommended.

4.3 Design value for the correction variable D

4.3.1 Design value for the correction variable D

The design value D_d for the correction variable D is determined from:

$$D_d = \exp(m(\ln D)) \cdot \exp(-t_{d,n-1}\, s(\ln D)\sqrt{1+1/n}) \quad (4.6)$$

with:

- $t_{d,n-1}$ inverse value of the standard central t-distribution for (n-1) degrees of freedom, corresponding to the probability $\Phi(-\alpha_d b)$.
- α_d influence coefficient for design value of D
- β target reliability index

If D is the dominating variable, α_d should be taken as $\alpha_R = 0.8$; otherwise $\alpha_d = 0.4\ \alpha_R = 0.32$. Values for $t_{d,\nu}$ can be obtained from Table 1.

Bolt Test Example (continued)

Let follow from the 30 test values, using (4.3) and (4.4):

$m(\ln D) = 0.18$
$s(\ln D) = 0.06$

As n is large and s(ln D) is small we have: $V(D) \approx 0.06$. It follows that f_u with $V = 0.07$ (see 3.1.3) is the dominating variable and the design value of D has to be found for a reliability index of $0.4\ \alpha_R\beta$.

Following from some calibration procedure, let $\beta = 3.8$ giving $0.4\ \alpha_R\beta = 1.22$. This corresponds to a probability $P = \Phi(-1.22) = 0.12$. Extrapolation from the table of $t_{d,\nu}$ ($\nu = 29$) leads to $t_{d,n-1} = 1.25$.

The design value of D is then given by:

$$D_d = \exp(0.18)\ \exp(-(1.25)(0.06)\sqrt{1.03}) = 1.08$$

4.4 Design values and characteristic values for basic variables

4.4.1 Design values based on distribution functions

Design values X_d shall be determined from the distribution functions of the basic variable as p-fractile with with:

$$p = \Phi(u_d)$$
$$u_d = \alpha_d\,\beta$$

- Φ standard normal distribution function
- α_d design value for influence coefficient
- β reliability index

If no specific information is available, $\alpha_d = \alpha_R = 0.8$ for the dominant basic variable and $\alpha_d = 0.4\ \alpha_R = 0.32$ otherwise.

Where no code constraints need to be observed and experience (including test results) gives no contradictory guidance, the type of distribution may be assumed lognormal, so that

$$X_d = \exp(\mu_{\ln X} - u_d\sigma_{\ln X}) = \mu_X \exp(-u_d V_X - V_X^2/2) \quad (4.7)$$

with:

- $\mu_{\ln X}$ mean value of ln X
- $\sigma_{\ln X}$ standard derivation of ln X
- μ_X mean value of X
- V_X coefficient of variation of X

$\alpha_d\beta$ →	1.28	1.64	2.33	2.58	3.08
$\phi(-\alpha_{db})$ →	0.10	0.05	0.01	0.005	0.001
ν = 1	3.08	6.31	31.82	63.66	318.31
2	1.89	2.92	6.97	9.93	22.33
3	1.64	2.35	4.54	5.84	10.21
4	1.53	2.13	3.75	4.60	7.17
5	1.48	2.02	3.37	4.03	5.89
6	1.44	1.94	3.14	3.71	5.21
7	1.42	1.89	3.00	3.50	4.78
8	1.40	1.86	2.90	3.36	4.50
9	1.38	1.83	2.82	3.25	4.30
10	1.37	1.81	2.76	3.17	4.14
20	1.33	1.72	2.53	2.84	3.55
30	1.31	1.70	2.46	2.75	3.38
∞	1.28	1.64	2.33	2.58	3.08

Table 1 Values of $t_{d,\nu}$

Comment:
Codes should include lists with distribution functions and their parameters, at least for the most important basic variables. Further, a code may provide conversion factors to deal with discrepancies between test and reality which also should be taken into account.

Bolt Test Example (continued)

The design values for the parameters d_b, t and f_u are given by:

$$d_{bd} = \mu(d_b)\ \exp(-0.4\ \ \alpha_R\beta\ V(d_b)) = 0.99\ \mu(d_b)$$

$$t_d = \mu(t)\ \ \exp(-0.4\ \ \alpha_R\beta\ V(t)) = 0.94\ \mu(t_d)$$

$$f_{ud} = \mu(f_u)\ \exp(\ \ -\alpha_R\beta\ V(f_u)) = 0.81\ \mu(f_u)$$

4.4.2 Characteristic values based on distribution functions

Characteristic values shall be calculated analogously to item 4.4.1. Where no code or other constrains need to be observed, the lower characteristic value should correspond to a probability of p = 5%.

Bolt Test Example (continued)

The characteristic (nominal) values for the bolt diameter and the plate thickness equal their mean values:

d_{bnom} = nominal bolt diameter = $\mu(d_b)$

t_{nom} = nominal plate thickness = $\mu(t)$

The characteristic value for the yield stress equals the 5% fractile:

f_{uk} = characteristic yield stress

= $\mu(f_u)\,(1-1.64V(f_u)) = 0.88\mu(f_u)$

4.4.3 Design and characteristic values based on codes

Characteristic values may be taken directly from the design codes.

Design values may be obtained by taking characteristic values and their safety factors from design codes:

$$X_d = X_k/\gamma_m \qquad (4.8)$$

wherein:
X_k is the characteristic value of the basic variable X
γ_m is the relevant partial safety factor - not considering model uncertainties

4.5 Design values and characteristic for the resistance

4.5.1 Design value

The design value R_d for the resistance is defined as the value corresponding with the $\Phi(-\alpha_R\beta)$ fractile, when Φ is the standard normal distribution-function, $\alpha_R = 0.8$ and β is the target reliability index.

Once the predictive distribution of D is known from the procedure as outlined in 4.2.3, the design value of the resistance R can be obtained:

(1) exactly by evaluating the distribution function of R on the basis of model (2.2) and the distribution functions for $\underline{X}$ and D; next the $\Phi(-\alpha_R\beta)$ fractile is taken with $\alpha_R = 0.8$. Note that the value $\alpha_R = 0.8$ is the standardized code value, which may be replaced by a more appropriate value based on calculations.

(2) approximately from:

$$R_d = D_d\ g_R\ (\underline{X}_d, \underline{w}, \underline{q}) \qquad (4.9)$$

with:
D_d design value of the correction variable D (see 4.3.1)
$\underline{X}_d$ design values of the basic variables $\underline{X}$ (see 4.4.1-4.4.3)

> **Bolt Test Example (continued)**
>
> The design value for R follows from:
>
> $$R_d = (1.08)(2.5)(0.99\ \mu(d_b))\ (0.94\ \mu(t_d))\ (0.81\ \mu(f_u))$$
>
> So:
>
> $$R_d = 2.04\ \ \mu(d_b)\ \mu(t_d)\ \mu(f_u)$$

4.5.2 Characteristic value - definition 1

If the characteristic value is defined as the value corresponding to a 5% fractile, the procedure is essentially the same as the one described in 4.5.1. In that case the test may lead to a partial safety factor, defined as

$$\gamma_m = R_d / R_k \tag{4.10}$$

4.5.3 Characteristic value - definition 2

The characteristic strength can also be defined as:

$$R_k = D_k\ g_R(\underline{X}_k, \underline{w}, \underline{q}) \tag{4.11}$$

with:
D_k characteristic or nominal value for the correction variable D
$\underline{X}_k$ characteristic or nominal values for the basic variables $\underline{X}$

This method also renders a partial coefficient of safety, as given by equation 4.10.

> **Bolt Test Example (continued)**
>
> Let R_k be calculated from:
>
> $$R_k = 2.5\ d_{bn}\ t_n\ f_{uk} = 2.5\mu(d_b)\,\mu(t)\,0.88\mu(f_u)$$
>
> This means that the characteristic (nominal) value for the correction variable D has been set equal to 1.0. The partial safety factor follows from:
>
> $$\gamma_m = \frac{R_k}{R_d} = \frac{2.5\ 0.88}{2.04} = \frac{2.20}{2.04} = 1.08$$

If there is a preference to use the standard value of γ_m from codes, a nominal value for D may follow from:

$$D_{nom} = \frac{R_d}{\gamma_m\ g_R(\underline{X}_k, \underline{w}, \underline{q})} \tag{4.12}$$

> **Bolt Test Example (continued)**
>
> Assume one wants to use $\gamma_m = 1.1$. In that case $D_{nom} = (2.04)(1.1)/(0.88)(2.5) = 1.02$ and the characteristic value for R is given as:
>
> $$R_k = 2.55\ d_{bn}\ t_n\ f_{uk}$$

Comment:
It should be clear that the methods outlined in 4.5.2 and 4.5.3 leads to combinations of characteristic values and partial safety factors, which will all result in the same design value for R, as required. The variety of possible methods is presented only because various applications in practice may lead to a preference for different approaches.

5. SUBSETS

5.1 General

Subsets of the specification space have to be considered if:
(a) specifications have been varied in an artificial way;
(b) specifications have been varied in a representative way, but reduction of V(D) is desirable.

5.2 Assessment of sub-sets for arbitrary varied specifications

- For each sub-set j (j = 1, 2, ...) mean and variance of ln D are determined according to item 4.2.3.
- where variation of $m_j(\ln D)$ and $s_j(\ln D)^2$ are readily explained by statistical uncertainties, a unique correction term may be adopted, i.e. $m(\ln D)$, $s(\ln D)^2$ and D_d are determined considering all $n = \Sigma n_j$ test results.

In cases of doubt, the following hypotheses should be tested with a significance level equal to 0.75:

$$m_1(\ln D) = m_2(\ln D) = \ldots\ldots = m_j(\ln D) \tag{a}$$
$$s_1(\ln D) = s_2(\ln D) = \ldots\ldots = s_j(\ln D) \tag{b}$$

- If (a) and (b) are accepted, a unique correction term may be adopted.

- If only (a) is accepted, then m(ln D) may be determined accounting for all $n = \Sigma n_j$ test results; $s_j(\ln D)$ is determined per sub-set from n_j test results and the design value for D_j is evaluated from

$$D_{d,j} = \exp\{m(\ln D)\}\exp\{-t_d(\nu_j)s_j(\ln D)\sqrt{1+1/n}\} \qquad (5.1)$$

i.e. from a central-t-distribution with $\nu_j = n_{j-1}$ degrees of freedom.

- If only (b) is accepted, then $m_j(\ln D)$ is determined per sub-set from n_j test results; s(ln D) is determined accounting for all $n = \Sigma n_j$ test results $s(\ln D)^2 = (1/\nu)(\Sigma\, \nu_j\, s_j(\ln D)^2)$ and the design value for D_j is evaluated from

$$D_{d,j} = \exp\{m_j(\ln D)\}\exp\{-t_d(\nu)s_j(\ln D)\sqrt{1+1/n}\} \qquad (5.2)$$

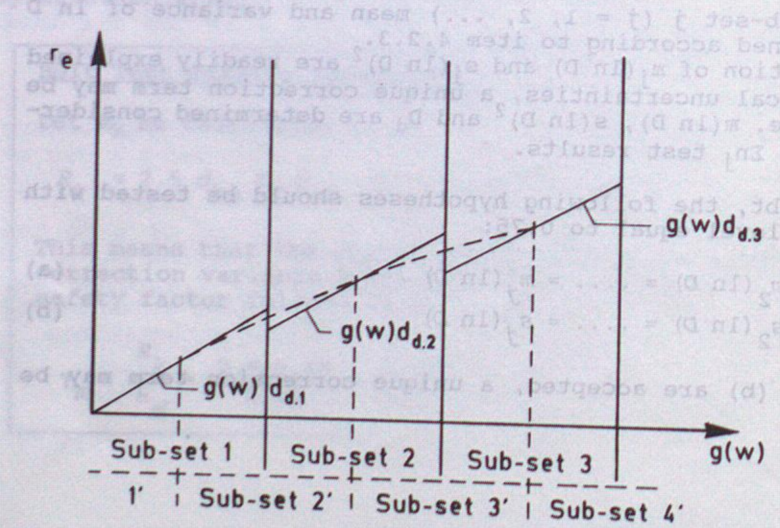
i.e. from a central-t-distribution with $\nu = \Sigma(n_j-1)$ degrees of freedom.

In place of adopting a constant variance (of ln D), other variance models may be considered as an alternative to hypothesis (b), e.g. adopting the variance as a function of a parameter w.

Comment 1:
Where different values for coefficients D_j per sub-set need to be introduced, consistency between sub-sets may be achieved by relating the values for D_j only to the "middle" of the sub-set; connecting the respective points eventually render new sub-sets for design purposes - see fig. 3 (dashed line).

Comment 2:
Testing of hypotheses might also be performed in a Bayesian way, leading to probabilities of truth for (a) and (b).

Fig.3 Sub-sets



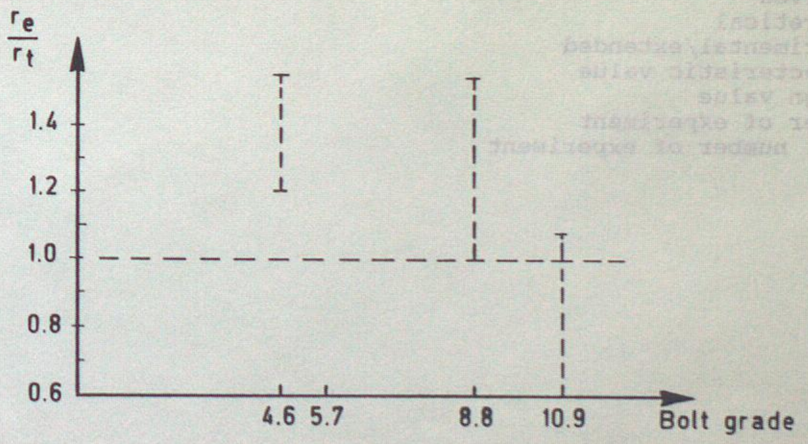
5.3 Improvement of the strength function by considering sub-sets of the test population

If the scatter of the r_e/r_t-values is regarded too high as to give economic design strength functions, the scatter may be reduced by correcting the strength functions, such that additional parameters not sufficiently contained in the strength functions are taken into account.

To make clear what parameters influence the scatter, the test results can be split up into sub-sets with respect to those parameters. Sub-sets should principally be defined by reasonable ranges of parameters. Where alternative definitions for sub-sets are considered, the definition rendering the smaller residual variance should be preferred. The further evaluation should be performed according to the case in (5.2) where (a) has been rejected and (b) has been accepted.

Comment:
As an illustration in fig. 4 the results of shear tests on bolt are given, split in sub-sets with respect to the bolt grade. Obviously the strength function in this case can be improved if the factor 0.7 in the strength function is modified and expressed as a function of the bolt grade (f_{ub}).

Fig. 4 Plot of r_e/r_t versus bolt grade for shear failure test with $g_R = 0.7\, f_u\, A_s$



5.4 Extrapolation

Where different correction terms per sub-set are required, extrapolation beyond the investigated range of design parameters is not permitted.

If the validity of a unique correction term has been established by hypothesis testing according to item 5.2, extrapolation by one-subset of approximately the "size" of sub-sets used for hypotheses testing may be envisaged.

NOTATION

D	correction variable
d_b	bolt diameter
f_u	ultimate strength of plate material
$g_R(...)$	calculation model for the resistance
m	sample mean
p	probability
$\underline{g}$	vector of load path parameters
R	resistance
s	sample standard deviation
t	plate thickness
t_v	standard central t-distributed variable with v degrees of freedom
u	standard normal variable ($\mu = 0$, $\sigma = 1$)
V	coefficient of variation
$\underline{w}$	vector of specifications
$\underline{X}$	vector of basic variables
α	influence coefficient
β	target reliability index
γ_m	safety factor
μ	mean
σ	standard deviation

Indices

nom	nominal
obs	observed
t	theoretical
e	experimental/extended
k	characteristic value
d	design value
i	number of experiment
n	total number of experiment