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Two-sided service markets: Effects of quality differentiation on market efficiency

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Determining the effects of quality differentiation on the efficiency of two-sided service markets is challenging. The presence of private information on both market sides and the heterogeneity of sellers can lead to substantial economic inefficiencies. Hence, this paper investigates how quality-differentiated sellers affect market efficiency from the perspective of mechanism design theory. First, we characterize second-best mechanisms for matching buyers and sellers. We then propose a heuristic algorithm for approximating the welfare-maximizing match outcomes. Based on empirical data, our simulation study suggests that an increased quality differentiation can reduce market efficiency; however, this inefficiency vanishes as the market size increases.

1 | INTRODUCTION

Advances in information technology have promoted the rise of two-sided service markets in many domains. Internet platforms such as Airbnb and Uber enable digital marketplaces where buyers and sellers trade on-demand services for money. Low entry barriers and reduced transaction costs attract a growing number of participants on both market sides. At the same time, market designers exploit emerging network effects to create and implement innovative business models. As of January 2020, investment opportunity tracker AngelList recorded 1614 start-up companies in the field of online marketplaces alone. Most recent manifestations of these two-sided marketplaces include peer-to-peer markets where individuals and small businesses exchange on-demand services, thus replacing long-term contracts by spot transactions (Einav, Farronato, & Levin, 2016). In many markets, sellers offer these services in a quality-differentiated fashion to distinguish themselves from their competition (Zervas, Proserpio, & Byers, 2017). Airbnb and Uber, for example, rely on reciprocal feedback and reputation systems to enable quality differentiation. Apart from these well-known platforms, markets for trading novel forms of services have also emerged. Energy services such as charging services for electric vehicles, for example, provide a value-oriented approach toward quality-differentiated electricity consumption (Lim, Mak, & Rong, 2014; Salah, Flath, Schuller, Will, & Weinhardt, 2017; Woo et al., 2014). These charging services are typically differentiated by the power output provided, thus directly affecting the speed of charging (He, Mak, Rong, & Shen, 2017).

Given the growing number of heterogeneous sellers and buyers on such marketplaces, it is increasingly important to better understand how the presence of sellers offering quality-differentiated services affects the efficiency of these two-sided markets. A key challenge arising in any market is the design of mechanisms that determine economically efficient outcomes. Economic theory corroborates that ex post efficiency cannot be achieved in the presence of two-sided private information (Myerson & Satterthwaite, 1983). Yet these inefficiencies disappear as markets become large—at least in settings where homogeneous sellers offer goods or services that only differ in price but not in exogenous quality (Cripps & Swinkels, 2006; Gresik & Satterthwaite, 1989; Rustichini, Satterthwaite, & Williams, 1994). When sellers offer quality-differentiated services, however, it is still unknown whether and in what way the market's efficiency might be affected by these sellers. Therefore, our research question can be framed as follows: In what way does the presence of quality-differentiated sellers affect the efficiency of two-sided service markets?

To address this research question, we characterize mechanisms for matching buyers and sellers that trade quality-differentiated on-demand services, where service quality is publicly known. We draw on mechanism design theory to derive a second-best mechanism that maximizes the expected social welfare subject to incentive compatibility constraints. For two-sided service markets, the social welfare is defined as the aggregated difference between valuation and cost, which depends on differentiated quality. The formal model of our proposal is informed by the work of Widmer and Leukel (2016) who characterize mechanisms for allocating electronic services from the perspective of a social planner. Although Widmer and Leukel derive rules and payments for the second-best matching, their model assumes quality to be an intrinsic part of the traders' utilities. Hence, the model does not integrate service quality that is publicly observable. In contrast, the objectives of this research are to (1) characterize mechanisms for two-sided markets that account for exogenous service quality, (2) propose a heuristic algorithm for determining the approximate welfare-maximizing match outcomes because analytic solutions are infeasible, and (3) study in what way market efficiency is affected by the presence of quality-differentiated sellers.

To validate our mechanism, we conduct a simulation study based on empirical data from two real-world market settings. These two settings reflect two different degrees of quality differentiation. The first setting considers quality-differentiated charging services for electric vehicles (buyers) offered by competing charging stations (sellers). This setting relies on actual sales prices and power output levels offered at 424 charging stations in Berlin, Germany. The second market setting considers quality-differentiated drivers (sellers) that are matched to potential riders (buyers) on a digital platform for ridesharing. Here, our experiments are based on a unique data set containing sales price and driver reputation in the form of numerical star rating from 307 offers listed on the European ridesharing platform BlaBlaCar. For both settings, we use our heuristic algorithm to determine the approximate welfare-maximizing match outcomes based on the associated real-world price distributions. Then, we study the asymptotic efficiency achieved by the second-best mechanism as well as the sellers' probabilities of sales and expected revenues for varying market sizes.

We find that the efficiency achieved by our mechanism exceeds 93% already for small markets exhibiting strong quality differentiation. This result holds even for vehicle charging platforms where quality levels of sellers are highly heterogeneous. The proposed mechanism absorbs strong quality differentiation among sellers in such a way that no more than 1.33 percentage points in efficiency losses must be tolerated compared with the benchmark without quality differentiation. We also find that the probability of sales and the expected revenues increase as sellers improve their quality. The reason for this increase is that the presence of quality-differentiated services forces buyers into competition for quality. Moreover, the probability of sales is nearly proportional to the expected revenues of sellers. Thus, our simulation study helps to explain how quality-differentiated sellers exploit two-sided private information to increase their probability of sales and expected revenue. Based on our findings, sellers can estimate their return on investment as a function of service quality. Market designers can also build on our results to assess potential efficiency losses that arise when two-sided private information is present on quality-differentiated service markets.

The remainder of this article is organized as follows. The next section discusses prior research on market mechanisms for quality-differentiated two-sided markets as well as applications. In Section 3, we describe our mechanism for matching buyers and sellers with exogenous quality. In Section 4, we study the mechanism's efficiency properties. Finally, we report on our simulation study, discuss the findings, and provide our conclusion in Section 5.

2 | THEORETICAL BACKGROUND

We review extant literature in mechanism design theory and discuss pertinent approaches in a set of real-world applications. We conclude the theoretical background by providing a summary.

2.1 | Mechanism design theory

Our study is related to the growing body of literature on matching mechanisms for two-sided markets in which agents are buyers and sellers. How market designers should match buyers and sellers efficiently is an important research area in mechanism design theory (Cavagnac, 2005; Kojima & Pathak, 2009; Maskin, 2008). A match outcome is efficient if there exists no other feasible match outcome that makes some participants better off without making other participants worse off (Holmström & Myerson, 1983). However, market designers typically do not know buyer valuation and seller cost in advance. As a result, inefficiencies and even market failure can occur (Akerlof, 1978; Hui, Saeedi, Shen, & Sundaresan, 2016; Samuelson, 1984). Therefore, matching mechanisms are needed that guarantee high efficiency despite the presence of two-sided private information.

Mechanism design theory prescribes a set of desired economic properties that must be retained by any mechanism (Börgers, 2015; Krishna, 2009). First, the mechanism must provide adequate incentives that induce strategic participants to reveal their true preferences (incentive compatibility). Second, participation in the mechanism must be voluntary for all buyers and sellers (individual rationality). Third, the mechanism must not run a permanent deficit in funds (budget balance). However, mechanism design theory corroborates that ex post efficiency is not attainable for mechanisms requiring these properties simultaneously (Delacrétaz, Loertscher, Marx, & Wilkening, 2019; Laffont & Maskin, 1979; Myerson & Satterthwaite, 1983). Because ex post efficiency is unattainable, the literature suggests to derive so-called *second-best mechanisms*, which maximize the expected social welfare while maintaining the three desired properties based on appropriate solution concepts (Bierbrauer, Ockenfels, Pollak, & Rückert, 2017; Gresik & Satterthwaite, 1989; Segal & Whinston, 2016). For matching markets, the expected social welfare is defined as the aggregated expected utilities of all buyers and sellers participating in the mechanism. The outcome of the emerging second-best mechanism can then be used to quantify inefficiencies that arise because of the presence of private information.

Classic research in designing two-sided mechanisms primarily focuses on matching buyers and sellers that trade homogeneous commodities (Cripps & Swinkels, 2006; Gresik & Satterthwaite, 1989; McAfee, 1992; Myerson & Satterthwaite, 1983; Rustichini et al., 1994). Although these mechanisms are clearly not ex post efficient, they achieve full efficiency for large markets at a rate that depends on the market size. Other research on two-sided matching markets develops mechanisms in which buyers and sellers have private information about their vertical quality characteristics that determine the match values (Damiano & Li, 2007; Gomes & Pavan, 2016; Johnson, 2013). Although these mechanisms internalize two-sided private information about quality, they do not study the efficiency achieved by second-best mechanisms that maximize the expected social welfare. Widmer and Leukel (2016) characterize rules and expected payments for allocating electronic services in the presence of two-sided private information from the perspective of a social planner. Based on the formal framework proposed by Johnson (2013), they study the efficiency of the associated second-best mechanism in a set of simulations. In our research, we extend the approach of Widmer and Leukel by integrating exogenous quality on the seller side of the market.

2.2 | Applications

Two-sided mechanisms for trading goods or services have been the subject of inquiry in a number of real-world applications. These applications include energy services, ridesharing, financial lending, and digital services such as cloud computing.

Smart energy systems enable consumers of energy to become producers and service providers. These developments facilitate peer-to-peer market models for energy services from existing markets where green electricity is offered by local farmers to advanced future quality-differentiated energy services like electricity storage, water heating, or electric vehicle charging (Alvaro-Hermana, Fraile-Ardanuy, Zufiria, Knapen, & Janssens, 2016; Parag & Sovacool, 2006). Tushar et al. (2018) provide an overview of game-theoretic approaches for trading energy services on peer-to-peer platforms. Khorasany, Mishra, and Ledwich (2018) review different market design concepts including the classification of participants for local energy trading. For electric vehicle charging, Kang et al. (2017) propose an iterative double auction mechanism for demand response by providing energy from the batteries to balance local electricity demand. Such platforms increasingly use two-sided auction formats to trade energy services among retailers and microgrids including local generation, demand response, and energy storage resources (Marzband, Javadi, Pourmousavi, & Lightbody, 2018).

With a substantial increase in the popularity of ridesharing platforms like Uber and Lyft, the efficient matching of drivers and riders on these two-sided platforms has become increasingly important. Recent research focuses on the design of matching mechanisms that internalize the cost share of drivers and private information about preferences (Ordóñez & Dessouky, 2017; Wang, Agatz, & Erera, 2017; Wang, Yang, & Zhu, 2018). Whereas these approaches aim at minimizing trip time, vehicle miles, and travel costs, they do not consider market efficiency from an economic perspective. Other works concerning peer-to-peer markets include financial lending and crowdfunding. Here, prior research focuses on screening the quality of borrowers (Iyer, Khwaja, Luttmer, & Shue, 2015) and studying the effects of different matching mechanisms on market participants, transaction outcomes, and social welfare (Wei & Lin, 2016). Yet these approaches do not assess the efficiency of markets with two-sided private information.

Bapna, Goes, and Gupta (2005, 2009) study the problem of allocating and pricing quality-differentiated digital services such as event livestreaming and video on demand. A monopolist offers a set of services differing in guaranteed quality levels to multiple customers who have differentiated valuations for these services. Widmer and Leukel (2016) compare the outcome of a profit-maximizing intermediary to that of a social planner in mechanisms that allocate digital services with differentiated quality levels. They provide a lower bound for the relative efficiency loss any second-best mechanism must tolerate because of private information about quality. Widmer and Leukel (2018) advance this model by defining a set of straightforward pricing schemes for digital services based on different pricing formats. Although their models internalize two-sided private information about quality, they do not allow for digital services that differ in exogenous quality.

Das, Du, Gopal, and Ramesh (2011) and Du, Das, and Ramesh (2012) study revenue-maximizing spot and forward prices for a cloud computing service provider that faces a known set of customers by considering the risk of demand stochasticity and unused storage capacity. Although their work analyzes the effects of providing spot markets for trading cloud computing services, differentiated quality metrics such as bandwidth or latency into the mechanism are not considered in their model. Zaman and Grosu (2013) examine combinatorial auctions for allocating quality-differentiated instances of virtual machines to a single data center as a seller. Because only one seller is present in the market, it remains unclear whether the emerging mechanism is efficient for multiple sellers competing in quality-differentiated service offers. Prior to the rise of cloud computing, markets for trading grid computing resources have received considerable attention in information systems research (Bapna, Das, Day, Garfinkel, & Stallaert, 2011; Bapna, Das, Garfinkel, & Stallaert, 2008; Schnizler, Neumann, Veit, & Weinhardt, 2008). In these studies, however, sellers do not offer quality-differentiated resources in the presence of two-sided private information.

2.3 | Summary and contributions

In summary, extant literature does not sufficiently explain how to efficiently match buyers and sellers on platforms where two-sided private information about valuation and cost is present and sellers are characterized by exogenous quality levels. Our work addresses this limitation and characterizes a second-best mechanism that matches buyers and sellers based on differentiated service quality in the presence of two-sided private information. Because analytic solutions to the optimization problem are infeasible, we also propose a heuristic algorithm that determines the approximate welfare-maximizing match outcomes for varying market sizes. Based on empirical data from real-world applications, we then study the effects of quality differentiation on efficiency of the two-sided service market and analyze the relationship between probability of sales and sellers' expected revenues.

3 | MODEL

We introduce a model providing definitions and formal notations for the design of a second-best mechanism that matches sellers and buyers based on exogenous seller quality as well as private information about valuation and costs. The model is informed by that of Widmer and Leukel (2016), which focuses on second-best mechanisms that internalize two-sided private information on quality. We extend this model by integrating publicly known quality on the seller side of the market.

3.1 | Demand and supply

We consider a setting where N buyers demand a single on-demand service from a set of M sellers, each of which offers exactly a single quantity of the service. Each seller $j \in \{1, \dots, M\}$ produces a one-dimensional quality $q_j > 0$, which is publicly observable through the platform. Although quality is public information, each seller has internal costs for supplying the service. The private cost information is given by σ_j for each seller j. σ_j is drawn from a probability density function $h(\sigma_j)$, being strictly positive on $[\underline{\sigma}, \bar{\sigma}]$ with cumulative distribution function $H(\sigma_j)$. On the demand side, the private valuation information of each buyer $i \in \{1, \dots, N\}$ is given by θ_i, which they have assigned to the service. θ_i is drawn from a probability density function $f(\theta_i)$, which is strictly positive on $[\underline{\theta}, \bar{\theta}]$ with cumulative distribution function $F(\theta_i)$. Similar to other approaches in mechanism design, we assume that $1 - F(\theta_i)$ and $H(\sigma_j)$ are log concave.

Because there are N buyers and M sellers on the market, let $\theta = (\theta_1, \dots, \theta_N)$ and $\sigma = (\sigma_1, \dots, \sigma_M)$ be their private information vectors, respectively. The vectors $\theta_{-i} = (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_N)$ and $\sigma_{-j} = (\sigma_1, \dots, \sigma_{j-1}, \sigma_{j+1}, \dots, \sigma_M)$ specify the private information except for i and j, respectively. Further, let $\mathbb{E}_{\theta_{-i},\sigma}$ denote the expectation over all private information conditional on buyer i's information, and let $\mathbb{E}_{\theta,\sigma_{-j}}$ denote the analogous expectation for seller j. The unconditional expectation over all private information is denoted by $\mathbb{E}$.

Each buyer assigns a valuation $v(\theta_i)$ to their requested service, which can be interpreted as their maximum willingness to pay. Assume that $v(\theta_i)$ is increasing and concave in θ_i. On the supply side, the seller infers a cost $c(\sigma_j)$ for providing their service. $c(\sigma_j)$ is increasing and convex in σ_j. Because quality is publicly known, buyers and sellers internalize their valuation and costs for quality in their private information.

Similar to other auction settings, we assume risk-neutral buyers and sellers with quasilinear utility functions. For describing a match between a buyer and a seller, let $x_{ij}(\theta, \sigma) \in [0, 1]$ denote the probability that buyer i is matched to seller j. For ease of the exposition, we use vectorized function arguments without further reference and, for instance, write $x_{ij}(\theta_i, \theta_{-i}, \sigma) \equiv x_{ij}(\theta_1, \dots, \theta_N, \sigma_1, \dots, \sigma_M)$ in any appropriate context.

Each buyer's expected utility function consists of their valuation weighted by the probability that they are matched to a seller of quality q_j, minus the payment they have to make for service consumption. Hence, i's expected utility is given by

$$U_B(\theta_i) = \mathbb{E}_{\theta_{-i},\sigma}\left[v(\theta_i)\sum_{j=1}^{M} q_j x_{ij}(\theta_i, \theta_{-i}, \sigma) - t_B(\theta_i, \theta_{-i}, \sigma)\right], \tag{1}$$

where $\mathbb{E}_{\theta_{-i},\sigma}[t_B(\theta_i, \theta_{-i}, \sigma)]$ is the expected payment made by i conditional on all other participants' private information. Similarly, the expected utility of each seller consists of the expected revenue accrued for service provisioning, minus the costs for producing quality q_j weighted by the probability that j is matched to i. Thus, j's expected utility is given by

$$U_S(\sigma_j) = \mathbb{E}_{\theta,\sigma_{-j}}\left[t_S(\theta, \sigma_j, \sigma_{-j}) - q_j c(\sigma_j)\sum_{i=1}^{N} x_{ij}(\theta, \sigma_j, \sigma_{-j})\right], \tag{2}$$

where $\mathbb{E}_{\theta,\sigma_{-j}}[t_S(\theta, \sigma_j, \sigma_{-j})]$ corresponds to the expected revenue accruing to j for providing the service to buyer i. The expected utility integrates a linear combination of quality and cost. That is, the seller

3.2 | Incentive compatibility and individual rationality

We consider a matching mechanism where each buyer i and each seller j submits a sealed bid to the platform operator corresponding to the willingness to pay and the cost, respectively. The operator collects all bids, matches asks to offers, and determines the associated payments. Similar to related work in mechanism design theory, we specify a direct revelation mechanism to describe the mechanism (Myerson, 1979). In a direct mechanism, all participants simultaneously report their private information to the operator who then dictates the matches in conjunction with the payments. In equilibrium, all participants will report truthfully. Thus, a direct revelation mechanism is defined by the matching variable and the payments; that is, $\{x_{ij}(\theta, \sigma), t_B(\theta, \sigma), t_S(\theta, \sigma)\}$ for all participants i and j.

We invoke the revelation principle to identify the optimal Bayesian Nash equilibrium from the set of all possible mechanisms

(Gibbard, 1973; Harris & Raviv, 1981; Myerson, 1981). A direct revelation mechanism is Bayesian incentive compatible if truthful bidding constitutes a Bayesian Nash equilibrium. Thus, in an incentive compatible mechanism, each participant's expected utility is maximized by submitting their true bid, conditional on the truthful bidding of others. Suppose that $\hat{\theta} \neq \theta_i$ is buyer i's submitted (not necessarily true) bid. Then the mechanism is (Bayesian) incentive compatible if and only if

$$U_B(\theta_i) \geq \mathbb{E}_{\theta_{-i},\sigma}\left[v(\theta_i)\sum_{j=1}^{M} q_j x_{ij}(\hat{\theta},\theta_{-i},\sigma) - t_B(\hat{\theta},\theta_{-i},\sigma)\right]. \tag{3}$$

Similarly, suppose $\hat{\sigma} \neq \sigma_j$ is seller j's submitted bid. Then,

$$U_S(\sigma_j) \geq \mathbb{E}_{\theta,\sigma_{-j}}\left[t_S(\theta,\hat{\sigma},\sigma_{-j}) - q_j c(\sigma_j)\sum_{i=1}^{N} x_{ij}(\theta,\hat{\sigma},\sigma_{-j})\right]. \tag{4}$$

Both inequalities must hold to account for incentive compatibility on both sides of the market. In addition to incentive compatibility, the mechanism must satisfy individual rationality, so that each participant willingly participates in the mechanism. In an individually rational mechanism, each participant's expected utility is greater than or equal to the utility of its outside option. We assume the outside option to be zero for all participants. Hence, the mechanism is individually rational if and only if $U_B(\theta_i) \geq 0$ for buyers i and $U_S(\sigma_j) \geq 0$ for sellers j.

The preceding characterization of incentive compatibility and individual rationality allows us to specify the format of our proposed mechanism in the following section.

4 | QUALITY-DIFFERENTIATED MATCHING

We first characterize incentive compatibility and individual rationality, followed by defining the mechanism's optimization problem. Then, we propose the heuristic-based matching algorithm used to solve the optimization problem.

4.1 | Characterizing incentive compatibility and individual rationality

Direct revelation mechanisms remove the participants' incentives to manipulate their bids by internalizing their potential strategic behavior into the mechanism itself. Hence, the direct mechanism implements the bids that participants would have submitted using their equilibrium strategies. Therefore, we adopt the approach of Gresik and Satterthwaite (1989), Myerson and Satterthwaite (1983), and Widmer and Leukel (2016) to define the following two functions. Let the virtual valuation of buyer i be given by

$$\psi_B(\theta_i) = v(\theta_i) - \frac{1-F(\theta_i)}{f(\theta_i)} v'(\theta_i). \tag{5}$$

The virtual valuation function enunciates the strategic behavior of buyers in an indirect mechanism implementation. Because $v(\theta_i)$ is increasing, $\psi_B(\theta_i)$ is strictly smaller than the actual valuation. Hence, buyers engage in bid manipulation by understating their true valuation (bid shading). In this way, buyers attempt to lower their payments to their benefit.

Similarly, each seller incurs a virtual cost of

$$\psi_S(\sigma_j) = c(\sigma_j) + \frac{H(\sigma_j)}{h(\sigma_j)} c'(\sigma_j). \tag{6}$$

Thus, using their equilibrium strategies, sellers have an incentive to overstate their true cost because $\psi_S(\sigma_j)$ is strictly greater than $c(\sigma_j)$. In such a case, sellers try to raise their expected compensation payments for the offered service by reporting a higher provision cost.

These virtual valuation and cost facilitate the design of a second-best mechanism for quality-differentiated matching of buyers and sellers. The following lemma characterizes the set of all incentive compatible and individually rational mechanisms for our matching problem.

Lemma 1. If $x_{ij}(\cdot,\cdot)$ is the probability of matching buyer i with seller j, then transfer functions $t_B(\cdot,\cdot)$ and $t_S(\cdot,\cdot)$ exist such that $\{x_{ij}(\cdot,\cdot), t_B(\cdot,\cdot), t_S(\cdot,\cdot)\}$ is incentive compatible and individually rational if and only if $\mathbb{E}_{\theta_{-i},\sigma}[x_{ij}(\theta_i,\cdot)]$ is nondecreasing, $\mathbb{E}_{\theta,\sigma_{-j}}[x_{ij}(\cdot,\sigma_j)]$ is nonincreasing, and

$$\sum_{i=1}^{N} U_B(\underline{\theta}) + \sum_{j=1}^{M} U_S(\bar{\sigma}) = \mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M} q_j(\psi_B(\theta_i) - \psi_S(\sigma_j)) x_{ij}(\theta,\sigma)\right] \geq 0. \tag{7}$$

Proof. Suppose $\{x_{ij}(\theta,\sigma), t_B(\theta,\sigma), t_S(\theta,\sigma)\}$ is incentive compatible. We start to derive our argument for sellers. For any cost pair $\hat{\sigma} \neq \sigma_j$, we must have

$$U_P(\sigma_j) \geq \mathbb{E}_{\theta,\sigma_{-j}}\left[t_S(\theta,\hat{\sigma},\sigma_{-j}) - q_j c(\sigma_j)\sum_{i=1}^{N} x_{ij}(\theta,\hat{\sigma},\sigma_{-j})\right] \text{ and} \tag{8}$$

$$U_P(\hat{\sigma}) \geq \mathbb{E}_{\theta,\sigma_{-j}}\left[t_S(\theta,\sigma_j,\sigma_{-j}) - q_j c(\hat{\sigma})\sum_{i=1}^{N} x_{ij}(\theta,\sigma_j,\sigma_{-j})\right]. \tag{9}$$

These two inequalities imply that

$$q_j(c(\hat{\sigma}) - c(\sigma_j))\mathbb{E}_{\theta,\sigma_{-j}}\left[\sum_{i=1}^{N}\left(x_{ij}(\theta,\sigma_j,\sigma_{-j}) - x_{ij}(\theta,\hat{\sigma},\sigma_{-j})\right)\right] \geq 0. \tag{10}$$

If $\hat{\sigma} \geq \sigma_j$, we also have $c(\hat{\sigma}) \geq c(\sigma_j)$ because of monotonicity. Therefore, we must have $\mathbb{E}_{\theta,\sigma_{-j}}[x_{ij}(\theta,\hat{\sigma},\sigma_{-j})] \leq \mathbb{E}_{\theta,\sigma_{-j}}[x_{ij}(\theta,t,\sigma_{-j})]$, so $\mathbb{E}_{\theta,\sigma_{-j}}[x_{ij}(\cdot,\sigma_j)]$ is nonincreasing.

Similarly, we can rearrange two incentive constraints for buyers as follows:

$$(v(\sigma_j) - v(\hat{\sigma}))\mathbb{E}_{\theta_{-i},\sigma}\left[\sum_{j=1}^{M} q_j\left(x_{ij}(\theta_i,\theta_{-i},\sigma) - x_{ij}(\hat{\theta},\theta_{-i},\sigma)\right)\right] \geq 0, \tag{11}$$

and so the quality-weighted matching function $\mathbb{E}_{\theta_{-i},\sigma}[q_j x_{ij}(\theta_i,\cdot)]$ must be nondecreasing.

Corollary 1 in Milgrom and Segal (2002) provides expressions for the indirect utility of each participant in any incentive compatible mechanism:

$$U_C(\theta_i) = U_C(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_i} v'(t)\mathbb{E}_{\theta_{-i},\sigma}\left[\sum_{j=1}^{M} q_j x_{ij}(t,\theta_{-i},\sigma)\right]dt \text{ and} \tag{12}$$

$$U_P(\sigma_j) = U_P(\bar{\sigma}) + q_j\int_{\sigma_j}^{\bar{\sigma}} c'(t)\mathbb{E}_{\theta,\sigma_{-j}}\left[\sum_{i=1}^{N} x_{ij}(\theta,t,\sigma_{-j})\right]dt, \tag{13}$$

where $U_C(\underline{\theta})$ and $U_P(\bar{\sigma})$ are the expected utilities evaluated at the lower and upper private value bounds, respectively. By substituting the indirect utilities (12) and (13) into the sum of all participants' expected utilities given in (1) and (2), we obtain an alternative expression for the expected social welfare defined within the maximization problem in (21):

$$\begin{aligned}&\sum_{i=1}^{N}\int_{\underline{\theta}}^{\bar{\theta}} U_C(\theta_i)f(\theta_i)d\theta_i + \sum_{j=1}^{M}\int_{\underline{\sigma}}^{\bar{\sigma}} U_P(\sigma_j)h(\sigma_j)d\sigma_j\\ &= \sum_{i=1}^{N}\int_{\underline{\theta}}^{\bar{\theta}}\left(U_C(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_i} v'(t)\mathbb{E}_{\theta_{-i},\sigma}\left[\sum_{j=1}^{M} q_j x_{ij}(t,\theta_{-i},\sigma)\right]dt\right)f(\theta_i)d\theta_i\\ &+ \sum_{j=1}^{M}\int_{\underline{\sigma}}^{\bar{\sigma}}\left(U_P(\bar{\sigma}) + q_j\int_{\sigma_j}^{\bar{\sigma}} c'(t)\mathbb{E}_{\theta,\sigma_{-j}}\left[\sum_{i=1}^{N} x_{ij}(\theta,t,\sigma_{-j})\right]dt\right)h(\sigma_j)d\sigma_j.\end{aligned} \tag{14}$$

Expression (14) is the expected social welfare expressed by the participants' indirect utilities. Therefore, (14) must equal the expected social welfare obtained by the participants' direct utilities in the maximand of (21). Equating these two expressions, followed by some basic algebraic manipulations, integration by parts as well as rearranging and collecting similar terms yield

$$\sum_{i=1}^{N} U_C(\underline{\theta}) + \sum_{j=1}^{M} U_P(\bar{\sigma}) = \mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M} q_j(\psi_C(\theta_i) - \psi_P(\sigma_j))x_{ij}(\theta,\sigma)\right]. \tag{15}$$

Because individual rationality holds, we must have $U_C(\underline{\theta}) \geq 0$ and $U_P(\bar{\sigma}) \geq 0$, which gives us expression (7) in Lemma 1.

Suppose now that $x_{ij}(\theta,\sigma)$ satisfies (7) and that $\mathbb{E}_{\theta_{-i},\sigma}[x_{ij}(\theta_i,\cdot)]$ is nondecreasing and $\mathbb{E}_{\theta,\sigma_{-j}}[x_{ij}(\cdot,\sigma_j)]$ is nonincreasing. Consider the following expected payments made by buyer i

$$\begin{aligned}\mathbb{E}_{\theta_{-i},\sigma}[t_B(\theta_i,\theta_{-i},\sigma)] = \; & v(\theta_i)\mathbb{E}_{\theta_{-i},\sigma}\left[\sum_{j=1}^{M} q_j x_{ij}(\theta_i,\theta_{-i},\sigma)\right]\\ & - \int_{\underline{\theta}}^{\theta_i} v'(t)\mathbb{E}_{\theta_{-i},\sigma}\left[\sum_{j=1}^{M} q_j x_{ij}(t,\theta_{-i},\sigma)\right]dt\end{aligned} \tag{16}$$

and the expected revenue of seller j

$$\begin{aligned}\mathbb{E}_{\theta,\sigma_{-j}}[t_S(\theta,\sigma_j,\sigma_{-j})] = \; & q_j c(\sigma_j)\mathbb{E}_{\theta,\sigma_{-j}}\left[\sum_{i=1}^{N} x_{ij}(\theta,\sigma_j,\sigma_{-j})\right]\\ & + q_j\int_{\sigma_j}^{\bar{\sigma}} c'(t)\mathbb{E}_{\theta,\sigma_{-j}}\left[\sum_{i=1}^{N} x_{ij}(\theta,t,\sigma_{-j})\right]dt.\end{aligned} \tag{17}$$

These expected transfers are obtained by equating the direct and indirect utilities, as well as setting the worst-off payoffs to zero; that is, $U_C(\underline{\theta}) = U_P(\bar{\sigma}) = 0$.

To check incentive compatibility of (16), observe that

$$\begin{aligned}U_C(\theta_i) - U_C(\hat{\theta}) &= \mathbb{E}_{\theta_{-i},\sigma}\Bigg[v(\theta_i)\sum_{j=1}^{M} q_j(x_{ij}(\theta_i,\theta_{-i},\sigma)\\ &\quad - x_{ij}(\hat{\theta},\theta_{-i},\sigma)) - (t_B(\theta_i,\theta_{-i},\sigma) - t_B(\hat{\theta},\theta_{-i},\sigma))\Bigg]\\ &= \mathbb{E}_{\theta_{-i},\sigma}\Bigg[v(\theta_i)\sum_{j=1}^{M} q_j\int_{\hat{\theta}}^{\theta_i}\frac{\partial}{\partial r}x_{ij}(r,\theta_{-i},\sigma)dr\\ &\quad - \Bigg(v(\theta_i)\sum_{j=1}^{M} q_j x_{ij}(\theta_i,\theta_{-i},\sigma) - v(\hat{\theta})\sum_{j=1}^{M} q_j x_{ij}(\hat{\theta},\theta_{-i},\sigma)\\ &\quad + \int_{\theta_i}^{\hat{\theta}} v'(r)\sum_{j=1}^{M} x_{ij}(r,\theta_{-i},\sigma)dr\Bigg)\Bigg]\\ &= \mathbb{E}_{\theta_{-i},\sigma}\Bigg[\sum_{j=1}^{M} q_j\int_{\hat{\theta}}^{\theta_i} v(\theta_i)\frac{\partial}{\partial r}x_{ij}(r,\theta_{-i},\sigma)dr\\ &\quad - \sum_{j=1}^{M} q_j\int_{\hat{\theta}}^{\theta_i} v(r)\frac{\partial}{\partial r}x_{ij}(r,\theta_{-i},\sigma)dr\Bigg]\\ &= \mathbb{E}_{\theta_{-i},\sigma}\left[\int_{\hat{\theta}}^{\theta_i}\sum_{j=1}^{M} q_j(v(\theta_i) - v(r))\frac{\partial}{\partial r}x_{ij}(r,\theta_{-i},\sigma)dr\right] \geq 0.\end{aligned} \tag{18}$$

The last inequality is a consequence of $\mathbb{E}_{\theta_{-i},\sigma}[x_{ij}(\theta_i,\cdot)]$ and $v(\cdot)$ being nondecreasing. Therefore, buyer i would do better reporting θ_i instead of $\hat{\theta}$. The proof of incentive compatibility for sellers is analogous.

Because we have assumed (7), the sum over all expected utilities evaluated at the lowest and highest private values, respectively, must be nonnegative. Further, Equations (12) and (13) imply that $U_C(\theta_i)$ is increasing in θ_i and $U_P(\sigma_j)$ is decreasing in σ_j. Because of these monotonicity properties and because of (7), it suffices to verify individual rationality for the buyer's lowest willingness to pay $\theta_i = \underline{\theta}$ and for the seller's highest costs $\sigma_j = \bar{\sigma}$. This yields $U_C(\theta_i) \geq 0$ and $U_P(\sigma_j) \geq 0$.

4.2 | Welfare-maximizing matches

From the set of incentive compatible and individually rational mechanisms characterized in Lemma 1, we now select a direct mechanism that maximizes the expected social welfare subject to budget balance. Ex ante budget balance ensures that all expected transfers made among the participants add up to zero. Hence, the ex ante budget balance constraint for our matching problem is defined as

$$\begin{aligned}&\sum_{i=1}^{N}\int_{\underline{\theta}}^{\bar{\theta}}\mathbb{E}_{\theta_{-i},\sigma}[t_B(\theta_i,\theta_{-i},\sigma)]f(\theta_i)d\theta_i\\ &- \sum_{j=1}^{M}\int_{\underline{\sigma}}^{\bar{\sigma}}\mathbb{E}_{\theta,\sigma_{-j}}[t_S(\theta,\sigma_j,\sigma_{-j})]h(\sigma_j)d\sigma_j = 0.\end{aligned} \tag{19}$$

If constraint (19) is violated, the mechanism must potentially be subsidized by external funds. Hence, to satisfy budget balance, this constraint must be integrated into the expected social welfare. The expected social welfare is just the sum of all participants' expected utilities given in (1) and (2):

$$\begin{aligned}
&\sum_{i=1}^{N}\int_{\underline{\theta}}^{\bar{\theta}}U_B(\theta_i)f(\theta_i)d\theta_i+\sum_{j=1}^{M}\int_{\underline{\sigma}}^{\bar{\sigma}}U_S(\sigma_j)h(\sigma_j)d\sigma_j\\
&=\sum_{i=1}^{N}\int_{\underline{\theta}}^{\bar{\theta}}\mathbb{E}_{\theta_{-i},\sigma}\left[v(\theta_i)\sum_{j=1}^{M}q_jx_{ij}(\theta_i,\theta_{-i},\sigma)-t_B(\theta_i,\theta_{-i},\sigma)\right]f(\theta_i)d\theta_i\\
&+\sum_{j=1}^{M}\int_{\underline{\sigma}}^{\bar{\sigma}}\mathbb{E}_{\theta,\sigma_{-j}}\left[t_S(\theta,\sigma_j,\sigma_{-j})-q_jc(\sigma_j)\sum_{i=1}^{N}x_{ij}(\theta,\sigma_j,\sigma_{-j})\right]h(\sigma_j)d\sigma_j\\
&=\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(v(\theta_i)-c(\sigma_j))x_{ij}(\theta,\sigma)\right].
\end{aligned}\tag{20}$$

The overall objective of the mechanism is the maximization of (20), subject to incentive compatibility and individual rationality. Therefore, our optimization problem is given by

$$\max_{x_{ij}}\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(v(\theta_i)-c(\sigma_j))x_{ij}(\theta,\sigma)\right]\tag{21}$$

subject to

$$\begin{aligned}
&\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(\psi_B(\theta_i)-\psi_S(\sigma_j))x_{ij}(\theta,\sigma)\right]\geq 0\ \text{(incentive compatibility)}\\
&0\leq\sum_j x_{ij}(\theta,\sigma)\leq 1\ \forall i\in\{1,\ldots,N\}\ \text{(feasibility for buyers)}\\
&0\leq\sum_i x_{ij}(\theta,\sigma)\leq 1\ \forall j\in\{1,\ldots,M\}\ \text{(feasibility for sellers)}\\
&x_{ij}\in\{0,1\}\ \text{(binary solutions only)}.
\end{aligned}\tag{22}$$

Mechanism design theory corroborates that ex post efficiency is unattainable when incentive compatibility and individual rationality constraints are imposed (Myerson & Satterthwaite, 1983). Therefore, we derive a second-best mechanism that maximizes the expected social welfare, subject to incentive compatibility, individual rationality, and budget balance.

To solve the optimization problem (21), we use the approach presented by Gresik and Satterthwaite (1989) as well as Myerson and Satterthwaite (1983). Based on the virtual valuation (5) and virtual cost (6), we define the following functions. For any $\alpha\in[0,1]$, let

$$\psi_B(\theta_i,\alpha)=v(\theta_i)-\alpha\frac{1-F(\theta_i)}{f(\theta_i)}v'(\theta_i)\tag{23}$$

and

$$\psi_S(\sigma_j,\alpha)=c(\sigma_j)+\alpha\frac{H(\sigma_j)}{h(\sigma_j)}c'(\sigma_j).\tag{24}$$

For varying values of α, these functions have the following interpretation. If $\alpha=0$, the mechanism would yield an ex post efficient outcome in which buyers and sellers are matched on the basis of their actual valuation and cost. Such ex post efficient mechanisms, however, do not exist. On the other hand, if $\alpha=1$, the mechanism maximizes the integral in inequality (7), thus maximizing the expected profits of a monopolist. Consequently, any second-best mechanism must choose the appropriate α between the boundary values 0 and 1 such that the budget balance constraint (19) is satisfied.

The Lagrangian of the optimization problem (21) is

$$\begin{aligned}
L&=\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(v(\theta_i)-c(\sigma_j))x_{ij}(\theta,\sigma)\right]\\
&+\lambda\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(\psi_B(\theta_i)-\psi_S(\sigma_j))x_{ij}(\theta,\sigma)\right]\\
&=\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j((v(\theta_i)+\lambda\psi_B(\theta_i))-(c(\sigma_j)+\lambda\psi_S(\sigma_j)))x_{ij}(\theta,\sigma)\right]\\
&=(1+\lambda)\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j\left(\psi_B\left(\theta_i,\frac{\lambda}{1+\lambda}\right)-\psi_S\left(\sigma_j,\frac{\lambda}{1+\lambda}\right)\right)x_{ij}(\theta,\sigma)\right].
\end{aligned}\tag{25}$$

Any function $x_{ij}(\theta,\sigma)$ that fulfills constraint (22) with equality and maximizes the Lagrangian for some $\lambda\geq 0$ must be a solution to the optimization problem (21). But the Lagrangian is maximized by $x_{ij}^{\alpha}(\theta,\sigma)$, when $\alpha=\lambda/(1+\lambda)$. Here, $x_{ij}^{\alpha}(\theta,\sigma)$ is a matching function that depends on α. By (7), constraint (22) is satisfied with equality if $\sum_{i=1}^{N}U_B(\underline{\theta})+\sum_{j=1}^{M}U_S(\bar{\sigma})=0$.

Similar to other mechanism design research, we assume regularity; that is, buyers' virtual valuations $\psi_B(\theta_i)$ and sellers' virtual costs $\psi_S(\sigma_j)$ are nondecreasing functions. Then an optimal $\alpha^*\in(0,1)$ exists for which the associated second-best mechanism maximizes the expected social welfare subject to incentive compatibility, individual rationality, and budget balance (Gresik & Satterthwaite, 1989). Let

$$G(\alpha)=\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{M}q_j(\psi_B(\theta_i)-\psi_S(\sigma_j))x_{ij}^{\alpha}(\theta,\sigma)\right].\tag{26}$$

Function $G(\alpha)$ is exactly the equality of constraint (7) in Lemma 1 for $x_{ij}(\theta,\sigma)=x_{ij}^{\alpha}(\theta,\sigma)$. Therefore, to obtain $\sum_{i=1}^{N}U_B(\underline{\theta})+\sum_{j=1}^{M}U_S(\bar{\sigma})=0$, we must solve for α in the equation

$$G(\alpha)=0.\tag{27}$$

The solution to (27) solves the optimization problem (21), thus yielding the optimal α^* for which the second-best mechanism is welfare maximizing. Because finding analytic solutions to (27) is infeasible given varying market sizes, we apply numerical methods for solving the optimization problem. The following section presents the heuristic algorithm used for finding the approximate solutions.

4.3 | Heuristic algorithm

For solving the optimization problem (21), we must calculate $\alpha^*\in(0,1)$ that solves $G(\alpha)=0$. We apply a hill climbing approach (Russell & Norvig, 2003) combined with a mixed-integer linear problem solver to find α^*. For identifying the initial set of $\boldsymbol{\alpha}_1$ used by the hill climbing approach, we build on the results obtained by Gresik and Satterthwaite (1989). Gresik and Satterthwaite determine the optimal $\alpha^*\in(0,1)$ for a second-best mechanism with uniformly distributed values in [0, 1] devoid of quality differentiation. Table 1 presents the optimal values of α^* for their mechanism.

In summary, the steps of our algorithm are as follows:

1. Construct the initial set $\boldsymbol{\alpha}_1$ based on the optimal values of α^* in Table 1 rounded up and down to one digit (after the decimal), respectively. For example, because Market Size 2 yields $\alpha^* = 0.2256$ in Table 1, we choose $\alpha_1 = \{0.2, 0.3\}$ as initial set for the hill climbing approach.
2. For each $\alpha \in \boldsymbol{\alpha}_d$, generate the two-sided private information θ_i and σ_j from the respective probability distribution k times.
3. For each generated θ_i and σ_j, solve the optimization problem stated in (21) using a mixed-integer linear problem solver.
4. Calculate the average absolute budget over all k solutions obtained in the previous step.
5. Choose the α_d^* for which the average absolute budget is closest to zero to satisfy the budget balance constraint in (19).
6. Once α_d^* has four decimal places, from all values of α_d^* obtained thus far, return that α_d^* for which the average budget is closest to zero.
7. To further refine the value of α_d^*, add one digit to construct the new set $\boldsymbol{\alpha}_{d+1}$.
8. Use the newly constructed set $\boldsymbol{\alpha}_{d+1}$ as input and repeat beginning in Step 1.

Algorithm 1: Hill climbing with mixed-integer linear problem solving

Data: Public quality vector $\boldsymbol{q}$; initial set $\boldsymbol{\alpha}_1$; number of iterations k
Result: α^*

1 initialization;
2 $d \leftarrow 1$;
3 **while** *true* **do**
4   **foreach** $\alpha \in \boldsymbol{\alpha}_d$ **do**
5   **for** $\iota = 1$ **to** k **do**
6       $\theta_i^\iota, \sigma_j^\iota \leftarrow$ generate random numbers from probability distribution;
      `// Mixed-integer linear problem solving`
7       $x_{ij}^{\alpha,\iota} \leftarrow$ solve problem (21) using $\{\theta_i^\iota, \sigma_j^\iota, \boldsymbol{q}\}$;
8   budget$^\iota \leftarrow$ calculate budget using optimal allocation $x_{ij}^{\alpha,\iota}$;
9   **end**
10   budgetMean $\leftarrow$ **mean**(budget$^\iota$);
11   **end**
  `// Hill climbing`
12   $\alpha_d^* \leftarrow \alpha \in \boldsymbol{\alpha}$ for which budgetMean is closest to 0;
13   digits $\leftarrow$ number of digits of α_d^*;
14   **if** *digits* > 4 **then**
15   **return** α_d^* for which budgetMean is closest to 0;
16   **end**
17   bound $\leftarrow 5 \cdot 10^{-\text{digits}-1}$;
18   $d \leftarrow d + 1$;
19   $\boldsymbol{\alpha}_d \leftarrow \left\{\alpha_d^* - \text{bound}, \alpha_d^* - \frac{\text{bound}}{2}, \alpha_d^* + \frac{\text{bound}}{2}, \alpha_d^* + \text{bound}\right\}$;
20 **end**

Algorithm 1 shows the high-level functionality of the hill climbing approach to find α^*. The algorithm takes as input an initial set of $\boldsymbol{\alpha}_1$ with only one digit. For each value of $\alpha \in \boldsymbol{\alpha}_d$, the optimization problem must be solved repeatedly many times, and average values must be calculated (for loop starting in Line 5). With each repetition, θ_i and σ_j are freshly and independently drawn from the underlying probability distribution (Line 6). Once the optimal allocations for each value of $\alpha \in \boldsymbol{\alpha}_d$ have been found, the algorithm calculates the arithmetic mean of the overall budget (Line 10). From among all values of $\boldsymbol{\alpha}_d$, the algorithm selects α_d^*, for which the average budget is closest to zero because the budget balance constraint (19) must be fulfilled (Line 12). Using this α_d^*, the algorithm refines the current best value by adding further digits to α_d^* to obtain α_{d+1}^* (Line 13). In Lines 17–19, a new set is created by constructing an interval around the current best value of α_d^*. For example, if $\alpha_d^* = 0.1$, the algorithm calculates a bound of $5 \cdot 10^{-2} = 0.05$, thus yielding a new set $\alpha_{d+1} = \{0.05, 0.075, 0.125, 0.15\}$ surrounding the given value of $\alpha_d^* = 0.1$. This newly created set of $\boldsymbol{\alpha}_{d+1}$ then serves as input for the next round of the algorithm (Line 4). The previous steps are repeated until α_d^* has four digits after the decimal (Line 14). Then, the algorithm returns from all α_d^* previously calculated that α_d^* for which the average budget is closest to zero.

TABLE 1 Efficiency of the second-best mechanism under the uniform distribution without quality differentiation as reported by Gresik and Satterthwaite (1989, p. 318)

Market size	α^*	Welfare(α^*)	Welfare(0)	Efficiency (%)
2	0.2256	0.3775	0.3999	94.37
3	0.1603	0.6257	0.6429	97.33
4	0.1225	0.8753	0.8889	98.47
6	0.0827	1.3750	1.3846	99.31
8	0.0622	1.8750	1.8823	99.61
10	0.0499	2.3750	2.3810	99.75
12	0.0416	2.8750	2.8800	99.83

4.4 | Illustrative example

To illustrate the optimization problem for matching buyers and sellers, we provide a simple example for a market setting in the domain of electric vehicle charging. Suppose two electric vehicles (buyers) seek to be matched to two available charging stations (sellers) at the same time. The Federal Network Agency of Germany reports that about 67% of all charging stations in Berlin offer a power output of 11 kW, whereas about 24% offer 22 kW (Federal Network Agency Germany, 2019). Hence, our example assumes that Station 1 offers a quality of 11, whereas Station 2 offers a quality of 22. That is, $q = (11, 22)$. Because higher power output implies faster charging times, higher power output indicates higher quality (and vice versa). For the sake of simplicity, we assume that both buyers' valuations $v(\theta_i) = \theta_i$ and both sellers' costs $c(\sigma_j) = \sigma_j$ are uniformly distributed on the interval [0, 1] for this example. In the subsequent evaluation section, however, we will draw valuation and cost from real-world probability distributions.

In the next step, we determine functions $\psi_B(\theta_i, \alpha)$ and $\psi_P(\sigma_j, \alpha)$ as defined in (23) and (24), respectively. Because θ_i and σ_j are uniformly distributed over $[0,1]$, these functions evaluate to $\psi_B(\theta_i,\alpha) = (1+\alpha)\theta_i$ and $\psi_P(\sigma_j,\alpha) = (1+\alpha)\sigma_j - \alpha$.

Using these definitions, we can now state the Lagrangian (25) for this specific example. First, we drop the constant $(1+\lambda)$. Then, we set $\alpha = \lambda/(1+\lambda)$ to obtain

$$\begin{aligned} L \ &= \mathbb{E}[11((1+\alpha)(\theta_1-\sigma_1)+\alpha)x_{11} + 22((1+\alpha)(\theta_1-\sigma_2)+\alpha)x_{12} \\ &\quad + 11((1+\alpha)(\theta_2-\sigma_1)+\alpha)x_{21} + 22((1+\alpha)(\theta_2-\sigma_2)+\alpha)x_{22}]. \end{aligned} \tag{28}$$

Because Table 1 reports an optimal $\alpha^* = 0.2256$ for a mechanism without quality differentiation, Algorithm 1 starts with an initial set $\alpha_1 = \{0.2, 0.3\}$. The algorithm then applies a mixed-integer linear problem solver to find matching function x_{ij}^{α} that maximizes Lagrangian (28) subject to feasibility

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{bmatrix} \le \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \tag{29}$$

Next, the solutions of the mixed-integer problem x_{ij}^{α} are substituted into constraint (22), which yields function $G(\alpha)$ defined in (26):

$$\begin{aligned} G(\alpha) \ &= \mathbb{E}\big[11(2(\theta_1-\sigma_1)+1)x_{11}^{\alpha} + 22(2(\theta_1-\sigma_2)+1)x_{12}^{\alpha} \\ &\quad + 11(2(\theta_2-\sigma_1)+1)x_{21}^{\alpha} + 22(2(\theta_2-\sigma_2)+1)x_{22}^{\alpha}\big]. \end{aligned} \tag{30}$$

We use Algorithm 1 to approximate the solutions to $G(\alpha) = 0$. Once the algorithm calculated $G(\alpha)$ for all $\alpha \in \boldsymbol{\alpha}_d$, we apply cubic interpolation to approximate function G. The fact that G is continuous in our setting is shown in Myerson and Satterthwaite (1983) as well as Gresik and Satterthwaite (1989). Figure 1 depicts the resulting approximation of G with its zero at $\alpha^* = 0.2413$.

Apart from determining the expected social welfare for $\alpha^* = 0.2413$, we also calculate the expected social welfare for $\alpha = 0$. As discussed in Section 4.2, $\alpha = 0$ characterizes the ex post efficient outcome function that matches buyers and sellers based on their true valuations and costs. The ratio between the outcome for $\alpha^* = 0.2413$ and the ex post outcome for $\alpha = 0$ provides a measure for the efficiency of the second-best mechanism. All steps are repeated $k = 10^5$ times, and average values are calculated. Table 2 presents the results obtained in this example.

TABLE 2 Example: Efficiency of the second-best mechanism under uniformly distributed variables

α^*	Welfare (α^*)	Welfare (0)	Absolute budget	Efficiency
0.2413	6.5128	6.9832	0.0011	93.26%

Our proposed second-best mechanism for this simple market with two buyers and two sellers achieves an efficiency of 93.26%. The mechanism distributes the full social surplus among all participants, leaving a negligible absolute budget of 0.0011 on average. In a similar market setting, Gresik and Satterthwaite (1989) find an efficiency of 94.37% at the optimal $\alpha^* = 0.2256$ (see Table 1). However, their model does not integrate exogenous quality levels of sellers. Hence, the presence of quality reduces market efficiency by about 1.1 percentage points for this simple market.

5 | EXPERIMENTAL EVALUATION

For studying the efficiency achieved by our proposed mechanism, we conduct a simulation study based on two real-world market settings. We report on the design and results of the experiments in this section.

5.1 | Experimental setup

Our simulation study considers two market settings that reflect the different quality characteristics sellers can have in real markets. First,

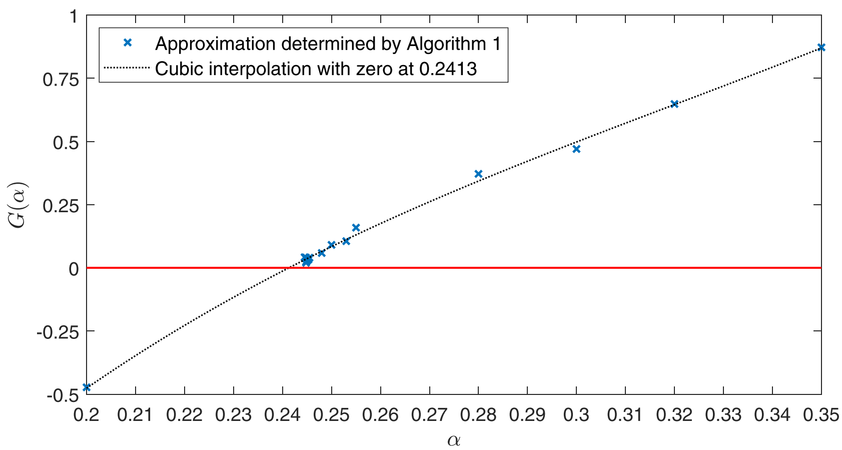

FIGURE 1 Approximation of $G(\alpha)$ with its zero at $\alpha^* = 0.2413$ [Colour figure can be viewed at wileyonlinelibrary.com]

we study a market where electric vehicles are matched to charging stations. Second, we consider matching drivers and riders on a ridesharing platform.

In the first market setting, our experiments for matching electric vehicles to charging stations rely on a data set published by the Federal Network Agency Germany (2019). This data set contains detailed information about a total of 424 public charging points for electric vehicles in Berlin, Germany, brought into service between May 20, 2009, and November 30, 2018. For each charging point, the data set provides the geographic location, the date of installation, the type of connector, and the power output in kilowatts (kW). Table 3 depicts the proportions for each differentiated power output of all 424 charging points in Berlin.

We assume that each charging station offers exactly one charging point at a given power output. Because higher power output levels result in faster charging times for electric vehicles, charging stations compete with each other based on differentiated power outputs. Hence, these power output levels correspond to the exogenous quality levels prescribed in our model.

For modeling the private information of electric vehicle users (buyers) and charging stations (sellers), we use actual electricity prices offered at the charging points of different operators in Germany (Statista, 2018). Table 4 shows these prices by operator. We determine the associated normal probability distribution using maximum likelihood estimation. This technique yields a normal distribution with mean $\mu = 0.3694$ and standard deviation $\sigma = 0.1214$. We assume that buyers' valuations and sellers' costs are independently drawn from this distribution. That is, our experiments for the electric vehicle charging market assume $\{\theta_i, \sigma_j\} \sim \mathcal{N}(0.3694, 0.0147)$ for all i, j.

In the second market setting, our experiments for matching drivers to riders on a ridesharing platform use a unique data set consisting of 307 car rides between two major German cities (i.e., Stuttgart and Munich) offered on BlaBlaCar from October 28 to November 3, 2018, and from February 14 to 21, 2019. We obtained these data manually from BlaBlaCar (2020). These data contain the price demanded for each ride and the driver reputation in the form of numerical star ratings. Based on the 307 prices for the rides in our data set ranging from €8.00 to €44.00, the associated normal probability distribution has mean $\mu = 12.26$ and standard deviation $\sigma = 2.97$. Hence, for the ridesharing market, we assume that private information is distributed according to $\{\theta_i, \sigma_j\} \sim \mathcal{N}(12.26, 8.82)$ for all i, j.

In the ridesharing market setting, the exogenous quality levels correspond to the numerical star ratings that drivers have accumulated from previous riders. On BlaBlaCar, average ratings typically range between 4.0 and 5.0 stars. Prior research implies that higher average ratings on digital marketplaces can lead to improved probability of sales and higher expected revenue for sellers (Ba & Pavlou, 2002; Mudambi & Schuff, 2010).

TABLE 4 Electricity prices at charging points of different operators in Germany (Statista, 2018)

Operator	Price (cent/kWh)
EnBW	55.4
Public Utility Munich	46.7
Allego New Motion	44.3
EWE	39.9
Innogy	39.0
Public Utility Düsseldorf	38.0
Power Grid Hamburg	29.5
Public Utility Dresden	27.3
Mainova	13.3

The two market settings considered in our study represent two different degrees of quality differentiation. On the one hand, quality levels of different charging stations in the electric vehicle setting exhibit much stronger quality differentiation than those in the ridesharing setting. For instance, whereas a midrange electric vehicle takes 3 h for recharging to 80% at a station offering 11-kW power output, the same vehicle only takes 1.5 h at 22 kW. Hence, a change in quality from 11 to 22 kW results in substantial time savings for electric vehicle users. On the other hand, the perceived impact of two consecutive average star ratings (e.g., 4.5 and 4.6) on ridesharing platforms may be difficult to discern in practice. Thus, the ridesharing setting provides an example for markets of weak quality differentiation.

For studying the asymptotic efficiency achieved by the proposed mechanism, we consider varying market sizes for both the electric vehicle charging market and the ridesharing market. To compare the efficiency of these two markets, we assume an equal number of buyers and sellers for both settings. First, we determine the market efficiency for $N = M = 2$, followed by an incremental increase of the market size up to $N = M = 12$. The reason for choosing these market sizes is that we restrict attention to matching electric vehicles and charging stations within bounded catchment areas in a city. Within sufficiently small areas, it is unlikely that market sizes drastically exceed 12 participants on each side. Similarly, we can also assume small market sizes in the ridesharing setting because it is unlikely that large numbers of drivers and riders simultaneously seek to trade ridesharing services for specific routes at particular departure times. On the other hand, the results of our simulation study presented in the subsequent sections imply that market sizes greater than 12 have virtually no effect on efficiency whatsoever.

For both market settings, we use two simple linear functions to model buyers' willingness to pay and sellers' costs. Buyer i's valuation function is given by $v(\theta_i) = \theta_i$, and seller j's cost function is $c(\sigma_j) = \sigma_j$. The associated virtual valuation and cost functions as defined in (23)

TABLE 3 Power outputs of charging points in Berlin (Federal Network Agency Germany, 2019)

Power output (kW)	3.7	11	22	43	50	350
Proportion (%)	7.08	66.75	23.58	0.94	1.18	0.47

and (24) are $\psi_C(\theta_i,\alpha)=\theta_i-\alpha\frac{1-F(\theta_i)}{f(\theta_i)}$ and $\psi_P(\sigma_j,\alpha)=\sigma_j+\alpha\frac{H(\sigma_j)}{h(\sigma_j)}$, respectively. Depending on the market setting, $f(\cdot)$ and $h(\cdot)$ are the probability density functions of the normal distribution $\mathcal{N}(0.3694, 0.0147)$ for electric vehicle charging and $\mathcal{N}(12.26, 8.82)$ for ridesharing. $F(\cdot)$ and $H(\cdot)$ are the associated distribution functions.

5.2 | Results

In this section, we report the results of our numerical simulations based on the exogenous quality levels of charging stations (i.e., power output) and drivers on the ridesharing platform (i.e., reputation in the form of numerical star ratings). Table 5 presents the results achieved by the second-best mechanism for the electric vehicle charging market. Table 6 depicts the efficiency in the ridesharing market. To compare efficiencies, Table 7 contains the benchmark efficiency achieved by the mechanism when no quality differentiation is present. Market size is defined as the number of participants on each side of the market (i.e., $N=M$). Column "α^*" is the value of $\alpha\in\boldsymbol{\alpha}_d$, for which the average budget is closest to zero. Column "Welfare(α^*)" depicts the social welfare achieved by the mechanism when using this α^*. Column "Welfare(0)" contains the social welfare the associated ex post efficient mechanism would achieve if it were to exist (see Section 4.2). Column "Absolute budget" depicts the absolute value of the budget that is left or required because of the numerical approximation determined by Algorithm 1 for the α^*. The last column presents the efficiency of the second-best mechanism, which is calculated as the ratio between Welfare(α^*) and Welfare(0).

For increasing market size, the values of α^* decrease in all market settings considered. Because of approximation inaccuracies, the absolute budget varies between 0 and 0.1034. If analytic solutions were feasible, the budget would exactly equate zero across all settings. Moreover, the efficiency increases from 93.92% (ridesharing with $N=M=2$) to 99.98% (benchmark with $N=M=9$) as the market size grows. The fact that efficiencies do not consistently increase in ascending order also results from inaccuracies due to the numerical approximation.

Comparing the efficiencies obtained in all three settings (electric vehicle, ridesharing, and benchmark without quality differentiation), it is apparent that lower degrees of quality differentiation yield higher efficiencies throughout our study. In contrast, the presence of strong quality differentiation in the market negatively affects efficiency. This

TABLE 5 Electric vehicle charging: Efficiency of the proposed second-best mechanism

Size	α^*	Welfare(α^*)	Welfare(0)	Absolute budget	Efficiency (%)
2	0.2335	1.69	1.78	0.0016	94.96
3	0.2000	3.93	4.07	0.0109	96.48
4	0.1535	5.16	5.29	0.0014	97.53
5	0.1256	7.47	7.59	0.0006	98.37
6	0.1250	7.14	7.23	0.1034	98.82
7	0.0930	9.96	10.05	0.0026	99.08
8	0.0795	10.85	10.91	0.0551	99.46
9	0.0690	12.06	12.11	0.0866	99.55
10	0.0655	13.21	13.26	0.0572	99.66
11	0.0600	15.67	15.72	0.0138	99.66
12	0.0555	16.90	16.91	0.0004	99.91

TABLE 6 Ridesharing: Efficiency of the proposed second-best mechanism

Size	α^*	Welfare(α^*)	Welfare(0)	Absolute budget	Efficiency (%)
2	0.2355	17.89	19.05	0.0015	93.92
3	0.1630	30.29	30.97	0.0010	97.81
4	0.1283	40.82	41.41	0.0126	98.56
5	0.1037	52.76	53.17	0.0212	99.23
6	0.0855	64.13	64.51	0.0634	99.41
7	0.0733	76.39	76.84	0.0040	99.41
8	0.0653	87.81	88.39	0.0120	99.34
9	0.0575	97.36	97.65	0.0334	99.70
10	0.0535	111.35	111.70	0.0831	99.69
11	0.0478	122.98	123.25	0.0863	99.78
12	0.0425	133.55	133.71	0.0363	99.88

TABLE 7 Benchmark: Efficiency of the proposed second-best mechanism without quality differentiation

Size	α^*	Welfare(α^*)	Welfare(0)	Absolute budget	Efficiency (%)
2	0.2333	3.73	3.96	0.0018	94.05
3	0.1545	6.15	6.27	0.0464	98.19
4	0.1248	8.51	8.62	0.0038	98.71
5	0.1000	10.88	10.98	0.0004	99.04
6	0.0830	13.28	13.34	0.0020	99.56
7	0.0730	15.63	15.70	0.0114	99.55
8	0.0628	18.03	18.06	0.0072	99.85
9	0.0544	20.41	20.42	0.0157	99.98
10	0.0500	22.81	22.82	0.0345	99.95
11	0.0453	25.14	25.15	0.0000	99.97
12	0.0416	27.50	27.59	0.0041	99.70

observation becomes more evident once we apply nonlinear curve fitting to the individual data points determined by our algorithm (cf. Figure 2). Hence, weaker quality differentiation particularly in small markets entails higher efficiency.

Figure 2 presents the results contained in Tables 5–7 graphically. It shows the asymptotic efficiency achieved by the second-best mechanism for the electric vehicle charging market (black $\times$) and the ridesharing market (blue $*$). The figure also shows the benchmark efficiency achieved by the mechanism when sellers are not differentiated by quality (red +). Whereas all individual data points are calculated by Algorithm 1, we use nonlinear curve fitting in the least squares sense to determine the associated best fits. For the electric vehicle charging market, the best fit is given by $1-\frac{0.14}{N^{1.36}}$ (black dash-dot line). For the ridesharing market, the best fit is $1-\frac{0.26}{N^{2.14}}$ (blue dashed line). Finally, the best fit for the benchmark is $1-\frac{0.31}{N^{2.41}}$ (red dotted line). All these efficiency lines increase toward the asymptote (green solid line) from below. However, the efficiency for the electric vehicle charging market is higher than that of the ridesharing market. The benchmark efficiency yields the highest values.

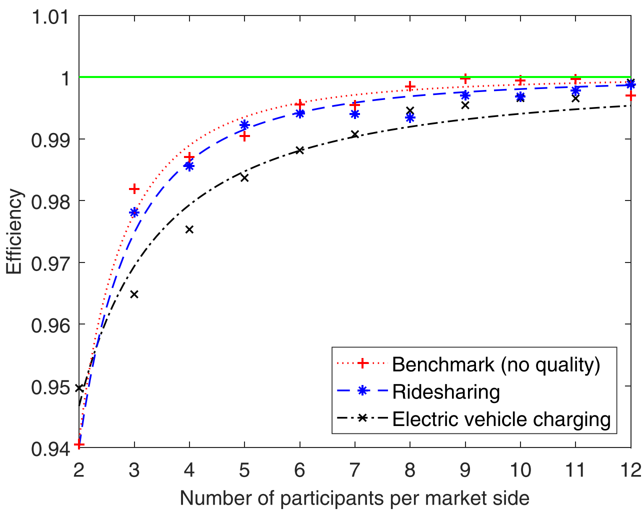

FIGURE 2 Efficiency of the second-best mechanism for three markets: electric vehicle charging, ridesharing, and benchmark (no quality) [Colour figure can be viewed at wileyonlinelibrary.com]

Apart from market efficiency, we also study the probability of sales and the expected revenues accruing to sellers in a fixed-sized market with 12 buyers and 12 sellers. Considering the market size as 12 enables us to study the probability of sales and the expected revenues as a function of varying quality levels. Hence, we assume that each of the 12 sellers offers a distinct quality level, which is publicly known. Table 8 shows the results obtained for the electric vehicle charging market. Here, the first charging station offers a quality of 3.7 kW, the second charging station offers 4.6 kW, and so on. Table 9 shows the respective results for a synthetic ridesharing market, where the quality levels of drivers range from 3.9 to 5.0 average star ratings.

Figure 3 depicts the contents of Table 8 graphically. The left axis represents the expected revenues charging stations accrue for providing electricity at different power outputs. The right axis indicates the probability of sales depending on the power output. For example, the charging station offering power output 3.7 kW is likely to sell its service with probability 38.62% and expect a revenue of 34.54 cent/kWh. Similarly, Figure 4 depicts the contents of Table 9 for the ridesharing market setting. Here, a driver with average star rating 3.9 is likely to sell the service with probability 46.43%, expecting a revenue of €12.17.

5.3 | Discussion and conclusion

The Internet has promoted the rise of exchange markets in many domains. Low entry and exit barriers as well as low transaction costs encourage sellers to launch businesses with a relatively short gestation period, thereby stimulating competition (Goel & Hsieh, 2002; Goldfarb & Tucker, 2019). Reduced search costs, the presence of quality-differentiated sellers, and remote accessibility attract an increasing number of buyers. Therefore, Internet-based platforms call for the design of marketplaces where buyers and sellers are matched appropriately.

A key challenge in designing such marketplaces is the choice of adequate market mechanisms. When two-sided private information is present in the market, the efficiency of such mechanisms is crucial but not well understood. Because inefficiencies might arise in such

TABLE 8 Electric vehicle charging: Probability of sales and expected revenues in a market with 12 electric vehicle users and 12 charging stations

Power output (kW)	Expected revenue (cent/kWh)	Probability of sales (%)
3.7	34.54	38.62
4.6	34.71	39.67
11	35.82	43.49
20	36.67	46.90
22	36.88	47.19
22.1	36.79	47.46
43	39.20	54.71
45	39.35	55.44
50	39.84	57.33
53	40.29	58.45
175	46.43	76.27
350	50.01	83.58

environments, market designers and platform owners should carefully assess these inefficiencies prior to making managerial decisions.

This research examines the effects of quality differentiation on the efficiency of service markets with two-sided private information. For this purpose, we develop a second-best mechanism that accounts for sellers' differentiated quality levels. We study the efficiency achieved by our mechanism as well as the probability of sales and the expected revenue accruing to sellers. For assessing potential inefficiencies, we conduct a simulation study based on empirical data from two real-world market settings. These two settings represent both strong quality differentiation (electric vehicle charging) as well as weak quality differentiation (ridesharing). Our research provides evidence that even in the presence two-sided private information and different degrees of quality differentiation on the seller side of the market, full efficiency can be achieved as the market size increases.

TABLE 9 Ridesharing: Probability of sales and expected revenues in a market with 12 riders and 12 drivers

Average star rating	Expected revenue (€)	Probability of sales (%)
3.9	12.17	46.43
4	12.18	46.65
4.1	12.20	46.83
4.2	12.19	47.25
4.3	12.23	47.64
4.4	12.25	47.99
4.5	12.30	48.98
4.6	12.35	49.06
4.7	12.39	49.77
4.8	12.45	50.62
4.9	12.53	51.86
5	12.62	53.08

5.3.1 | Contributions to theory

Our research makes three important contributions to the growing body of literature studying the effects of quality differentiation on the efficiency of two-sided service markets (Bapna et al., 2005; Bockstedt & Goh, 2011; Widmer & Leukel, 2016).

The first contribution that we make is a theoretically sound model of a matching mechanism that allows for exogenous quality information from third-party platforms. This model helps to better understand the reciprocal effects between exogenous quality levels and private information about valuation and cost of service provisioning. By validating our proposed mechanism in a simulation study, we demonstrate its usefulness for designing two-sided service markets. From a theoretical perspective, our model can be seen as an extension of the model proposed by Widmer and Leukel (2016). Although Widmer and Leukel derive a second-best mechanism for studying market efficiency properties, they do not integrate publicly known quality on the seller side of the market. Moreover, Widmer and Leukel build their simulation study on artificial data only. In contrast, we integrate exogenous quality into the model to examine the effects of different degrees of quality differentiation on the efficiency of the market. We also use empirical data from two real-world market settings to validate our mechanism. Unlike Widmer and Leukel, we analyze the correlation between the probability of sales and the sellers' expected revenues.

We show that quality differentiation affects the efficiency of two-sided service markets. Although these inefficiencies arise because of the presence of private information, extending the model by exogenous quality further exacerbates the market's inefficiencies. The reason for these additional inefficiencies might be that buyers have greater incentives to understate their true valuation whenever they observe significant differentials between the sellers' quality levels. Thus, buyers can increase their utility by receiving high-quality services at lower prices. This increase in utility, however, negatively affects the overall efficiency. On the other hand, we also show that the impact of differentiated quality on market efficiency is at most 1.33 percentage points compared with the benchmark without quality differentiation in all the market settings considered. Hence, the proposed mechanism absorbs different degrees of quality differentiation (weak and strong) by significantly mitigating efficiency losses. We also demonstrate that the probability of sales is nearly proportional to the sellers' expected revenues. The reason for this proportion is that buyers prefer high-quality services for which they are willing to pay higher prices. In contrast, whereas buyers compete for higher quality, sellers are indifferent about which buyer receives their services as long as they are matched to any buyer and receive their payments.

Second, we contribute a heuristic algorithm for determining the approximate welfare-maximizing match outcomes generated by our second-best mechanism subject to incentive compatibility constraints. In two-sided markets with varying numbers of buyers and sellers in cross-market relationships, it is infeasible to derive the analytic solutions to the optimization problem. Hence, our algorithm uses a hill climbing approach combined with a mixed-integer linear problem solver to find the associated numerical solutions. The algorithm

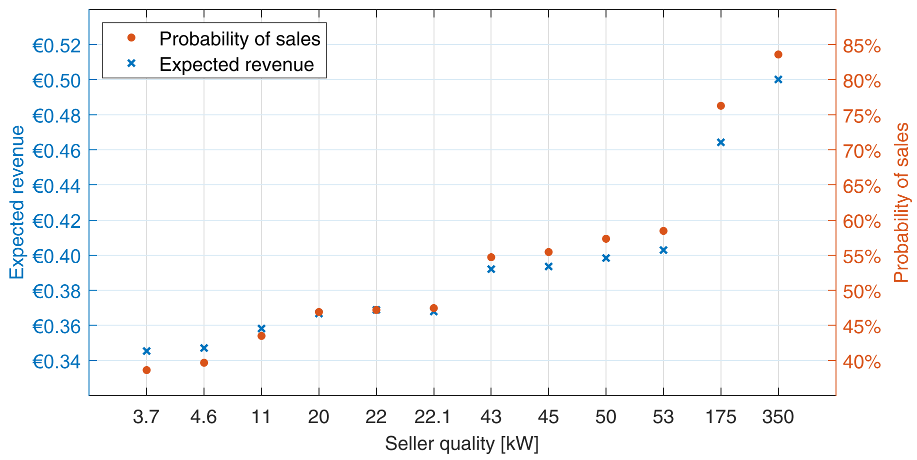

FIGURE 3 Electric vehicle charging: Expected revenues and probability of sales for sellers in a market with 12 sellers and 12 buyers [Colour figure can be viewed at wileyonlinelibrary.com]

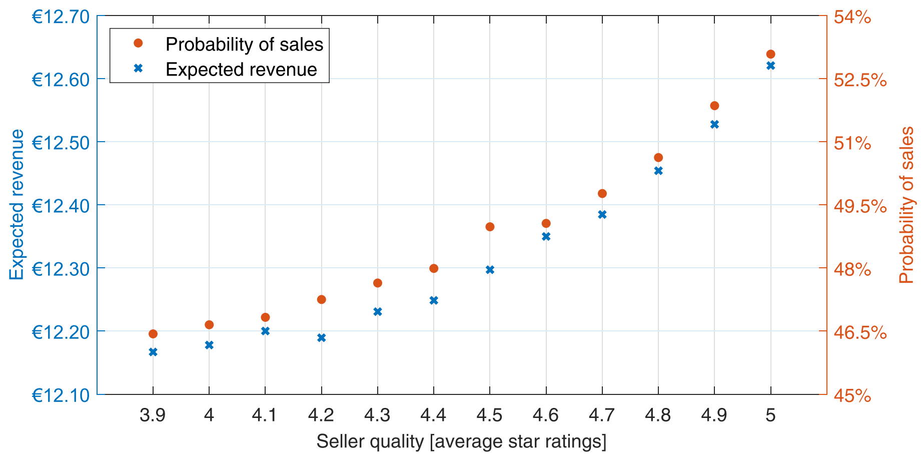

FIGURE 4 Ridesharing: Expected revenues and probability of sales for sellers in a market with 12 sellers and 12 buyers [Colour figure can be viewed at wileyonlinelibrary.com]

outlines all the steps necessary to assess the efficiency properties of markets in which sellers offer quality-differentiated services, thus facilitating the applicability to domains other than electric vehicle charging and ridesharing.

Third, our simulation study suggests that the mechanism's inefficiency disappears as the market size increases. This finding is consistent with related research in mechanism design theory, which corroborates that double auctions achieve full efficiency in large markets (Cripps & Swinkels, 2006; Reny & Perry, 2006; Rustichini et al., 1994; Yoon, 2001). Whereas these double auctions assume the presence of two-sided private information, they do not consider exogenous quality on the seller side of the market. Thus, it is not clear a priori whether the efficiency properties of double auctions continue to hold when exogenous seller quality is integrated into the model. We demonstrate that, although exogenous quality negatively affects the mechanism's efficiency in small markets, these inefficiencies vanish once the market size becomes larger. Because several peer-to-peer marketplaces have emerged that face the problem of matching few buyers to few sellers, it is important to understand the effects of quality differentiation on efficiency in small markets.

5.3.2 | Implications for practice

From a managerial perspective, our findings related to the efficiency of quality-differentiated service matching have important implications. First, our results help managers in designing two-sided service markets that allow for exogenous quality information from peer-to-peer platformssuch as BlaBlaCar. As such, our proposal supports market designers to align the incentives of buyers and sellers with the underlying information system. The fundamental question in designing incentive-aligned information systems is whether the participants make an effort to contribute correct information and whether they can gain from distorting their true information (Ba, Stallaert, & Whinston, 2001). For designing two-sided service markets, managers must provide adequate incentives for buyers and sellers to ensure participation and liquidity (Tham, Sojli, & Skjeltorp, 2017). Especially in the presence of private information, our proposal can help managers to obtain accurate estimates about the participants' strategies to distort their true information. Whereas prior research has offered various measures for quantifying market efficiency (Belloni, Lopomo, & Wang, 2016; Bichler, Shabalin, & Ziegler, 2013; Candogan & Pekeč, 2018), our study advances these estimates by internalizing different degrees of quality differentiation on two-sided service markets. Designing such markets without considering seller quality differentiation bears the risk of neglecting the emerging network effects on market efficiency and social welfare.

The rise of two-sided marketplaces in the sharing economy requires market designers to carefully evaluate how direct and indirect network effects might influence the strategic behavior of participants (Etzion & Pang, 2014) and the overall social welfare (Benjaafar, Kong, Li, & Courcoubetis, 2018; Jiang & Tian, 2016). For instance, national government agencies have recently promoted financial incentives for electric vehicle adoption and charging infrastructure development in a wide range of government policies (Sierzchula, Bakker, Maat, & Van Wee, 2014). Federal policy makers for electric vehicle promotion face the problem of designing optimal policies that incorporate indirect network effects due to the interdependence between electric vehicle adoption and charging station investment (Li, Tong, Xing, & Zhou, 2017). Our research can support these policy makers in assessing cross-market effects of quality-differentiated charging stations on the sharing economy.

When strong quality differentiation is present on the seller side, market designers should be cognizant of the fact that more participants might be necessary to compensate for the emerging efficiency losses. In such market settings, the short-term and long-term consequences of quality-differentiated service provisioning should be carefully weighed. Our study sheds light on the magnitude of the potential inefficiencies that arise when participants have private information and sellers offer quality-differentiated services in different market settings. Moreover, sellers can use our analysis to evaluate their performance in the market. Especially in vertical service markets, it is important to characterize pricing strategies and policy implications emerging from different selling formats (Zhang, Joseph, & Subramaniam,2014). Because the probability of sales is nearly proportional to the expected revenues in our study, sellers can estimate their return on investment as a function of service quality. Improving quality might improve sellers' capabilities to attract more buyers and realize higher payoffs.

5.4 | Limitations

The results of our research should be interpreted in light of its limitations.Although our simulation study is based on empirical data providing actual prices and quality levels in real market settings for electric vehicle charging and ridesharing, we have assumed that two-sided private information about valuation and cost is normally distributed. The choice of normally distributed variables is commonly applied in related research for providing benchmark results in different market settings (Egri & Váncza, 2013; Overby & Forman, 2014; Prasad & Rao, 2014). However, these distributions might not necessarily reflect real-world valuation and cost structures on two-sided markets such as peer-to-peer platforms. On the other hand, because our analysis concerns studying ex ante efficiency, our theoretical model does not provide full details on possible implementation formats for quality-differentiated markets such as Vickrey mechanisms (Widmer & Leukel, 2018) or position auctions (Johnson, 2013).

5.5 | Future research

Based on our findings, future research can be pursued in at least two directions. First, our simulation study could be extended by including more realistic valuation and cost structures of buyers and sellers. A possible extension could focus on environments that do not depend on publicly known probability distributions (Loertscher & Marx, 2019). For assessing the emerging network effects under consideration of quality differentiation, more quantitative studies are needed that integrate the interdependence of two-sided private information and exogenous quality levels. Second, our theoretical model could be advanced by studying real-world matching rules and pricing schemes. Addressing such indirect implementation issues would yield additional insights in understanding the strategic bidding behavior of quality-differentiated sellers in the presence of two-sided private information.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in "Charging station map" at https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Energie/Unternehmen_Institutionen/HandelundVertrieb/Ladesaeulen/Ladesaeulenkarte_Datenbankauszug21.xlsx (Federal Network Agency Germany, 2019). The data that support the findings of this study are openly available in "BlaBlaCar" at https://www.blablacar.de/ (BlaBlaCar, 2020).

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