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Time variant reliability for structures with nonlinear behaviour

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The paper describes a method for the probabilistic safety assessment of structural components under time-dependent stochastic structural properties and random loadings as well as nonlinear structural behavior. Since no gradient information is available, a mixed solution strategy is applied using both gradient based FORM/SORM and adaptive conditional sampling. The results of a time dependent reliability analysis for the linear structure are used as the starting point for the adaptive sampling scheme for the nonlinear structure. Hence, the reliability analysis is split into a time-invariant part involving all non-ergodic variables at the critical time and a time-variant part yielding up-crossing rates. The loads may be modelled as stationary or non-stationary Gaussian processes or rectangular wave renewal processes. As an example, a pressure vessel is analyzed to demonstrate that the method is feasible for realistic, large scale finite element models.

The probabilistic safety assessment of structural components with respect to fatigue or catastrophic failure requires the consideration of time-dependent stochastic structural properties and random loadings as well as nonlinear structural behavior, in particular elastoplasticity and creep. In this case, classical methods i.e. reliability algorithms based on FORM/SORM coupled with a suitable nonlinear Finite element method (FEM) code fail, because gradient information may be either very difficult and expensive to obtain or simply not available. However, a mixed solution strategy is possible by performing the necessary probability integrations only in part by rigorous FORM/SORM and by adaptive conditional sampling otherwise. The latter requires only simple function calls to the FEM code. Unfortunately, any adaptive sampling scheme depends very much on the starting vector especially for higher dimension of the basic uncertainty vector. It is proposed to perform first a possibly line dependent reliability analysis for the linear structure yielding the critical combination of stochastic variables. The results can then be used as the starting point for the adaptive sampling scheme for the non-linear analysis. If it can be assumed that the sensitivities vary only insignificantly with time, the reliability analysis can be split into a time invariant part involving all non-ergodic variables at the critical time and a time variant part yielding up-crossing rates. Thereby, the loads may be modelled as stationary or non-stationary Gaussian processes or rectangular wave renewal processes. Their parameters may also be ergodic sequences.

The work is based on the combination of the reliability analysis program COMREL, developed by the RCP GmbH, and the Technical University München – with the finite element code PERMAS – developed by INTES GmbH, Stuttgart which allows the calculation of the reliability of mechanical components. The development of the PERMAS module RA was initiated and sponsored by Forschungszentrum Jülich GmbH, Jülich. So far, the features of the PERMAS-RA code covered time-independent and time-dependent analysis analysis for linear static and eigenvalue analysis[1-4]. The current work is based on the extension of the code with respect to nonlinear structural behaviour.[5]

As an example, the reliability of a pressure vessel of a nuclear power plant under transient extreme loading is analyzed to demonstrate that the method is feasible for realistic large scale convergence towards the true reliability estimate.

TIME-VARIANT RELIABILITY

Theory and concepts for the computation of time-invariant reliability are now well known and can be performed efficiently and reliably. Computationally feasible approaches to time-variant reliability problems at present are all of asymptotic nature. They rest on the construction of a counting process for the exits of the structural state function into the failure domain. The intensity parameter of this Poisson

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process is determined from the outcrossing rate of the load effect process through the possibly time-variant limit state function. The mean number of exits into the failure domain has to be determined by time integration of the outcrossing rate. The calculation of the outcrossing rates is a non-trivial task. At present, solutions for differentiable Gaussian vector processes and rectangular wave renewal processes are available[6,7]. A second difficulty usually arises when assuring the Poissonian nature (lack of memory) of the outcrossings under the presence of time invariant or at least non-ergodic basic variables.

Consider the general task of estimating the probability $P_f(t)$ such that a realization $z(\tau)$ of a random state vector $Z(\tau)$ representative for a given problem, enters the failure domain $V = \{z(\tau) \mid g(z(\tau), \tau) \leq 0, 0 \leq \tau \leq t\}$. $g(.)$ is the limit state function. $Z(\tau)$ may conveniently be separated into three components as :

$$Z^T(\tau) = [R^T \quad Q^T(\tau) \quad S^T(\tau)] \tag{1}$$

where R is a vector of random variables that are either independent or deterministically dependent of time t, $Q(\tau)$ is a slowly varying ergodic random vector sequence and $S(\tau)$ is a vector, not necessarily stationary but sufficiently mixing random process variables, having fast fluctuations as compared to $Q(\tau)$.

The following formula has been established in part by making use of the ergodicity theorem[8]

$$P_f(t) \approx 1 - E_R[\exp(-E_Q[E[N_S^+(t \mid R,Q)]])]$$
$$\leq E_R\,[E_Q\,[E\,[N_S^+(t \mid R,Q)]]] \tag{2}$$

Herein, $E[N_S^+(t \mid R, Q)] = \int_0^t \nu^+(\tau \mid R, Q)\, d\tau$ (3)

is the mean value of exits into the failure domain, and

$$\nu^+(\tau \mid R,Q) = \lim_{\Theta \to 0} \frac{1}{\Theta} P(\{S(\tau) \in V(R, Q, \tau)\}$$
$$\cap \{S(\tau + \Theta) \in \overline{V}(R, Q, \tau + \Theta)\}) \tag{4}$$

the outcrossing rate. Equation 2 is a rather good approximation for the stationary case but must be considered as a first approximation whenever $S(\tau)$ is non-stationary or the limit state function exhibits strong dependence on τ. However, the bound given is strict but close to the exact result only for very small failure probabilities. Therefore, it is used throughout.

The local outcrossing rate can be computed by FORM/SORM. The same methodology is also applied for the time integration Eq. (3). Substantially more difficult is the expectation operation with respect to the non ergodic R variables as those involve the uncertain system properties. This expectation can be performed either by crude Monte Cardo integration or with importance sampling.

ADAPTIVE AND CONDITIONAL SAMPLING

Adaptive Sampling

Comparative studies show that the conventional Monte Cardo method leads to a very high number of samples when the failure probability is small and/or X is of large dimension. Adaptive sampling starts with the mean of the sampling probability density function in the origin of the U-space. Each time a sample falls into the failure domain, the density function is shifted into the sampling point in the U-space or some other preselected point. The idea is to continuously update the sampling density function leading to the optimal one. Hence we have

$$P_f = \int_{g(x) \leq 0} dF(x) \approx \frac{1}{N} \sum_{i=1}^{N} \Delta_{g(x_i) \leq 0} \frac{f_X(x_i)}{h(x)\,(x_i)} \tag{5}$$

where $h_X(x)$ is the sampling density and $f_X(x)$ the density of X which is assumed to exist. If $h_X(x)$ is appropriately chosen, i.e. placed in the important region, the coefficient of variation of the probability estimate decrease rapidly. If the algorithm searches for the important region during sampling, this is called adaptive Monte Carlo. The sampling density is the density of a standard normal variable with the initial mean either at zero or at an explicitly defined starting solution. Whenever sampling results in a value of the state function which is absolutely smaller than the previous one, that point will be used as the new mean of the sampling density. The probability estimate and its coefficient of variation are updated during the whole sampling process. Updating of the sampling density usually can be recognised by jumps in both the probability estimate and the coefficient of variation. Adaptive Monte Carlo does not require differentiability of the state function.

This updating process asymptotically converges to the probability density function at the failure point. As the function is biased towards the failure point, the necessary number of samples is reduced since a large number of simulations occur close to the solution point.

The main drawback of the adaptive sampling method is its inherent danger to over represent a sub-region of the failure domain, if the sampling density was biased to a local minimum of the failure function during the first sampling. It was noticed in many problems, that the efficiency of adaptive sampling schemes depends strongly on the starting conditions[9]. Therefore, the sampling should be repeated with different starting sets X_0, if the failure function is highly nonlinear.

Conditional sampling

The conditional sampling scheme used here is a combination of a sampling technique and the FORM/SORM algorithm. With the set of basic variables separated into the subsets X and Y and the failure domain $D = \{y \mid g_Y(y) \leq 0\}$, the probability of failure is calculated as:

$$P_f = \int P_f(D \mid x) f_X(x)\, dx \tag{6}$$

Here the conditional failure probability $P_f(D \mid x)$ can be determined by FORM/SORM. The condition is removed by sampling. The variance of this estimate can be shown to be:

$$var(\tilde{\tilde{P}}_f) = \frac{1}{N(N-1)} \sum_{i=1}^{N} (P_f(D) \mid X = x_i)^2 \tag{7}$$

The variance is unbiased, too. Hence, the accuracy of the failure probability estimate can be judged from its coefficient of variation. It is also important to notice that this estimate is rather inaccurate if N is not much larger than $1/\tilde{P}_f$.

Criteria for the Number of Samples

In practice, the maximum number of samples N_{max} and a target value for the coefficient of variation (30%), are defined by the user. Samples are generated until all of the following conditions are fulfilled.

1. Atleast 10% of the N_{max} samples have been performed.
2. The coefficient of variation of the failure probability is below the target value for atleast 1% of the N_{max} samples. Only samples exceeding minimum number of samples defined in (1)) above are counted, provided that they have been successful and lie in the failure domain.

NONLINEAR STRUCTURAL BEHAVIOR

Since no gradient information is needed, the sampling method can be used to assess structures with any kind of nonlinear behavior (nonlinear material, contact analysis or geometrical nonlinearities). In the present study, an elastoplastic material law with a temperature dependent yield stress and a linear hardening model has been applied.

The solution procedure used in the finite element code is the initial strain method, working with the original stiffness. The material constitutive model is considered in the analysis through the pseudo load vectors $\{\Delta Q^p\}$ and $\{\Delta Q^c\}$ which are computed from the inelastic strain increments $\{\Delta \epsilon^p\}$ and $\{\Delta \epsilon^c\}$ for plasticity and creep. The incremental relation is:

$$[K]\ \{\Delta u\} = \{\Delta P\} + \{\Delta Q^p\} + \{\Delta Q^c\} \tag{8}$$

where $[K]$ is the original stiffness matrix, $\{\Delta u\}$ is the resulting incremental displacement vector and $\{\Delta P\}$ is the external load increment. The solution of Eq. (8) does not require an update of the stiffness matrix, hence, the Cholesky factor must be computed only once. This method is efficient but is restricted to applications where the plastic zones do not lead to a global failure of the structure. In our case, however, this is no severe restriction since for the safety of a critical structure like a pressure vessel only limited plastic regions are acceptable.

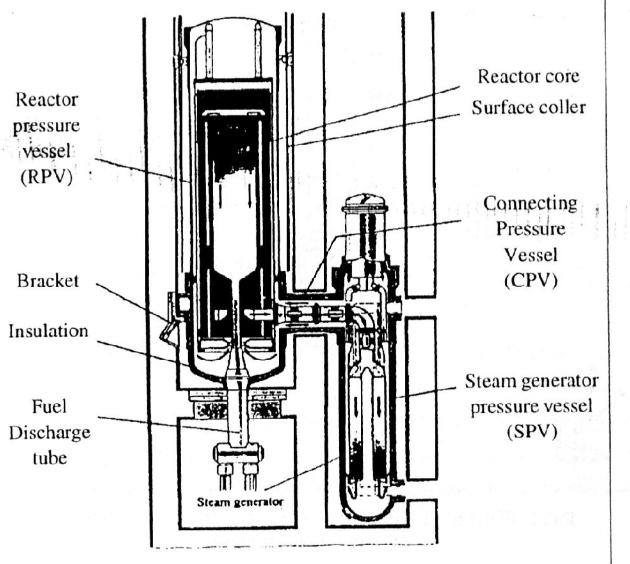

FIG.1 REACTOR PRESSURE VESSEL ASSEMBLY

EXAMPLES

The example of a pressure vessel of a High Temperature Reactor has been chosen because the probabilistic behavior of the system has been studied[10,11] under various aspects. The reactor pressure vessel (RPV) is part of the primary circuit pressure boundary of the HTR 200 module (Fig. 1). The primary circuit consists of the reactor pressure vessel (RPV), the steam generator pressure vessel (SPV), the connecting pressure vessel (CPV) and the fuel discharge tube.

Material: The pressure vessels should be made of the temperature–resistant steel 20 Mn Mo Ni 55 which is also used for KWU-PWR reactor pressure vessels. The material properties are temperature dependent (R_{p02} at 350°C = 390 N/mm^2, linear hardening constant = 50000N/mm^2).

Loads: The expected lifetime of the structure is 40 years, and, the following loading conditions are assumed:

1. 500 shut-down/start-up sequences.
2. About 30,000 minor state changes with pressure and temperature variations during a day.
3. Five pressure tests of the cold reactor (every 8 years) with 77bar.

In the finite element model, six different loading cases are considered:

1. The dead weight of the structure including internal components.
2. The internal pressure under normal operation of 60 bar.
3. The temperature distribution under normal operation.
4. An additional accidental internal pressure increase of 10 bar.
5. An additional accidental rise of temperature.
6. The overload due to the pressure tests every 8 years.

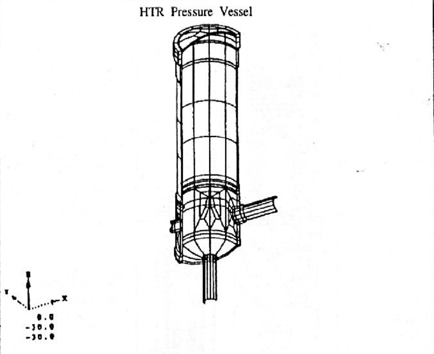

FIG.2 FINITE ELEMENT MODEL OF RPV, CPV AND FUEL DISCHARGE TUBE

Leading cases 2–6 should be modelled as stochastic processes. Loads 2 and 3 as well as loads 4 and 5 are highly correlated with a correlation coefficient of 0.8.

Finite element discretization: The example is restricted to the reactor pressure vessel in order to keep the finite element model (Fig.2) into a moderate size. Since many nonlinear computations are performed during the simulation, the mesh is rather coarse but higher order elements are used:

Finite elements	266	(27-node hexahedron)
Nodal points	3492	
Degrees of freedom	9916	

Stochastic model: The stochastic model consists of 9 different basic variables:

- Five basic variables describe the material parameters of the different parts of the pressure vessel. To be on the safe side, only a small correlation with a coefficient of 0.5 is assumed between these variables.
- Four basic variables define the different loading conditions. Of these, the temperature and internal pressure under normal operation are correlated with a correlation coefficient of 0.8.

Time-variant analysis: The following combinations are studied:

Combination	Participating Loads	Rate/year
A	1+2+3	12.5
B	1+2+3+4+5	750.0
C	6	0.13

Nonlinear Analysis: For the nonlinear analysis, the loads are assumed to be time invariant stochastic variables. The limit state function is defined in terms of the maximum equivalent stress anywhere in the structure.

$$f(X) = \bar{\sigma}_{limit} - \bar{\sigma}_m \qquad (9)$$

where $\bar{\sigma}_{limit}$ is the maximum allowable equivalent stress and $\bar{\sigma}_m$ is the highest stress anywhere in the structure. $\bar{\sigma}_{limit}$ is an uncertain variable, with the mean value slightly higher than the mean plastic yield stress of the material. This has the effect that small plastic zones with an equivalent plastic strain of up to 0.02% are tolerated.

Computing Resources: The computer runs have been performed on a DEC-Alpha 3000 Model 800 workstation, using about 40 MB of central memory. One static analysis of the structure needs about 1 minute, a nonlinear analysis between 2 and 6 minutes.

Results: The failure probabilities P_f are computed as:

Combination	P_f	No. of Iterations
A	$6.0 * 10^{-6}$	514
B	$2.8 * 10^{-4}$	500
C	$5.4 * 10^{-7}$	2145

The goal of the present work was not to study the specific results for the reactor pressure vessel but the applicability of the implemented algorithm. The behavior of the adaptive sampling scheme for cases A and C shown in Fig. 3.

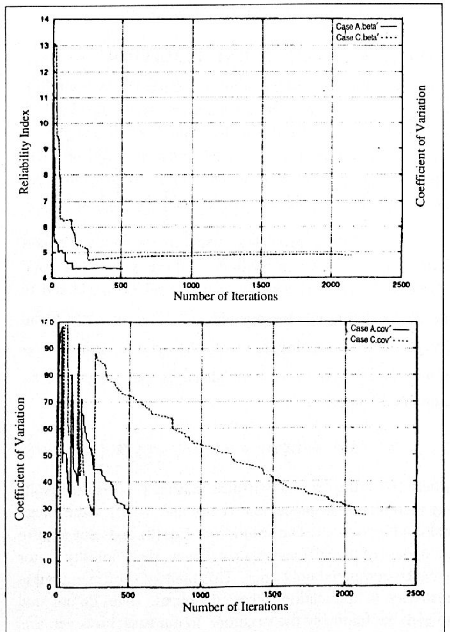

FIG.3 RELIABILITY INDEX AND ITS COEFFICIENT OF VARIATION DURING SIMULATION

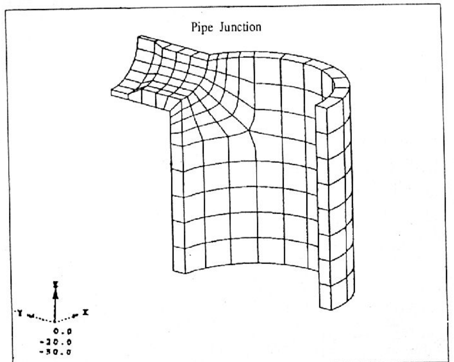

FIG.4 FINITE ELEMENT MODEL OF A PIPE JUNCTION

The estimated coefficient of variation as well as the predicted reliability index show characteristic jumps whenever the sampling density is adapted.

Considerable savings in the number of necessary samples are achieved, if the starting solution is chosen closer to the limit state rather than the mean values of the X variables. The starting solution must be chosen carefully in order to avoid numerical instabilities from samples that are close to the U-space bounds.

Pipe Junction

For the pipe junction shown in Fig.4 in the results of different methods for calculating the reliability are compared.

The finite element model of the pipe junction has 4757 degrees of freedom (27-node hexahedrons). The stochastic model consists of 3 basic variables: the internal pressure (normal distribution), the Young's modulus (lognormal distribution) and the yield limit (lognormal distribution). The limit state function is the difference between the yield limit of the material and the maximum Von. Mises stress resulting from the internal pressure.

The results clearly show, that the beta indices are close together, the FORM and SORM method need the shortest computing times and the adaptive Monte Carlo method starting at the beta-point resulting from FORM/SORM method saves considerably number of samples.

Method	Beta	P_f	Simulations	Computing time
FORM	6.627	1.72 * 10–11	–	134 s
FORM	6.639	1.59 * 10–11	10	552 s
SORM	6.641	1.56 * 10–11	–	208 s
SORM	6.641	1.57 * 10–11	10	515 s
AMC*	6.641	1.57 * 10–11	524	5390 s
AMC**	6.651	1.47 * 10–11	128	1329 s

Note: AMC Adaptive Monte Carlo: AMC* Start with mean values of basic variables ; AMC** Start at beta-point of FORM

CONCLUSIONS

The Adaptive Monte Carlo Method to compute the time-variant reliability of structures with nonlinear material behavior is suitable for any type of nonlinearities where the FORM/SORM method fail due to the lack of gradient information. The number of samples necessary slightly increases with the reliability index and the number of uncertain variables. The authors have implemented an efficient computer program to allow the treatment of realistic structures without the need for large-scale supercomputers.

The adaptive simulation scheme is more efficient if a starting solution is chosen which is closer to the limit state than the mean values of the uncertain variables Due to the large number of independent finite element computations, the method is also well suited for the application of parallel computers.

Adaptive or conditional sampling is only one of the methods under study. An especially effective combination of sampling methods and response surfaces is being developed and will be reported in the near future.

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