Journal of Structural Engineering
Vol. 31, No.1, April–June 2004 pp. 35–49

No. 31-6

Optimization of sustainable Civil Engineering infrastructures

R. Rackwitz*

Technical facilities should be optimal with respect to benefits and cost. Cost-benefit analysis is the framework in which our considerations are valid. It is found that the only replacement strategy fulfilling the requirement of sustainability is systematic reconstruction after failure. If the structure is deteriorating a suitable inspection and maintenance strategy has to be adopted. Repairs must be planned already in the design phase and must be such that they correspond to renewal, i.e. re-establish the (stochastically) initial state. Some suitable objective functions are developed based on a renewal model. Special techniques for solving the optimization problem are also presented. Technical and natural risks are perceived individually and on a societal level rather irrationally and subjectively. Only if society acts rationally in controlling involuntary and anonymous risks from the natural and technical environment in an affordable and efficient manner can it gain better overall life quality in the long run. The strategies to reduce risks have been discussed but no specific recommendations were possible because most societies lack a clear statement of their ethical preferences.

The notion of life cycle engineering probably has emerged in the military field where, apart from first installation, inspection and maintenance and, finally, removal and replacement of a system formed the major cost. Later, it was also used in civil engineering when the societies recognized that the infrastructure in a country or region had not only to be built but maintained and finally renewed because failures occured not only due to extreme external events but primarily due to wear out, deterioration and, in some cases, obsolescence. The strategies adopted by engineers to manage the life cycle of a structure were widely technical, i.e. by improving reliability and durability and by proposing design solution which should enable a longer time span of full use, possibly with other uses than initially foreseen. This is, no doubt, an important aspect of life cycle engineering. But not the only one and, most likely, not even the most important one.

In addition, there is growing awareness of the fact that our world is a limited world in the sense that it has only limited non-renewable natural resources, even limited renewable resources like water and limited arable land, for example. This lead the co-called Brundland Commission (UN Commission on Environment and Development) to conclude in 1987 in their famous report "Our Common Future" that a sustainable development is defined as a development "that meets the needs of the present without compromising the ability of future generations to meet their own needs". In the mean time one can say that this statement has widely become a new ethical standard. The immediate implications for the planning, design and operation of civil engineering infrastructures are clear: Save energy, save non-renewable resources and find out about re-cycling of building materials, do not pollute the air, water or soil with toxic substances, save or even regain arable land, do not interfere into the natural water household to an extent disproportionate to the adverse effects of such an interference, and much more.

For civil engineering infrastructures, but not only for those, there is a third aspect and that is the financial aspect. It is assumed that civil engineering infrastructures are financed by the public via taxes, public charges or other. It is in any case the citizen who pays and, of course, also enjoys the benefits derived from their existence. More precisely, intergenerational equity is the core of the new ethical standard the Brundland Commission has set; here with particular reference to the financial aspects of planning, designing, maintaining and replacing civil engineering infrastructure. Our generation must not leave the burden of maintenance or replacement of too short-lived structures to future generations, it must not use more of the financial resources than are really available but it can use only those which are available and affordable in a sustainable manner and discounting with its many myopic aspects must be done with utmost care. In this sense, civil engineering structures should be optimal not only from a technological point of view but also from a sustainability point of view.

The paper will first review a renewal model for setting up suitable objective functions for cost-benefit analysis. After

*Prof. Dr.- Ing., Institut fuer Baustoffe und Konstrukton, Technische Universitaet, Muenchen, Arcisstr, 21, D-80290, Muenchen. Technical University Munich, Munich, Germany

presenting the basic model it is extended to different failure modes and obsolescence, to deteriorating facilities and to a simple case of inspection and maintenance. Then, a societal risk acceptance criterion is summarized and some thoughts about sustainable and inter-generationally acceptable public interest rates are given. This is the basis for developing some optimization tools. Two examples illustrate the theory.

COST-BENEFIT OPTIMAL TECHNICAL FACILITIES

Already in 1971 Rosenblueth and Mendoza[1] proposed optimization with respect to benefits and cost as the final goal of setting up structural codes and for the design and operation of structures. A technical facility is financially optimal if the following objective is maximized:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}) \tag{1}$$

It is assumed that all quantities in Eq. (1) can be measured in monetary units. **p** is the vector of all safety relevant parameters or actions. $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. The quantities $B(\mathbf{p})$, $C(\mathbf{p})$ but especially $D(\mathbf{p})$ involve uncertainties. Statistical decision theory then dictates that expected values are to be taken[2]. In the following it is assumed that $B(\mathbf{p})$, $C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of **p**. The cost as well as the benefits may differ for the different parties involved. The different parties, e.g., the owner, the builder, the user and society, may also have different economic objectives. A facility makes sense only if $Z(p)$ is positive within certain parameter ranges for all parties involved. In this paper, however, we will primarily focus on an optimization for and in the name of the public.

In view of sustainability one has to distinguish between at least four replacement strategies:

- **the facility is given up after service or failure**
- **the facility is systematically replaced after failure**
- **the facility is renewed (repaired) after deterioration**
- **the facility is renewed due to obsolescence.**

Further, we distinguish between facilities which can **fail upon completion or never** and facilities which can **fail at a random point in time** later due to service loads, extreme external disturbances or deterioration. The option "failure upon completion or never" implies that loads and resistances on the facility are time-invariant which is not considered further. Also, the option "facility given up after service or failure" is not considered further because infrastructure facilities must remain functioning for all forseeable future in accordance with the sustainability requirement. Finally, reconstruction times are assumed to be negligibly short as compared with the normal life time of the facility for simplicity. Appropriate extensions for finite renewal times and failure at construction are available[3].

THE RENEWAL MODEL

Discounting

The facility has to be optimized during design and construction at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted down to the decision point. For analytical convenience continuous discounting is assumed which is accurate enough for all practical purposes. Let $\gamma(t)$ be a function of time. If damage $D(t)$ occurs at time t its present value $D(0)$ can be calculated from the elementary differential equation

$$\frac{dD(t)}{dt} = -\gamma(t)D(t) \tag{2}$$

whose solution is:

$$D(0) = D(t)\exp\left[-\int_0^t \gamma(\tau)d\tau\right]$$

For a constant time-averaged discount rate $\gamma = \frac{1}{t_s}\int_0^{t_s}\gamma(\tau)d\tau$ with $t_s > t$ some reference time, one simply has $D(0) = D(t)\exp[-\gamma t]$. Discount rates are understood as real rates net of any taxes. If a interest rate γ' for discrete discounting is given it is simply $\gamma = \ln(1+\gamma')$.

Basic Renewal Model

Assume random events in time forming a renewal process. The times between failure (renewal) events have identical distribution function $F(t, \mathbf{p})$, $t \geq 0$, with probability density $f(t, \mathbf{p})$ and are independent. The independence assumption needs to be verified carefully. In particular, one has to assume that loads and resistances in the system are independent for consecutive renewal periods and there is no change in the design rules after the first and all consecutive failures (renewals). Even if designs change failure time distributions must remain the same. The objective function for systematic reconstruction obviously is in full generality

$$Z(\mathbf{p}) = \int_0^\infty b(t)e^{-\int_0^t \gamma(\tau)d\tau}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^\infty \int_0^\infty e^{-\int_0^t \gamma(\tau)d\tau} f_n(t, \mathbf{p})dt \tag{3}$$

with $b(t)$ the benefit per time unit and $f_n(t, \mathbf{p})$ the density of the time to the n-th renewal. It is assumed that construction cost $C(\mathbf{p})$ are without cost of financing. Financing cost can, however, be included easily. H is the monetary loss in case of failure including direct failure cost, demolition cost, cost of removal of debris, loss of business and other indirect cost and, of course, the cost to reduce the risk to human life and limb. Therefore, it is useful to decompose H into physical losses H_M and losses H_F associated with losses of human life and limb. Also, it is assumed that $C(\mathbf{p})$ and H are independent of time.

For constant benefit per time unit $b(t) = b$ and a constant discount rate γ the objective function simplifies greatly, especially because one can make use of the convolution theorem for Laplace transforms in the damage term*

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^\infty be^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^\infty \int_0^\infty e^{-\gamma t} f_n(t, \mathbf{p})dt \\ &= \int_0^\infty be^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^\infty f^*(\gamma, \mathbf{p})^{n-1} f^*(\gamma, \mathbf{p}) \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)r^*(\gamma, \mathbf{p}) \end{aligned} \tag{4}$$

where $f^*(\gamma, \mathbf{p})$ is the Laplace transform of $f(t, \mathbf{p})$ and $r^*(\gamma, \mathbf{p})$ is the Laplace transform of the renewal density (renewal intensity) $r(t, \mathbf{p}) = \sum_{k=1}^\infty f_k(t, \mathbf{p})$.

If failures occur according to a Poisson process with occurence rate $\lambda(\mathbf{p})$ Eq. (4) simplifies to[1]:

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda(\mathbf{p})}{\gamma} \tag{5}$$

because $f^*(\gamma, \mathbf{p}) = \frac{\lambda(\mathbf{p})}{\gamma+\lambda(\mathbf{p})}$ for $f(t, \mathbf{p}) = \lambda(\mathbf{p})\exp[-\lambda(\mathbf{p})t]$. This result is especially relevant because the parameter $\lambda(\mathbf{p})$ may be replaced asymptotically by the stationary outcrossing rate $\nu^+(\mathbf{p})$ frequently used in time-variant structural reliability analysis[4]. If $\nu^+(\mathbf{p})$ depends on an uncertain parameter vector **R** and/or a random sequence **Q** one should use $E_{\mathbf{R},\mathbf{Q}}[\nu^+(\mathbf{p}, \mathbf{R}, \mathbf{Q})]$ instead.

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{E_{\mathbf{R},\mathbf{Q}}[\nu^+(\mathbf{p}, \mathbf{R}, \mathbf{Q})]}{\gamma} \tag{6}$$

It is seen that continuous discounting and continuous failure models lead to relatively simple, analytical results. Completely parallel results, however, can be obtained for discrete failure models and discrete discounting[5].

If, further, at an extreme Poissonian loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ one obtains for independent failure events[6,7]:

$$\begin{aligned} r^*(\gamma, \mathbf{p}) &= \sum_{n=1}^\infty f^*(\gamma)P_f(\mathbf{p})[f^*(\gamma)R_f(\mathbf{p})]^{n-1} \\ &= \frac{P_f(\mathbf{p})f^*(\gamma)}{1 - R_f(\mathbf{p})f^*(\gamma)} = \frac{\lambda P_f(\mathbf{p})}{\gamma} \end{aligned} \tag{7}$$

*Laplace transforms are defined by $f^*(\gamma) = \int_0^\infty e^{-\gamma t} f(t)dt$ and there is $0 \leq f^*(\gamma) \leq 1$ if $f(t), t \geq 0$, is a probability density for which $f^*(0) = 1$ and $f^*(\infty) = 0$. In the transformed space there is $h^*(\gamma) = f(\gamma)^* g^*(\gamma)$ for $h(t) = \int_0^t f(t-\tau)g(\tau)d\tau$, an operation necessary to determine $f_n(t) = \int_0^t f_{n-1}(t-\tau)f(\tau)d\tau$.

with $R_f(\mathbf{p}) = 1 - P_f(\mathbf{p})$ and λ the occurence rate of the disturbing event. An important asymptotic result for arbitrary failure models is[8]

$$\lim_{t\to\infty} r(t, \mathbf{p}) = \lim_{\gamma\to 0} \gamma r^*(\gamma, \mathbf{p}) = \frac{1}{E[T(\mathbf{p})]} \tag{8}$$

where $E[T(\mathbf{p})]$ is the mean time between renewals.

The precise details of this and more general renewal models can be found in[9]. Many other objective functions can be formulated. For example, serviceability failure, obsolescence, aging, deterioration and inspection and maintenance, finite renewal times, repeated renewal during construction and finite service times can be dealt with. Benefit and damage term can be functions of time[3,5,9–11]. Also, multiple failure modes have been considered[3]. Some more important cases especially relevant for life cycle costing and sustainability are considered next.

Independent failure modes and different failure causes including obsolescence

Assume for the moment two independent failure modes, denoted by "V_1" and "V_2", respectively, each requiring renewal after failure. The times between renewals then are distributed as $F(t) = 1 - (1 - F_{V_1}(t))(1 - F_{V_2}(t)) = 1 - \bar{F}_{V_1}(t)\bar{F}_{V_2}(t)$. The corresponding density is $f(t) = f_{V_1}(t)\bar{F}_{V_2}(t) + f_{V_2}(t)\bar{F}_V(t)$ and its Laplace transform is $f^{**}(\gamma, \mathbf{p}) = f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma)$. It follows that

$$D(\mathbf{p}) = \frac{(C_1(\mathbf{p}) + H_1)f^{**}_{V_1|\bar{V}_2}(\gamma) + (C_2(\mathbf{p}) + H_2)f^{**}_{V_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma))} \tag{9}$$

This equation is derived as follows: Let $\theta_i = t_i - t_{i-1}$ be the times between renewals with density $f_{V_1,V_2}(t)$ and, for example, C_{V_1} and C_{V_2} the cost associated with the two types of renewals. Then, the expected cost is

$$\begin{aligned} D &= E\left[\sum_{n=1}^\infty (C_{V_1} + C_{V_2})\exp\left[-\gamma\sum_{k=1}^n \theta_k\right]\right] \\ &= \sum_{n=1}^\infty E[\exp(-\gamma\theta)]^{n-1} E[(C_{V_1} + C_{V_2})\exp[-\gamma\theta]] \\ &= \frac{E[(C_{V_1} + C_{V_2})\exp[-\gamma\theta]]}{1 - E[\exp(-\gamma\theta)]} \\ &= \frac{C_{V_1} f^{**}_{V_1|\bar{V}_2} + C_{V_2} f^{**}_{V_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma))} \end{aligned}$$

One can generalize to more (independently) caused renewals:

$$\begin{aligned} D(\mathbf{p}) &= \frac{\sum_{i=1}^s C_i(\mathbf{p}) f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma)}{1 - \sum_{i=1}^s f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma)} \\ &\leq \frac{\sum_{i=1}^s C_i(\mathbf{p}) f^*_{V_i}(\gamma)}{1 - \sum_{i=1}^s f^*_{V_i}(\gamma)} \end{aligned} \tag{10}$$

with $f(t) = \sum_{i=1}^{s} f_i(t) \prod_{j\neq i} \bar{F}_j(t)$ and, therefore $f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma) = \int_0^\infty \exp[-\gamma t] f_i(t) \prod_{j\neq i} \bar{F}_j(t)dt$. Here, we distinguish between ordinary Laplace transforms $f^*(\gamma)$ for densities and modified Laplace transforms $f^{**}(\gamma)$ for which $f^{**}(\gamma) \leq f^*(\gamma)$. Frequently, the upper bound can be used and this has also been proposed in[12]. For independent Poissonian failure modes one can show that the upper bound is the exact result.

Obsolescence occurs if the technical facility no more fulfills its function. For example, a bridge may become too narrow for the increasing traffic, a fabrication hall is replaced because the machinery inside this hall has to be modernized and restructured, certain vehicles are put out of service because they become too uncomfortable, too uneconomical or unserviceable because of outdated equipment. Usually, this happens inspite of full system integrity. In fact, most facilities will be replaced not because they fail or deteriorate but because they become obsolete. Unfortunately, very few data are available about this well-known fact[13]. Obsolescence is almost always completely independent of the system state. But this is just the case dealt with in the foregoing section where one of the failure modes, i.e. cause for renewal, is treated as obsolescence. With A denoting all cost for demolishment and removal of debris it is:

$$D(\mathbf{p}) = \frac{(C(\mathbf{p}) + H) f^{**}_{V_1|\bar{A}_2}(\gamma) + (C(\mathbf{p}) + A) f^{**}_{A_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{A}_2}(\gamma) + f^{**}_{A_2|\bar{V}_1}(\gamma))} \tag{11}$$

Multiple mode failures (series systems) with stationary failure models or even non-stationary failure models with dependent modes can also be considered[3,11].

Deteriorating Structures and Numerical Laplace Transforms

Deteriorating structures are characterized by an increasing hazard function $\rho(t) = \frac{f(t)}{1-F(t)}$. Let the failure probability at a given time t be computed by FORM or SORM so that

$$P_f(t) = P(T \leq t) = F_T(\mathbf{p}, t) = \Phi(-\beta(\mathbf{p}, t)) C_{\mathrm{SORM}}$$

where

$$\beta(\mathbf{p}, t) = \|\mathbf{u}^*\| = \min\{\|\mathbf{u}\|\} \text{ for } g(\mathbf{u}, \mathbf{p}, t) \leq 0$$

with $g(\mathbf{u}, t)$ a monotonically decreasing state function, $\mathbf{u} = \mathbf{T}(\mathbf{x})$ a vector of independent standard normal variables and $\mathbf{x}$ the vector of uncertain variable in the original space. $\mathbf{u} = \mathbf{T}(\mathbf{x})$ is a unique probability distribution transformation. C_{SORM} is the second-order correction which usually is neglected. The mean time to failure is

$$E[T(\mathbf{p})] = \int_0^\infty (1 - F_T(\mathbf{p}, t)dt = \int_0^\infty \Phi(\beta(\mathbf{p}, t))dt$$

It can be shown that the density of the time to failure is to first order

$$f_T(\mathbf{p}, t) = -\varphi(\beta(\mathbf{p}, t))\frac{d\beta(\mathbf{p}, t)}{dt} = -\varphi(\beta(\mathbf{p}, t))\frac{\frac{\partial}{\partial t} g(\mathbf{u}^*, \mathbf{p}, t)}{\|\nabla_{\mathbf{u}} g(\mathbf{u}^*, \mathbf{p}, t)\|}$$

so that the Laplace transform can be determined numerically from

$$f^*(\gamma, \mathbf{p}) \approx \Delta \sum_{j=0}^{m} w_j \exp[-\gamma t_j] f_T(\mathbf{p}, t_i) \tag{12}$$

with w_j are the weights of a suitable integration formula (trapezoid, Simpson, Newton, ...) and Δ is an appropriate spacing in time.

Inspection and Repair of Aging Components

In the literature maintenance cost frequently have been assumed to increase continuously with time. More realistic in the structures area is the case where maintenance cost are the sum of inspection and possible repair cost. Assume inspections at regular intervals $a, 2a, 3a, \ldots$ Repairs occur only at these points in time (or with some delay, say at $a+\Delta, 2a+\Delta, 3a+\Delta, \ldots$). Inspections and repairs occur only if renewals have not occurred before due to obsolescence or failure. Assume further that repairs, if undertaken, restore the properties of a component to its original (stochastic) state, i.e. repairs are equivalent to renewals. Inspection and repair times are assumed negligibly short. Of course, it makes only sense to consider aging components with increasing risk function $\rho(t)$.

A renewal (repair) occurs either after failure or at times $a, 2a, 3a, \ldots$. Renewal (repair) times are assumed negligibly short and $\Delta = 0$. In[14] this is denoted by age replacement. Then, we obviously have following the derivation of Eq. (9)[15]

$$Z(\mathbf{p}, a) = B - C(\mathbf{p}) - \frac{(C(\mathbf{p}) + H) f_V^{***}(\gamma, \mathbf{p}, a) + I_1(\mathbf{p}) \exp[-\gamma a]\bar{F}_V(\mathbf{p}, a)}{1 - (f_V^{***}(\gamma, \mathbf{p}, a) + \exp[-\gamma a]\bar{F}_V(\mathbf{p}, a))} \tag{13}$$

with $I_1(\mathbf{p}) < (C(\mathbf{p}) + H)$ the cost of repair, $\bar{F}_V(\mathbf{p}, a)$ the probability of survival up to a and $f^{***}_{X|\bar{Y}}(\gamma, \mathbf{p}, a) = \int_0^a \exp[-\gamma t] f_X(t) \bar{F}_Y(t)dt$ the incomplete, modified Laplace transform of $f_X(t)$. Note that $f^*(\gamma) = \frac{\exp[-\gamma a]}{1-\exp[-\gamma a]}$ is the Laplace transform of a deterministic density function $f(t) = \delta(a)$.

If there are regular inspections there is not necessarily a repair because inspections are uncertain (or the signs of deterioration are vague). Denote the failure model for the aging component by "V" whereas "A" stands for any other (independent) failure mode (or obsolescence as another cause for renewal). Then, inspection and repair cost must also be included in the damage term:

$$Z(\mathbf{p}, a) = B(\mathbf{p}, a) - C(\mathbf{p}) - D(\mathbf{p}, a) \tag{14}$$

Including now one failure mode "V" with subsequent renewal and obsolescence "A"

$$D(\mathbf{p}, a) = \frac{ND}{D} \tag{15}$$

$$\begin{aligned} ND = {} & (C(\mathbf{p}) + A)(f^{***}_{A|\bar{V}}(\gamma, a) + A11) + (C(\mathbf{p}) + H) \\ & \times (f^{***}_{V|\bar{A}}(\gamma, \mathbf{p}, a) + A12) + I_0((1 - P_R(a)) \\ & \times \exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p},a) + A21) + (I_0 + I_1(\mathbf{p})) \\ & \times (P_R(a) \exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p}, a) + A22) \end{aligned}$$

$$D = 1 - \begin{pmatrix} f^{***}_{A|\bar{V}}(\gamma, a) + A11 + f^{***}_{V|\bar{A}}(\gamma, \mathbf{p}, a) + A12 \\ + P_R(a) \exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p}, a) + A22 \end{pmatrix}$$

$$A11 = \sum_{n=2}^{\infty} \prod_{j=1}^{n-1} (1 - P_R(ja)) \times f^{****}_{A|\bar{V}}(\gamma, \mathbf{p}, (n-1)a \leq t \leq na)$$

$$A12 = \sum_{n=2}^{\infty} \prod_{j=1}^{n-1} (1 - P_R(ja)) \times f^{****}_{V|\bar{A}}(\gamma, \mathbf{p}, (n-1)a \leq t \leq na)$$

$$A21 = \sum_{n=2}^{\infty} (1 - P_R(na)) \prod_{j=1}^{n-1} (1 - P_R(ja)) \times \exp[-\gamma(na)]\bar{F}_A(na)\bar{F}_V(\mathbf{p}, na)$$

$$A22 = \sum_{n=2}^{\infty} P_R(na) \prod_{j=1}^{n-1} (1 - P_R(ja)) \times \exp[-\gamma na]\bar{F}_A(na)\bar{F}_V(\mathbf{p}, na)$$

and where:

$P_R(a)$ = probability of repair after inspection increasing in a

$\bar{P}_R(a) = 1 - P_R(a)$ = probability of no repair after inspection

a = deterministic inspection interval

I_0 = cost per inspection

$I_1(\mathbf{p})$ = repair cost including inspection cost

$$f^{***}_{X|\bar{Y}}(\gamma, \mathbf{p}, a) = \int_0^a \exp[-\gamma t] f_X(t)\bar{F}_Y(t)dt$$

$\leq \int_0^a \exp[-\gamma t] f_X(t)dt$ = incomplete, modified Laplace transform of $f_X(t)$

$$f^{****}_{X|\bar{Y}}(\gamma, \mathbf{p}, (n-1)a \leq t \leq na) = \int_{(n-1)a}^{na} \exp[-\gamma t] f_X(t) \bar{F}_Y(t)dt$$

Here, one has to extend the renewal interval to $2a, 3a, \ldots$ if an inspection is not followed by repair. The terms $A11$, $A12$, $A21$ and $A22$ vanish for $P_R(a) \to 1$ and are significant only for relatively small a. Note that the renewal cost $C(\mathbf{p})$ can also be different in the two cases. If the benefit is constant in time we simply have $B(\mathbf{p}, a) = \frac{b}{\gamma}$. The case of non-constant benefit $b(t)$ is dealt with in[3].

The repair probability depends on the magnitude of a suitable damage indicator. For cumulative damage phenomena $P_R(a, \mathbf{p})$ increases with a. For example, $P_R(a, \mathbf{p}) = P(S(a, X, \mathbf{p}) > s_c)$ with $S(a, X, \mathbf{p})$ a monotonically increasing damage indicator and X a random variable taking into account of all uncertainties during inspection. Frequently, the length of inspection intervals is taken as an optimization parameter. The case without inspection and $P_R(a, \mathbf{p}) = 1$ is already dealt with in the literature[5,15]. Repair after inspection is interpreted as preventive renewal (replacement of an aging component after a finite time of use a). Renewal after failure is called corrective renewal. It must be mentioned that optimal inspection/repair intervals do not always exist. Preventive renewals must, in fact, be substantially cheaper than corrective renewals. Also, the repair probability must be sufficiently high at a.

Discussion

The renewal model applied herein turned out to be very powerful in deriving suitable objective functions. As mentioned many more refinements and extensions are possible. They are all based on the concept of systematic reconstruction which clearly is a sustainable concept. The assessment of cost appears to be easy and straightforward for construction cost and direct physical damage cost. More difficult is the quantification of indirect failure cost such as loss of use by the public, loss of business, etc. Even more difficult but related is the assessment of the benefit derived from a civil engineering infrastructure. But most difficult is the quantification of the losses in human life and limb in case of failure. Some results on this subject will be summarized in the next section. Similarly difficult is the assessment of the appropriate interest rate. An attempt has also been made to do this.

ADDITIONAL CONSTRAINTS BASED ON SOCIETAL CRITERIA FOR RISK ACCEPTANCE

At this point we may ask whether monetarily optimal facilities automatically lead to acceptable facilities. Clearly, the monetary value of a human life must be discussed in this context and whether the failure probability corresponding to an optimal solution is also acceptable from a societal point of view. Let us first define risk. Graham/Wiener[16] define: Risk = The chance of an adverse outcome to human health, the quality of life, or the quality of the environment. Other definitions have also been proposed. But it is important that risk now is understood as something far more general than just risk to human life and limb. Other aspects of life quality are also involved. Risk control is not only the scientific understanding and management of a risk but according to common understanding its cost efficiency and its affordability to the individual as well as to society. And it should be clear that only involuntary risks to human life and limb from technical installations or the natural environment by an anonymous member of society can be discussed. The cost of any risk reduction is carried by the public, i.e. by us all via taxes or public charges.

Recently, interesting concepts have been proposed for the assessment of public risk acceptance[17–24]. In essence, they set out from a composite social indicator, the *societal life quality index*, also to be interpreted as a utility function which encompasses three important indicators of life quality, that is life expectancy, consumption (income net of taxes) and the time necessary to raise the total income by paid work, i.e. the time not available for leisure. It is beyond the scope of this paper to present the concepts and derivations in detail. Instead we refer to the cited literature. In[22] the following version has been derived

$$L_{\bar{E}} = \frac{g^q}{q} \int_0^{a_u} e_d(a,\zeta,\rho,n)h(a,n)da = \frac{g^q}{q}\bar{E} \tag{16}$$

with

$$\bar{E} = \int_0^{a_u} e_d(a,\zeta,\rho,n)h(a,n)da \tag{17}$$

$$e_d(a,\zeta,\rho,n) = \frac{\exp((\rho+\zeta-n)a)}{\ell(a)} \int_a^{a_u} \exp\left[-\int_0^t (\mu(\tau)+(\rho+\zeta-n))d\tau\right] dt \tag{18}$$

$$h(a,n) = \frac{\exp[-na]\ell(a)}{\int_0^{a_u}\exp[-na]\ell(a)da} \tag{19}$$

In these formulae $g \approx 0.6\,\text{GDP}$ is the part of the GDP available for risk reduction interventions (approximately the part available for private use), $q = \frac{w}{1-w}$ a risk aversion parameter with w the life working time as a fraction of life expectancy at birth $e(0) = \int_0^{a_u}\ell(a)da$ with survival probability $\ell(a) = \exp[-\int_0^a \mu(t)dt]$ at age a and $\mu(t)$ the age dependent mortality obtainable from life tables, $e_d(a,\zeta,\rho,n)$ the "discounted" remaining life expectancy given that a person has survived until age a, ρ the socalled time preference rate, n the population growth rate, ζ the rate of economic growth and $h(a,n)$ the density of the distribution of ages in a (stable) population[25]. Dividing Eq. (16) by the marginal utility $u'(g) = g^{q-1}$ gives the so-called *societal value of a statistical life*

$$SVSL = \frac{g}{q}\bar{E} \tag{20}$$

The value of a statistical life has nothing to do with the amounts which have to be paid to the surviving dependents after an event as compensation by insurance or the social system. And it is not the (monetary) value of a human being. "*. . . the value of human life is infinite and beyond measure, . . .*" (according to[26] representing also the views of the other world religions). Moreover, it is the monetary amount a society is willing to pay and can afford for risk reduction.

Using Eq. (16) a small relative change in the societal life quality index can be assessed as

$$\frac{dL_{\bar{E}}}{L_{\bar{E}}} = \frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}}{\bar{E}}$$

so that the requirement $dL_{\bar{E}} \geq 0$ leads to a general acceptance criterion

$$\frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}}{\bar{E}} \geq 0 \tag{21}$$

The change in age-averaged, discounted life expectancy can be expressed in terms of a change in (crude) mortality m as

$$\frac{d\bar{E}}{\bar{E}} \approx \frac{\frac{d}{dx}\bar{E}(x)|_{x=0}x}{\bar{E}} = -\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}dm \tag{22}$$

The *societal willingness to pay* is finally defined as

$$dC_Y = -dg = g\frac{1}{q}\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}dm = G_{x\bar{E}}(\zeta,\rho,n)dm \tag{23}$$

The demographic constant $c_{x\bar{E}}(\zeta,\rho,n)$ depends on the mortality reduction scheme x of a particular intervention, for example whether the intervention reduces mortality proportional to age dependent mortality or simply as a constant at all ages. In the following only constant mortality changes denoted by scheme Δ will be considered. The acceptability criterion Eq. (21) or Eq. (23) is *necessary, affordable* and *efficient* from a societal point of view[18].

Application to technical objects requires that the mortality change is expressed in terms of changes in the failure rate. Let dm be proportional to the increment in the mean failure rate $dh(p)$ i.e. it is assumed that the process of failures and renewals is already in a stationary state that is for $t \to \infty$ (see Eq. (8)). Rearrangement and introducing the incremental cost and the failure rate as a function of a (scalar) parameter p yields

$$\frac{dC_Y(p)}{dr(p)} \geq -k\frac{c_{\Delta\bar{E}}(\zeta,\rho,n)}{m}g\frac{1}{q} = -kG_{\Delta\bar{E}}(\zeta,\rho,n) \tag{24}$$

where

$$dm = kdr(p), \quad 0 < k \leq 1 \tag{25}$$

the proportionality constant k relating the changes in mortality to changes in the failure rate. More specifically, k is the probability of being killed in a failure event. Note that for any reasonable risk reducing intervention there is necessarily $dr(p)/dp < 0$. $k(0 \leq k \leq 1)$ must be determined by careful failure consequence analysis.

The Life Saving Cost (LSC) or Implied Cost of Averting a Fatality (ICAF) can be obtained from the equality of Eq. (21) after replacing $\bar{E}$ by $e = e(0)$, separation and integration from g to $g + \Delta g$ and e to $e + \Delta e$, i.e. the cost $\Delta C = -\Delta g$ per year to extend a person's life by Δe is:

$$\Delta C = -\Delta g = g\left[1-\left(1+\frac{\Delta e}{e}\right)^{-\frac{1}{q}}\right]$$

Because ΔC is a yearly cost and the (undiscounted) LSC has to be spent for safety related investments into technical projects at the decision point $t = 0$, one should multiply by $e_r = \Delta e$ and

$$\text{LSC}(e_r) = g\left[1-\left(1+\frac{e_r}{e}\right)^{-\frac{1}{q}}\right]e_r \tag{26}$$

follows. The societal equality principle prohibits to differentiate with respect to special ages within a group. The conditional (remaining) life expectancy given that the person has survived up to age a is:

$$e(a) = \int_a^{a_u}\frac{\ell(t)}{\ell(a)}dt = \frac{1}{\ell(a)}\int_a^{a_u}\exp\left[-\int_0^t \mu(\tau)d\tau\right]dt \tag{27}$$

Therefore, averaging the remaining life expectancy over the age distribution leads to the Societal Life Saving Cost (SLSC)

$$\text{SLSC} = \int_0^{a_u} \text{LSC}\,(e(a))h(a,n)da \tag{28}$$

where $h(a,n)$ is the density of the age distribution of the population with n its population growth rate. In countries with a fully developed social system SLSC is approximately the amount to support the (not working) surviving dependants of an event by the social system, mostly by redistribution. If no social system is present, it is useful to think of the amount an insurance should cover after an event. For example, if $GDP \approx$ 25 000 PPP US\$ and thus, $g \approx$ 15 000 PPP US\$, $e \approx 77$ years and $w \approx 0.15$, one calculates SLSC $\approx$ 600 000 PPP US\$.

The criterion Eq. (24) is derived for safety-related regulations for a larger group in a society or the entire society. For a specific project it makes sense to apply criterion (24) to the specific group exposed. Therefore, the "life saving cost" of a technical project with N_{PE} potential endangered persons is:

$$H_F = \text{SLSC}\,k\,N_{PE} \tag{29}$$

The monetary losses in case of failure are decomposed into $H = H_M + H_F$ in formulations of the type Eq. (1) with H_M all losses not related to human life and limb.

Criterion (24) changes accordingly into:

$$\frac{dC_Y(p)}{dr(p)} \geq -G_{\Delta\bar{E}}(\zeta,\rho,n)kN_{PE} \tag{30}$$

All quantities in Eq. (30) are related to one year. For a particular technical project all design and construction cost, denoted by $dC(p)$ must be raised at the decision point $t = 0$. The yearly cost must be replaced by the erection cost $dC(p)$ at $t = 0$ on the left hand side of Eq. (30) and discounting is necessary. The method of discounting is the same as for discharging an annuity. If the public is involved $dC_Y(p)$ may be interpreted as cost of societal financing of $dC(p)$ such that $dC_Y(p) = dC(p)\frac{\gamma\exp[\gamma t_s]}{\exp[\gamma t_s]-1}$. The (real) interest rate to be used must then be a societal interest rate. Otherwise the interest rate is the market rate. g in $G_{\Delta\bar{E}}(\zeta,\rho,n)$ also grows in the long run approximately exponentially with rate $\zeta - n$, the rate of effective economic growth in a country (see[27] for an empirical verification). It can be taken into account by discounting. The acceptability criterion for individual technical projects then is (discount factor for discounted erection cost moved to the right hand side):

$$\frac{dC(p)}{dr(p)} \geq -\frac{\exp[\gamma t_s]-1}{\gamma\exp[\gamma t_s]}G_{\Delta\bar{E}}(\zeta,\rho,n)\frac{\zeta\exp[\zeta t_s]}{\exp[\zeta t_s]-1}kN_{PE}$$
$$\underset{t_s\to\infty}{\rightarrow} -G_{\Delta\bar{E}}(\zeta,\rho,n)kN_{PE}\frac{\zeta}{\gamma} \tag{31}$$

where t_s is service time. For $\zeta \to 0$ as well as $\gamma \to 0$ we have the interesting limiting result for arbitrary t_s:

$$\frac{dC(p)}{dr(p)} \geq \underset{\zeta\to 0,\gamma\to 0}{\rightarrow} -G_{\bar{E}}(\zeta,\rho,n)kN_{PE} \tag{32}$$

Here, a slight inconsistency is encountered because there is double discounting with respect to g by γ and with respect to $\frac{c_x\bar{E}}{m}$ in $G_{\Delta\bar{E}}(\zeta,\rho,n)$ by $\zeta + \rho - n$ although the effect of discounting is relatively small. Alternatively, discounting can be performed with the same rate in Eq. (31) so that the effect of discounting cancels. Generalizing now to a vectorial parameter $\mathbf{p}$ we have

$$\nabla_p C(\mathbf{p}) + G_{\Delta\bar{E}}(\zeta,\rho,n)kN_{PE}\nabla_p r(\mathbf{p}) \geq 0 \tag{33}$$

which is easily seen to be equivalent to the solution of the following optimization task

$$\text{Minimize:} \quad Z'(\mathbf{p}) = C(\mathbf{p}) + G_{\Delta\bar{E}}(\zeta,\rho,n)kN_{PE}r(\mathbf{p}) \tag{34}$$

Equation (33) is seen to be the optimality condition $\nabla_p Z'(\mathbf{p}) = 0$ of the (unconstrained) optimization problem Eq. (34). Equation (34) allows solving for vectorial parameter $\mathbf{p}$. A solution to Eqs. (33) or (34) can always be found because $\nabla_p C(\mathbf{p})$ usually grows approximately linearly in $\mathbf{p}$ whereas $\nabla_p r(\mathbf{p})$ decays exponentially.

PUBLIC INTEREST RATES

Two Bounds on Public Interest Rates

In accordance with economic theory benefits and (expected) cost should be discounted by the same rate as done above. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for an optimization in the name of the public is difficult. The requirement that the objective function must be non-negative leads immediately to the conclusion that the interest rate must have an upper bound $\gamma_{\max}$ depending on the benefit rate $b = \beta C(\mathbf{p})$ (see[10]). For the model in Eq. (7) we have

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda P_f(\mathbf{p})}{\gamma} = 0 \qquad (35)$$

and, therefore, by solving for γ and given (optimal) $\mathbf{p} = \mathbf{p}^*$

$$\gamma < \gamma_{\max} < \beta - \lambda P_f(\mathbf{p})\left(1 + \frac{H}{C(\mathbf{p})}\right) \qquad (36)$$

implying $\gamma < \beta$ for $\lambda P_f(\mathbf{p}) \ll \beta$. It follows that the benefit rate β must be slightly larger than $\gamma_{\max}$. From Eq. (35) one also concludes that there must be $\gamma > 0$ because the limit $\gamma \to 0^+$ is $\pm\infty$ or at least undefined. The quantification of public interest rates $\gamma_{\max} < \beta$ is discussed below.

Public Interest Rates Based on Economic Growth Theory

The cost for saving life years in Eq. (29) enters into the objective function (4) and with it the question of discounting those cost arises. Also, a discount rate may or may not be present in Eq. (31). At first sight discounting of human lives is not in agreement with our moral value system. However, a number of studies summarized in[28] and[17] express a rather clear opinion based on ethical and economical arguments and on public opinion. The cost for saving life years must be discounted at the same rate as other investments. Otherwise serious inconsistencies cannot be avoided. The arguments are as follows: the ethical argument is essentially based on the categorical imperative of Kant[29]: *Act only on the maxim which you can will to be a universal law*. Because discounting follows from opportunity cost as a fact of life "*... future generations must be treated in the same way as we want to be treated today*".

In view of the time horizon of some 20 to more than 100 years (i.e. several generations) it should be a long-term average. It should be net of inflation and taxes. In the private sector a long term real interest rate is roughly identical to the (maximum) return rate one could get from an investment. But can the public also adopt such a strategy? The public does not make financial profit except by its economical growth. And what is financial profit of public life saving interventions? Clearly, the interest rates should be close to the long-term economic growth rate (per capita) as this is the rate with which a member of the public becomes more wealthy. In the economics literature this is sometimes called the "natural interest rate". Traditionally, it has been argued that public investments should be financed within some mean residual life expectancy of the population, i.e. within 40 to 50 years. For larger financing horizons the burden of financing would be left in part to the next generation. If this time is viewed as the time of amortization of a public investment, rates of 2 to 2.5% are implied.

There have been ongoing but somewhat inconclusive discussions when discounting public investments into health care (see, for example[30]). Recently, further discussions have taken place in the context of sustainable development, long term public investments in general and intergenerational justice-aspects which appear very relevant in our context. Discounting for sustainability should at least be consistent with discounting for risk reduction investments. Weinstein/Stason[31] and others require that interest rates for life saving investments should be the same as for other cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed, for example, in[32] and[33], based primarily on ethical grounds in the context of CO_2 – induced global warming, nuclear waste disposals, depletion of natural resources, etc. A life is simply worth saving with the same effort now and in the future. In this case the rationale of our basic optimization model Eq. (4) together with Eq. (31) and part of the considerations in chapter 3 break down.

Due to the requirement $\beta > \gamma_{\max}$ stated just below Eq. (36), the interest rate is strongly related to the benefit a society earns from the various activities of its members, i.e. its real economic growth per capita (see also[19] where the public benefit and interest rate has been set equal to the growth rate). The United Nations Human Development Report 2001[34] gives values between 1.2 and 1.9% for industrialized countries during 1975–1998. If one considers the last 100 years and the data in[27] for some selected countries one determines a growth rate $\zeta = \frac{\ln(g_{1992}/g_{1870})}{1992-1870}100$ of about 1.8% for Western Europe, the so-called Western Offshots, USA, Canada and Australia, and Japan. For Southern Europe, Latin America and Asia one finds from the same data similar growth rates $\bar{\zeta} \approx 1.7\%$, for Eastern Europe still $\bar{\zeta} \approx 1.4\%$ but for Africa only $\bar{\zeta} \approx 0.9\%$. The growth data in[27] are per capita. Therefore, the growth rate of the respective economy is by the population growth rate larger.

Modern economic growth theory widely applied to sustainability financing can provide more insight. Nordhaus[35] and others (see[36] for an overview but also the other papers in Energy Policy, 23, 3/4, 1995) follow the classical Ramseyan approach (see[37–39]) for optimal stable economic growth in perfect markets

$$\gamma = \rho + \epsilon\zeta > 0 \qquad (37)$$

where γ is the real market interest rate, ρ the rate of pure time preference of consumption, $\epsilon > 0$ the elasticity of marginal consumption (income) and ζ the consumption (income) growth rate. Clearly, the subjective element is the quantity ρ. Nordhaus[35] obtains $\gamma \approx 0.05$, Arrow[40] estimates $\gamma \approx 0.03$. In many other studies for sustainable development discount rates γ cluster around 5%. All those values are close to the real market rates or a little smaller. However, there are many authors in economics as well as philosophical and political sciences including Ramsey who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds[32,33,41] while it is considered fully acceptable for intragenerational discounting. On the other hand, intergenerational equity arguments in Arrow[40] indicate that there should be $\rho > 0$ in order to remove an "*... incredible and unacceptable strain on the present generation*". Rabl[41], who sets $\rho = 0$, argues that

there must be $0 < \gamma < \epsilon\zeta$ in the framework of long-term public investments. One can also show that there must be $\rho > n$ in the underlying theory. In[22] the following bounds have been proposed:

$$n + \zeta(1 - \epsilon) < \rho < \gamma \le \gamma_{\max} < \beta = n + \epsilon\zeta \qquad (38)$$

Values for ρ and β are presented in table 1. It is then possible to compute $\gamma_{\max} < \beta$ from Eq. (36). $\gamma_{\max}$ usually is only insignificantly (1 to 20%) smaller than β depending on the specific case at hand, i.e. the particular sensitivities of $C(\mathbf{p})$ and $h(\mathbf{p})$ with respect to $\mathbf{p}$. The interest rates $\gamma_{\max}$ implied by the value of β are considerably lower the usual real market interest rates in developed countries.

In the literature the adequacy of the Ramseyan model is sometimes questioned. For example, so-called overlapping generation models or generation adjusted discounting models are advocated instead. The main idea is to discount for living generations at the rate in Eq. (38) with $\rho > 0$ but diminish the rate for all yet unborn generations down to $\zeta - n$ or even lower, thus facilitating the transition into a sustainable state of economy[41–43]. Unfortunately, the convolution theorem for Laplace transforms used, for example, in Eq. (4) no more holds for time-dependent discount rates complicating the setting up of suitable objective functions. Instead, one has to use Eq. (3). But it is not expected that those refinements, if properly justified, change our results significantly.

Some further precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. Natural resources will be depleted and arable land will become scarce. Many raise serious doubts whether the foreseeable demographic changes (aging populations and negative population growth in industrial countries) and the increasing scarcity of non-renewable natural resources and other environmental concerns can be compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will actually happen. But there is an important mathematical result which may guide our choice. Weitzman[44] and others showed that the far-distant future should be discounted at the lowest possible rate ≥ 0 if there are different possible scenarios each with a given probability of being true. Exactly this strategy has been pursued in the foregoing. It should be emphasized that lowest possible interest rates so far have been chosen only for the subjective part ρ of the real interest rate γ. Taking a small ρ also is conservative for the demographic constants in Eqs. (20) or (23). Finally, it must be remembered that these rates should be used only when setting safety standards in the various fields, when investing into public health care programs, etc., by Eqs. (30) or (31). They have very little to do with the rates the owner, the operator or the user would have to acquire from the financial market and which must be used when optimizing technical facilities with objectives (with or without life saving cost Eq. (29) included) of the type Eq. (4). Some more discussion on discounting is provided in[22].

Some numerical values for various economic and demographic quantities entering Eq. (33) or Eq. (34) are given in the following table[22].

TABLE 1

SOCIAL INDICATORS FOR SOME COUNTRIES

Country	GDP[1)], g[2)]	ζ[3)]	m[4)]	n[5)]	e	q[6)]	ρ	γ	SLSC[7,8)]	$G_{\Delta\bar{E}}$[8)]	SVSL
USA	34260, 22030	1.8	0.87	0.90	76	0.22	1.5	3.0	$6.7\cdot10^5$	$5.0\cdot10^6$	$1.8\cdot10^6$
Germany	25010, 14460	1.9	1.04	0.27	78	0.17	0.6	2.1	$5.6\cdot10^5$	$4.9\cdot10^6$	$2.2\cdot10^6$
Poland	9030, 5630	1.6	1.00	−0.03	73	0.19	0.3	1.2	$2.0\cdot10^5$	$1.7\cdot10^6$	$9.0\cdot10^5$
Switzerland	29000, 17700	1.9	0.88	0.27	79	0.17	0.8	2.3	$7.0\cdot10^5$	$5.4\cdot10^6$	$2.4\cdot10^6$
UK	23500, 15140	1.3	1.07	0.23	78	0.19	0.5	1.5	$5.7\cdot10^5$	$3.6\cdot10^6$	$2.3\cdot10^6$
India[9)]	2340, 1560	5.0	0.87	1.6	62	0.33	3.8	6.0	$5.5\cdot10^4$	$8.9\cdot10^4$	$1.1\cdot10^5$
Japan	26460, 15960	2.7	0.83	0.17	80	0.20	0.7	2.4	$6.1\cdot10^5$	$4.4\cdot10^6$	$1.6\cdot10^6$
Australia	25370, 15750	1.2	0.72	0.99	78	0.21	1.2	1.9	$6.5\cdot10^5$	$5.4\cdot10^6$	$1.9\cdot10^6$

[1)]in PPPUS$[45], [2)]private consumption in PPPUS$[34], [3)]average yearly economic growth per capita in % for 1870–1992 after[27], [4)]crude mortality (2000) in %[46], [5)]population growth (2000) in %[46], [6)]estimates based on[47] including 1 hour travel time per working day and a life working time of 45 years, [7)]SLSC computed with g and age-averaged life expectancies, [8)] computed from recent period life tables, Δ indicates constant additive mortality changes, [9)]data in part unreliable, abridged life table from[48], growth rate ζ for 1975–2000, q estimated

Unfortunately, the data for India but also for Poland appear to be in part inconsistent. The large discount rates together with the high q for India make the values of $G_{\Delta\bar{E}}$ and $SVSL$ especially small.

There have been many attempts to estimate this quantity indirectly, mostly by estimating the cost of some life saving operation like limiting highway speed, installing smoke detectors in homes or using seat belts in cars[30,49]. Also, the compensation in risky jobs by higher wages has been used as well as surveys with respect to hypothetical risky situations, so-called contingent valuation studies. The values reported for industrialized countries are between less than 1 Mill. US$ and more than 10 Mill. US$, i.e. more than 2 to 20 times as much as the (undiscounted) value of average lost earnings in case of a fatal accident at mid life. The study in[50] summarizes many other investigations. For comparison, $G_{\Delta\bar{E}}(\zeta, \rho, n)$ and $SVSL$ should be calculated with the full GDP, i.e. the values in Table 1 should be multiplied by GDP/g. In Fig. 1 some results collected in[50] are presented graphically showing the large scatter of the estimates. The correlation coefficient as a crude measure of the dependence of VSL on income or GDP is also given in the figure. The figure contains in the lower left corner three estimates under Indian conditions.

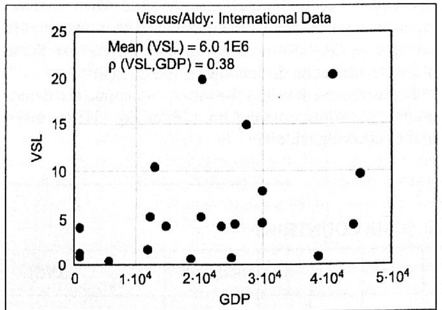

FIG. 1. ESTIMATES OF THE VALUE OF A STATISTICAL LIFE (VSL) ACCORDING TO[50]. GDP AND VSL GIVEN IN US$ NOT CORRECTED FOR PURCHASING POWER PARITY

NUMERICAL TECHNIQUES OF OPTIMIZATION

Principles of a One-Level Approach

Let $\mathbf{p}$ be a parameter vector which enters the cost function and the limit state function $g(\mathbf{u}, \mathbf{p}) = 0$. Benefit, construction and damage function as well as the limit state function(s) are differentiable in $\mathbf{p}$ and $\mathbf{u}$. The conditions for the application of FORM/SORM hold. In the so-called β—point $\mathbf{u}^*$ the optimality conditions (Kuhn-Tucker conditions) are[51]:

$$g(\mathbf{u}, \mathbf{p}) = 0$$

$$\frac{\mathbf{u}}{\|\mathbf{u}\|} = -\frac{\nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})}{\|\nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})\|} \qquad (39)$$

The geometrical meaning of (39) is that the gradient of $g(\mathbf{u}, \mathbf{p}) = 0$ is perpendicular to the vector of direction cosines of $\mathbf{u}^*$. The idea is elaborated in[51] now is to use these conditions as constraints in the cost optimization problem thus avoiding a bi-level optimization. It will turn out that this concept is crucial for further numerical analysis as described below.

It is important to reduce the set of the gradient conditions in the Kuhn-Tucker conditions by one. Otherwise the system of Kuhn-Tucker conditions is over determined. It is also important that the remaining Kuhn-Tucker conditions are retained under all circumstances, for example, if one or more gradient Kuhn-Tucker conditions become colinear with one or more of the other constraints possibly included in the cost-benefit optimization task. Otherwise the so-called β-point conditions are not fulfilled.

Formulations for Time-Variant Problems

In the simplest stationary, one-component case we have:

$$Z(\mathbf{p}) = B - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F) \cdot \frac{\lambda P_f(\mathbf{p})}{\gamma} \qquad (40)$$

subject to:

$$g(\mathbf{u}, \mathbf{p}) = 0$$

$$u_i \|\nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})_i \|\mathbf{u}\| = 0; \quad i = 1, \ldots, n-1$$

$$h_k(\mathbf{p}) \leq 0, k = 1, \ldots, q$$

$$\lambda P_f(\mathbf{p}) \leq h_{\text{admissible}}$$

or

$$\nabla_p C(\mathbf{p}) + G_{x\bar{E}}(\zeta, \rho, n) k N_{PE} \nabla_p \nu^+(\mathbf{p}) \geq 0$$

depending on whether a reliability constraint $h_{\text{admissible}}$ is imposed exogenously or criterion (33) is used.

The same scheme applies to the full Laplace transform of non-stationary problems.

$$Z(\mathbf{p}) \approx B - C(\mathbf{p}) - (C(\mathbf{p}) + H) \cdot \frac{f^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \qquad (41)$$

$$g(\mathbf{u}_j, \mathbf{p}, t_j) = 0 \text{ for } j = 0, 1, \ldots, m$$

$$u_{i,j} \|\nabla_{\mathbf{u}} g(\mathbf{u}_j, \mathbf{p}, t_j)\| + \nabla_{\mathbf{u}} g(\mathbf{u}_j, \mathbf{p}, t_j)_i \|\mathbf{u}_j\| = 0$$

$$i = 1, \ldots, n-1; \; j = 0, \ldots, m$$

$$h_\ell(\mathbf{p}) \leq 0, \ell = 1, \ldots, q$$

$$\frac{1}{E[T(\mathbf{p})]} \leq h_{\text{admissible}}$$

or

$$\nabla_p C(\mathbf{p}) + G_{x\bar{E}}(\zeta, \rho, n) k N_{PE} \nabla_p \left(\frac{1}{E[T(\mathbf{p})]}\right) \geq 0$$

Here, the failure rate criterion must use the asymptotic failure rate Eq. (8). Equation (12) is used resulting in a large number of equality constraints which may cause numerical difficulties in extreme cases.

EXAMPLES

Random Demand versus Random Capacity[22]

The example has already been given in[19] in somewhat different form and with modified parameters. We take a rather simple case of a single-mode system where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modelled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances $S_i, i = 1, 2, \ldots$. The resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independent and log-normally distributed with mean equal to unity and coefficient of variation V_S so that p can be interpreted as central safety factor. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln((1+V_R^2)(1+V_S^2))}}\right) \qquad (42)$$

An appropriate objective function then is with $b = b(\mathbf{p})$:

$$Z(p) = \frac{b}{C_0\gamma} - \left(1 + \frac{C_1}{C_0}p^a\right) - \left(1 + \frac{C_1}{C_0}p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right)\frac{\lambda P_f(p)}{\gamma} \qquad (43)$$

The criterion (31) has the form:

$$\frac{d}{dp}(C_0 + C_1 p^a) \geq -G_{x\bar{E}}(\rho, n) k N_{PE} \frac{d}{dp}(\lambda P_f(p)) \qquad (44)$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6, C_1 = 10^4, a = 1.25, H_M = 3 \cdot C_0, V_R = 0.2, V_S = 0.3$, and $\lambda = 1[1/\text{year}]$. The LQI-data is $e = 77, GDP = 25000, g = 15000, m = 0.01$ and two extreme demographic constants $C_{\delta\bar{E}} = 25$ or $C_{\pi\bar{E}} = 75, w = 0.15, N_{PE} = 100, k = 0.1$ so that $H_F = SLSC\, kN_{PE} = 8.4 \cdot 10^6, G_{\delta\bar{E}}(\rho, n)kN_{PE} = 2.1 \cdot 10^7$ and $G_{\pi\bar{E}}(\rho, n)kN_{PE} = 6.2 \cdot 10^7$. The value of N_{PE} is chosen relatively large for demonstration purposes. Monetary values are in US$. Optimization is performed for the public and for the owner separately.

For the public $b_S = \beta C_0$ with $\beta = 0.02$ from table 1 and $\gamma_S = 0.0185$ determined from Eq. (36) are chosen. Benefit and discount rate are chosen such that the public does not make direct profit from an economic activity of its members. Optimization including the cost H_F gives $p_S^* = 4.35$, the corresponding failure rate is $1.2 \cdot 10^{-5}$. Criterion (31) is already fulfilled for $p_l = 3.34$ and $p_u = 3.68$, respectively, corresponding to yearly failure rates of $2.5 \cdot 10^{-4}$ and $9.1 \cdot 10^{-5}$, respectively, but $Z_S(p_l)/C_0$ and $Z_S(p_u)/C_0$ being already negative. It is notable that although the two demographic constants $C_{\pi\bar{E}}$ differ by a factor of three acceptability limits are close together. It is also interesting to see that in this case the public can do better in adopting the optimal solution rather than just realizing the facility at its acceptability limit (see Fig. 2).

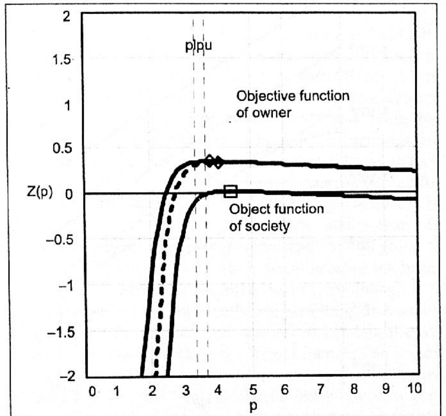

FIG. 2. OBJECTIVE FUNCTION FOR SOCIETY AND OWNER (WITH AND WITHOUT LIFE SAVING COST)

The owner uses some typical values of $b_O = 0.07C_0$ and $\gamma_O = 0.05$ and does or does not include life saving cost. If he includes life saving cost the objective function is shifted to the right (dotted line). The calculations yield $p_O^* = 3.76$ and $p_O^* = 4.03$, respectively, and the corresponding failure rates are $7.1 \cdot 10^{-5}$ and $3.2 \cdot 10^{-5}$. The $SLQI$-based acceptability criterion limits the owner's region for reasonable designs. Inclusion of life saving cost has relatively little influence on the position of the optimum.

It is noted that the stochastic model and the variability of capacity and demand also play an important role for the magnitude and location of the optimum as well as on the position of the acceptability limit. The specific marginal cost (rate of change) of a safety measure and its effect on a reduction of the failure rate are equally important.

This example also allows to derive risk-consequence curves by varying the number of fatalities in an event (see Fig. 3). With the same data as before but $SLSC = 7 \cdot 10^5$ and $G_{x\bar{E}}(\rho, n) = 4 \cdot 10^6$ for $N_F = 1$ we first vary the cost effectiveness of the safety measure. Here, only the ratio C_1/C_0 is changed. The upper bounds (solid lines) are derived from Eq. (44) and the lower bounds (dashed lines) corresponds to the societal optimum according to Eq. (43) ($b_S = 0.02C_0, \gamma_S = 0.0185$).

Most realistic is probably a ratio of $C_1/C_0 = 0.001$. The failure rate of approximately 10^{-4} per year for $N_F = 1$ corresponds well with the controllable crude mortality of the same magnitude as mentioned earlier. In Fig. 4 the mortality regimes are varied indicating that this is of only moderate influence. In this figure the region between the upper bound(s) and the lower curve derived from the societal optimum may be interpreted as ALARP-region (ALARP = As Low As Reasonably Practicable).

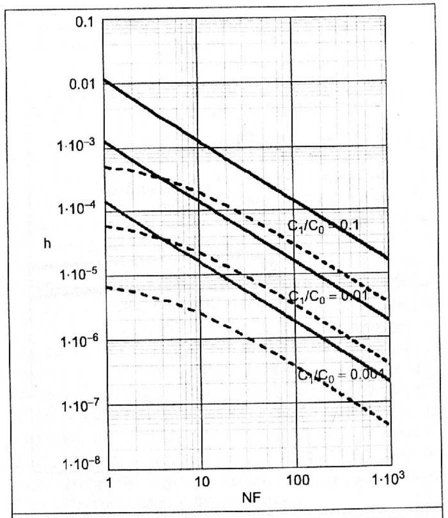

FIG. 3. ACCEPTABLE FAILURE RATE OVER NUMBER OF FATALITIES FOR DIFFERENT C_1/C_0 DASHED LINES CORRESPOND TO OPTIMAL SOLUTION FOR THE PUBLIC

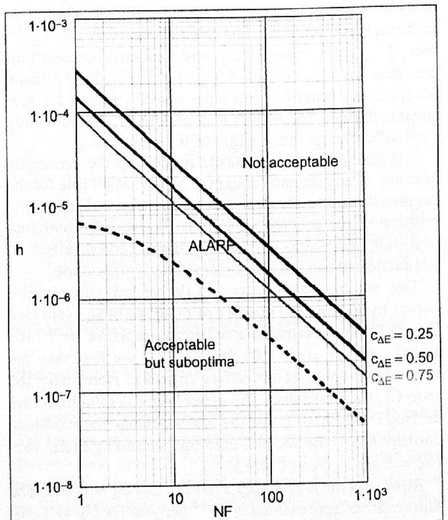

FIG. 4. ACCEPTABLE RISK FOR DIFFERENT MORTALITY REGIMES

Note that in these figures the failure rate is given by λP_f and the number of fatalities is given by $N_F = kN_{PE}$. Therefore, these figures cover the full range of λ and P_f and k and N_{PE}, respectively.

Optimal Replacement of a Reinforced Concrete Structure Subject to Chloride Corrosion in Warm Sea Water[52]

Following[53] a simplified failure criterion for chloride corrosion in the splash zone in warm sea water is:

$$C_{cr} - C_s\left(1 - \mathrm{erf}\left(\frac{c}{2\sqrt{Dt}}\right)\right) \leq 0$$

where C_{cr} = critical chloride content, C_s = surface chloride content, c = concrete cover and D = diffusion parameter. The stochastic model is

Variable	Distr. function	Parameters
C_{cr}	Uniform	0.125, 0.175
C_s	Uniform	0.2, 0.4
c	Log-normal	m_c, 1
D	Uniform	0.1, 0.315

The uniform distributions reflect the large uncertainty in the variables. The units are chosen such that t is in years. Inspection are performed at regular intervals a. They are followed by renewals (repairs) with probability $P_R(a) = 1 - \exp[-a_R a^2]$. The optimization variables are the mean concrete cover m_c and the length a of the inspection interval. Erection cost are $C(m_c) = C_0 + C_1 m_c^2$, inspection cost are $I_0 = 0.1C_0$, repair cost are $I_1 = 0.5C_0$ and we have $C_0 = 10^6$, $C_1 = 10^4$, $H = 10C_0$, $b = 015C_0$, $\gamma = 0.03$ and $a_R = 0.01$ the solution is $a^* = 66$ and $m_c^* = 6.5$. It turns out that preventive repairs should be performed every 66 years which saves up to 30% of the cost. These results comply well with practical experience with such structures. The contributions to the total damage cost are shown in Fig. 5. Relatively small variations in the repair model or in the cost factors will, however, result in cases where it is better not to inspect and repair but just wait for failure. It is noted that for the given failure model no mean time to failure exists.

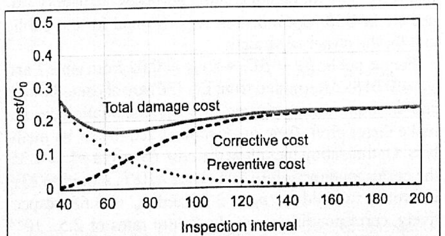

FIG. 5. TOTAL COST FOR REGULAR INSPECTIONS AND RENEWALS

CONCLUSION

Technical facilities should be optimal with respect to benefits and cost. Cost-benefit analysis is the framework in which our considerations are valid. It is found that the only replacement strategy fulfilling the requirement of sustainability is systematic reconstruction after failure. If the structure is deteriorating a suitable inspection and maintenance strategy has to be adopted. Repairs must be planned already in the design phase and must be such that they correspond to renewal, i.e. re-establish the (stochastically) initial state. Some suitable objective functions are developed based on a renewal model. Special techniques for solving the optimization problem are also presented.

The risk of failure and especially the risk to human life and limb must be limited. The *life quality index* is presented. A general risk acceptability criterion is derived. The $SLSC$ (societal life saving cost = implied cost of averting a fatality) to be used in optimization as live saving or compensation cost and the *societal willingness-to-pay* based on the *societal value of a statistical life* or on the *societal life quality index* are derived. The acceptability criterion, which is *necessary, affordable* and *efficient* from a societal point of view, depends on the marginal cost to reduce the risk, the corresponding marginal decrease in risk, the GDP, the life working time and on demographic factors obtainable from life tables. For example, key parameters such as the societal life saving cost ($SLSC$) and the societal value of a statistical life ($SVSL$) or an equivalent quantity derived from the life quality index turn out to cluster around 600 000 PPPUS$ and 4.0 Mill. PPPUS$, respectively, with very little variation for industrialized countries. They can be appreciably smaller in developing countries with smaller GDP, higher economic and population growth rates and different demographic characteristics. It also appears remarkable that cost-benefit optimal facilities usually provide more safety than the acceptability criterion in most cases if life saving cost are included in the analysis.

If time is involved all monetary quantities need to be discounted down to the decision point. Discount rates γ must be long term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long term averages from the financial market for his cost-benefit analysis the assessment of interest rates for investments of the public into risk reduction is more difficult. The classical Ramsey model decomposes the output growth rate into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long term population growth rate if used for the determination of parameters in the acceptability criterion. The output growth rate (= public interest rate) on the other hand should be smaller than the sum of the population growth rate and the long term growth rate of a national economy which is around 2% for most industrial countries which is also intergenerationally acceptable from an ethical point of view. All in all, discounting plays an important role in public cost-benefit considerations but is less important for a public risk acceptability criterion. It is also shown that given a certain output growth rate there is a corresponding maximum real interest rate in order to maintain non-negativity of the objective function in the public's interest.

Technical and natural risks are perceived individually and on a societal level rather irrationally and subjectively. Risk control should be affordable and efficient. A lack of efficiency in risk control has been shown in a number of studies, among others in the study of[49], indicating that many if not most public risk reduction interventions are highly inefficient. Some others can be shown to be, in fact, no more affordable thus taking away resources needed for other risk reducing projects and/or reducing life quality in the sense that other components of life quality than life expectancy are inedaquately diminuished. A third group of risks is inadequately taken into account because the benefits from an undertaking appear to be overwhelming.

Only if society acts rationally in controlling involuntary and anonymous risks from the natural and technical environment in an affordable and efficient manner can it gain better overall life quality in the long run. The strategies to reduce risks have been discussed but no specific recommendations were possible because most societies lack a clear statement of their ethical preferences. While there is no dispute over everybody's right to live and enjoy life and the equality principle some other principles are, if at all, not explicitly spoken out. There are several open basic questions. Those are:

- Does the public allow for economic societal myopia, that is discounting by the rate of time preference?
- Is intergenerational equity an ethical imperative?
- Is sustainability an ethical imperative?
- Is affordability a principle of public decision making?

REFERENCES

1. Rosenblueth, E., Mendoza, E., Reliability Optimization in Isostatic Structures, *J. Eng. Mech. Div.*, ASCE, 97, EM6, 1971, pp. 1625–1642.
2. Von Neumann, J., Morgenstern, A., *Theory of Games and Economical Behavior*, Princeton University Press, 1943.
3. Streicher, H., Rackwitz, R., *Objective functions for reliability-oriented structural optimization*, Proc. Workshop on Reliability-based Optimization, Warsaw, September, 2003.
4. Rackwitz, R.: Computational techniques in stationary and non-stationary load combination - A review and some extensions, *J. of Struct. Engg*, SERC, V. 25, No. 1, 1998, pp. 1–20.
5. Van Noortwijk, J.M., *Cost-based Criteria for Obtaining Optimal Design Decisions*, in: Proc. ICOSSAR 01, Newport Beach 23–25 June, Structural Safety and Reliability, Corotis et al. (Eds.), Sweets & Zeitlinger, Lisse, 2001.

6. Hasofer, A.M., Design for Infrequent Overloads, *Earthquake Eng. and Struct. Dyn.*, 2, 4, 1974, pp. 387–388.

7. Rosenblueth, E., Optimum Design for Infrequent Disturbances, *J. Struct. Div.*, ASCE, 102, ST9, 1976, pp. 1807–1825.

8. Cox, D.R., *Renewal theory*, Methuen, London, 1962.

9. Rackwitz, R., Optimization - The Basis of Code Making and Reliability Verification, *Struct. Safety*, 22, 1, 2000, pp. 27–60.

10. Hasofer, A.M., Rackwitz, R., *Time-dependent Models for Code Optimization*, in: Proc. ICASP'99, (eds. R.E. Melchers & M. G. Stewart), Balkema, Rotterdam, 1, 2000, pp. 151–158.

11. Streicher, H., Rackwitz, R., Time-variant Reliability-oriented Structural Optimization and a Renewal Model for Life-cycle Costing, accepted for publication in *J. Prob. Engg. Mech.*, 2004.

12. Kuschel, N., Rackwitz, R., *A new Approach for Structural Optimization of Series Systems*, Proc. of the ICASP8 Conference, Sydney, 12–15 Dec., 1999, (ed. R.E. Melchers & M. G. Stewart), Balkema, Rotterdam, 2, 2000, pp. 987–994.

13. Iizuka, H., A Statistical Study on Life Time of Bridges, *Struct. & Earthquake Eng. J.*, JSCE, 5, 1, 1988, pp. 51–60.

14. Barlow, R.E., Proschan, F., *Mathematical Theory of Reliability*, Wiley, New York, 1965.

15. Fox, B., Age Replacement with Discounting, *Oper. Res.*, 14, 1966, pp. 533–537.

16. Graham, J.D., Wiener, J.B., Risk vs. Risk, *Tradeoffs in Protecting Health and the Environment*, Harvard University Press, Cambridge, MA, 1995.

17. Lind, N.C., *Target Reliabilities from Social Indicators*, in: Proc. ICOSSAR93, Balkema, 1994, pp. 1897–1904.

18. Nathwani, J.S., Lind, N.C., Pandey, M.D., Affordable Safety by Choice: *The Life Quality Method*, Institute for Risk Research, University of Waterloo, Waterloo, Canada, 1997.

19. Rackwitz, R., Optimization and Risk Acceptability Based on the Life Quality Index, *Struct. Safety*, 24, 2002, pp. 297–331.

20. Lind, N.C., Social and Economic Criteria of Acceptable Risk, *Rel. Engg and Sys. Safety*, 78, 2002, pp. 21–25.

21. Pandey, M.D., Nathwani, J.S., Canada Wide Standard for Particulate Matter and Ozone: Cost-Benefit Analysis using a Life-Quality Index, *J. Risk Analysis*, 23, 1, 2003, pp. 55–68.

22. Rackwitz, R., Acceptable Risks and Affordable Risk Control for Technical Facilities and Optimization, to be published in *J. Rel. Engg and Sys. Safety*, 2003.

23. Rackwitz, R., Discounting for Optimal and Acceptable Technical Facilities Involving Risks, accepted for publication in *HERON*, 2003.

24. Rackwitz, R., Optimal and Acceptable Technical Facilities Involving Risks, accepted for publication in *Risk Analysis*, 2003.

25. Keyfitz, N., *Applied Mathematical Demography*, Springer, New York, 1985.

26. Jakobovits, I., *Jewish Medical Ethics*, New York, 1975.

27. Maddison, A., *Monitoring the World Economy*, 1820–1992, OECD, Paris, 1995.

28. Pate-Cornell, M.E., Discounting in Risk Analysis: Capital vs. Human Safety, *Proc. Symp. Structural Technology and Risk*, University of Waterloo Press, Waterloo, ON, Canada, 1984.

29. Kant, I., *Die Grundlegung der Metaphysik der Sitten*, 2nd. ed., Riga, 1786.

30. Viscusi, W.K., *Discounting Health Effects on Medical Decision, in: Valuing Health Care, Costs, Benefits and Effectiveness of Pharmaceuticals and Other Medical Technologies*, F.A. Sloan (ed), Cambridge University Press, 1996, pp. 125–147.

31. Weinstein, M.C. and Stason, W.B., Foundation of Cost-effectiveness Analysis for Health and Medical Practices, *New Engl. J. Medi.* 296(31), 1977, pp. 716–721.

32. Broome, J., *Counting the Cost of Global Warming*, White Horse Press, Cambridge, UK, 1992.

33. Schelling, Thomas C., *Intergenerational Discounting, Energy Policy*, 23, 4/5, 1995, pp. 395–401.

34. United Nations, Human Development Report 2001, www.undp.org/hdr2001, 2001.

35. Nordhaus, W.D., *Managing the Global Commons*, MIT Press, Cambridge, Mass., 1994.

36. Toth, F.L., *Discounting in Integrated Assessments of Climate Change*, Energy Policy, 23, 3/4, 1995, pp. 403–409.

37. Ramsey, F.P., A Mathematical Theory of Saving, *Eco. J.*, 38, 152, 1928, pp. 543–559.

38. Solow, R.M., *Growth Theory*, Clarendon Press, Oxford, 1970.

39. Barro, R.J., Sala-i-Martin, X., *Economic Growth*, McGraw-Hill, New York, 1995.

40. Arrow, K.J., et al., *Intertemporal Equity, Discounting and Economic Efficiency*, in: (Eds.) Bruce, J.P., et al.,: Climate Change 1995, Economic and Social

Dimensions of Climate Change, Cambridge, Cambridge University Press, 1996, pp. 125–144.

41. Rabl, A., Discounting of Long Term Cost: What Would Future Generations Prefer Us to Do?, *Ecological Economics*, 17, 1996, pp. 137– 145.

42. Bayer, S., Generation-adjusted Discounting in Long-term Decision-making, *Int. J. Sust. Dev.*, 6, 1, 2002, pp. 133–149.

43. Gerlagh, R., van der Zwaan, B.C.C., The Effects of Ageing and an Environmental Trust Fund in an Overlapping Generations Model on Carbon Emission Reductions, *Ecological Economics*, 36(2), 2001, pp. 311–326.

44. Weitzman, M.L., Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate, *Journal of Environmental Economics and Management*, 36, 1998, pp. 201–208.

45. World Development Indicators database, www.worldbank.org/data/, 2001.

46. CIA-factbook 2001, www.cia.gov/cia/publications/factbook/

47. OECD, *Employment Outlook and Analysis*, Paris, 2001.

48. Lopez, A.D., Salomon,J., Ahmad, O., Murray, C.JH., Mafat, D., Life Tables for 191 countries: Data, Methods and Results, http://www3.who.int/whosis/discussion_paper/pdf/paper09.pdf, 2001.

49. Tengs, T.O., Adams, M.E., Pliskin, J.S., Safran, D.G., Siegel, J.E., Weinstein, M.C., Graham, J.D., Five-Hundred Life-Saving Interventions and Their Cost-Effectiveness, *Risk Analysis*, 15, 3, 1995, pp. 369–390.

50. Viscusi, W.K., Aldy, J.E., The Value of a Statistical Life: A Critical Review of Market Estimates Throughout the World, *Risk & Uncertainty*, 27, 1, 2003, pp. 5–76.

51. Kuschel, N., Rackwitz, R., Two Basic Problems in Reliability-Based Structural Optimization, *Math. Meth. Oper. Res.*, 46, 1997, pp. 309–333.

52. Streicher, H., Rackwitz, R., *Renewal Models for Optimal Life-cycle Cost of Aging Civil Infrastructures*, 3rd Int. IABMAS Workshop on Life-Cycle Cost Analysis and Design of Civil Infrastructure Systems, Lausanne, March 2003, pp. 24–26.

53. Rackwitz, R., Balaji Rao, K., *Numerical Computation of Mean Failure Times for Locally Non-Stationary Failure Models*, Proc. of the ICASP8 Conference, Sydney, 12–15 Dec., 1999, (ed. R.E. Melchers & M. G. Stewart), Balkema, Rotterdam, V. 1, 2000, pp. 159–165.

54. Friis Hansen, F., Madsen, H.O., *A Comparison of Some Algorithms for Reliability-Based Structural Optimization and Sensitivity Analysis*, Proc. 4th IFIP WG 7.5 Conf. Munich, edited by R. Rackwitz and P. Thoft-Christensen, Springer Verlag Berlin, 1992, pp. 443–451.

4A