Risk Analysis, Vol. 24, No. 3, 2004

Optimal and Acceptable Technical Facilities Involving Risks

Ruediger Rackwitz*

Economic cost-benefit optimization of technical facility requires suitable "life saving cost" and/or an appropriate acceptance criterion if human life and limb are at risk. Traditionally, acceptance criteria implicit in codes of practice, standards, or regulations for well-defined fields of application are calibrated against past and present practice. This is all but satisfying. It is unclear whether present rules are already optimal. Extrapolations into new fields of application are extremely difficult. Direct cost-benefit analysis is proposed as an alternative. Based on the recently proposed "life quality index" (LQI), a rational acceptance criterion and so-called life saving cost are derived. The classical life quality index is reviewed, modified, and imbedded in modern economics theory. The results are then applied to technical facilities. The relation between optimization and the LQI-based acceptance criterion is discussed. The relevant economics literature is reviewed with respect to discount rates applicable for long-term investments into risk reduction. They should be as low as possible according to a recent mathematical result. Modern economic growth theory decomposes the output growth rate into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long-term population growth rate. The public benefit rate (output growth rate) on the other hand should be smaller than the sum of the population growth rate and the long-term growth rate of a national economy, which is around 2% for most industrial countries. Accordingly, the rate of time preference of consumption is about 1%, which is also intergenerationally acceptable from an ethical point of view. Given a certain output growth rate there is a corresponding maximum financial interest rate in order to maintain nonnegativity of the objective function at the optimum. Finally, a simple demonstration example is added.

KEY WORDS: Discounting; life quality index; optimum technical facilities; reliability; risk acceptability

1. INTRODUCTION

Economic cost-benefit optimization of technical facility, be it a vehicle, a structural facility like a building or a bridge, or an industrial installation, requires suitable "life saving cost" and/or an appropriate acceptance criterion if human life and limb are at risk. Traditionally, acceptance criteria implicit in codes of practice, standards, or regulations for well-defined fields of application are calibrated against past and present practice. This is all but satisfying. It is unclear whether present rules are already optimal. Extrapolations into new fields of application are extremely difficult. Direct cost-benefit analysis is a much better and rational basis. This will briefly be reviewed in order to set the framework for the acceptability consideration (Section 2). Based on the recently proposed "life quality index" (LQI),[34] a rational acceptance criterion can be derived (Section 3). The classical life quality index is then modified and imbedded in modern economics theory based on References 42 and 57 (Section 4). It is applied to technical facilities (Section 5). The

*Technical University Munich, Munich, Germany, Arcisstr. 21, D-80290 München; tel: +49-89-289-23050; fax: +49-89-289-23046; rackwitz@mb.bv.tum.de.

0272-4332/04/0100-0675$22.00/1

relation between optimization and the LQI-based acceptance criterion is discussed (Section 6). The relevant economics literature is reviewed with respect to discount rates applicable for long-term investments into risk reduction. Discount rates can be based on findings in modern economic growth theory but should be as low as possible (Section 7). Finally, a simple demonstration example is added (Section 8).

2. OPTIMAL TECHNICAL FACILITIES

A technical facility is optimal if the following objective is maximized:[55]

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}). \tag{1}$$

For the purpose of this article it is assumed that all quantities in Equation (1) can be measured in monetary units. Here $\mathbf{p}$ is the vector of all safety relevant parameters, $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction, and $D(\mathbf{p})$ is the cost in case of failure. While $B(\mathbf{p})$ and $C(\mathbf{p})$ can be considered as nearly deterministic, $D(\mathbf{p})$ generally is uncertain. Then, statistical decision theory dictates that expected values are to be taken.[35] In the following it is assumed that $B(\mathbf{p})$, $C(\mathbf{p})$, and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$. It is reasonably assumed that $C(\mathbf{p})$ increases whereas $D(\mathbf{p})$ decreases in each component of $\mathbf{p}$. The cost may differ for the different parties involved, e.g., the owner, the builder, the user, and society. The erection of a facility makes sense only if $Z(\mathbf{p})$ is positive within certain parameter ranges for all parties involved. Their intersection defines reasonable facilities (public or other subsidizing excluded).

The facility has to be optimized during design at the decision point, i.e., at time $t = 0$. Therefore, all cost needs to be discounted. A continuous discounting function is assumed, which is accurate enough for all practical purposes:

$$\delta(t) = \exp[-\gamma t], \tag{2}$$

where γ is the interest rate. For example, if failure occurs at time t (in years) with consequences D_0, the discounted damage is $D(t) = D_0 \exp[-\gamma t]$. If a yearly discount rate γ' is defined for discrete discounting, we have $\gamma = \ln(1 + \gamma')$.

In general, one has to distinguish between two replacement strategies at least, one where *the facility is given up after service or failure* and one where *the facility is systematically replaced after failure*. Further, we distinguish between facilities that *fail upon completion or never* and facilities that *fail at a random point in time* much later due to service loads, extreme external disturbances, or deterioration. The first option implies that demands on the facility are time-invariant. Reconstruction times are assumed to be negligibly short. At first sight there is no particular preference for either of the replacement strategies. For infrastructure facilities, the second category is a natural strategy. Facilities used only once, e.g., special auxiliary construction structures, boosters for space transport vehicles, or devices exploiting limited deposits, might fall into the first category.

For simplicity, the objective function is only derived for a special case. At an extreme disturbance of random magnitude (e.g., flood, wind storm, earthquake, explosion) with density of independent, identically distributed interarrival times $f(t)$ (independent) failure occurs with probability $P_f(\mathbf{p})$. The facility can fail at the first, second, third, etc. disturbance and will then be reconstructed or it survives. The density of the kth interarrival time is obtained from $f_k(t) = \int_0^\infty f_{k-1}(t-\tau) f(\tau) d\tau$. Therefore, the density of times to the nth failure is

$$g_n(t, \mathbf{p}) = \sum_{k=1}^{n} f_k(t) P_f(\mathbf{p})(1 - P_f(\mathbf{p}))^{k-1}. \tag{3}$$

In order to set up a suitable objective function of the type in Equation (1) one substracts construction cost and expected damage cost and reconstruction cost from the benefit, all discounted down to the decision point. For constant benefit per time unit $b(t, \mathbf{p}) = b(\mathbf{p})$ one obtains

$$Z(\mathbf{p}) = \int_0^\infty b e^{-\gamma t} dt - C(\mathbf{p}) - (C(\mathbf{p}) + H) \times \sum_{n=1}^{\infty} \int_0^\infty e^{-\gamma t} g_n(t, \mathbf{p})\, dt.$$

Taking Laplace transforms[1] [24,54]

$$g^*(\gamma, \mathbf{p}) = \sum_{n=1}^{\infty} f^*(\gamma) P_f(\mathbf{p})[f^*(\gamma)(1 - P_f(\mathbf{p}))]^{n-1}, \tag{4}$$

[1] Laplace transforms are defined by $f^*(\gamma) = \int_0^\infty e^{-\gamma t} f(t)\, dt$ and there is $0 \le f^*(\gamma) \le 1$ if $f(t)$ is a probability density and $f^*(0) = 1$ and $f^*(\infty) = 0$. In the transformed space there is $h^*(\gamma) = f(\gamma)^* g^*(\gamma)$ for $h(t) = \int f(t-\tau) g(\tau) d\tau$, an operation necessary to determine $f_n(t)$ because $f_n^*(\gamma) = f_{n-1}^*(\gamma) f^*(\gamma)$.

results in

$$Z(p) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H) \times \frac{P_f(\mathbf{p}) f^*(\gamma)}{1 - (1 - P_f(\mathbf{p})) f^*(\gamma)} = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H) h^*(\gamma, \mathbf{p}). \tag{5}$$

It may sometimes be realistic to change to a modified renewal process, in which case the density $f^*(\gamma)$ of the first disturbance in Equation (3) has to be replaced by $f_1^*(\gamma)$. If, in particular, the loading events follow a stationary Poisson process with intensity λ we have

$$h^*(\gamma, \mathbf{p}) = \frac{\lambda P_f(\mathbf{p})}{\gamma}, \tag{6}$$

because $f^*(\gamma) = \frac{\lambda}{\gamma + \lambda}$ for $f(t) = \lambda \exp[-\lambda t]$. It is noted here that the memoryless nature of a Poisson process for the disturbances implies that $f^*(\gamma) = f_1^*(\gamma)$. $h^*(\gamma, \mathbf{p})$ is the Laplace transform of the renewal density (renewal intensity) $h(t, \mathbf{p}) = \sum_{n=1}^{\infty} g_n(t)$. H is the monetary loss in case of failure, including direct failure cost, loss of business and, of course, the cost to reduce the risk to human life and limb.

For the renewal density and its Laplace transform there is an important asymptotic result:[16]

$$\lim_{t \to \infty} h(t, \mathbf{p}) = \lim_{\gamma \to 0} \gamma h^*(\gamma, \mathbf{p}) = \frac{1}{E[T_f(\mathbf{p})]}, \tag{7}$$

where $E[T_f(\mathbf{p})]$ is the mean time between renewals (or failures).

The precise details of the renewal model can be found in Reference 47. Many other objective functions can be formulated. For example, nonconstant benefit, serviceability failure, obsolescence, aging, deterioration, inspection and maintenance, and finite service times can be dealt with.[47] Also, multiple mode failures (series systems) can be considered.[61]

3. RATIONAL SOCIOECONOMICALLY-BASED RISK ACCEPTANCE CRITERIA—THE LIFE QUALITY INDEX

In this section some important developments in the so-called social indicator approach, especially by[32] Nathwani *et al.*[34] and Pandey & Nathwani,[42] are reviewed. This approach was initially developed as an alternative to the well-known human development index (HDI)[65] used as a measure of "how well a nation serves the well-being of its citizens" but it turned out also to be an excellent basis for deriving risk acceptance criteria. An attempt is made to support its various assumptions and hypotheses by some empirical evidence. It is also compared with earlier similar developments in health-related economics by Shepard and Zeckhauser[57] and others.

Any argumentation with respect to risk acceptability must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere, including everyone's right to life, the right of a free development of her/his personality, and the democratic equality principle. It is clear that only involuntary risks, i.e., risks to which the public is exposed involuntarily from its technical and natural environment, can reasonably be discussed here. Risk reduction is a primary concern of society, but not the only one because risk reduction generally involves cost. Thus, the cost expended for risk reduction must be balanced against competing needs in view of limited resources.

Cantril[13] and similar more recent studies conclude from empirical studies that long life and wealth are among the primary concerns of humans in a modern society—among others, as there are good family relationship, personal well-being, a good cultural, and ecological environment, etc., all parameters, that define the "quality of life." Life expectancy at birth (mean time from birth to death) e is the area under the survivor curve (survival function) $\ell(a) = \exp[-\int_0^a \mu(t)\, dt]$, i.e.,

$$e = e(0) = \int_0^{a_u} \ell(a)\, da = \int_0^{a_u} \exp\left[-\int_0^a \mu(t)\, dt\right] da, \tag{8}$$

where a_u = largest age considered and $\mu(a)$ = age dependent mortality or force of mortality. It makes sense to adjust it for times in poor health and times in hospital or homes for elderly people so that the "quality adjusted" (disability adjusted) life expectancy e_{QALY} is about 90% of e.[73]

Another suitable indicator of the quality of life is the gross domestic product (GDP) per capita and year. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. Its absolute value and growth rate are measures for the productivity of a society. It not only provides the infrastructure of a country, its social structure, its cultural and educational offers, and its ecological conditions, among others, but also the means for the individual enjoyment of life by consumption. In most developed countries about $60 \pm 5\%$ of the GDP is used privately, $20 \pm 5\%$ by the state (e.g., for military, police, jurisdiction, and education) and the rest for investments. Most importantly in our context, it creates the possibilities to "purchase" additional life years through better medical care, improved safety in road traffic, more safety in or around building facilities, more safety from hazardous technical activities, more safety from natural hazards, etc. In our context it does not matter whether those investments into "life saving" are carried out individually and voluntarily or enforced by regulation, or by the state via taxes. Neither the share for the state nor the investments into depreciating production means can be reduced appreciably because they form the conditions for the individual to enjoy life in high quality, now and in the future. Therefore, only the part for private use is available for risk reduction. Then, the part available for risk reduction is $g \approx 0.6$ GDP. The exact share for risk reduction must be determined separately for each country or group in a country and requires great care. The public must decide how much it is willing to spend on risk reduction and how much it is willing to give up of other public services.

Let

$$L = L(a, b, \ldots, e, \ldots) \tag{9}$$

be a composite social indicator with $a, b, \ldots, e, \ldots$ certain social indicators.(32) Further, let it be differentiable so that

$$dL = \frac{\partial L}{\partial a}da + \frac{\partial L}{\partial b}db + \cdots + \frac{\partial L}{\partial e}de + \cdots. \tag{10}$$

If only the two factors mentioned before, that is g and e, are considered, dL vanishes for:

$$dL = 0 \Rightarrow \frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}}, \tag{11}$$

lying that a change in e should be compensated for an appropriate change in g if L is required to be stant. Assume that L is the product of a function (as a measure of the quality of life) and another ction of the time $t = (1 - w)e$ to enjoy life (as a asure of the quantity of life) where w is the time to spend in paid work. Also, assume that the quantity chosen such that L is maximized. This appears to reasonable assumption because most work is dull, boring, troublesome, and sometimes dangerous. One also can draw on a historical argument. In 1870 the yearly time spent in work was 2,900 hours, in 1950 still 2,000, but at present only 1,600 on average. Simultaneously, life expectancy rose from 45 to almost 80 years

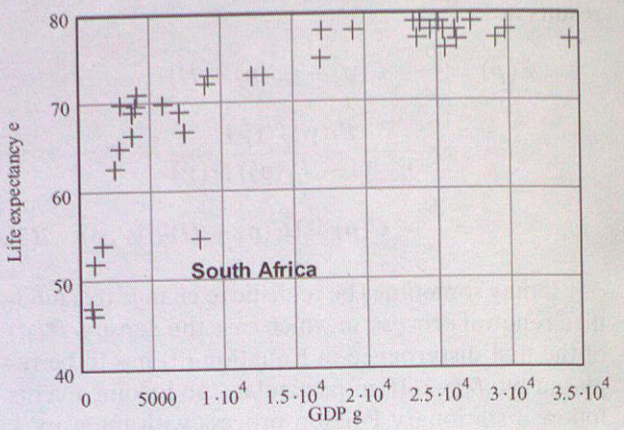

Fig. 1. Life expectancy versus GDP in different countries.(64)

and the GDP increased from some 2,000 PPPUS\$[2] well beyond 20,000 PPPUS\$.(37) Higher life quality, therefore, was not only achieved through longer lives and higher consumption but also by significantly more leisure time.

Some elegant mathematical derivation in Reference 34 leads to the traditional form of the *LQI*:

$$L_w = \frac{g^w}{q}e^{1-w}(1-w)^{1-w}, \tag{12}$$

where $q = w/(1 - w)$. For later convenience we take the $1/(1 - w)$th root so that:

$$L = \frac{g^q}{q}e(1-w). \tag{13}$$

The fraction of time w of e necessary for paid work varies between 0.12 and 0.25 (see Reference 48 for estimates of w for different countries but also References 40 and 37). Nathwani *et al.* (1997) start from a simple product $L = f(g)h(t)$ with $t = (1 - w)e$ and where t is the fraction of life devoted to leisure and *we* the fraction of life devoted to paid work. Thus, the LQI is a product of a function $f(g)$ measuring life quality and a function $h(t)$ measuring the duration of enjoyment of life. $f(g)$ and $h(t)$ are assumed to be independent and monotonically increasing functions. Income and life expectancy are highly correlated across countries (see Fig. 1). However, they are only weakly (and positively) correlated within countries with a developed social welfare system. This is demonstrated by Fig. 2 where it should be noted that the incomes in the highest quantile differ from those in the lowest

2 All monetary values are given in international US\$ adjusted for purchasing power parity according to the World Bank.(74)

Germany

Relative Income Position	Life Expectancy Men	Life Expectancy Female
1. Quartile	77	82
2. Quartile	82	85
3. Quartile	81	84
4. Quartile	83	86

Gini-coefficient= 0.30

Canada

Relative Income Position	Life Expectancy
1. Quintile	74.0
2. Quintile	76.9
3. Quintile	77.5
4. Quintile	78.1
5. Quintile	78.5

Gini-coefficient= 0.315

Fig. 2. Life expectancy versus income for Germany in year 2000(52) and for Canada in year 1986.(41)

quantile by a factor of 7–10, roughly the same range of incomes as the GDPs in Fig. 1. The product of two independent factors of the LQI in a given country, therefore, is justified in good approximation.

Defining relative changes in the LQI by $\frac{dL}{L} = \frac{g}{f(g)}\frac{df(g)}{dg}\frac{dg}{g} + \frac{t}{h(t)}\frac{dh(t)}{dt}\frac{dt}{t} = k_g\frac{dg}{g} + k_t\frac{dt}{t}$ and setting $k_g/k_t = const.$ according to the universality requirement in Reference 34, one finds two differential equations: $k_g \equiv \frac{g}{f(g)}\frac{df(g)}{dg} = r$ and $k_t \equiv \frac{t}{h(t)}\frac{dh(t)}{dt} = s$ with solutions $f(g) = g^r$ and $h(t) = t^s = ((1 - w)\ell)^s$. Assume then that $g \propto c\,\ell\,w$ where c is the productivity of work (GDP/working hours). Fig. 3 shows that the assumption of proportionality between GDP and productivity is in excellent agreement with data.

"*Presumably, people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work.*"(34) Consequently, people who work, possibly together with their families, optimize work and leisure time, i.e., their LQI. From $\frac{dL}{dw} = 0$ one determines $r = s\frac{w}{1-w}$, which, without loss

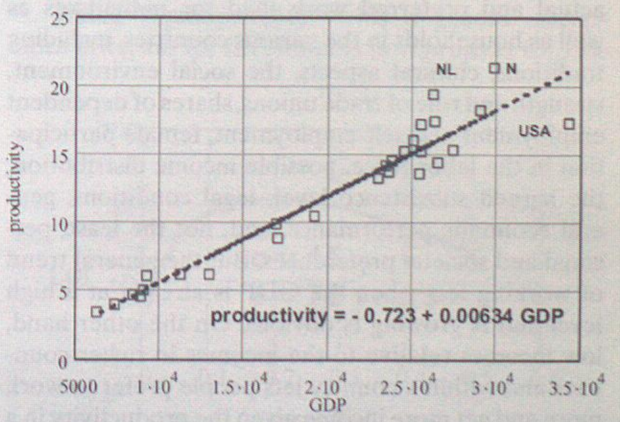

Fig. 3. Productivity versus GDP for various countries.

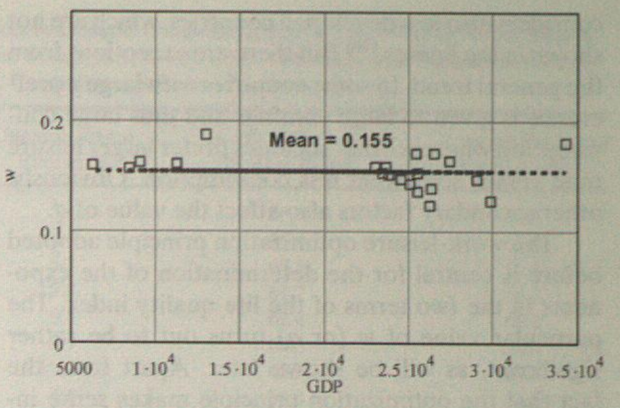

Fig. 4. Life working time for various countries according to Reference 40.

of generality, together with $r + s = 1$ results in $L = g^w e^{1w}(1 - w)^{1-w} \approx g^w e^{1-w}$. w can be assumed to be almost constant so that the factor $(1 - w)^{1-w}$ can be dropped in many applications. The index fulfills the boundary conditions $L = 0$ for $g = 0$ and $e = 0$. and should be interpreted as a utility functio $u(g, e, w)$ of an anonymous person. Additiona divide g^w by $q = \frac{w}{1-w}$, which gives Equatior Dividing g^w by q removes a minor inconsiste the original form for all practical purposes be persons with the same g and e but larger w have higher life quality. The position of the opti remains unchanged by these operations. In Figs 5 the quantities w and q are plotted for some in alized countries. Both w and q cluster closely around the global mean. However, it can be observed that societies with larger g generally work less, whereas people in countries with smaller g work more in order to increase utility of consumption—especially if one

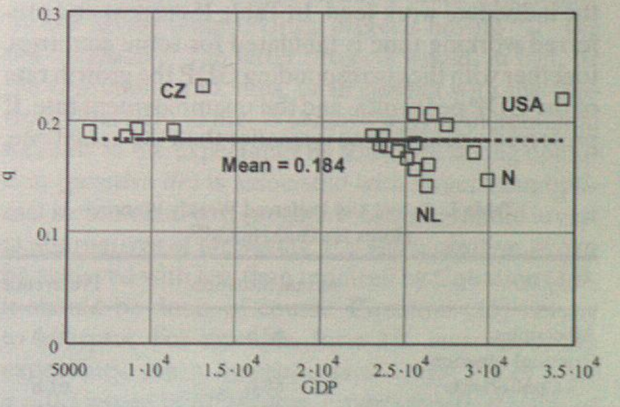

Fig. 5. Parameter q for various countries according to Reference 40.

considers also less developed countries, which are not shown in the figures.[48] But there are exceptions from the general trend. In some countries with large g preference is given to large earnings and thus large consumption whereas other societies prefer larger leisure time versus somewhat less consumption. Obviously, other secondary factors also affect the value of q.

The work-leisure optimization principle adopted before is central for the determination of the exponents in the two terms of the life quality index. The particular value of w (or q) turns out to be rather significant, as will be shown later. Apart from the fact that the optimization principle makes sense intuitively, at least in the long run, and is clearly supported by the above-mentioned historical development of wealth, life expectancy, and working time, the question is whether one can find other empirical evidence. Some indirectly supporting material is given in Reference 8 for European countries. For example, Bielenski *et al.*[8] show by a representative inquiry of up to 3,000 people in each of the different countries that people tend to prefer less work time in countries with a high GDP but would prefer more work and thus more income in countries with lower GDP (and larger unemployment rate and/or less work volume offered by the economy). Dependent full-time employees in countries with high GDP prefer to reduce their weekly working time from some 38.5 hours down to 34–36 hours. Self-employed people would like to reduce their work load from 48 hours down to 38 hours. The dependence between income and preferred weekly working time can clearly be seen from the analysis in Reference 8 on a household level (see Table I).

In countries with a high GDP (and low unemployment rate) there is a tendency to redistribute the available work volume among a larger labor force (especially females with part-time work) in order to reduce the individual work load. In Table II current and preferred working time is tabulated for some countries, together with the corresponding GDP, the growth rate of the GDP per capita, and the unemployment rate. If preferred working time is smaller than the actual one, the income level is supposed to remain constant, if it is larger more income is desired. These tendencies are also supported by recent official labor statistics.[40] In Reference 21 it is stated that in some countries full-time employees voted for weekly working times not below 35 hours.

The decline in working hours for full-time work, which can be traced back for over a century, is slowed down but still exists in most countries in recent years.[40] But further reductions in yearly working hours are still to be expected in many countries once the transition from purely full-time work to part-time work is realized given good economic performance. For two countries the trend is reversed, which are the United States and Sweden, but for two completely different reasons. For the United States a growing part of overtime work in a fast-growing economy (growth rate 2.8%) has led to a slight increase in working time since 1980 (without increasing part-time work). In Sweden it was just the large increase in part-time work and a transition from part-time work back to full-time work together with rather flexible (legalized) rules for individual work-time preferences. Therefore, Sweden is in line with the hypothesis while the development in the United States is exceptional.

The Netherlands and, in part Norway, are also exceptional in that recent sources give a rather low w due to a large proportion of part-time employment (up to 30%) but also due to the fact that the statistics contain only dependent employment. The relatively high value of w for the United States appears partially to be due to the household survey technique as opposed to the establishment survey technique used in most other countries,[39,40] but higher preference for large earnings might also serve as an explanation.

Many, partially interacting factors determine the actual and preferred work load for individuals as well as households in the various countries, including traditions, cultural aspects, the social environment, strength and role of trade unions, shares of dependent employment and self-employment, female participation in the labor force, possible income distribution, the agreed subsistence level, legal conditions, general economic performance, and, not the least, personal and societal preferences. But the general trend of working less when the GDP is already at a high level and is growing is obvious. On the other hand, low incomes relative to the incomes in richer countries and within a country lets people prefer to work more and get more income given the productivity in a country. There are good reasons to believe that many countries are already at or close to the optimum given their specific conditions (mainly productivity level), especially because the life quality index shows a rather flat optimum if plotted against w for given $r = 0.16$ and $s = 1 - r$ (see Fig. 6). And there appears to be sufficient evidence that the work-leisure optimization principle is, in fact, effective in general and in the long run.

Table I. Actual and Preferred Weekly Working Hours for Households[8]

	Actual Situation	Preference
All couples	62 h	61 h
Financial situation		
• Comfortable	66 h	61 h
• Adequate	59 h	61 h
• Difficult	53 h	64 h

Table II. Actual and Preferred Weekly Working Hours of Both Partners in a Couple (Household) With at Least One of the Partners in Paid Employment.[8] Actual Working Hours of Both Partners Together (Unemployed = 0). GDP After Reference 74, Growth Rate of GDP/Capita in 1975–2000 After Reference 65, Unemployment Rate After Reference 14

Country	GDP in PPP US$	Growth Rate GDP Per Capita in %	Unemployment Rate in %	Average Current Weekly Hours	Average Preferred Weekly Hours
Austria	26310	2.0	5.4	66.6	62.1
Belgium	27500	2.2	8.4	65.4	62.0
Denmark	25500	1.6	5.3	68.5	61.8
Finland	22900	2.0	9.8	67.7	66.3
France	24470	1.7	9.7	62.4	66.2
Germany	25010	1.9	9.9	60.8	59.6
Greece	16900	0.9	11.3	65.1	67.3
Ireland	25470	4.0	4.1	61.8	58.3
Italy	23400	2.1	10.4	58.0	58.9
Luxembourg	36400	3.9	2.7	58.0	55.8
Netherlands	26170	1.8	2.6	58.3	55.9
Portugal	17000	2.9	4.3	59.1	70.8
Spain	19300	2.2	14.0	54.4	66.0
Sweden	23770	1.4	6.0	69.3	65.9
United Kingdom	23500	2.0	5.5	66.4	58.9
Norway	29760	2.6	3.0	66.4	66.2

In summary, it is remarkable that all fundamental assumptions and hypotheses in the original developments by Nathwani *et al.*[34] are well supported by data, at least to the extent one can expect in this difficult field. If used as an alternative for the human development index (HDI),[64] the LQI should be appropriately normalized.

Using Equation (11) yields a general acceptance criterion for investments into projects for risk reduction:

$$\frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} \geq -\frac{g}{e}\frac{1}{q}, \qquad (14)$$

or by rearrangement:

$$\frac{dg}{g} + \frac{1}{q}\frac{de}{e} \geq 0. \qquad (15)$$

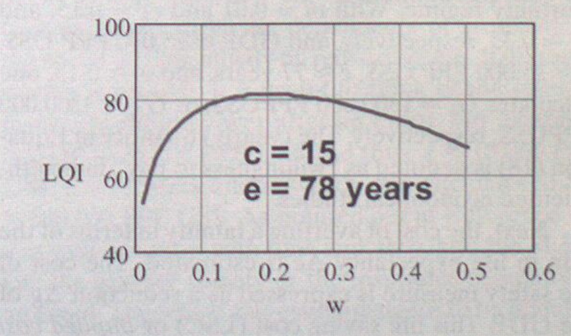

Fig. 6. LQI over working time fraction for given exponents.

Criteria of the type in Equation (15) remain unaffected by multiplicative constants such as the productivity c or multiplicative corrections of life expectancy as proposed in health-related economic studies in order to adjust life expectancy for life times in bad health. The equality in Equation (15) gives an indication of what is *necessary* and *affordable* to a society for life saving undertakings; projects having "<" are not admissible. The latter projects would, in fact, be life consuming and, thus, be in conflict with the constitutional right to life. Whenever a given incremental increase in life expectancy by some life saving operation (positive de) is associated with larger than optimal incremental cost (negative dg), one should invest in alternatives of life saving. If a given positive de can be achieved with less than required by Equation (15), it should be done, of course. Equation (15) is easy to interpret. For example, for a 1% increase in life expectancy, yearly investments of about 5% of g for $q = 0.2$ would be affordable. From a practical point of view it is important that all quantities on the left-hand side of Equation (15) are easily available and can be updated any time. The democratic equality principle dictates that average values for g, e, and w have to be taken. Any deviations from average values for any specific group of people need to be justified carefully if Equation (15) is applied to projects with involuntary risks. It is important to note that the criterion in Equation (15) is independent of any benefit other than life extension. Much further discussion is provided in References 31 and 34.

Life quality clearly has more dimensions than GDP, life expectancy, and leisure time. Values such as personal well-being, good family relationships, a healthy ecological environment, and many other values cannot be measured by the a little ambitiously named life quality index. However, we only intend to derive a criterion helping to balance conflicting aims.

Practical application of Equation (15) in a life saving operation is not always easy (see, however, the many examples in Reference 34). In general, the cost involved in some life saving operations can be estimated easily. The estimation of the effect of a life saving operation is more difficult but there are good approximations if life saving operations result in certain forms of small changes of age-dependent mortality rates. Let crude mortality be changed by dm. For a (small) uniform proportional change, i.e., $dm = \delta m$ or $\delta = dm/m$ in age-dependent mortality $\mu(a)$, i.e., $\mu_\delta(a) = \mu(a)(1+\delta)$, the change in de/e by expanding it into a McLaurin series and retaining only the linear term is:[30]

$$\begin{aligned}\frac{de}{e} &\approx \frac{\frac{d}{d\delta}\int_0^{a_u}\exp\left[-\int_0^a(\mu(\tau)(1+\delta))d\tau\right]da\;|_{\delta=0}}{\int_0^{a_u}\ell(a)da}\delta \\ &= \frac{\frac{d}{d\delta}\int_0^{a_u}\ell(a)^{1+\delta}da\;|_{\delta=0}}{\int_0^{a_u}\ell(a)da}\delta \\ &= -\frac{\int_0^{a_u}\ln(\ell(a))\ell(a)da}{\int_0^{a_u}\ell(a)da}\delta \\ &= -C_\delta\,dm = -c_\delta\frac{dm}{m}, \end{aligned} \tag{16}$$

where $c_\delta \approx 0.15$ (developed countries) to more than 0.5 (some developing countries) depending on the age structure and life expectancy of the group and therefore $C_\delta \approx 15$ (see Reference 48 for more details). Although this scheme has been used most in demographic sciences,[30] it places the majority of the profit of a mortality reduction on older people. Also, note that $c_\delta = C_\delta m$ but m and c_δ are not independent quantities.

Alternatively, one can assume that a (small) change $dm = \Delta$ in crude mortality distributes equally as a constant at all ages. Then, $\mu(a)$ changes into $\mu_\Delta(a) = \mu(a) + \frac{\Delta}{a_u}$ and one has

$$\begin{aligned}\frac{de}{e} &\approx \frac{\frac{d}{d\Delta}\int_0^{a_u}\exp\left[-\int_0^a(\mu(\tau)+\frac{\Delta}{a_u})d\tau\right]da\;|_{\Delta=0}}{\int_0^{a_u}\ell(a)\,da}\Delta \\ &= -\frac{\int_0^{a_u}\frac{a}{a_u}\ell(a)\,da}{\int_0^{a_u}\ell(a)\,da}\Delta \\ &= -C_\Delta dm = -c_\Delta\frac{dm}{m}, \end{aligned} \tag{17}$$

with $c_\Delta = C_\Delta m$. In this case the constants c_Δ are around $0.35 (C_\Delta \approx 35)$ for developed countries. For a given dm the changes in $\frac{de}{e}$ become roughly twice as large as in Equation (16). This must be expected because a constant change of $\mu(a)$ in young ages has substantially more effect on life expectancy than in older ages. For technical applications, e.g., in structural reliability, industrial hazard protection, flood protection, earthquake-resistant design, etc., this is probably the most realistic and fair regime.

Other mortality reduction regimes can be thought of. For example, one can consider age dependent mortality regimes if a change in mortality only affects those older than 60 years or any other age group as might be relevant in health-related public investments. The selection of the appropriate mortality regime turns out to be rather important in applications and must be suitably chosen in the context of a specific application. It should also be mentioned that choosing other mortality reduction regimes than Equation (17) can raise serious ethical questions because certain age groups profit more than others.

Using Equation (16) or (17) in Equation (15) leads to the yearly cost of a risk-reducing intervention:

$$dC_Y = -dg = g\frac{1}{q}\frac{c_x}{m}\,dm = g\frac{1}{q}C_x\,dm = G_x\,dm. \tag{18}$$

The index "x" stands for either "δ" or "Δ" or any other mortality regime. With $m = 0.01$ and $c_\delta = 0.15$, and $c_\Delta \approx 0.35$, respectively, and GDP $\approx$ 25,000 PPP US\$, $g \approx$ 15,000 PPP US\$, $e \approx 77$ years, and $w \approx 0.15$, one calculates $G_\delta \approx$ 1300,000 PPPUS\$, or $G_\Delta \approx$ 3200,000 PPPUS\$, respectively. The (yearly) quantity in Equation (18) is denoted as "willingness to pay" in health-oriented economical studies.

Next, the cost of averting a fatality in terms of the gain in life expectancy Δe is estimated. The cost of the safety measure is expressed as a reduction Δg of the GDP. This life saving cost (LSC) or *implied cost of averting a fatality* (ICAF) can be obtained from

the equality of Equation (15) after separation and integration from g to $g + \Delta g$ and e to $e + \Delta e$, i.e., the cost $\Delta C = -\Delta g$ per year to extend a person's life by Δe is

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{-\frac{1}{q}}\right].$$

Because ΔC is a yearly cost and the (undiscounted) LSC has to be spent for safety-related investments into technical projects at the decision point $t = 0$, one should multiply by $e_r = \Delta e$ and

$$LSC(e_r) = g\left[1 - \left(1 + \frac{e_r}{e}\right)^{-\frac{1}{q}}\right]e_r, \tag{19}$$

follows. The societal equality principle prohibits differentiating with respect to special ages within a group. The conditional (remaining) life expectancy given that the person has survived up to age a is

$$e(a) = \int_a^{a_u}\frac{\ell(t)}{\ell(a)}dt = \frac{1}{\ell(a)}\int_a^{a_u}\exp\left[-\int_0^t\mu(\tau)d\tau\right]dt. \tag{20}$$

Therefore, averaging the remaining life expectancy over the age distribution in a population leads to the societal life saving cost (SLSC):

$$SLSC = \int_0^{a_u} LSC(e(a))\,h(a,n)\,da \approx LSC\left(\frac{e}{2}\right), \tag{21}$$

where $h(a, n)$ is the density of the age distribution of the population with n its population growth rate. It depends on the particular social, taxation, and legal system of a country whether g or the full GDP has to be used in Equation (21). The density of the age distribution can be obtained from life tables. For a stable population it is given by

$$h(a,n) = \frac{\exp[-na]\ell(a)}{\int_0^{a_u}\exp[-na]\ell(a)\,da}. \tag{22}$$

In countries with a fully developed social system, SLSC is approximately the amount to support the (not working) surviving dependents of an event by the social system, mostly by redistribution. If no social system is present, it is useful to think of the amount insurance should cover after an event. For example, if GDP $\approx$ 25,000 PPP US\$ and thus, $g \approx$ 15,000 PPP US\$, $e \approx 77$ years, and $w \approx 0.15$, one calculates SLSC $\approx$ 600,000 PPP US\$. As pointed out in Reference 48, g as well as e grow with time and, thus, also the SLSC, i.e., the cost for society or the insurance company in an event. Therefore, this estimate needs to be updated from time to time.

4. DISCOUNTING AND AGE AVERAGING

Health-related macroeconomics has developed similar concepts starting with the seminal work by Usher.[66] Denote by $c(\tau) > 0$ the consumption rate at age τ and by $u[c(\tau)]$ the utility derived from consumption. Individuals tend to undervalue a prospect of future consumption as compared to that of present consumption. This is taken into account by some discounting. The lifetime utility for a person at age a until she/he attains age $t > a$ then is

$$\begin{aligned}U(a,t) &= \int_a^t u[c(\tau)]\exp\left[-\int_a^\tau\rho(\theta)d\theta\right]d\tau \\ &= \int_a^t u[c(\tau)]\exp[-\rho(\tau - a)]d\tau, \end{aligned} \tag{23}$$

for constant $\rho(\theta) = \rho$. There is evidence that a constant ρ is only a crude approximation but we will maintain it throughout the article. Note that discounting is with respect to utility and not with respect to consumption. It is assumed that consumption is not delayed, i.e., incomes are not transformed into bequests. ρ should be conceptually distinguished from a financial interest rate and is referred to as rate of time preference of consumption. The rate has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc. Its existence in human behavior has been widely demonstrated in human ethology and economics.[20,45] It is partially justified because there is uncertainty about one's future. Exponential population growth with rate n should be considered, replacing ρ by $\rho - n$, taking into account that families are by a factor $\exp[nt]$ larger at a later time $t > 0$. Approximate exponential population growth for the last 100 years can be verified from the data collected in Reference 37. The economics literature also states that if no such "discounting" is applied, more emphasis is placed on the well-being of future generations rather than on improving welfare of those alive at present, assuming economic growth. Future generations are wealthier. Therefore, one should add the real, exponential growth rate ζ or think of ρ as including ζ. Exponential economic growth at a constant rate can again be verified from the data in Reference 37 for at least the last 100 years. Economical growth will be considered explicitly in contrast to Reference 49. A rate $\rho + \zeta > n$ is necessary for Equation (23) to converge if future generations are included, i.e., if the utility integral must be extended to $t \to \infty$. ρ is reported to be between 1% and 4% for health-related investments, with a tendency to lower values.[67] Empirical estimates reflecting pure consumption behavior vary considerably but are in part significantly larger (see, for example, References 45 and 29). The numerical value of ρ will be discussed in detail below.

The expected remaining present value lifetime utility at age a (conditional on having survived until a) then is (see References 17, 2, 57, 53, 19, 28)

$$\begin{aligned} L(a) = E[U(a)] &= \int_a^{a_u} \frac{f(t)}{\ell(a)} U(a,t)\,dt \\ &= \int_a^{a_u} \frac{f(t)}{\ell(a)} \int_a^t u[c(\tau)] \\ &\quad \times \exp[-(\rho+\zeta-n)(\tau-a)]d\tau\,dt \\ &= \frac{1}{\ell(a)} \int_a^{a_u} u[c(t)] \\ &\quad \times \exp[-(\rho+\zeta-n)(t-a)]\ell(t)\,dt \\ &= u[c]e_d(a,\zeta,\rho,n), \end{aligned} \tag{24}$$

where $f(t)\,dt = (\mu(\tau)\exp[-\int_0^t \mu(\tau)d\tau])\,dt$ is the probability of dying between age t and $t + dt$ computed from life tables. Also, a constant consumption rate c independent of t has been introduced, which can be shown to be optimal under perfect market conditions.[57] Note that $L(a)$ is finite throughout due to $a_u < \infty$. The "discounted" life expectancy $e_d(a, \zeta, \rho, n)$ at age a can be computed from

$$\begin{aligned} e_d(a,\zeta,\rho,n) &= \frac{\exp((\rho+\zeta-n)a)}{\ell(a)} \\ &\quad \times \int_a^{a_u} \exp\left[-\int_0^t (\mu(\tau)+(\rho+\zeta-n))d\tau\right] dt. \end{aligned} \tag{25}$$

"Discounting" affects $e_d(a, \zeta, \rho, n)$ primarily when $\mu(\tau)$ is small (i.e., at young age) while it has little effect for larger $\mu(\tau)$ at higher ages. It is important to recognize that "discounting" by ρ is initially with respect to $u[c(\tau)]$ but is formally included in the life expectancy term. Clearly, there is $e_d(0, 0, \rho, 0) \leq e$ for $\rho > 0$. For the moment it is assumed that the mortalities $\mu(\tau)$ and, therefore, also the survival probabilities $\ell(\tau)$, do not change over time, for example, due to further progress in medical sciences.

For $u[c]$ we select a simple isoelastic power function

$$u[c] = \frac{c^q - 1}{q}, \tag{26}$$

with $0 \leq q \leq 1$, widely used in economics implying constant relative risk aversion (CRRA) according to Arrow-Pratt. The elasticity of marginal utility, i.e., the change in the slope of the utility function of consumption is: $\epsilon = -\frac{c\frac{d^2u(c)}{d^2c}}{\frac{du(c)}{dc}}$. $\epsilon > 0$ defines risk aversion, $\epsilon = 0$ risk neutrality and $\epsilon < 0$ risk proneness. For the utility function in Equation (26) we have constant $\epsilon = 1 - q$. The form of Equation (26) reflects the reasonable assumption that marginal utility $\frac{du[c]}{dc} = c^{q-1}$ decays with consumption c. $u[c]$ is a concave function since $\frac{du[c]}{dc} > 0$ for $q \geq 0$ and $\frac{d^2u[c]}{dc^2} < 0$ for $q < 1$. The power function form of Equation (26) is also the result of the derivations for Equation (13). For simplicity, we take $c = g \gg 1$.

Shepard & Zeckhauser[57] now define the "value of a statistical life" at age a by converting Equation (24) into monetary units in dividing it by the marginal utility $\frac{du(c(t))}{dc(t)} = u'[c(t)]$:

$$\begin{aligned} VSL(a) &= \int_a^{a_u} \frac{u[c(t)]}{u'[c(t)]} \\ &\quad \times \exp[-(\rho+\zeta-n)(t-a)t]\frac{\ell(t)}{\ell(a)}\,dt \\ &= \frac{u[c]}{u'[c]}\frac{1}{\ell(a)} \\ &\quad \times \int_a^{a_u} \exp[-(\rho+\zeta-n)(t-a)]\ell(t)\,dt \\ &= \frac{g}{q}\frac{1}{\ell(a)} \\ &\quad \times \int_a^{a_u} \exp[-(\rho+\zeta-n)(t-a)]\ell(t)\,dt \\ &= \frac{g}{q} e_d(a,\zeta,\rho,n), \end{aligned} \tag{27}$$

because $\frac{u[c(t)]}{u'[c(t)]} = \frac{g}{q}$. It is seen that $VSL(a)$ decays with age as $e_d(a, \zeta, \rho, n)$. The "willingness to pay" has been defined as

$$WTP(a) = VSL(a)\,dm. \tag{28}$$

Obviously, the mortality regime of Equation (17) is assumed in Equation (28) but a generalization to other mortality regimes should be possible. In analogy to Pandey and Nathwani,[42] and here we differ from the related economics literature, these quantities are averaged over the age distribution $h(a, n)$ in a stable population in order to take proper account of the composition of the population exposed to natural-event-type hazards and event-type hazards in and from

technical objects. This defines the "societal value of a statistical life":

$$SVSL = \frac{g}{q}\bar{E}(\zeta,\rho,n), \tag{29}$$

with the age-averaged, discounted life expectancy:

$$\bar{E}(\zeta,\rho,n) = \int_0^{a_u} e_d(a,\zeta,\rho,n)h(a,n)\,da. \tag{30}$$

The "societal willingness to pay" follows as

$$SWTP = SVSL\,dm. \tag{31}$$

For $\rho = 0$ the age-averaged "discounted" life expectancy $\bar{E}(\zeta, \rho, n)$ is a quantity that is about 60% of e and considerably less than that for larger ρ. The elasticity of $SVSL$ with respect to income is one. Analysis shows that $SVSL$ and with it $\bar{E}(\zeta, \rho, n)$ strongly decay with increasing $\zeta + \rho - n$.

The numerical value $q = \frac{w}{1-w}$ may be derived from the work-leisure optimization principle underlying Equation (13). Using this principle, one obtains $q \approx 0.2$ from estimates of w in Reference 48 and elsewhere. It agrees well with estimates used, for example, in References 2, 57 and 19.

Inspection of Equation (24) with Equation (26) and integrating over the age distribution $h(a, n)$, however, reveals exactly Equation (13) with e replaced by $\bar{E}(\zeta, \rho, n)$. It has been called *Societal Life Quality Index* (SLQI) by Pandey and Nathwani.[42]

$$L_{q\bar{E}} = \frac{g^q}{q}\int_0^{a_u} e_d(a,\zeta,\rho,n)h(a,n)\,da = \frac{g^q}{q}\bar{E}(\zeta,\rho,n). \tag{32}$$

It is to be emphasized that the SLQI, like the original LQI, is not a monetary quantity and has dimension "$(\text{US\$})^q(\textit{years})$". If divided by the marginal utility $u'(c)$ in order to convert it into a monetary quantity it coincides with Equation (29).

The reasoning in Equation (15) offers the possibility to arrive at a slightly different criterion for the willingness to pay. Define a new coefficient relating changes in mortality to changes in averaged "discounted" life expectancies for given mortality regimes, similar to Equation (16) or (17):

$$\begin{aligned} \frac{d\bar{E}}{\bar{E}} &\approx \frac{\frac{d}{dx}\bar{E}(x)\mid_{x=0} x}{\bar{E}} \\ &= -C_{x\bar{E}}(\zeta,\rho,n)dm = -\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\,dm. \end{aligned} \tag{33}$$

The coefficients $C_{x\bar{E}}(\zeta, \rho, n)$ for averaged "discounted" life expectancies turn out to be somewhat larger than those computed with "undiscounted" and not averaged life expectancies. The coefficients $C_{x\bar{E}}(\zeta, \rho, n)$ are all decreasing while $\zeta + \rho - n$ increases, but at different speeds depending on demographic characteristics. In Equation (33), discounting is applied in the numerator as well as in the denominator for all mortality reduction schemes. Therefore, the effect of discounting on $C_{x\bar{E}}(\zeta, \rho, n)$ is only moderate in contrast to the effect of discounting on SVSL.

Application of the reasoning in Nathwani *et al.*[34] leads to the same form as in Equation (15) with e replaced by $\bar{E}$:

$$\begin{aligned} \frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}}{\bar{E}} &\approx \frac{dg}{g} - \frac{1}{q}C_{x\bar{E}}(\zeta,\rho,n)\,dm \\ &\approx \frac{dg}{g} - \frac{1}{q}\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\,dm \geq 0. \end{aligned} \tag{34}$$

Rearrangement then produces a formula also expressing the "willingness to pay":

$$\begin{aligned} dC_Y &= g\frac{1}{q}C_{x\bar{E}}(\zeta,\rho,n)\,dm. \\ &= g\frac{1}{q}\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\,dm = G_{x\bar{E}}(\zeta,\rho,n)\,dm. \end{aligned} \tag{35}$$

$G_{x\bar{E}}(\zeta, \rho, n)$ in Equation (35) contains implicitly or explicitly crude mortality, which in this context can also be called background mortality, i.e., the specific mortality in a group due to other causes of death including those of natural death. It is remarkable that in both cases of Equation (29) and Equation (35) the societal willingness to pay is proportional to the mortality reduction, to the amount g of GDP available for risk reduction and some demographic constant (either $\bar{E}(\zeta, \rho, n)$ or $G_{x\bar{E}}(\zeta, \rho, n)$) and inversely proportional to the risk aversion parameter q or life working time fraction.

It must be admitted that two fundamental results of our approach, the proportionality of the willingness to pay and the level of mortality reduction, respectively, are not in full agreement with an empirical study.[4] But more empirical evidence is needed before crucial assumptions as those underlying Equations (31) or (35) are discarded.

For the same data as used for SLSC above, i.e., $g \approx 15{,}000$ PPP US\$, $e \approx 77$ years, $w \approx 0.15$, and $m \approx 0.01$, $n \approx 0.003$, $\zeta \approx 0.019$, $\rho \approx 0.006$, a European life table and, therefore, $C_{\delta\bar{E}}(\zeta, \rho, n) \approx 22$, the constant $G_{\delta\bar{E}}(\zeta, \rho, n)$ is 1.9×10^6 PPP US\$ for the mortality regime in Equation (16). If one adopts the mortality regime in Equation (17) we have $C_{\Delta\bar{E}}(\zeta, \rho, n) \approx 46$ and $G_{\Delta\bar{E}}(\zeta, \rho, n) \approx 3.9 \times 10^6$ PPP US\$. These values are to be compared with SVSL $= 1.9 \times 10^6$

PPP US$. Remember that discounting affects the SVSL estimates to greater extent than the constants $G_{x\bar{E}}(\zeta, \rho, n)$ $G_{x\bar{E}}$ and SVSL are four- to six-times the (undiscounted) residual life time earnings. Neglecting discounting altogether gives $C_{\Delta\bar{E}}(0, 0, 0) \approx 50$ and, therefore, $G_{\Delta\bar{E}}(0, 0, 0) \approx 4.2 \times 10^6$ PPP US$, which is close to $G_{\Delta\bar{E}}(\zeta, \rho, n)$ and SVSL $= 3.5 \times 10^6$ PPP US$. This suggests to circumvent the difficult question of discounting by the time preference rate and ignore discounting quite generally when assessing the demographic constants. No age averaging and no discounting results in $G_\Delta \approx 3.2 \times 10^6$ PPP US$, which is at the lower end of the estimates.

Both lines of thought, the economical and the LQI approach, have a good conceptual and theoretical basis. They complement each other. In particular, the derivations for Equation (13) justify the power function form in Equation (26) and let Equation (32) be interpreted as an expected remaining present value lifetime utility for all those alive at $t = 0$. Neither criterion in Equations (35) nor (31) depend on any benefit other than risk reduction or life extension. In most applications, clear support for decisions can be reached by using either of the approaches, even the one without discounting and age averaging. But it is believed that age averaging is generally necessary for the technical applications we have in mind because the risk reduction intervention is to be executed at $t = 0$ for all those living now. The concept of discounting future utilities by $(\zeta + \rho - n)$ may be debatable as the subjective time preference rate ρ is concerned but not with respect to the population and economic growth. The SLQI-based approach explicitly combines three important human concerns, that is high life expectancy, high consumption, and an optimized time available for the development of one's personality off the time for paid work. Criterion in Equation (35), having in mind its derivation, also tells us that larger expenses for risk reduction are inefficient and smaller expenses are not admissible in view of the constitutional right for life. Furthermore, the criterion of Equation (35) is affordable from a societal point of view. The "willingness to pay" according to Equation (35) should replace the one in Equation (18) except for cases in which the more general and probably more realistic concept leading to Equation (35) does not apply. Some further discussion is provided in Reference 50.

Similar adjustments with respect to discounting for the SLSC appear unnecessary. The compensation costs calculated approximately by the SLSC become real in an adverse event and have to be carried by the social system or insurance or both. Also, double discounting is to be avoided if SLSC are used in equations of the type of Equation (5).

5. APPLICATION TO TECHNICAL FACILITIES

The findings can be applied to safety regulations for structures and other technical facilities. It can reasonably be assumed that the life risk in and from such facilities is uniformly distributed over the ages, sexes, and economic status of those affected, assuming that everybody uses or is exposed to such facilities and, therefore, is also exposed to possible fatal accidents. It is also assumed that there is a constant stream of safety-related activities in time. The total cost of a safety-related regulation per member of the group and year is:

$$dg = -dC_Y(\mathbf{p}) = -\frac{1}{N}\sum_{i=1}^{n} dC_{Y,i}(\mathbf{p}), \qquad (36)$$

where n is the total number of objects under discussion, each with incremental yearly cost $dC_{Y,i}$ and N is the group size. Inserting into Equation (34) gives

$$\frac{-dC_Y(\mathbf{p})}{g} + \frac{1}{q}\left(-c_{x\bar{E}}\frac{dm}{m}\right) \geq 0. \qquad (37)$$

Let dm be proportional to the asymptotic failure rate $dh(\mathbf{p})$ $(t \to \infty)$ (see Equation (7)). Then, one finds by rearrangement:

$$\frac{dC_Y(\mathbf{p})}{dh(\mathbf{p})} \geq -k\frac{c_{x\bar{E}}}{m}g\frac{1}{q} = -k\, G_{x\bar{E}}, \qquad (38)$$

where $dm = kdh(\mathbf{p})$, $0 < k \leq 1$, the proportionality constant k relating the changes in mortality to changes in the failure rate and $G_{x\bar{E}} = \frac{1}{q}\frac{c_{x\bar{E}}}{m}g$. The constant k may be interpreted as a person's probability of actually being killed in case of failure. Note that for any reasonable intervention there is necessarily $dh(\mathbf{p}) < 0$. Also, it is independent of any benefit rate and is essentially an efficiency criterion as it relates incremental investments into life saving to incremental reductions of the failure rate.

A criterion like Equation (38) derived for safety-related regulations for a larger group in a society or the entire society can also be applied to individual technical projects. The constant $G_{x\bar{E}}$ and, similarly, the SLSC have been derived from general considerations of changes in mortality by changes in safety-related but costly measures implemented in a regulation, code, or standard by the public. $G_{x\bar{E}}$ as well as SLSC were related to one anonymous person. For a specific project it makes sense to apply the criterion in Equation (38) to the whole group exposed. One may think of a number of technical projects all subject to the same regulation and each with N_F potential fatalities in an exposed group of size N. Therefore, the "life saving cost" of a technical project with N_F potential fatalities is:

$$H_F = SLSCkN_F. \qquad (39)$$

This quantity must be used in equations of the type Equation (5) as part of the failure cost. The criterion in Equation (38) changes into:

$$\frac{dC_Y(\mathbf{p})}{dh(\mathbf{p})} \geq -k\, G_{x\bar{E}}\, N_F = -K_F, \qquad (40)$$

where the abbreviation $K_F = kG_{x\bar{E}}N_F = k\frac{c_{x\bar{E}}}{m}g\frac{1}{q}N_F$ is used. All quantities in Equation (38) are related to one year. They apply to a safety-related regulation under the assumption that there is steady-state building or production activity. For a particular technical project, all cost, denoted by $dC(\mathbf{p})$, must be raised at the decision point $t = 0$. The yearly cost in Equation (38) must be replaced by the erection cost at $t = 0$ on the left-hand side of the equation. The method of discounting is the same as for discharging an annuity. For infinite discounting time $(t \to \infty)$ consistent with the strategy of systematic reconstruction there is $dC_Y(\mathbf{p}) = dC(\mathbf{p})\gamma$. $dC_Y(\mathbf{p})$ may be interpreted as cost of societal financing of $dC(\mathbf{p})$. But the right-hand side, that is, g, also grows, at least with rate $\zeta - n$. But as mentioned discounting is already present in $c_{x\bar{E}}$. In order to avoid double discounting it is, therefore, proposed to discount with same rate on both sides of Equation (40). Then, the effect of discounting cancels and the acceptance criterion for individual technical projects is

$$\frac{dC(\mathbf{p})}{dh(\mathbf{p})} \geq -K_F. \qquad (41)$$

The same derivations apply to the purely economic concept with $G_{x\bar{E}}$ replaced by SVSL.

N_F as well as k must be estimated taking account of the average number of persons endangered by the event, the severity and suddenness of failure, possible availability and functionality of rescue systems, etc. N_F and k also depend on the cause of failure and, frequently, are dependent on the safety measures themselves. The estimation of realistic values of N_F and k might be the most difficult tasks in actual practical applications. It is typically the subject of risk analysis or, more precisely, failure consequence analysis. A sufficiently general formula for determining the probabiltiy adopted from[15] k is

$$k = \frac{N_F}{N_{PE}}, \qquad (42)$$

with

$$N_F = N_{PE}M_2M_3[M_4 + (1 - M_4)M_5],$$

where

- $N_F =$ expected number of fatalities in facility
- $N_{PE} =$ number of people in facility potentially endangered in event
- $M_2 =$ probability of being present in facility in event
- $M_3 =$ probability of being trapped in event (e.g., no escape)
- $M_4 =$ probability of being killed during failure
- $M_5 =$ probability of dying after failure (e.g., before rescue or in hospital).

6. OPTIMIZATION FOR TECHNICAL COMPONENTS

For the special task in Equation (5) with Equation (6) we have:

$$\text{maximize: } Z(\mathbf{p}) = \frac{b(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\frac{\lambda P_f(\mathbf{p})}{\gamma},$$
$$\text{subject to: } \quad f_k(\mathbf{p}) \leq 0, k = 1, \ldots, q \qquad (43)$$
$$\nabla_p C(\mathbf{p}) + K_F \nabla_p(\lambda P_f(\mathbf{p})) \geq 0,$$

where the first condition represents some restrictions on the vector $\mathbf{p}$ of optimization variables. The second condition represents the LQI-acceptability criterion written out for vectorial parameter $\mathbf{p}$ and $t \to \infty$. The failure consequences are now decomposed into direct cost H_M (including indirect failure cost such as loss of business, service, etc.) and life saving cost H_F, defined in Equation (39). Technical details for the solution of Equation (43) are summarized in Reference 60.

The formulation of Equation (43) includes the SLQI-criterion of Equation (41). Assume that the conditions $f_k(\mathbf{p}) \leq 0$ are not active in the solution point. At the optimum there must be $\nabla_p Z(\mathbf{p}) = \mathbf{0}$, i.e., for $\mathbf{p} = \mathbf{p}^*$

$$\nabla_p C(\mathbf{p}) + \left[(C(\mathbf{p}) + H_M + H_F)\nabla_p\left(\frac{\lambda P_f(\mathbf{p})}{\gamma}\right)\right] = 0, \qquad (44)$$

which is to be compared with the equality of Equation (41) written out for vectorial parameter **p** and $t \rightarrow \infty$:

$$\nabla_p C(\mathbf{p}) + K_F \nabla_p(\lambda P_f(\mathbf{p})) = 0. \tag{45}$$

If there is $(C(\mathbf{p}) + H_M + H_F)/\gamma \geq K_F$ the optimal solution for Equation (43) will automatically fulfill the SLQI-criterion of Equation (41). It can be shown that this is frequently the case under conditions of interest. Optimal structures are almost always safer than the SLQI criterion would require.

7. SOCIETAL DISCOUNT RATES

The cost for saving life years in Equation (41) also enters into the objective function of Equation (5) in the form of Equation (39) and with it the question of discounting those costs arises. In accordance with economic theory, benefits and (expected) cost, whatever types of benefits and cost are considered, should be discounted by the same rate as done above. Different parties, e.g., the owner, operator, or the public, may use different rates, however. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for an optimization in the name of the public is difficult. The requirement that the objective function must be nonnegative leads immediately to the conclusion that the interest rate must have an upper bound γ_{max} depending on the average benefit rate $b(\mathbf{p}) = \beta C(\mathbf{p})$ (see References 25, 48). For the model in Equation (5) we have:

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\frac{\lambda P_f(\mathbf{p})}{\gamma} = 0, \tag{46}$$

and, therefore, by solving for γ

$$\gamma < \gamma_{max} < \beta - \lambda P_f(\mathbf{p})\left(1 + \frac{H_M + H_F}{C(\mathbf{p})}\right), \tag{47}$$

implying $\gamma_{max} < \beta$ for $\lambda\, P_f(\mathbf{p}) \ll \beta$. The right-hand side of inequality in Equation (47) depends on **p** and, therefore, we could solve for a maximum interest rate γ_{max} by maximizing it. It turns out that the solution vector $\hat{\mathbf{p}}$ is very close or numerically identical to the solution vector $\mathbf{p}^*$ for the optimization task in Equation (5). In good approximation one can set $\hat{\mathbf{p}} \approx \mathbf{p}^*$. It follows that the benefit rate β must be slightly larger than γ_{max}. From Equation (46) one also concludes that there must be $\gamma > 0$.

At first sight, discounting of human lives is not in agreement with our moral value system. However, a number of studies summarized in References 43 and 32 express a rather clear opinion based on ethical and economical arguments. The cost for saving life years must be discounted at the same rate as other investments, especially in view of the vision that our present value system is expected to be maintained for future generations, a goal that is supported by empirical studies on human preferences quoted in Reference 32. Otherwise serious inconsistencies cannot be avoided.

What should then the societal interest rate be? It is clear that it is different from the interest rates on the financial market. In view of the time horizon of some 20 to more than 100 years (i.e., several generations), it should be a long-term average. It should be net of inflation and taxes. Weinstein and Stason[72] and others require that interest rates for life saving investments should be the same as for other costs and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed in References 56, 12, 15, and 18, based primarily on ethical grounds in the context of CO_2-induced global warming, nuclear waste disposal, depletion of natural resources, etc. In this case, the rationale of our basic optimization model in Equation (5) and part of the considerations in Section 4 break down. But it is beyond the author's grasp to imagine an economic world without discounting. There have been ongoing but somewhat inconclusive discussions when discounting public investments into health care (see, for example, Reference 67). As already mentioned, those discussions have been revived recently in the context of sustainable development, long-term public investments in general, and intergenerational justice—aspects that appear particularly relevant in our context. Our choices of discount rates for technical objects should at least be consistent with those for a sustainable economic development and should equally fulfill the requirement of intergenerational equity. Therefore, in the following, the main stream of arguments is reviewed.

Due to the requirement $\beta > \gamma_{max}$ stated just below Equation (47), the interest rate is strongly related to the benefit a society earns from its various activities, i.e., its real economic growth. The growth rate measures the success of all activities of a society—among them also activities for saving lives. It is sometimes called the "natural interest rate" and mirrors technological progress. In most developed industrial countries the growth rate was about 2% over the last 50 years. The United Nations Human Development Report 2001[64] gives values between 1.2% and 1.9% for industrialized countries during 1975–1998. If one considers the last 120 years and the data in Reference 37 for some selected countries, one determines a growth rate $\zeta = \frac{\ln(g_{1992}/g_{1870})}{1992-1870} 100$ for exponential growth of about 1.8%.

Some more insight can be gained from modern economic growth theory. Nordhaus[36] and others follow the classical Ramseyan approach (see References 51, 59, 10, and 6) for optimal economic growth:

$$\gamma = \rho + \epsilon\, \zeta > 0, \tag{48}$$

where ρ is the rate of pure time preference of consumption, $\epsilon > 0$ is the elasticity of marginal consumption (income), and ζ is the consumption (income) growth rate. Here, a perfect market with stable growth is assumed. In such a market γ equals the real growth rate of the total output of goods and services and this is set equal to the real market interest rate. With $\rho \approx 0.03$ and $\zeta \approx 0.02$ as well as $\epsilon = 1$ Nordhaus[36] obtains $\gamma \approx 0.05$. Arrow[1] estimates $\gamma \approx 0.03$ assuming $\rho \approx 0.01$, $\zeta \approx 0.012$, and $\epsilon = 1.5$ (!), but with a tendency to larger values. In many other studies for sustainable development, discount rates cluster around 5% (see Reference 63 for a review). All those values are close to the real market rates or only a little smaller. Solow,[59] who presumes $\rho \approx 0.01$ to 0.02, adds a convergence condition for the (infinite) utility integral

$$\rho + \epsilon\zeta > n + \zeta \tag{49}$$

to Equation (48). The term $\epsilon\ \zeta$ is generally undisputed but there are many authors in economics as well as in the philosophical and political sciences, including Ramsey, who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds[56,12,46,26,7] while it is considered fully acceptable for intragenerational discounting. Also, positive rates $\rho > 0$ are shown to be not mandatory for investments into health care (see, for example, Reference 3). Intergenerational and intragenerational rates of pure time preference, if greater than zero, should be the same if strongly counterintuitive results are to be avoided.[18] On the other hand, intergenerational equity arguments in Reference 1, while fully accepting zero time preference rates from a moral point of view, indicate that there should be $\rho > 0$ in order to remove an "*incredible and unacceptable strain on the present generation.*" Rabl,[46] who sets $\rho = 0$, argues that there must be $0 < \gamma < \epsilon\ \zeta$ in the framework of long-term public investments. However, Rabl neglects demographic aspects. As mentioned earlier, we must have $\rho > n$ and, therefore, with $\rho \approx n$ at least $0 < \gamma < n + \epsilon\zeta$. On the basis of the Solow condition, Equation (49), one can, in fact, justify a rate ρ even slightly larger than n. One derives

$$n + \zeta(1 - \epsilon) < \rho < \gamma \leq \gamma_{max} < \beta$$
$$= n + \epsilon\zeta \text{ or } \beta = n + \zeta, \tag{50}$$

with preference for the smaller upper bound resulting from $\rho = n$. The larger upper bound is obtained by using $\rho = n + \zeta\ (1 - \epsilon)$ in Equation (50). Remarkably, the parameter ϵ then drops out in Equation (50). It appears that ρ is small enough to be acceptable in view of the controversy about the rate of time preference of consumption. Also, the values for β appear reasonable and acceptable, maybe with the exception of the extremes. It is then possible to compute $\gamma_{max} < \beta$ from Equation (47). γ_{max} usually is only insignificantly (1–20%) smaller than β depending on the specific case at hand, i.e., the particular sensitivities of $C(\mathbf{p})$ and $h(\mathbf{p})$ with respect to **p**. The interest rates γ_{max} implied by the value of β are considerably lower, around 1.7%, than the usual real market interest rates.

The above considerations based on a simple, ideal, steady state Ramseyan growth model in a closed economy can at least define the range of benefit and interest rates as well as reasonable rates of pure time preference to be used in long-term investments into life saving operations. It is believed that the steady state assumption of the Ramsey model is not too far from reality in developed countries. Also, the assumption of an infinite time horizon is consistent with our general setting. Historical long-term population and economic growth rates cannot be questioned. The value of ϵ varies very little, between 0.75 and 0.85, say. Only the pure time preference rate ρ to be used in Equation (24) and possibly in Equation (50) can be subject to discussion. It is suggested to take the lowest possible value, which is $\rho = n + \zeta\ (1 - \epsilon) > 0$. Of course, our considerations do not exclude larger rates for the time preference of consumption in special projects if there are no potential intergenerational conflicts.

In the literature, the adequacy of the Ramseyan model is sometimes questioned. For example, the so-called overlapping generation models or generation adjusted discounting models are advocated instead (see Reference 10 for theoretical considerations and References 7 and 23 for applications). The main idea is to discount for living generations at the rate in Equation (48) but diminish the rate for all yet unborn generations down to $n + \epsilon\zeta > 0$ or even lower. Other extensions and/or modifications have been proposed.

But it is not expected that those models, if properly justified, change our results significantly.

Some precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. In many industrialized countries a very small or negative population growth rate is expected for the future, accompanied by larger life expectancies and a significant change in the age structure, implying decreasing national labor forces and requiring new social structures. At the same time, the world population will grow by 1.2% each year, mainly in developing countries. Natural resources will be depleted and arable land will become scarce. Many people raise serious doubts whether those demographic changes and the increasing scarcity of natural resources can be fully compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will happen. But there is an important mathematical result that may guide our choice. Weitzman[71] and others showed that the far-distant future should be discounted at the lowest possible rate if there are different possible scenarios each with a given probability of being true.

Table III collects some relevant data for countries for which sufficiently reliable economic and demographic data are available. The data can vary depending on the type and date of the sources used. The life tables are all recent period life tables from national statistical offices or from Reference 11. n, m, e, and w are taken at their present values. The age distribution $h(a, n)$ in Table III is determined from period life tables. Stable populations are assumed. Because the largest age a_u considered in the life tables is around 110 years, this is also the time span over which our considerations are valid. The economic growth rate ζ has been averaged over the years 1870–1992. It certainly would be misleading to take only averages over the last 50 years or less. The values for ρ, β, $SLSC$, $G_{\delta E}$, and SVSL calculated from these data using Equations (50), (35), and (29) are given in Table III. $G_{\Delta E}$ is given in Reference 75 and are not repeated here. Instead, the values G_Δ (Equation (18)) and $G_{\Delta E}$ (Equation (35)) are presented in Table 3 together with the other data. The demographic constants $C_{\Delta x}$ can be calculated by multiplying the value given in Table 3 by the corresponding q/g. It is seen that $G_{\Delta E}$ generally is largest followed by the undiscounted and not age-averaged coefficient G_Δ. The largest uncertainties are possibly due to the part of GDP effectively available for risk reduction and due to the life working time estimates. As suggested earlier, the part of the GDP available for risk reduction is taken as that available for private use. The Scandinavian countries have comparatively low values due to a smaller share g of their GDP for private use in the official sources. Some adjustments are

Table III. Social Indicators for Some Countries: (a) after Reference 74 in PPPUS\$, (b) Private consumption in PPPUS\$ According to Reference 64 (c) Economic Growth in % for 1870–1992 After Reference 37, (d) Crude Mortality (2000) in %,[14] (e) Population Growth (2000) in %,[14] (f) Estimates Based on Reference 40 Including One Hour Travel Time Per Working Day and a Life Working Time of 45 Years, (g) SLSC Computed with g and Age-Averaged Life Expectancies

Country	$GDP^{a,b}$	ζ^c	m^d	n^e	e	w^f	ρ	β	$SLSC^g$	$G_\Delta, G_{\Delta E}$	$SVSL$
Canada	27,330, 16,040	2.0	0.73	0.99	78	0.17	1.4	2.6	6.7×10^5	$4.0 \times 10^6, 4.8 \times 10^6$	2.0×10^6
USA	34260, 22030	1.8	0.87	0.90	77	0.18	1.3	2.3	8.7×10^5	$4.1 \times 10^6, 4.9 \times 10^6$	2.5×10^6
Austria	26,310, 14790	1.8	0.98	0.24	77	0.14	0.6	1.7	5.8×10^5	$3.4 \times 10^6, 4.2 \times 10^6$	2.3×10^6
Czech Rep.	12,900, 6,730	1.5	1.08	−0.07	73	0.19	0.3	1.2	2.2×10^5	$9.8 \times 10^5, 1.3 \times 10^6$	7.4×10^5
Denmark	25,500, 12,900	1.8	1.09	0.30	77	0.14	0.6	1.8	5.0×10^5	$2.7 \times 10^6, 3.3 \times 10^6$	2.0×10^6
Finland	22,900, 12,100	1.8	0.98	0.16	77	0.16	0.5	1.9	4.5×10^5	$2.4 \times 10^6, 3.0 \times 10^6$	1.6×10^6
France	24,470, 14,660	1.9	0.91	0.37	78	0.15	0.7	1.9	5.8×10^5	$3.4 \times 10^6, 4.2 \times 10^6$	2.1×10^6
Germany	25,010, 14,460	1.9	1.04	0.27	78	0.14	0.6	1.9	5.6×10^5	$3.6 \times 10^6, 4.5 \times 10^6$	2.1×10^6
Ireland	25,470, 12,610	1.5	0.81	1.12	76	0.15	1.4	2.3	5.2×10^5	$3.2 \times 10^6, 3.8 \times 10^6$	2.0×10^6
Italy	23,400, 14,460	1.9	1.01	0.07	79	0.15	0.4	1.6	5.5×10^5	$2.7 \times 10^6, 3.4 \times 10^6$	1.9×10^6
Netherlands	26,170, 15,470	1.5	0.87	0.55	78	0.12	0.8	1.8	6.4×10^5	$4.5 \times 10^6, 5.6 \times 10^6$	3.0×10^6
Norway	29,760, 14,149	2.1	0.98	0.49	78	0.13	0.8	2.3	5.8×10^5	$3.5 \times 10^6, 4.3 \times 10^6$	2.3×10^6
Sweden	23,770, 12,620	1.9	1.06	0.02	79	0.15	0.3	1.6	4.7×10^5	$2.4 \times 10^6, 3.1 \times 10^6$	1.7×10^6
Switzerland	29,000, 17,700	1.9	0.88	0.27	79	0.15	0.6	1.8	7.0×10^5	$4.3 \times 10^6, 5.3 \times 10^6$	2.5×10^6
UK	23,500, 15,140	1.3	1.07	0.23	78	0.16	0.5	1.3	5.7×10^5	$2.7 \times 10^6, 3.4 \times 10^6$	2.3×10^6
Japan	26,460, 15,960	2.7	0.83	0.17	80	0.17	0.7	2.3	6.0×10^5	$3.4 \times 10^6, 4.1 \times 10^6$	1.6×10^6
Australia	25,370, 15,750	1.2	0.72	0.99	78	0.17	1.2	1.9	6.5×10^5	$4.2 \times 10^6, 5.2 \times 10^6$	2.4×10^6

necessary so that the quantity g includes all health expenditures. Although the work-leisure principle outlined before may still be valid, in general it appears that the accounting of life working time must also be improved for our purposes. Additional material on working times can be found in References 27, 37, 22, and 39. Such adjustments are expected to be less than 5–10% in the values of G_x.

There have been many attempts to estimate this quantity indirectly (among the rich literature for this subject see, for example, References 67, 68, 62, 33, 58 for a collection of governmental stipulations), mostly by estimating the cost of some life saving operation like limiting highway speed,[5] installing smoke detectors in homes, or using seat belts in cars. Also, the compensation in risky jobs by higher wages has been used,[69] as well as surveys with respect to hypothetical risky situations, so-called contingent valuation studies.[44] The values reported are between less than 1 million US\$ and more than 10 million US\$, i.e., more than 2–20 times as much as the (undiscounted) value of average lost earnings in case of a fatal accident at mid life. The studies in References 70, 38, and 9 are so-called meta-analyses, i.e., analyses across several other studies. For comparison, $G_{\Delta E}(\zeta, \rho, n)$ and SVSL should be calculated with the full GDP, i.e., the values in Table III should be multiplied by GDP/g. In Figs. 7 and 8, some results collected in References 70 and 38 are presented graphically showing the large scatter of the estimates. The estimates in Reference 38 based on wage differentials in the United States are significantly smaller. The correlation coefficient as a crude measure of the dependence of VSL on income or GDP is also given in the figures.

For public risk reduction interventions the interest rates, i.e., ρ as well as γ to be inferred from β, shown in Table III appear low enough to be acceptable, especially in view of the large uncertainties when assessing the quantities k and N_{PE}. Note that larger βs tend to occur whenever the population and/or economic growth rates are also larger. In general, the βs are smaller than presently used for public investments, which are between 2% and 7%.

There is another sustainability aspect. Period life tables have been used for Table III. So-called cohort life tables certainly would be better as they reflect the common trend toward larger life expectancies and more compact age distributions. Time- and age-dependent mortalities can be obtained by extrapolating from a sequence of historical period life tables so that $\mu_{t,y}(a) = \mu_y(a)b(a)^{\vartheta+a-y}$ where y is the reference year, i.e., the last year for which a period table

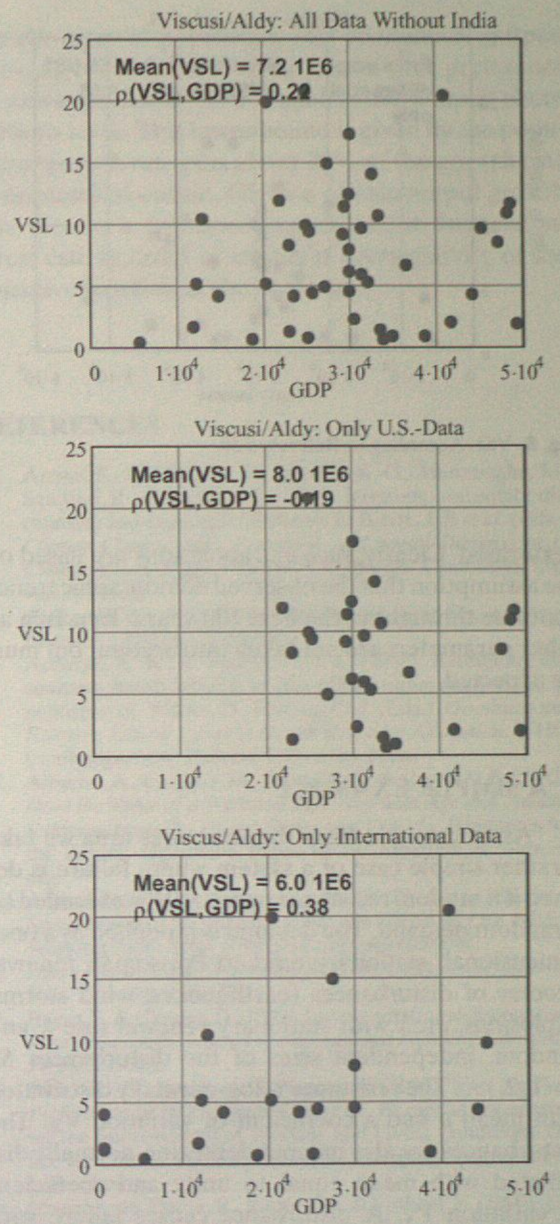

Fig. 7. VSL according to Reference 70.

is available and $\vartheta \leqslant y$ is the year of birth. Unfortunately, cohort life tables exist only for a few countries. Cohort life tables, for example, yield 7% larger life expectancies at present. Example calculations indicated that the results for the constants $G_{x E}$ or SVSL differ at most by ± 20% from those obtained for period life tables. Results are collected in Table IV for six countries for which an uninterrupted sequence of period life tables is available for at least the last 50 years so that extrapolations for the age-dependent mortalities can be

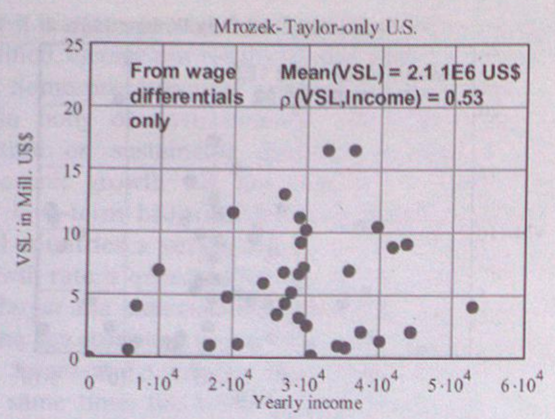

Fig. 8. VSL According to Reference 38.

performed. Clearly, such extrapolations are based on the assumption that the observed demographic trends continue throughout the next 100 years. Trends in all other parameters are not taken into account but must be expected.

8. A SIMPLE EXAMPLE

As an example from the structures area we take a rather simple case of a system where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modelled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances S_i, $i = 1, 2, \ldots$. The resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independently log-normally distributed with mean equal to unity and coefficient of variation V_S. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln\left((1+V_R^2)(1+V_S^2)\right)}}\right). \quad (51)$$

An appropriate objective function then is with $b = b(\mathbf{p})$

$$Z(p) = \frac{b}{C_0\gamma} - \frac{C_1}{C_0}p^a - \left(\frac{C_1}{C_0}p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right)\frac{\lambda P_f(p)}{\gamma}. \quad (52)$$

The criterion in Equation (41) has the form

$$\frac{d}{dp}(C_0 + C_1 p^a) \geq -K_F \frac{d}{dp}\lambda P_f(p). \quad (53)$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_M = 3C_0$, $V_R = 0.2$, $V_S = 0.3$, and $\lambda = 1\,[1/year]$. The *LQI*-data is $e = 77$, $GDP = 25000$, $m = 0.01$, $C_{xE} = 25$, $w = 0.16$, $N_F = 10$, and $k = 0.1$ so that $H_F = 5.1 \times 10^5$ and $K_F = 1.1 \times 10^8$ for $\gamma_S = 0.0188$. Monetary values are in US$. Optimization is performed for the public and for the owner separately.

For the public, $b = 0.02C_0$ and $\gamma_S = 0.0188$ are chosen following Reference 48. In particular, they are chosen such that the public does not make direct profit from an economic activity of its members. Optimization including the cost H_F gives $p^* = 4.11$, the corresponding failure rate is 2.5×10^{-5}. $Z_S(p^*)/C_0 \approx 0.016$ and $Z_S(p)$ is positive in the interval $p^* = [4.00, 4.28]$ (see Fig. 9). Criterion (41) is already fulfilled for $p_{\lim} = 3.84$ corresponding to a yearly failure rate of 5.5×10^{-5} but $Z_S(p_{\lim})/C_0$ being already negative.

The owner uses some typical values of $b = 0.07C_0$ and $\gamma_O = 0.05$ and does not include life saving cost. The calculations yield $p^* = 3.76$, the corresponding failure rate is 7.1×10^{-5}. The LQI-based acceptability criterion limits the owner's region for reasonable designs with $Z_O(p^*)/C_0 > 0$. At the optimum it is $Z_O(p^*)/C_0 = 0.342$ and $Z_O(p)$ is positive in the interval $p = [2.43, 19.18]$. Note that from the public's perspective $p_{\lim} = 3.84$, which requires the owner to go a little beyond his or her optimum but $Z_O(p_{\lim})/C_0 > 0$. The optimum $Z_O(p^*)/C_0$ for the user is larger than that for the public due to the larger interest rate, a higher benefit, and because life saving cost H_F are not considered. It also leads to slightly less safe structures, which again is primarily caused by the higher interest rate.

Table IV. Social Indicators for Some Countries, e in Year 2100. SLSC, $G_{\Delta E}$, and SVSL Computed from Cohort Life Tables Based on Reference 11. Notations as in Table III

Country	GDP,g	ζ	m	n	e	w	ρ	β	$SLSC$	$G_{\Delta E}$	$SVSL$
USA	34,260, 22,030	1.8	0.87	0.90	86	0.17	1.3	2.3	9.9×10^5	5.7×10^6	2.9×10^6
France	24,470, 14,660	1.9	0.91	0.37	85	0.14	0.7	1.9	8.2×10^5	6.1×10^6	2.7×10^6
Germany	25,010, 14,460	1.9	1.04	0.27	85	0.13	0.6	1.9	8.0×10^5	5.4×10^6	2.8×10^6
Japan	26,460, 15,960	2.7	0.83	0.17	92	0.16	0.7	2.3	6.9×10^5	4.9×10^6	1.7×10^6
Sweden	23,770, 12,620	1.9	1.06	0.02	82	0.14	0.3	1.6	4.9×10^5	3.3×10^6	1.9×10^6
Switzerland	29,000, 17,700	1.9	0.88	0.27	85	0.14	0.6	1.8	7.5×10^5	5.7×10^6	2.6×10^6

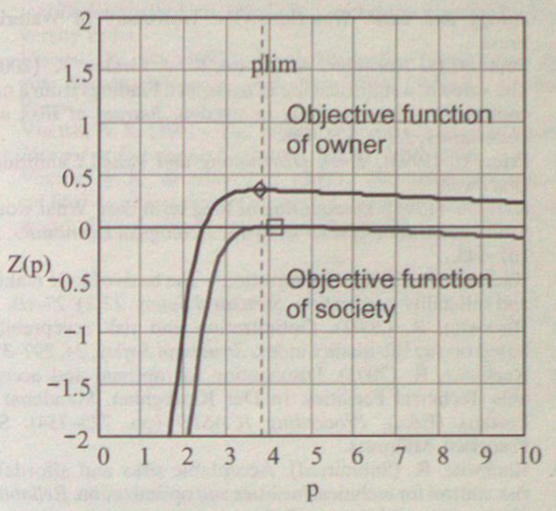

Fig. 9. Objective functions for owner and society.

9. CONCLUSIONS

Cost-benefit optimization is a powerful tool for designing risky technical projects. But this requires to include realistic life saving cost and a suitable acceptance criterion. Both life saving cost and the acceptance criterion can be derived on the basis of the life quality index (LQI). Life saving cost are identical with the SLSC (implied cost of averting a fatality). Application to technical facilities is presented. It is shown that the societal life quality index fits very well into general economic approaches of health care and life saving. Optimization with life saving cost included requires suitable societal discount rates. The (real) discount rate for life saving operations is discussed in the framework of modern economical growth theory. It is found that societal interest rates in industrial countries should be slightly smaller than the sum of the growth rates of population and economical output, between 0.6% and 2.6%. The rate of time preference of consumption may be bounded to the below at about 0.0% to 1.4%. This lower bound is given by the population growth rate plus about 20% of the growth rate of economical output. Given a certain output growth rate there is a corresponding maximum financial interest rate in order to maintain nonnegativity of the objective function at the optimum.

REFERENCES

1. Arrow, K. J., Cline, W. R., Maeler, K.-G., Munasinghe, M., Squitieri, R., & Stiglitz, J. E. (1996). Intertemporal equity, discounting and economic efficiency. In Bruce, J. P. *et al.* (Eds.), *Climate Change 1995, Economic and Social Dimensions of Climate Change* (pp. 125–144) Cambridge: Cambridge University Press.
2. Arthur, W. B. (1981). The economics of risks to life. *American Economic Review, 71*, 54–64.
3. Acharya, A. K., & Murray, C. J. L. (2002). Rethinking discounting health benefits in cost effectiveness analysis. To be published in Wikler, D., Murray, CLJ. (Eds.), *Goodness and Fairness: Ethical Issues in Health Resource Allocation*. WHO, Cambridge, MA: Harvard University Press.
4. Alberini, A., Cropper, M., Krupnik, A., & Simon, N. B. (2002). *Does the Value of a Statistical Life Vary with Age and Health? Evidence from the United States and Canada*. Resources for the Future, Discussion Paper 02-19. Available at www.rff.org.
5. Ashenfelter, O., & Greenstone, M. (2002). *Using Mandated Speed Limits to Measure the Value of a Statistical Life*. NBER Working Paper 9094, www.nber.org/papers/w9094.
6. Barro, R. J., & Sala-i-Martin, X. (1995). *Economic Growth*. New York: McGraw-Hill.
7. Bayer, S., & Cansier, D. (1999). Intergenerational discounting: A new approach. *Journal of International Planning Literature, 14*(3), 301–325.
8. Bielenski, H., Bosch, G., & Wagner, A. (2002). *Working Time Preferences in 16 European Countries*. European Foundation for the Improvement of Working and Living Conditions, Office for Official Publications ot the European Communities, Luxembourg.
9. Blaeij, A. de, Florax, R., Rietfeld, P., & Verhoef, E. (2000). *The Value of Statistical Life in Road Safety*. Tinbergen Institute, TI 2000-089/3.
10. Blanchard, O. J., & Fischer, S. (1989). *Lecture on Macroeconomics*. Cambridge: MIT Press.
11. Berkeley Mortality Database. Available at www.mortality.org.
12. Broome, J. (1992). *Counting the Cost of Global Warming*. Cambridge, UK: White Horse Press.
13. Cantril, H. (1965). *The Pattern of Human Concerns*. New Brunswick, NJ:, Rutgers University Press.
14. CIA-Factbook. (2001). Available at www.cia.gov/cia/publications/factbook/.
15. Coburn, A., & Spence, R. (1992). *Earthquake Protection*. Chichester: J. Wiley & Sons.
16. Cox, D. R. (1962). *Renewal Theory*. London: Methuen.
17. Conley, B. C. (1976). The value of human life in demand for safety. *American Economic Review, 66*(1), 45–55.

18. Cowen, T. (2001). *What is the Correct Intergenerational Discount Rate?* Working paper, Center for Study of Public Choice, George Mason University, Fairfax, VA 22030.
19. Cropper, M. L., & Sussman, F. G. (1990). Valuing future risks to life. *Journal of Environmental Economics and Management*, *19*, 160–174.
20. Cropper, M. L., Aydede, S. K., & Portney, P. R. (1992). Rates of time preference for saving lives. *American Economics Review*, *82*(2), 469–472.
21. European Industrial Relations Observatory. (2001). *Working Time Developments—Annual Update* 2001.
22. Evans, J. M., Lippoldt, D. C., & Marianna, P. (2001). *Trends in Working Hours in OECD Countries*, *45*. OECD, Paris.
23. Gerlagh, R., & v.d.Zwaan, B. (2001). The effects of aging and an environmental trust fund in an overlapping generations model on carbon emission reductions. *Ecological Economics*, *36*, 311–326.
24. Hasofer, A. M. (1974). Design for infrequent overloads. *Earthquake Engineering and Structure Dynamics*, *2*(4), 387–388.
25. Hasofer, A. M., & Rackwitz, R. (2000). Time-dependent models for code optimization. In R. E. Melchers & M. G. Stewart (Eds.), *Proceedings of ICASP'99* (pp. 151–158). Rotterdam: Balkema.
26. Heal, G. (1997). Discounting and climate change: An editorial comment. *Climate Change*, *37*, 335–343.
27. International Labour Office. (1993). *World Labour Report 1993*. Geneva.
28. Johannesson, M., Johansson, P-O., & Löfgren, K-G. (1997). On the value of changes in life expectancy: Blips versus parametric changes. *Journal of Risk and Uncertainty*, *15*, 221–239.
29. Kapteyn, A., & Teppa, F. (2002). *Hypothetical Intertemporal Consumption Choices*. Working paper, CentER, Tilburg University, Netherlands.
30. Keyfitz, N. (1985). *Applied Mathematical Demography*. New York: Springer.
31. Lind, N. C., Nathwani, J. S., & Sidall, E. (1993). *Managing Risks in the Public Interest*. Institute for Risk Research, University of Waterloo, Waterloo, Canada.
32. Lind, N. C. (1994). Target reliabilities from social indicators. In *Proceedings of ICOSSAR93* (pp. 1897–1904). Balkema.
33. Lutter, R., & Morrall, J. F. (1994). Health-health analysis, a new way to evaluate health and safety regulation. *Journal of Risk and Uncertainty*, *8*, 43–66.
34. Nathwani, J. S., Lind, N. C., & Pandey, M. D. (1997). *Affordable Safety by Choice: The Life Quality Method*. Institute for Risk Research, University of Waterloo, Waterloo, Canada.
35. von Neumann, J., & Morgenstern, A. (1943). *Theory of Games and Economical Behavior*. Princeton, NJ: Princeton University Press.
36. Nordhaus, W. D. (1994). *Managing the Global Commons*. Cambridge, MA: MIT Press.
37. Maddison, A. (1995). *Monitoring the World Economy 1820–1992*. Paris: OECD.
38. Mrozek, J. R., & River, L. O. (2001). *What Determines the Value of Life? A Meta-Analysis*. Department of Economics, Georgia State University.
39. OECD. (1998). *Employment Outlook and Analysis*. Paris: OECD.
40. OECD. (2001). *Employment Outlook and Analysis*. Paris: OECD.
41. Nathwani, J. S., & Pandey, M. D. (1997). Measurement of socio-economic inequality using the life quality index. *Social Indicators Research*, *39*, 187–202.
42. Pandey, M. D., & Nathwani, J. S. (2002). Canada wide standard for particulate matter and ozone: Cost–benefit analysis using a life-quality index. *Journal of Risk Analysis*, *23*(1), 55–68.
43. Pate-Cornell, M. E. (1984). *Discounting in Risk Analysis: Capital vs. Human Safety, Proc. Symp. Structural Technology and Risk*. Waterloo, ON: University of Waterloo Press.
44. Persson, U., Norinder, A., Hjalte, K., & Gralen, K. (2001). The value of a statistical life in transport: Findings from a new contingent valuation study in sweden. *Journal of Risk and Uncertainty*, *23*(2), 121–134.
45. Price, C. (1993). *Time, Discounting and Value*. Cambridge: Blackwell.
46. Rabl, A. (1996). Discounting of long term cost: What would future generations prefer us to do? *Ecological Economics*, *17*, 137–145.
47. Rackwitz, R. (2000). Optimization—The basis of code making and reliability verification. *Structural Safety*, *22*(1), 27–60.
48. Rackwitz, R. (2002). Optimization and risk acceptability based on the life quality index. *Structural Safety*, *24*, 297–331.
49. Rackwitz, R. (2003). Discounting for optimal and acceptable Technical Facilities. In Der Kiureghian, Mandanat & Pestana (Eds.), *Proceeding ICASP9* (pp. 725–734). San Francisco, Millpress.
50. Rackwitz, R. (Submitted). Acceptable risks and affordable risk control for technical facilities and optimization. *Reliability Engineering and Systems Safety*.
51. Ramsey, F. P. (1928). A mathematical theory of saving. *Economic Journal*, *38*, 152, 543–559.
52. Reil-Held, A. (2000). *Einkommen und Sterblichkeit in Deutschland: Leben Reiche länger?* Institut für Volkswirtschaftslehre und Statistik, Universität Mannheim, Mannheim.
53. Rosen, S. (1988). The value of changes in life expectancy. *Journal of Risk and Uncertainty*, *1*, 285–304.
54. Rosenblueth, E. (1976). Optimum design for infrequent disturbances. *Journal of Structure Divison*, ASCE, *102*, ST9, 1807–1825.
55. Rosenblueth, E., & Mendoza, E. (1971).Reliability optimization in isostatic structures. *Journal of England Mechanical Division*, ASCE, *97*, EM6, 1625–1642.
56. Schelling, T. C. (1995). Intergenerational discounting. *Energy Policy*, *23*(4/5), 395–401.
57. Shepard, D. S., & Zeckhauser, R. J. (1984). Survival versus consumption. *Management Science*, *30*(4), 423–439.
58. Skjong, R., & Ronold, K. O. (2002). So much for safety. In *Proceedings OMAE 2002*, Paper 28451.
59. Solow, R. M. (1970). *Growth Theory*. Oxford: Clarendon Press.
60. Streicher, H., & Rackwitz, R. (2002). Structural optimization—A one level approach. In Jendo, Dolinsky, & Kleiber (Eds.), *Proceedings of Workshop on Reliability-Based Optimization*. (pp. 151–171). Warsaw: Institute of Technological Research.
61. Streicher, H., & Rackwitz, R. (2004). Time-variant reliability-oriented structural optimization and a renewal model for life-cycle costing. *Probabilistic Engineering Mechanics*, *19*, 171–183.
62. Tengs, T. O., Adams, M. E., Pliskin, J. S., Safran, D. G., Siegel, J. E., Weinstein, M. C., & Graham, J. D. (1995). Five-hundred life-saving interventions and their cost-effectiveness. *Risk Analysis*, *15*(3), 369–390.
63. Toth, F. L. (1995). Discounting in integrated assessment of climate change. *Energy Policy*, *23*(4/5), 403–409.
64. United Nations, Human Development Report. (2001). www.undp.org/hdr2001.
65. United Nations, Human Development Report. (2002). www.undp.org/hdr2002.
66. Usher, D. (1994). An imputation to the measure of economic growth for changes in life expectancy. In *National Accounting and Economic Theory: The Collected Papers of Dan Usher Volume* 1 (pp. 105–144). U.K.: Edward Elgar Publishing Ltd.
67. Viscusi, W. K. (1996). Discounting health effects on medical decision. In F. A. Sloan (Ed.), *Valuing Health Care, Costs, Benefits and Effectiveness of Pharmaceuticals and Other Medical Technologies* (pp. 125–147). Cambridge: Cambridge University Press.

68. Viscusi, W. K. (1992). *Fatal Tradeoffs: Public and Private Responsibilities for Risk*. New York: Oxford University Press.
69. Viscusi, W. K. (1993). The valuation of risks to life and health. *Journal of Economic Literature*, *XXXI*, 1912–1946.
70. Viscusi, W. K., & Aldy, J. E. (2003). The value of a statistical life: A critical review of market estimates throughout the world. *Risk & Uncertainty*, *27*(1), 5–76.
71. Weitzman, M. L. (1998). Why the far-distant future should be discounted at its lowest possible rate. *Journal of Environmental Economics and Management*, *36*, 201–208.
72. Weinstein, M. C., & W. B. Stason, (1977). Foundation of cost-effectiveness analysis for health and medical practices. *New England Journal of Medicine*, *296*(31), 716–721.
73. World Health Organization. (1999). *World Health Report*.
74. World Bank. (2001). *World Development Indicators Database*.
75. Rackwitz, R. (forthcoming). Discounting for optimal and technical facilities involving risks. *Heron*.