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The effect of discounting, different mortality reduction schemes and predictive cohort life tables on risk acceptability criteria

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Abstract

Technical facilities should be optimal with respect to benefits and cost. Optimization of technical facilities involving risks for human life and limb require an acceptability criterion and suitable discount rates both for the public and the operator depending on for whom the optimization is carried out. The life quality index is presented and embedded into modem socio-economic concepts. A general risk acceptability criterion is derived. The societal life saving cost to be used in optimization as life saving or compensation cost and the societal willingness-to-pay based on the societal value of a statistical life or on the societal life quality index are developed. Different mortality reduction schemes are studied. Also, predictive cohort life tables are derived and applied. Discount rates γ must be long-term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long-term averages from the financial market for his cost–benefit analysis the assessment of interest rates for investments of the public into risk reduction is more difficult. The classical Ramsey model decomposes the real interest rate (=output growth rate) into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is proposed to use a relatively small interest rate of 3% implying a rate of time preference of consumption of about 1%. This appears intergenerationally acceptable from an ethical point of view. Risk-consequence curves are derived for an example.

Keywords: Optimum technical facilities; Life quality index; Risk acceptability; Discounting

1. Introduction

Technical facilities should be optimal with respect to their performance, but also with respect to benefits and cost. They also must be acceptable with respect to the losses of human life and limb and this is essentially the answer to the old question: how safe is safe enough? Practice has responded so far in a widely empirical way, which is not satisfactory because it results in a large scatter of requirements and related expenses. Risk reduction involves cost and this means that a monetary valuation of human lives is unavoidable. This paper attempts to find an answer based on a few broadly acceptable principles of the public interest.

The paper is organized as follows: Section 2 defines lifetime utilities of consumption and introduces discounting. The so-called life quality index is used to define an appropriate utility function. This leads to the so-called societal value of a statistical life from which one form of the societal willingness-to-pay is derived. However, we prefer to derive the societal willingness-to-pay directly from the societal life quality index where age averaging is performed. Different mortality reduction schemes and the use of predictive cohort life tables are then studied. Those concepts are applied to technical facilities in Section 3. Section 4 discusses societal discount rates. In Section 5 the numerical constants in rational acceptance criteria are calculated for a number of countries. Then, the concepts are applied in several illustrations. It is demonstrated that risk-consequence curves can calculated on the basis of the general risk acceptability criterion.

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2. Rational socio-economically based risk acceptance criteria

2.1. General

Risk is commonly understood as consequence times occurrence probability in technical contexts that is the potential for some expected loss. Here, we are interested in the risk to human life and health. Risk is perceived individually or collectively in a rather subjective and emotional manner. People often respond to risk in a seemingly irrational way—at least from the viewpoint of experts. Many find it difficult to contemplate risks in terms of probability and consequences. Some researchers attribute this to the cognitive limitations of human beings [55]. Sociological research has identified some major factors when judging risks: severity and dread, the degree of knowledge about risks, the controllability and the catastrophe potential, i.e. the number of people endangered in an event. Only the first factor is really important [56]. Daily risks with limited consequences, for example when driving a car is more easily accepted than low probability—large consequence risks like air transport. Perception, judgment and tolerance of risks depend on individual education, media and politics. They depend on the nature, frequency and exposure to risks. Individual perception of involuntary risks appears to be highly dependent on personal benefits but is almost independent if this is highly beneficial on a societal level. Voluntary life-style risks are grossly underestimated. Risks from the natural environment are frequently underestimated, risks from the technical environment are overestimated. Individual risk perception can deviate largely from estimates by experts and authorities. And large discrepancies between individuals on the one side and experts and authorities on the other side exist what is acceptable and what not. The individual feels hardly responsible for the cost involved in public risk reduction although it is he who pays for it in one or the other way. The cost-effectiveness of public life-saving interventions ranges from a few cents to billions of dollars per life-year saved [58]. But this situation only reflects the vagueness and ambiguity how risks are perceived and judged. Consequently, the various measures of risk control are non-uniform and not systematic. They are guided randomly by personal or group interests and the media. Professional engineering ethics, in particular, would demand that risks are quantifiable and judged from broadly acceptable principles of the public interest. Risk management in the name and for the public should be rational, transparent, communicable and cost-effective.

The question of limiting the risks to human lives is essentially the question of how much society is willing to pay and can afford to 'reduce the probability of premature death by some intervention changing the behavior and/or technology of individuals or organizations' [58]. Further, any argumentation must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere including everyone's right to live, the right of a free development of her/his personality and the democratic equality principle. It is clear that only involuntary risks, i.e. risks to which an anonymous member of society is exposed from its technical or natural environment, can reasonably be discussed here.

Cantril [12] and many similar, more recent studies conclude from empirical studies that long life (in good health) and wealth are among the primary concerns of humans in a modern society. Let long life be measured by life expectancy at birth or at age a. Another suitable indicator of the quality of life, the measure of wealth, is the real gross domestic product (GDP) per capita and year. It can be considered as a surrogate measure of wealth-related aspects of quality of life in a given society. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. It provides the infrastructure of a country, its social structure, its cultural and educational offers, its ecological conditions among others but also the means for the individual enjoyment of life by consumption in its various forms. In most developed countries about $60\pm10\%$ of the GDP is used privately, $20\pm10\%$ by the state (e.g. for military police, jurisdiction, education, etc.) and the rest for (re-)investments. The GDP also creates the possibilities to 'purchase' additional life years through better medical care, improved safety in road and railway traffic, more safety in or around building facilities, more safety from hazardous technical activities, more safety from natural hazards, etc. It does not matter whether those investments into 'life saving' are carried out individually, voluntarily or enforced by regulation or by the state via taxes or other charges. There are ongoing discussions about the way in which the GDP is calculated. For example, sustainability aspects or not economically accountable contributions to individual incomes are not properly taken into account (see, for example, [53]). Nevertheless, the GDP is, at present, the best estimate for the wealth production in a country. If it is assumed that neither the share for the state (without transfer payments) nor the investments into depreciating production means can be reduced, only the part for private use is available for risk reduction in first approximation. Therefore, the part available for risk reduction is $g\approx0.6$ GDP or a little less as a lower bound. An upper value for g in terms of the GDP is obtained by leaving the investments untouched but take the government share without any transfer and subsidizing payments giving $g\approx0.7$ GDP or a little larger. An upper bound is the GDP itself, of course. The exact share for risk reduction must be determined separately for each country or group in a country.

Finally, we try to estimate the size of risk reductions we are going to discuss. Overall crude mortality (per year) is about 0.01 in industrial countries but only 3 in 10,000 are not due to natural causes. If one subtracts from this number those deaths, which are induced by voluntary risky activities (sports and some traffic accidents) and those, which are unavoidable such as house accidents, climbing stairs, etc. then, the reduction of a mortality of about 0.0002 or less is the subject of our study. It may be a little larger because certain risks typical for industrialized countries like air pollution are not separated out in the usual statistical records.

2.2. Lifetime utilities, value of a statistical life and willingness-to-pay

Health-related macroeconomics has developed important concepts starting with the seminal work by Usher [61]. Denote by $c(\tau)>0$ the consumption rate at age τ and by $u[c(\tau)]$ the utility derived from consumption. Individual tend to value a prospect of future consumption less than one of present consumption. This is taken into account by some discounting. The life time utility for a person at age a until she or he attains age $t>a$ then is

$$U(a,t)=\int_a^t u[c(\tau)]\exp\left[-\int_a^\tau \gamma(\theta)\mathrm{d}\theta\right]\mathrm{d}\tau = \int_a^t u[c(\tau)]\exp[-\gamma(\tau-a)]\mathrm{d}\tau \qquad (1)$$

for constant $\gamma(\theta)=\gamma$. There is evidence that the assumption of a constant γ is only a crude approximation but we will maintain it throughout the paper for simplicity. Note that discounting is with respect to utility and not with respect to consumption.

On average, families are by a factor $\exp[nt]$ larger at a later time $t>0$ where n is the population growth rate. Approximate exponential population growth for the last 100 years can be verified from the data collected in [36] if the data are appropriately adjusted for emigration and immigration effects. Future generations are also wealthier because there is economic growth. Long-term exponential economic growth at a constant rate ζ can again be verified from the data in [36] for at least the last 100 years. Following economic theory (for example [57]) the time preference rate is decomposed as $\gamma=\rho+\varepsilon\delta$, where for the moment $\varepsilon\approx1$. ρ is denoted by pure time preference rate of consumption. It should be conceptually distinguished from a financial interest rate. The rate has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc. Its existence in human behavior has been widely demonstrated in human ethology and economics [16, 43] although it is hardly defensible from an ethical point of view. It is partially justified because there is uncertainty about one's future. Therefore, if related to an average individual, Eq. (1) needs to be modified into

$$\begin{aligned} U(a,t) &= \int_a^t u[c(\tau)]\exp[n\tau]\exp[-\zeta\tau]\exp[-\rho(\tau-a)]\mathrm{d}\tau \\ &= \int_a^t u[c(\tau)]\exp[-(\rho+\zeta-n)(\tau-a)]\mathrm{d}\tau \\ &= \int_a^t u[c(\tau)]\exp[-(\rho+\delta)(\tau-a)]\mathrm{d}\tau \end{aligned} \qquad (2)$$

where $\delta=\zeta-n$ is the per capita growth rate. A rate $\rho+\zeta>n$ is necessary for Eq. (1) to converge if future generations are included, i.e. if the utility integral must be extended to $t\to\infty$. ρ is reported to be between 1 and 4% for health related investments, with tendency to lower values [63]. Empirical estimates reflecting pure consumption behavior vary considerably but are in part significantly larger (see, for example, [43], p. 100, and [26]). The numerical value of ρ will be discussed in detail below.

Next we need to introduce a few notions of life statistics. Life expectancy at birth (mean time from birth to death) e is the area under the survivor curve (survival function) $l(a)=\exp\left[-\int_0^a \mu(t)\mathrm{d}t\right]$s, i.e. [27]

$$e=e(0)=\int_0^{a_u} l(a)\mathrm{d}a \qquad (3)$$

where a_u=largest age considered and $\mu(a)$=age dependent mortality or force of mortality obtainable from so-called period life tables available in each country. Period life tables are tables collecting mortalities as the number of deaths at age a divided by the number of survivors at age a in a large group, usually a cohort of 100,000 individuals. $\mu(a)$ is the same as the hazard function and $l(a)$ is reliability in classical reliability theory. The remaining life expectancy given that the individual has survived up to age a is

$$e_c(a)=\frac{e(a)}{l(a)}=\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)\mathrm{d}\tau\right]\mathrm{d}t \qquad (4)$$

The density of the age distribution of a population can also be obtained from life tables. For a stable population it is given by

$$h(a,n)=\frac{\exp[-na]l(a)}{\int_0^{a_u}\exp[-na]ls(a)\mathrm{d}a} \qquad (5)$$

where n is the population growth rate. In a stable population the mortalities do not change with time. A stationary population is obtained for $n=0$ so that $h(a)=l(a)/e(0)$.

The expected remaining present value life time utility at age a (conditional on having survived until a) then is [4,54, 51,15]

$$
\begin{aligned}
L(a) = E[U(a)] &= \int_a^{a_u} \frac{f(t)}{l(a)} U(a,t)\mathrm{d}t \\
&= \int_a^{a_u} \frac{f(t)}{l(a)} \int_a^t u[c(\tau)]\exp[-(\rho+\delta)(\tau-a)]\mathrm{d}\tau\mathrm{d}t \\
&= \frac{1}{l(a)} \int_a^{a_u} u[c(t)]\exp[-(\rho+\delta)(t-a)]l(t)\mathrm{d}t \\
&= u[c]e_\mathrm{d}(a,\rho,\delta)
\end{aligned} \tag{6}
$$

where $f(t)\mathrm{d}t = \left(\mu(\tau)\exp\left[-\int_0^t \mu(\tau)\mathrm{d}\tau\right]\right)\mathrm{d}t$ is the probability of dying between age t and $t+\mathrm{d}t$ computed from life tables. The expression in the third line is obtained upon integration by parts. Also, a constant consumption rate c independent of t has been introduced which can be shown to be optimal under perfect market conditions [54]. Note that $L(a)$ is finite throughout due to $a_u < \infty$. The 'discounted' life expectancy $e_\mathrm{d}(a,\rho,\delta)$ at age a can be computed from

$$
e_\mathrm{d}(a,\rho,\delta) = \int_a^{a_u} \exp\left[-\int_a^t (\mu(\tau) + (\rho+\delta))\mathrm{d}\tau\right]\mathrm{d}t \tag{7}
$$

'Discounting' affects $e_\mathrm{d}(a,\rho,\delta)$ primarily when $\mu(\tau)$ is small (i.e. at young age) while it has little effect for larger $\mu(\tau)$ at higher ages. It is important to recognize that 'discounting' by $\rho+\delta$ is initially with respect to $u[c(\tau)]$ but is formally included in the life expectancy term.

For $u[c]$ we select a simple isoelastic power function

$$
u[c] = \frac{c^q - 1}{q} \tag{8}
$$

with $0 \le q \le 1$ widely used in economics implying constant relative risk aversion (CRRA) according to Arrow–Pratt. The form of Eq. (8) reflects the reasonable assumption that marginal utility $\mathrm{d}u[c]/\mathrm{d}c = c^{q-1}$ decays with consumption c. $u[c]$ is a concave function (to the below) since $\mathrm{d}u[c]/\mathrm{d}c > 0$ for $q \ge 0$ and $\mathrm{d}^2u[c]/\mathrm{d}c^2 < 0$ for $q < 1$. As discussed extensively in the economics literature, a small q implies that higher lifetime utility is primarily obtained from living longer while a high q implies that higher lifetime utility is primarily gained from consumption. The value of q is further discussed below. For simplicity, we take $c = g \gg 1$ and, therefore

$$
u[g] \approx \frac{g^q}{q} \tag{9}
$$

We are now faced with the problem of choosing an appropriate value for q. This is essentially the question of how much to sacrify of the utility of consumption and other aspects of life quality in favor of some more life years, which can be achieved by some payment for risk reduction. Income is produced by work, all considered at a national level. More income will be produced by more work but this will leave less time for leisure. It will be proportional to productivity of work defined as production (in monetary terms) per time unit of work. The individual can increase leisure time by either increasing life expectancy by risk reduction or by reducing the time spent in economic production, which generally means smaller income. Some recent ideas to apply these aspects are repeated here in all brevity. Nathwani et al. [34] hypothesized that "...people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work" (denoted as work–leisure optimization principle) and defined a measure for life quality as $L = f(s)h(t)$ where s is income, $t = (1-\omega)e$ is leisure time, ω $(0 < \omega < 1)$ the fraction of life expectancy lost due to (paid) work and $f(s)$ and $h(t)$ two unknown function of these quantities. This quantity is denoted by life quality index (LQI) in Nathwani et al. [34]. Defining relative changes in the LQI by

$$
\frac{\mathrm{d}L}{L} = \frac{s}{f(s)}\frac{\mathrm{d}f(s)}{\mathrm{d}s}\frac{\mathrm{d}g}{g} + \frac{t}{h(t)}\frac{\mathrm{d}h(t)}{\mathrm{d}t}\frac{\mathrm{d}t}{t} = k_s\frac{\mathrm{d}s}{s} + k_t\frac{\mathrm{d}t}{t}
$$

and setting $k_s/k_t = \text{const.}$ according to the universality requirement (indifference of the relative impact of s and t on life quality with respect to the actual values of s and t), one finds two differential equations

$$
k_s \equiv \frac{s}{f(s)}\frac{\mathrm{d}f(s)}{\mathrm{d}s} = c_1
$$

and

$$
k_t \equiv \frac{t}{h(t)}\frac{\mathrm{d}h(t)}{\mathrm{d}t} = c_2
$$

with solutions $f(s) = s^{c_1}$ and $h(t) = t^{c_2} = ((1-\omega)e)^{c_2}$. Assume further that $s \propto p\ \omega$ where p is the productivity of work. According to the work–leisure optimization principle people who work, possibly together with their families, optimize work and leisure time, i.e. their LQI. From the first-order condition $\mathrm{d}L/\mathrm{d}\omega = 0$ one determines $c_1 = c_2\ \omega/1-\omega$ which together with $c_1 + c_2 = 1$ results in $L = g^\omega e^{1-\omega}(1-\omega)^{1-\omega} \approx g^\omega e^{1-\omega}$ where $g = p\omega$ the yearly consumption rate (or, in this context, the part of the production available for risk reduction). Assume that the present observable value of ω is the optimum value $\omega = w$. In [41] another form is proposed by taking the $(1-w)$th root and dividing the result by $q = w/1-w$ so that

$$
L_q = \frac{g^q}{q}e(1-w) \approx \frac{g^q}{q}e \tag{10}
$$

Dividing g^q by q removes a minor inconsistency because persons with the same g and e but larger w would have higher life quality. In the derivations by Nathwani et al. [34] $s = pe\omega$ could also be assumed and the LQI takes a slightly different form $L = (g^w/w)e(1-w)^{1-w}$. There appears to be no strong argument to prefer one or the other form and we will work with Eq. (10) in the rest of the paper. Some more discussion, especially about the validity of the work–leisure optimization principle and its empirical verification can be found in [47] and [48]. Note that the term g^q/q has exactly the same functional form as Eq. (9). The actual value of q is found to be between 0.1 and more than 0.15. Nathwani et al. [34] estimate $w = 0.125$. This agrees well with an estimate $q = 0.2$ used in [4,54,15,32]. In Table 4 below the work–leisure optimization principle is applied systematically to several countries. It can be observed quite generally that societies with larger g generally work less whereas people in countries with smaller g work more in order to increase utility of consumption. But there are exceptions from the general trend. In some countries with large g preference is given to large earnings and thus large consumption whereas other societies prefer larger leisure time versus somewhat less consumption. Obviously, other secondary factors also affect the value of w. This is also supported by recent labor statistics [38,39]. Comparing Eq. (6) with Eq. (9) inserted for $a = 0$ with Eq. (10) reveals that the latter is nothing else than a lifetime utility without discounting with a deterministic lifetime equal to life expectancy at birth.

Shepard/Zeckhauser [54] now define the 'value of a statistical life' at age a by converting Eq. (6) into monetary units in dividing it by the marginal utility $\mathrm{d}u(c)/\mathrm{d}c = u'[c] = g/q$:

$$
\begin{aligned}
\mathrm{VSL}(a) &= \int_a^{a_u} \frac{u[c(t)]}{u'[c(t)]}\exp[-(\rho+\delta)(t-a)]\frac{l(t)}{l(a)}\mathrm{d}t \\
&= \frac{u[c]}{u'[c]}\frac{1}{l(a)}\int_a^{a_u}\exp[-(\rho+\delta)(t-a)]l(t)\mathrm{d}t \\
&= \frac{g}{q}\frac{1}{l(a)}\int_a^{a_u}\exp[-(\rho+\delta)(t-a)]l(t)\mathrm{d}t \\
&= \frac{g}{q}e_\mathrm{d}(a,\rho,\delta)
\end{aligned} \tag{11}
$$

It is seen that $\mathrm{VSL}(a)$ decays with age as $e_\mathrm{d}(a,\rho,\delta)$. To assign a value to human life on whatever basis is a very controversial issue. In fact, a monetary value of life does not exist. "... the value of human life is infinite and beyond measure, ..." [23]. Such strong ethical statements certainly relate to an individual. Here, the 'value of a statistical life' must rather be understood as a formal constant in a relation expressing the societal monetary amount to reduce a mortality risk by unity. The 'willingness-to-pay' has been defined as

$$
\mathrm{WTP}(a) = \mathrm{VSL}(a)\mathrm{d}m \tag{12}
$$

$\mathrm{d}m$ is a (small) reduction in (crude) mortality. In analogy to Pandey/Nathwani [41], and here they differ from the related economics literature, these quantities are averaged over the age distribution $h(a,n)$ in a stable population in order to take proper account of the composition of the population exposed to natural hazards and hazards in and from technical objects. In health-related economics the 'societal value of a statistical life' is defined by

$$
\overline{\mathrm{SVSL}} = \frac{g}{q}\bar{E}(\rho,\delta) \tag{13}
$$

with the age-averaged, discounted life expectancy:

$$
\bar{E}(\rho,\delta) = \int_0^{a_u} e_\mathrm{d}(a,\rho,\delta)h(a,n)\mathrm{d}a \tag{14}
$$

The 'societal willingness-to-pay' follows as:

$$
\overline{\mathrm{SWTP}} = \overline{\mathrm{SVSL}}\ \mathrm{d}m \tag{15}
$$

For $\rho = 0$ the averaged 'discounted' life expectancy $\bar{E}(\rho,n)$ is a quantity which is about 60% of $e(0)$ and considerably less than that for larger $\rho+\delta$.

Table 1 shows the $\overline{\mathrm{SVSL}}$ for some selected countries as a function of $\rho+\zeta$ indicating the importance of a realistic assessment of ρ.

A reasoning absolutely parallel to the one for Eq. (10) leads to a modified life quality index. It has been called Societal Life Quality Index (SLQI) by Pandey/Nathwani [41]

$$
L_{q\bar{E}} = \frac{g^q}{q}\int_0^{a_u} e_\mathrm{d}(a,\rho,\delta)h(a,n)\mathrm{d}a = \frac{g^q}{q}\bar{E}(\rho,\delta) \tag{16}
$$

It is to be emphasized that the SLQI, like the original LQI, is not a monetary quantity and has dimension '(US\$)^w(years)'. It should interpret as a utility function. If divided by the marginal utility $u'(c)$ in order to convert it into a monetary quantity it coincides with Eq. (13). The rest of this section is devoted to applications and the study of various parameters.

2.3. Willingness-to-pay and risk acceptability from the life quality index

Shepard/Zeckhauser and others [54,29,30,34] derived the willingness-to-pay from the condition that a change in life expectancy and the corresponding change in consumption balance each other by keeping the expected lifetime utility (or the remaining undiscounted and not age-averaged lifetime utility) constant. With $e = e(0)$ this is expressed as

$$
\mathrm{d}L = \frac{\partial L}{\partial e}\mathrm{d}e + \frac{\partial L}{\partial g}\mathrm{d}g = 0 \tag{17}
$$

After rearrangement

Table 1
$\overline{\mathrm{SVSL}}$ 10^6 in PPP US\$ for some countries for various $\rho+\zeta$ (from recent complete life tables from national statistical offices), population growth rates n according to [13]

		France	Germany	Japan	USA
$e(0)$ (years)		78	78	80	77
n (%)		0.37	0.27	0.17	0.90
g (PPP US\$)		14,660	14,460	15,960	22,030
q		0.119	0.116	0.133	0.148
$\rho+\zeta$	0%	5.90	5.69	5.40	8.75
	1%	4.43	4.31	4.08	6.41
	2%	3.46	3.38	3.21	4.90
	3%	2.79	2.74	2.60	3.87
	4%	2.31	2.28	2.17	3.16

$$\mathrm{WTP} = \mathrm{d}g = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} de$$

and by inserting Eq. (10) this leads to

$$\mathrm{d}g = -\frac{g}{q}\frac{\mathrm{d}e}{e} \quad \text{or} \quad \frac{\mathrm{d}g}{g} + \frac{1}{q}\frac{\mathrm{d}e}{e} \geq 0 \tag{18}$$

Criteria of the type (18) remain unaffected by multiplicative constants such as the productivity p or multiplicative corrections of life expectancy as proposed in health related economic studies in order to adjust life expectancy for lifetimes in bad health (which very well enter numerically into the LQI). Also, those criteria are independent of monotone transformations. The equality in (18) gives an indication of what is *necessary* and *affordable* to a society for life saving undertakings; projects having '<' are not admissible. The latter projects would, in fact, be life consuming and, thus, be in conflict with the constitutional right to life. Whenever a given incremental increase in life expectancy by some life saving operation (positive de) is associated with larger than optimal incremental cost (negative dg) one should invest into alternatives of life saving. If a given positive de can be achieved with less than required by Eq. (18) it should be done, of course. Eq. (18) is easy to interpret. For example, for a 1% increase in life expectancy yearly investments of about 5% of g for $q=0.2$ would be affordable. From a practical point of view it is important that all quantities in Eq. (18) are easily available and can be updated any time. The democratic equality principle dictates that average values for g,e and w have to be taken. Any deviations from average values for any specific group of people need to be justified carefully if Eq. (18) is applied to projects with involuntary risks. It is important to note that the simple criterion (18) is independent of any benefit derived from the life saving undertaking other than life saving and in so far is also independent of any discounting.

The indifference relation Eq. (17) remains valid for age-dependent, discounted expected lifetime utilities Eq. (6) and, consequently, also after age averaging. In this case we have for the societal willingness-to-pay

$$\frac{\mathrm{d}g}{g} + \frac{1}{q}E_{\mathrm{A}}\left[\frac{\mathrm{d}e_{\mathrm{d}}(A,\rho,\delta)}{e_{\mathrm{d}}(A,\rho,\delta)}\right] \geq 0 \text{ or}$$

$$\mathrm{d}g \geq -\frac{g}{q}E_{\mathrm{A}}\left[\frac{\mathrm{d}e_{\mathrm{d}}(A,\rho,\delta)}{e_{\mathrm{d}}(A,\rho,\delta)}\right] \tag{19}$$

2.4. Mortality reduction regimes

The direct quantification of de/e or of $E_{\mathrm{A}}[\mathrm{d}e_{\mathrm{d}}(A,\rho,\delta)/e_{\mathrm{d}}(A,\rho,\delta)]$ is difficult but there are good approximations if life saving operations result in certain forms of small changes of age-dependent mortality rates. We start with the assumption that crude mortality is changed by dm. For a (small) uniform proportional change $\mathrm{d}m=\pi m$ in age dependent mortality $\mu(a)$, i.e. $\mu_\pi(a)=\mu(a)(1+\pi)$, the change in de/e by expanding it into a McLaurin series and retaining only the linear term can be approximated by [27]

$$\frac{\mathrm{d}e}{e} \approx \frac{\frac{\mathrm{d}}{\mathrm{d}\pi}\int_0^{a_u}\exp\left[-\int_0^a(\mu(\tau)(1+\pi))\mathrm{d}\tau\right]\mathrm{d}a|_{\pi=0}}{\int_0^{a_u} l(a)\mathrm{d}a}\pi$$

$$= \frac{\frac{\mathrm{d}}{\mathrm{d}\pi}\int_0^{a_u} l(a)^{1+\pi}\,\mathrm{d}a|_{\pi=0}}{\int_0^{a_u} l(a)\mathrm{d}a}\pi = \frac{\int_0^{a_u}\ln(l(a))l(a)\mathrm{d}a}{\int_0^{a_u} l(a)\mathrm{d}a}\pi$$

$$= -C_\pi\,\mathrm{d}m = -c_\pi\frac{\mathrm{d}m}{m} \tag{20}$$

where $c_\pi \approx 0.15$ (developed countries) to more than 0.5 (some developing countries) depending on the age structure and life expectancy of the group (see [46] for more details). For $m \approx 0.01$ we have $C_\pi \approx 15$. Although this scheme has been used most in demographic sciences [27] it places the majority of the profit of a mortality reduction on older people.

Alternatively, one can assume that a (small) change $\mathrm{d}m=\Delta$ in crude mortality distributes equally as a constant at all ages. Then, $\mu(a)$ changes into $\mu_\Delta(a)=\mu(a)+\Delta$ and one has

$$\frac{\mathrm{d}e}{e} \approx \frac{\frac{\mathrm{d}}{\mathrm{d}\Delta}\int_0^{a_u}\exp\left[-\int_0^a(\mu(\tau)+\Delta)\mathrm{d}\tau\right]\mathrm{d}a|_{\Delta=0}}{\int_0^{a_u} l(a)\mathrm{d}a}\Delta$$

$$= -\frac{\int_0^{a_u} al(a)\,\mathrm{d}a}{\int_0^{a_u} l(a)\,\mathrm{d}a}\Delta = -C_\Delta\,\mathrm{d}m = -c_\Delta\frac{\mathrm{d}m}{m} \tag{21}$$

with $c_\Delta = C_\Delta m$. In this case the constants c_Δ are around 0.35 ($C_\Delta \approx 35$) for developed countries. For a given dm the changes in de/e become roughly twice as large as in Eq. (20). This must be expected because a constant change of $\mu(a)$ in young ages has substantially more effect on life expectancy than in older ages. For technical applications, e.g. in structural reliability, industrial hazard protection, flood protection, earthquake-resistant design, etc. this is probably the most realistic and intragenerationally fair regime.

As a third general mortality regime assume that age-dependent mortality changes proportional to some function $f(a)$, i.e. with $\mathrm{d}m=\nu$ according to $\mu_\nu(a)=\mu(a)+\nu f(a)$

$$\frac{\mathrm{d}e}{e} \approx \frac{\frac{\mathrm{d}}{\mathrm{d}\nu}\int_0^{a_u}\exp\left[-\int_0^a(\mu(\tau)+\nu f(\tau))\mathrm{d}\tau\right]\mathrm{d}a|_{\nu=0}}{\int_0^{a_u} l(a)\mathrm{d}a}\nu$$

$$= -\frac{\int_0^{a_u}\int_0^a f(x)\mathrm{d}x l(a)\,\mathrm{d}a}{\int_0^{a_u} l(a)\mathrm{d}a}\nu = -C_\nu\,\mathrm{d}m = -c_\nu\frac{\mathrm{d}m}{m} \tag{22}$$

with $c_\nu = C_\nu m$. For example, if we choose the age distribution we have $f(a)=h(a,n)$ and, therefore, $\int_0^a h(x,n)\mathrm{d}x = H(a,n)$. Then, $c_\nu \approx 0.5$ ($C_\nu \approx 50$) implies that for given dm the relative change in life expectancy de/e is about three times as much or more than for a proportional change as in Eq. (20). This mortality reduction scheme may be valid for highly infectious epidemics where available drugs have to be distributed evenly to all age groups. Here, all get treatment but the young profit most from the intervention with respect to life expectancy. The influence of the particular age distribution can be significant. Eq. (20) and the like are valid for positive as well as negative but small dm.

Table 2
Dependence of $C_{\pi\bar{E}}$ and $C_{\Delta\bar{E}}$ on rate $\rho+\zeta$

		France	Germany	Japan	USA
$e(\bar{e})$ (years)		78 (37.9)	78 (38.3)	80 (39.9)	77 (34.0)
n (%)		0.37	0.27	0.17	0.90
m (%)		0.91	1.04	0.83	0.87
	0%	27, 30	22, 29	26, 29	33, 32
	1%	23, 26	19, 25	22, 25	27, 28
$\rho+\zeta$	2%	19, 22	16, 21	19, 21	22, 24
	3%	16, 19	14, 19	16, 19	19, 21
	4%	14, 17	12, 16	14, 16	16, 18

Other mortality reduction regimes can be thought of. For example, one can also consider age dependent mortality regimes if a change in mortality only affects those older than 60 years or any other age group as might be relevant in health-related public investments. The selection of the appropriate mortality regime turns out to be rather important in applications and must be suitably chosen in the context of specific applications.

Similar demographic constants can also be developed for age-averaged and discounted life expectancies. The formulae are complicated. Their general form is

$$E_{\mathrm{A}}\left[\frac{\frac{\mathrm{d}}{\mathrm{d}x}e_{\mathrm{d}}(A,\rho,\delta,x)|_{x=0}x}{e_{\mathrm{d}}(A,\rho,\delta)}\right] \approx -C_x(\rho,\delta)x \tag{23}$$

where 'x' stands for the particular mortality reduction scheme. For example, for a mortality reduction scheme reducing mortality by a constant small quantity Δ, i.e. $\mu_\Delta(a)=\mu(a)+\Delta$ one finds

$$C_{\Delta\bar{E}}(\rho,\delta) = -\int_0^{a_u}\frac{\int_a^{a_u}(t-a)\exp\left[-\left(\int_a^t\mu(\tau)\mathrm{d}\tau+(\rho+\delta)(t-a)\right)\right]\mathrm{d}t}{\int_a^{a_u}\exp\left[-\left(\int_a^t\mu(\tau)\mathrm{d}\tau+(\rho+\delta)(t-a)\right)\right]\mathrm{d}t}h(a,n)\mathrm{d}a \tag{24}$$

The coefficients $C_{x\bar{E}}(\rho,n)$ are all decreasing while $\rho+\zeta$ increases, but at different speed depending on demographic characteristics. Table 2 shows the coefficients $C_{x\bar{E}}(\rho,\delta)$ ($x=\pi$ or Δ) for some countries.

Table 2 demonstrates the significant but rather complex influence of demographic factors and discount rates. For information the mean age $\bar{e}$ of the population is also given from which the residual mean life expectancy can be calculated. $C_{\pi\bar{E}}$ is generally smaller than $C_{\Delta\bar{E}}$ because it places more weight on elderly people. Both coefficients decay with $\rho+\zeta$ and the demographic differences vanish for large $\rho+\zeta$. Comparing the results with the results in Table 1 indicates that the influence of $\rho+\zeta$ is significantly larger in Table 1 than in Table 2 for all considered mortality regimes. To illustrate this further consider the additive mortality reduction scheme Eq. (21) but now the mortality reduction affects only those under 18 years, between 18 and 60 and above 60, respectively. Such strategies might be suitable for certain risk reduction interventions in pollution control of water or atmosphere. For example, for the USA one determines coefficients $C_{\Delta\bar{E}}$ of 9.1, 8.1 and 0.7, respectively. These values, of course, add up to the value for a constant mortality change at all ages.

2.5. Predictive cohort tables

So far period life tables have been used. As described previously they are an accurate account of the mortalities in a cohort at year y. Alternatives are so-called generation or cohort tables. Cohort (or generation) life tables ideally are constructed by counting the number of deaths in a group born at time $t=0-a_u$ at each age a. They certainly provide more information as they reflect the common trend towards larger life expectancies and more compact age distributions. Unfortunately, cohort life tables do exist only for a few countries. Some cohort life tables, for example, yield 7% larger life expectancies at present. But cohort tables can be constructed if a sufficiently long sequence of period tables is available. It is even possible to construct predictive cohort tables if the trends in mortalities in the period tables over the years is extrapolated into the future assuming that the trends observed in the past persist also into the future. Time-and age-dependent mortalities have been found to follow rather accurately the following exponential function

$$\mu_{\vartheta,y}(a) = \mu_y(a)b(a)^{\vartheta+a-y} \tag{25}$$

where y is the reference year, i.e. the last year for which a period table is available and $\vartheta \lesseqgtr y$ is the year of birth. The coefficients $b(a)$ must be determined by an appropriate regression analysis. Table 3 compares for constant ρ the results for period and predictive cohort tables for the two mortality reduction schemes in Eqs. (20) and (21), respectively. For information, the coefficients without any discounting and age-averaging, for age-averaging only and for discounting only are also given.

It is seen that the differences in the coefficients for period and predictive life tables are relatively small, at least for the countries considered. Discounting only reduces the coefficients significantly whereas age-averaging only increases

Table 3
Comparison of demographic constants from period and predictive cohort tables

	France	Germany	Japan	USA
δ (%)	1.9	1.9	2.7	1.8
ρ (%)	1	1	1	1
From period tables	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$
Proportional mortality change	17, 6, 32, 21	14, 5, 29, 19	16, 4, 34, 21	20, 7, 34, 22
Constant mortality change	40, 25, 24, 16	40, 25, 23, 16	41, 22, 24, 15	39, 25, 25, 17
From predictive cohort tables	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$	C_x, C_{xd}, C_{xA}, $C_{x\bar{E}}$
Proportional mortality change	13, 3, 26, 18	12, 4, 25, 16	11, 2, 26, 16	16, 6, 28, 18
Constant mortality change	44, 23, 25, 17	44, 23, 26, 17	47, 25, 26, 16	44, 28, 28, 19

C_x, no discounting and no age-averaging; C_{xd}, discounting only; C_{xA}, age-averaging only, $C_{x\bar{E}}$ discounting and age-averaging.

them. Here again, one observes considerable differences between the two mortality reduction schemes. However, applying both age-averaging and discounting levels out the differences.

2.6. Discussion

Both lines of thought, the economical (SVSL) and the LQI approach (SLQI), have a good conceptual and theoretical basis. They complement each other. In particular, the derivations for Eq. (10) justify the power function form in Eq. (8) and lets Eq. (16) be interpreted as an expected remaining present value life time utility for all those alive at $t=0$. Neither criterion (19) nor (15) depend on any benefit other than risk reduction or life extension. In most applications clear support for decisions can be reached by using either of the approaches, even the one without discounting and age averaging. Age averaging is generally necessary for the technical applications we have in mind because the risk reduction intervention is to be executed at $t=0$ for all those living now. The concept of discounting future utilities by $(\rho+\delta)$ may be debatable as the subjective time preference rate ρ is concerned but not with respect to the population and economic growth. The SLQI-based approach explicitly combines three important human concerns, that is high life expectancy, high consumption and an optimized time available for the development of one's personality off the time for paid work. Criterion (19), having in mind its derivation, also tells us that larger expenses for risk reduction are inefficient and smaller expenses are not admissible in view of the constitutional right for life. In particular, criterion (19) is affordable from a societal point of view. Eq. (19) deserves the name 'societal willingness-to-pay' even in a more direct sense than Eq. (15) as it is the result of some optimization of time of work to raise the income and leisure time given a certain productivity of the economy. Insofar the SLQI-concept appears to be somewhat richer and more suitable for our purposes than the purely economic approach leading to Eqs. (13) and (15).

There is a certain dilemma arising from the actual unequal distribution of wealth and life expectancy in a society. A certain group in a society may benefit from safety interventions more than another. Then, it should be fair that the 'gainers' compensate the 'losers' so that their LQI is at least maintained. For example, in projects where certain groups of people must take higher risks, voluntarily or involuntarily, it should be fair to provide compensation by higher incomes or more leisure time. One even may follow a requirement in [33] which states that the 'gainers' should still have some left over. Similar 'solidarity' principles should also apply if only a certain group or region in society is exposed to some hazards. Much further discussion is provided in [28,34,41,47].

Life quality clearly has more dimensions than GDP, life expectancy and leisure time. Values such as personal well-being, good family relationships, a healthy ecological environment, cultural heritage and many other values cannot be measured by the life quality index. However, we only intend to derive a criterion helping to balance the conflicting aims of life extension and loss of consumption in a rational manner.

So far, we concentrated on the willingness-to-pay for averting fatalities and neglected the cost implied by injuries. This appears justified as the latter are relatively small. For instance, the study in [19] suggests that, for the United States, the cost of injury can be taken as 1000 US\$/person and 10,000 US\$/person for minor and serious injury, respectively. In [64] somewhat larger numbers up to 100,000 US\$/person are given. These numbers are by orders of magnitude smaller than those determined on the basis of the LQI and by other approaches.

3. Application to technical facilities

It can reasonably be assumed that the life risk in and from technical facilities or from environmental hazards is uniformly distributed over the age and sex of those affected. Also, it is assumed that everybody uses such facilities

and, therefore, is exposed to possible fatal accidents. For simplicity, the design parameter p is exp (assumed to be a scalar. Inserting into Eq. (19) gives:

$$\frac{-\mathrm{d}C_Y(p)}{g} + \frac{1}{q}(-C_{x\bar{E}}(\rho,\delta)\mathrm{d}m) \geq 0$$

Let $\mathrm{d}m$ be proportional to the mean failure rate $\mathrm{d}h(p)$, i.e. it is assumed that the process of failures and renewals is already in a stationary state that is for $t\to\infty$. Rearrangement yields

$$\frac{\mathrm{d}C_Y(p)}{\mathrm{d}h(p)} \geq -kC_{x\bar{E}}(\rho,\delta)g\frac{1}{q} = -kK_{x\bar{E}}(\rho,\delta) \qquad (26)$$

where $K_{x\bar{E}}(\rho,\delta) = C_{x\bar{E}}(\rho,\delta)\frac{g}{q}$ and

$$\mathrm{d}m = k\,\mathrm{d}h(p), \quad 0<k\leq 1 \qquad (27)$$

and the proportionality constant k relates the changes in mortality to changes in the failure rate. Note that for any reasonable risk reducing intervention there is necessarily $\mathrm{d}h(p)/\mathrm{d}p<0$.

The criterion Eq. (26) is derived for safety-related regulations for a larger group in a society or the entire society. Can it also be applied to individual technical projects? $K_{x\bar{E}}(\rho,\delta)$ was related to one anonymous person. For a specific project it makes sense to apply criterion (26) to the specific group exposed. Criterion (26) changes accordingly into:

$$\frac{\mathrm{d}C_Y(p)}{\mathrm{d}h(p)} \geq -K_{x\bar{E}}(\rho,\delta)kN_{\mathrm{PE}} \qquad (28)$$

N_{PE} is the number of potentially endangered persons.

N_{PE} as well as k must be estimated taking account of the number of persons endangered by the event, the cause of failure, the severity and suddenness of failure, possibly availability and functionality of rescue systems, etc. The constant k may be interpreted as a person's probability of actually being killed in case of failure. It can vary between less than 1/10,000 and 1. For example, estimates in [14] show $k=0.01$–0.1 for earthquakes, $k=10^{-6}$ to 0.1 for floods in [24], $k=0.1$ for building fires in [20] and $k=0.02$ or less to 0.7 for large fires in road tunnels in [40]. In practice the estimation of N_{PE} and k is the subject of a detailed risk analysis or, better, failure consequence analysis. In general, an estimate can only be made for specific projects. It should be noted that the probability k is not necessarily a constant. It can depend on the cause and the severity (strength) of the event and the type of failure. Further discussions of the methodology to determine the parameters N_{PE} and k as well as the particular mortality regime associated with a particular hazard are beyond the scope of this paper.

The same derivations apply to the purely economic concept with $K_{x\bar{E}}(\rho,\delta)$ replaced by $\overline{\mathrm{SVSL}}$.

4. Societal discount rates

Discounting of utilities has played a major role in the developments. At first sight discounting of human lives is not in agreement with our moral value system. However, a number of studies summarized in [42] and [29] express a rather clear opinion based on ethical and economical arguments and on public opinion. The cost for saving life years must be discounted at the same rate as other investments. Otherwise serious inconsistencies cannot be avoided. The arguments are as follows: the ethical argument is essentially based on the categorical imperative of Kant [25]: Act only on the maxim which you can will to be a universal law. Because discounting follows from opportunity cost as a fact of life "...future generations must be treated in the same way as we want to be treated today". Bordley [9] applied economic reasoning and obtained the same result for life risk reduction policies, showing that discounted longevity as a measure of benefits can be deduced from a utility-maximization model. Cropper et al. [16] and others showed that the public also discounts life risks like other investments by asking what fraction of a life saved today counts the same as a life 5, 10,...,100 years in the future. Interestingly, they found a discount rate over 10% for short time horizons of less than 10 years but a rate of some 3.5% for 100 years. Subjective discounting appears to be a fact as mentioned earlier.

What should then the societal interest rate be? In view of the time horizon of some 20 to more than 100 years (i.e. several generations) it should be a long-term average. It should be net of inflation and taxes. For $\gamma=0.075$, 1\$ benefit (or loss) in 100 years is presently worth less than 0.1 cent, which appears unacceptable if human lives (in present and future generations) are concerned. But $\gamma=0.015$ gives 0.23 \$, which lets us feel a little more comfortable, yet still unsatisfied. The long time horizon generally suggests using small rates. In the private sector a long-term real interest rate is roughly identical to the (maximum) return rate one could get from an investment. But can the public also adopt such a strategy? The public does not make financial profit except by its economical growth and what is the profit of life saving interventions? Clearly, the interest rates should be close to the long-term economic growth rate per capita as this is the rate with which a member of the public becomes wealthier. In the economics literature this is sometimes called the 'natural interest rate'. There is also the financing aspect. Traditionally, it has been argued that public investments should be financed within some mean residual life expectancy of the population, i.e. within 40–50 years. For larger financing horizons the burden of financing would be left in part to the next generation. If this time is viewed as the time of amortization of a public investment, rates of 2–2.5% are implied. In an interesting paper exploring the hypothesis that human time preferences are in evolutionary equilibrium Rogers [50] came to the conclusion that the real interest should be $\gamma \approx \ln(2)/T + n$ where T is the mean life expectancy or generation time, also resulting in $\gamma \approx 0.02$.

There have been ongoing but somewhat inconclusive discussions when discounting public investments into health care (see, for Example, [63]). Recently, further discussions have been taken place in the context of sustainable development, long-term public investments in general and intergenerational justice—aspects which appear very relevant in our context. Discounting for sustainability should at least be consistent with discounting for risk reduction investments. Weinstein/Stason [66] and others require that interest rates for life saving investments should be the same as for other cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed, for example, in [11] and [52], based primarily on ethical grounds in the context of CO_2-induced global warming, nuclear waste disposals, depletion of natural resources, etc. A life is simply worth saving with the same effort now and in the future.

Public interest rates are strongly related to the benefit a society earns from the various activities of its members, i.e. its real economic growth per capita (see also [46] where the public benefit and interest rate has been set equal to the growth rate). The United Nations Human Development Report 2001 [60] gives values between 1.2 and 1.9% for industrialized countries during 1975–1998. If one considers the last 120 years and the data in [36] for some selected countries one determines a real growth rate (per capita) $\delta = (\ln(GDP_{1992}/GDP_{1870})/(1992-1870))100$ of about 1.8% which is also the growth rate for Western Europe, USA, Canada, Australia and Japan.

Some more insight can be gained from modern economic growth theory and sustainability financing. Nordhaus [35] and others (see [59] for an overview but also the other papers in Energy Policy, 23, 3/4, 1995) follow the classical Ramseyan approach (see [49,57,6]) for optimal stable economic growth in perfect markets

$$\gamma = \rho + \varepsilon\delta > 0 \qquad (29)$$

where γ is the real market interest rate, ρ the rate of pure time preference of consumption, $\varepsilon > 0$ the elasticity of marginal consumption (income) and δ the consumption (income) growth rate. Clearly, the subjective element is the quantity ρ. With $\rho \approx 0.03$ and $\delta \approx 0.02$ as well as $\varepsilon = 1$ Nordhaus [35] obtains $\gamma \approx 0.05$. Arrow [3] estimates $\gamma \approx 0.03$ assuming $\rho \approx 0.01$, $\delta \approx 0.012$ and $\varepsilon = 1.5$ (!), however, with tendency to larger values. In many other studies for sustainable development discount rates γ cluster around 5%. However, there are many authors in economics as well as philosophical and political sciences including Ramsey [49] who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds [52,11,44] while it is considered fully acceptable for intragenerational discounting. For example, Rabl [44], who sets $\rho = 0$, argues that there must be $0 < \gamma < \delta$ in the framework of long-term public investments. On the other hand, intergenerational equity arguments in Arrow [3] indicate that there should be $\rho > 0$ in order to remove an '...incredible and unacceptable strain on the present generation'.

The above considerations based on a simple, ideal, steady state Ramseyan growth model in a closed economy can at least define the range of benefit and interest rates as well as reasonable rates of pure time preference to be used in long-term investments into life saving operations. It is believed that the steady state assumption of the Ramseyan model is not too far from reality in developed countries. Historical long-term population and economic growth rates cannot be questioned but there is considerable uncertainty about the future taking account of sustainability aspects. The value of ε varies very little, say between 0.75 and 0.85. Only the pure time preference rate ρ to be used in Eq. (1) can be subject to discussion and choice.

Nevertheless, in the literature the adequacy of the Ramseyan model is sometimes questioned. So-called overlapping generation models or generation adjusted discounting models are advocated instead. The main idea is to discount for living generations at the rate in Eq. (29) with $\rho > 0$ but diminish the rate for all yet unborn generations down to δ or even lower, thus facilitating the transition into a sustainable state of economy [44,7]. But it is not expected that those refinements change our results significantly. Some further precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. Natural resources will be depleted and arable land will become scarce. Many raise serious doubts whether the foreseeable demographic changes (aging populations and negative population growth in industrial countries) and the increasing scarcity of non-renewable natural resources and other environmental concerns can be compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will actually happen. But there is an important mathematical result which may guide our choice. Weitzman [65] and others showed that the far-distant future should be discounted at the lowest possible rate ≥ 0 if there are different possible scenarios each with a given probability of being true. Exactly this strategy should be pursued. It should be emphasized that lowest possible interest rates so far are chosen only for the subjective part ρ of the real interest rate γ.

It is obvious that the results about the appropriate public interest rate for long-term investments are not yet fully conclusive and still controversial. More research and discussion is necessary. As an intermediate compromise it is suggested to tentatively take $\rho = 0.01$ resulting in $\gamma \approx 0.03$ for the computations of the willingness-to-pay below.

5. Some results

Table 4 collects some relevant data for countries for which sufficiently reliable economic and demographic data are available. The data can vary depending on the type and date of the sources used. Although the best possible has been made out of the available data, some uncertainties and ambiguities remain, mainly due to differences in the way statistical data are taken in different countries. The results cover most industrialized countries including some extremes. They show the complex interaction of past and present economic conditions with demographic factors. They should be considered as preliminary estimates, especially if one wishes to compare across countries.

The life tables are all recent period life tables from national statistical offices or from [10]. n, m, e and w are taken at their present values but slow demographic changes could, in principle, be considered. The age distribution $h(a,n)$ in Table 4 is determined from period life tables. Stable populations are assumed. Because the largest age a_u

Table 4
Social indicators for some countries

Country	GDP[a], g[b]	δ[c]	m[d]	n[e]	e	q[f]	$K_\pi, K_\Delta, K_{\pi\bar{E}}, K_{\Delta\bar{E}}$[g]	$\overline{SVSL}$[g]
Canada	27,330, 16,040	2.0	0.73	0.99	78	0.13	2.5, 4.9, 2.1, 2.3	2.7
USA	34,260, 22,030	1.8	0.87	0.90	77	0.15	2.9, 5.8, 2.5, 2.8	3.3
Austria	26,310, 14,790	1.8	0.98	0.24	77	0.11	1.8, 5.2, 1.8, 2.4	2.8
Bulgaria	6200, 4400	1.3	1.45	−1.14	70	0.15	0.4, 1.1, 0.4, 0.5	0.6
Czech Rep.	12,900, 6730	1.5	1.08	−0.07	73	0.17	0.6, 1.5, 0.6, 0.7	0.9
Denmark	25,500, 12,900	1.8	1.09	0.30	77	0.11	1.6, 4.6, 1.6, 2.1	2.5
Finland	22,900, 12,100	1.8	0.98	0.16	77	0.13	1.4, 3.7, 1.3, 1.6	1.9
France	24,470, 14,660	1.9	0.91	0.37	78	0.12	2.0, 4.9, 1.9, 2.2	2.6
Germany	25,010, 14,460	1.9	1.04	0.27	78	0.12	1.7, 5.0, 1.6, 2.3	2.6
Ireland	25,470, 12,610	1.5	0.81	1.12	76	0.13	1.8, 3.8, 1.6, 2.0	2.4
Italy	23,400, 14,460	1.9	1.01	0.07	79	0.12	1.5, 4.8, 1.5, 2.1	2.5
Hungary	11,200, 6870	1.2	1.32	−0.32	71	0.14	0.7, 1.8, 0.7, 0.9	1.0
Netherlands	26,170, 15,470	1.5	0.87	0.55	78	0.10	2.4, 6.1, 2.4, 3.0	3.6
Norway	29,760, 14,149	2.1	0.98	0.49	78	0.10	1.9, 3.3, 1.7, 2.5	2.9
Poland	9030, 5630	1.6	1.00	−0.03	73	0.14	0.7, 1.5, 0.7, 0.7	0.8
Russia	8377, 5440	1.2	1.34	−0.35	66	0.16	0.6, 1.2, 0.6, 0.6	0.8
Sweden	23,770, 12,620	1.9	1.06	0.02	79	0.12	1.3, 4.2, 1.4, 1.9	2.2
Switzerland	29,000, 17,700	1.9	0.88	0.27	79	0.12	2.3, 6.2, 2.2, 2.8	3.2
UK	23,500, 15,140	1.3	1.07	0.23	78	0.13	1.6, 4.7, 1.7, 2.3	2.8
Japan	26,460, 15,960	2.7	0.83	0.17	80	0.13	1.9, 4.9, 1.7, 2.0	2.2
Australia	25,370, 15,750	1.2	0.72	0.99	78	0.14	2.2, 4.5, 2.1, 2.4	3.0

[a] After [67] in PPP US$.
[b] Private consumption in PPP US$ according to [60].
[c] Economic growth in % for 1870–1992 after [36].
[d] Crude mortality (2000) in % [13].
[e] Population growth (2000) in % [13].
[f] Estimates based on [21,36,18,39] including 1 h travel time per working day and a life working time of 40–45 years.
[g] In 10^6 PPP US$.

considered in the life tables is around 110 years this is also the time span over which our considerations are valid. It is known in demographics that life expectancies and age structures have changed and most likely will change. Extrapolations into the future would be required but this is not done here because only very few predictive cohort life tables are available.

The economic growth rate δ has been averaged over the years 1870–1992. It certainly would be misleading to take only averages over the last 50 years or less. The values for $K_\pi, K_\Delta, K_{\pi\bar{E}}, K_{\Delta\bar{E}}$ and $\overline{\text{SVSL}}$ calculated from these data. The demographic constants C_x can be calculated by multiplying the value given in Table 4 by the corresponding q/g. It is seen that K_Δ generally is largest followed by the undiscounted and not age-averaged coefficient K_π. The largest uncertainties are possibly due to the part of GDP effectively available for risk reduction and due to the life working time estimates. As mentioned earlier the part of the GDP available for risk reduction is taken as that available for private use. It must be considered as a lower bound. The Scandinavian countries have comparatively low values due to a smaller share g of their GDP for private use in the official sources. Some adjustments are necessary so that the quantity g really includes all health expenditures and what is available for risk reduction.

The Netherlands and, in part also Norway, are exceptional in that recent sources give a rather low w due to a large proportion of part time employment but also due to the fact that the statistics contain only dependent employment. Factors like the unemployment rate, the productivity level of the labor force, the size of the households, the specific legal and social system must also be considered. The relatively high value of w for the USA appears partially to be due to the household survey technique as opposed to the establishment survey technique used in most other countries [38,39] but as mentioned, higher preference for large earnings might also serve as an explanation. Although the work–leisure principle outlined before may still be valid in general it appears that the accounting of life working time must be improved for our purposes and other factors need additional consideration. One hour travel time per working day and a work lifetime of 40–45 years is considered as in [34] which appears rather on the conservative side. Also, w varies with time. Over the past 50 years it fell down from $w \approx 0.15$ to less than $w \approx 0.1$. But a variable w is not covered by the theory outlined above (see [22]). If other forms of Eq. (10) are preferred it is easy to adjust the table entries. Such adjustments are expected to be less than 5–10% in the values of K_x.

The value of $K_{\Delta\bar{E}}$ is probably most relevant for risk reduction interventions with respect to technical and natural hazards. Ignoring discounting, population and economic growth generally gives larger values. Selecting larger values of ρ for $K_{x\bar{E}}$ will result in smaller values as demonstrated in Table 2. The results for the Eastern European countries are also relatively consistent.

The estimates for $K_{\Delta\bar{E}}(\rho, \delta)$ and $\overline{\text{SVSL}}$ are in good agreement with several other estimates in the literature based on various empirical concepts such as compensating wage differentials in the labor market, contingent valuation studies or implicit VSL in road traffic regulations (see, for example, [62,31,64,5,8,1] and many others). The studies [64,37,8] are so-called meta-analyses, i.e. analyses across several other studies. For comparison, $K_{\Delta\bar{E}}(\rho, \delta)$ and $\overline{\text{SVSL}}$ should be calculated with the full GDP, i.e. the values in Table 4 should be multiplied by 1.6. Most of those estimates are between 100,000 and over 10 mill. PPP US$ with a clustering around 5–6 mill. PPP US$.

6. Illustration examples

6.1. Examples from air pollution

In the following application the criteria (19) or (15) are illustrated for some man-made types of air pollution. For all three examples one should bear in mind that the dose–effect relationships usually are rather uncertain, at most within an order of magnitude.

As a first application of Eq. (19) assume a regulation for cleaning the flue gas from health endangering pollutants in fossil fuel power plants. Such plants roughly produce one third of the total pollution, another third is emitted by transport (trucks, etc.) and another third by industry and other sources including natural sources (sand storms). It has been estimated that control of fine particulate matter emission could extend life expectancy by 250 days in Central Europe which is the long-term goal of the European Commission [2]. Using some average data $g = 15{,}000$ EURO and $q = 0.15$ one determines

$$\mathrm{d}C = \frac{g}{q}\frac{\mathrm{d}e}{e} = \frac{15000}{0.15}\frac{250/3}{77 \times 365} \approx 300\ \text{EURO}$$

for an once-only payment for the flue gas cleaning installation(s) per person assuming for simplicity that once the flue gas cleaning installation(s) are built no further loss of life years occurs. This gives

$$\mathrm{d}C_Y = \mathrm{d}C\frac{(1+r)^t((1+r)-1)}{((1+r)^t-1)} \approx 38.9\ \text{EURO}$$

at a discount rate of $r = 0.05$ during $t = 10$ years (the assumed life time of a flue gas cleaning installation). This extra cost of 38.9 EURO/person is approximately 5–10% of the yearly bill for electrical current of an average household ($\approx$2.3 persons/household). The extra cost for flue gas cleaning is affordable and necessary from the view point of the SLQI. The electricity companies may increase their prices accordingly to cover the investments into flue gas cleaning.

Another typical technical endangerment caused by the human preference for automobile transport is benzene.

Other minor sources in building materials also exist. Benzene is primarily released during operation by combustion and refueling but also by evaporation from automobiles. For 1 μg/m^3 per day one has estimated an increased mortality of about $\mathrm{d}m_0 = 1.5 \times 10^{-5}$ (primarily due to leukaemia). The present mean exposure to this ubiquitous carcinogen is estimated in the order of 25 μg/m^3 per day. To bring it down to an acceptable, technically realizable level of 5 μg/m^3 per day the benzene content in the fuel and benzene evaporation during filling by vapor recovery nozzles has to be limited, and appropriate three-way catalytic converters have to be introduced. This yields with $C \approx C_\Delta \approx 30$ an affordable once-only payment of $\mathrm{d}C = (g/q)C\Delta_{\text{Benzene}}\,\mathrm{d}m_0 \approx 1000$ PPPUS per person ($\Delta_{\text{Benzene}} =$ 20 μg/m^3). An average catalytic converter costs 500 PPP US$ and there are about 0.5 cars per person. The other cost for additional petrol station installations are estimated as negligible. Therefore, the limit of 5 μg/m^3 set in [17] can be reached at the cost as suggested by criterion (19) if the extra cost for engines without pinging requiring only less than 1% of benzene in the petrol is also negligible.

Asbestos (chrysotile) is known to cause different forms of cancer primarily for workers in production, manipulation and removal of asbestos-containing building materials. However, it is ubiquitous and the average content of tumorigenic asbestos fibres (less than 5 μm long) in air has been estimated as 200 fibres/m^3. The World Health Organization (WHO) has estimated a mortality of the general public of 2×10^{-6} per 100 fibres/m^3 leading to 6.5 PPP US$ for a once-only payment for complete removal of polluting asbestos materials or a yearly payment of 0.5 PPP US$ per person at a discount rate of 2% for 25 year. This is to be compared with the actual investments which in most industrialized countries appear to be much higher.

These three examples are meant as illustrations. Due to large uncertainties in the dose–mortality relationships and the exposure dose these calculations and the corresponding conclusions can only be considered as first crude estimates.

6.2. Structural reliability

This example has already been given in [46] in somewhat different form and with modified parameters. As an example from the structures area we take a rather simple case of a single-mode system where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modeled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances S_i, $i = 1, 2, \ldots$. The resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independent and log-normally distributed with mean equal to unity and coefficient of variation V_S so that p can be interpreted as central safety factor. A disturbance causes failure with probability:

$$P_\mathrm{f}(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_\mathrm{S}^2}{1+V_\mathrm{R}^2}}\right\}}{\sqrt{\ln((1+V_\mathrm{R}^2)(1+V_\mathrm{S}^2))}}\right) \tag{30}$$

An appropriate objective function then is with benefit per unit time b [45]:

$$Z(p) = \frac{b}{C_0\gamma} - \left(1 + \frac{C_1}{C_0}p^a\right) - \left(1 + \frac{C_1}{C_0}p^a + \frac{H_\mathrm{M}}{C_0} + \frac{H_\mathrm{F}}{C_0}\right)\frac{\lambda P_f(p)}{\gamma} \tag{31}$$

The criterion (28) has the form:

$$\frac{\mathrm{d}}{\mathrm{d}p}(C_0 + C_1 p^a) \geq -K_{x\bar{E}}(\rho, n)kN_{\mathrm{PE}}\frac{\mathrm{d}}{\mathrm{d}p}(\lambda P_f(p)) \tag{32}$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_\mathrm{M} = 3 \times C_0$, $V_\mathrm{R} = 0.2$, $V_\mathrm{S} = 0.3$, and $\lambda = 1$ [1/year]. The LQI-data are $e = 77$, GDP$=25{,}000$, $g = 15{,}000$, $m = 0.01$ and an average demographic constant $C_{x\bar{E}} = 25$, $q = 0.125$, $N_{\mathrm{PE}} = 100$, $k = 0.1$ so that $H_\mathrm{F} = \text{SLSC}\ kN_{\mathrm{PE}} \approx 6 \times 10^6$ and $K_{x\bar{E}}(\rho, n)kN_{\mathrm{PE}} \approx 3.0 \times 10^7$ The quantity SLSC is the so-called societal life saving cost and equals approximately the lost earnings if an accident would occur at mid-life. The value of N_{PE} is chosen relatively large for demonstration purposes. Monetary values are in US$. Optimization is performed for the public and for the owner separately.

For the public $b_\mathrm{S} = \beta C_0$ with $\beta = 0.032$ and $\gamma_\mathrm{S} = 0.03$ are chosen. Benefit and discount rate are chosen such that the public does not make direct profit from an economic activity of its members. Optimization including the cost H_F gives $p_\mathrm{S}^* = 4.21$, the corresponding failure rate is 1.8×10^{-5}. Without life saving cost one obtains $p_\mathrm{S}^* = 3.92$ with failure rate 4.4×10^{-5}. Criterion (28) is already fulfilled for $p_{\lim} = 3.45$ corresponding to yearly failure rates of 1.8×10^{-4}, but $Z_\mathrm{S}(p_l)/C_0$ and $Z_\mathrm{S}(p_u)/C_0$ being already negative. It is also interesting to see that in this case the public can do better in adopting the optimal solution rather than just realizing the facility at its acceptability limit.

The owner uses some typical values of $b_0 = 0.07C_0$ and $\gamma_0 = 0.05$ and does or does not include life saving cost. If he includes life saving cost the objective function is shifted to the right (solid line) as shown in Fig. 1. The calculations yield $p_0^* = 4.05$ and $p_0^* = 3.76$, respectively, and the corresponding failure rates are 3.0×10^{-5} and 7.1×10^{-5}. The SLQI-based acceptability criterion limits the owner's region for reasonable designs. Inclusion of life saving cost has relatively little influence on the position of the optimum.

It is noted that the stochastic model and the variability of capacity and demand also play an important role for the magnitude and location of the optimum as well as for the position of the acceptability limit. The specific marginal cost (rate of change) of a safety measure and its effect on a reduction of the failure rate are equally important.

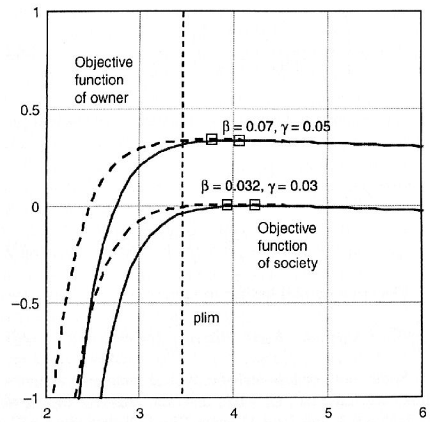

Fig. 1. Objective function for society and owner (solid lines with and dashed lines without life saving cost).

This example also allows to derive risk-consequence curves by varying the number of fatalities in an event. With the same data as before and SLSC$=7.5\times10^5$ and $K_{\lambda\bar{E}}(p,n)$ $=3\times10^6$ for $N_F=1$ we vary the cost effectiveness of the safety measure. Here, only the ratio C_1/C_0 is changed. In Fig. 2 the upper bounds (solid lines) are derived from Eq. (32) and the lower bounds (dashed lines) correspond to the societal optimum according to Eq. (31) ($b_S=0.032C_0$, $\gamma_S=$ 0.03).

Most realistic is probably a ratio of $C_1/C_0=0.001$. In Fig. 2 the region between the upper bound(s) and the lower curve derived from the societal optimum may be interpreted as ALARP-region (ALARP=As Low As Reasonably Practicable). It is seen that the acceptable limits are lines with approximately slope -1 in the log–log-plot.

Note that in these figures the failure rate is given by λP_f and the number of fatalities is given by $N_F=kN_{PE}$. Therefore, these figures cover the full range of λ and P_f and k and N_{PE}, respectively.

7. Summary and conclusions

The life quality index is presented and embedded into modern socio-economic concepts. A general risk acceptability criterion is derived. The societal willingness-to-pay based on the societal value of a statistical life or on the societal life quality index are derived. The acceptability criterion, which is necessary, affordable and efficient from a societal point of view, depends on the marginal cost to reduce the risk, the corresponding marginal decrease in risk, the GDP, the life working time and on demographic factors

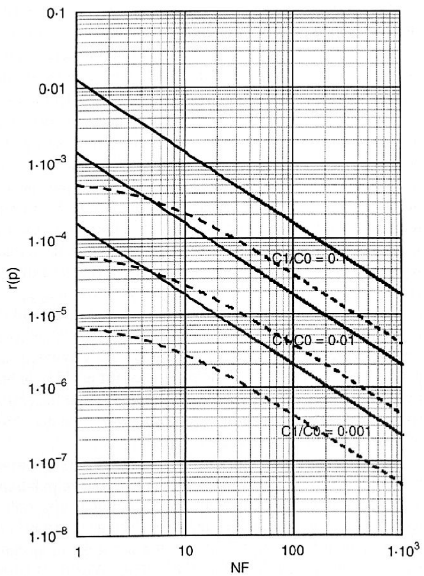

Fig. 2. Acceptable failure rate over number of fatalities for different C_1/C_0. Dashed lines correspond to optimal solution for the public.

obtainable from period or cohort life tables. For example, key parameters such as the constant derived from the societal life quality index (SLQI) and the societal value of a statistical life (SVSL) turn out to cluster around 2.3 and 2.7 mill. PPP US$, respectively, with very little variation for industrialized countries. Because future risks are considered most of the time-dependent factors have been investigated and taken into account up to a time horizon of approximately 100 years.

If time is involved all monetary quantities need to be discounted down to the decision point. Discount rates γ must be long-term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long-term averages from the financial market for his cost–benefit analysis the assessment of interest rates for investments of the public into risk reduction is more difficult. The classical Ramsey model decomposes the output growth rate into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is proposed to use a relatively small interest rate of 3% composed of about 2% economic growth and a rate of time preference of consumption of about 1%. This appears intergenerationally acceptable from an ethical point of view.

Application of the new concepts is illustrated by a number of simple examples.

Technical and natural risks are perceived individually and on a societal level rather irrationally and subjectively. Frequently, risks are communicated to and from the public in such a way that regulatory bodies or other authorities rarely can apprehend fully the nature, magnitude and severity of specific, recognized risks. Accordingly, they hardly are in a position to respond rationally by efficient risk control measures. A lack of efficiency has been shown in a number of studies, among others in the study of [58], indicating that many if not most public risk reduction interventions are highly inefficient. Some others can be shown to be, in fact, no mote affordable thus taking away resources needed for other risk reducing projects and/or reducing life quality in the sense that other components of life quality than life expectancy are inadequately diminished. A third group of risks is inadequately taken into account because the benefits from an undertaking appear to be overwhelming.

Exactly here is where this study attempts to give an answer. Only if society acts rationally in controlling involuntary and anonymous risks from the natural and technical environment in an affordable and efficient manner can it gain better overall life quality in the long run.

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