Conditional copula simulation for systemic risk stress testing

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Abstract

Since the financial crisis of 2007-2009 there is an active debate of regulators and academic researchers on systemic risk, with the aim of preventing similar crises in the future or at least reducing their impact. A major determinant of systemic risk is the interconnectedness of the international financial market. We propose to analyze interdependencies in the financial market using copulas, in particular using flexible vine copulas, which overcome limitations of the popular elliptical and Archimedean copulas. To investigate contagion effects among financial institutions, we develop methods for stress testing by exploiting the underlying dependence structure. New approaches for Archimedean and, especially, for vine copulas are derived. In a case study of 38 major international institutions, 20 insurers and 18 banks, we then analyze interdependencies of CDS spreads and perform a systemic risk stress test. The specified dependence model and the results from the stress test provide new insights into the interconnectedness of banks and insurers. In particular, the failure of a bank seems to constitute a larger systemic risk than the failure of an insurer.

JEL classification: C53, G28

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1 Introduction

Dealing with the lessons learned from the financial crisis, the discussion about systemic risk has become more and more important. The collapse of Lehman Brothers in 2008 showed that the sudden and uncontrolled breakdown of a global financial company not only affected other financial institutions and seriously endangered the stability of the global financial sector but also had a great impact on the real economy of several countries around the world. As a result, the Financial Stability Board (FSB) developed guidelines to assess the systemic importance of financial institutions, markets, and instruments. The FSB defines systemic risk as "*the risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy*" [Financial Stability Board et al., 2009]. Furthermore, an institution, market, or instrument is regarded as systemic if "*its failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion*" on the financial system and/or the real economy.

The systemic relevance of an institution can be assessed based on several criteria that have been identified by the FSB. The three most important are size, lack of substitutability, and interconnectedness: Financial institutions whose "*distress or disorderly failure, because of their*

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size, complexity, and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity" [Financial Stability Board, 2011] are called systemically important. These institutions will face additional regulatory measures to reduce the systemic risk imposed by them. The Basel Committee on Banking Supervision [2011] and the International Association of Insurance Supervisors [2012] have developed methodologies to determine globally systemically important banks and insurers, respectively. The assessment methodology for insurers differs to that used for banks, since it takes into account the fundamental differences in the business models of banks and insurance companies. While a systemic classification of insurers has not been published yet, a list of globally systemically important banks is released on a yearly basis. In 2012, there were 28 banks on this list [Financial Stability Board, 2012].

Despite the popular expression "too big to fail", it has been argued in recent literature that the interconnectedness of an institution is much more important in the assessment of systemic risk: Cont and Moussa [2010] and Cont et al. [2013] find that the impact of the failure of an institution strongly depends on the interdependencies among institutions and less on its size. Similarly, Markose et al. [2012] observe in their analysis of interconnectedness in the US banking sector that only a few major institutions play a dominant role in terms of network centrality and connectivity. With respect to contagion in the US insurance sector, Park and Xie [2011] evaluate the impact of reinsurer downgradings on US property-casualty insurers and conclude that a systemic crisis caused by reinsurance transactions is rather unlikely. Billio et al. [2012] analyze the interdependencies among financial institutions from different sectors using principle component analysis and Granger-causality networks and detect an interesting asymmetry in the financial system, as banks are more likely to transmit shocks than insurers, hedge funds or broker-dealers. Hence, in light of this research, it is more appropriate to speak of systemically important institutions as "too (inter-)connected to fail".

The exploration of contagion and interconnectedness is also the topic of this article. We propose to use copulas to analyze interdependencies in the global financial market, notably in the banking as well as in the insurance sector and not in both sectors in isolation, as it is often done. In doing so, we aim to find out whether there are significant differences in the dependence structure among banks and among (re-)insurers. As a statistical tool for dependence modeling, copulas allow for an accurate analysis beyond linear correlations and common multivariate Gaussian distributions. Therefore, we not only consider the popular classes of Archimedean and elliptical copulas, but also the more recently proposed vine copulas (see Kurowicka and Joe [2011] and Czado et al. [2013] for recent overviews). Such vine copulas allow to take into account tail and asymmetric dependencies and therefore overcome limitations of the elliptical copulas that are typically used in larger dimensional dependence analysis. Vine copulas may also provide more parsimonious parameterizations of multivariate distributions and therefore constitute useful models for a flexible dependence analysis (see also Brechmann and Czado [2011]).

Stress testing is an important tool for the assessment and classification of systemic risk. The systemic relevance of an institution is decisively determined by the potential impact of its failure on other institutions. It is therefore crucial to analyze such stress situations in the market by taking into account the interdependence among the institutions. Statistically speaking, we are interested in the following situation: Let $\boldsymbol{X} := (X_1, ..., X_d)'$ be a random vector of risk quantities. Then we are interested in the case $\boldsymbol{X}_{-i}|X_i = x_i$, $i \in \{1, ..., d\}$, where $\boldsymbol{X}_{-i}$ denotes the random vector $\boldsymbol{X}$ without the ith component and the event $\{X_i = x_i\}$ corresponds to a stress situation. For instance, let X_i be the company value, then a stress situation occurs when x_i is very small. Clearly, such an analysis requires the availability of the conditional distribution of $\boldsymbol{X}_{-i}|X_i = x_i$, given the specific underlying dependence model. As this distribution is typically not known in closed form, conditional simulation algorithms are needed for the scenario analysis. While these are straightforward and well-known in the case of elliptical copulas, we derive appropriate methods for Archimedean and vine copulas.

The methodology developed in this article is used in a case study of 38 important financial institutions from all over the world, among them 20 insurers and 18 banks. Their credit default swap spreads, as market-based indicators of the credit worthiness, are statistically analyzed and appropriate multivariate dependence models are constructed. A stress testing exercise then provides insights into the systemic relevance of the different institutions. We detect differences among regional markets and, in addition, among the banking and the insurance sector. Interestingly, the classification of globally systemically important banks is hardly reflected in the data. Furthermore, the analysis reveals new results regarding the classification of insurers, which, however, can not yet be compared to an official classification

The remainder of the paper is structured as follows. Section 2 provides the necessary methodological background on copulas and vine copulas in particular. Conditional copula simulation for the classes of elliptical, Archimedean and vine copulas is then treated in Section 3. The case study is presented in Section 4. Section 5 concludes.

2 Copulas

The statistical notion of dependence is closely related to the concept of copulas. In the first place, a d-dimensional copula simply is a multivariate distribution function on $[0,1]^d$ with uniformly distributed marginals. According to the theorem by Sklar [1959] any multivariate distribution is however directly linked to a copula. Let $\boldsymbol{X} = (X_1, ..., X_d)' \sim F$ with marginal distributions $F_1, ..., F_d$, then Sklar [1959] shows that

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)), \quad x_1, ..., x_d \in (\mathbb{R} \cup \{-\infty, \infty\})^d, \tag{2.1}$$

where C is a d-dimensional copula. That is, Sklar's Theorem (2.1) allows to decompose any multivariate distribution in terms of its margins and a copula that specifies the between-variable dependence. If $\boldsymbol{X}$ is a continuous random vector, then the copula C is unique and the multivariate density f of $\boldsymbol{X}$ can be decomposed as

$$f(x_1, ..., x_d) = c(F_1(x_1), ..., F_d(x_d)) f_1(x_1) ... f_d(x_d), \tag{2.2}$$

where c is the copula density and $f_1, ..., f_d$ are the marginal densities of f. More details on copulas in general can be found in the comprehensive reference books by Joe [1997] and Nelsen [2006]. Here, we concentrate on the popular classes of elliptical and Archimedean copulas as well as the more recently proposed vine copulas, which are also known as pair-copula constructions.

If F is an elliptical distribution function (see Fang et al. [1990] and McNeil et al. [2005]), then the associated copula C is also called elliptical. More precisely, an elliptical copula is defined through inversion of Sklar's Theorem (2.1) as

$$C(u_1, ..., u_d) = F(F_1^{-1}(u_1), ..., F_d^{-1}(u_d)), \quad u_1, ..., u_d \in [0,1],$$

where F is elliptical and $F_1, ..., F_d$ are the corresponding margins. The most popular examples of elliptical copulas are the Gaussian copula with correlation matrix $R = (\rho_{ij})_{i,j=1,...,d} \in [-1,1]^{d \times d}$ and the Student's t copula with association matrix $R \in [-1,1]^{d \times d}$ and $\nu > 2$ degrees of freedom. In addition to being reflection symmetric (if $(U_1, U_2)' \sim C$, then it also holds that $(1-U_1, 1-U_2)' \sim C$), the Gaussian copula is tail independent, while the Student's t copula exhibits symmetric lower and upper tail dependence [Embrechts et al., 2002].

Another important class of copulas are Archimedean copulas. For a generator function $\varphi : [0,1] \to [0,\infty)$, whose inverse φ^{-1} is d-monotone on $[0,\infty)$ (see McNeil and Nešlehová [2009]), a d-dimensional Archimedean copula is defined as

$$C(u_1, ..., u_d) = \varphi^{-1}\left(\varphi(u_1) + ... + \varphi(u_d)\right), \quad u_1, ..., u_d \in [0,1]. \tag{2.3}$$

Popular Archimedean copulas are the Clayton, Gumbel and Frank copulas, which possess different properties to those of the elliptical copulas such as asymmetric tail dependence. From Definition (2.3) it is however clear that Archimedean copulas imply exchangeability: If $\boldsymbol{U}$ is distributed according to an Archimedean copula, any permutation of the components of $\boldsymbol{U}$ follows the same Archimedean copula. Since this may be rather restrictive in larger dimensional applications, Archimedean copulas are primarily used in the bivariate case or as building blocks of so-called vine copulas.

Vine copulas represent a more general approach to construct flexible multivariate copulas, which recently gained increasing attention in the literature (see Kurowicka and Joe [2011] and Czado et al. [2013] for recent overviews). While a special case has already been discussed by Joe [1996], Bedford and Cooke [2001, 2002] independently construct a general class of multivariate distributions. These so-called regular vine distributions only depend on bivariate and univariate distributions. We illustrate the concept here with a three-dimensional example.

Let $\boldsymbol{X} = (X_1, X_2, X_3)' \sim F$ with density f. This density can be decomposed by conditioning into

$$f(x_1, x_2, x_3) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2). \tag{2.4}$$

Using Sklar's Theorem (2.2), it follows that

$$\begin{aligned} f_{2|1}(x_2|x_1) = \frac{f_{1,2}(x_1, x_2)}{f_1(x_1)} &= \frac{c_{1,2}(F_1(x_1), F_2(x_2)) f_1(x_1) f_2(x_2)}{f_1(x_1)} \\ &= c_{1,2}(F_1(x_1), F_2(x_2)) f_2(x_2), \end{aligned} \tag{2.5}$$

where $C_{1,2}$ is the bivariate copula of the variable pair (X_1, X_2). In the same way, it holds that

$$\begin{aligned} f_{3|1,2}(x_3|x_1, x_2) = \frac{f_{2,3|1}(x_2, x_3|x_1)}{f_{2|1}(x_2|x_1)} &= \frac{c_{2,3;1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) f_{2|1}(x_2|x_1) f_{3|1}(x_3|x_1)}{f_{2|1}(x_2|x_1)} \\ &= c_{2,3;1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) f_{3|1}(x_3|x_1) \\ &\overset{(2.5)}{=} c_{2,3;1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) c_{1,3}(F_1(x_1), F_3(x_3)) f_3(x_3), \end{aligned}$$

where

$$F_{2|1}(x_2|x_1) = C_{2|1}(F_2(x_2)|F_1(x_1)) := \frac{\partial C_{1,2}(F_1(x_1), F_2(x_2))}{\partial F_1(x_1)} \tag{2.6}$$

and similarly for $F_{3|1}$. Here, $C_{1,3}$ is the copula of the variable pair (X_1, X_3) and $C_{2,3;1}$ that of the pair (X_2, X_3) given the first variable X_1. Further, $C_{2|1}$ denotes the conditional distribution function of U_2 given U_1 when $(U_1, U_2)' \sim C_{1,2}$.

To summarize, we have decomposed the density f of $\boldsymbol{X}$ into its marginal densities and the three bivariate copulas $C_{1,2}$, $C_{1,3}$ and $C_{2,3;1}$ with densities $c_{1,2}$, $c_{1,3}$ and $c_{2,3;1}$, respectively. For the three-dimensional copula of $\boldsymbol{X}$ this means that its density is formed as the product of these bivariate copulas, of which two are unconditional and one is conditional on another variable. Note that it is typically assumed that the conditional copula $C_{2,3;1}$ only depends on X_1 through the arguments $F_{2|1}$ and $F_{3|1}$. More details on this so-called simplifying assumption can be found in Hobæk Haff et al. [2010], Stöber et al. [2012] and Acar et al. [2012].

Due to this construction in terms of a cascade of bivariate copulas, vine copulas are also called pair-copula constructions (PCCs) as introduced by Aas et al. [2009]. The main virtue of such PCCs is that each bivariate copula can be chosen independently from different copula classes such as elliptical or Archimedean copulas. In this way, a wide range of different dependence structures can be captured using vine copulas.

The above decomposition into a product of bivariate copulas can be extended to the general multivariate case. This however requires different choices regarding the order of the variables

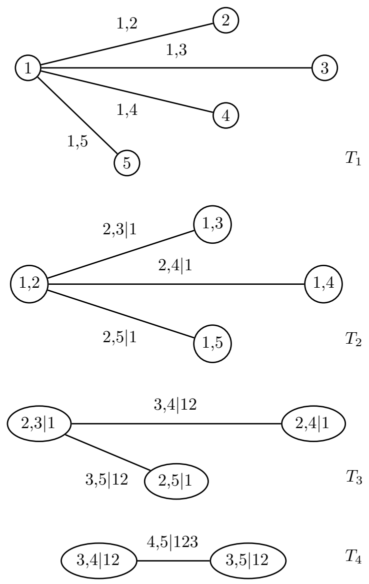

Figure 1: C-vine tree sequence with edge labels for five variables.

in the decomposition as made in (2.4). Bedford and Cooke [2001, 2002] introduce vines as a graphical tool to organize different PCCs and state general conditions in which cases such a d-dimensional PCC of $d(d-1)/2$ bivariate copulas is in fact a valid decomposition. This yields the class of regular vine copulas as investigated further by Kurowicka and Cooke [2006] and Dißmann et al. [2013]. Here, we only concentrate on the sub-class of canonical vine (C-vine) copulas, which yield an appealing model for our purpose of systemic risk stress testing.

A C-vine is characterized by an ordering of the variables. Without loss of generality let $1, ..., d$ be the ordering. This ordering defines the sequence of conditioning in the PCC: First we condition on variable 1, then on variable 2, and so on. Let $\boldsymbol{U} = (U_1, ..., U_d)' \sim C$, where C is a C-vine copula with density c. Then this yields the following C-vine copula density decomposition:

$$\begin{aligned} c(u_1, ..., u_d) &= \prod_{\ell=2}^{d} c_{1,\ell}(u_1, u_\ell) \\ &\times \prod_{j=2}^{d-1} \prod_{k=1}^{d-j} c_{j,j+k;1,...,j-1}(C_{j|1,...,j-1}(u_j|u_1, ..., u_{j-1}), C_{j+k|1,...,j-1}(u_{j+k}|u_1, ..., u_{j-1})), \end{aligned} \tag{2.7}$$

where the index of the first product in the second term runs over the conditioning variables and the index of the second product over the partners of variable j in the conditioned pair $(j, j+k)$. As in (2.6), the conditional distribution functions $C_{j+k|1,...,j-1}$ of U_{j+k} given $U_1, ..., U_{j-1}$ can be obtained recursively for $j = 2, ..., d-1$ and $k = 0, ..., d-j$ using

$$\begin{aligned} C_{j+k|1,...,j-1}&(u_{j+k}|u_1, ..., u_{j-1}) := F_{U_{j+k}|U_1,...,U_{j-1}}(u_{j+k}|u_1, ..., u_{j-1}) \\ &= h_{j+k|j-1;1,...,j-2}\left(C_{j+k|1,...,j-2}(u_j|u_1, ..., x_{j-2}) | C_{j-1|1,...,j-2}(u_{j-1}|u_1, ..., u_{j-2})\right), \end{aligned} \tag{2.8}$$

where

$$h_{j+k|j-1;1,...,j-2}(v|u) := \frac{\partial C_{j-1,j+k;1,...,j-2}(u, v)}{\partial u}.$$

For $j = 2$ it holds that $h_{2+k|1} = C_{2+k|1}$.

This decomposition can be represented graphically in a C-vine tree sequence (see Figure 1). Each edge is uniquely associated to one bivariate copula in the decomposition: Those in the first tree correspond to the first term in (2.7), while those in trees $T_j,\ j \geq 2$, are associated with the first product in the second term of (2.7). This graphical representation also conveniently illustrates the order of the variables: In the first tree, variable 1 plays a pivotal role. Then in the second tree, variable 2 takes on this pivotal role, since all possible pairs of variable 2 with the remaining variables are modeled conditionally on variable 1, and similarly for all other trees. The order of the variables can therefore be seen as an importance ordering of the most relevant variables among variables $1, ..., d$. Motivated by the graphical representation, the pivotal variable in each tree is also called root node.

3 Conditional simulation

As noted in the introduction, we are interested in the following situation for systemic risk stress testing. Let $\boldsymbol{X} := (X_1, ..., X_d)' \sim F$ be a continuous random vector and let $\boldsymbol{X}_{-i}$ denote the subvector of $\boldsymbol{X}$ having the ith component removed, $i \in \{1, ..., d\}$. Then what is the distribution $F_{-i|i}$ of $\boldsymbol{X}_{-i}|X_i = x_i$? If the event $\{X_i = x_i\}$ corresponds to an extreme situation, this distribution describes the impact of the ith variable being stressed.

Using the stressed distribution $F_{-i|i}$, we are then interested in calculating quantities like expected value $E(\boldsymbol{X}_{-i}|X_i = x_i)$ and variance $Var(\boldsymbol{X}_{-i}|X_i = x_i)$ to assess size and variability of the impact. Since this may not be feasible in closed form, one often has to resort to statistical simulation from $F_{-i|i}$ to calculate Monte Carlo estimates of the quantities of interest.

A general approach can be formulated using the transformation by Rosenblatt [1952], which is an extension of the univariate probability integral transform. Let $F_{j|1,...,j-1}$ denote the conditional distribution function of $X_j|X_1 = x_1, ..., X_{j-1} = x_{j-1}$ for $j = 1, ..., d$ (for $j = 1$ the conditioning set is empty). Then

$$W_j := F_{j|1,...,j-1}(X_j|X_1, ..., X_{j-1}), \quad j = 1, ..., d, \tag{3.1}$$

define independent and identically distributed uniform random variables.

Without loss of generality let $i = 1$. That is, the aim is to sample from $F_{-1|1}$, the conditional distribution of $\boldsymbol{X}_{-1}|X_1 = x_1$. For this we draw a sample $w_2, ..., w_d$ of independent, uniformly distributed random variables. By inverting the Rosenblatt transformation (3.1), the values

$$x_j := F^{-1}_{j|1,...,j-1}(w_j|x_1, ..., x_{j-1}), \quad j = 2, ..., d,$$

define a sample from $F_{-1|1}$. In other words, one iteratively samples from the distribution of $X_j|X_1 = x_1, ..., X_{j-1} = x_{j-1}$ for $j = 2, ..., d$.

This approach is very appealing if the conditional distribution functions $F_{j|1,...,j-1}$ can be determined in closed form. In the case of a C-vine copula as the underlying copula of $\boldsymbol{X}$, this will in fact be possible. Before that, we discuss the popular classes of elliptical and Archimedean copulas. While the conditional simulation of elliptical copulas is well-known, the procedure in the case of Archimedean copulas is more challenging and we derive a new approach here.

Note that if $U_j := F_j(X_j)$ for $j = 1, ..., d$, then it is equivalent to sample from the distribution of $\boldsymbol{X}_{-i}|X_i = x_i$ or that of $\boldsymbol{U}_{-i}|U_i = u_i$ where $u_i := F_i(x_i)$, since samples from the latter can be transformed back to the original level of the data by applying the inverse distribution function $F_j^{-1},\ j = 1, ..., d$. We therefore concentrate on the case $\boldsymbol{U}_{-i}|U_i = u_i$. Without loss of generality we further let $i = 1$.

3.1 Elliptical copulas

For conditional simulation from elliptical copulas it is advantageous to transform the random variables by the respective inverse distribution functions. That is, for the Gaussian copula set

$Y_j := \Phi^{-1}(U_j),\ j = 1, ..., d$, and $y_1 := \Phi^{-1}(u_1)$, where Φ is the standard normal distribution function, and for the Student's t copula set $Y_j := F_t^{-1}(U_j|\nu),\ j = 1, ..., d$, and $y_1 := F_t^{-1}(u_1|\nu)$, where $F_t(\,\cdot\,|\nu)$ is the univariate Student's t distribution function with ν degrees of freedom. Then one draws samples $(y_2, ..., y_d)'$ from the corresponding conditional distribution function of $\boldsymbol{Y}_{-1}|Y_1 = y_1$ with appropriate parameters. These samples are finally transformed by $u_j = \Phi(y_j)$ or $u_j = F_t(y_j|\nu)$, respectively, for $j = 2, ..., d$.

For the multivariate Gaussian case the conditional distribution of $\boldsymbol{Y}_{-1}|Y_1 = y_1$ is well-known [Kotz et al., 2004]. Let $\boldsymbol{Y} \sim N_d(\boldsymbol{\mu}, \Sigma)$ with mean vector $\boldsymbol{\mu} \in \mathbb{R}^d$ and covariance matrix $\Sigma = (\sigma_{ij})_{i,j=1,...,d} \in \mathbb{R}^{d\times d}$. Further let $\boldsymbol{\mu} = (\mu_1, \boldsymbol{\mu}'_{-1})'$ and

$$\Sigma = \begin{pmatrix} \sigma_{11} & \boldsymbol{\sigma}'_1 \\ \boldsymbol{\sigma}_1 & \Sigma_{(-1,-1)} \end{pmatrix},$$

where $\boldsymbol{\sigma}_1 := (\sigma_{12}, ..., \sigma_{1d})'$ and $\Sigma_{(-1,-1)}$ denotes the covariance matrix Σ with first row and first column removed. Then $\boldsymbol{Y}_{-1}|Y_1 = y_1$ is again Gaussian with modified mean vector and covariance matrix:

$$\boldsymbol{Y}_{-1}|Y_1 = y_1 \sim N_{d-1}(\widetilde{\boldsymbol{\mu}}, \widetilde{\Sigma}),$$

where

$$\widetilde{\boldsymbol{\mu}} = \boldsymbol{\mu}_{-1} + \boldsymbol{\sigma}'_1\Sigma^{-1}_{(-1,-1)}(y_1 - \mu_1) \quad \text{and} \quad \widetilde{\Sigma} = \Sigma_{(-1,-1)} - \boldsymbol{\sigma}_1\boldsymbol{\sigma'}_1/\sigma_{11}. \tag{3.2}$$

Since the conditional distribution is hence known in closed form, expectation and variance are given explicitly and simulation is only required if non-standard quantities need to be obtained. For the conditional copula simulation set $\boldsymbol{\mu} = \boldsymbol{0}$ and $\Sigma = R$, where R is the corresponding correlation matrix.

The conditional distribution in the case of a multivariate Student's t distribution is also known in closed form. Let $\boldsymbol{Y} \sim t_d(\Sigma, \nu)$ with association matrix $\Sigma \in \mathbb{R}^{d\times d}$ and $\nu > 2$ degrees of freedom. The d-dimensional Student's t distribution function with association matrix $\Sigma \in \mathbb{R}$ and ν degrees of freedom is denoted by $F_{t_d}(\,\cdot\,|\Sigma, \nu)$. Then (see, e.g., Kotz and Nadarajah [2004])

$$F_{\boldsymbol{Y}_{-1}|Y_1}(\boldsymbol{y}_{-1}|y_1) = F_{t_{d-1}}\left(\sqrt{\frac{\nu+1}{\nu + y_1^2/\sigma_{11}}}\left(\boldsymbol{y}_{-1} - y_1\frac{\boldsymbol{\sigma}_1}{\sigma_{11}}\right)\middle|\,\widetilde{\Sigma}, \nu+1\right),$$

where $\widetilde{\Sigma}$ is defined in (3.2). That is, a sample from $F_{\boldsymbol{Y}_{-1}|Y_1}$ can be drawn by sampling $(\widetilde{y}_2, ..., \widetilde{y}_d)'$ from $t_d(\widetilde{\Sigma}, \nu+1)$ and then setting $y_j = \widetilde{y}_j\sqrt{(\nu + y_1^2/\sigma_{11})/(\nu+1)} + y_1\sigma_{1j}/\sigma_{11}$. To conditionally sample from a Student's t copula, set $\Sigma = R$ as before.

3.2 Archimedean copulas

According to Mesfioui and Quessy [2008], the conditional distribution of $U_j|U_1 = u_1, ..., U_{j-1} = u_{j-1}$ for $j = 2, ..., d$, in the case of $\boldsymbol{U}$ being distributed according to an Archimedean copula C, is given by

$$C_{j|1,...,j-1}(u_j|u_1, ..., u_{j-1}) = \frac{(\varphi^{-1})^{(j-1)}\big(\sum_{i=1}^{j}\varphi(u_i)\big)}{(\varphi^{-1})^{(j-1)}\big(\sum_{i=1}^{j-1}\varphi(u_i)\big)}.$$

Conditional inverse sampling using the Rosenblatt transformation (3.1) hence requires inversion of $(\varphi^{-1})^{(j-1)}$ for $j = 2, ..., d$, which may be numerically rather challenging, although explicit functional expressions of $(\varphi^{-1})^{(j-1)}$ for common Archimedean generators are provided in Hofert et al. [2012]. We therefore derive an alternative conditional sampling strategy.

Here, we use a trick and introduce the following variable: $Z := C(U_1, ..., U_d) \in [0, 1]$, which is the contour level of a copula. The variable Z is univariate and known to be distributed according to the so-called Kendall distribution function F_Z, which can be determined in terms of the Archimedean generator (see Barbe et al. [1996]). Now, instead of directly sampling from the conditional distribution of $U_j|U_1 = u_1, ..., U_{j-1} = u_{j-1}$ when using the Rosenblatt transformation (3.1), the idea is to iteratively sample z from $Z|U_1 = u_1$ and use this to sample u_j from $U_j|Z = z, U_1 = u_1, ..., U_{j-1} = u_{j-1}$ for $j = 2, ..., d$. That is, first the contour level Z is sampled given the event $\{U_1 = u_1\}$ and then the remaining variables are obtained given this contour level and the event $\{U_1 = u_1\}$. This approach is beneficial, since the distribution $F_{U_j|Z,U_1,...,U_{j-1}}$ of $U_j|Z = z, U_1 = u_1, ..., U_{j-1} = u_{j-1}$ is known in closed form as (see Brechmann [2012])

$$F_{U_j|Z,U_1,...,U_{j-1}}(u_j|z, u_1, ..., u_{j-1}) = \left(1 - \frac{\varphi(u_j)}{\varphi(z) - \sum_{i=1}^{j-1} \varphi(u_i)}\right)^{d-j}. \tag{3.3}$$

The inversion of this conditional distribution function is straightforward, and it is therefore numerically very efficient to use it for the conditional inverse sampling strategy using the Rosenblatt transformation (3.1).

Hence, the open question is how to sample from $Z|U_1 = u_1 \sim F_{Z|U_1}$. For this, we begin with decomposing the density $f_{Z|U_1}$ corresponding to $F_{Z|U_1}$ as

$$f_{Z|U_1}(z|u_1) = f_{U_1|Z}(u_1|z) f_Z(z), \tag{3.4}$$

which holds, since U_1 is uniform, that is $f_{U_1}(u_1) = 1,\ u_1 \in (0, 1)$. The density f_Z of Z is derived by Barbe et al. [1996] as

$$f_Z(z) = \frac{(-1)^{d-1}}{(d-1)!} \varphi(z)^{d-1} \varphi'(z) (\varphi^{-1})^{(d)}(\varphi(z)). \tag{3.5}$$

Further, since Expression (3.3) also holds for $j = 1$ (with an empty conditioning set), the density $f_{U_1|Z}$ of $U_1|Z = z$ is given by

$$f_{U_1|Z}(u_1|z) = -(d-1) \left(1 - \frac{\varphi(u_1)}{\varphi(z)}\right)^{d-2} \frac{\varphi'(u_1)}{\varphi(z)}. \tag{3.6}$$

Combining Equations (3.4), (3.5) and (3.6) then yields

$$f_{Z|U_1}(z|u_1) = \frac{1}{(d-2)!} \left(\varphi(u_1) - \varphi(z)\right)^{d-2} \varphi'(u_1) \varphi'(z) (\varphi^{-1})^{(d)}(\varphi(z)),$$

and

$$F_{Z|U_1}(z|u_1) = \int_0^z f_{Z|U_1}(y|u_1) dy \overset{x=\varphi(y)}{=} \frac{1}{(d-2)!} \varphi'(u_1) \int_1^{\varphi^{-1}(z)} \left(\varphi(u_1) - x\right)^{d-2} (\varphi^{-1})^{(d)}(x) dx. \tag{3.7}$$

This last expression can then be used for conditional inverse sampling from $Z|U_1 = u_1$. This means, in contrast to inversion of $(\varphi^{-1})^{(j-1)}$ for $j = 2, ..., d$, as in direct conditional inverse sampling of Archimedean copulas, the only numerically challenging step of this newly proposed strategy is inversion of $F_{Z|U_1}$, which is given in (3.7).

3.3 C-vine copulas

Simulation from a C-vine copula is straightforward using the Rosenblatt transformation (3.1) and the conditional distribution functions $C_{j|1,...,j-1}$ given in (2.8). The general sampling algorithm for C-vine copulas can be found in Aas et al. [2009]. Since this sampling strategy makes use of the ordering of the variables in the C-vine, it is straightforward to conditionally sample from $\boldsymbol{U}_{-1}|U_1 = u_1$. However, in contrast to elliptical and Archimedean copulas, the cases of

$\boldsymbol{U}_{-i}|U_i = u_i$ for $i > 1$ need to be considered explicitly, since the variables of a C-vine copula cannot simply be reordered.

Now, let $i > 1$. Clearly, for $j > i$ the sampling strategy of $U_j|U_1, ..., U_i, ..., U_{j-1}$ does not change. The question hence is how to sample from $U_1|(U_i = u_i)$, $U_2|(U_1 = u_1, U_i = u_i)$,..., $U_{i-1}|(U_1 = u_1, ..., U_{i-2} = u_{i-2}, U_i = u_i)$. This means that we need to compute the corresponding distribution functions $C_{j|1,...,j-1,i}$ for $1 \leq j < i$. It holds that

$$\begin{aligned} &C_{j|1,...,j-1,i}(u_j|u_1, ..., u_{j-1}, u_i) \\ &\quad = h_{j|i;1,...,j-1}\left(C_{j|1,...,j-1}(u_j|u_1, ..., u_{j-1})|C_{i|1,...,j-1}(u_i|u_1, ..., u_{j-1})\right). \end{aligned} \tag{3.8}$$

Both arguments, $C_{j|1,...,j-1}$ and $C_{i|1,...,j-1}$, can be computed recursively according to Equation (2.8).

To make things more concrete, we consider an illustrative example with $d = 5$ and $i = 4$. That is, we need to determine the conditional distribution functions $C_{1|4}$, $C_{2|1,4}$ and $C_{3|1,2,4}$, while $C_{5|1,2,3,4}$ is the same as in standard C-vine copula simulation. The first term, $C_{1|4} = h_{1|4}$, is straightforwardly given as derivative with respect to the second argument of the copula $C_{1,4}$, which is part of the decomposition (2.7) and therefore known. According to Equation (3.8) we further obtain

$$C_{2|1,4}(u_2|u_1, u_4) = h_{2|4;1}\left(C_{2|1}(u_2|u_1)|C_{4|1}(u_4|u_1)\right) = h_{2|4;1}\left(h_{2|1}(u_2|u_1)|h_{4|1}(u_4|u_1)\right), \tag{3.9}$$

where the known copulas $C_{1,2}$, $C_{1,4}$ and $C_{2,4;1}$ are used. By using Equation (3.8) we also compute $C_{3|1,2,4}$ as

$$C_{3|1,2,4}(u_3|u_1, u_2, u_4) = h_{3|4;1,2}\left(C_{3|1,2}(u_3|u_1, u_2)|C_{4|1,2}(u_4|u_1, u_2)\right), \tag{3.10}$$

where $C_{3|1,2}$ and $C_{4|1,2}$ are computed as in (2.8). In particular, the copula $C_{3,4;1,2}$ as part of the decomposition (2.7) is used. Sampling using the Rosenblatt approach is then feasible: Let w_1, w_2, w_3 and w_5 be independent, uniformly distributed samples. Then, we obtain

$$\begin{aligned} u_1 &= h^{-1}_{1|4}(w_1|u_4), \\ u_2 &= h^{-1}_{2|1}(h^{-1}_{2|4;1}(w_2|h_{4|1}(u_4|u_1))|u_1), \\ u_3 &= h^{-1}_{3|1}(h^{-1}_{3|2;1}(h^{-1}_{3|4;1,2}(w_3|h_{4|2;1}(h_{4|1}(u_4|u_1)|h_{2|1}(u_2|u_1)))|h_{2|1}(u_2|u_1))|u_1), \end{aligned}$$

and finally u_5 by recursively inverting the terms in (3.9), (3.10) and (2.8), respectively.

More generally the sampling algorithm can be written down as outlined in the following. This extends the C-vine simulation algorithm by Aas et al. [2009] from where notation is adopted.

Algorithm 3.1 (Conditional C-vine copula simulation)**.** To generate a sample from a C-vine copula given that the ith variable is equal to u_i, proceed as follows.

Let $V = (v_{j,k})_{j,k=1,...,d}$ be an auxiliary array.

Obtain independent uniformly distributed samples $w_1, ..., w_{i-1}, w_{i+1}, ..., w_d$.

$v_{i,1} = u_i$

for $j \leftarrow 1, ..., i-1, i+1, ..., d$

    $v_{j,1} = w_j$

    if $j < i$ **then**

        $v_{j,1} = h^{-1}_{j|i;1,...,j-1}(v_{j,1}|v_{i,j})$

    end if

    if $j > 1$ **then**

for $k \leftarrow j-1, ..., 1$

$v_{j,1} = h^{-1}_{j|k;1,...,k-1}(v_{j,1}|v_{k,k})$

end for

end if

$u_j = v_{j,1}$

if $j < d$ **then**

for $\ell \leftarrow 1, ..., j-1$

$v_{j,\ell+1} = h_{j|\ell;1,...,l-1}(v_{j,\ell}|v_{\ell\ell})$

end for

end if

if $j < i$ **then**

$v_{i,j+1} = h_{i|j;1,...,j-1}(v_{i,j}|v_{j,j})$

end if

end for

Return samples $u_1, ..., u_{i-1}, u_{i+1}, ..., u_d$.

4 Systemic risk analysis and stress testing

The purpose of our case study on interconnectedness in the financial market is threefold. First, using appropriate statistical dependence models we carefully analyze the interdependencies among major financial institutions in the banking as well as the insurance sector and point out differences between these two sectors. Second, the developed statistical models are used to stress test the global financial market in order to obtain new insights with respect to the assessment and classification of systemically important institutions. Third, as we use credit default swap spreads for our analyses, we also investigate whether such data is actually useful to analyze systemic risk. The developed methodology is however independent of data, which means that the first two questions can be investigated using the same tools but different data.

Recently, there has been active research on the connection of credit default swaps and systemic risk. Credit default swaps (CDS) are bilateral credit derivative contracts that allow the trading of default risks of an underlying corporate or government entity. Since the payoff of a CDS contract is caused by the default on debt, CDS spreads are a market-based indicator of the credit worthiness of the reference entity. Rising CDS spreads indicate growing default expectations of the other market participants regarding the referenced entity. In fact, Hull et al. [2004] and Norden and Weber [2004] found that there is statistical evidence for the CDS market actually anticipating later rating announcements by the credit rating agencies.

The relationship between CDS and systemic risk seems obvious: If there is a systemic event in the market, default expectations of relevant institutions should rise, which is then reflected in increasing CDS spreads. Authors have therefore developed measures of systemic risk that are directly based on CDS spreads or the default probabilities derived from these (see for instance Acharya et al. [2011], Huang et al. [2009], and Giglio [2011]). CDS spreads have also been used to examine interdependencies among financial institutions: see Markose et al. [2012], Rahman [2009], Kaushik and Battiston [2012], and Chen et al. [2012]. None of the authors however use copulas to account for non-standard interdependencies among the institutions. This is one aim of our study.

As data for our statistical analyses we use senior CDS spreads with a maturity of five years observed from January 4, 2006 to October 25, 2011 ($n = 1371$ daily observations), which are obtained from Bloomberg. In the attempt of a balanced selection of companies regarding their geographical region and sectoral belonging, we select 38 companies from the financial sector for the analysis of their interdependence structure. Among these are 18 banks and 20 (re-)insurers from different countries in three major geographical regions (abbreviations are shown in brackets):

- *Systemically important banks according to the Financial Stability Board [2012] (15):*
  - *Europe:* Banco Bilbao Vizcaya Argentaria (BBVA), Banco Santander (BS), Barclays, BNP Paribas, Deutsche Bank (DB), Royal Bank of Scotland (RBS), Société Générale (SG), Standard Chartered (SC), UBS, Unicredit
  - *USA:* Citigroup, Goldman Sachs (GS), JP Morgan Chase (JPM)
  - *Asia-Pacific:* Bank of China (BoC), Sumitomo Mitsui

  Note that at the time of this analysis Banco Bilbao Vizcaya Argentaria and Standard Chartered had not yet been officially classified as systemic; see Financial Stability Board [2011].
- *Not systemically important banks (3):*
  - *Europe:* Intesa Sanpaolo
  - *Asia-Pacific:* Kookmin Bank, Westpac Banking
- *(Re-)Insurers (20):*
  - *Europe:* Aegon, Allianz, Assicurazioni Generali, Aviva, AXA, Hannover Rück (HR), Legal & General (LG), Munich Re (MR), Prudential, SCOR, Swiss Re (SR), Zurich Insurance
  - *USA:* ACE, Allstate, American International Group (AIG), Chubb, Hartford Financial Services, XL Group
  - *Asia-Pacific:* QBE Insurance, Tokio Marine (TM)

4.1 Model specification

For our analyses we use daily log returns of the CDS spreads. To deal with the serial dependence in the time series as well as the between-series dependence, we employ the popular copula-GARCH approach (see, e.g., Liu and Luger [2009]): When univariate time series are modeled by appropriate GARCH models, dependence is captured among the residuals of the time series, which can be regarded as approximately independent and identically distributed samples of the respective innovations distribution. Typically this approach is carried out in two steps. First, the marginal time series are estimated. Then, the margins are fixed and solely the between-series dependence is analyzed. This approach is called inference functions for margins [Joe and Xu, 1996].

The time series of the log returns of the CDS spreads show common features of financial time series such as autocorrelation, leptokurtosis (heavy tails) and volatility clustering (see [Hendrich, 2012, Table 4.2]). To remove these characteristics, we apply appropriate time series models. While often (ARMA-)GARCH models with (skewed) Student's t innovations provide good fits for financial time series (see, e.g., Chu et al. [2010] for an application to the iTraxx CDS index), this is not the case here. Hence, for each of the 38 time series we separately consider extended GARCH models, such as the asymmetric exponential GARCH by Nelson [1991] or GARCH-in-mean by Engle et al. [1987], as well as non-standard innovations distributions like the generalized error, the generalized hyperbolic and the normal inverse Gaussian. All model fits are then carefully checked using a range of goodness-of-fit tests such as the Ljung-Box, the

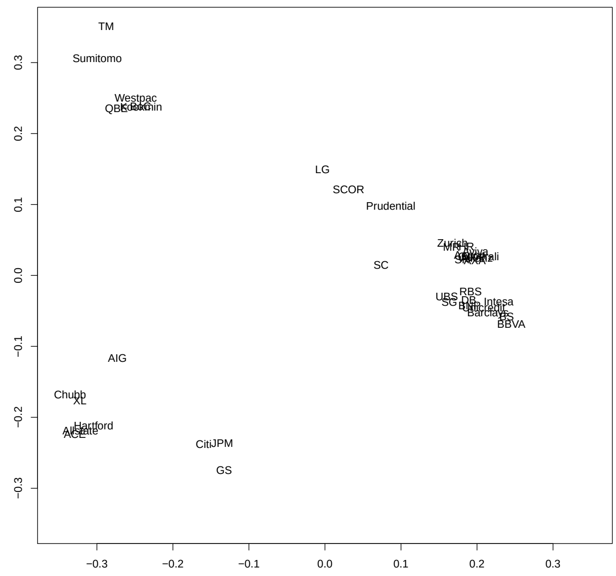

Figure 2: Multidimensional Kruskal-Shephard scaling of the institutions according to the dissimilarity measure $1 - \widehat{\tau}_{j,k}$.

	EU banks	EU ins.	US banks	US ins.	AP banks	AP ins.
EU banks	0.36-0.67					
EU ins.	0.25-0.46	0.29-0.60				
US banks	0.23-0.29	0.19-0.29	0.44-0.47			
US ins.	0.16-0.24	0.15-0.24	0.19-0.27	0.21-0.43		
AP banks	0.08-0.20	0.10-0.22	0.07-0.17	0.10-0.19	0.12-0.29	
AP ins.	0.09-0.18	0.10-0.19	0.08-0.15	0.09-0.19	0.14-0.31	0.14-0.14

Table 1: Ranges of empirical Kendall's τ values $\widehat{\tau}_{j,k}$ within and between sectors in the different regions.

ARCH-LM or the Nyblom stability test. More details on the fitting process can be found in Hendrich [2012, Section 5.1].

After adequately removing the serial dependence in each of the 38 univariate time series, we investigate dependence among the residuals $e_{\ell j},\ j = 1, ..., 38,\ \ell = 1, ..., n$. As we fix the estimated margins, we set $\widehat{u}_{\ell j} := \widehat{F}_j(e_{\ell j})$, where $\widehat{F}_j$ is the estimated innovations distribution of the $j$the time series. To get a first impression of the interdependencies among the different institutions, we calculate empirical Kendall's τ values, $\widehat{\tau}_{j,k}$, for all pairs $j, k = 1, ..., 38,\ j < k$, and use multidimensional Kruskal-Shephard scaling to embed the institutions in the plane according to the dissimilarity measure $1 - \widehat{\tau}_{j,k}$ (see, e.g., Hastie et al. [2009]). This means that the closer two institutions are to each other, the stronger is the dependence of their CDS spreads. The resulting plot is shown in Figure 2. Ranges of the empirical Kendall's τ values per geographical region and sector are shown in Table 1.

The multidimensional scaling shows that there is significant geographical clustering present among the CDS spreads: European institutions can be found on the right of Figure 2, US institutions in the lower left corner and institutions from the Asia-Pacific region in the upper left corner. Within these regions there also is a clear separation of banks and insurers observable. Hence, all pairs of companies within either one of the sectors show the strongest dependencies. Another interesting fact is that the banks that have not been officially classified as systemically

Copula	Log lik.	# Par.	AIC	BIC
Gumbel	8640.22	1	-17278.45	-17273.22
Gaussian	18326.53	703	-35247.07	-31575.09
Student's t	19915.88	704	-38423.76	-34746.56
C-vine	20393.29	488	-39810.58	-37261.61

Table 2: Log likelihoods, numbers of parameters, AIC and BIC values of the copulas estimated by maximum likelihood.

important do not play a significantly different role than the other banks. The classification is not reflected here.

This exploratory look at the data illustrates that there are considerably different relationships among the institutions depending on the geographical region and the sector. Such heterogeneous dependencies cannot be appropriately captured using an Archimedean copula, which assumes exchangeability of all variables. While elliptical copulas are more appropriate for this purpose, they are still somewhat restrictive by imposing symmetric tail dependence. In the literature, it is however often observed that in times of crisis the dependence of joint negative events increases. For CDS spreads this means that one may expect the presence of upper tail dependence, which reflects the joint probability of extreme upward jumps in the expected default probabilities. Such dependence characteristics can be accounted for using a vine copula.

We select a C-vine copula in a sequential way that was proposed by Czado et al. [2012]. The institution with the highest sum of absolute empirical Kendall's τ values to the other institutions is selected as the first root node. Then, using the AIC, appropriate bivariate copulas (first term in the decomposition (2.7)) are selected from the following list: non tail dependent Gaussian, symmetric tail dependent Student's t, lower tail dependent Clayton, upper tail dependent Gumbel and non tail dependent Frank as well as rotations by 90°, 180° (survival copula) and 270° degrees of the tail asymmetric copulas. To obtain a more parsimonious model, the independence copula is also taken into account after performing an independence test of each pair. As second root node the institution with the maximal sum of absolute empirical Kendall's τ values after removing dependence on this first pivotal variable (using (2.8)) is then selected. Appropriate bivariate copulas for the conditioned pairs ($j = 2$ in the first product of the second term of (2.7)) are again selected according to the AIC. This selection procedure is then carried forward for all remaining root nodes. In this way, all required bivariate copulas forming the building blocks of the C-vine copula are selected. Estimation then proceeds by joint maximum likelihood over all copula parameters. Note that we also fitted a more general regular vine copula as described in Dißmann et al. [2013]. The model however did not improve over the C-vine copula, so that we do not consider it any further here (see [Hendrich, 2012, Section 6.3.5] for more details).

Table 2 shows the maximum likelihood fits of Gaussian, Student's and C-vine copulas for our 38-dimensional data set. In addition, the fit of an upper tail dependent 38-dimensional Gumbel copula is shown for comparison, illustrating the inappropriateness of an exchangeable Archimedean copula here. According to AIC and BIC the C-vine copula can be regarded as the best model. In addition to a higher log likelihood compared to the elliptical copulas, it also benefits from a smaller number of parameters, which is achieved by using the independence copula for certain conditional pairs. As noted above, the C-vine copula may also better account for potential asymmetry in the dependence structure. In fact, three upper tail dependent bivariate Gumbel copulas were selected for the bivariate copulas of the first C-vine tree specifying unconditional dependence. Overall, almost 50% of the selected copulas are non-elliptical. This indicates that a purely elliptical approach to measuring systemic interdependencies falls short of adequately capturing all relevant dependence characteristics, in particular the asymmetric tail behavior, which is, of course, especially important in the analysis of stress situations. The Student's t copula however does not assume tail independence as the Gaussian copula does. In fact, the estimated degrees of freedom are 14.71 and therefore clearly indicate the presence of non-Gaussian dependence.

The ordering of the institutions itself is less important here, since it is strongly driven by the number of institutions selected among certain regions and sectors and therefore does not directly provide an ordering of systemic importance. It is hence not surprising that the European institutions Allianz, BNP Paribas and Zurich Insurance are selected as the first three pivotal variables. Hartford Financial Services and JP Morgan Chase are then the first US institutions in the ordering, while QBE Insurance is the first institution from the Asia-Pacific region.

Interestingly, neither the exploratory analysis in Figure 2 nor the fitted models show that interdependencies involving systemically important banks are structurally different from those involving institutions that have not been classified as systemic. An interesting finding however is that the dependence of US banks and European institutions is determined to be higher than that of US insurers and European institutions. This indicates the, maybe not surprising, fact that especially the US banking sector plays a systemically important role in the financial market. This is in line with findings of Billio et al. [2012]. To obtain a more differentiated view on the systemic importance of specific institutions and sectors, we conduct a stress testing exercise of the global CDS market.

4.2 Stress testing and classification

According to the Financial Stability Board et al. [2009], a systemic crisis is defined as the distress of a whole system caused by the failure of one institution and the subsequent spreading of malfunction from one company to another. Hence, we now aim to further investigate the possibility of contagion among the institutions in our sample. We perform a simulation study to exploit the modeled dependence structure. More precisely, we assume a stress situation for one of the institutions and simulate the resulting impact on the remaining institutions. In particular, we are interested to find out whether there are significant differences regarding the type of the institution that is stressed.

The fictitious stress situation that we analyze is a severe drop in the credit-worthiness of one particular institution. Assuming that the market works properly, this would result in a sharp increase of the CDS spreads for the company in question, since the market participants expect its default and require higher risk premiums. Such an increase, in turn, would be reflected in large residuals of the fitted time series models for the log returns of the CDS spreads and thus in quantiles of the respective distributions that are close to one. This means we are able to directly work on the copula level and not on the original level of the data. For our simulation study we assume that the variable of interest, $U_i, i \in \{1, ..., 38\}$, takes on the predefined quantile value of $u_i = 0.99$. Given this stress situation, we then use the methods developed in Section 3 to simulate the impact on the remaining institutions in terms of quantiles of their innovations distributions. That is, we draw samples from the distribution of $\boldsymbol{U}_{-i}|U_i = u_i$. This simulation is repeated $N = 10,000$ times for each institution and for each of the different copulas that have been fitted in the previous section. We denote the samples conditioned on U_i being stressed by $\widetilde{u}_{\ell,j|i},\ j \in \{1, ..., 38\} \setminus \{i\},\ \ell = 1, ..., N$.

As an illustration, Figure 3 shows the mean impact per sector and region in the case of JP Morgan Chase and Hartford Financial Services being stressed: For sector s (within a specific region) with members M_s define

$$\widetilde{\mu}_{s|i} := \frac{1}{N} \sum_{\ell=1}^{N} \widetilde{u}_{\ell,s|i}, \quad \text{where} \quad \widetilde{u}_{\ell,s|i} := \frac{1}{|M_s \setminus \{i\}|} \sum_{j \in M_s \setminus \{i\}} \widetilde{u}_{\ell,j|i}. \tag{4.1}$$

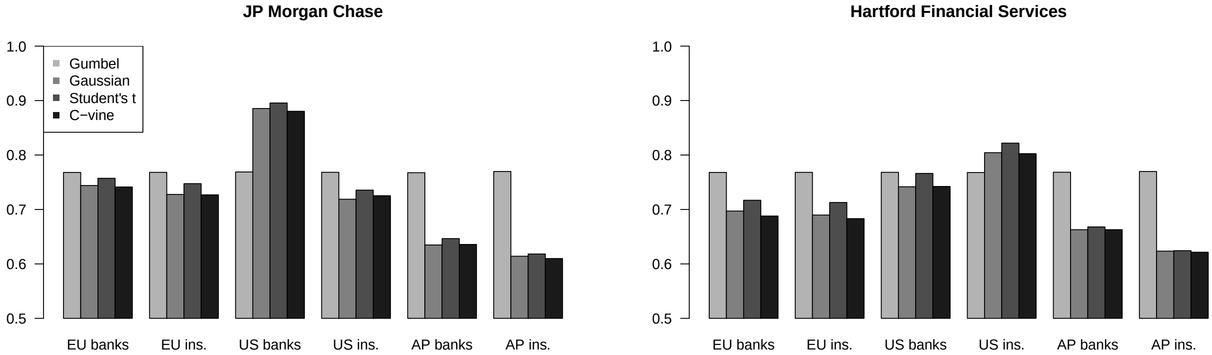

Figure 3: Mean impact $\widetilde{\mu}_{s|i}$ (4.1) per sector and region in the case of JP Morgan Chase (left panel) and Hartford Financial Services (right panel) being stressed according to different copulas. An impact of 0.5 corresponds to independence, since this is the mean of a uniform random variable.

This gives an indication which sectors are most strongly influenced by stress to institution i. Of course, this is only informative if the underlying copula is non-exchangeable: The results obtained when using the (exchangeable) Gumbel copula are shown here only for comparison, since they imply that each sector is impacted in the same way, an obviously incorrect statement. As expected, JP Morgan Chase most strongly impacts the US banking sector and Hartford Financial Services the US insurance sector as shown by the elliptical copulas and the C-vine copula. Interestingly, a stress to JP Morgan Chase influences the US banking sector as well as European banks and insurers equally strongly, while the impact of stress to Hartford Financial Services is stronger on US banks than on European institutions. This underlines the previous statement that US banks play a systemically important role in the global financial market.

Comparing the results of elliptical and C-vine copulas, we observe that the tail dependence implied by the Student's t copula increases the mean values in comparison to the Gaussian case. The C-vine copula lies somewhere in the middle, since it is more flexible in accounting for different types of (tail) dependence. Due to this flexibility, we will concentrate on the C-vine copula in the following.

To further investigate the question which of the sectors in the market is most systemic, we compute the mean impact of one sector s_1 on another s_2 as

$$\widetilde{\mu}_{s_2|s_1} := \frac{1}{|M_{s_1}|} \sum_{i \in M_{s_1}} \widetilde{\mu}_{s_2|i}. \tag{4.2}$$

The resulting values are shown in Table 3. They confirm the previous findings about the systemic role of US banks and also show that the impact of a stressed bank is, in general, stronger than that of a stressed insurer. This is quite interesting in light of the argumentation of the Geneva Association [2010] claiming that insurers should not be treated as being similarly systemic as banks.

Finally, we move to the question of a possible classification of systemically important institutions. Here, we concentrate on the two largest sectors: European banks and insurers with eleven and twelve members, respectively. Among these institutions we not only consider the mean impacts, $\widetilde{\mu}_{\text{EU-banks}|i}$ and $\widetilde{\mu}_{\text{EU-ins.}|i}$, but also the corresponding confidence intervals to better assess the differences in the conditional simulations. For this we compute empirical quantiles from $\widetilde{u}_{\ell,s|i}$ (see (4.1)). The results are shown in Figure 4. According to our analysis the systemically most important banks are (in this order) Barclays, Banco Santander, BNP Paribas, Banco Bilbao Vizcaya Argentaria and Unicredit. The ranking of insurers is Allianz, Aviva, Assicurazioni Generali, Zurich Insurance and Aegon. The differences among the simulated values are however quite small and the confidence intervals largely overlap. It should also be noted that, by the

	Stress situation in					
Impact on	EU banks	EU ins.	US banks	US ins.	AP banks	AP ins.
EU banks	0.87	0.83	0.73	0.67	0.65	0.62
EU ins.	0.83	0.87	0.72	0.68	0.66	0.64
US banks	0.74	0.73	0.88	0.72	0.63	0.60
US ins.	0.68	0.69	0.73	0.79	0.64	0.62
AP banks	0.65	0.66	0.63	0.64	0.68	0.69
AP ins.	0.63	0.65	0.61	0.62	0.69	0.62

Table 3: Mean impact (4.2) per sector (rows) of another sector being stressed (columns) according to the C-vine copula.

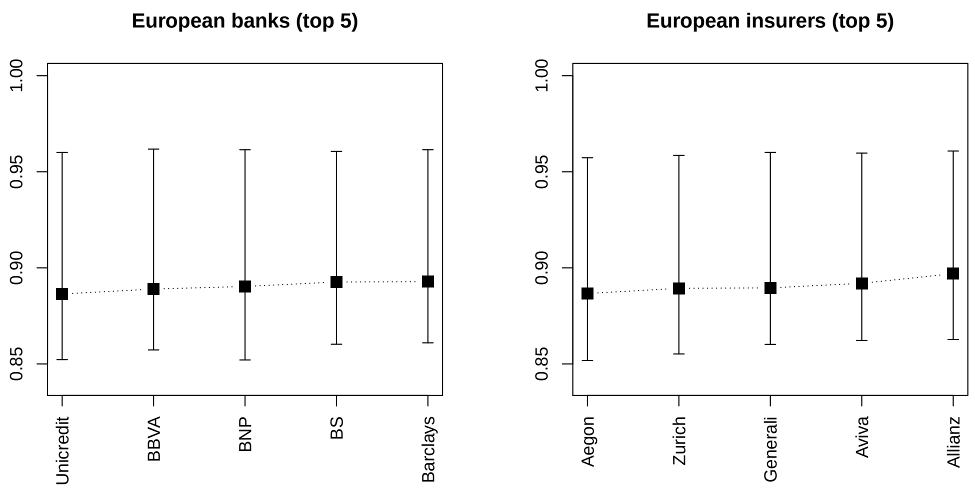

Figure 4: Top five banks and insurers impacting the European banking and insurance sectors, respectively, in case of a stress situation. Mean values and 50% confidence bounds according to the C-vine copula are shown.

time of this analysis, Banco Bilbao Vizcaya Argentaria had not been officially classified as systemically important by the Financial Stability Board [2011]. The 2012 classification [Financial Stability Board, 2012] however included the bank and is therefore in line with our analysis. This however indicates that either a systemic risk analysis should not solely be based on CDS spreads or that the classification of the Financial Stability Board [2011, 2012] does not appropriately take into account the observed interdependence among default probabilities as reflected by CDS spreads.

This partly answer the question whether CDS spreads are actually useful for systemic risk analysis. As a market-based indicator of the credit worthiness of an institution they certainly contain important information to be taken into account. However, we found that dependencies in the CDS market are strongly driven by geographical regions, which hinders a global classification of systemically important institutions. The removal of this geographical dependence in a copula framework is a prerequisite for further attempts to classify institutions using CDS spreads and subject of ongoing research.

5 Conclusion

We propose a copula-based approach to the analysis of interdependencies among financial institutions for systemic risk measurement. For the purpose of stress testing the market, we develop necessary conditional simulation procedures. In particular, we derive new methods for Archimedean and, especially, for vine copulas. The application of these techniques in the analysis of the CDS spreads of 38 major international banks and insurers gives new insights into their interconnectedness and the closely related question of systemic importance. In the dependence analysis we find evidence of non-elliptical structures, especially of asymmetric tail behavior, which is crucial to take into account in stress situations. We also find that banks are systemically more important than insurers. Particularly US banks strongly influence the international financial market. The question whether CDS spreads are actually useful for systemic risk analysis cannot be answered entirely: As a market-based indicator of the credit worthiness of an institution they contain important information. However, they should not be the sole source of information for the assessment of systemic relevance.

We finally also take a first step to a classification of institutions according to the performed stress test. It should nevertheless be kept in mind that the results also depend on the selected sample, although this includes major institutions of the global financial market. The proposed methodology, in particular the stress testing approach, is however not limited to the presented case study but can easily be applied to other relevant data. A major purpose of such investigations certainly should be the further assessment and classification of systemically important institutions according to some appropriate systemic risk measure (see, e.g., Adrian and Brunnermeier [2010], Acharya et al. [2011], and Bernard et al. [2013]).

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