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Searching for New Physics with Rare Decays and CP Violation

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Abstract. After an overture and a non-technical exposition of the relevant theoretical framework, that together offer hopefully a grand picture of the present status of flavour physics, I will collect 20 Goals for this important field. Most of them could be reached already in the next decade. Taken together these goals could be considered as a systematic search for New Physics with the help of flavour- and CP-violating decays of K, D, B mesons and leptons. Also electric dipole moments and $(g-2)_\mu$ play an important role in this program. In this context we will discuss very briefly several extensions of the Standard Model like models with Minimal Flavour Violation, general MSSM, Littlest Higgs Model with T parity and Randall-Sundrum models. This presentation is not meant to be a comprehensive review of flavour physics but rather a personal view on this fascinating field and an attempt to collect those routes that with the help of oncoming experiments should allow us to reach a much deeper understanding of flavour physics at very short distance scales.

1. Overture

The main goal of elementary particle physics is to search for physics laws at very short distance scales. From the Heisenberg uncertainty principle [1] we know that to test scales of order 10^{-18}m we need the energy of approximately 200 GeV. With approximately $E = 4\,\text{TeV}$, effectively available at the LHC, we will be able to test distances as short as $5 \cdot 10^{-20}$m. Unfortunately, it is unlikely that we can do better before 2046 through high energy collider experiments. On the other hand flavour-violating and CP-violating processes are very strongly suppressed and are governed by quantum fluctuations that allow us to test energy scales as high as 200 TeV corresponding to short distances in the ballpark of 10^{-21}m. Even shorter distance scales can be tested, albeit indirectly, in this manner. Consequently frontiers in testing ultrashort distance scales belong to flavour physics or more concretely to very rare processes like particle-antiparticle mixing, rare decays of mesons, CP violation and lepton flavour violation. Also electric dipole moments and $(g-2)_\mu$ belong to these frontiers even if they are flavour conserving. While such tests are not limited by the available energy, they are limited by the available precision. The latter has to be very high as the Standard Model (SM) has been until now very successful and finding departures from its predictions has become a real challange. Let us than have a brief look at the SM and its flavour part.

1.1. Standard Model

The Standard Model of strong and electroweak interactions, considered these days as a low energy effective quantum field theory, based on the spontaneously broken gauge symmetry

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q, \tag{1}$$

describes low energy phenomena in terms of 28 parameters that have to be determined from experiment. Two of these parameters ($\alpha_{\rm QCD}$, $\theta_{\rm QCD}$) are related to strong interactions and four to the electroweak gauge boson sector and Higgs sector. The remaining 22 parameters reside in the flavour sector: 6 quark masses, 6 lepton masses, 4 parameters of the CKM matrix [2, 3] and 6 parameters of the PMNS matrix [4, 5].

At first sight it would appear that whereas the success of the SM in describing the data in the strong interaction sector and electroweak gauge boson sector is very profound, the corresponding success in the flavour sector is rather obvious in view of so many free parameters. Yet in the case of the CKM picture of flavour changing interactions, combined with the GIM mechanism [6] that governs flavour changing neutral current (FCNC) processes in the SM, such a view would totally misrepresent the facts.

Indeed, once all quark and lepton masses are determined, there are only four free parameters of the CKM matrix to our disposal and in terms of them all existing data in the flavour sector can be properly described within experimental and theoretical uncertainties. Indeed, bearing in mind a few hints for the departures from the CKM picture of flavour and CP violation, to which we will return later on,

- all leading decays of K, D, B_d^0 and B_s^0 mesons, that have been measured, are correctly described,
- suppressed transitions in the SM, like $K^0-\bar K^0$ mixing, $B_d^0-\bar B_d^0$ mixing and $B_s^0-\bar B_s^0$ mixing have been not only found at the suppressed level, but even at the predicted order of magnitude and in fact even better than that,
- CP-violating observables in K_L, $K^\pm$, B_d^0 and $B^\pm$ decays agree well with the existing data and
- the best measured semi-rare (radiative) B-decays: $B\to X_s\gamma$, $B\to X_s l^+l^-$ and $B_d\to K^*\gamma$ all turned out to have branching ratios close to the SM predictions.

Table 1. Approximate SM values and experimental upper bounds for selected branching ratios and the neutron electric dipole moment d_n.

	$B_s\to\mu^+\mu^-$	$K_L\to\pi^0\nu\bar\nu$	$K_L\to\mu e$	$\mu\to e\gamma$	d_n
SM	$3\cdot10^{-9}$	$3\cdot10^{-11}$	10^{-40}	10^{-54}	10^{-32} e cm.
Exp Bound	$6\cdot10^{-8}$	$6\cdot10^{-8}$	10^{-12}	10^{-11}	$5\cdot10^{-26}$ e cm.

But this is not the whole story, as a number of very strongly suppressed branching ratios within the SM are also consistent with experiment: the corresponding decays have not been observed yet. Examples are collected in Table 1, where we compare approximate SM values with the experimental upper bounds. Clearly there is still a lot of room for new physics (NP) contributions.

However, one of the very suppressed decays has been seen. It is $K^+\to\pi^+\nu\bar\nu$ which in the SM is predicted to have the branching ratio $Br(K^+\to\pi^+\nu\bar\nu)=(8.5\pm0.7)\cdot10^{-11}$ [7]. Seven events have been found implying $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (17 \pm 11) \cdot 10^{-11}$ [8], on the high side but still consistent with the SM value.

In spite of all these successes the situation is certainly not satisfactory. Indeed,

- the neutral Higgs boson has not been found yet,
- the Higgs mass m_H is plagued by quadratic divergences present in the one-loop contributions to the Higgs propagator with internal top quark, gauge boson and Higgs exchanges. Within the SM there is no protective symmetry that would keep $m_H = \mathcal{O}(v_{ew})$ and if we want to assure this in the presence of a cut-off as high as Λ_{Planck}, fantastic fine tuning of SM parameters has to be made, which is obviously very unnatural,
- the hierarchical structures of quark and lepton masses and of their flavour violating interactions parametrized by the CKM and PMNS matrices remain a mystery, which at least from my point of view has not been satisfactorily uncovered in spite of intensive efforts during the last 30 years. But there are some interesting advances which we will discuss later on.

There are clearly other issues like the quantization of electric charge, the number of quark and lepton generations, the number of space dimensions, dark matter and dark energy but I will not address them here.

1.2. Going Beyond the SM

The problems listed above are not an invention of this decade but have been with us for more than 30 years. Already in 1970's NP beyond the SM has been postulated with the hope that it will help us to solve some, possibly all, of the SM problems. Over 30 years many extensions of the SM have been proposed. These are GUTs, left-right symmetric models, technicolour, extended technicolour, petite unification models, supersymmetry, Little Higgs models, Z'-models, models with extra space dimensions, new versions of technicolour, like top colour, strongly interacting Higgs models, Higgsless models and string theories. One could continue this list but let me stop here. Each of these models while solving some of the SM problems, predicted new particles and new interactions that one could test directly in high energy collider experiments and indirectly through quantum fluctuations, which govern electroweak precision tests (EWPT), FCNC processes and very importantly CP-violating transitions, which could be flavour violating or not.

The budget of all these efforts is well known: except for the neutrino masses and the related neutrino oscillations no sign of new physics has been convincingly identified yet, even if some small deviations from the SM expectations have been possibly seen here and there. We will return to them in the course of this writing.

While this situation is admittedly rather disappointing, not everthing is lost. The tremendous efforts of experimental groups in measuring a vast number of observables as precisely as presently possible, similar efforts by phenomenologically minded quantum field theorists to work out detailed predictions of the SM and of its most favourite extensions and last but certainly not least efforts by less phenomenologically minded quantum field theorists to suggest new directions and construct new models, has very well prepared our community for possible discoveries. With the recent start of LHC, these discoveries could for the first time be really just around the corner and this is clearly very exciting not only for young generations but in particular for those like me, who waited for these discoveries 30 years or more.

In the preceding paragraph I have put on equal footing the efforts of experimentalists, more phenomenologically minded QFTist and less phenomenologically minded QFTist, because I think all three groups of physicists are equally important in the search for new physics. While quite often the first and the last of these groups is credited more for progress in particle physics than the second group, one should realize that without working out detailed predictions of the

SM and of its many extensions, which took place already three decades, it would not be possible to assess the level at which the SM works and in which direction one should look in order to find NP. It is indeed the combined effort of these three groups of physicists with different interests and also different skills that will one day enable us to find out what is going on at and beyond the TeV scale.

1.3. Minimal Flavour Violation

The simplest class of extensions of the SM are the models in which all flavour and CP violating processes are governed, as in the SM, by the structure of Yukawa matrices or equivalently by the CKM matrix. In the so-called constrained Minimal Flavour Violation (CMFV) framework [9, 10, 11] also the operator structure of the decay amplitudes is the same as in the SM. This is the case of models with one Higgs doublet in which the top Yukawa coupling dominates. More generally [12, 13], as formulated elegantly with the help of global symmetries and the spurion technique [14], the operator structure in MFV models can differ from the SM one if two Higgs doublets are present and bottom and top Yukawa couplings are of comparable size. A well known example is the MSSM with MFV and large $\tan\beta$.

The MFV approach is simple and offers an elegant explanation to the fact that the CKM framework works so well even if new physics is required to be present at scales $\mathcal{O}(1\,\text{TeV})$. But one has to admitt that it is a rather pessimistic approach to new physics. The deviations from the SM expectations in CP conserving processes amount in the case of CMFV to at most 50% at the level of the branching ratios [15, 16, 17]. More generally in the MFV framework only in cases where scalar operators are becoming important and helicity suppression in decays like $B_s \to \mu^+\mu^-$ is lifted, enhancements of the relevant branching ratios by more than a factor of 2 and even one order of magnitude relative to the SM are still possible. However, independently of whether it is CMFV or MFV, the CP violation in this class of models is SM-like and in order to be able to distinguish among various models in this class high precision will be required which calls for experiments like Super-Belle, Super-B and $K \to \pi\nu\bar{\nu}$ experiments.

One should also emphasize that MFV in the quark sector does not offer the explanation of the size of the observed baryon-antibaryon asymmetry in the universe (BAU) and it does not address the hierarchy problem related to the quadratic divergences in the Higgs mass. Similarly the hierarchies in the quark masses and quark mixing angles remain in this framework unexplained. For this reason there is still potential interest in non-MFV new physics scenarios to which we will now turn our attention.

1.4. Most Popular Non-MFV Extensions of the SM

The search for new physics at 1 TeV scale is centered already for three decades around the hierarchy problem, be it the issue of quadratic divergences in the Higgs mass, the disparity of the electroweak, GUT and Planck scales or the doublet-triplet splitting in the context of SU(5) GUTs. The three most promising and most popular directions which aim to solve at least some of these problems are as follows:

a) Supersymmetry

In this approach the cancellation of divergences in m_H is achieved with the help of new particles of different spin-statistics than the SM particles: supersymmetric particles. For this approach to work, these new particles should have masses below 1 TeV, otherwise the fine tuning of parameters cannot be avoided. One of the important predictions of the simplest realization of this scenario, the MSSM with R-parity, is the light Higgs with $m_H \leq 130\,\text{GeV}$ and one of its virtues is its perturbativity up to the GUT scales. The uggly feature of the MSSM is a large number of parameters residing dominantly in the soft sector that has to be introduced in the process of supersymmetry breaking. Constrained versions of the MSSM can reduce the number of parameters significantly. The same is true in the case of the MSSM with MFV. An excellent review can be found in [18].

b) Little Higgs Models

In this approach the cancellation of divergences in m_H is achieved with the help of new particles of the same spin-statistics. Basically the SM Higgs is kept light because it is a pseudogoldstone boson of a spontaneously broken global symmetry. Thus the Higgs is protected from aquiring a large mass by a global symmetry, although in order to achieve this the weak gauge group has to be extended and the Higgs mass generation properly arranged (*collective symmetry breaking*). The dynamical origin of the global symmetry in question and the physics behind its breakdown is not specified. But in analogy to QCD one could imagine a new strong force at scales $\mathcal{O}(10-20\,\text{TeV})$ among new very heavy fermions that bind together to produce the SM Higgs. In this scenario SM Higgs is analogous to the pion. At scales well below 5 TeV Higgs is considered as elementary particle but at 20 TeV its composite structure should be seen. At these high scales one will have to cope with non-perturbative strong dynamics, and an unknown ultraviolet completion with some impact on low energy predictions of Little Higgs models has to be specified. The advantage of these models, relative to supersymmetry, is a much smaller number of free parameters. Excellent reviews can be found in [19, 20].

c) Extra Space Dimensions

When the number of space dimensions is increased, new solutions to the hierarchy problems are possible. Most ambitious proposals are models with a warped extra dimension first proposed by Randall and Sandrum (RS) [21] which provide geometrical explanation of the hierarchy between the Planck scale and the EW scale. Moreover, when the SM fields, except the Higgs field, are allowed to propagate in the bulk [22, 23, 24], these models naturally generate the hierarchies in the fermion masses and mixing angles [24, 22] while simultaneously suppressing FCNC transitions with the help of the so-called RS-GIM mechanism [25, 26]. Yet, in these models FCNC processes appear already at tree level and in some cases, as discuss below, some fine-tuning of parameters in the flavour sector is necessary in order to achieve consistency with the data.

In models with extra dimensions using the AdS/CFT duality, between the higher-dimensional physics and strongly-coupled, purely four-dimensional physics, one can also get new insight in the ideas of TeV-scale strong dynamics present for instance in Technicolour models and alike. Finally, one interesting mechanism in the context of extra dimensional models is Gauge-Higgs Unification (see [27] and references therein). Here, the Higgs can be regarded as the 5th component of a gauge field in D=5 dimensions and its mass is protected by the gauge symmetry.

d) Flavour Blind MSSM

Finally, I would like to mention here a MFV framework that goes beyond the usual models with MFV. It is MSSM with MFV in which new *flavour conserving* but CP-violating phases are present. This framework, to be denoted by FMSSM in what follows, has already been discussed in [28, 29, 30, 31]. Recently a new analysis has been presented in [32], where a number of correlations between various flavour conserving and flavour violating observables both CP-violating has been pointed out. Some of these results will be presented below.

1.5. Signatures of New Physics in Collider and Low Energy Processes

The directions for new physics just described imply specific collider signatures that in principle could allow us to distinguish between them. The new particles predicted by these models have masses typically in the range from 200 GeV to 2000 GeV and could be discovered at LHC. They can also contribute through virtual exchanges in box, penguin and tree diagrams to FCNC processes and loop diagrams to electroweak precision observables (EWPO). The interplay between direct NP searches, EWPO and FCNC will be crucial for the identification of the right extension of the SM.

a) Supersymmetry

Here in addition to light Higgs, squarks, sleptons, gluinos, charginos and neutralinos, also charged Higgs particles $H^\pm$ and additional neutral scalars are expected. When R-parity is imposed, there is a dark matter candidate. All these particles can contribute to FCNC transitions. New sources of flavour and CP violation come from the misalignement of quark and squark mass matrices and similar new flavour and CP-violating effects are present in the lepton sector. Some of these effects can be strongly enhanced at large $\tan\beta$.

b) Little Higgs Models

Here in contrast to the MSSM, new heavy gauge bosons $W_H^\pm$, Z_H and A_H in the case of the so-called littlest Higgs model without [33] and with T-parity [34, 35] are expected. Restricting our discussion to the model with T-parity (LHT), the masses of $W_H^\pm$ and Z_H are typically $\mathcal{O}(700\,\text{GeV})$. A_H is significantly lighter with a mass of a few hundred GeV and being the lightest particle with odd T-parity can play the role of the dark matter candidate. Concerning the fermion sector, there is a new very heavy T-quark necessary to cancel the quadratic divergent contribution of the ordinary top quark to m_H and a copy of all SM quarks and leptons, required by T-parity. These mirror quarks and mirror leptons interact with SM particles through the exchange of $W_H^\pm$, Z_H and A_H gauge bosons that in turn implies new flavour and CP-violating contributions to decay amplitudes that are governed by new mixing matrices in the quark and lepton sector. These matrices can have very different structure than the CKM and PMNS matrices. The mirror quark and leptons can have masses in the range 500-1500 GeV and could be discovered at the LHC. As we will see their impact on FCNC processes can be sometimes spectacular. Reviews on flavour physics in the LHT model can be found in [36, 37].

c) Extra Dimensions

Here the obvious signatures are the lightest Kaluza-Klein particles, the excited sisters and brothers of the SM particles. When KK-parity is present, like in models with universal extra dimensions, then also a dark matter candidate is present. In models with warped extra dimensions and protective custodial symmetries to avoid problems with electroweak precision tests (EWPT) and FCNC processes, the gauge group is generally larger than the SM gauge group and similarly to the LHT model new heavy gauge bosons are present. However, even in models with custodial symmetries these gauge bosons must be sufficiently heavy ($2-3\,\text{TeV}$) in order to be consistent with EWPT.

The models with a warped extra dimension (RS models) address also the issue of the hierarchies of fermion masses and mixings [24, 22]which in this framework implies FCNC transitions at the tree level [38, 25, 26, 39]. The so-called RS-GIM mechanism [25, 26] combined with additional custodial protection of flavour violating Z couplings allows yet to achieve the agreement with existing data without a considerable fine tuning of parameters [40, 41]. Most recent reviews on the latter work that have been presented at this conference can be found in [42, 43].

1.6. Little Hierarchy Problem

As we have seen, stabilization of the Higgs mass under radiative corrections requires NP at scales $\mathcal{O}(1\,\text{TeV})$. Yet EWPT performed first at LEP/SLC and extended at Tevatron imply that NP, unless properly screened, can only appear at scales 5-10 TeV or higher. The situation is much worse in FCNC processes. There the masses of new particles carrying flavour and having $\mathcal{O}(1)$ couplings cannot contribute at tree level unless their masses are larger than 1000 TeV or even more.

Thus in order to keep the solutions to the Higgs mass discussed above alive, protective symmetries must be present in order to suppress NP effects to EWPO and FCNC processes in spite of NP being present at scales $\mathcal{O}(1\,\text{TeV})$ or lower. In this context the custodial SU(2) symmetry in the case of EWPT should be mentioned. In the framework of the LHT model this symmetry is quarantied by the T-parity. For the FCNC processes we need generally a GIM mechanism which forbids tree level contributions. If this mechanism is violated and FCNC transitions occur already at tree level other protections are necessary. In RS models the RS-GIM mechanism [25, 26] plays an important role and the recently pointed out custodial protection for flavour violating Z couplings [40, 41].

In this context MFV is very popular as models with MFV can naturally satisfy the existing FCNC constraints. While this framework will play a distinguished role below, we will also present the results coming from the non-MFV scenarios discussed above.

1.7. EWPT versus FCNC Processes

Already at this stage it is important to emphasize the different roles of EWPT and Flavour Physics in constraining the extensions of the SM. Indeed, we know already that possible signals of NP in EWPO have to be very small, at most few % or smaller. Experiments at LEP, SLC and Tevatron reached already very high precision and they can be primarly used to bound the parameters of a given extension of the SM.

On the other hand, many observables in Flavour Physics, both in the quark and lepton sector, have not been measured or measured very poorly. This will change soon through improved measurements at Belle and Tevatron but in particular through experiments at LHC and future Super-Belle at KEK, Super Flavour Facility in Rome as well as dedicated K-physics and charm physics experiments. Also experiments related to electric dipole moments will be very important in this enterprise. In other words Flavour Physics only now enters the precision era and moreover spectacular deviations from SM predictions in this sector are clearly still possible if new physics exists at the TeV scale. The interplay of such effects with EWPT and direct searches for NP at Tevatron and LHC will not only be very exciting but simultaneously should tell us which direction to go above the TeV scale.

1.8. The 2026 Vision

In the next two decades we should be able to make a dramatic progress in understanding flavour physics. In particular the following decays could be observed for the first time and in certain cases their branching ratios could be precisely measured. These are:

- The mixing induced CP asymmetry $S_{\psi\phi}$ in B_s decays, the rare decays $B_{s,d} \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0 l^+ l^-$ and $B \to K^*\nu\bar{\nu}$ of which only $K^+ \to \pi^+\nu\bar{\nu}$ has been observed to date. A tiny direct CP violation in $B \to X_s\gamma$ could be found much larger than expected. Also CP violation in charm decays could be discovered. If we are lucky also $K_L \to \mu e$ could be seen.
- Lepton Flavour Violating (LFV) processes like $\mu \to e\gamma$, $\tau \to \mu\gamma$ and the decays to three leptons like $\mu \to eee$ could be discovered and a non-vanishing CP phase in the lepton sector measured.
- Various electron dipole moments could be measured for the first time or their upper bounds improved by several orders of magnitude. Also the issue of the observed $(g-2)_\mu$ anomaly is likely to be clarified well before 2026.

Additional observables, that already now put severe constraints on the SM and its extensions, will be much more precisely measured and the corresponding hadronic uncertainties reduced through improved lattice calculations. These are:

- ε_K, ΔM_s and ΔM_s, for which excellent data exist but theoretical uncertainties amount to roughly 20%.
- improved measurements of $B \to X_s\gamma$ and $B \to X_s l^+ l^-$ that already played an important role in constraining various extensions of the SM. However more accurate data and improved theory would still be very helpful.

- the measurements of CP averaged symmetries and CP asymmetries resulting from angular distributions in $B_d \to K^*(\to K\pi)\mu^+\mu^-$ with the former being generalizations of the well known forward-backward asymmetry in the decays $B \to X_s l^+ l^-$ and $B \to K^* l^+ l^-$.
- the improved measurement of $B^+ \to \tau^+ \nu$ and improved tests of universality in tree level decays $K^+ \to l^+ \nu$ and $B^+ \to l^+ \nu$ that could possibly signal LFV effects in meson decays.

Thus it is to be expected that in 2026 we will have many new numbers to play with, with the hope to identify the correct model. Not all these numbers will be equally important. Moreover from my point of view global fits may only give a global view of the situation, possibly hiding some interesting new effects that take place only locally. With this in mind let me outline my goals for the rest of this writing.

2. Intermezzo: Our Goals for the Next Pages

The first goal for this writing is to summarize in simple terms the present theoretical framework for weak decays and to propose a classification of different extensions of the SM with respect to their flavour structure by means of a 2×2 matrix to be called *Flavour Matrix* in what follows. Having this classification at hand it will be straightforward to see the differences between CMFV, general MFV, LHT, the MSSM with new flavour violating interactions (GMSSM) and RS models. Each of them can be placed in a particular entry of the flavour matrix. In this context we will also count the number of new flavour parameters of the extensions listed above and conclude that without the study of numerous correlations between various observables, that would reduce the number of parameters, it will be difficult to rule out the non-MFV scenarios in questions and to distinguish between them.

However, the main goal of this paper is a collection of 20 goals for the LHC era, which if reached should give us a very deep insight into flavour dynamics at very short distance scales.

We should strongly emphasize that we do not intend here a comprehensive review of Flavour Physics. Such a review, written by a hundred of flavour experts has appeared one year ago [44, 45] and moreover, extensive studies of the physics at future flavour machines and other visions can be found in [46, 47]. We would rather like to paint a picture of Flavour Physics in general terms and collect various strategies for the exploration of this fascinating field that hopefully will turn out to be useful in the coming years. In this context we will recall present puzzles in Flavour Physics that could turn out to be the first hints of NP and on various occasions we will present the predictions of the scenarios mentioned above. Last but certainly not least let me cite two excellent text books on CP violation and flavour physics [48, 49], where many fundamentals of this field are clearly explained and other extensions of the SM and other observables are discussed in detail.

3. Theoretical Framework

3.1. Preliminaries

The starting point of any serious analysis of weak decays in the framework of a given extension of the SM is the basic Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM}(g_i, m_i, V_{\rm CKM}^{ij}) + \mathcal{L}_{\rm NP}(g_i^{\rm NP}, m_i^{\rm NP}, V_{\rm NP}^{ij}), \tag{2}$$

where $(g_i, m_i, V_{\rm CKM}^{ij})$ denote the parameters of the SM and $(g_i^{\rm NP}, m_i^{\rm NP}, V_{\rm NP}^{ij}) \equiv \varrho_{\rm NP}$, the additional parameters in a given NP scenario.

Our main goal then is to identify in weak decays the effects decribed by $\mathcal{L}_{\rm NP}$ in the presence of the background from $\mathcal{L}_{\rm SM}$. In the first step one derives the Feynman rules following from (2), which allows to calculate Feynman diagrams. But then we have to face two challenges:

- our theory is formulated in terms of quarks, but experiments involve their bound states: K_L, $K^\pm$, B_d^0, B_s^0, $B^\pm$, etc,

- NP takes place at very short distance scales $10^{-19} - 10^{-18}$ m, while K_L, $K^\pm$, B_d^0, B_s^0, $B^\pm$ live at $10^{-16} - 10^{-15}$ m.

The solution to these challenges is well known. One has to construct an effective theory relevant for experiments at low energy scales. Operator Product Expansion (OPE) and Renormalization Group (RG) methods are involved here. They allow to separate the perturbative short distance (SD) effects, where NP is present, from long distance (LD) effects for which non-perturbative methods are necessary. Moreover RG methods allow an efficient summation of large $\log(\mu_{\rm SD}/\mu_{\rm LD})$. A detailed exposition of these techniques can be found in [50, 51] and fortunately we do not have to repeated them here. At the end of the day the formal expressions involving matrix elements of local operators and their Wilson coefficients can be cast into the following *Master Formula for Weak Decays* [52].

3.2. Master Formula for Weak Decays

The master formula in question reads:

$$\mathrm{A(Decay)} = \sum_i B_i \eta^i_{\rm QCD} V^i_{\rm CKM} F_i(m_t, \varrho_{\rm NP}), \tag{3}$$

where B_i are non-perturbative parameters representing hadronic matrix elements of the contributing operators, $\eta_i^{\rm QCD}$ stand symbolically for the renormalization group factors, $V^i_{\rm CKM}$ denote the relevant combination of the elements of the CKM matrix and finally $F_i(m_t, \varrho_{\rm NP})$ denote the loop functions resulting in most models from box and penguin diagrams but in some models can also represent tree level diagrams if such diagrams contribute. $\varrho_{\rm NP}$ denotes symbolically all parameters beyond m_t, in particular the set $(g_i^{\rm NP}, m_i^{\rm NP}, V_{\rm NP}^{ij})$. It turns out to be useful to factor out $V^i_{\rm CKM}$ in all contributions in order to see transparently the deviations from MFV.

In the SM only a particular set of parameters B_i is non-vanishing, the functions F_i are *real* and the only flavour and CP violations enters through the CKM factors $V^i_{\rm CKM}$.

In the CMFV models the only modification relative to the SM are new values of the functions F_i that in addition to the SM contributions receive contribution from new particle exchanges introducing new parameters $\varrho_{\rm NP}$. These functions are *real* and *flavour independent* so that the CKM matrix still governs all flavour and CP-violating transitions.

In more general MFV models, new parameters B_i, corresponding to new operators, can contribute. However, each term in the sum in (3) has still the property of terms in the CMFV models so that even in the presence of new operators the CKM matrix still governs all flavour and CP-violating transitions.

In the simplest non-MFV models, the basic operator structure of CMFV models remains but the functions F_i in addition to real SM contributions can contain new flavour parameters and new complex phases. Consequently the CKM matrix ceases to be the only source of flavour and CP violation.

Finally, in most general non-MFV models, new operators (new B_i parameters) contribute and the functions F_i in addition to real SM contributions can contain new flavour parameters and new complex phases.

Obviously this classification of different classes of models corresponds to a 2 × 2 matrix but before presenting this matrix let us briefly discuss the essential ingredients in our master formula.

Clearly without a good knowledge of non-perturbative factors B_i, no precision studies of Flavour Physics will be possible, unless the non-perturbative uncertainties can be reduced or even removed by taking suitable ratios of observables. In certain rare cases it is also possible to measure the relevant hadronic matrix elements entering rare decays by using leading tree level decays. Examples of such fortunate situations are certain mixing induced CP asymmetries and the branching ratios for $K \to \pi\nu\bar{\nu}$ decays. Yet, in many cases one has to face the direct evaluation of B_i. While lattice, QCD-sum rules, Light-cone sum rules and large-N methods made significant progress in the last 20 years, the situation is clearly not satisfactory and one should hope that new advances in the calculations of B_i parameters will be made in the LHC era, in order to adequately use improved data. Recently an impressive progress in calculating the parameter $\hat{B}_K$, relevant for CP violation in $K^0 - \bar{K}^0$ mixing has been made and we will discuss its implications later on.

An important progress has also been made in organizing the dominant contributions in non-leptonic two body B meson decays and decays like $B \to V\gamma$ with the help of QCD factorization approach, SCET and Perturbative QCD approach.

Concerning the factors $\eta_i^{\rm QCD}$, an impressive progress has been made during the last 18 years. The 1990's can be considered as the era of NLO QCD calculations. Basically the NLO corrections to all relevant decays and transitions have been calculated already in the last decade [50], with a few exceptions, like the width differences $\Delta\Gamma_{s,d}$ in the $B^0_{s,d} - \bar{B}^0_{s,d}$ systems that were completed only in 2002 [53]. This decade can be considered as the era of NNLO calculations. In particular one should mention here the NNLO calculations of QCD corrections to $B \to X_s l^+ l^-$ [54, 55, 56, 57, 58], $K^+ \to \pi^+\nu\bar{\nu}$ [59, 60], and in particular to $B_s \to X_s\gamma$ [61] with the latter one being by far the most difficult one. Also important steps towards a complete calculation of NNLO corrections to non-leptonic decays of mesons have been made in [62].

The final ingredients of our master formula, in addition to $V^i_{\rm CKM}$ factors, are the loop functions F_i resulting from penguin and box diagrams with top quark, $W^\pm$, Z^0, heavy new gauge boson and heavy new fermion and scalar exchanges. They are known at one-loop level in several extensions of the SM, in particular in the two Higgs doublet model (2HDM), littlest Higgs model without T parity (LH), the ACD model with one universal extra dimension (UED), MSSM with MFV and non-MFV violating interactions, FBMSSM, LHT model, Z'-models, RS models, left-right symmetric models, model with the fourth sequential generation of quarks and leptons. Moreover in the SM $\mathcal{O}(\alpha_s)$ corrections to all relevant one loop functions are known. It should also be mentioned that in the loop functions in our master formulae one can conveniently absorb tree level FCNC contributions present in particular in RS models.

3.3. The Flavour Matrix

The preceding discussion suggests to exhibit different extensions of the SM in a form of a 2×2 matrix shown in Fig.1. Let us briefly describe the four entries in this matrix.

The element (1,1) or the class A represents the models with CMFV [9, 10, 11]. The SM, the versions of Two-Higgs doublet models (2HDM) with low $\tan\beta$, the Littlest Higgs Model without T-parity (LH) and the ACD model [63] with a universal fifth flat extra dimension belong to this class. We will see below that this class of models does not allow for large deviations from the SM predictions.

The elements (1,1) and (1,2) or classes A and B taken together, the upper row of the flavour matrix, represent the class of models with MFV at large. A nice formulation of this approach using spurion technology and flavour symmetries is given in [14]. Basically the new effect in the (1,2) entry relative to (1,1) alone is the appearance of new operators with different Dirac structures that are strongly suppressed in the CMFV framework but can be enhanced if $\tan\beta$ is large or equivalently if Y_d cannot be neglected. 2HDM with large $\tan\beta$ belongs to this class. Until recently it was believed that the MSSM only with large $\tan\beta$ corresponds to the entry (1,2) but the analysis in [64] has shown that even at low $\tan\beta$ Y_d cannot be neglected when the parameter μ in the Higgs sector is large and gluino contributions become important. We will see below that the presence of new operators, in particular scalar operators, allows to lift the helicity suppression of certain rare decays like $B_s \to \mu^+\mu^-$, resulting in very different predictions than found in CMFV models.

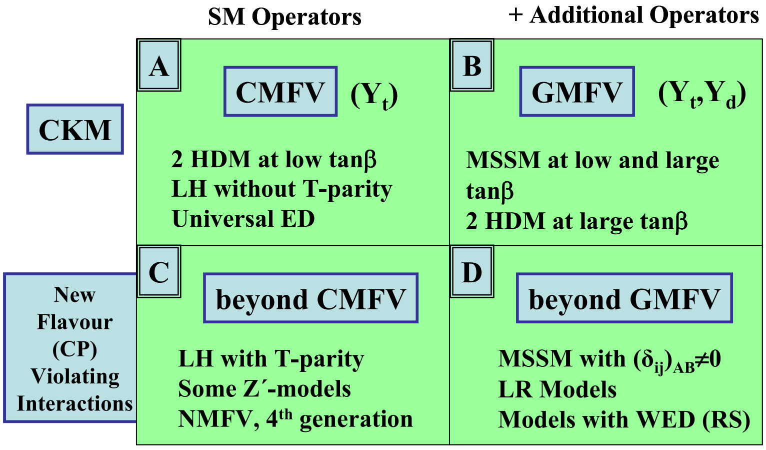

Figure 1. The Flavour Matrix

The FBMSSM scenario carrying new complex phases that are flavour conserving represents a very special class of MFV models in which the functions F_i become complex quantities in contrast to what we stated previously but as these new phases are flavour conserving a natural place for FBMSSM is the upper row of the flavour matrix.

A very interesting class of models is the one represented by the entry (2,1) or the class C. Relatively to CMFV it contains new flavour violating interactions, in particular new complex phases, forecasting novel CP-violating effects that may significantly differ from those present in the CMFV class. As there are no new operators relatively to the SM ones, no new B_i-factors and consequently no new non-perturbative uncertainties relative to CMFV models are present. Therefore predictions of models belonging to (2,1) entry suffer generally from smaller non-perturbative uncertainties than models represented by the second column in the flavour matrix in Fig. 1.

When discussing the (2,1) models, it is important to distinguish between models in which new physics couples dominantly to the third generation of quarks, basically the top quark, and models where there is a new sector of fermions that can communicate with the SM fermions with the help of new gauge interactions. Phenomenological approaches with enhanced Z-penguins [65, 66, 67], NMFV approach of [68] and usual Z'-models [69] belong to the first subclass of (2,1), while the LHT model represents the second subclass.

Finally there is the most complicated class of models represented by the entry (2,2) or the class D in which not only new flavour violating effects but also new operators are relevant. The MSSM with flavour violation coming from the squark sector and RS models are likely to be the most prominent members of this class of models. In the MSSM a useful parametrization of the new effects is given by δ_{ij}^{AB} with $i, j = 1, 2, 3$ and $A, B = L, R$ in the context of the so-called mass insertion approach [70, 71]. In the RS models FCNC transitions take place already at tree level and the pattern of flavour violation in these models generally differs from the LHT model and MSSM [41]. Left-right symmetric models belong also to this class. Spurion technology for this class of models has been developed by Feldmann and Mannel [72].

3.4. The Role of Correlations between Observables

The models with non-MFV interactions introduce many new parameters. As an example we list in Table 2 the number of additional real and complex parameters in the quark sector. In

Table 2. Additional physical parameters in the quark sector of various NP scenarios compared with the number of quark parameters in the SM.

Model	Real Parameters	Phases
GMSSM	36	27
FBMSSM	6	1
LHT	7	3
RS	18	9
SM	9	1

the case of GMSSM and RS models the number of new parameters is truely spectacular, while it is more modest in the case of FMSSM and LHT. Clearly in view of so many parameters the distinction between various scenarios will only be possible through the analysis of very many observables and correlations between them.

Table 3. Examples of the several classes of correlations.

Class	Correlated decays/observables		
1	$K_L \to \pi^0\nu\bar{\nu}$	$\longleftrightarrow$	$K^+ \to \pi^+\nu\bar{\nu}$
	$K_L \to \pi^0\nu\bar{\nu}$	$\longleftrightarrow$	$B \to X_{s,d}\nu\bar{\nu}$
	$B \to X_s\nu\bar{\nu}$	$\longleftrightarrow$	$B \to X_d\nu\bar{\nu}$
2	$K_L \to \pi^0\mu^+\mu^-$	$\longleftrightarrow$	$K_L \to \pi^0 e^+e^-$
	$K_L \to \mu^+\mu^-$	$\longleftrightarrow$	$B_s \to \mu^+\mu^-$
	$B_s \to \mu^+\mu^-$	$\longleftrightarrow$	$B_d \to \mu^+\mu^-$
3	$K_L \to \pi^0\nu\bar{\nu}$	$\longleftrightarrow$	$K_L \to \pi^0\mu^+\mu^-\ (e^+e^-)$
	$K^+ \to \pi^+\nu\bar{\nu}$	$\longleftrightarrow$	$K_L \to \mu^+\mu^-$
	$K_L \to \pi^0\nu\bar{\nu}$	$\longleftrightarrow$	$B_s \to \mu^+\mu^-$
	$B \to X_s\nu\bar{\nu}$	$\longleftrightarrow$	$B_s \to \mu^+\mu^-$
4	$B_{s,d} \to \mu^+\mu^-$	$\longleftrightarrow$	$\Delta M_{s,d}$
	$K \to \pi\nu\bar{\nu}$	$\longleftrightarrow$	$S_{\psi K_S}$
5	$S_{\psi\phi}$	$\longleftrightarrow$	A^s_{SL}

Now, in CMFV models very stringent correlations between various observables exist [9, 10, 11]. Indeed, after the CKM elements have been factored out, the rest of the decay amplitudes show universal (flavour blind) behaviour so that relations between various branching ratios in the K system, between various branching ratios in the B system and even between branching ratios in K and B systems exist. We collect these correlations in Table 3. Details behind these classification can be found in [41]. In models with non-MFV interactions these correlations are generally violated due to the non-universality of NP effects in K, B and D decays. In FMSSM interesting correlations between CP violation in flavour violating and CP violation in flavour conserving processes exist. We will return to them later on.

4. 20 Goals in Flavour Physics for the Coming Years

4.1. Strategy

As already advertised in our 2026 vision in Section 1.8 the coming two decades will bring new numbers for various branching ratios and other observables which will allow to reduce the number of candidates for the best extension of the SM and in any case will allow the reduction of the parameter space in a given model. In view of this it is important to develop a strategy for an efficient test of various NP scenarios. In principle, one could proceed, as done often in the literature, with a global fit of the parameters of a given scenario. Many analyses of such an approach can be found these days in the literature. While such analyses have certain virtues, I think that it is more efficient to select a number of observables that are:

- Sensitive to NP contributions.
- Theoretically under control.
- Vanishing or strongly suppressed in the SM.
- Allowing clear cut conclusions about a given extension of the SM.

In this spirit I will present my personal 20 goals in flavour physics for the next two decades, which when reached will give us a very deep insight in flavour physics at very short distance scales. I would like to stress that the order in which these goals will be listed does not correspond to any priority list, although some of these goals are more important than the others. This will be clear from my presentation. Moreover, the importance of each goal will likely change with time.

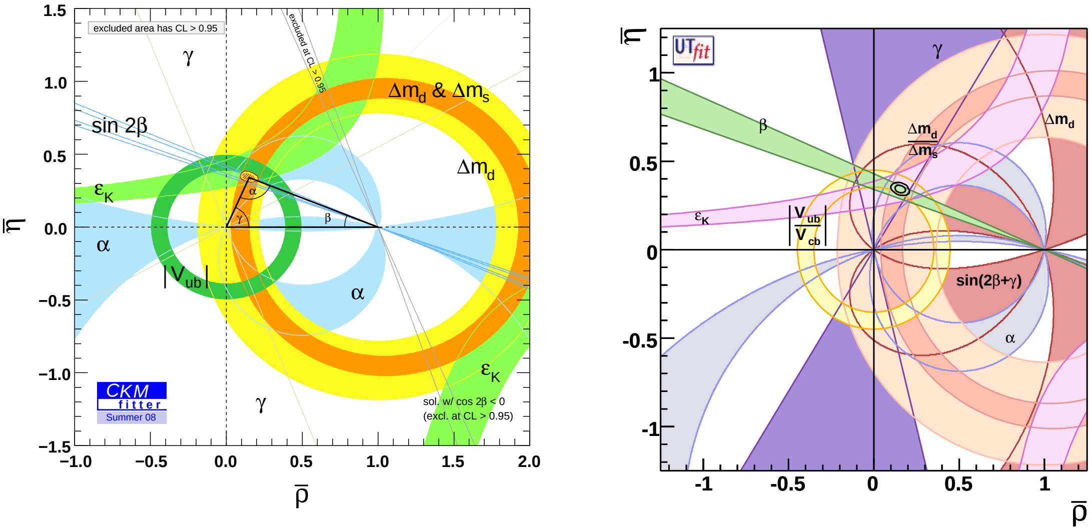

Figure 2. Unitarity triangle fits by CKMfit (left) and UTfit (right) collaborations in 2008.

4.2. A Quick Look at the Status of the CKM Matrix

The success of the CKM description of flavour violation and in particular CP violation can be best seen by looking at the so-called Unitarity Triangle (UT) in Fig.2. The extensive analyses of UTfit and CKMfit collaborations [73, 74] show that the data on $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$, ε_K, ΔM_d, ΔM_s and the CP-asymmetry $S_{\psi K_S}$, that measures the angle β in the UT, are compatible with each other within theoretical and experimental uncertainties.

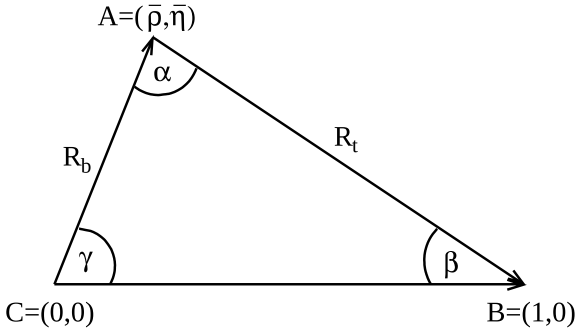

Figure 3. Unitarity Triangle

Moreover the angles α and γ of the UT determined by means of various non-leptonic decays and sophisticated strategies are compatible with the ones extracted from Fig. 2.

While this agreement is at first sight impressive and many things could already have turned out to be wrong, but they did not, one should remember that only very few theoretically clean observables have been measured precisely sofar. Basically only three parameters relevant for the CKM matrix have been measured accurately:

$$|V_{us}| = 0.2255 \pm 0.0010, \qquad |V_{cb}| = (41.2 \pm 1.1) \cdot 10^{-3}, \qquad \beta = \beta_{\psi K_S} = (21.5 \pm 1.0)^\circ, \tag{4}$$

and as we will stress later on the precision on $|V_{cb}|$ is not yet fully sufficient in order to make precision test in rare K decays and also to calculate precisely the CP-violating parameter ε_K. Moreover, β measured through the asymmetry $S_{\psi K_S}$ is the true β only in models with CMF and MFV and if scenarios (2,1) and (2,2) in the flavour matrix are at work the measured $\beta_{\psi K_S}$ is really

$$\beta_{\psi K_S} = \beta_{\rm true} + \phi_d^{\rm new}, \tag{5}$$

where $\phi_d^{\rm new}$ is a new phase coming for instance from the mirror quark sector in the LHT model or the squark mass matrix in the GMSSM.

The remaining quantities used in the UTfits: ΔM_d, ΔM_d, ε_K, $|V_{ub}|$ suffer from hadronic uncertainties and moreover except for the last one could be polluted by new physics contributions.

One should also emphasize that until now only $\Delta F = 2$ FCNC processes could be used in the UTfits. The measured $B \to X_s\gamma$ and $B \to X_s l^+l^-$ decays and their exclusive counterparts are sensitive to $|V_{ts}|$ that has nothing to do with the plot in Fig. 2. This underlines the importance of the measurements of the branching ratios for rare $\Delta F = 1$ FCNC processes such as $K^+ \to \pi^+\nu\bar\nu$, $K_L \to \pi^0\nu\bar\nu$, $K_L \to \pi^0 l^+l^-$, $B_s \to \mu^+\mu^-$, $B_d \to \mu^+\mu^-$ and $B \to K^*\nu\bar\nu$. We will return to them later on. As it will still take some time before the branching ratios for these strongly suppressed decays will be known with high accuracy, we have to improve the status of the observables that are already now available to us or will be available in a few years from now. This brings me to the first goal of our program.

4.3. Goal 1: CKM Matrix from Tree Level Decays

Precise measurements of the side R_b and of the angle γ in the UT of Fig. 3 by means of tree level decays, that are independent of any new physics to a good approximation, are undisputably very important.

This would allow to determine the CKM matrix without NP pollution, with the four fundamental flavour parameters being

$$|V_{us}|, \qquad |V_{ub}|, \qquad |V_{cb}|, \qquad \gamma \tag{6}$$

and to construct the reference unitarity triangle (RUT) [75] by means of ($\lambda = |V_{us}|$)

$$R_b = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|, \tag{7}$$

and γ.

This is indeed a very important goal as it would give us immediately the true values of R_t and β in Fig. 3 by simply using

$$R_t = \sqrt{1 + R_b^2 - 2R_b \cos\gamma}, \qquad \cot\beta = \frac{1 - R_b\cos\gamma}{R_b \sin\gamma}. \tag{8}$$

Table 4. The "true" and CMFV values of four observables discussed in the text.

Model	γ	R_b	$\sin 2\beta$	R_t
TRUE	$(80 \pm 20)^\circ$	0.40 ± 0.04	0.718 ± 0.072	1.01 ± 0.01
CMFV	$(63 \pm 4)^\circ$	0.36 ± 0.02	0.671 ± 0.024	0.90 ± 0.03

It is clearly very tempting to use already now the present "true" values

$$\gamma_{\rm true} = (80 \pm 20)^\circ, \qquad (R_b)_{\rm true} = 0.40 \pm 0.04 \tag{9}$$

to find by means of (8) the true values $(R_t)_{\rm true}$ and $\sin 2\beta_{true}$ in Table 4.

On the other hand within CMFV one has universally [11]

$$(R_t)_{\rm CMFV} \approx 0.90 \ \left[\frac{\xi}{1.21}\right] \sqrt{\frac{17.8/{\rm ps}}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.507/{\rm ps}}}, \qquad (\sin 2\beta)_{\rm CMFV} = (\sin 2\beta)_{\psi K_S}, \tag{10}$$

where

$$\xi = \frac{\sqrt{\hat B_{B_s}} F_{B_s}}{\sqrt{\hat B_{B_d}} F_{B_d}} = 1.21 \pm 0.04. \tag{11}$$

Using then

$$R_b = \sqrt{1 + R_t^2 - 2R_t \cos\beta}, \qquad \cot\gamma = \frac{1 - R_t \cos\beta}{R_t \sin\beta}, \tag{12}$$

one can find $(R_b)_{\rm CMFV}$ and $\gamma_{\rm CMFV}$ given in Table 4. A detailed presentation of this exercice can be found in [11]. Other useful trigonometric relations between sides and angles of the UT have been collected in [76].

Looking at Table 4 we observe slight differences between the true values in the first row and the CMFV values in the second row of this table, but these differences are not yet significant. Yet the pattern of differences between the values in the first and second raw in Table 4 indicates that a new *negative* phase $\phi_d^{\rm new} = -(2 \pm 3)^\circ$ would improve the situation as now one could obtain

$$S_{\psi K_{\rm S}} = \sin(2\beta_{\rm true} + 2\phi^{\rm new}) = 0.671 \pm 0.024\ . \tag{13}$$

It is evident from this exercise that improved determinations of $|V_{ub}|$ and γ would help to find out whether CMFV and even MFV at large is in trouble or not. As a preparation for a

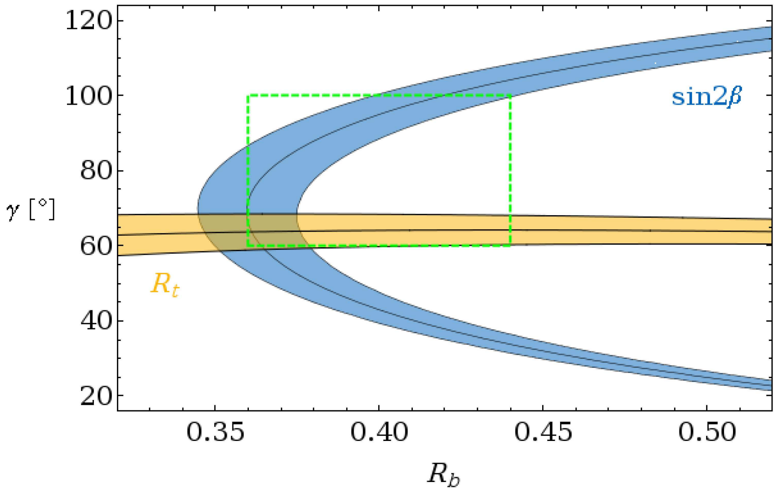

Figure 4. Various constraints in the $R_b - \gamma$ plane as explained in the text.

future determination of these two important quantities we show in Fig. 4 the full situation in the $R_b - \gamma$ plane [77], where the *blue* region is the region allowed in the framework of MFV. It is determined by the measured value of $(\sin 2\beta)_{\psi K_S}$. The *yellow* region corresponds to $(R_t)_{\rm CMFV}$ of (10) and the overlap of *blue* and *yellow* regions to CMFV represented by the values of $(R_b)_{\rm CMFV}$ and $\gamma_{\rm CMFV}$ in Table 4. Finally the white square represents the pure tree level determination of (9).

It is clear from this plot that once the determination of γ and R_b will be improved and consequently the white square gets smaller, we will be able to see whether already this very limited set of measurements implies the necessity for going beyond CMFV or even beyond MFV. Needless to say, even if CMFV and MFV would survive this test, it is by no means guarantied that they would survive the tests in $\Delta F = 1$ processes like rare K and B decays that will be considered in other goals on our list. Still this discussion clearly demonstrates the importance of our first goal and the related second goal to which we will now turn our attention.

4.4. Goal 2: Improved Lattice Calculations

To clarify the situation with respect to possible hints for new physics in the $(\bar\varrho, \bar\eta)$ plane [78, 79], it is essential that the determination of R_t by means of ΔM_d and ΔM_s is improved not only through more accurate values of ξ but also through more accurate values of $F_{B_s}\sqrt{\hat B_{B_s}}$ and $F_{B_d}\sqrt{\hat B_{B_d}}$ for which the present lattice values read [80]

$$F_{B_s}\sqrt{\hat B_{B_s}} = 270(30)\,{\rm MeV}, \qquad F_{B_d}\sqrt{\hat B_{B_d}} = 225(25)\,{\rm MeV}. \tag{14}$$

This would allow to find out whether $\Delta M_{d,s}$ in the SM agree with the data or not. Presently there is an agreement but the theoretical uncertainty of roughly 20% does not allow any definite conclusions. There is a hope that in the coming decade these uncertainties could be reduced down to 5%. In this context the lower bound on $\Delta M_{d,s}$ from CMFV [81] should be mentioned. This bound implies that $\Delta M_{d,s}$ is very likely to be enhanced in the CMFV models relative to the SM values.

4.5. Goal 3: Precise Prediction for ε_K

Until the discovery of CP violation in B_d system, the CP-violating parameter ε_K played the crucial role in the test of CP violation, but after the precise measurements of $\sin 2\beta$ and of the ratio $\Delta M_d/\Delta M_s$ its role in the CKM fits declined because of the large error in the parameter $\hat{B}_K$. Also for this reason the size of CP violation in K and B systems are commonly declared to be compatible with each other within the SM. This situation could change soon for the following reasons:

- Improved lattice calculations of $\hat{B}_K$. In particular a recent simulation with dynamical fermions results in $\hat{B}_K = 0.72 \pm 0.04$ [82, 83] that should be improved in the coming months,
- Inclusion of additional corrections to $\varepsilon_{\rm K}$ [84] that were usually neglected in the literature in view of the 20% error on $\hat{B}_{\rm K}$. As this parameter is now much better known it is mandatory to include them. Effectively these new corrections can be summarized by an overall factor in $\varepsilon_{\rm K}$: $\kappa_\varepsilon = 0.92 \pm 0.02$ [84].
- As pointed out in [84] the decrease of $\hat{B}_{\rm K}$ relative to previous lattice results together with κ_ε being significantly below unity implies within the SM a tension between very precisely measured values of $S_{\psi K_{\rm S}}$ and $\varepsilon_{\rm K}$.

Indeed $S_{\psi K_{\rm S}} = \sin 2\beta = 0.671 \pm 0.024$ implies within the SM the value of $|\varepsilon_{\rm K}|$ that is typically 20% below its very precise experimental value [84, 85]

$$\frac{|\varepsilon_K|_{\rm SM}}{|\varepsilon_K|_{\rm exp}} = 0.80 \pm 0.11 \tag{15}$$

If confirmed by more precise values of $\hat{B}_K$ and more precise values of CKM parameters, in particular $|V_{cb}|$, which enters roughly as $|V_{cb}|^4$ in $\varepsilon_{\rm K}$, this could signal new physics in $\varepsilon_{\rm K}$.

But this is not the only possibility. Alternatively, no new physics in $\varepsilon_{\rm K}$ would imply $\sin 2\beta = 0.88 \pm 0.11$ [86, 84]. This could only be made consistent with the measured value of $S_{\psi K_{\rm S}}$ by introducing a new phase $\phi_{\rm new}$ in $B_d^0 - \bar{B}_d^0$ mixing so that using (13) with $\phi_{\rm new} \approx -9^\circ$ an agreement with the measured value of $S_{\psi K_{\rm S}}$ could be obtained. Other possibilities are discussed in [84]. Personally I would bet on significant new physics in $\varepsilon_{\rm K}$ or new physics in both $\varepsilon_{\rm K}$ and $B_d^0 - \bar{B}_d^0$ mixing as $\sin 2\beta = 0.88 \pm 0.11$ would imply $|V_{ub}|$ that is significantly larger than the one obtained from tree level decays.

Finally one should emphasize that presently ε_K is the only FCNC process from the K system that enters the UTfits and decreasing the error in $\hat{B}_K$ below 5% would also be very important for the UTfits. As such the $\sin 2\beta - \varepsilon_K$ consistency test is presently the only available test of CP violation across different generations.

4.6. Goal 4: CP Violation in the $\boldsymbol{B_s}$ *System*

The tiny complex phase of the element V_{ts} in the CKM matrix precludes any sizable CP violating effects in the decays of the B_s mesons within the SM and models with MFV. In particular very clean mixing induced asymmetry $S_{\psi\phi}$ is predicted to be

$$(S_{\psi\phi})_{\rm SM} = \sin(2|\beta_s|) \approx 0.04, \tag{16}$$

with $-\beta_s$ being the phase of V_{ts}. In the presence of new physics (16) is modified as follows [11]

$$S_{\psi\phi} = \sin(2|\beta_s| - 2\phi_s^{\rm new}), \tag{17}$$

where $\phi_s^{\rm new}$ is a new phase in the $B_s^0 - \bar{B}_s^0$ mixing.

Already in 2006 Lenz and Nierste [87], analyzing D0 and CDF data pointed out some hints for a large phase $\phi_s^{\rm new}$. In 2008 new hints appeared, emphasized in particular by the UTfit collaboration [88]. However, the most recent messages from CDF and D0 [89] imply a 2.7σ

deviation from the SM prediction and we have to wait for higher statistics in order to conclude that NP is at work here [90]. As the central value of the measured $S_{\psi\phi}$ is around 0.4, that is one order of magnitude larger than the SM value, the confirmation of this high value in the future would be a spectacular confirmation of non-MFV interactions at work. As demonstrated recently such large values can easily be found in a RS model [40] and the same comment applies to the GMSSM. The most likely values for $S_{\psi\phi}$ in the LHT do not exceed 0.2 [91] and finding this asymmetry as high as 0.4 would be in favour of RS and GMSSM.

Clearly a sizable $S_{\psi\phi}$ is not the only manifestation of CP violation in the B_s system but presently it is the most prominent one as it can be measured accurately at LHCb and is theoretically rather clean. The last point has been recently discussed in detail in [92], where proposals have been made towards the reduction of possible hadronic uncertainties in the extraction of $S_{\psi\phi}$ from future data.

4.7. Goal 5: Resolution of the $(\sin 2\beta)_{\phi K_S}$ *Puzzle*

The angle β has been measured in several other decays, in particular in penguin dominated decays like $B \to \phi K_S$ or $B \to \eta' K_S$ with the result that it is generally smaller than $(\sin 2\beta)_{\psi K_S}$ putting the SM and MFV in some difficulties. Clarification of this disagreement is an important goal for the LHC era. While this tension became weaker with time, the theoretically clean asymmetry $S_{\phi K_S}$ still remains to be significantly smaller than the expected value of approximately 0.67:

$$S_{\phi K_S} = 0.44 \pm 0.17. \tag{18}$$

This tension cannot be resolved at LHCb and its resolution will remain as one of the important goals for Super Belle at KEK and later Super-B machine in Rome. We will see later that the desire to explain the value in (18) in the framework of FBMSSM will have interesting implications for other CP violating observables like the direct CP asymmetry in $B \to X_s\gamma$ and electric dipole moments.

4.8. Goal 6: Non-Leptonic Two Body B Decays

The best information on CP violation in the B system to date comes from two body non-leptonic decays of B_d and $B^{\pm}$. While until now these decays dominated this field, LHCb will extend these studies in an important manner to B_s and B_c decays. This is clearly a challenging field not only for experimentalist but in particular for theorists due to potential hadronic uncertainties. Yet, in the last ten years an impressive progress has been made in measuring many channels, in particular $B \to \pi\pi$ and $B \to \pi K$ decays, and developing a number of methods like QCD Factorization, Perturbative QCD approach, SCET and more phenomenological approaches based on flavour symmetries. Excellent reviews of this subject have been given recently by Buchalla [93], Fleischer [94] and Silvestrini [95]. They contain a lot of useful material. I think this field will continue to be important for the search of NP and the tests of the CKM parameters in view of very many channels whose branching ratios should be measured in the next decade with a high precision. This is also a place where the structure of QCD effects in the interplay with weak interactions can be studied very well and the correlations of the lessons gained from this field with those coming from theoretically cleaner decays discussed subsequently will undoubtly enrich our view on flavour physics. On the other hand in view of potential hadronic uncertainties present in these decays this field in my opinion will not provide definite answers about NP if the latter contributes only at the level of 10% or less to the relevant branching ratios and CP asymmetries.

4.9. Goal 7: Improved Measurements of Radiative Decays

The radiative decays $B \to X_s\gamma$ and $B \to X_s l^+l^-$ and their exclusive counterparts $B \to K^*\gamma$ and $B \to K^* l^+ l^-$ played already a very important role in constraining various scenarios of NP. Somewhat disappointingly the agreement between the measurements of inclusive decays and the SM predictions is very good.

Beginning with $B \to X_s\gamma$, the recent calculations of NNLO corrections to this decay give [61]

$$Br(B \to X_s\gamma) = (3.15 \pm 0.23) \cdot 10^{-4}, \tag{19}$$

which is lower than quoted several years ago. If in addition some non- perturbative contributions are included the reference [96] finds $Br(B \to X_s\gamma) = (2.98 \pm 0.26) \cdot 10^{-4}$ which is 1.4σ lower than the combined measurement from BaBar and Belle

$$Br(B \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.10 \pm 0.03) \cdot 10^{-4}. \tag{20}$$

While the result in [96] remains controversial, finding the SM value significantly below the data would have interesting consequences as we will mention below.

A big achievement of theorists and experimentalists is the reduction of the total uncertainties in the measurement and the SM prediction for this branching ratio below 10%. This is unique in the field of FCNC processes at present, as in all other processes either theory or experiments are not accurate enough. As such, $B \to X_s\gamma$ puts already now severe constraints on NP. With the SM value being slightly below experiment, models that provide enhancements of $Br(B \to X_s\gamma)$ relative to the SM value are clearly favoured and those which can only provide a suppression are on the way to being ruled out. An improved measurement of this branching ratio at the level of 5% combined with a better understanding of non-perturbative uncertainties and the reduction of the parametric uncertainties in (19) would clearly result in a very powerful test of the SM and its extensions.

The situation with the inclusive decay $B \to X_s l^+ l^-$ is not as impressive but still the agreement between the theory and data is good. A review can be found in [97]. On the experimental side we have

$$Br(B \to X_s l^+ l^-)_{\rm exp} = \begin{cases} (1.6 \pm 0.5) \cdot 10^{-6} & (\text{low } q^2), \\ (4.4 \pm 1.3) \cdot 10^{-6} & (\text{high } q^2), \end{cases}$$

whereas the SM results for these two ranges of q^2 at the NNLO level read [98]

$$Br(B \to X_s l^+ l^-)_{\rm SM} = \begin{cases} (1.6 \pm 0.1) \cdot 10^{-6} & (\text{low } q^2), \\ (2.3 \pm 0.8) \cdot 10^{-6} & (\text{high } q^2). \end{cases}$$

While these agreements of the SM with the data are from the point of view of the search for NP rather disappointing, the hopes still exist that something new will be seen in the forward-backward asymmetry for which the value of q^2 at which it crosses zero can be very precisely predicted: $\hat{s}_0 = 3.50 \pm 0.12\,\text{GeV}^2$. In models with MFV the value of $\hat{s}_0$ increases linearly with $\sqrt{Br(B \to X_s\gamma)}$. This correlation, pointed out in [99] in the context of a ACD model [63] has been verified in the MSSM with MFV at low $\tan\beta$. Interestingly, in the FBMSSM where CP-violating but flavour conserving phase are added to the MSSM, sizable violation of this correlation is found [100].

Other hopes are conntected with the direct CP asymmetry in $B \to X_s\gamma$, which in the SM is predicted to be very small

$$A_{CP}(B \to X_s\gamma)_{SM} = 0.004 \pm 0.002, \tag{21}$$

while the data still allow for one order of magnitude enhancements:

$$A_{CP}(B \to X_s\gamma)_{EXP} = 0.004 \pm 0.036. \tag{22}$$

In this context a correlation between the asymmetry $S_{\phi K_S}$ discussed previously and $A_{CP}(B \to X_s\gamma)$ has been found in the framework of FMSSM [32]. The desire to explain the suppression of $S_{\phi K_S}$ with respect to $S_{\psi K_S}$ implies an order of magnitude enhancement of $A_{CP}(B \to X_s\gamma)$ over the SM value. Moreover, the sign of this asymmetry is uniquely predicted in this model to be positive and finding it to be negative in future data would bring this scenario in difficulties.

4.10. Goal 8: A Goldmine of Observables: $\boldsymbol{B \to K^* l^+ l^-}$

In the difficult times at financial markets a goldmine is a very useful thing to have. Such a goldmine is provided by the exclusive decay $B \to K^*(\to K\pi)l^+l^-$ which will be studied in detail at LHCb. Indeed various CP averaged symmetries and CP asymmetries resulting from angular distributions offer 24 observables which will provide an impressive amount of experimental numbers that will help to distinguish between various NP scenarios. Model independent analyses of [101, 102] have been recently generalized in [100], where also specific models like MFV, the MSSM with MFV, the FBMSSM, the LHT and the GMSSM have been analyzed. Moreover a number of correlations have been identified. Several of the identified CP averaged observables can be considered as generalizations of the well known forward-backward asymmetries in $B \to X_s l^+l^-$ and $B \to K^* l^+ l^-$. The pattern of the zeros in these CP averaged observables characteristic for a given model should be useful in identifying the correct model or at least bound severly its parameters. One of the important result of these studies is that new CP-violating phases will produce clean signals in CP-violating asymmetries. Clearly, it will be very exciting to monitor the upcoming LHC, Belle upgrade and eventually Super-B factory in this and the next decade to see whether the angular observables discussed in [101, 102, 100] will give a hint for any of the extensions of the SM analyzed there and NP in general.

4.11. Goal 9: Measurement of $Br(B_{s,d} \to \mu^+\mu^-)$

One of the main targets of flavour physics in the coming years will be the measurement of the branching ratio for the highly suppressed decay $B_s \to \mu^+\mu^-$. Hopefully also the even more suppressed decay $B_d \to \mu^+\mu^-$ will be discovered as well. These two decays are helicty suppressed in the SM and CMFV models. Their branching ratios are proportional to the squares of the corresponding weak decay constants that suffer still from sizable uncertainties as discussed in the context of the goal 2. However using simultaneously SM expressions for very well measured mass differences $\Delta M_{s,d}$ this uncertainty can be eliminated [103] leaving as the only TH uncertainty in $Br(B_{s,d} \to \mu^+\mu^-)$ the uncertainties in the hadronic parameters $\hat{B}_{B_s}$ and $\hat{B}_{B_d}$ that are already known quite well from lattice calculations [80] and enter the branching ratios linearly. Therefore the SM predictions are rather precise

$$Br(B_s \to \mu^+\mu^-) = (3.6 \pm 0.3) \cdot 10^{-9}, \qquad Br(B_d \to \mu^+\mu^-) = (1.1 \pm 0.1) \cdot 10^{-10}. \tag{23}$$

Comparing these predictions with the 95% C.L. upper limits from CDF and D0

$$Br(B_s \to \mu^+\mu^-) \leq 6 \cdot 10^{-8}, \qquad Br(B_d \to \mu^+\mu^-) \leq 2 \cdot 10^{-8}, \tag{24}$$

it is clear that a lot of room is still left for NP contributions.

Large contributions to the branching ratios in question can come from neutral scalar exchanges in which case new scalar operators are generated and the helicity suppression is lifted. Thus large enhancements of $B_{s,d} \to \mu^+\mu^-$ are only possible in the models placed in the entries (1,2) and (2,2) of the flavour matrix in Fig. 1. The prime example is the MSSM at large $\tan\beta$, where still in 2002 $Br(B_s \to \mu^+\mu^-)$ could be as large as 10^{-6}. The impressive progress by CDF and D0 collaborations, the decrease of the corresponding upper bound by two orders of magnitude, totally excluded this possibility but there is still a hope that a clear signal of NP will be seen in these decays.

In the MSSM with MFV and large $\tan\beta$ there is a strong correlation between $Br(B_{s,d} \to \mu^+\mu^-)$ and ΔM_s [104, 105, 106, 107] implying that the enhancement of these branching ratios with respect to the SM is correlated with the suppression of ΔM_s below the SM value. In fact the MSSM with MFV was basically the only model that "predicted" the suppression of ΔM_s below the SM prediction that seemed to be the case just after the discovery of $B_s^0 - \bar{B}_s^0$ mixing. Meanwhile the lattice values for weak decay constants changed and there is no suppression within theoretical uncertainties observed in the data. With the decrease of the experimental upper bound on $Br(B_{s,d} \to \mu^+\mu^-)$ the predicted suppression amounts to at most 10% and it will require considerable reduction of the lattice uncertainties in the evaluation of ΔM_s in the SM before the correlation in question can be verified or falsified by experiment. In the MSSM with non-MFV interactions the correlation discussed here is absent.

Now, irrespectively of large uncertainties in the separate SM predictions for $B_{s,d} \to \mu^+\mu^-$ and $\Delta M_{s,d}$, there exists a rather precise relation between these observables that can be considered as one of the TH cleanest predictions of CMFV. This *golden relation* reads [103]

$$\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)} = \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}\, r\,, \tag{25}$$

with $r = 1$ in CMFV models but generally different from unity. For instance in the LHT model one finds $0.3 \leq r \leq 1.6$ [108], while in the RS model with custodial protection $0.6 \leq r \leq 1.3$ [41].

It should be stressed that the ratio $\hat{B}_{B_d}/\hat{B}_{B_s} = 1.00 \pm 0.03$ [80] constitutes the only TH uncertainty in (25). The remaining entries can be obtained directly from experimental data. The right hand side is already known rather precisely: 33.7 ± 1.3, but it will still take some time before the left hand side will be known with comparable precision unless NP enhances both branching ratios by an order of magnitude.

4.12. Goal 10: Precise measurement of $\boldsymbol{Br(B^+ \to \tau^+\nu)}$

The next goal on our list is the not so rare decay $B^+ \to \tau^+\nu$. Its branching ratio has been measured by Belle and BaBar collaborations with the result

$$Br(B^+ \to \tau^+\nu) = (1.4 \pm 0.4) \cdot 10^{-4}. \tag{26}$$

In the SM this decay proceeds through a tree diagram with a $W^\pm$ exchange with the prediction

$$Br(B^+ \to \tau^+\nu)_{\rm SM} = (0.95 \pm 0.20) \cdot 10^{-4}. \tag{27}$$

This branching ratio is proportional to $F_B^2 |V_{ub}|^2$. The first uncertainty can be significantly reduced by using the experimental value of ΔM_s. The uncertainty due to V_{ub} is more serious but using the best known value one ends with the result in (27).

The agreement of the SM with the data is good but the large experimental error and significant TH preclude any clear cut conclusions at present. Interestingly the result in (27) can be significantly modified by the tree level exchange of a light charged Higgs. As addressed long time ago by Hou [109] and in modern times calculated first by Akeroyd and Recksiegel [110], and later by Isidori and Paradisi [111], one has in the MSSM with MFV and large $\tan\beta$

$$\frac{Br(B^+ \to \tau^+\nu)_{\rm MSSM}}{Br(B^+ \to \tau^+\nu)_{\rm SM}} = \left[1 - \frac{m_B^2}{m_{H^\pm}^2} \frac{\tan^2\beta}{1 + \epsilon \tan\beta}\right]^2, \tag{28}$$

with ϵ collecting the dependence on supersymmetric parameters. This means that in the MSSM this decay can be strongly suppressed unless the choice of model parameters is such that the second term in the parenthesis is larger than 2, which is not very likely when all constraints, in particular from $B \to X_s\gamma$ and $B_s \to \mu^+\mu^-$ are taken into account. Improvement in the experimental data for the branching ratio in question to be expected from Belle in coming years and later from its upgrade and from Super B in the next decade, as well as improvements on $|V_{ub}|$, are required to have a clear picture. Recent summaries of the $H^\pm$ physics can be found in [112, 113].

Having already branching ratios for $B \to X_s\gamma$, $B \to X_s l^+l^-$, $B_{s,d} \to \mu^+\mu^-$ and $B^+ \to \tau^+\nu_\tau$ at hand one can ask how powerful they are when considered simultaneously. Many analyses of this type can be found in the literature. Here I would like to mention only the analysis of a very interesting $SO(10)$-GUT model of Dermisek and Raby [114] which gives a successful description of quark and lepton masses, of the PMNS matrix and of all elements of the CKM matrix except possibly of $|V_{ub}|$ that is found to be $3.2\cdot 10^{-3}$, definitely a bit too low. Yet as shown in [115], this model fails to describe simultaneously the data on the rare decays in question with supersymmetric particles in the reach of the LHC. This is mainly due to the $\tan\beta = 50$ required in this model. It can be shown that this is a problem of most GUTs with Yukawa unification [116]. Possible solutions to this problem have been suggested in that paper. This discussion demonstrates that flavour physics can have a significant impact not only for physics at the LHC scales but also indirectly for much shorter scales connected with GUTs.

4.13. Goal 11: Tests of $\boldsymbol{\mu - e}$ *and* $\boldsymbol{\mu - \tau}$ *Universalties*

Lepton flavour violation (LFV) and the related breakdown of universality can be tested in meson decays by studying the ratios [117, 118]

$$R_{\mu e} = \frac{Br(K^+ \to \mu^+\nu)}{Br(K^+ \to e^+\nu)}, \qquad R_{\mu\tau} = \frac{Br(B^+ \to \mu^+\nu)}{Br(B+ \to \tau^+\nu)}, \tag{29}$$

where it is understood that summation over different neutrino flavours is made. In the first case the theory is very precise, a 0.1% uncertainty. $R_{\mu e}$ with a precision of 0.5% should be soon available from CERN and will constitute an important test of $\mu - e$ universality. The ratio $R_{\mu\tau}$ is even more sensitive to NP contributions but it will still take some time before it will be known with good precision.

4.14. Goal 12: Rare Kaon Decays and $\boldsymbol{b \to s\nu\bar\nu}$ *Transitions*

Undoubtly among the top highlights of flavour physics in the next decade will be the measurements of the branching ratios of two *golden modes* $K^+ \to \pi^+\nu\bar\nu$ and $K_L \to \pi^0\nu\bar\nu$. $K^+ \to \pi^+\nu\bar\nu$ is CP conserving while $K_L \to \pi^0\nu\bar\nu$ is governed by direct CP violation. Both decays are dominated in the SM and its many extensions by Z penguin contributions. It is well known that these decays are theoretically very clean and are known in the SM including NNLO QCD corrections [60]. Reviews of these two decays can be found in [119, 120]. The most recent predictions read [7]

$$Br(K^+ \to \pi^+\nu\bar\nu)_{\rm SM} = (8.5 \pm 0.7)\cdot 10^{-11}, \tag{30}$$

$$Br(K_L \to \pi^0\nu\bar\nu)_{\rm SM} = (2.8 \pm 0.6)\cdot 10^{-11}, \tag{31}$$

where the errors are dominated by parametrical uncertainties, in particular by the CKM parameters and in the case of $K^+ \to \pi^+\nu\bar\nu$ by the value of the charm quark mass.

Once measured these decays will provide a very clean determination of the angle β in the UT as some parametric uncertainties, in particular the value of $|V_{cb}|$, cancel out in this determination [121]. This implies another *golden relation* of the MFV [121, 122]

$$(\sin 2\beta)_{S_{\psi K_S}} = (\sin 2\beta)_{K_L \to \pi^0\nu\bar\nu} \tag{32}$$

which can be strongly violated in models with new flavour and CP-violating interactions, such as the LHT model [108] and RS models analyzed in [41].

In the LHT and RS models $Br(K_L \to \pi^0\nu\bar{\nu})$ can be enhanced by a factor of 5 and the $Br(K^+ \to \pi^+\nu\bar{\nu})$ by a factor of 2-3 which is sufficient to reach the central experimental value [8]

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = (17.3^{+11.5}_{-10.5}) \cdot 10^{-11}\,. \tag{33}$$

The experimental upper bound on $Br(K_L \to \pi^0\nu\bar{\nu})$ is still by more than two orders of magnitude above the SM value in (31) but the present upper bound from E391a at KEK [123] of $Br(K_L \to \pi^0\nu\bar{\nu}) \leq 6.7 \cdot 10^{-8}$ should be significantly improved in the coming years with the hope to observe the first events in the first half of the next decade.

The rare decays $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$ are dominated by CP-violating contributions [124, 125, 126, 127, 128]. The dominant indirect CP-violating contributions are practically determined by the measured decays $K_S \to \pi^0\ell^+\ell^-$ and the parameter ε_K. Consequently these decays are not as sensitive as $K_L \to \pi^0\nu\bar{\nu}$ to NP contributions that are present here only in subleading direct CP-violation. Yet in models like the LHT model [108] and the RS scenario analyzed in [41], which contain new sources of CP violation, enhancements of the branching ratios by a factor of $1.5 - 2.0$ over the SM values [127]

$$Br(K_L \to \pi^0 e^+e^-)_{\rm SM} = 3.54^{+0.98}_{-0.85}\left(1.56^{+0.62}_{-0.49}\right) \cdot 10^{-11}\,, \tag{34}$$

$$Br(K_L \to \pi^0 \mu^+\mu^-)_{\rm SM} = 1.41^{+0.28}_{-0.26}\left(0.95^{+0.22}_{-0.21}\right) \cdot 10^{-11} \tag{35}$$

can be found. The values in parentheses correspond to the destructive interference between direct and indirect CP violating contributions.

There is a linear correlation between $Br(K_L \to \pi^0 e^+e^-)$ and $Br(K_L \to \pi^0\mu^+\mu^-)$ that has been first investigated in [125, 126, 127]. Such a correlation is common to all models with no scalar operators contributing to the decays in question [125, 126, 127] and has been found explictely in the RS and LHT models [108, 41].

The present experimental bounds

$$Br(K_L \to \pi^0 e^+e^-) < 28 \cdot 10^{-11} \quad [129]\,, \qquad Br(K_L \to \pi^0 \mu^+\mu^-) < 38 \cdot 10^{-11} \quad [130] \tag{36}$$

are still by one order of magnitude larger than the SM predictions.

Also the B decays with $\nu\bar{\nu}$ in the final state provide a very good test of modified Z penguin contributions [131, 132], but their measurements appear even harder than of the rare K decays just discussed. A recent analysis of these decays within the SM and several NP scenarios can be found in [133].

4.15. Goal 13: Calculations of $\boldsymbol{B_6}$ and $\boldsymbol{B_8}$ in $\boldsymbol{\varepsilon'/\varepsilon}$

One of the important actors of the 1990s in flavour physics was the ratio ε'/ε that measures the size of the direct CP violation in $K_L \to \pi\pi$ relative to the indirect one described by ε_K. In the SM ε' is governed by QCD penguins but receives also an important distructive contribution from electroweak penguins that is generally much more sensitive to NP than the QCD penguin contribution.

Here the problem is the strong cancellation of QCD penguin contributions and electroweak penguin contributions to ε'/ε and in order to obtain useful predictions the precision on the corresponding hadronic parameters B_6 and B_8 should be at least 10%. Such a precision seems to be unrealistic for the coming five years from the present perspective. This is really a pitty, as the calculations of short distance contributions to this ratio (Wilson coefficients of QCD and electroweak penguin operators) are very advanced and the present experimental value from NA48 [134] and KTeV [135]

$$\varepsilon'/\varepsilon = (16.7 \pm 1.6) \cdot 10^{-4} \tag{37}$$

could have a considerable impact on possible enhancements of rare K decays if the relevant hadronic matrix elements were under control. Consequently an important goal for lattice groups is a precise calculation of B_6 and B_8. Reviews of ε'/ε can be found in [136, 137].

4.16. Goal 14: CP Violation in Charm Decays

Charm decays have been for many years shadowed by the successes of K decays and B decays, although a number of experimental groups and selected theorists made considerable efforts to study them. This is due to the GIM mechanism being very effective in suppressing the FCNC transitions in this sector, long distance contributions plugging the evaluation of the ΔM_D and insensitivity to top physics in the loops. However, large $D^0 - \bar{D}^0$ mixing discovered in 2007 [138, 139, 140] and good prospects for the study of CP violation in these decays at Super Belle and Super B gave a new impetus to this field. The main targets here are:

- Dedicated studies of CP Violation in D decays that is predicted to be very small in the SM and is TH much cleaner that the ΔM_D.
- Dedicated studies of $D^+ \to \mu^+\nu_\mu$, $D^+ \to \tau^+\nu_\tau$ and $D_s \to \tau^+\nu_\tau$ with higher experimental and lattice accuracy with the aim to study charge Higgs effects.

Excellent reviews can be found in [141, 142]. Various other aspects are discussed in [143, 144, 145, 146, 147].

4.17. Goal 15: Search for CP Violation in the Lepton Sector and $\boldsymbol{\theta_{13}}$

Clearly the discovery of CP violation in the lepton sector would be a very important mile stone in particle physics for many reasons. In particular the most efficients explanations of BAU these days follow from leptogenesis. While in the past the necessary size of CP violation was obtained from new sources of CP violation at very high see-saw scales, the inclusion of flavour effects, in particular in the resonant leptogenesis, gave hopes for the explanation of BAU using only the phases in the PMNS matrix. This implies certain conditions on the parameters of this matrix, that is the relevant Dirac phase, two Majorana phases and in analogy to the element V_{ub} in the CKM matrix, the angle θ_{13} in the PMNS matrix. As I am am not an expert in this field I have nothing profound to add here beyond what is already known in the literature. Most recent review can be found in [148].

4.18. Goal: 16 Lepton Flavour Violation (LFV)

The non-vanishing neutrino masses and neutrino oscillations as well as see-saw mechanism for the generation of the neutrino masses gave an impressive impetus to the study of flavour violation in the lepton sector in the last ten years. In the SM with right-handed Dirac neutrinos, the smallness of neutrino masses implies tiny branching ratios for LFV processes. For instance

$$Br(\mu \to e\gamma)_{\rm SM} \approx 10^{-54}, \tag{38}$$

more than 40 orders of magnitude below the 90% C.L. upper bound from the MEGA Collaboration

$$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}. \tag{39}$$

The prospects for the measurements of LFV processes with much higher sensitivity than presently available in the next decade look very good. In particular the MEG experiment at PSI [149] should be able to test $Br(\mu \to e\gamma)$ at the level of $\mathcal{O}(10^{-13} - 10^{-14})$, and the Super Flavour Factory [46] is planned to reach a sensitivity for $Br(\tau \to \mu\gamma)$ of at least $\mathcal{O}(10^{-9})$. The planned accuracy of SuperKEKB of $\mathcal{O}(10^{-8})$ for $\tau \to \mu\gamma$ is also of great interest. Very important will also be an improved upper bound on $\mu - e$ conversion in Ti. In this context the dedicated J-PARC experiment PRISM/PRIME [150] should reach the sensitivity of $\mathcal{O}(10^{-18})$, i. e. an improvement by six orders of magnitude relative to the present upper bound from SINDRUM II at PSI [151].

Now the GMSSM, the LHT model and the RS models are capable in reaching the bound in (39) and in fact this bound puts already rather stringent constraints on the parameters of these models. For instance in the case of the LHT model the mixing matrix in the mirror lepton sector has to be either very hierarchical, at least as hierarchical as the CKM matrix or the mirror-lepton spectrum has to be quasi-degenerate [152, 153]. Analogous constraints exist in other models.

In order to distinguish various NP scenarios that come close to the bound in (39) it will be essential to study a large set of decays to three leptons in the the final state. Indeed while in the MSSM [154, 155, 156, 157, 158] the dominant role in the decays with three leptons in the final state and in $\mu - e$ conversion in nuclei is played by the dipole operator, in [152, 153] it was found that this operator is much less relevant in the LHT model, with Z^0 penguin and box diagrams being the dominant contributions. This implies a striking difference of various ratios of branching ratios $Br(l_i \to 3l_j)$ over $Br(l_i \to l_j\gamma)$ in the MSSM, where they are typically $\mathcal{O}(10^{-2} - 10^{-3})$ and in the LHT model, where they are $\mathcal{O}(1)$ [152].

There exist also interesting correlations between leptogenesis and lepton flavour violation but this is beyond the scope of this presentation. Additional correlations relevant for lepton flavour violation exist as we will see in our next goal.

4.19. Goal 17: Clarification of the $(g-2)_\mu$ Anomaly

The anomalous magnetic moment of the muon, even if flavour and CP conserving played in this decade an important role in the tests of the extensions of the SM. The SM prediction reads [159]

$$a_\mu^{\rm SM} = 11659\ 1785\ (51) \cdot 10^{-11} \tag{40}$$

and the experimental value from BNL [160]

$$a_\mu^{\rm exp} = 11659\ 2080\ (63) \cdot 10^{-11}. \tag{41}$$

This is a 3.6σ discrepancy and one of the most prominent puzzles in particle physics in this decade.

MSSM with large $\tan\beta$ and scalars with masses below 400 GeV is capable to reproduce the experimental value of a_μ provided the μ parameter in the Higgs Lagrangian has a specific sign, positive in my conventions. Moreover an interesting correlation between the amount of necessary shift Δa_μ and the value of $Br(\tau \to \mu\gamma)$ and $Br(\mu \to e\gamma)$ exists [161], so that within the MSSM at large $\tan\beta$ these two branching ratios could be as high as $4 \cdot 10^{-9}$ and $3 \cdot 10^{-12}$, respectively and in the reach of dedicated experiments in the coming years.

On the other hand the LHT fails to reproduce the data in (41) and a_μ in this model is within the uncertainties indistiguishable from its SM value [152]. Apparently there is no visible correlation between NP in a_μ and LFV in this model.

Thus if the data in (41) remain, they would favour MSSM over LHT. Unfortunately $a_\mu^{\rm SM}$ contains an amount of hadronic uncertainies that can be removed to some extent with the help of e^+e^- data. The recent data from BaBar differ significantly from those used to obtain the value in (40) so that with these new data the anomaly in question decreases down to 0.9σ. The clarification of this modification is an important goal for the coming years.

4.20. Goal 18: Electric Dipole Moments

So far CP violation has only been observed in flavour violating processes. In the SM CP violation in flavour conserving processes is very strongly suppressed as best expressed by the SM values of electric dipole moments of neutron and electron that amount to [162]

$$d_n \approx 10^{-32}\ \text{e cm.} \qquad d_e \approx 10^{-38}\ \text{e cm.} \tag{42}$$

This should be compared with the present experimental bounds [163, 164]

$$d_n \le 2.9 \cdot 10^{-26}\ \text{e cm.} \qquad d_e \le 1.7 \cdot 10^{-27}\ \text{e cm.} \tag{43}$$

They should be improved in the coming years by 1-2 orders of magnitude.

In the last two decades several NP models have been ruled out or put into difficulties through steady, albeit slow, improvements in the upper bounds on electric dipole moments. The MSSM is still alive in this respect. In particular as recently shown within FBMSSM, the desire to explain the measured anomalous value of $S_{\phi K_S}$ in this model implies d_e and d_n in the ballpark of $5 \cdot 10^{-28}$e cm. and $8 \cdot 10^{-28}$e cm., respectively [32]. It will be interesting to see whether this correlation will be seen in the future data.

4.21. Goal 19: Flavour Violation at High Energy

Our presentation deals mainly with tests of flavour and CP violation in low energy processes. However, at the LHC it will be possible to investigate these phenomena also in high energy processes, in particular in top quark decays. Selected recent analyses on flavour physics in high energy processes can be found in [165, 166, 167, 168, 169, 170, 171].

4.22. Goal 20: Construction of a New Standard Model (NSM)

Finally, in view of so many parameters present in basically all extensions of the SM like the MSSM, the LHT and RS models it is unlikely from my point of view that any of the models studied presently in the literature will turn out in 2026 to be the new model of elementary particle physics. On the other hand various structures, concepts and ideas explored these days in the context of specific models may well turn out to be included in the NSM that is predictive, consistent with all the data and giving explanation of observed hierarchies in fermion masses and mixing matrices. While these statements may appear to be very naive, the fact is that the construction of the NSM is the main goal of elementary physics and every theorist has a dream that the future NSM will carry her (his) name.

5. Final Messages and 5 Big Questions

The study of flavour physics in conjuction with direct collider searches for new physics, with electroweak precision tests and cosmological investigations will undoubtly lead one day to a NSM. Whether it will happen in 2026 or only in 2046 it is not clear at present. Afterall 35 years passed since the completion of the present SM and no fully convinvcing candidate for the NSM exists in the literature. On the other hand in view of presently running and oncoming experiments, the next decade could be like 1970's in which practically every year a new important discovery has been made. Even if by 2026 a NSM may not exist yet, it is conceivable that we will be able to answer the following crucial questions by then:

- Are there any fundamental scalars?
- Are there any new fundamental fermions like vector-like fermions or the 4th generation of quarks and leptons?
- Are there any new gauge bosons leading to new forces at very short distance scales and an extended gauge group?
- What are the precise patterns of interactions between the gauge bosons, fermions and scalars with respect to flavour and CP Violation?

- Can the answers to these four questions help us in understanding BAU and other fundamental cosmological questions?

There are of course many other profound questions related to grand unification, gravity and string theory and to other aspects of elementary particle physics and cosmology but from my point of view I would really be happy if in 2026 satisfactory answers to the five questions posed above were available.

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