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Multi-fidelity Optimization of an Acoustic Metamaterial using Model Order Reduction and Machine Learning

Problem Description

Metamaterials are structures that are artificially designed to show properties which cannot be observed in conventional materials. Here, acoustic metamaterials that have a stop band are investigated. This is a frequency range where no waves can propagate freely. A common design strategy to generate such structures is repeating a unit cell infinitely often in 2 dimensions according to a rectangular lattice:

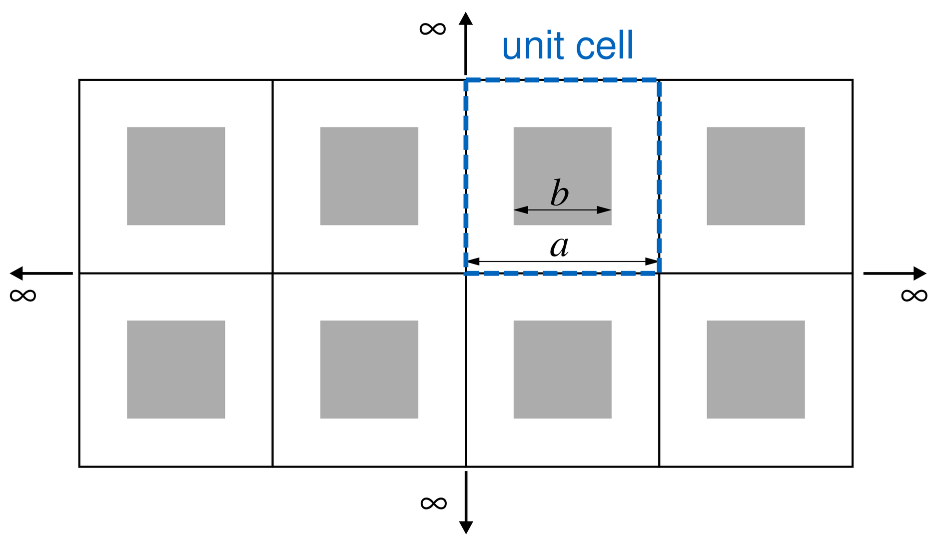

Figure 1: Schematic sketch of a metamaterial generated by periodic repetition of a unit cell.

Due to the periodicity it is sufficent to regard only one unit cell, which is discretized using the Finite Element Method (FEM). According to Bloch's theorem [Bloch 1929] the discretized undamped equation of motion then becomes the so-called dispersion eigenvalue problem with respect to the angular frequency ω and the wavenumbers, which are contained in the wave propagation constants (WPCs) μ_x and μ_y:

$$(\mathbf{K}(\mu_x,\mu_y) - \omega^2 \mathbf{M}(\mu_x,\mu_y))\mathbf{q} = \mathbf{0}, \qquad (1)$$

with $\mathbf{K}$ being the stiffness matrix, $\mathbf{M}$ the mass matrix and $\mathbf{q}$ the vector of degrees of freedom (DOFs). To obtain the stop band (SB), all values along the so-called irreducible Brillouin contour (IBC) shown in the top right corner of Fig. 2 need to be prescribed for the WPCs and equation (1) has to be solved for the frequency. This results in the dispersion curve, where a SB appears as gap in frequency:

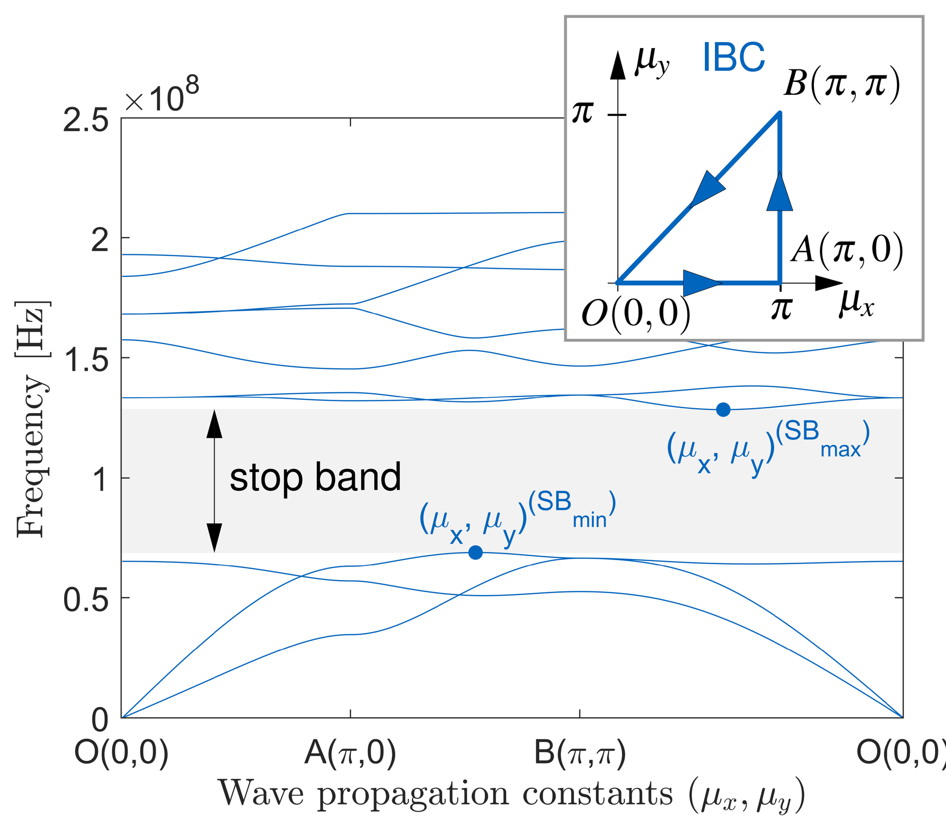

Figure 2: Dispersion curve of investigated metamaterial for $a = 6.7\,\mathrm{mm}$ and $b = 5\,\mathrm{mm}$ and its irreducible Brillouin contour.

Low-fidelity Models

Computing the dispersion curve is a computationally expensive task since an eigenvalue problem has to be solved multiple times. Therefore, three models of different lower fidelity can be derived:

- Reduced Order Model (ROM): Krattiger and Hussein [2018] proposed a projection-based, modal method in which the full order model (FOM) is projected onto the first few eigenmodes of the interior DOFs and the first few eigenmodes of the boundary DOFs.
- Surrogate Modeling: Radial Basis Function (RBF) interpolation with cubic kernel functions is used to predict the width and center of the SB for unsampled parameters.
- Relevant Wave Propagation Constants (RWPC): As can be seen in Fig. 2 the SB can be computed by solving the dispersion eigenvalue problem only twice, namely for $(\mu_x,\mu_y)^{SB,min}$ and for $(\mu_x,\mu_y)^{SB,max}$, which are in the following called the relevant WPCs. To predict these values for unsampled parameter points, the following machine learning model is trained for all four $\mu_\bullet^{(*)}$:

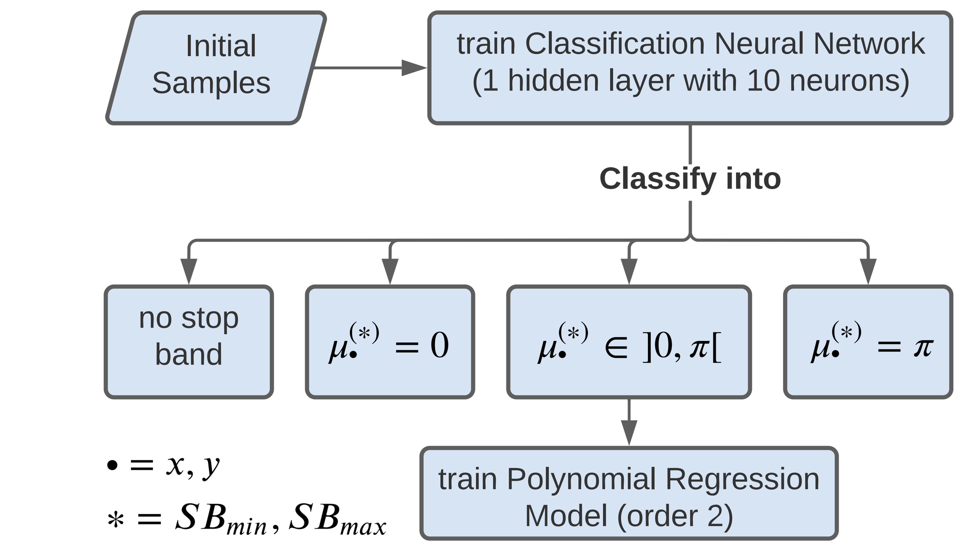

Figure 3: Flow chart of machine learning model for prediction of relevant WPCs.

Multi-fidelity Optimization

The objective of the optimization is to maximize the SB width, while the SB center is constrained to lie in a given frequency interval. As design variables the geometric parameters a and b of the unit cell from Fig. 1 are used with $b \in [2,9]\,\mathrm{mm}$ and $\frac{b}{a} \in [0.2, 0.9]$. The three models of different fidelity are combined in the optimization as follows:

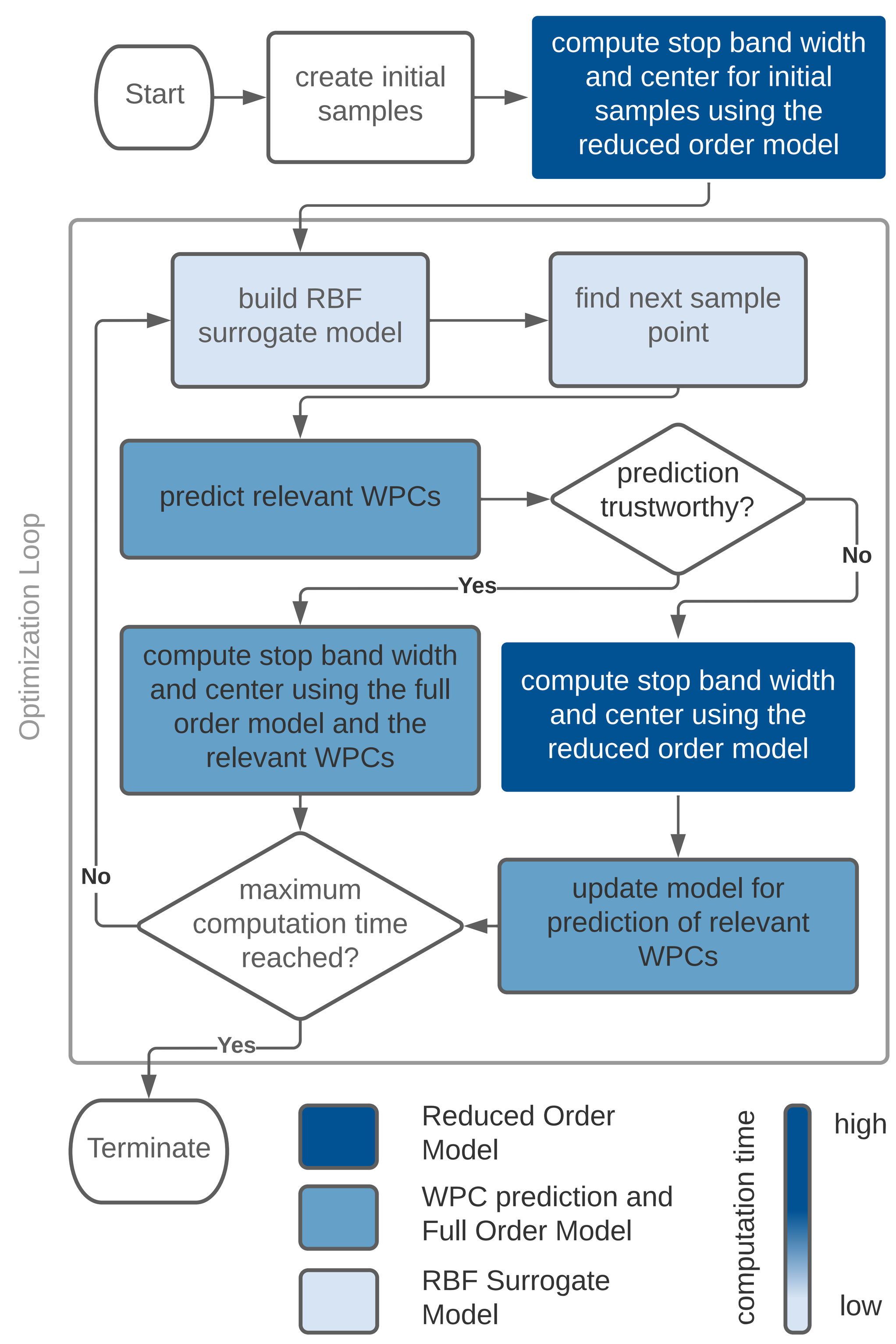

Figure 4: Flow chart of the optimization procedure.

The initial samples are hereby created using Latin Hypercube Sampling (LHS). The method used to find the next sample point is the trust-region based adaptive radial basis function algorithm by Liu et al [2021].

Results

To assess the accuracy of the three models of lower fidelity, 10 samples are generated using LHS as training data. For the test data, $b = 3\,\mathrm{mm}$ is fixed and the fraction $\frac{b}{a}$ is varied linearly from 0.2 to 0.9 in 50 samples. Fig. 5 shows the prediction of the machine learning model for $\mu_y^{(SB_{min})}$. It can be seen that the neural network classifies almost all test samples correctly and the polynomial regression model approximates the true response very well, especially for lower values of $\frac{b}{a}$. Fig. 6 shows the accuracy and the required time for the SB computation of all available models averaged over the 50 test samples in a logarithmic scale. Using the prediction of the relevant WPCs is both more accurate and faster than the ROM.

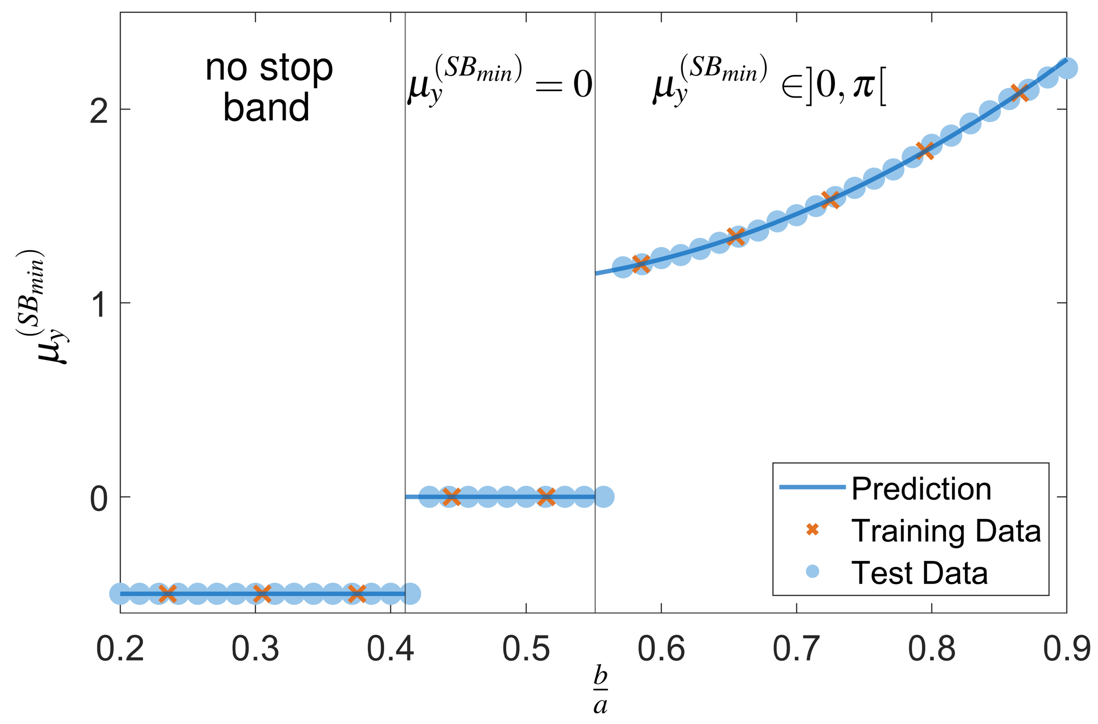

Figure 5: Prediction of $\mu_y^{(SB_{min})}$.

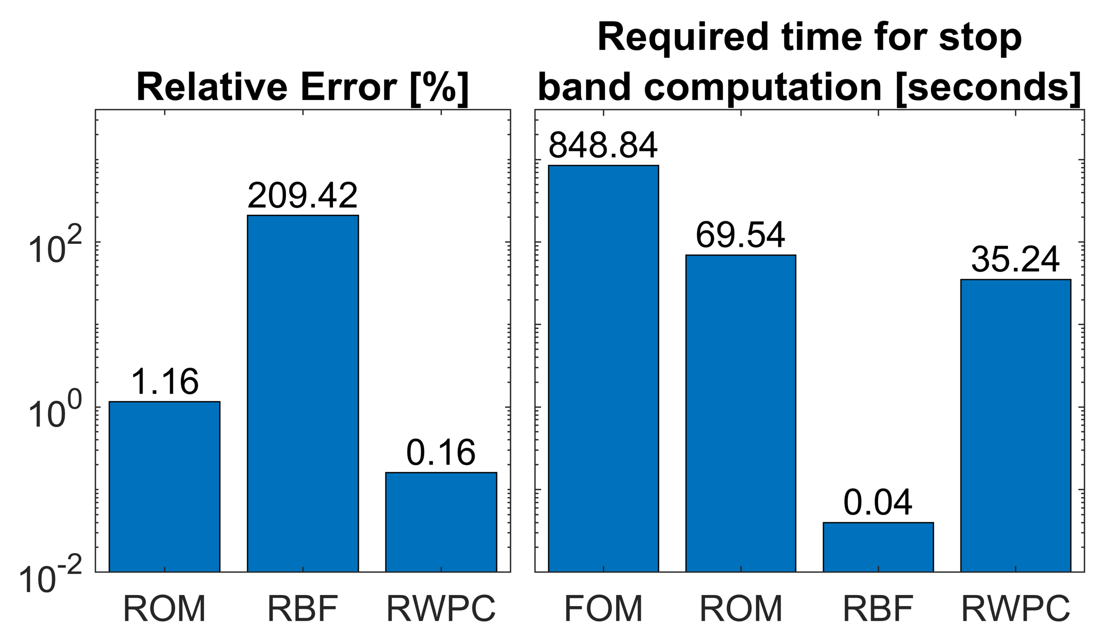

Figure 6: Accuracy and required computation time of the stop band for full order model and the three models of lower fidelity shown in a logarithmic scale.

These results are also reflected in the optimization. When using all three models of lower fidelity, the optimization converges twice as fast compared to when using only the ROM and the Surrogate Model. Additionally, the true objective function value computed with the full order model is higher for the solution found by the optimization with all three models of lower fidelity compared to the solution found when using only the ROM and the Surrogate Model.

Conclusion

A new model of low fidelity to compute the stop band of an acoustic metamaterial is derived and succesfully combined with two other models of low fidelity to accelerate an optimization. In future work, the procedure will be tested on higher-dimensional input spaces.

References

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