# BLIND NOISE AND CHANNEL ESTIMATION

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## ABSTRACT

In the classical methods for blind channel identification (Subspace method, TXK, XBM) [1, 2, 3], the additive noise is assumed to be spatially white or known to within a multiplicative scalar. When the noise is non-white (colored or correlated) but has a known covariance matrix, we can still handle the problem through prewhitening. However, there are no techniques presently available to deal with completely unknown noise fields. It is well known that when the noise covariance matrix is unknown, the channel parameters may be grossly inaccurate. In this paper, we assume the noise spatially correlated, and we apply this assumption for blind channel identification. We estimate the noise covariance matrix without any assumption except its structure which is assumed to be a band-Toeplitz matrix. The performance evaluation of the developed method and its comparison to the modified subspace approach (MSS) [4] are presented.

## 1. INTRODUCTION

One common problem in signal transmission through any channel is the additive noise. In general, additive noise is generated internally by components such as resistors, and solid-state devices used to implement the communication system. This is sometimes called thermal noise or Johnson noise. Other sources of noise and interference may arise externally to the system, such as interference from the other users. When such noise and interference occupy the same frequency band at the desired signal, its effect can be minimized by proper design of the transmitted signal and its demodulator at the receiver. The effects of noise may be minimized by increasing the power in the transmitted signal. However, equipment and other practical constraints limit the power level in the transmitted signal [5].

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The classical model used in communication systems supposes on the one hand that the power of the noise is identical on each sensor, and on the other hand that there is no noise space/time correlation. However, this situation is seldom met, which involve a clear degradation of the performances of the subspace methods. Here, we recall some well-known methods which treat the noise problem in array processing for direction-of-arrival estimation. In fact, in recent years, there has been a growing interest in the problem of techniques with the objective of decreasing the signal to noise ratio resolution threshold or the spatially colored noise [6, 7, 8, 9, 10]. The ambient noise is unknown in practice, therefore modeling or its estimation are necessary. The methods developed for this problem are very few and there are no definitive solution. There are some practical methods; in [11] two methods are obtained by optimization of criterion and by using AR or ARMA modeling of noise. In [7] the spatial correlation matrix of noise is modeled by the known Bessel functions. As in [6] the ambient noise covariance matrix is modeled by a sum of hermitian matrices known up to multiplicative scalar. In [8] this estimate is obtained by measuring the array covariance matrix when no signals are present. This procedure assumes that the noise is not changing in function of time, which is not fulfilled in several domain applications. Another possibility [8] arises when the correlation structure is known to be invariant under a translation or rotation. The so-called differencing covariance technique can be then applied to reduce the noise influence. In this method, two identical translated and/or rotated measurements of the array covariance matrix are required and assumes the invariance of the noise covariance matrix, while the source signals change between the two measurements. The estimate noise covariance matrix is eliminated by a simple subtraction. Furthermore, this method cannot be applied when the source covariance matrix satisfies the same invariance property or when only one measurement is available. In [7] a particular modeling structure noise covariance matrix, which takes into account the characteristic noise relative to its origins, is given. Recently, a maximum posteriori approach (MAP) has been developed in [10]; this method can only be applied in the case of a linear array. In [9], the method called "Instrumental Variable" (IV) is used to reduce the noise without estimated it; this estimator considers that the noise is temporally independent. One technique based to the $MDL$ criterion has been developed in [12] for detection and localization of the signals in the presence of unknown noise; this estimator is asymptotically biased [12]. However, the study of the noise for blind channel identification is very limited. In [4], a modified subspace method (MSS) for blind identification in the presence of unknown correlated noise has been presented, indeed one use some matrices, for a time lag when the noise is absent. The object of this correspondence is to improve the blind channel identification in the presence of a correlated noise by whitening the received data. The noise is assumed spatially correlated. The structure of the paper is as follows. In the section II, we present the studied problem and in section III, we describe the noise covariance matrix model used in this study and its estimation by the proposed algorithm, we apply the noise estimation for blind channel identification using the subspace method. We present, in the section IV, some simulation results and performance comparisons.

## 2. PROBLEM FORMULATION

Consider $L$ FIR channels driven by a common source. The output vector of the $i$th channel can be written as:

$$\mathbf{r}_i(k) = \mathcal{H}^{(i)}\mathbf{s}(k) + \mathbf{n}_i(k), \quad (1)$$

where, $\mathbf{r}_i(k)$ is the output sequence of the $i$th channel, $\mathbf{s}_i(k)$ is the input sequence and $\mathbf{n}_i(k)$ is the noise sequence on the $i$th channel.
$\mathbf{r}_i(k) = [r_i(k) \quad r_i(k+1) \quad ... \quad r_i(k+N-1)]$,
$\mathbf{s}(k) = [s(k-M) \quad s(k-M+1) \quad ... \quad s(k+M-1)]$,
$\mathbf{n}_i(k) = [n_i(k) \quad n_i(k+1) \quad ... \quad n_i(k+N-1)]$.

$$\mathcal{H}^{(i)} = \begin{pmatrix} h_0^{(i)} & h_1^{(i)} & \cdots & h_M^{(i)} & \cdots & \cdots & 0 \\ 0 & h_0^{(i)} & h_1^{(i)} & \cdots & h_M^{(i)} & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & h_0^{(i)} & h_1^{(i)} & \cdots & h_M^{(i)} \end{pmatrix},$$

where, $h_k^{(i)}$ is the impulse response of the $i$th channel, $M$ is the maximum order of the $L$ channels and $N$ is the width of the temporal window. $\mathcal{H}^{(i)}$ is of dimension $(N \times (N+M))$.

Then we have,

$$\mathbf{r}(k) = \mathcal{H}\mathbf{s}(k) + \mathbf{n}(k), \quad (2)$$

$$\begin{pmatrix} \mathbf{r}_1(k) \\ \vdots \\ \mathbf{r}_L(k) \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1 \\ \vdots \\ \mathcal{H}_L \end{pmatrix} \mathbf{s}(k) + \begin{pmatrix} \mathbf{n}_1(k) \\ \vdots \\ \mathbf{n}_L(k) \end{pmatrix}.$$

The matrix $\mathcal{H}$ is known as the $(LN \times (N+M))$ filtering matrix, which has the full rank $(N+M)$ under the following assumptions: the $L$ channels do not share a common zero and $N \geq (M+1)$.
The blind identification problem is to find $\mathcal{H}$ from the sequence,

$$\{\mathbf{r}(k) \quad \text{for} \quad k = 1, 2, ..., K\}.$$

The subspace method [1] exploits the sample covariance matrix of all channel outputs: $\mathbf{\Gamma} = E[\mathbf{r}\mathbf{r}^+]$,
$\mathbf{\Gamma} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{r}(k)\mathbf{r}^+(k)$, where $K$ is the number of samples and $^+$ denotes the conjugate transpose. Assume that the signals and the additive noise are independent, stationary and ergodic zero mean complex valued random processes, and as $K$ becomes large, this matrix has the asymptotical structure: $\mathbf{\Gamma} = \mathcal{H}\mathbf{\Gamma}_s\mathcal{H}^+ + \mathbf{\Gamma}_n$, with $\mathbf{\Gamma}_n = E[\mathbf{n}\mathbf{n}^+]$ the noise covariance matrix and $\mathbf{\Gamma}_s = E[\mathbf{s}\mathbf{s}^+]$ is the signal covariance matrix.
*The goal of blind channel identification and equalization is to identify $\mathcal{H}$ (channel identification) and to estimate $\mathbf{s}(k)$ from $\mathbf{r}(k)$ (channel equalization).*
The subspace blind channel identification procedure [1] consists on the estimation of the $(LN \times 1)$ vector $\mathbf{h}$ of channel coefficients from the observation vector. Indeed, this approach is based on the eigendecomposition of the data covariance matrix,

$$\mathbf{\Gamma} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & \\ & \mathbf{\Lambda}_n \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^+.$$

The subspace method yields an estimate $\widehat{\mathcal{H}}$ of $\mathcal{H}$ by solving the equation: $\mathbf{U}_n^+\widehat{\mathcal{H}} = \mathbf{0}$, in a least square sense (where $\widehat{\mathcal{H}}$ is subject to the same structure as $\mathcal{H}$). This estimate is uniquely (up to a constant scalar) equal to $\mathcal{H}$. From [1], we have:

$$\mathbf{U}_n^+\mathcal{H} = \mathbf{h}^+\mathcal{U}_n = \mathbf{0}, \quad (3)$$

with $\mathcal{U}_n$ is the $(L(M+1) \times (N+M))$ matrix obtained by stacking the $L$ filtering matrices $\mathcal{U}_n^{(l)}$.
$\mathcal{U}_n = \left[\mathcal{U}_n^{(0)T} ... \mathcal{U}_n^{(L-1)T}\right]^T$, where,

$$\mathcal{U}_n^{(l)} = \begin{pmatrix} u_1^{(l)} & u_2^{(l)} & \cdots & u_N^{(l)} & \cdots & \cdots & 0 \\ 0 & u_1^{(l)} & u_2^{(l)} & \cdots & u_N^{(i)} & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & u_1^{(l)} & u_2^{(l)} & \cdots & u_N^{(l)} \end{pmatrix},$$

and $\mathbf{h} = \left[\bar{\mathbf{h}}^{(0)}, ..., \bar{\mathbf{h}}^{(L-1)}\right]$, with $\bar{\mathbf{h}}^{(i)} = \left[h_0^{(i)}, ..., h_M^{(i)}\right]^T$.
The optimization system derived in [1] is:

$$\widehat{\mathbf{h}} = arg \min_{\|\mathbf{h}\|=1} \mathbf{h}^+ \mathcal{U}_{ss} \mathbf{h}, \tag{4}$$

where,

$$\mathcal{U}_{ss} = \sum_{i=1}^{LN-M-N-1} \mathcal{U}_n^{(i)} \mathcal{U}_n^{(i)+}$$

is the filtering noise projection matrix.
The noise is assumed Gaussian, complex and spatially correlated. Its real and imaginary part are supposed independents, Gaussian with, $E[\mathbf{n}_l] = \mathbf{0}$, $E[\mathbf{n}_l \mathbf{n}_l^T] = \mathbf{0}$, and $E[\mathbf{n}_l \mathbf{n}_l^+] = \mathbf{\Gamma}_n$. $\mathbf{\Gamma}_n$ is the noise covariance matrix, the superscripts "*" and "+" denote conjugate and conjugate transpose, respectively. We consider the noise covariance matrix is band, defined by:

$$\mathbf{\Gamma}_n(i,m) = \begin{cases} 0, & for \quad | i-m | > K \\ \rho_i, & for \quad | i-m | < K \quad and\ i \neq m \\ \sigma_i^2, & for \quad i = m \end{cases}$$

Where $\rho_i = \bar{\rho}_i + j\tilde{\rho}_i, i = 1, \ldots, K$, $\rho_i$ are complex variables, $j^2 = -1$, $\sigma^2$ are the noise variance at each receiver, and $K$ is the spatially noise correlation length.

$$\mathbf{\Gamma}_n = \begin{pmatrix} \sigma_1^2 & \rho_{12} & \cdots & \rho_{1K} & \cdots & 0 \\ \rho_{21}^* & \sigma_2^2 & \rho_{23} & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ 0 & \cdots & \rho_{ij}^* & \sigma_i^2 & \cdots & 0 \\ \vdots & \ddots & \cdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \rho_{(LN)K}^* & \cdots & \sigma_{LN}^2 \end{pmatrix}.$$

Two manners to give back observation covariance matrix a noise-free matrix: either by subtraction of the noise covariance matrix, $\mathcal{H}\mathbf{\Gamma}_s\mathcal{H}^+ = \mathbf{\Gamma} - \mathbf{\Gamma}_n$; then we have then a "clean" observation covariance matrix; however, we can obtain a negative matrix if $\mathbf{\Gamma}_n$ is bad-estimated.
Or by whitening; in this case we find again the classical model of communication systems $\left(\mathbf{\Gamma}_n^{-\frac{1}{2}}\mathbf{\Gamma}\mathbf{\Gamma}_n^{-\frac{1}{2}}\right)$. However, this processing is most robust but needs more computational load.
From the data matrix $\mathbf{\Gamma} = \mathcal{H}\mathbf{\Gamma}_s\mathcal{H}^+ + \mathbf{\Gamma}_n$, the goal of the first part of this paper is to estimate the noise covariance matrix $\mathbf{\Gamma}_n$ and in the second part, we estimate, blindly, $\mathcal{H}$ from the "clean" obtained matrix $[\mathcal{H}\mathbf{\Gamma}_s\mathcal{H}^+]$ using the subspace method [1].

## 3. BLIND NOISE ESTIMATION (BNE)

In many applications such as communication systems, it is reasonable to assume the correlation is decreasing along the receivers. That is a widely used model for a colored noise. The correlation rate $\rho$ is decreasing when the distance between two receivers increases.
In this study, we consider the noise covariance matrix band-Toeplitz with the diagonal values are decreasing, so-called *decreasing band-Toeplitz*. It is the unique assumption to estimate the noise covariance matrix.
The BNE algorithm from the noise covariance matrix estimation is summarized in the following steps:
*Step 1:* - Estimation and eigendecomposition of the received covariance matrix $\mathbf{\Gamma}$; $\widehat{\mathbf{\Gamma}} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{r}_t \mathbf{r}_t^+$, with $T$ is the number of independent realizations; $\widehat{\mathbf{\Gamma}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^+$, where, $\mathbf{\Lambda} = diag[\lambda_1, \ldots, \lambda_{LN}]$, and $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{LN}]$; $\lambda_i$ and $\mathbf{u}_i$ are the eigenvalues and the eigenvectors of the observation covariance matrix, respectively.
- Initialization of the noise covariance matrix : $\widetilde{\mathbf{\Gamma}}_n = \mathbf{0}$.
*Step 2:* - Calculation of the matrix: $\mathbf{W}_{N+M} = \mathbf{U}_S \mathbf{\Lambda}_S^{1/2}$, with $\mathbf{U}_S = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{N+M}]$ is the matrix of $(N+M)$ eigenvectors corresponding to the $(N+M)$ eigenvalues, and $\mathbf{\Lambda}_S = diag[\lambda_1, \ldots, \lambda_{N+M}]$ is the matrix of $(N+M)$ eigenvalues.
- Calculation of the matrix: $\mathbf{\Delta} = \mathbf{W}_{N+M}\mathbf{W}_{N+M}^+$.
*Step 3:* Calculation of: $\widetilde{\mathbf{\Gamma}}_n^{(1)} = K\_band\left[\widehat{\mathbf{\Gamma}} - \mathbf{\Delta}\right]$, with $\widetilde{\mathbf{\Gamma}}_n^{(1)}$ is the band noise covariance matrix at first iteration, and $K\_band[.]$ designates the matrix band with $(K+1)$ is the bandwidth.
*Step 4:* Eigendecomposition of the matrix: $\left[\widehat{\mathbf{\Gamma}} - \widetilde{\mathbf{\Gamma}}_n^{(1)}\right] = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^+$. The new matrices $\mathbf{\Delta}$ and $\widetilde{\mathbf{\Gamma}}_n^{(2)}$ are, again, estimated in *step 2* and *step 3*. These iterations are repeated until the improvement of $\widetilde{\mathbf{\Gamma}}_n^{(i)}$.
*Stop test:* The algorithm is stopped when the distance between $\widetilde{\mathbf{\Gamma}}_n^{(i)}$ and $\widetilde{\mathbf{\Gamma}}_n^{(i+1)}$ becomes less then some value $\varepsilon$. We define the distance between $\widetilde{\mathbf{\Gamma}}_n^{(i)}$ and $\widetilde{\mathbf{\Gamma}}_n^{(i+1)}$ as $\| \widetilde{\mathbf{\Gamma}}_n^{(i+1)} - \widetilde{\mathbf{\Gamma}}_n^{(i)} \|_F$, the Frobenius norm of the matrix $\left(\widetilde{\mathbf{\Gamma}}_n^{(i+1)} - \widetilde{\mathbf{\Gamma}}_n^{(i)}\right)$.
The estimate noise covariance matrix $\widetilde{\mathbf{\Gamma}}_n$ is obtained when the algorithm is stopped.
The matrix $\widetilde{\mathbf{\Gamma}}_n$ is used to "denoise" the received data. In fact, the free-noise received covariance matrix is $\widetilde{\mathbf{\Gamma}} = \widehat{\mathbf{\Gamma}} - \widetilde{\mathbf{\Gamma}}_n$ or $\widetilde{\mathbf{\Gamma}} = \left(\widetilde{\mathbf{\Gamma}}_n^{-\frac{1}{2}}\widehat{\mathbf{\Gamma}}\widetilde{\mathbf{\Gamma}}_n^{-\frac{1}{2}}\right)$. This "clean" matrix

is used to estimate the channel matrix. In order to evaluate its performance, we apply the subspace method [1]. Indeed, Moulines et al. [1], showed that if the subchannels don't share common zeros, $\mathbf{h}$ is uniquely determined by the noise subspace $\widetilde{\mathbf{U}}_n$, the subspace estimator is given by:

$\widehat{\mathbf{h}} = arg \min_{||\mathbf{h}||=1} \mathbf{h}^+ \widehat{\mathcal{U}}_{ss}\mathbf{h}$, where $\widehat{\mathcal{U}}_{ss}$ is the filtering noise projection matrix estimated from the ''clean" data covariance matrix. This estimator does not require the knowledge of the source covariance as long as $\mathbf{\Gamma}_s > 0$. We also compare our result to the modified subspace (MSS) method [4].

## 4. PERFORMANCE EVALUATION

To demonstrate the efficiency of the proposed algorithm, some computer simulations have been conducted. In the following simulations, we take the parameters described in [1], in fact the number of virtual channels is $L = 4$; the width of the temporal window is $N = 10$; the degree of the ISI is $M = 4$, the channel coefficients are given by [1]:

| $h_0$ | $h_1$ | $h_2$ | $h_3$ |
|---|---|---|---|
| -0.049+0.359j | 0.443-0.0364j | -0.211-0.322j | 0.417-0.030j |
| 0.482+0.569j | 1 | -0.199-0.918j | 1 |
| -0.556+0.587j | 0.921-0.194j | 1 | 0.873-0.145j |
| 1 | 0.189-0.208j | -0.284-0.524j | 0.285+0.309j |
| -0.171+0.061j | -0.087-0.054j | 0.136-0.190j | -0.049+0.161j |

Table 1: **Four virtual complex channels.**

for all these simulations, the number of data samples used to estimate each $\mathbf{h}$ ranges from 100 to 1000 in steps of 100.

The root mean-square error ($RMSE$) defined, below, is employed as a performance measure of the input estimates:

$RMSE = \frac{1}{||\mathbf{H}||}\sqrt{\frac{1}{K}\sum_{i=1}^{K} \| \mathbf{H}_i - \mathbf{H} \|^2}$, where $K$ is the number of trials (100 in our cases) and $\mathbf{H}_i$ is the estimate of the inputs from the $ith$ trial.

The signal to noise ratio ($SNR$) is defined as:

$SNR = 10\log_{10} \frac{E\{||\mathbf{Hs}(k)||^2\}}{E\{||\mathbf{n}(k)||^2\}}$. We define the Frobenius norm of estimation error (EE) of the noise covariance matrix as:$EE = \| \mathbf{\Gamma} - (\mathbf{H}\mathbf{\Gamma}_s\mathbf{H}^+ + \mathbf{\Gamma}_n) \|_F$ .

We compare the presented algorithm with the existing methods such as the modified subspace approach (MSS) [4]. This comparison is based on the root mean square error of the channel matrix estimates. We recall, this approach in the following: Let $\mathbf{\Gamma}(\tau) = \mathcal{H}\mathbf{J}(\tau)\mathcal{H}^+ + \mathbf{\Gamma}_n(\tau)$, where $\mathbf{J}(\tau)$ is the $(N+M) \times (N+M)$ shift matrix. In [4], one assumes that $\mathbf{\Gamma}_n(\tau) = \mathbf{0}$ as long as $\tau \geq N$. Therefore, we have the relation $\mathbf{\Gamma}(\tau) = \mathcal{H}\mathbf{J}(\tau)\mathcal{H}^+$ for $\tau \geq N$. At the time lag $\tau = N$, $\mathbf{\Gamma}(N) = \mathcal{H}\left(\mathbf{J}(N) + \mathbf{J}(N)^+\right)\mathcal{H}^+$, the matrix $\mathbf{\Gamma}(N)$ is used to estimate the channel parameters.

The Figures (1a and 1b) present the root square-mean error (RMSE) of the parameters estimates for a band-Toeplitz noise covariance matrix and the Frobenius norm of estimation of error (EE) of the noise covariance matrix versus number of samples.

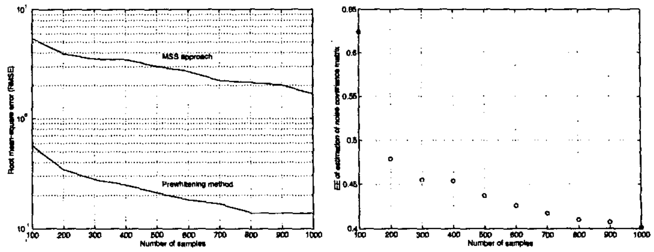


Figure 1: **(a) Root square-mean error (RMSE) of the parameters estimates (band-Toeplitz noise covariance matrix). (b) Frobenius norm of estimation of error (EE) of the noise covariance matrix (band-Toeplitz noise covariance matrix) versus number of samples**

In the case of a band noise covariance matrix with a correlation length $K = 4$, we have Figures (2a and 2b), versus $SNR$ between 0 dB to 16 dB.

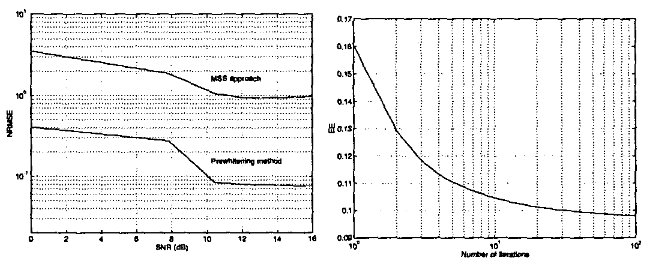


Figure 2: **(a) Root mean-square error of the parameters estimates (band-Toeplitz noise covariance matrix ($K = 4$)) versus SNR. (b) Frobenius norm of the estimation of error (EE) of noise covariance matrix as a function of number of iterations.**

We study, the influence of the correlation length versus the error of the noise covariance matrix estimation Figure (3a) and the channel parameters Figure (3b). In fact, the correlation length varies between $K = 1$ and $K = 4$, with $SNR = 3$ dB.

The normalized error ($NE$) is defined by, $NE = \frac{||\mathbf{H}_i - \mathbf{H}||}{||\mathbf{H}||}$.

We consider the noise covariance matrix band, and we estimate the normalized error and the Frobenius norm versus of different scenarios of the channel matrix (Figures (4a and 4b).

These simulations show that the processing which consists to first estimation of the noise covariance matrix and prewhitening the observation has many advantages, is more efficient then the modified subspace (MSS) approach [4]. The use of the denoised subspace

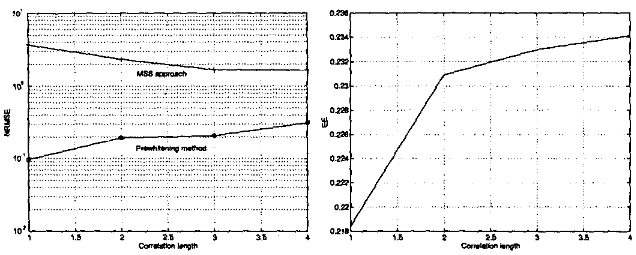


Figure 3: (a) Root mean-square error of the parameters estimates versus correlation length. (b) Frobenius norm of the estimation of error (EE) of noise covariance matrix as a function of correlation length.

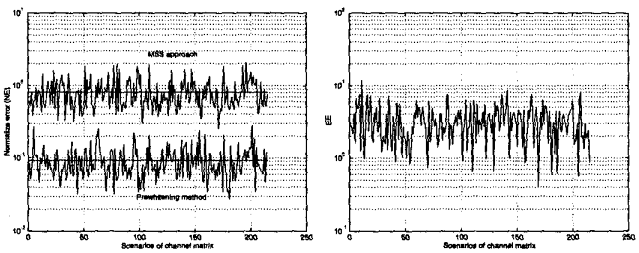


Figure 4: (a) Normalized error (NE) of the parameters estimates versus scenarios of channel matrix when the noise covariance matrix is band. (b) Frobenius norm of the estimation of error (EE) of band noise covariance matrix as a function of scenarios of channel matrix.

method presented in this paper becomes interesting in the case of low $SNR$ and when the noise covariance matrix is band. When the length correlation increases, the interest of the estimation of the noise increases also. Several computer simulations confirm these conclusions.

This algorithm can be, also, applied, naturally, for other blind channel identification methods such as XBM, TXK ...[2, 3] disregard of the system type used.

## 5. CONCLUSION

To estimate, blindly, the noise than the channel parameters, an algorithm was presented. We have considered a spatially correlated noise, with only the assumption that the matrix noise is band-Toeplitz, than by an iterative algorithm using the eigenstructure, we have estimated the noise parameters. In order to use a "clean" data for the the estimation of the channel matrix, the estimated noise matrix was used for "prewhitening" the observations. The subspace approach was, than, applied for the blind estimation of the channel parameters.

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