Downlink Beamforming for FDD Mobile Radio Systems Based on Spatial Covariances

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Abstract — **A SDMA mobile radio system based on the estimation of spatial covariances has been proposed recently. The beamforming vectors of all users are jointly calculated such that all terminals receive their required signals with a given signal-to-interference-and-noise ratio (SINR). This approach may suffer from the frequency gap between the uplink and downlink mode in 3rd generation FDD systems. To this end, in this paper we consider the combination of a least squares approximation of beampatterns and the proposed power minimization beamforming to overcome the difficulties of FDD in mobile communication systems. Simulation results of an artifical scenario are presented.**

I. Downlink Beamforming Based on Spatial Covariances

The proposed concepts for third generation mobile radio systems, which has been selected by ETSI SMG recently, are the TD-CDMA concept for *time-division duplex* (TDD) and the WCDMA concept for *frequency-division duplex* (FDD) systems [1]. Adaptive antenna arrays exploit the inherent spatial diversity of the mobile radio channel and perform spatial interference suppression. In [2] a robust SDMA system was proposed which is based on the estimation of spatial covariance matrices at the base station (BS) rather than on the explicit knowledge of the different directions of arrival (DOAs) of the user signals. The k-th user-specific covariance matrix

$$\boldsymbol{C}_k = E\left[|\hat{\boldsymbol{x}}_k \hat{\boldsymbol{x}}^H|\right] \tag{1}$$

is estimated by means of the array receive vectors

$$\hat{\boldsymbol{x}}_k(t) = \sqrt{\hat{P}_k} \sum_{l=1}^{L_k} \hat{h}_{kl} \hat{\boldsymbol{a}}_{kl} \hat{s}_k(t-\tau_{kl}) + \boldsymbol{n}(t) \quad \in \quad \mathcal{C}^M, \tag{2}$$

where L_K denotes the number of dominant wavefronts of the k-th user. Throughout the paper, uplink and downlink variables are marked by the symbols $(\hat{\cdot})$ and $(\check{\cdot})$, respectively. The

$$\hat{h}_{kl}(\psi_{kl}, \theta_{kl}, \hat{\lambda}), \hat{\boldsymbol{a}}_{kl}(\psi_{kl}, \theta_{kl}, \hat{\lambda}) \tag{3}$$

denote the channel transfer function of amplitude and phase and the array steering vector of the M element antenna array with the azimuth ψ_{kl} and elevation θ_{kl} of the l-th wavefront, and the uplink carrier wavelength $\hat{\lambda}$. The uplink baseband signal for the mobile k is expressed as

$$\hat{s}_k(t) = \sum_{m=-\infty}^{\infty} b_k(m) c_k(t - mT_k), \tag{4}$$

$$c_k(\tau) = \sum_{q=1}^{Q_k} d_{kq} p(\tau - qT_c) \tag{5}$$

with $\hat{P}_k$ denoting the RF power of the transmitted signal. The channel is generally frequency selective so that the time delays τ_{kl} are different for all paths $l = 1, \ldots, L_k$. The further parameters are the modulated (e.g. QPSK) symbols

$$b_k(m) \in \{1, j, -1, -j\}, \tag{6}$$

the spreading code $c_k(t)$ of length $T_k = Q_k T_c$, and the chip-waveform $p : \tau \in \mathcal{C} \rightarrow \mathcal{C}$ (e.g. pulse of square-root raised cosine spectrum). Then, assuming all paths $l = 1, \ldots, L_k$ are uncorrelated, the medium-term estimation of $\boldsymbol{C}_k$ equals

$$\tilde{\boldsymbol{C}}_k = \sum_{l=1}^{L_k} |\hat{h}_{kl}|^2 \hat{\boldsymbol{a}}_{kl} \hat{\boldsymbol{a}}_{kl}^H. \tag{7}$$

Assuming the system uses the same carrier frequencies for uplink and downlink transmission, which results in $\hat{\lambda} = \check{\lambda}$ and $\hat{\boldsymbol{a}}_{kl} = \check{\boldsymbol{a}}_{kl}$, the downlink beamforming problem has been formulated as power minimization problem [2]:

$$\underset{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K}{\text{minimize}} \left\{ P = \sum_{k=1}^{K} \|\boldsymbol{w}_k\|_2^2 \right\} \tag{8}$$

subject to

$$\frac{S_{kk}}{\text{SINR}} = N_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} S_{kl}, \quad k = 1, \ldots, K. \tag{9}$$

The S_{kl} describes the expectation of the signal power received by the k-th user and created by the beamforming weight vector $\boldsymbol{w}_l \in \mathcal{C}^M$ of user l. Here, the S_{kl} is given by

$$S_{kl} = \boldsymbol{w}_l^H \boldsymbol{C}_k \boldsymbol{w}_l \quad \in \quad \mathcal{R}_+. \tag{10}$$

The noise power N_k is due to intercell interference, but not by receiver-inherent thermal or quantization noise. Instead of cancelling all intracell interference the power minimizing beamformer minimizes the expectation of the array power while maintaining a given SINR for all users.

However, in the WCDMA concept, where the frequency gap between uplink ($\hat{f} = 1990$ MHz) and downlink ($\check{f} = 2175$ MHz) is much larger than in the GSM system, the presented approach might experience a degradation because of the frequency dependence of the spatial covariance matrices. That spatial signatures estimated on the uplink cannot be used for downlink transmission has been also demonstrated in [3, 4].

II. Approximating the Beampattern

The *beamforming* $\mathcal{C}^M \to \mathcal{C}$ by means of antenna arrays is mathematically equal to the *Fourier transform* of the complex weighting $\boldsymbol{w}$ of corresponding antenna elements into the complex domain of the electromagnetic far field. For the uniform linear antenna array the *beampattern* is denoted by

$$\boldsymbol{w} \mapsto \mathcal{A}(\boldsymbol{w}, \psi) = \sum_{m=1}^{M} w_i e^{j2\pi m \frac{d}{\lambda} \cos\psi} \quad (11)$$

$$= \boldsymbol{a}(\psi, \lambda)^T \boldsymbol{w} \quad (12)$$

with the distance d between adjacent elements. Furthermore, only the azimuth ψ is considered; the elevation θ is not considered. If the weight factors of all elements are equally constant to one, the above directional beampattern has the explicit form

$$\frac{\sin(\pi M \cos\psi/2)}{M \sin(\pi \cos\psi/2)} \quad (13)$$

which is known as *Dirichlet's kernel.*

In [5] two approaches have been proposed to transpose second order statistics of the channel from uplink to downlink frequency. We focus on the first approach which is based on the *least squares approximation* of the array manifold at $\hat{f}$ by means of beamforming at $\check{f}$, i.e. minimizing the error

$$\int_{\psi=0}^{\pi} \left(\mathcal{A}(\hat{\boldsymbol{w}}, \psi) - \mathcal{A}(\check{\boldsymbol{w}}, \psi)\right)^2 d\psi. \quad (14)$$

The second method in [5] presents a theoretical concept for circular arrays which is not considered in this paper.

Sampling the array manifold every $\frac{\pi}{P}$ degrees in radiant, the least squares approximation in (14) results in

$$\min_{\check{\boldsymbol{w}}} \left\{ \|\hat{\boldsymbol{w}}^T \hat{\boldsymbol{A}} - \check{\boldsymbol{w}}^T \check{\boldsymbol{A}}\|_2^2 \right\}, \quad (15)$$

$$\check{\boldsymbol{w}} = (\check{\boldsymbol{A}}^* \check{\boldsymbol{A}}^T)^{-1} \check{\boldsymbol{A}}^* \hat{\boldsymbol{A}}^T \hat{\boldsymbol{w}}, \quad (16)$$

respectively, with

$$\hat{\boldsymbol{A}} = (\boldsymbol{a}(\psi_1, \hat{\lambda}), \boldsymbol{a}(\psi_2, \hat{\lambda}), \ldots, \boldsymbol{a}(\psi_P, \hat{\lambda})), \quad (17)$$

$$\check{\boldsymbol{A}} = (\boldsymbol{a}(\psi_1, \check{\lambda}), \boldsymbol{a}(\psi_2, \check{\lambda}), \ldots, \boldsymbol{a}(\psi_P, \check{\lambda})), \quad (18)$$

and

$$\psi_\mu = (\mu - 1)\frac{\pi}{P}, \mu = 1, \ldots, P. \quad (19)$$

III. Drawing the Power Minimization Over the Frequency Gap

Using the power minimization approach of (8)-(9), based on second order statistic from the uplink, requires the transposition of the spatial covariances (cf. 7) in (10) by

$$\hat{\boldsymbol{C}}_k \mapsto \check{\boldsymbol{C}}_k = \boldsymbol{T}^H \hat{\boldsymbol{C}}_k \boldsymbol{T}. \quad (20)$$

The matrix

$$\boldsymbol{T} = (\hat{\boldsymbol{A}}^* \hat{\boldsymbol{A}}^T)^{-1} \hat{\boldsymbol{A}}^* \check{\boldsymbol{A}}^T \quad (21)$$

expresses the reverse approximation of (16), this is

$$\hat{\boldsymbol{w}} = \boldsymbol{T} \check{\boldsymbol{w}}. \quad (22)$$

In other words, the power minimization problem (8)-(9) can be drawn over the frequency gap by a coordinate shift of the uplink parameters by means of the relevant downlink parameters $\check{\boldsymbol{w}}$ and the operator $\boldsymbol{T}$. To this end, the downlink beamforming problem at $\check{f}$ is reformulated as

$$\min_{\check{\boldsymbol{w}}} \left\{ \check{\boldsymbol{w}}^H \boldsymbol{T}^H \boldsymbol{T} \check{\boldsymbol{w}} \right\} \quad (23)$$

subject to

$$\check{\boldsymbol{w}}^H \check{\boldsymbol{D}}_k \check{\boldsymbol{w}}^H = 1, \quad k = 1, \ldots, K \quad (24)$$

with stacking the downlink beamforming vectors $\check{\boldsymbol{w}}_k$ into a complex $(M \cdot K) \times 1$ vector $\check{\boldsymbol{w}}$, and

$$\check{\boldsymbol{D}}_k = \begin{pmatrix} -\frac{\check{\boldsymbol{C}}_k}{N_k} & & & & \boldsymbol{0} \\ & \ddots & & & \\ & & \frac{\check{\boldsymbol{C}}_k}{N_k \,\mathrm{SINR}} & & \\ & & & \ddots & \\ \boldsymbol{0} & & & & -\frac{\check{\boldsymbol{C}}_k}{N_k} \end{pmatrix} \quad (25)$$

$\in \mathcal{C}^{M \cdot K \times M \cdot K}$.

IV. Simulation Experiments

The experiments are based on an artificial scenario of $K = 2$ users and $M = 7$ antenna elements of a uniform linear antenna array (ULA). Moreover, each MS receives the BS via three dominant paths with DOAs at (▲= 60, 40, 0) degree and (△= 50,20,10) degree, for both users respectively. An angular spread of 10 degree is considered for each path.

The presented results in this paper are calculated by the direct solution of the nonlinear optimization problem (23)-(24) by means of standard methods from mathematical optimization [6]. Of course, the suboptimal techniques for downlink beamforming with the power minimizer (cf. [2]), which are less demanding in number of complex floating-point operations, are still applicable for the new optimization problem (23)-(24).

Figure 1 presents the beampatterns of the power minimization beamforming at downlink frequency $\check{f}$ for two different approaches (The solid line (—) in Figure 1 shows the beampattern for $\check{f} = \hat{f}$, and SINR $= 10$dB, and serves as a reference (REF)):

- Approximation of the beampattern at downlink frequency $\check{f}$ by means of the least squares approach for the power minimizer (SINR $= 10$dB, cf. (23)-(24)).
- Using the beamforming vector $\check{\boldsymbol{w}} \leftarrow \hat{\boldsymbol{w}}$, after power minimization (SINR $= 10$dB) at the uplink frequency $\hat{f}$, i.e. don't care about the frequency gap. In other words, using the uplink beamforming vector at the downlink frequency (beamforming vector reuse (REUSE)).

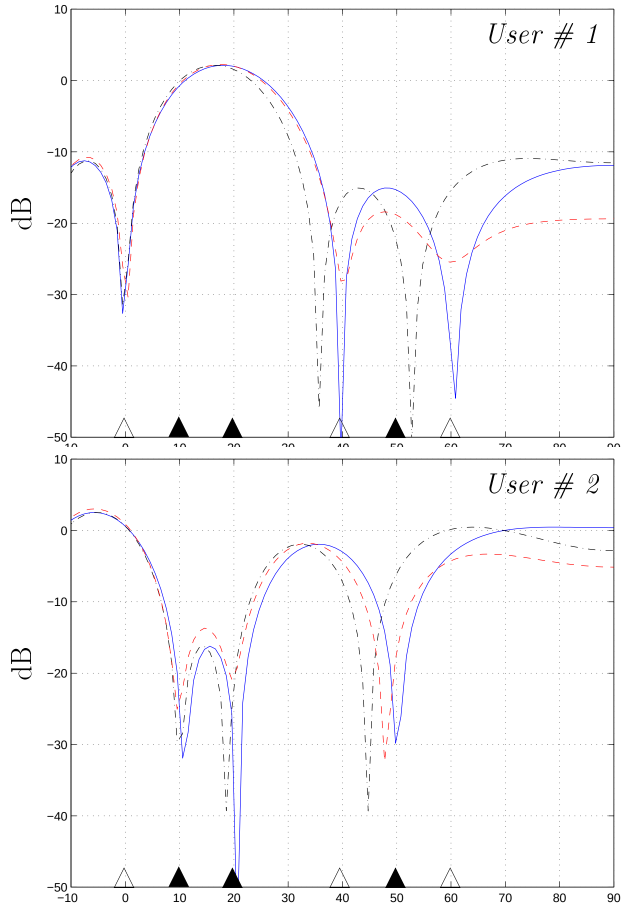

Figure 1: Downlink beamforming for user #1 and user #2 at $\check{f}$.

Whereas the approximated beampattern $(--)$ nearly performs like the reference pattern, the REUSE version $(-.-.)$ produces significant interference between the users. Note the mutual interference of the users at the azimuths of 50 and 60 degree. This is also confirmed by the effective SNIR of each user (Table 1) which is calculated by means of the complete knowledge of the spatial covariances at the downlink frequency $\check{f}$. The averaged transmitted power

$$\overline{\check{\boldsymbol{w}}_k^H \check{\boldsymbol{w}}_k} / \overline{N_k} \tag{26}$$

for each user is approximately equal (11.894dB and 11.885dB) for the LS and the beamforming vector REUSE, respectively.

V. Conclusion

A new technique has been introduced that combines the power minimization beamforming [2] with the least squares approximation of the downlink beampattern [5].

Table 1: SINR for the downlink beamforming at $\check{f}$.

SINR	REF	LS	REUSE
1	10.00 dB	9.67 dB	6.83 dB
2	10.00 dB	9.64 dB	9.68 dB

The results show that the effects of the frequency gap ($\hat{f}/\check{f} = 0.915$) between uplink and downlink frequency in FDD-WCDMA radio systems must not be neglected, in particular for scenarios of asymmetric wavefronts of users. The approximated power minimizer ensures the desired SINR of each user without increase of the transmitted power at the base station.

VI. References

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