Transmit Matched Filter and Transmit Wiener Filter for the Downlink of FDD DS-CDMA Systems

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Abstract

We present space-time transmit filters for FDD DS-CDMA systems based on partial channel state information, i. e. we do not take into account the channel coefficients. Although the FDD transmit zero-forcing filter seems to be the most intuitive approach, we focus on the FDD transmit matched filter and the FDD transmit Wiener filter. Similarly to the respective receive filters and TDD transmit filters, the FDD matched filter maximizes the desired signal portion at the receiver and is optimum for low signal-to-noise-ratio scenarios, whereas the FDD transmit Wiener filter takes into account the noise power at the receiver and is therefore able to find an optimum trade-off between signal maximization and interference suppression. Additionally, we show that the FDD transmit matched filter is a type of Eigenbeamforming. The simulation results reveal the excellent performance of the two FDD transmit filters. The FDD transmit Wiener filter even outperforms the TDD transmit matched filter for high signal-to-noise-ratio which is based on the instantaneous channel properties.

I Introduction

The conventional approach to deal with the distortion caused by the channel is receive processing, where a receive filter is adapted to the properties of the channel and the *a priori* known modulation operation performed at the transmitter. If the receive filter has to remove *intracell interference*, the resulting receiver is very complex. Thus, receive processing is especially disadvantageous for the downlink, since the *mobile stations* (MSs) have to be low cost and simple as possible. In *time division duplex* (TDD) systems, the *base station* (BS) can estimate the instantaneous channel impulse response during the reception in the uplink and is able to design transmit filters which suppress the interference at the receiver (e. g. [1]), maximize the received signal portion due to the desired signal (e. g. [2, 3]), or find a trade-off between interference suppression and signal maximization [4, 5]. However, the instantaneous downlink channel state information is not available at the BS in *frequency division duplex* (FDD) systems, but the *long-term* channel properties (path delays, directions of departure, and variances of the channel coefficients) are independent from frequency (e. g. [6]) and can therefore be estimated during the reception in the uplink to be able to design FDD transmit filters for the downlink. Note that the spatial signature can be easily transformed from uplink to downlink frequency without estimating the direction of departure [7]. Montalbano et al. [8] presented an FIR transmit filter for FDD *direct sequence code division multiple access* (DS-CDMA) systems which removes the intracell interference under the assumption of rake receivers at the MSs and by exploiting the *long-term* channel properties. In [9], Forster et al. developed a similar transmit filter for GSM in frequency domain.

We derive space-time FDD transmit filters for the downlink of DS-CDMA systems based on *long-term* channel properties which maximize the desired signal portion at the receiver or minimize the modified *mean square error* (MSE). In Section II, we explain the system model based on *long-term* channel parameters which is crucial for FDD and is the main difference to TDD or *short-term* transmit processing. We review the FDD *transmit zero-forcing filter* (TxZF) in Section III to understand the derivation of the FDD *transmit matched filter* (TxMF) and the FDD *transmit Wiener filter* (TxWF) in Section IV and V, respectively. In Section VI, we present the simulation results which show the superiority of the FDD TxMF and FDD TxWF compared to the FDD TxZF.

II System Model

A Based on All Channel Parameters

The output signal of the rake demodulator for the m-th symbol of MS k can be expressed as

$$\hat{s}_k^{(m)} = \sum_{n=0}^{\chi_k-1} \sum_{f=1}^{F_k} h_{k,f}^* d_k^{(m),*}[n] x_k[n + m\chi_k + \nu_{k,f}], \qquad (1)$$

where $(\bullet)^*$, χ_k, F_k, and $d_k^{(m)}[n]$ denote complex conjugate, the spreading factor, the number of rake fingers, and the spreading code of the m-th symbol for MS k which is composed of the *orthogonal variable spreading factor* short code $o_k[n]$ of length χ_k and the BS-specific pseudo-noise long code $g[n]$, i. e. $d_k^{(m)}[n] = g[n+m\chi_k]o_k[n]$, respectively. One slot consists of M_k symbols for MS k and $N_c = \chi_k M_k$ chips. With the Rayleigh fading path coefficient $h_{k,q}$, the path delay $\nu_{k,q}$, and the steering vector $\boldsymbol{a}_{k,q} \in \mathbb{C}^{N_a}$ of the q-th path connecting the BS and MS k in the downlink the receive signal can be written as

$$x_k[n] = \sum_{q=1}^{Q_k} h_{k,q} \boldsymbol{a}_{k,q}^{\mathrm{T}} \boldsymbol{y}[n - \nu_{k,q}] + \eta_k[n]. \qquad (2)$$

Here, $(\bullet)^{\mathrm{T}}$, Q_k, and $\eta_k[n]$ denote transposition, the number of paths, and the additive noise of MS k, respectively. Since N_a antenna elements are deployed at the BS, the transmit signal $\boldsymbol{y}[n]$ is an N_a-dimensional complex vector.

B Based on Long-Term Channel Parameters

The *short-term* channel parameters $h_{k,q}$ are not available at the base station in FDD systems, since the channel coefficients $h_{k,q}$ strongly depend on frequency [6] and vary fast. On the other hand, the *long-term* channel parameters, i. e. directions of departure (to compute the steering vectors $\boldsymbol{a}_{k,q}$), delays $\nu_{k,q}$, and path powers $\sigma^2_{h_{k,q}}$, are independent from frequency [6] and stay constant over a relatively long time period compared to the time of one slot, because the geometrical properties of the channel change slowly. To separate *short-term* and *long-term* channel properties we plug Equation (2) into (1) and get:

$$\hat{s}_k^{(m)} = \sum_{f=1}^{F_k} \sum_{q=1}^{Q_k} h_{k,f}^* h_{k,q} s_{k,f,q}^{(m)} + \sum_{f=1}^{F_k} h_{k,f}^* \eta_{k,f}^{(m)}.$$

Note that $s_{k,f,q}^{(m)}$ is the portion of the estimate $\hat{s}_k^{(m)}$ which propagated over the q-th path and the f-th rake finger, where we dropped the channel coefficient $h_{k,q}$ and the rake finger weight $h_{k,f}^*$:

$$s_{k,f,q}^{(m)} = \sum_{n=0}^{\chi_k - 1} d_k^{(m),*}[n] \boldsymbol{a}_{k,q}^{\mathrm{T}} \boldsymbol{y}[n + m\chi_k + \nu_{k,f} - \nu_{k,q}].$$

The base station can compute the signal $s_{k,f,q}^{(m)}$ but is not able to compute the rake output $\hat{s}^{(m)}$. Thus, the base station has to consider each summand $h_{k,f}^* h_{k,q} s_{k,f,q}^{(m)}$ separately. The key idea of FDD transmit processing is to optimize

$$\tilde{s}_{k,f,q}^{(m)} = s_{k,f,q}^{(m)} \cdot \begin{cases} \sqrt{2}\sigma^2_{h_{k,f}} & q = f \\ \sigma_{h_{k,f}} \sigma_{h_{k,q}} & \text{otherwise} \end{cases} \qquad (3)$$

instead of the rake output signal $\hat{s}_k^{(m)}$. As $\tilde{s}_{k,f,q}^{(m)}$ is independent from the channel coefficients $h_{k,q}$, it only depends on *long-term* channel properties, but has the same power as the summand $h_{k,f}^* h_{k,q} s_{k,f,q}^{(m)}$:

$$\mathrm{E}\left[\left|\tilde{s}_{k,f,q}^{(m)}\right|^2\right] = \mathrm{E}\left[\left|h_{k,f}^* h_{k,q} s_{k,f,q}^{(m)}\right|^2\right],$$

since $\mathrm{E}[|h_{k,q}|^2] = \sigma^2_{h_{k,q}}$, $\mathrm{E}[h_{k,q} h_{k,f}^*] = 0, f \neq q$, and $\mathrm{E}[\bullet]$ denotes expectation. When we include

$$\tilde{\eta}_{k,f}^{(m)} = \sigma_{h_{k,f}} \eta_{k,f}^{(m)} = \sigma_{h_{k,f}} \sum_{n=0}^{\chi_k - 1} d_k^{(m),*}[n] \eta_k[n + \nu_{k,f}],$$

whose power equals the power of the noise portion $h_{k,f}^* \eta_{k,f}^{(m)}$ of the f-th rake finger, we can define following signal

$$\tilde{u}_{k,f,q}^{(m)} = \tilde{s}_{k,f,q}^{(m)} + \tilde{\eta}_{k,f}^{(m)}, \qquad (4)$$

which has the same power as the f-th rake finger's output due to the q-th path and whose desired value is

$$u_{k,f,q}^{(m)} = \begin{cases} s_k^{(m)} & q = f \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

Thus, we try to suppress the portions of $\hat{s}_k^{(m)}$ which are superposed incoherently ($q \neq f$), whereas we attempt to set the portions which are added up coherently to the value of the desired symbol $s_k^{(m)}$. To end up with a compact formulation we collect the desired symbols of MS k in

$$\boldsymbol{s}_k = \left[s_k^{(0)}, \ldots, s_k^{(M_k-1)}\right]^{\mathrm{T}} \in \mathbb{C}^{M_k}$$

and put the rake finger signals into the $F_k Q_k M_k$ vector

$$\tilde{\boldsymbol{u}}_k = \left[\tilde{\boldsymbol{u}}_{k,1,1}^{\mathrm{T}}, \ldots, \tilde{\boldsymbol{u}}_{k,F_k,1}^{\mathrm{T}}, \tilde{\boldsymbol{u}}_{k,1,2}^{\mathrm{T}}, \ldots, \tilde{\boldsymbol{u}}_{k,F_k,Q_k}^{\mathrm{T}}\right]^{\mathrm{T}},$$

where $\tilde{\boldsymbol{u}}_{k,f,q} \in \mathbb{C}^{M_k}$ is defined similar to $\boldsymbol{s}_k$. With the transmit signal and the receive noise

$$\boldsymbol{y} = \left[\boldsymbol{y}^{\mathrm{T}}[0], \ldots, \boldsymbol{y}^{\mathrm{T}}[N_c - 1]\right]^{\mathrm{T}} \in \mathbb{C}^{N_a N_c} \quad \text{and}$$
$$\boldsymbol{\eta}_k = \left[\eta_k[0], \ldots, \eta_k[N_c + \nu_{\max} - 1]\right]^{\mathrm{T}} \in \mathbb{C}^{N_c + \nu_{\max}},$$

respectively, the *long-term* equivalents of the rake finger signals of MS k read as

$$\tilde{\boldsymbol{u}}_k = \boldsymbol{X}_k \boldsymbol{y} + \boldsymbol{V}_k \boldsymbol{\eta}_k, \qquad (6)$$

where we introduced

$$\begin{aligned}
\boldsymbol{V}_k &= \boldsymbol{F}_k \left(\mathrm{vec}\left(\mathrm{diag}\left(\mathbf{1}_{Q_k}\right)\right) \otimes \mathbf{1}_{N_c + \nu_{\max}}\right), \\
\boldsymbol{F}_k &= \left(\mathbf{1}_{F_k Q_k} \otimes \boldsymbol{D}_k^{\mathrm{H}}\right) \left(\mathbf{1}_{Q_k} \otimes \boldsymbol{T}_k\right), \\
\boldsymbol{X}_k &= \left(\mathrm{diag}\left(\mathrm{vec}\left(\boldsymbol{\Xi}_{F_k \times Q_k}\right)\right) \otimes \mathbf{1}_{M_k}\right) \boldsymbol{F}_k \boldsymbol{H}_k, \quad \text{and} \\
\nu_{\max} &= \max_{k,f,q} \nu_{k,f,q}.
\end{aligned}$$

Here, $\mathbf{1}_M$, vec $(\bullet)$, '$\otimes$', and $(\bullet)^{\mathrm{H}}$ denote the $M \times M$ identity matrix, vectorization by stacking the columns, the Kronecker product, and conjugate transpose, respectively. Note that vec $(\mathrm{diag}\left(\mathbf{1}_{Q_k}\right))$ is the all ones vector of length Q_k. The $N_c \times M_k$ block Toeplitz modulator matrix

$$\boldsymbol{D}_k = \mathrm{diag}\left(g[0], \ldots, g[N_c - 1]\right) \left(\mathbf{1}_{M_k} \otimes \boldsymbol{o}_k\right)$$

includes the short code $\boldsymbol{o}_k = \left[o_k[0], \ldots, o_k[\chi_k - 1]\right]^{\mathrm{T}}$. We put the *long-term* equivalents of the rake fingers into

$$\boldsymbol{T}_k = \left[\sigma_{h_{k,1}} \boldsymbol{S}^{\mathrm{T}}_{(\nu_{k,1}, N_c, \nu_{\max})}, \ldots, \sigma_{h_{k,F_k}} \boldsymbol{S}^{\mathrm{T}}_{(\nu_{k,F_k}, N_c, \nu_{\max})}\right]^{\mathrm{T}}$$

with the selection matrix

$$S_{(L,M,N)} = \left[\mathbf{0}_{M\times L}, \mathbf{1}_M, \mathbf{0}_{M\times N-L}\right] \in \mathbb{R}^{M\times M+N}$$

and the $M \times N$ zero matrix $\mathbf{0}_{M\times N}$. The *long-term* equivalent of the channel is simply

$$\boldsymbol{H}_k = \left[\boldsymbol{H}_{k,1}^{\mathrm{T}}, \ldots, \boldsymbol{H}_{k,Q_k}^{\mathrm{T}}\right]^{\mathrm{T}} \quad \text{with}$$
$$\boldsymbol{H}_{k,q} = \sigma_{h_{k,q}} \boldsymbol{S}_{(\nu_{k,q},N_c,\nu_{\max})}^{\mathrm{T}} \otimes \boldsymbol{a}_{k,q}^{\mathrm{T}}$$

and $\Xi_{M\times N}$ is a $M \times N$ matrix with $\sqrt{2}$ on the main diagonal and the other elements are set to 1 to fulfill Equation (3). If we define $\boldsymbol{u}_k$ similar to $\tilde{\boldsymbol{u}}_k$, the desired rake finger signals (cf. Equation 5) can be expressed as

$$\boldsymbol{u}_k = \boldsymbol{\Psi}_k \boldsymbol{s}_k \in \mathbb{C}^{F_k Q_k M_k}, \tag{7}$$

where $\boldsymbol{\Psi}_k = \mathrm{vec}\left(\boldsymbol{S}_{(0,F_k,Q_k-F_k)}\right) \otimes \mathbf{1}_{M_k}$. We put the symbols, rake finger signals, and noise of all K users into

$$\boldsymbol{s} = \left[\boldsymbol{s}_1^{\mathrm{T}}, \ldots, \boldsymbol{s}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{tot}}},$$
$$\tilde{\boldsymbol{u}} = \left[\tilde{\boldsymbol{u}}_1^{\mathrm{T}}, \ldots, \tilde{\boldsymbol{u}}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{F_{\mathrm{tot}}}, \quad \text{and}$$
$$\boldsymbol{\eta} = \left[\boldsymbol{\eta}_1^{\mathrm{T}}, \ldots, \boldsymbol{\eta}_K^{\mathrm{T}}\right] \in \mathbb{C}^{K(N_c+\nu_{\max})},$$

respectively, to get an expression for all rake finger signals based on *long-term* channel parameters:

$$\tilde{\boldsymbol{u}} = \boldsymbol{X}\boldsymbol{y} + \boldsymbol{V}\boldsymbol{\eta} \quad \text{and} \quad \boldsymbol{u} = \boldsymbol{\Psi}\boldsymbol{s} \in \mathbb{C}^{F_{\mathrm{tot}}}, \tag{8}$$

where we defined $\boldsymbol{u}$ similar to $\tilde{\boldsymbol{u}}$. The number of symbols and rake finger signals for all K MSs can be written as

$$M_{\mathrm{tot}} = \sum_{k=1}^{K} M_k \quad \text{and} \quad F_{\mathrm{tot}} = \sum_{k=1}^{K} F_k Q_k M_k,$$

respectively. To complete Equation (8) we have to define

$$\boldsymbol{V} = \mathrm{blockdiag}\left(\boldsymbol{V}_1, \ldots, \boldsymbol{V}_K\right) \in \mathbb{C}^{F_{\mathrm{tot}}\times K(N_c+\nu_{\max})},$$
$$\boldsymbol{X} = \left[\boldsymbol{X}_1^{\mathrm{T}}, \ldots, \boldsymbol{X}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{F_{\mathrm{tot}}\times N_s N_c}, \quad \text{and}$$
$$\boldsymbol{\Psi} = \mathrm{blockdiag}\left(\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_K\right) \in \mathbb{R}^{F_{\mathrm{tot}}\times M_{\mathrm{tot}}}.$$

The aim of linear transmit processing is to find the transmit filter $\boldsymbol{P}$ whose output is the transmit signal

$$\boldsymbol{y} = \boldsymbol{P}\boldsymbol{s}. \tag{9}$$

When we plug above equation into Equation (8), the resulting *long-term* equivalents of the rake finger signals read as

$$\tilde{\boldsymbol{u}} = \boldsymbol{X}\boldsymbol{P}\boldsymbol{s} + \boldsymbol{V}\boldsymbol{\eta}. \tag{10}$$

Equation (10) is used by the base station to optimize the rake demodulator outputs (cf. Equation 1).

III FDD Transmit Zero-Forcing Filter

Similar to the TDD TxZF the FDD TxZF removes all interference. Hence, the TxZF sets the noise-free portion $\boldsymbol{X}\boldsymbol{P}\boldsymbol{s}$ of $\tilde{\boldsymbol{u}}$ in Equation (10) to $\beta\boldsymbol{\Psi}\boldsymbol{s}$, where $\beta \in \mathbb{R}$ is the gain of the filter chains of transmit filter, channels, and receive filters. Additionally, the TxZF uses the available transmit power E_{tr} and maximizes the gain β [4]:

$$\left[\boldsymbol{P}_{\mathrm{ZF}}, \beta_{\mathrm{ZF}}\right] = \arg\min_{\boldsymbol{P},\beta} \beta^{-2} \tag{11}$$
$$\text{s.t.: } \boldsymbol{X}\boldsymbol{P} = \beta\boldsymbol{\Psi} \quad \text{and} \quad \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}.$$

With the covariance matrix of the symbols $\boldsymbol{R}_s = \mathrm{E}[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}]$ the resulting FDD TxZF can be written as

$$\boldsymbol{P}_{\mathrm{ZF}} = \beta_{\mathrm{ZF}} \boldsymbol{X}^{\mathrm{H}} \left(\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}}\right)^{-1} \boldsymbol{\Psi} \in \mathbb{C}^{N_s N_c \times M_{\mathrm{tot}}} \quad \text{and}$$
$$\beta_{\mathrm{ZF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\left(\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}}\right)^{-1} \boldsymbol{\Psi}\boldsymbol{R}_s\boldsymbol{\Psi}^{\mathrm{T}}\right)}}. \tag{12}$$

IV FDD Transmit Matched Filter

The FDD TxMF maximizes the desired signal portion in the rake finger signals (cf. Equations 8 and 10), uses the available transmit power E_{tr}, and neglects the interference as the TDD TxMF [3]:

$$\boldsymbol{P}_{\mathrm{MF}} = \arg\max_{\boldsymbol{P}} \mathrm{Re}\left(\mathrm{E}\left[(\boldsymbol{\Psi}\boldsymbol{s})^{\mathrm{H}} \tilde{\boldsymbol{u}}\right]\right) \tag{13}$$
$$\text{s.t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}.$$

The solution of the above optimization is the FDD TxMF:

$$\boldsymbol{P}_{\mathrm{MF}} = \beta_{\mathrm{MF}} \boldsymbol{X}^{\mathrm{H}} \boldsymbol{\Psi} \in \mathbb{C}^{N_s N_c \times M_{\mathrm{tot}}} \quad \text{and} \tag{14}$$
$$\beta_{\mathrm{MF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{R}_s\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{X}\right)}}.$$

Note that the symbols for MS k are transformed by the TxMF based only on the properties of the k-th channel:

$$\boldsymbol{y}_{\mathrm{MF}} = \beta_{\mathrm{MF}} \sum_{k=1}^{K} \boldsymbol{y}_{\mathrm{MF},k} = \beta_{\mathrm{MF}} \sum_{k=1}^{K} \boldsymbol{X}_k^{\mathrm{H}} \boldsymbol{\Psi}_k \boldsymbol{s}_k.$$

With some equalities from linear algebra (e. g. [10]) we yield for the portion of MS k:

$$\boldsymbol{y}_{\mathrm{MF},k} = \sum_{f=1}^{F_k} \sqrt{2}\sigma_{h_{k,f}}^2 \left(\mathbf{1}_{N_c} \otimes \boldsymbol{a}_{k,f}^*\right) \boldsymbol{D}_k.$$

Above result shows that the FDD TxMF spreads the symbols with $\boldsymbol{D}_k$ and then performs a type of *Eigenbeamforming* (cf. [11]), i. e. the FDD TxMF is a spatial filter based on the eigenvector $\boldsymbol{a}_{k,f}$ of the spatial covariance matrix for the respective channel tap. An alternative interpretation of the FDD TxMF follows from

$$\mathrm{E}\left[\sum_{f=1}^{F_k} h_{k,f}^* \sum_{q=1}^{Q_k} h_{k,q}\boldsymbol{a}_{k,q}^{\mathrm{T}}\delta[n-\nu_{k,q}+\nu_{k,f}]\right] = \\ = \sum_{f=1}^{F_k} \sigma_{h_{k,f}}^2 \boldsymbol{a}_{k,f}^{\mathrm{T}}\delta[n].$$

Hence, the FDD TxMF matches the *long-term* average of the channel and the rake demodulator.

V FDD Transmit Wiener Filter

In a recent report [4] (see also [5]), we presented the TDD TxWF which minimizes the *modified MSE*. We allow the FDD TxWF to have a gain β similar to the FDD TxZF and get for the modified MSE

$$\varepsilon(\boldsymbol{P},\beta) = \mathrm{E}\left[\|\boldsymbol{u}-\beta^{-1}\tilde{\boldsymbol{u}}\|_2^2\right].$$

The FDD TxWF minimizes above modified MSE and uses the available transmit power E_{tr}:

$$[\boldsymbol{P}_{\mathrm{WF}},\beta_{\mathrm{WF}}] = \arg\min_{\boldsymbol{P},\beta} \varepsilon(\boldsymbol{P},\beta), \tag{15}$$
$$\text{s.t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}.$$

After forming the Lagrangian function and setting its derivative with respect to $\boldsymbol{P}$ to zero, we obtain

$$\boldsymbol{P}(\xi') = \beta(\xi')\left(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{X}+\xi'\mathbf{1}_{N_aN_c}\right)^{-1}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{\Psi} \quad \text{and}$$
$$\beta(\xi') = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\left(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{X}+\xi'\mathbf{1}_{N_aN_c}\right)^{-2}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{\Psi}\boldsymbol{R}_s\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{X}\right)}}.$$

Since the choice of $\beta(\xi')$ guarantees the constraint of Equation (15), we only have to minimize the modified MSE with respect to $\xi' \in \mathbb{R}$:

$$\xi' = \arg\min_{\xi} \varepsilon(\boldsymbol{P}(\xi),\beta(\xi)).$$

The minimizer can be found by setting the derivative of the modified MSE with respect to ξ to zero and reads as

$$\xi' = \frac{\mathrm{tr}\left(\boldsymbol{V}\boldsymbol{R}_\eta\boldsymbol{V}^{\mathrm{H}}\right)}{E_{\mathrm{tr}}} = \frac{\sum_{k=1}^{K}\mathrm{tr}\left(\boldsymbol{V}_k\boldsymbol{R}_{\eta_k}\boldsymbol{V}_k^{\mathrm{H}}\right)}{E_{\mathrm{tr}}},$$

where $\boldsymbol{R}_\eta = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}]$ and $\boldsymbol{R}_{\eta_k} = \mathrm{E}[\boldsymbol{\eta}_k\boldsymbol{\eta}_k^{\mathrm{H}}]$ denote the noise covariance matrices for all K MSs and MS k, respectively. Therefore, we have found the FDD TxWF

$$\boldsymbol{P}_{\mathrm{WF}} = \beta_{\mathrm{WF}}\tilde{\boldsymbol{P}} \quad \text{and} \quad \beta_{\mathrm{WF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\tilde{\boldsymbol{P}}\boldsymbol{R}_s\tilde{\boldsymbol{P}}^{\mathrm{H}}\right)}} \quad \text{with}$$
$$\tilde{\boldsymbol{P}} = \left(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{X}+\frac{\mathrm{tr}\left(\boldsymbol{V}\boldsymbol{R}_\eta\boldsymbol{V}^{\mathrm{H}}\right)}{E_{\mathrm{tr}}}\mathbf{1}_{N_aN_c}\right)^{-1}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{\Psi}. \tag{16}$$

Note that $\boldsymbol{P}_{\mathrm{WF}}$ depends upon $\mathrm{tr}(\boldsymbol{V}^{\mathrm{H}}\boldsymbol{R}_\eta\boldsymbol{V})$ which is the *long-term* average of the noise powers observed at the K rake filter outputs and cannot be estimated by the transmitting BS. Consequently, the optimum FDD TxWF can only be designed, when the MSs feedback the noise powers to the BS. However, as we can expect that the *long-term* properties change very slowly, the noise powers will also be valid over a long time period and the necessary feedback rate is small. We observe that the FDD TxWF in Equation (16) is an FDD TxMF $\boldsymbol{X}^{\mathrm{H}}\boldsymbol{\Psi}$ followed by a transformation which converges to a weighted identity matrix for high receive noise and acts as an interference canceller for vanishing receive noise. Thus, the FDD TxWF reduces to the FDD TxMF for low *signal-to-noise-ratio* (SNR), whereas it behaves like the FDD TxZF for high SNR.

VI Simulation Results

We compared the FDD transmit filters by applying them to the downlink of a CDMA system with $N_c = 64$ chips in one slot. All $K = 4$ MSs have one antenna element and use scrambled orthogonal spreading codes of length $\chi_k = 4$. Therefore, each MS receives 16 symbols per slot. The BS is equipped with a uniform linear array consisting of $N_a = 4$ antenna elements. The channels connecting the BS and the MSs have a maximum delay $\nu_{\max} = 5$ chips and the number of rake fingers is equal to the number of paths, i. e. $F_k = Q_k$. The presented results are the average of 100 *long-term* channel realizations (steering vectors, path delays, powers of paths), where we simulated 100 *short-term* channel realizations (path coefficients) per *long-term* channel realization.

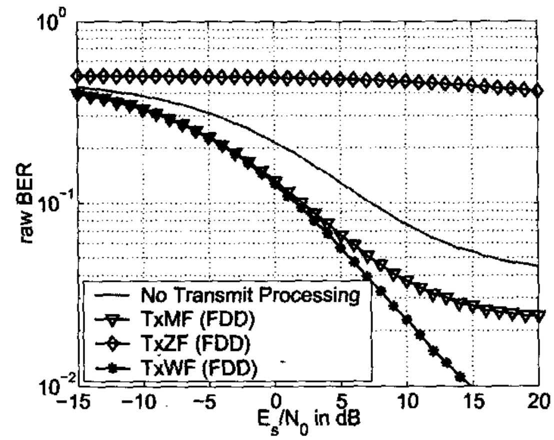

Figure 1: FDD transmit filters for $Q_k = 2$ paths

Figure 1 and 2 show the uncoded *bit error ratio* (BER) versus the ratio of transmit power per symbol and receive noise power per chip (SNR) for the three FDD transmit filters, when the channels have $Q_k = 2$ and $Q_k = 3$ paths, respectively. As expected the FDD TxMF outperforms the FDD TxZF for low SNR, but saturates for high SNR, since it neglects interference. The FDD TxZF benefits from the interference suppression for high SNR (cf. Figure 2). However, the interference suppression leads to a low gain β_{ZF} and the TxZF is bad for low SNR or is even unable to reach the performance of the other transmit filters for the simulated SNR values (cf. Figure 1). The FDD TxWF finds the optimum trade-off between signal power maximization and interference suppression. We can observe in both figures that the FDD TxWF converges to the FDD TxMF for low SNR and to the FDD TxZF for high SNR (cf. Figure 2). We also included the results for a

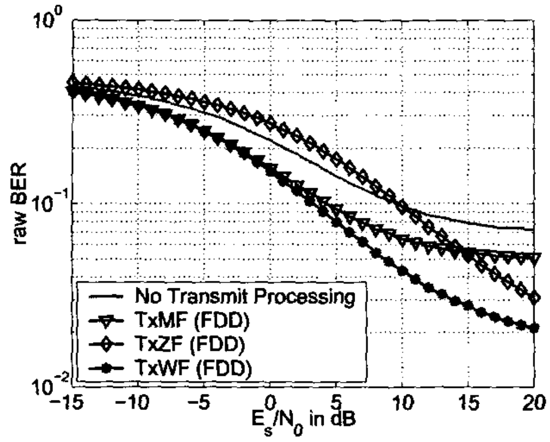

Figure 2: FDD transmit filters for $Q_k = 3$ paths

system without transmit processing and one transmit antenna element. As this system is even outperformed by the interference limited TxMF although it uses the same transmit power we can follow that exploiting the *long-term* channel properties for the design of space-time FDD transmit filters is always advantageous.

The FDD transmit filters are compared with the TDD transmit filters for $Q_k = 2$ paths in Figure 3, where we can see the impact of utilizing only *long-term* instead of *short-term* channel properties to design the transmit filter. As TDD transmit filters are based on the *short-term* properties of the channel, TDD transmit filters are able to exploit the diversity of the channel, whereas the FDD transmit filters cannot fully utilize the diversity offered by the channel and are worse than the respective TDD transmit filters which exhibit steeper curves. However, the FDD TxWF outperforms the TDD TxMF for high SNR which is interference limited, because FDD TxWF leads to interference suppression although it is based on partial channel state information.

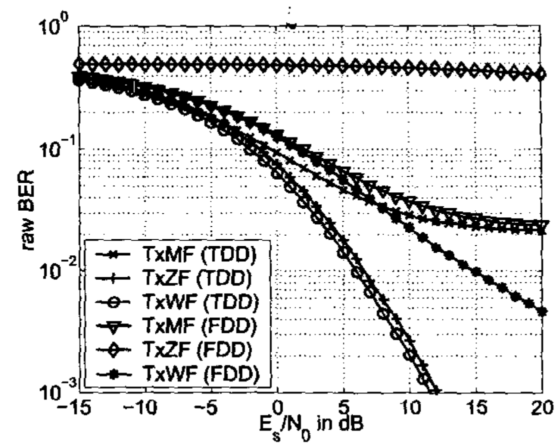

Figure 3: Comparison with TDD transmit filters

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