MULTI-STAGE MMSE DECISION FEEDBACK EQUALIZATION FOR EDGE

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ABSTRACT

Compared to wired channels, time dispersive radio channels possess more frequent nulls in their spectral characteristics. Thus, the performance of linear filters to compensate intersymbol interference degrades dramatically. The well-known nonlinear *Decision Feedback Equalizer* (DFE) is one approach to improve this behavior. However, in systems with observations of high dimensionality, the optimum DFE structure is computational intensive.

In this paper, we apply the method of the *Multi-Stage Wiener Filter* (MSWF) to a conventional *Minimum Mean Square Error* (MMSE) DFE in order to reduce computational complexity. The application of the new algorithm to an *Enhanced Data rates for GSM Evolution* (EDGE) system demonstrates the ability to outperform the even more computational intensive linear *Wiener Filter* (WF).

1. INTRODUCTION

The *Wiener Filter* (WF) [1] estimates an unknown signal from the observation signal in the *Minimum Mean Square Error* (MMSE) sense exploiting only second order statistics. In applications with observations of high dimensionality, the required inversion of the covariance matrix of the observation vector implies high computational complexity.

Goldstein et al. developed an computationally cheap approximation of the WF, the so-called *Multi-Stage Wiener Filter* (MSWF, [2]). More recently, Honig et al. [3] showed that the MSWF is the solution of the Wiener-Hopf equation in the *Krylov subspace* of the covariance matrix of the observation vector and the cross-correlation vector between the observation and the desired signal. Therefore, the *Lanczos algorithm* may be used to compute the reduced rank filter weights [4]. Note that compared to the MSWF, methods based on eigen subspaces like the *Principal Component Method* [5] or the more sophisticated *Cross Spectral Method* [6] are suboptimum when the sample support is low.

Nevertheless, all mentioned methods to reduce complexity are based on the optimum linear filter. In mobile communication systems, nonlinear processing is necessary [7] because radio channels possess nulls in their spectral characteristics. The *Decision Feedback Equalizer* (DFE) introduced by Austin [8] is one possible nonlinear approach but has high computational complexity as well as the WF.

Zoltowski et al. [9] applied a *Conjugate Gradient algorithm* with decision feedback and structured channel estimation to Digital TV which employs a 8-VSB modulation scheme with no memory. Sun et al. [10] analyzed the asymptotic performance of a Krylov subspace based DFE in a multi-user and multi-antenna system. Our contribution is to apply the ideas of the MSWF to the *Minimum Mean Square Error* (MMSE) implementation of the DFE and analyze the performance of its application to an *Enhanced Data rates for GSM Evolution* (EDGE) system which suffers from severe intersymbol interference due to pulse shaping as well as multipath. Since EDGE implies a modulation technique with memory, the problem of applying the MSWF to a DFE for EDGE is different from the application to a DFE for Digital TV.

The next section reviews briefly the MSWF. Before the derivation of the Multi-Stage DFE in Section 4, we introduce the DFE in Section 3. Finally, in Section 5, we present simulation results of the application to an EDGE system. Throughout the paper the covariance matrix of a vector $\boldsymbol{x}[n]$ is denoted by $\boldsymbol{R}_{\boldsymbol{x}} = \mathrm{E}\{\boldsymbol{x}[n]\boldsymbol{x}^{\mathrm{H}}[n]\}$, the cross-correlation between the vectors $\boldsymbol{x}[n]$ and $\boldsymbol{y}[n]$ is $\boldsymbol{R}_{\boldsymbol{x},\boldsymbol{y}} = \mathrm{E}\{\boldsymbol{x}[n]\boldsymbol{y}^{\mathrm{H}}[n]\}$, the cross-correlation between a vector $\boldsymbol{x}[n]$ and a scalar $d[n]$ is $\boldsymbol{r}_{\boldsymbol{x},d} = \mathrm{E}\{\boldsymbol{x}[n]d^*[n]\}$, and the variance of a scalar $d[n]$ is $\sigma_d^2 = \mathrm{E}\{|d[n]|^2\}$. The operation '$(\cdot)^{\mathrm{H}}$' denotes conjugate transpose.

2. MULTI-STAGE WIENER FILTER

The *Wiener Filter* (WF) minimizes the *mean square error*

$$\xi_0(\boldsymbol{w}) = \sigma_{d_0}^2 - 2\,\mathrm{Re}\left\{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{r}_{\boldsymbol{x}_0,d_0}\right\} + \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{x}_0}\boldsymbol{w}, \qquad (1)$$

i. e. the variance of the error $d_0[n] - \hat{d}_0[n]$, where the estimate $\hat{d}_0[n] = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}_0[n]$ is obtained by applying the linear

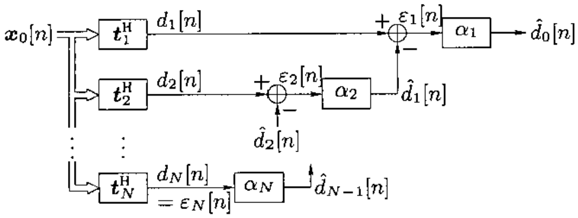

Fig. 1. MSWF as Filter Bank

filter $\boldsymbol{w} \in \mathbb{C}^N$ to the observation signal $\boldsymbol{x}_0[n] \in \mathbb{C}^N$. This design criterion leads to the *Wiener-Hopf equation*

$$\boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{w}_0 = \boldsymbol{r}_{\boldsymbol{x}_0,d_0}, \quad (2)$$

whose solution, the WF $\boldsymbol{w}_0$, achieves the *Minimum Mean Square Error* (MMSE) $\xi_0(\boldsymbol{w}_0) = \sigma_{d_0}^2 - \boldsymbol{r}_{\boldsymbol{x}_0,d_0}^{\mathrm{H}} \boldsymbol{R}_{\boldsymbol{x}_0}^{-1} \boldsymbol{r}_{\boldsymbol{x}_0,d_0}$.

Figure 1 sketches the block diagram of the *Multi-Stage Wiener Filter* (MSWF, [2]), an alternative representation of the WF. The first filter $\boldsymbol{t}_1$ is the normalized matched filter $\boldsymbol{r}_{\boldsymbol{x}_0,d_0}/\|\boldsymbol{r}_{\boldsymbol{x}_0,d_0}\|_2$ and the i-th filter $\boldsymbol{t}_i$ maximizes the real part of the correlation between its output $d_i[n]$ and the output $d_{i-1}[n]$ of the previous filter $\boldsymbol{t}_{i-1}$. If we restrict the filters $\boldsymbol{t}_i$ to be orthonormal, the i-th filter can be computed via the following optimization [4]:

$$\begin{aligned} \boldsymbol{t}_i &= \arg\max_{\boldsymbol{t}} \mathrm{E}\left\{\mathrm{Re}\left\{d_i[n] d_{i-1}^*[n]\right\}\right\} \\ \text{s.t.:} \quad & \boldsymbol{t}^{\mathrm{H}}\boldsymbol{t} = 1 \quad \text{and} \quad \boldsymbol{t}^{\mathrm{H}}\boldsymbol{t}_k = 0, \quad 1 \leq k < i. \end{aligned} \quad (3)$$

The solution is the *Arnoldi iteration* (e. g. [11])

$$\boldsymbol{t}_i = \frac{\left(\prod_{k=1}^{i-1} \boldsymbol{P}_k\right) \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{t}_{i-1}}{\left\|\left(\prod_{k=1}^{i-1} \boldsymbol{P}_k\right) \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{t}_{i-1}\right\|_2} \in \mathbb{C}^N, \quad (4)$$

with the projector $\boldsymbol{P}_k = \mathbf{1}_N - \boldsymbol{t}_k \boldsymbol{t}_k^{\mathrm{H}}$ onto the space orthogonal to $\boldsymbol{t}_k$ and the $N \times N$ identity matrix $\mathbf{1}_N$. Since $\boldsymbol{R}_{\boldsymbol{x}_0}$ is Hermitian, we can use the *Lanczos algorithm*

$$\boldsymbol{t}_i = \frac{\boldsymbol{P}_{i-1} \boldsymbol{P}_{i-2} \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{t}_{i-1}}{\|\boldsymbol{P}_{i-1} \boldsymbol{P}_{i-2} \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{t}_{i-1}\|_2}, \quad (5)$$

which leads to a tridiagonal covariance matrix $\boldsymbol{R}_{\boldsymbol{d}}$ of the pre-filtered observation vector $\boldsymbol{d}[n] = [d_1[n], \ldots, d_N[n]]^{\mathrm{T}}$. The scalar WFs α_i estimate the output of the previous filter $d_{i-1}[n]$ from the error signal $\varepsilon_i[n]$.

The MSWF of rank D is obtained by neglecting the signal $\hat{d}_D[n]$. Thus, the D-dimensional observation vector $\boldsymbol{d}^{(D)}[n] = \boldsymbol{T}^{(D),\mathrm{H}} \boldsymbol{x}_0[n]$, where the pre-filter matrix $\boldsymbol{T}^{(D)} = [\boldsymbol{t}_1, \ldots, \boldsymbol{t}_D] \in \mathbb{C}^{N \times D}$ implies only the first D filter vectors. The reduced dimension WF $\boldsymbol{w}_{\boldsymbol{d}}^{(D)} \in \mathbb{C}^D$ estimates $d_0[n]$ from $\boldsymbol{d}^{(D)}[n]$, and the rank D approximation of the full-dimensional WF $\boldsymbol{w}_0^{(D)} = \boldsymbol{T}^{(D)} \boldsymbol{w}_{\boldsymbol{d}}^{(D)} \in \mathbb{C}^N$ can be expressed as

$$\boldsymbol{w}_0^{(D)} = \boldsymbol{T}^{(D)} \left(\boldsymbol{T}^{(D),\mathrm{H}} \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{T}^{(D)}\right)^{-1} \boldsymbol{T}^{(D),\mathrm{H}} \boldsymbol{r}_{\boldsymbol{x}_0,d_0}, \quad (6)$$

which yields to the mean square error

$$\xi_0\left(\boldsymbol{w}_0^{(D)}\right) = \sigma_{d_0}^2 - \boldsymbol{w}_0^{(D),\mathrm{H}} \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{w}_0^{(D)}. \quad (7)$$

Note that the rank D MSWF is equivalent [3, 4] to the solution of the Wiener-Hopf equation in the D-dimensional *Krylov subspace* $\mathcal{K}^{(D)}(\boldsymbol{R}_{\boldsymbol{x}_0}, \boldsymbol{r}_{\boldsymbol{x}_0,d_0})$ where $\mathcal{K}^{(D)}(\boldsymbol{A}, \boldsymbol{b}) = \mathrm{span}\{[\boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \ldots, \boldsymbol{A}^{(D-1)}\boldsymbol{b}]\}$.

3. MMSE DECISION FEEDBACK EQUALIZER

The *Decision Feedback Equalizer* (DFE, [8, 7]) considers not only the observation signal $\boldsymbol{x}_0[n]$ but also prior symbol decisions $\tilde{\boldsymbol{d}}_0[n-1] = \boldsymbol{\delta}[n-1] * \tilde{d}_0[n]$ with $\tilde{d}_0[n] = Q(\hat{d}_0[n])$, to estimate the desired signal $d_0[n]$, i. e. $\hat{d}_0[n] = \boldsymbol{g}^{\mathrm{H}} \boldsymbol{x}_0[n] + \boldsymbol{f}^{\mathrm{H}} \tilde{\boldsymbol{d}}_0[n-1]$. The operation '$*$' denotes convolution, the function $Q(\cdot)$ quantization or hard decision, and $\boldsymbol{\delta}[n-1] = [\delta[n-1], \ldots, \delta[n-M]]^{\mathrm{T}} \in \{0,1\}^M$.

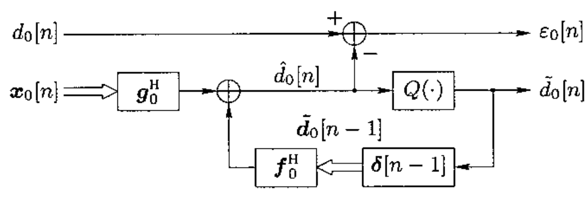

Fig. 2. MMSE Decision Feedback Equalizer

In this paper, we restrict ourselves to the WF version of the DFE depicted in Figure 2. Thus, the forward filter $\boldsymbol{g}_0 \in \mathbb{C}^N$ and the feedback filter $\boldsymbol{f}_0 \in \mathbb{C}^M$ are the results of the minimization of the mean square error $\xi_{\mathrm{DFE}}(\boldsymbol{g}, \boldsymbol{f}) = \mathrm{E}\{|d_0[n] - \hat{d}_0[n]|^2\}$, i. e.

$$\{\boldsymbol{g}_0, \boldsymbol{f}_0\} = \arg\min_{\{\boldsymbol{g},\boldsymbol{f}\}} \xi_{\mathrm{DFE}}(\boldsymbol{g}, \boldsymbol{f}). \quad (8)$$

For the solution, we assume that the previous M symbols have been decided correctly: $\tilde{\boldsymbol{d}}_0[n-1] = \boldsymbol{d}_0[n-1]$. Thus, $\boldsymbol{r}_{\tilde{\boldsymbol{d}}_0,d_0} = \mathbf{0}$, and $\boldsymbol{R}_{\tilde{\boldsymbol{d}}_0} = \sigma_{d_0}^2 \mathbf{1}_M$ and the mean square error can be written as

$$\begin{aligned} \xi_{\mathrm{DFE}}(\boldsymbol{g}, \boldsymbol{f}) = \sigma_{d_0}^2 &- 2\,\mathrm{Re}\left\{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{r}_{\boldsymbol{x}_0,d_0}\right\} + \boldsymbol{g}^{\mathrm{H}} \boldsymbol{R}_{\boldsymbol{x}_0} \boldsymbol{g} \\ &+ 2\,\mathrm{Re}\left\{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0} \boldsymbol{f}\right\} + \sigma_{d_0}^2 \boldsymbol{f}^{\mathrm{H}} \boldsymbol{f}. \end{aligned} \quad (9)$$

We define $\boldsymbol{R}_{\bar{\boldsymbol{x}}_0} = \boldsymbol{R}_{\boldsymbol{x}_0} - \sigma_{d_0}^{-2} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0}^{\mathrm{H}}$ and get the forward and feedback filters

$$\boldsymbol{g}_0 = \boldsymbol{R}_{\bar{\boldsymbol{x}}_0}^{-1} \boldsymbol{r}_{\boldsymbol{x}_0,d_0} \quad \text{and} \quad \boldsymbol{f}_0 = -\sigma_{d_0}^{-2} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0}^{\mathrm{H}} \boldsymbol{g}_0, \quad (10)$$

respectively, achieving the MMSE of the DFE

$$\xi_{\mathrm{DFE}}(\boldsymbol{g}_0, \boldsymbol{f}_0) = \sigma_{d_0}^2 - \boldsymbol{r}_{\boldsymbol{x}_0,d_0}^{\mathrm{H}} \boldsymbol{R}_{\bar{\boldsymbol{x}}_0}^{-1} \boldsymbol{r}_{\boldsymbol{x}_0,d_0}. \quad (11)$$

If we use Equation (10), we get the following expression for the estimate: $\hat{d}_0[n] = \boldsymbol{g}_0^{\mathrm{H}}(\boldsymbol{x}_0[n] - \sigma_{d_0}^{-2} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0} \tilde{\boldsymbol{d}}_0[n-$

$1]) = \boldsymbol{g}_0^{\mathrm{H}} \bar{\boldsymbol{x}}_0[n]$. This leads to an alternative structure of the DFE shown in Figure 3. There, the forward filter estimates the desired signal $d_0[n]$ from the transformed observation $\bar{\boldsymbol{x}}_0[n]$, where the part of the interference in the original observation vector $\boldsymbol{x}_0[n]$ caused by the transmission of prior symbol decisions $\tilde{\boldsymbol{d}}_0[n-1]$ is removed.

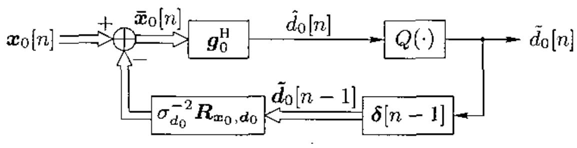

Fig. 3. Alternative DFE Structure

4. MULTI-STAGE MMSE DECISION FEEDBACK EQUALIZER

The previous defined matrix $\boldsymbol{R}_{\bar{\boldsymbol{x}}_0}$ is the covariance matrix of the transformed observation $\bar{\boldsymbol{x}}_0[n]$. Besides, it can easily be shown that $\boldsymbol{r}_{\boldsymbol{x}_0,d_0} = \boldsymbol{r}_{\bar{\boldsymbol{x}}_0,d_0}$. Thus, the forward filter $\boldsymbol{g}_0$ in Equation (10) is the solution of the Wiener-Hopf equation $\boldsymbol{R}_{\bar{\boldsymbol{x}}_0}\boldsymbol{g}_0 = \boldsymbol{r}_{\bar{\boldsymbol{x}}_0,d_0}$, and therefore the WF to estimate the desired signal $d_0[n]$ from the transformed observation $\bar{\boldsymbol{x}}_0[n]$.

Now, we approximate the WF $\boldsymbol{g}_0$ by the rank D MSWF as described in Section 2 and get the rank D *Multi-Stage Decision Feedback Equalizer* (MSDFE) with the forward filter

$$\boldsymbol{g}_0^{(D)} = \bar{\boldsymbol{T}}^{(D)} \left(\bar{\boldsymbol{T}}^{(D),\mathrm{H}} \boldsymbol{R}_{\bar{\boldsymbol{x}}_0} \bar{\boldsymbol{T}}^{(D)} \right)^{-1} \bar{\boldsymbol{T}}^{(D),\mathrm{H}} \boldsymbol{r}_{\boldsymbol{x}_0,d_0}, \quad (12)$$

and the feedback filter

$$\boldsymbol{f}_0^{(D)} = -\sigma_{d_0}^{-2} \boldsymbol{R}_{\boldsymbol{x}_0,\boldsymbol{d}_0}^{\mathrm{H}} \boldsymbol{g}_0^{(D)}. \quad (13)$$

The columns of the pre-filter matrix $\bar{\boldsymbol{T}}^{(D)}$ are the base vectors of the Krylov subspace $\mathcal{K}^{(D)}(\boldsymbol{R}_{\bar{\boldsymbol{x}}_0}, \boldsymbol{r}_{\boldsymbol{x}_0,d_0})$ and may be computed by the Lanczos algorithm as shown in Section 2. The rank D MSDFE achieves the mean square error (cf. Equation 9)

$$\xi_{\mathrm{DFE}} \left(\boldsymbol{g}_0^{(D)}, \boldsymbol{f}_0^{(D)} \right) = \sigma_{d_0}^2 - \boldsymbol{g}_0^{(D),\mathrm{H}} \boldsymbol{R}_{\bar{\boldsymbol{x}}_0} \boldsymbol{g}_0^{(D)}. \quad (14)$$

5. APPLICATION TO AN EDGE SYSTEM

In the following, we consider an EDGE system with 8PSK modulation and *Laurent pulse shaping*. The Laurent impulse is a linearized GMSK impulse [12] which has a duration of five symbol times. Thus, we have severe intersymbol interference even without channel distortion. The symbol time $T = 3.69\,\mu$s and the two antennae of the mobile station receive the signal of a base station which propagates over Rayleigh multipath fading channels with a delay spread of $\tau_{\max} = 10\,\mu$s or three symbol times. Note that the second antenna is not really necessary but it makes the problem of equalization easier but also increases dimensionality by a factor of two. We assume a constant channel during one burst with 148 symbols (excluding guard symbols).

The MSWF for a linear equalizer and the MSDFE are used as equalizer filters for the received signal at the mobile station. We sample two times during one symbol duration. If we use the MSWF, we take 20 samples of each antenna to build the space-time observation vector $\boldsymbol{x}_0[n]$, thus, its dimension $N = 40$ and if we choose the MSDFE, we take only 15 samples, i.e. $N = 30$, because of the additional $M = 10$ feedback taps. We keep the total number of taps $N + M$ constant in order to compare fairly both equalizers ($M = 0$ for the MSNWF).

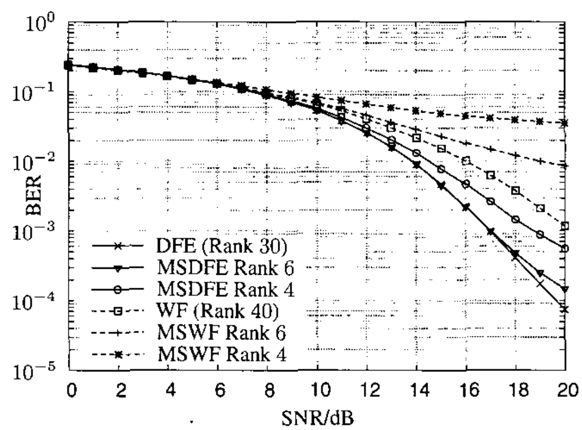

Fig. 4. BER for known channel

Figure 4 plots the raw *Bit Error Rate* (BER) over the *Signal to Noise Ratio* (SNR) in dB for either the MSWF or the MSDFE with different ranks D. We assume perfect channel knowledge at the mobile station. Firstly, the nonlinear MSDFE outperforms the corresponding linear MSWF with the same rank D. Moreover, the MSDFE with rank 4 is even better than the optimum WF despite the enormous reduction in computational complexity. Secondly, the rank 6 MSDFE is a good approximation of the optimum DFE whereas the BER of the rank 6 MSWF is still higher than the BER of the optimum WF at the same SNR.

This fact is further confirmed by Figure 5 which shows the BER of the MSWF and the MSDFE over the rank D at SNR $= 15$ dB. It can be seen that subject to D, the MSDFE converges much faster to the optimum than the MSWF. This is due to the fact that the observation vector of the MSDFE ($N = 30$) has less dimensions than the observation vector of the MSWF ($N = 40$). Note that despite the smaller forward filter length, the rank D MSDFE has a lower BER than the rank D MSWF for all D.

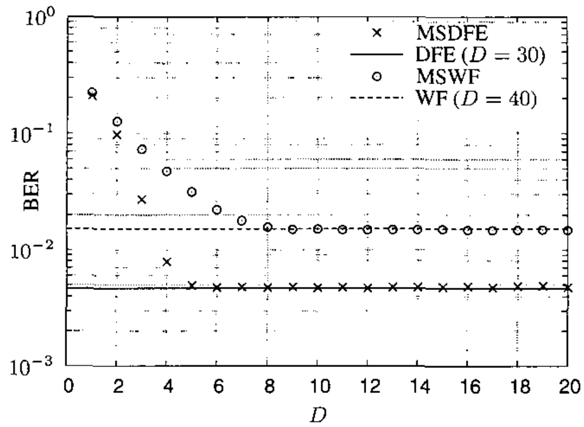

Fig. 5. BER for known channel at SNR = 15 dB

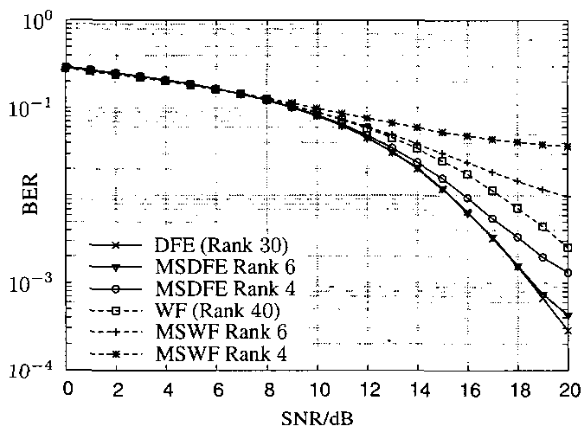

Fig. 6. BER for estimated channel

Finally, we see the BER over SNR for either the MSWF or the MSDFE in Figure 6, where the necessary statistics is derived from a channel matrix estimated via *least squares method* [13] from the 26 training symbols of a burst. The results are slightly worse than the simulation results for known channel in Figure 4, but again, the MSDFE beats the MSWF in performance or computational complexity.

Besides lowering computational complexity, the MSWF improves performance in cases of low sample support where there are not enough training symbols or data snapshots to average over in estimating either the cross-correlation vector or the covariance matrix. However, the simulation examples presented in this paper had adequate sample support so that we focus mainly on the reduction in computational complexity.

6. CONCLUSIONS

In this paper, we derived a reduced rank DFE based on the MSWF. Simulation results of an application to an EDGE system showed that the MSDFE outperforms the even more computational intensive optimum linear WF. Besides, less dimensions D are necessary to achieve the performance of the corresponding full rank optimum filter, i. e. the DFE for the MSDFE and the WF for the MSWF.

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