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Physical Layer Characterization of a Multi-User MISO System by Efficient Outage Probabilities

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Abstract— **In bottom-up cross-layer optimization, each layer offers a set of feasible operation points to the layer above, enabling the application to decide which parameter setup is optimum. In this paper, the capabilities of the physical layer in a multi-user MISO system are described in terms of the users' outage probabilities. Optimum descriptions are given by sets of efficient outage probability tuples. We first derive a closed-form expression for a user's outage probability in the presence of multi-user interference. Next, the set of efficient outage probability tuples under a simplified SDMA transmit strategy is derived. We show how a good approximate description of the efficient set is found by equidistant sampling. Finally, the performance of the proposed transmit strategy is compared with an orthogonal multiple access scheme, demonstrating that optimum mode selection at the physical layer can lead to significant performance gains at the application layer.**

I. Introduction

A key challenge in the design of future wireless communication systems is the efficient provision of a multitude of different applications, such as voice, data and real-time video. Different applications have different requirements. Optimum performance can only be achieved if the choice of operation point in each layer takes into account the properties of the different applications. This observation leads directly to the concept of cross-layer optimization. As we want to preserve the layered architecture, we do not consider a global decision function that jointly optimizes the parameters of all layers for determining the overall optimum operation point. We also do not regard a *top-down* approach, where the upper layers formulate QoS targets lower layers are supposed to meet with minimum effort (e.g. [1]). The optimum solution may thereby not always be achieved, if the upper layers offer some flexibility with respect to the QoS targets. Instead a *bottom-up* approach ([2], [3]) is applied, where, beginning at the physical layer, each layer offers a set of operation points, the *efficient set*, to the the layer above, which processes these points to determine its own efficient set. Finally, the application chooses which operation point fits its needs best and communicates this choice to the lower layers, which can then adjust their parameters accordingly. As thereby each layer possesses no specific knowledge about the upper layers and their requirements, an operation point is considered as efficient, if one user can only be further improved by deteriorating at least another one. This definition implies no single optimum solution but results in a set of solution points. *Multi-objective optimization* (MOO) (e.g. [4]) provides the mathematical framework for determining these efficient sets. In this paper the physical layer of a *multi-user multiple-input single-output* (MU-MISO) system is analyzed with respect to this approach. We assume that only second order statistics are known at the transmitter. As in this case outage probabilities are a suitable information-theoretic performance measure [5], we take the users' outage probabilities to characterize the physical layer in the efficient set. After introducing the system model in Section II and explaining the problem setup in Section III, we derive and simplify a closed form expression for the outage probabilities in a flat-fading Rayleigh environment, when *multiple access interference* (MAI) occurs, in Section IV. We then confine our considerations to a two user scenario and for that it is shown in Section V how the efficient set can be analytically determined. In Section VI orthogonal multiple access schemes are applied to suppress interference, and outage probabilities and efficient set are determined in that case. As both efficient sets contain an infinite number of elements and a finite discrete representation is desired for the bottom-up approach [6], a way to sample the outage probabilities nearly equidistantly is presented in Section VII. Finally the physical layer will be incorporated into a simple application scenario in Section VIII.

Notation: $(\bullet)^{\mathrm{T}}$, $(\bullet)^{\mathrm{H}}$, $\mathrm{E}(\bullet)$, and $\mathrm{tr}(\bullet)$ denote transposition, conjugate transposition, expectation and trace of a matrix, respectively.

II. System model

We consider a MU-MISO system with M_{t} transmit antennas and K non-cooperating receivers, as shown in Figure 1. The

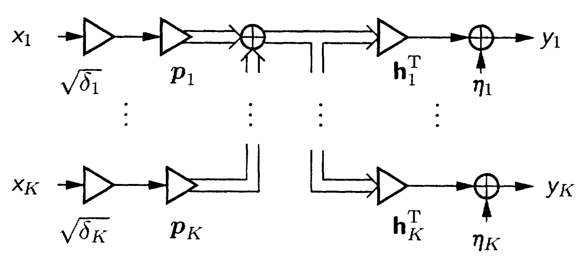

Figure 1. System model

symbols x_k, where $k = 1, ..., K$ denotes the user index, result from separate coding with Gaussian codebooks for each user. Thereby it is assumed that the codes are capacity-achieving. Furthermore all x_k are independent and identically distributed (i.i.d) Gaussian variables with unit variance. They are multiplied with the square root of the corresponding transmit power δ_k first, whereby the total transmit power is limited to P_{tr}, i.e. $\sum_{k=1}^{K} \delta_k \leq P_{\mathrm{tr}}$. $\boldsymbol{p}_k \in \mathbb{C}^{M_t \times 1}$ are the normalized filter vectors with $\|\boldsymbol{p}_k\|_2 = 1$. The wireless link from the base station to user k, for which we assume flat Rayleigh block fading, is modeled by multivariate circularly symmetric Gaussian variables $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{R}_k)$. The signals are corrupted by additive white Gaussian noise $\eta_k \sim \mathcal{CN}(0; \sigma_k^2)$.

III. Problem Setup

With partial channel knowledge available at the transmitter, i.e. the covariance matrices $\boldsymbol{R}_k$ are known, we seek to minimize the outage probabilities $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_K)$ of all users. We thereby assume that the rates $R_1, \ldots, R_K$, which are measured in *bits per channel use* (bpcu), are fixed and given. The optimum transmission strategy would result from a joint optimization of the filter vectors $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K$ and power allocation $\delta_1, \ldots, \delta_K$. However, while the transmission strategy that minimizes outage probability is known for single-user systems [7], for multi-user systems the optimum transmission strategy is, to the authors' knowledge, an open problem. In the following, the problem is substantially simplified by choosing the filter vectors *a priori* and only optimizing the power allocation. Motivated by the results for single-user systems [7] , we choose the filter vectors $\boldsymbol{p}_k$ such that $\boldsymbol{p}_k^*$ corresponds to the strongest eigenvalue of $\boldsymbol{R}_k$. That is also known as long-term eigenbeamforming. Hence the transmit powers δ_k remain as parameters the physical layer can adjust to minimize outage probabilities. That leads to the following MOO problem:

$$\hat{\boldsymbol{\varepsilon}} = \min_{\delta_1, \ldots, \delta_K} (\boldsymbol{\varepsilon}) \quad \text{s.t.} \quad \sum_{k=1}^{K} \delta_k \leq P_{\mathrm{tr}} \quad \text{and} \quad \delta_k \geq 0 \quad \forall k.$$

IV. Computation of Outage Probabilities

The outage probability ε_k is defined as the probability that the given rate R_k can not be reliably transmitted over an instantaneous channel realization. Hence, the probability, that the mutual information $I_k(x_k, y_k)$ is smaller than R_k has to be computed:

$$\varepsilon_k = \mathsf{P}\left(I_k(x_k, y_k) \leq R_k\right) \tag{1}$$

Taking into account that the users' symbols are coded independently with Gaussian codebooks, the interference can be considered as additional Gaussian noise and the mutual information can be obtained from

$$I_k(x_k, y_k) = \log_2 \left(1 + \frac{\delta_k \mathbf{h}_k^{\mathrm{H}} \boldsymbol{P}_k \mathbf{h}_k}{\sigma_k^2 + \sum_{\substack{i=1 \\ i \neq k}}^{K} \delta_i \mathbf{h}_k^{\mathrm{H}} \boldsymbol{P}_i \mathbf{h}_k} \right),$$

where

$$\boldsymbol{P}_i = \boldsymbol{p}_i^* \boldsymbol{p}_i^{\mathrm{T}} \in \mathbb{C}^{M_t \times M_t}$$

are Hermitian and positive semidefinite matrices. Inserting these results into (1) and applying some transformations yields:

$$\varepsilon_k = \mathsf{P}\left(\mathbf{h}_k^{\mathrm{H}} \boldsymbol{P}_k' \mathbf{h}_k \leq (2^{R_k} - 1)\sigma_k^2\right),$$

with

$$\boldsymbol{P}_k' = \delta_k \boldsymbol{P}_k - \sum_{\substack{i=1 \\ i \neq k}}^{K} \delta_i \boldsymbol{P}_i (2^{R_k} - 1).$$

Hence, the cumulative distribution function (cdf) of the variable $y_k = \mathbf{h}_k^{\mathrm{H}} \boldsymbol{P}_k' \mathbf{h}_k$ has to be evaluated at the point $Y_k = (2^{R_k} - 1)\sigma_k^2$, where y_k is a quadratic form in normal random variables. For this purpose two linear transforms have to be applied such that y_k can be written as a linear combination of statistically independent normal complex variables $(\hat{\mathbf{h}}_k)_j$ with zero mean and unit variance:

$$y_k = \sum_{j=1}^{M_t} \lambda_{j,k} |(\hat{\mathbf{h}}_k)_j|^2.$$

$(\hat{\mathbf{h}}_k)_j$ is the j-th element of the transformed variable vector $\hat{\mathbf{h}}_k = \boldsymbol{V}_k^{\mathrm{H}} \boldsymbol{R}_k^{-\frac{1}{2}} \mathbf{h}_k \in \mathbb{C}^{M_t \times 1}$, $\lambda_{j,k}$ are the eigenvalues of the matrix $\boldsymbol{R}_k^{\frac{1}{2}} \boldsymbol{P}_k' \boldsymbol{R}_k^{\frac{1}{2}}$ and $\boldsymbol{V}_k$ contains the corresponding eigenvectors and is orthonormal. Due to the fact, that $\boldsymbol{R_k}^{\frac{1}{2}}$, $\boldsymbol{P}_k'$ and therefore $\boldsymbol{R}_k^{\frac{1}{2}} \boldsymbol{P}_k' \boldsymbol{R}_k^{\frac{1}{2}}$ are Hermitian matrices, this transformation is always possible, and all $\lambda_{j,k}$ are real. The vector $\hat{\mathbf{h}}_k$ is still circularly symmetric as it results from an linear transformation of the circularly symmetric channel vector $\mathbf{h}_k$. Therefore real and imaginary part of $(\hat{\mathbf{h}}_k)_j$ are statistically independent normal random variables with variance 0.5 and

$$y_k = \sum_{j=1}^{M_t} \lambda_{j,k} \left[\left(\mathrm{Re}\{(\hat{\mathbf{h}}_k)_j\} \right)^2 + \left(\mathrm{Im}\{(\hat{\mathbf{h}}_k)_j\} \right)^2 \right]$$

turns out to be a weighted sum of central chi-square-distributed random variables. In a final transformation the variances of the Gaussian variables underlying the chi-square-distributed random variables are set to 1. Then Lemma 4.3b.1 in [8] can be applied and $\varepsilon_k = \mathsf{P}(y_k \leq Y_k)$ can be computed according to:

$$\varepsilon_k = 1 - \sum_{j=1}^{s_k} \frac{\lambda_{j,k}^{M_t - 1}}{\prod_{\substack{\ell=1 \\ \ell \neq j}}^{M_t} (\lambda_{j,k} - \lambda_{\ell,k})} \exp\left(-(2^{R_k} - 1) \frac{\sigma_k^2}{\lambda_{j,k}} \right). \tag{2}$$

In Equation (2) $\lambda_{j,k}$ have been sorted, such that the s_k positive eigenvalues are indexed by $j = 1, \ldots, s_k$. For the rare case of not distinct positive eigenvalues $\lambda_{j,k}$ Equation (2) can not be applied. A general closed form expression for ε_k can then be derived from Lemma 4.3b.3 in [8]. Note that (2) provides no explicit relationship between the parameters δ_k and the outage probabilities.

If the number of transmit antennas M_t is greater or equal to the number of users K and the filter vectors $\boldsymbol{p}_i$ are linearly independent, (2) can be simplified by applying the following transformation:

$$\boldsymbol{R}_k^{\frac{1}{2}} \boldsymbol{P}_k' \boldsymbol{R}_k^{\frac{1}{2}} = \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{D}_k \boldsymbol{S}_k, \tag{3}$$

with the quadratic matrices

$$\boldsymbol{D}_k = \mathrm{diag}\left(-\delta_1 \frac{Y_k}{\sigma_k^2}, \ldots, \delta_k, \ldots, -\delta_K \frac{Y_k}{\sigma_k^2}, 0, \ldots, 0 \right) \in \mathbb{C}^{M_t \times M_t}$$

and

$$\boldsymbol{S}_k = \left[\boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{p}_1, ..., \boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{p}_K, \boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{Q}_k\right]^{\mathrm{T}} \in \mathbb{C}^{M_t \times M_t}.$$

$\boldsymbol{Q}_k \in \mathbb{C}^{M_t \times (M_t - K)}$ is chosen such that $\boldsymbol{S}_k$ is non-singular. Now, the "Hermitian analogon of *Sylvester's Law of Inertia*" (c.f. [9], p.116) can be applied, which states that in any diagonalization of a complex matrix, the number of positive and negative entries in the diagonal matrix is the same. Therefore the signature of $\boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{P}'_k\boldsymbol{R}_k^{\frac{1}{2}}$ can be obtained from Equation (3): As $Y_k = (2^{R_k} - 1)\sigma_k^2 > 0$ and all powers $\delta_i > 0$, $\boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{P}'_k\boldsymbol{R}_k^{\frac{1}{2}}$ exhibits one positive and $K-1$ negative eigenvalues, while $M_t - K$ eigenvalues are equal to zero. Applying this result to Equation (2) yields

$$\varepsilon_k = 1 - \frac{\lambda_{1,k}^{K-1}}{\prod\limits_{\substack{\ell=1 \\ \ell \neq j}}^{K} (\lambda_{1,k} - \lambda_{\ell,k})} \exp\left(-(2^{R_k} - 1)\frac{\sigma_k^2}{\lambda_{1,k}}\right). \quad (4)$$

For the rest of the paper we mainly restrict our considerations to the case of $K = 2$ users for sake of simplicity. For $K = 2$, (4) reads as:

$$\varepsilon_k = 1 - \frac{\lambda_{1,k}}{(\lambda_{1,k} - \lambda_{2,k})} \exp\left(-(2^{R_k} - 1)\frac{\sigma_k^2}{\lambda_{1,k}}\right), \quad (5)$$

where $\lambda_{1,k}$ is the positive and $\lambda_{2,k}$ the negative eigenvalue. Now the fact that $\boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{P}'_k\boldsymbol{R}_k^{\frac{1}{2}}$ is a matrix of rank two can be exploited to compute its eigenvalues and it is possible to derive expressions showing explicitly the dependencies between the eigenvalues and transmit powers δ_i. To simplify notation, the expressions are only given for user 1, for user 2 an exchange of the indices 1 and 2 is necessary:

$$\lambda_{1/2} = \frac{1}{2}\Big[\delta_1\rho_1 - \delta_2(2^{R_1} - 1)\rho_{1,n} \pm \sqrt{\delta_1^2 a + \delta_2^2(2^{R_1} - 1)^2 b - \delta_1\delta_2(2^{R_1} - 1)c}\Big],$$

with $\rho_1 = \mathrm{E}\left[|\mathbf{h}_1^{\mathrm{T}}\boldsymbol{p}_1|^2\right]$, $\rho_{1,n} = \mathrm{E}\left[|\mathbf{h}_1^{\mathrm{T}}\boldsymbol{p}_2|^2\right]$, $a = f(\boldsymbol{R}_1, \boldsymbol{p}_1)$, $b = f(\boldsymbol{R}_1, \boldsymbol{p}_2)$ and $c = f(\boldsymbol{R}_1, \boldsymbol{p}_1, \boldsymbol{p}_2)$.
With these expressions, a direct computation of outage probabilities from transmit powers is possible. This is needed in Section VII when sampling the efficient set.

V. Determination of the Efficient Set

After deriving the relationship between the parameters of the physical layer and the outage probabilities in the previous section, those parameter setups still need to be determined which lead to efficient outage probabilities. Before that we summarize the basic concepts of multi-objective optimization in the first subsection. Readers familiar with MOO can skip it and immediately continue with Subsection B.

A. Basics of Multi-objective Optimization

If, as mentioned in Section III, the minimization of the vector-valued cost function $\boldsymbol{f} : \boldsymbol{\delta} \longmapsto \boldsymbol{\varepsilon} = \boldsymbol{f}(\boldsymbol{\delta})$, $\mathbb{R}^K \rightarrow \mathbb{R}^K$ is considered, it is necessary to make a statement about the optimality of a point $\boldsymbol{f}(\boldsymbol{\delta})$. For this purpose an order relation on $\mathbb{R}^K$ is required [10]. We use the following definition of a strict order "$\prec$":

$$\boldsymbol{\varepsilon}^1 \prec \boldsymbol{\varepsilon}^2 \iff \varepsilon_k^1 \leq \varepsilon_k^2 \;\; \forall k \;\; \wedge \;\; \exists k : \varepsilon_k^1 < \varepsilon_k^2$$

Note that "$\prec$" is a partial order, i.e. there exist elements $\boldsymbol{\varepsilon}^1, \boldsymbol{\varepsilon}^2$ with $\boldsymbol{\varepsilon}^1 \neq \boldsymbol{\varepsilon}^2$, for which neither $\boldsymbol{\varepsilon}^1 \prec \boldsymbol{\varepsilon}^2$ nor $\boldsymbol{\varepsilon}^2 \prec \boldsymbol{\varepsilon}^1$ is true.

Let $\mathcal{G}$ and $\boldsymbol{f}(\mathcal{G})$ denote the feasible set and the image of the feasible set under $\boldsymbol{f}$, respectively. Here $\mathcal{G}$ is given by the transmit power constraint: $\mathcal{G} = \left\{\boldsymbol{\delta} \in \mathbb{R}_{0,+}^K : \|\boldsymbol{\delta}\|_1 \leq P_{\mathrm{tr}}\right\}$. Now assume that there exist points $\boldsymbol{\varepsilon}^1, \boldsymbol{\varepsilon}^2 \in \boldsymbol{f}(\mathcal{G})$ that are smaller (with respect to $\prec$) than any $\boldsymbol{\varepsilon} \in \boldsymbol{f}(\mathcal{G})$ they can be compared to, but for which neither $\boldsymbol{\varepsilon}^1 \prec \boldsymbol{\varepsilon}^2$ nor $\boldsymbol{\varepsilon}^2 \prec \boldsymbol{\varepsilon}^1$ is true. There is no reason for differentiating between $\boldsymbol{\varepsilon}^1$ and $\boldsymbol{\varepsilon}^2$, and thus both are considered valid solutions of the minimization. Based on this generalized concept of optimality, two sets are defined: The efficient set

$$\mathcal{E} = \{\boldsymbol{\varepsilon} \in \boldsymbol{f}(\mathcal{G}) \mid \nexists \boldsymbol{\varepsilon}' \in \boldsymbol{f}(\mathcal{G}) : \boldsymbol{\varepsilon}' \prec \boldsymbol{\varepsilon}\},$$

which contains all the points that are smaller than all points they can be compared to, and the corresponding Pareto set

$$\mathcal{P} = \{\boldsymbol{\delta} \in \mathcal{G} \mid \boldsymbol{f}(\boldsymbol{\delta}) \in \mathcal{E}\}.$$

Points $\boldsymbol{\delta} \in \mathcal{P}$ are called Pareto optimal. Based on these two sets, we can define the operators min and argmin for multi-objective optimization as follows:

$$\min_{\boldsymbol{\delta} \in \mathcal{G}} \boldsymbol{f}(\boldsymbol{\delta}) \equiv \mathcal{E}, \quad \operatorname*{argmin}_{\boldsymbol{\delta} \in \mathcal{G}} \boldsymbol{f}(\boldsymbol{\delta}) \equiv \mathcal{P}.$$

B. Determination of the Pareto Set

Applying these mathematical definitions to the current system, the Pareto set $\mathcal{P}$ for $K = 2$ users is given by

$$\mathcal{P} = \left\{(\delta_1, \delta_2) \in \mathbb{R}_{0,+}^2 : \delta_1 + \delta_2 = P_{\mathrm{tr}}\right\},$$

and the efficient set can be obtained by plugging the Pareto optimal transmit powers into Equation (5).
Proof:
We now consider a second method to compute the outage probability ε_1: Recalling from Section IV that

$$\varepsilon_1 = \mathsf{P}\left(\log_2\left[1 + \frac{\delta_1 \tilde{h}_{1,1}}{\sigma_1^2 + \delta_2 \tilde{h}_{1,2}}\right] \leq R_1\right),$$

where $\tilde{h}_{1,1} = |\mathbf{h}_1^{\mathrm{T}}\boldsymbol{p}_1|^2$ and $\tilde{h}_{1,2} = |\mathbf{h}_1^{\mathrm{T}}\boldsymbol{p}_2|^2$, ε_1 can be computed with the joint probability density function $\mathsf{p}(\tilde{h}_{1,1}, \tilde{h}_{1,2})$ according to:

$$\varepsilon_1 = \int_0^{\infty}\int_0^{\gamma_1} \mathsf{p}(\tilde{h}_{1,1}, \tilde{h}_{1,2})\, \mathrm{d}\tilde{h}_{1,1}\, \mathrm{d}\tilde{h}_{1,2}. \quad (6)$$

In Equation (6) only

$$\gamma_1 = (2^{R_1} - 1)\frac{1}{\delta_1}(\sigma_1^2 + \delta_2 \tilde{h}_{1,2})$$

depends on the transmit powers. Furthermore, as $0 \leq \mathrm{p}(\tilde{h}_{1,1}, \tilde{h}_{1,2}) \leq 1$, ε_1 is monotonically increasing in γ_1. Therefore it is sufficient to consider γ_1 instead of ε_1 to determine $\mathcal{P}$. The same is true for ε_2 and $\gamma_2 = (2^{R_2} - 1)\frac{1}{\delta_2}(\sigma_2^2 + \delta_1 \tilde{h}_{2,1})$. Consequently, Pareto optimality of (δ_1, δ_2) is achieved if one of the following inequalities cannot be fulfilled by feasible transmit powers δ_1' and δ_2', where in at least one inequality "<" must hold:

$$\frac{(\sigma_1^2 + \delta_2' \tilde{h}_{1,2})}{\delta_1'} \leq \frac{(\sigma_1^2 + \delta_2 \tilde{h}_{1,2})}{\delta_1}. \tag{7}$$

$$\frac{(\sigma_2^2 + \delta_1' \tilde{h}_{2,1})}{\delta_2'} \leq \frac{(\sigma_2^2 + \delta_1 \tilde{h}_{2,1})}{\delta_2}. \tag{8}$$

First, "<" is achieved in (7) by choosing $\delta_1' = \delta_1 + \Delta\delta_1 > \delta_1$. Plugging δ_1' into (8) and transforming the inequality yields:

$$\delta_2' \geq \frac{\sigma_2^2 + (\delta_1 + \Delta\delta_1)\tilde{h}_{2,1}}{\sigma_2^2 + \delta_1 \tilde{h}_{2,1}} \cdot \delta_2. \tag{9}$$

Taking into account, that $\Delta\delta_1$ was chosen to be positive, δ_2' must be greater than δ_2, as the factor it is multiplied with in (9) is greater than one. If $\delta_1 + \delta_2 = P_{\mathrm{tr}}$, then, due to the power constraint, the inequality $\delta_2' \leq \delta_2 - \Delta\delta_1$ must hold and hence $\delta_2' < \delta_2$, what violates (9). Similarly it can be shown that (7) cannot be fulfilled if δ_2 is increased while δ_1 must be reduced. For the special case $\delta_1 = 0$, $\varepsilon_1 = 1$ and (8) can only be fulfilled if $\delta_2 < P_{\mathrm{tr}}$. Finally it is proved that for $\delta_1 + \delta_2 < P_{\mathrm{tr}}$ there always exits a tuple (δ_1', δ_2') that fulfills the inequalities mentioned above. Let $\delta_1' = \delta_1 + \Delta\delta_1$ and $\delta_2' = \delta_2 + \Delta\delta_2$ with $\Delta\delta_1, \Delta\delta_2 > 0$. Plugging δ_1' and δ_2' into (7) and (8) results in the inequality:

$$\frac{\delta_2}{\frac{\sigma_2^2}{\tilde{h}_{2,1}} + \delta_1} \leq \frac{\Delta\delta_2}{\Delta\delta_1} \leq \frac{\frac{\sigma_1^2}{\tilde{h}_{1,2}} + \delta_2}{\delta_1}.$$

By an appropriate choice of $\Delta\delta_1 > 0$ and $\Delta\delta_2 > 0$, this inequality can always be fulfilled and therefore $\varepsilon_1(\delta_1', \delta_2') < \varepsilon_1(\delta_1, \delta_2)$ and $\varepsilon_2(\delta_1', \delta_2') < \varepsilon_2(\delta_1, \delta_2)$.

VI. User separation with orthogonal multiple access schemes

So far we have considered the case that the users are separated in space by eigenbeamforming, what implies a certain amount of interference. That can be totally avoided by employing an orthogonal multiple access scheme, such as OFDMA, TDMA or FDMA. In that case each user can only exploit $1/K$-th of the available time or frequency resources. That means the rate sent over K time slots or carrier frequencies in the SDMA scenario described in the previous sections must now be transmitted over one slot or carrier. An outage now occurs if $I_k(x_k, y_k) \leq KR_k$. In turn the users' transmit powers are concentrated to this slot or carrier frequency, i.e. compared to the SDMA scenario $K\delta_k$ is available. Making these adoptions, outage probabilities are then computed the same way as in Section IV. Furthermore, as $\boldsymbol{P}_k' = K\delta_k \boldsymbol{R}_k^{\frac{1}{2}} \boldsymbol{p}_k^* \boldsymbol{p}_k^{\mathrm{T}} \boldsymbol{R}_k^{\frac{1}{2}}$ is a matrix of rank one, $\lambda_{1,k} = K\delta_k \operatorname{tr}(\boldsymbol{R}_k \boldsymbol{P}_k)$ and $\lambda_{2,k} = 0$. Inserting into Equation (5) yields:

$$\varepsilon_k = 1 - \exp\left(-\frac{(2^{KR_k} - 1)\sigma_k^2}{K\delta_k \operatorname{tr}(\boldsymbol{R}_k \boldsymbol{P}_k)}\right). \tag{10}$$

The Pareto set $\mathcal{P}$ for this transmission strategy contains all transmit powers δ_k, that sum up to P_{tr}. Thus, for $K = 2$ the Pareto set is identical to the one described in Section V. According to (10) outage probabilities now do not anymore depend on transmit powers of other users and therefore, as long as the transmit power budget is not fully exploited, one user can always be improved without affecting other users. Otherwise increasing the power of one user implies reducing that of one other, which results in a higher outage probability of that user.

We have now introduced two different transmission strategies for the system described in Section II. Which transmission strategy leads to better outage probabilities depends on the spatial scenario, i.e. the strategy described in this section performs worse if eigenbeamforming can separate users well in space. Therefore it is proposed to choose the best transmission strategy as the case arises. However, as this can not be done analytically, the choice of transmission strategy has to be conducted after sampling the outage probabilities of both strategies, as described in the next section.

VII. Sampling the efficient set

In the context of bottom-up approach in cross-layer optimization the upper layer is not interested in functional relationships between outage probabilities and the parameters of the physical layer. As proposed in [6] it is rather desirable to find a finite set of outage probabilities that can be offered to the upper layer. It is furthermore important that these sampled outage probabilities are spaced apart nearly equidistantly in the $(\varepsilon_1, \varepsilon_2)$ – plane, as they are used by upper layers for their own optimization processes. This is achieved for both transmission strategies as follows: Using the Pareto optimality criterion $\delta_2 = P_{\mathrm{tr}} - \delta_1$, ε_1 and ε_2 can be parameterized by δ_1 only. Starting with an initial low (or high) value for δ_1, this transmit power is then in each sampling step i incremented by

$$\Delta\delta_1(i) = \frac{d}{\sqrt{\left(\left.\frac{\partial\varepsilon_1}{\partial\delta_1}\right|_{\delta_1(i-1)}\right)^2 + \left(\left.\frac{\partial\varepsilon_2}{\partial\delta_1}\right|_{\delta_1(i-1)}\right)^2}} \tag{11}$$

and inserted into the corresponding formulas for the outage probabilities. Thereby d denotes the desired distance of two neighboring samples in the $(\varepsilon_1, \varepsilon_2)$–plane:

$$d = \sqrt{\Delta\varepsilon_1^2 + \Delta\varepsilon_2^2}.$$

Approximating $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ by first order Taylor expansion according to:

$$\Delta\varepsilon_1 = \frac{\partial\varepsilon_1(\delta_1)}{\partial\delta_1}\Delta\delta_1, \quad \Delta\varepsilon_2 = \frac{\partial\varepsilon_2(\delta_1)}{\partial\delta_1}\Delta\delta_1$$

leads to Equation (11). Note that for this approximation the explicit dependency between the outage probabilities and transmit powers derived in Section IV for $K = 2$ users is necessary. The result of this sampling process is shown for the SDMA scenario with $d = 0.01$ in Figure 2. The transmit signal to noise ratio (SNR), defined as $\text{SNR} = 10\log_{10}\left(\frac{P_{tr}}{\sigma_1^2+\sigma_2^2}\right)$, was set to 20 dB, each rate R_1 and R_2 was fixed to 3 bpcu and the covariance matrices were generated from a random spatial scenario. Thereby a Laplacian power angle profile (PAP) was used with an angular spread of 10°.

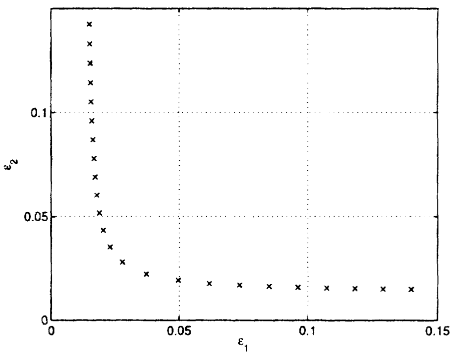

Figure 2. Sampled outage probabilities of the efficient set with $d = 0.01$

VIII. Application Scenario

In order to illustrate our results we choose a simple two-layer model, where the sampled outage probabilities are directly reported to the application layer. Motivated by video transmission, the application layer's cost function is, as in [2], the overall all distortion

$$D = \alpha D_1 + (1-\alpha)D_2,$$

where $\alpha \in [0,1]$ denotes a weighting factor adjustable to assign different priorities to the users. For Gaussian sources with variance one, the source distortion of each user, caused by lossy coding schemes, is given by 2^{-2R_k}. Assuming no measures for error concealment at the receiver, each user's total distortion D_k, which is in analogy to [3] compounded of the source and the channel loss distortion, can be computed according to:

$$D_k = 2^{-2R_k}(1-\varepsilon_k) + \varepsilon_k.$$

Figure 3 shows the cost function D minimized with respect to the sampled outage probabilities ε_1 and ε_2 in dependence of α averaged over 10000 spatial scenarios. For each of those a random direction of departure was generated from a uniform distribution over $]-\pi;\pi]$ and together with a Laplacian PAP and an angular spread of 10° used to compute the covariance matrix of a uniform linear array with $M_t = 4$ antennas. The rates were fixed to $R_1 = R_2 = 3$ bpcu and the SNR was, as in Section VII, set to 20 dB, whereby $\sigma_1^2 = \sigma_2^2$. For the upper curve an orthogonal multiple access (MA) scheme was applied only. For the lower curve the optimum transmission mode was selected, i.e. eigenbeamforming was used, when improvements were possible compared to the MA scheme. This could be observed in more than 50% of the scenarios.

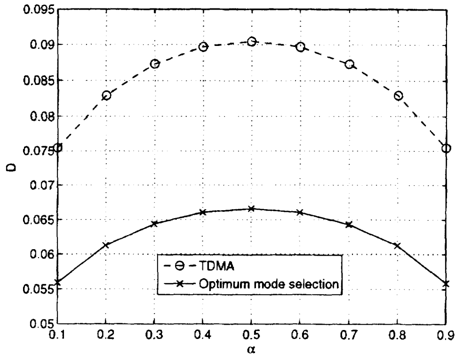

Figure 3. Overall distortion D averaged over 10000 channel realizations

IX. Conclusions

In the paper we gave explicit relationships between the parameters of the physical layer in a MU-MISO system and the outage probabilities. The efficient set needed for a bottom-up approach in cross-layer optimization was determined and sampled. We thereby considered two different modes for user separation and demonstrated that a combination of both leads to improvements in system performance.

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