HYBRID TRANSMIT WAVEFORM DESIGN BASED ON BEAMFORMING AND ORTHOGONAL SPACE-TIME BLOCK CODING

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ABSTRACT

In this paper, we derive a hybrid of *Beam-Forming* (BF) and *Space-Time Block Coding* (STBC), where the space-time code is transmitted over the beams generated by the steering vectors corresponding to the channel path directions. This is for the practical case where the transmit array may have adequate information on the departure angles of the dominant paths between transmitter and receiver, but unreliable information on the associated complex path gains. We compute analytically the *Signal-to-Noise Ratio* (SNR) of the proposed hybrid for the specific case of a two-path channel model and using the orthogonal Alamouti code, and compare the result to the SNR of optimal *Linear Precoding* (LP) and the theoretically possible SNR of *Orthogonal STBC* (OSTBC). Simulation results show that the performance of the BF/STBC hybrid can be very close to LP—under certain conditions—or even better in the practical case where there are phase estimation errors in the path gain estimates employed at the transmitter.

1. INTRODUCTION

Transmit antenna diversity is a powerful technique that provides resilience to fading. To this point, transmit diversity techniques can primarily be partitioned into either *Linear Precoding* (LP) or *Space-Time Block Coding* (STBC) [1, 2]. Space-time codes can provide full diversity advantage without employing *Channel State Information* (CSI) at the transmitter, while LP provides superior error rate performance at the expense of complete transmitter CSI.

Recent research has shown that there can be performance and implementation benefits when STBC is combined with *Beam-Forming* (BF) [3, 4, 5, 6, 7, 8]. These techniques, which we refer to as hybrid BF and STBC, transmit a space-time code designed for M antennas over N beams with $N > M$. The space-time code matrix is mapped to the larger array by BF vectors, which provide improved performance by adapting the space-time code matrix to current channel conditions. Hybrid techniques can operate using statistical [4, 7], partial [3, 6], or quantized CSI [5, 8].

While statistical techniques provide excellent error rate performance, tracking of the spatial correlation matrix can be difficult to implement. The channel can often be modeled as a sum of array response vectors of varying gain at different *Angles of Departure* (AoDs) from different point sources [9]. When the number of incoming rays is relatively small, the AoDs can be easily tracked. Knowledge of the AoDs and corresponding response vectors gives a partial, but reliable, description of the channel subspace structure.

In this paper, we present hybrid BF and STBC techniques that adapt a transmitted space-time code to the current channel conditions using knowledge of the array response vector for each channel path. We show that these techniques can provide performance close to LP which requires perfect CSI at the transmitter. Furthermore, when the receiver's channel estimate is incorrect hybrid BF and STBC techniques can actually outperform LP.

This paper is organized as follows. Section 2 overviews the channel and system under configuration. Prior work on LP, BF, *Eigen-BF* (EBF), and STBC is reviewed in Section 3. Section 3 also presents our ray-based hybrid technique. Section 4 presents several simulation results. We conclude in Section 5.

Throughout the paper, vectors and matrices are denoted by lower case bold and capital bold letters, respectively. The matrix $\boldsymbol{I}_n$ is the $n \times n$ identity matrix. The operation $\mathrm{E}\{\cdot\}$ denotes expectation, $(\cdot)^*$ conjugate complex, $(\cdot)^{\mathrm{T}}$ transpose, $(\cdot)^{\mathrm{H}}$ Hermitian, i. e., conjugate transpose, and $\|\cdot\|_2$ the Euclidean norm. We use $\mathrm{tr}\{\boldsymbol{A}\}$ as the trace of the matrix $\boldsymbol{A}$ and $\mathrm{Re}\{z\}$ as the real part of the complex number z. All random processes are assumed to be zero-mean and stationary. The variance of the scalar process $x[n]$ is denoted by $\sigma_x^2 = \mathrm{E}\{|x[n]|^2\}$.

2. SYSTEM MODEL

We consider a system with N antennas at the transmitter and one antenna at the receiver. The received signal $y[n] \in \mathbb{C}$ may be written as

$$y[n] = \boldsymbol{h}^{\mathrm{T}} \boldsymbol{x}[n] + \nu[n], \quad (1)$$

where $\boldsymbol{h} \in \mathbb{C}^N$ is the channel vector, $\boldsymbol{x}[n] \in \mathbb{C}^N$ is the transmitted signal at time index n, and $\nu[n] \in \mathbb{C}$ is complex-valued *Additive White Gaussian Noise* (AWGN) with variance σ_ν^2. For a fair comparison amongst the different methods explained in the following section, we define the total transmit power $P = \mathrm{E}\{\|\boldsymbol{x}[n]\|_2^2\}$ to be the same for all transmit strategies.

For illustrative purposes, the channel is assumed to have two paths with the AoDs θ_1 and θ_2, respectively, i. e.,

$$\boldsymbol{h} = g_1 \boldsymbol{a}(\theta_1) + g_2 \boldsymbol{a}(\theta_2), \tag{2}$$

where $g_1 \in \mathbb{C}$ and $g_2 \in \mathbb{C}$ are path weights, and $\boldsymbol{a}(\theta) = [1, \exp(\mathrm{j}\,\mu), \ldots, \exp(\mathrm{j}\,(N-1)\mu)] \in \mathbb{C}^N$ is the *steering vector* corresponding to the AoD $\theta \in [-\pi/2, \pi/2]$. The variable $\mu = 2\pi d \sin\theta/\lambda$ denotes the *spatial frequency* of a *uniform linear array* with elements spaced by d and wavelength λ.

In the sequel, we investigate different transmit strategies to generate the transmitted signal $\boldsymbol{x}[n]$. Although, we consider only the two-path channel model due to space limitations, the extension of the following derivations to channels with an arbitrary number of paths is straight forward.

3. HYBRID BF AND STBC TECHNIQUES

3.1. Optimal LP

We denote LP [10] to be the optimal transmit strategy using full CSI. The transmitted signal may be written as

$$\boldsymbol{x}[n] = \boldsymbol{p} s[n], \tag{3}$$

with the *precoder* $\boldsymbol{p}$ and the symbol sequence $s[n] \in \mathbb{M}$ with modulation alphabet $\mathbb{M}$ and variance σ_s^2. The precoder is chosen to maximize the *Signal-to-Noise Ratio* (SNR) $\gamma = \sigma_{\hat{s}}^2|_{\nu[n]=0} / \sigma_{\hat{s}}^2|_{s[n]=0}$ at the output of the receiver which results in

$$\boldsymbol{p} = \sqrt{\frac{P}{\sigma_s^2 \|\boldsymbol{h}\|_2^2}} \boldsymbol{h}^*, \tag{4}$$

under the given power constraint and under the assumption that the receiver is an arbitrary scalar weight, i. e., $\hat{s}[n] = w y[n]$. It can easily be shown that the SNR $\gamma_{\mathrm{LP}} = P \|\boldsymbol{h}\|_2^2 / \sigma_\nu^2$ is given by

$$\gamma_{\mathrm{LP}} = \left(N \sum_{i=1}^{2} |g_i|^2 + 2\,\mathrm{Re}\left\{ g_1^* g_2 \boldsymbol{a}_1^{\mathrm{H}} \boldsymbol{a}_2 \right\} \right) \frac{P}{\sigma_\nu^2}, \tag{5}$$

using the channel model of Section 2 and the shortcut notation $\boldsymbol{a}_i = \boldsymbol{a}(\theta_i)$, $i \in \{1, 2\}$.

Note that the chosen precoder is also optimal in the *Maximum Likelihood* (ML) sense due to the given AWGN scenario [2].

3.2. BF/STBC Hybrid

In this section, we derive a hybrid of BF and STBC based on the orthogonal Alamouti code [11] in the case where only partial CSI, i. e., the AoDs but not the path weights, is available at the transmitter. We transmit

$$\boldsymbol{x}[n] = \sqrt{\frac{P}{2N\sigma_s^2}} \boldsymbol{x}'[n], \tag{6}$$

where the signal $\boldsymbol{x}'[n]$ at time index $n = 2k$ and $n = 2k+1$ is defined as

$$\boldsymbol{x}'[2k] = \boldsymbol{a}_1^* s[2k] + \boldsymbol{a}_2^* s[2k+1] \quad \text{and} \tag{7}$$

$$\boldsymbol{x}'[2k+1] = -\boldsymbol{a}_1^* s^*[2k+1] + \boldsymbol{a}_2^* s^*[2k], \tag{8}$$

respectively. Again, the prefactor in Equation (6) is due to the transmit power constraint. If we compose the receive vector $\boldsymbol{y}[k] = [y[2k], y^*[2k+1]]^{\mathrm{T}} \in \mathbb{C}^2$ and analogous, the noise vector $\boldsymbol{\nu}[k] = [\nu[2k], \nu^*[2k+1]]^{\mathrm{T}} \in \mathbb{C}^2$ and the symbol vector $\boldsymbol{s}[k] = [s[2k], s[2k+1]]^{\mathrm{T}} \in \mathbb{M}^2$, we can rewrite the system model of Section 2 as

$$\boldsymbol{y}[k] = \boldsymbol{H}\boldsymbol{s}[k] + \boldsymbol{\nu}[k], \tag{9}$$

with the resulting channel matrix

$$\boldsymbol{H} = \sqrt{\frac{P}{2N\sigma_s^2}} \begin{bmatrix} \boldsymbol{h}^{\mathrm{T}}\boldsymbol{a}_1^* & \boldsymbol{h}^{\mathrm{T}}\boldsymbol{a}_2^* \\ \boldsymbol{h}^{\mathrm{H}}\boldsymbol{a}_2 & -\boldsymbol{h}^{\mathrm{H}}\boldsymbol{a}_1 \end{bmatrix} \in \mathbb{C}^{2\times 2}. \tag{10}$$

To get the estimate $\hat{\boldsymbol{s}}[k]$ of the symbol vector, we apply the receiver $\boldsymbol{W}$ to the receive vector $\boldsymbol{y}[k]$, i. e., $\hat{\boldsymbol{s}}[k] = \boldsymbol{W}\boldsymbol{y}[k]$. Using the *Zero-Forcing* (ZF) optimization criterion

$$\boldsymbol{W} = \underset{\boldsymbol{\mathcal{W}}}{\operatorname{argmin}}\, \mathrm{tr}\left\{ \boldsymbol{\mathcal{W}}^{\mathrm{H}} \boldsymbol{\mathcal{W}} \right\} \quad \text{s. t.} \quad \boldsymbol{\mathcal{W}}\boldsymbol{H} = \boldsymbol{I}_2, \tag{11}$$

yields

$$\boldsymbol{W} = \frac{2N\sigma_s^2}{P} \left(\sum_{i=1}^{2} \left| \boldsymbol{h}^{\mathrm{H}} \boldsymbol{a}_i \right|^2 \right)^{-1} \boldsymbol{H}^{\mathrm{H}}. \tag{12}$$

Note that this solution is equal to ML detection since the columns of the channel $\boldsymbol{H}$ are mutually orthogonal.

Finally, the SNR $\gamma_{\mathrm{H}} = P \sum_{i=1}^{2} |\boldsymbol{h}^{\mathrm{H}}\boldsymbol{a}_i|^2 / (2N\sigma_\nu^2)$ can be written as

$$\gamma_{\mathrm{H}} = \left(\frac{N^2 + \left|\boldsymbol{a}_1^{\mathrm{H}}\boldsymbol{a}_2\right|^2}{2N} \sum_{i=1}^{2} |g_i|^2 + 2\,\mathrm{Re}\left\{ g_1^* g_2 \boldsymbol{a}_1^{\mathrm{H}} \boldsymbol{a}_2 \right\} \right) \frac{P}{\sigma_\nu^2}. \tag{13}$$

Note that $0 \leq \left|\boldsymbol{a}_1^{\mathrm{H}}\boldsymbol{a}_2\right|^2 \leq N^2$, i. e., the SNR γ_{H} of the BF/STBC hybrid is equal to the SNR γ_{LP} of LP if and only if the steering vectors are identical, i. e., $\boldsymbol{a} = \boldsymbol{a}_1 = \boldsymbol{a}_2$. This observation can easily be verified regarding the resulting single-path case: The influence of the resulting path weight

$g = g_1 + g_2$ can be fully equalized at the receiver without performance loss. On the other hand, the worst SNR performance of the hybrid is given if the AoDs are such that the steering vectors are orthogonal to each other, i. e., $\boldsymbol{a}_1 \perp \boldsymbol{a}_2$, if the path weights g_1 and g_2 are uncorrelated random variables. In this case, the mean of the SNR of the hybrid scheme is $3\,\mathrm{dB}$ less compared to the one of LP, i. e., $\mathrm{E}\{\gamma_{\mathrm{LP}}\} = 2\,\mathrm{E}\{\gamma_{\mathrm{H}}\}$, as can be seen by comparing Equations (5) and (13).

The hybrid can also be derived using EBF [4, 7, 12] where $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ in Equations (7) and (8) are chosen to be the eigenvectors corresponding to the two largest eigenvalues of the channel auto-correlation matrix $\boldsymbol{R}_{\boldsymbol{h}} = \mathrm{E}\{\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}\}$. If the eigenvectors are assumed to be normalized, the prefactor of Equation (6) has to be changed to $\sqrt{P/(2\sigma_s^2)}$ and the following dependent equations has to be adopted adequately which is not shown in this paper due to space limitations.

3.3. OSTBC

Let us now assume no CSI at the transmitter. The transmit signal $\boldsymbol{x}[n]$ can then be defined as

$$\boldsymbol{x}[n] = \sqrt{\frac{P}{N\sigma_s^2}}\boldsymbol{x}'[n], \tag{14}$$

where the signals $\boldsymbol{x}'[n]$, $n \in \{Nk, \ldots, N(k+1)-1\}$, are chosen by *Orthogonal STBC* (OSTBC). Although such full-rate codes only exist for $N = 2$ in the case of complex symbols (cf. [13]), i. e., $\mathbb{M} \subset \mathbb{C}$, a theoretical SNR can be computed as an upper bound for the SNR of full-rate STBC using N transmit antennas or for lower-rate OSTBC. The combination of the resulting channel matrix and the ZF receiver yields an identity matrix scaled by $\sqrt{P/(N\sigma_s^2)}\,\|\boldsymbol{h}\|_2^2$. Due to the noise amplification of the ZF receiver, the SNR $\gamma_{\mathrm{OSTBC}} = P\,\|\boldsymbol{h}\|_2^2\,/(N\sigma_\nu^2)$. Finally, it can be rewritten as

$$\gamma_{\mathrm{OSTBC}} = \left(\sum_{i=1}^{2} |g_i|^2 + \frac{2}{N}\,\mathrm{Re}\left\{g_1^* g_2 \boldsymbol{a}_1^{\mathrm{H}}\boldsymbol{a}_2\right\}\right)\frac{P}{\sigma_\nu^2}, \tag{15}$$

using Equation (2). If g_1 and g_2 are assumed to be uncorrelated random variables, it follows that $\mathrm{E}\{\gamma_{\mathrm{LP}}\} = N\,\mathrm{E}\{\gamma_{\mathrm{OSTBC}}\}$, i. e., there is a $10\lg N\,\mathrm{dB}$ lost when using OSTBC instead of LP.

4. SIMULATION RESULTS

Figure 1 depicts the averaged *Bit Error Probability* (BEP) over the Transmit-SNR (Tx-SNR) $10\lg(P/\sigma_\nu^2)\,\mathrm{dB}$ computed via $\mathrm{BEP} = 1/2\,\mathrm{erfc}(\sqrt{\gamma})$ using the SNRs γ derived in Section 3 and assuming QPSK transmission over four transmit antennas ($N = 4$) spaced by $d = \lambda/2$. The AoDs are uniformly distributed between $-\pi/2$ and $\pi/2$ and the path weights are complex Gaussian distributed with variance one. It can be seen that LP is best since it has full CSI available at the transmitter. OSTBC without any CSI performs $10\lg 4\,\mathrm{dB} \approx 6\,\mathrm{dB}$ worse than LP. Whereas the EBF/STBC hybrid is $3\,\mathrm{dB}$ away from LP, the proposed BF/STBC hybrid is even closer for small Tx-SNR values.

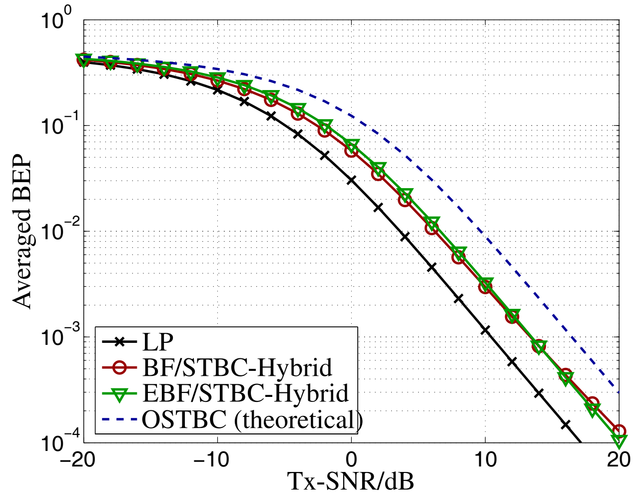

Fig. 1. Comparison of different transmit strategies averaging over uniformly distributed AoDs and Gaussian distributed path weights

Figure 2 shows the averaged BEP dependency of LP and the BF/STBC hybrid on the AoDs. For $\theta_1 = \theta_2 = 0$, both performs equal as derived in Subsection 3.2. Whereas the worst SNR performance is given for orthogonal steering vectors, i. e., $\sin\theta_2 = 1/2$, the averaged BEP performance is not minimum in this case, especially for high SNR. This is due to the non-linear relationship between the BEP and γ based on the complementary error function. Note that the higher slope in the case of non-collinear steering vectors ($\theta_2 \neq 0$) is due to the diversity gain.

In the sequel, we consider estimation errors in the phases of the path weights available at the transmitter. The amplitudes of the path weights and the AoDs are assumed to be known perfectly since they can be estimated very accurately due to their slow-varying characteristic. The receiver is assumed to have perfect CSI. We consider the channel of Equation (2) with the AoDs $\theta_1 = 0$ and $\theta_2 = \arcsin(1/4)$, and the path weights $g_1 = 1$ and $g_2 = 0.7$. The transmitter uses the estimates $\hat{g}_1 = g_1$ and $\hat{g}_2 = g_2\exp(\mathrm{j}\,\alpha)$ with the phase mismatch α. Figure 3 shows the results for $\alpha \in \{0, \pi/4, \pi/2, 3\pi/4\}$. Again, in the case of no mismatch ($\alpha = 0$), the performance of the hybrid scheme is between the one of LP and OSTBC. As can be seen, the phase mismatch α in the CSI play a crucial rule in the behavior of LP. On the other hand, OSTBC and the hybrid scheme are not affected by the mismatch since OSTBC requires no

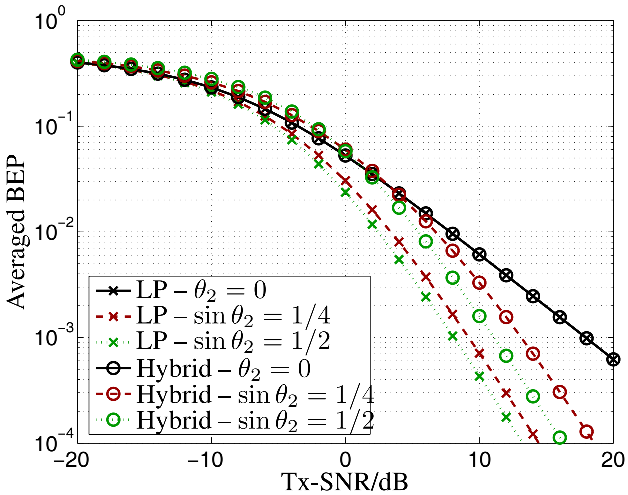

Fig. 2. Comparison between LP and BF/STBC hybrid with $\theta_1 = 0$ and different θ_2 averaging over Gaussian distributed path weights

CSI at the transmitter and the hybrid BF and STBC technique is only based on partial CSI, i. e., the AoDs which are known perfectly at the transmitter. Thus, LP tends to perform worse then the BF/STBC hybrid and is even outperformed by OSTBC when the mismatch becomes large.

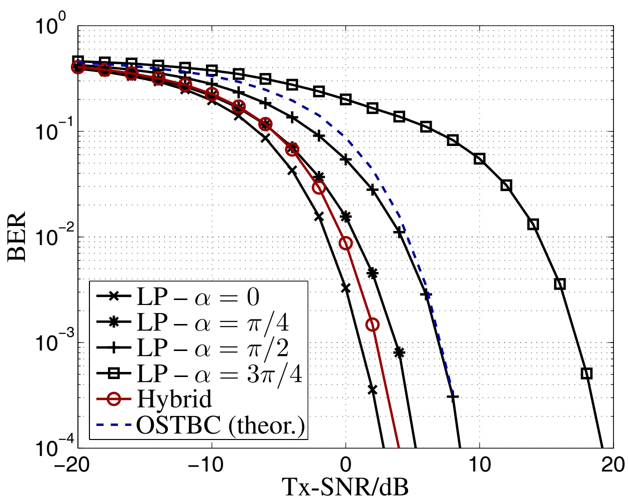

Fig. 3. Comparison of different transmit strategies with $\theta_1 = 0$, $\sin\theta_2 = 1/4$, $g_1 = 1$, $g_2 = 0.7$, and different phase mismatchs α

5. CONCLUSIONS

In this paper, we presented hybrid BF and OSTBC techniques based on the steering vectors corresponding to the channel path directions. We derived the SNR of the proposed hybrid scheme and compared it to the SNR of LP and a theoretical bound of OSTBC. Simulation results based on the averaged BEP showed that the BF/STBC hybrid outperforms the EBF/STBC hybrid for low Tx-SNR. Moreover, BER investigations assuming a fixed channel scenario revealed that LP is beaten by the proposed hybrid scheme and even by OSTBC if the path weights are estimated at the transmitter.

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