ROBUST PRECODING FOR MULTIUSER MISO SYSTEMS WITH LIMITED-FEEDBACK CHANNELS

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***Abstract*—In this paper we focus on *Multi User Multiple Input Single Output* (MU-MISO) systems where one centralized transmitter with multiple antennas serves several decentralized single-antenna receivers. Precoding is an attractive way to combat multiuser interferences because it reduces cost and power consumption in the user equipment. When implementing precoding, however, the Channel State Information (CSI) should be available at the base station. In Frequency Division Duplex (FDD) systems, the CSI is sent from the receivers by means of a feedback channel whose data rate is often severely limited. Thus, CSI is imperfect because it is affected by the errors caused by channel estimation, feedback delay, truncation, and quantization of the Karhunen-Loève coefficients. In this paper, we explain how to model these errors which is the basic premise to design both robust linear precoding and robust Tomlinson Harashima precoding, whose performance will be compared.**

I. INTRODUCTION

Preequalization at transmission to combat the interuser interference is necessary in a *Multi-User* (MU) system with decentralized receivers. Different *Linear Precoding* (LP) solutions can be obtained by means of a joint optimization of the receive and transmit filters according to different criteria. This optimization has been extended to obtain the *Tomlinson-Harashima Precoding* (THP) solution, whose performance is clearly higher for perfect *Channel State Information* (CSI). Therefore, the design of these systems is already known for the ideal case where CSI is perfectly known at the transmitter. However, the situation is different where there is erroneous CSI. Additionally, the application of the SINR criterion is questionable since it is unclear up to now how to include the uncertainties in the SINR in a systematic way. Consequently, it is inevitable to resort to an MSE criterion together with these precoding schemes for the case of partial CSI.

Most work on precoding with erroneous CSI has mainly focused on *Time Division Duplex* (TDD) systems. Contrarily, in this work we focus on the more extended case of *Frequency Division Duplex* (FDD) systems where the transmitter cannot obtain the CSI from the received signals, even under the assumption of perfect calibration, because the channels are not reciprocal. Instead, the receivers estimate their channels and send the CSI back to the transmitter by means of a feedback channel. Since the data rate of the feedback channels is often limited [1], the CSI must be compressed to ensure that the tight scheduling constraints are satisfied. To limit the CSI sent to the transmitter we will use a truncation of the *Karhunen-Loève* (KL) decomposition in this paper that is optimum in the sense that it provides dimensionality reduction based on the channel's covariance matrix with the smallest possible MSE.

The following sources of errors are considered for the proposed precoding designs: channel estimation, truncation of the KL transform, quantization of the KL coefficients, and feedback channel delay. With the obtained error model, we develop robust schemes, one for LP and another one for THP, that take into account the statistical properties of the errors in the filters design.

This paper is organized as follows. Sections II and III describe the signal and channel models, respectively, and in Section IV, the models for the CSI error sources are developed. Section V presents our robust designs. Illustrative computer simulations are presented in Section VI and some concluding remarks are made in Section VII.

II. SYSTEM MODEL

We consider a MU-MISO system with N_t transmit antennas and K single antenna receivers as depicted in the Fig. 1. The precoder generates the transmit signal $\boldsymbol{x}$ from all data symbols $\boldsymbol{u} = [u_1, \ldots, u_K]$ belonging to the different users $1, \ldots, K$. The signal x_ℓ from transmit antenna ℓ propagates over the channel with the coefficient $h_{k,\ell}$ to the k-th receiver, superimposes with the signals of the other transmit antennas, and is perturbed by the additive white Gaussian noise n_k with variance σ_n^2, i.e.,

$$y_k = \sum_{\ell=1}^{N_\mathrm{t}} h_{k,\ell} x_\ell + n_k = \boldsymbol{h}_k^\mathrm{T} \boldsymbol{x} + n_k \qquad (1)$$

where $(\bullet)^\mathrm{T}$ denotes transpose and $\boldsymbol{h}_k = [h_{k,1}, \ldots, h_{k,N_\mathrm{t}}]^\mathrm{T} \in \mathbb{C}^{N_\mathrm{t} \times 1}$ represents the flat fading vector channel corresponding to the k-th user. The transmit signal $\boldsymbol{x}$ must satisfy an average total transmit power constraint, i.e., $E[\|\boldsymbol{x}\|_2^2] = E_\mathrm{tx}$.

In THP, the received signal y_k is then multiplied by a gain control factor $1/\beta$. As you can see in Fig. 1, a modulo operator is applied to the weighted received signal to remove the ambiguities introduced by the precoder. The resulting estimate of u_k is denoted by $\hat{u}_k$. At the transmitter, the feedforward filter $\boldsymbol{F}^\mathrm{H}$ suppresses parts of the interference linearly, whereas the feedback loop with the strictly lower triangular feedback filter $\boldsymbol{B}^\mathrm{H} - \mathbf{I}$ subtracts the remaining interferences non-linearly. Note that $(\bullet)^\mathrm{H}$ indicates conjugate transpose. Since the order of precoding has an effect on performance, the data signal $\boldsymbol{u}$ is reordered by means of the

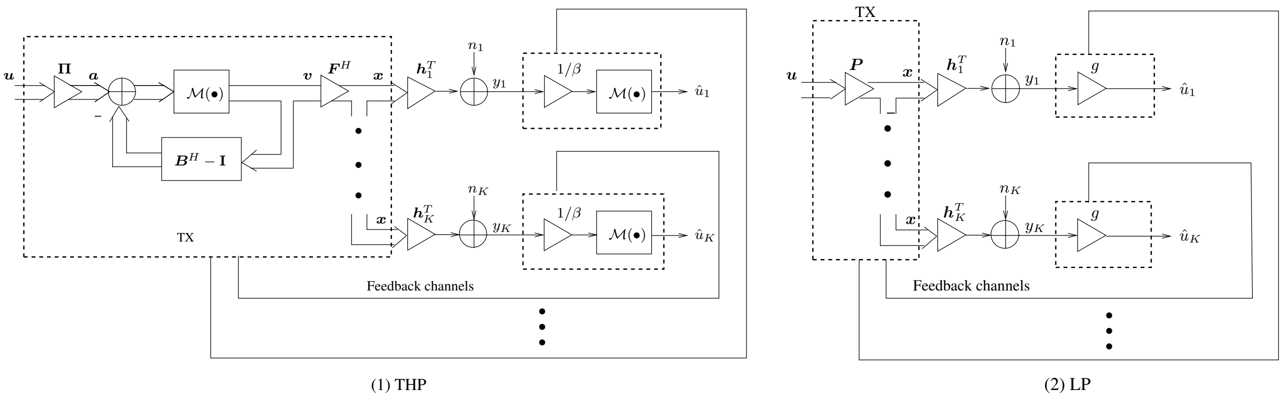

Fig. 1. Block diagram of the precoding schemes: THP and LP.

permutation filter $\boldsymbol{\Pi}$ [2]. The signal $\boldsymbol{\Pi u}$ is passed through the feedback loop, where the modulo operator $\mathcal{M}(\bullet)$ limits the amplitude of $\boldsymbol{v}$ and thus, the power of the transmit signal $\boldsymbol{x}$.

In LP, as you can see too in Fig. 1, the data symbols $\boldsymbol{u}$ are passed through the transmit filter $\boldsymbol{P}$ to form the transmit signal $\boldsymbol{x} = \boldsymbol{Pu}$. After multiplying by the gain control g, we get the estimate $\hat{u}_k = g y_k$.

III. Channel Model

We model the k-th user's channel vector as a vector of zero-mean circularly symmetric complex Gaussian distributed random variables, i.e., $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{C}_{h,k})$, where $\boldsymbol{C}_{h,k}$ is the covariance matrix of the k-th user's channel. The channels of the different users are statistically independent.

In the q-th time slot, our model for the k-th user channel vector is

$$\boldsymbol{h}_k(q) = \boldsymbol{C}_{h,k}^{1/2} \boldsymbol{h}_{\mathrm{w},k}(q). \tag{2}$$

with $\boldsymbol{h}_{\mathrm{w},k}(q)$ being a vector of stationary circularly symmetric complex white Gaussian processes (with unit variance elements) and where $(\bullet)^{1/2}$ represents the Cholesky decomposition. According to the modified Jakes model described in [3], temporal channel correlations are modeled by $\boldsymbol{h}_{\mathrm{w},k}(q)$ whereas the spatial correlations are introduced by the multiplication by $\boldsymbol{C}_{h,k}^{1/2}$ [4].

Notice that, according to our model, the channel $\boldsymbol{h}_k$ is stationary because $\boldsymbol{h}_{\mathrm{w},k}$ is stationary. But realistic channels are often non-stationary and the channel's covariance matrix has to be tracked in real situations. However, since the covariance matrix changes very slowly compared to the channel itself, it is realistic to assume that it is constant and perfectly known at both the receiver and the transmitter.

IV. Imperfect CSI

In realistic situations, the CSI that is available at the transmitter is not perfectly known. In this case, it is a matter of discussion what kind of information has to be sent to the transmitter and the way of recovering it from the receiver side. In the system that we propose in this paper, we start estimating the channel at the receivers using the observations of the pilot symbols. Then, we project the resulting channel estimation onto the eigenvectors of the channel's covariance matrix to obtain the Karhunen-Loève transformation of the channel vector which optimally provides a dimensionality reduction with the smallest possible MSE. The coefficients of the truncated KL expansion are then quantized prior to transmission over the feedback channel which introduces a delay. Taking into account the delay of the feedback channel as an additional error source, the partial CSI is then used at the transmitter to reconstruct the channel vector and to design the filters.

Along this section we will assume that the signals and errors are uncorrelated.

A. Statistical model for channel estimation errors

We use the heuristic estimator based on N_{tr} pilot symbols per time slot q so that the least-squares channel estimates are

$$\boldsymbol{h}_{\mathrm{LS},k}(q) = \boldsymbol{S}^{\dagger}\boldsymbol{y}_k(q) = \boldsymbol{h}_k(q) + \boldsymbol{S}^{\dagger}\boldsymbol{n}(q) = \boldsymbol{h}_k(q) + \boldsymbol{n}_{\mathrm{LS},k}(q) \tag{3}$$

with $\boldsymbol{S}^{\dagger} = (\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{S}^{\mathrm{H}}$ and $\boldsymbol{S} \in \mathbb{C}^{N_{\mathrm{tr}} \times N_{\mathrm{t}}}$ containing the training symbols, $\boldsymbol{n}_k(q) \in \mathbb{C}^{N_{\mathrm{tr}} \times 1}$ being the AWGN with variance σ_n^2, and where

$$\boldsymbol{n}_{\mathrm{LS},k}(q) \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma_n^2(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}). \tag{4}$$

B. Statistical model for Karhunen–Loève errors

The eigenvalue decomposition of $\boldsymbol{C}_{h,k}$ reads as

$$\boldsymbol{C}_{h,k} = E[\boldsymbol{h}_k(q)\boldsymbol{h}_k^{\mathrm{H}}(q)] = \sum_{i=1}^{r_k} \lambda_{k,i}\boldsymbol{v}_{k,i}\boldsymbol{v}_{k,i}^{\mathrm{H}} = \boldsymbol{V}_k^{'}\boldsymbol{\Lambda}_k\boldsymbol{V}_k^{'\mathrm{H}} \tag{5}$$

where r_k is the rank of $\boldsymbol{C}_{h,k}$ and $\boldsymbol{v}_{k,i}$ and $\lambda_{k,i}$ are, respectively, the i-th eigenvector (or i–th column of the matrix $\boldsymbol{V}_k^{'}$) and the i-th eigenvalue of $\boldsymbol{C}_{h,k}$ (or the i–th entry of the diagonal matrix $\boldsymbol{\Lambda}_k$).

Applying the KL transform, the channel vector in the time slot q can be obtained as $\boldsymbol{V}_k^{'}\boldsymbol{c}_k(q)$, where $\boldsymbol{c}_k(q)$ are the coefficients of the KL transform given by $\boldsymbol{c}_k(q) = \boldsymbol{V}_k^{'\mathrm{H}}\boldsymbol{h}_{\mathrm{LS},k}(q)$. No errors are added to our channel estimation if all the coefficients of the KL transform are employed. To compress the channel information and taking into account the good energy compaction properties of the KL decomposition, we can approximate the vector channels $\boldsymbol{h}_k(q)$ by

$$\begin{aligned}\boldsymbol{h}_{\mathrm{KL},k}(q) = \boldsymbol{V}_k\boldsymbol{c}_k(q) &= \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{h}_{\mathrm{LS},k}(q) \\ &= \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{h}_k(q) + \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{n}_{\mathrm{LS},k}(q)\end{aligned} \tag{6}$$

where $\boldsymbol{V}_k = [\boldsymbol{v}_{k,1}, \ldots, \boldsymbol{v}_{k,r}, \boldsymbol{0}_{N_{\mathrm{t}} \times N_{t-r}}]$ and r denotes the number of KL coefficients sent from the receiver after truncation. The noise $\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{n}_{\mathrm{LS},k}(q)$ and the signal $\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{h}_k(q)$

$$
\begin{aligned}
&\boldsymbol{h}_k(D) = \boldsymbol{h}_{\mathrm{Q},k}(D) + \boldsymbol{n}_{\mathrm{Q},k}(D) + \boldsymbol{n}_{\mathrm{KL},k}(D) + \boldsymbol{h}_k^{''}(D)\\
&\quad \text{where } \boldsymbol{h}_{\mathrm{Q},k}(D) \text{ is the quantized version of } \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{h}_{\mathrm{LS},k}(D)\\
&\quad \text{and}\\
&\qquad \boldsymbol{C}_{\mathrm{Q},k}(D) = E[\boldsymbol{n}_{\mathrm{Q},k}(D)\boldsymbol{n}_{\mathrm{Q},k}^{\mathrm{H}}(D)] = \tfrac{\Delta^2}{6}\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\\
&\qquad \boldsymbol{n}_{\mathrm{KL},k}(D) \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0}, \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\left(\sigma_n^2(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1} + 2\left(1 - J_0\left(2\pi\tfrac{f_{\mathrm{D},k}}{f_{\mathrm{slot}}}D\right)\right)\boldsymbol{C}_{h,k}\right)\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\right)\\
&\qquad \boldsymbol{h}_k^{''}(D) \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0}, (\boldsymbol{I} - \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}})\boldsymbol{C}_{h,k}(\boldsymbol{I} - \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}})\right)
\end{aligned}
$$

TABLE I
THE ERRORS MODEL.

lie in the same subspace spanned by the columns of $\boldsymbol{V}_k$. Therefore, $\boldsymbol{h}_{\mathrm{KL},k}(q)$ gives us no information about the properties of $\boldsymbol{h}_k(q)$ lying in range$(\boldsymbol{V}_k)^{\perp}$.

The resulting error contribution due to the KL truncation reads as

$$
\begin{aligned}
\boldsymbol{h}_k^{''}(q) &= (\boldsymbol{I} - \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}})\boldsymbol{h}_k(q)\\
&\sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, (\boldsymbol{I} - \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}})\boldsymbol{C}_{h,k}(\boldsymbol{I} - \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}})). \quad (7)
\end{aligned}
$$

Note that $\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{n}_{\mathrm{LS},k}(q)$ is orthogonal to $\boldsymbol{h}_k^{''}(q)$.

So, we have

$$\boldsymbol{h}_k(q) = \boldsymbol{h}_k^{'}(q) + \boldsymbol{h}_k^{''}(q) \quad (8)$$

with

$$\boldsymbol{h}_k^{'}(q) = \boldsymbol{h}_{\mathrm{KL},k}(q) + \boldsymbol{n}_{\mathrm{KL},k}(q) \quad (9)$$

and $\boldsymbol{n}_{\mathrm{KL},k}(q) \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma_n^2\boldsymbol{V}\boldsymbol{V}^{\mathrm{H}}(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{V}\boldsymbol{V}^{\mathrm{H}})$.

C. Statistical model for quantization errors

The uniform quantizer is the most common of the scalar quantizers whose principle is rather simple. After the normalization of the KL coefficients $c_{k,i}(q)$, the process of quantization is as follows. Before transmission, we choose representants to construct an initial set of codebooks that are stored at both transmitter and receiver. The receivers perform a search to find for the components (real and imaginary parts) of the KL coefficients obtained in each time slot the element in the codebook that is closest. Then, the corresponding codebook index is fed back to the transmitter. Finally, the transmitter simply looks at its codebook and builds the precoder parameters from the selected codeword [5]. Therefore, $\boldsymbol{h}_{\mathrm{KL},k}$ lies somewhere in the respective cell, i.e.,

$$\boldsymbol{h}_{\mathrm{KL},k}(q) = \boldsymbol{h}_{\mathrm{Q},k}(q) + \boldsymbol{n}_{\mathrm{Q},k}(q) \quad (10)$$

where $\boldsymbol{h}_{\mathrm{Q},k}$ is the representant and where $\boldsymbol{n}_{\mathrm{Q},k}(q)$ is assumed uniformly distributed over the cell, for simplicity reasons.

Remember that $\boldsymbol{h}_{\mathrm{KL},k}(q)$ only lies in the subspace spanned by the columns of $\boldsymbol{V}_k$. Under the assumption that also the quantizer works in this subspace, i.e., $\boldsymbol{h}_{\mathrm{Q},k}(q) = \boldsymbol{V}_k\boldsymbol{h}_{\mathrm{Q},k}^{'}(q)$, we can follow that $\boldsymbol{n}_{\mathrm{Q},k}(q)$ lies also in the subspace spanned by the columns of $\boldsymbol{V}_k$. Therefore, we get the rank deficient covariance matrix for the quantization error

$$\boldsymbol{C}_{\mathrm{Q},k} = \boldsymbol{V}_k\boldsymbol{C}_{\mathrm{Q}'}\boldsymbol{V}_k^{\mathrm{H}} \quad (11)$$

where $\boldsymbol{C}_{\mathrm{Q}'} = \frac{\Delta^2}{6}\boldsymbol{I}_{N_{\mathrm{t}}\times N_{\mathrm{t}}}$. Since the KL coefficients are uncorrelated and consequently, the quantization errors are too, the variance is $2E[|\epsilon|^2]$, where ϵ is the error in the uniform quantizer and Δ the quantizer step size [6].

D. Statistical model for feedback delay errors

The transmission over the feedback channel introduces a delay of D slots. This delay can equivalently be modeled as follows. The estimator gets outdated training data, i.e., the observation of the estimator is delayed by D slots. Then, the respective feedback channel has no delay. The LS estimate for delayed training data reads as

$$\boldsymbol{h}_{\mathrm{LS},k}(D) = \boldsymbol{h}_k(0) + \boldsymbol{n}_{\mathrm{LS},k}(D) \quad (12)$$

where $\boldsymbol{n}_{\mathrm{LS},k}(D)$ has the same statistical properties as described above in (4). Clearly,

$$
\begin{aligned}
\boldsymbol{h}_{\mathrm{LS},k}(D) &= \boldsymbol{h}_k(D) + \boldsymbol{h}_k(0) - \boldsymbol{h}_k(D) + \boldsymbol{n}_{\mathrm{LS},k}(D)\\
&= \boldsymbol{h}_k(D) + \boldsymbol{n}_{\mathrm{LS},k}^{'}(D) \quad (13)
\end{aligned}
$$

being $\boldsymbol{n}_{\mathrm{LS},k}^{'}(D) = \boldsymbol{h}_k(0) - \boldsymbol{h}_k(D) + \boldsymbol{n}_{\mathrm{LS},k}(D)$. With the properties of $\boldsymbol{h}_k(q)$ and $\boldsymbol{h}_{\mathrm{w},k}(q)$ showed in Section III, and taking into account that

$$
\begin{aligned}
\boldsymbol{C}_{h_{\mathrm{w}},k}(D) &= E[\boldsymbol{h}_{\mathrm{w},k}(q)\boldsymbol{h}_{\mathrm{w},k}^{\mathrm{H}}(q - D)]\\
&= J_0\left(2\pi\frac{f_{\mathrm{D},k}}{f_{\mathrm{slot}}}D\right)\boldsymbol{I}_{N_{\mathrm{t}}\times N_{\mathrm{t}}} \quad (14)
\end{aligned}
$$

where J_0 denotes the zero–th order Bessel function of the first kind, $f_{\mathrm{D},k}$ is the *maximum* Doppler frequency, and f_{slot} the slot rate, we obtain

$$
\begin{aligned}
E[\boldsymbol{e}_D\boldsymbol{e}_D^{\mathrm{H}}] &= 2\boldsymbol{C}_{h,k} - E[\boldsymbol{h}_k(0)\boldsymbol{h}_k^{\mathrm{H}}(D)] - E[\boldsymbol{h}_k(D)\boldsymbol{h}_k^{\mathrm{H}}(0)]\\
&= 2\left(1 - J_0\left(2\pi\frac{f_{\mathrm{D},k}}{f_{\mathrm{slot}}}D\right)\right)\boldsymbol{C}_{h,k}. \quad (15)
\end{aligned}
$$

Here $\boldsymbol{e}_D = \boldsymbol{h}_k(0) - \boldsymbol{h}_k(D)$.

Hence, the new LS error has the property

$$\boldsymbol{n}_{\mathrm{LS},k}^{'}(D) \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0}, \boldsymbol{C}^{'}\right) \quad (16)$$

with $\boldsymbol{C}^{'} = \sigma_n^2(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1} + 2\left(1 - J_0\left(2\pi\frac{f_{\mathrm{D},k}}{f_{\mathrm{slot}}}D\right)\right)\boldsymbol{C}_{h,k})$.

Therefore, at the end, we find the model of the errors described in Table I.

V. ROBUST DESIGNS

The performance of precoding schemes strongly degrades due to imperfect CSI at the transmitter. However, better performance can be obtained if the error modeling made in Section IV is used to design average robust precoding schemes [7].

Our model for the channel matrix is given by

$$\boldsymbol{H}(q) = \hat{\boldsymbol{H}}(q) + \boldsymbol{\Theta}_{\mathrm{T}}(q) \quad (17)$$

where $\hat{\boldsymbol{H}}(q)$ is the quantized version of the channel matrix and $\boldsymbol{\Theta}_{\mathrm{T}}(q)$ is the error matrix.

To obtain the corresponding filters for each scheme, we minimize the MSE averaged over $\boldsymbol{\Theta}_{\mathrm{T}}$ under the transmit energy constraint. Thus, for THP, the *robust transmit Wiener THP filters* can be written as

$$\begin{aligned} \beta &= \sqrt{\frac{E_{\mathrm{tx}}}{\mathrm{tr}\left(\hat{\boldsymbol{H}}^{\mathrm{H}}\hat{\boldsymbol{H}} + \boldsymbol{C}_{\boldsymbol{\Theta}_{\mathrm{T}}} + \frac{\mathrm{tr}(\boldsymbol{R}_{\mathrm{n}})}{E_{\mathrm{tx}}}\boldsymbol{I}\right)^{-2}\hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{s}}\hat{\boldsymbol{H}}}} \\ \boldsymbol{B}^{\mathrm{H}} &= \boldsymbol{L}^{-1} \\ \boldsymbol{F}^{\mathrm{H}} &= \beta\boldsymbol{A}^{-1}\hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{L}^{\mathrm{H}}\boldsymbol{D} \end{aligned} \quad (18)$$

where 'tr' denotes the trace operator, symbols and noise are white, i.e., $\boldsymbol{R}_{\mathrm{s}} = \boldsymbol{I}$ and $\boldsymbol{R}_{\mathrm{n}} = \sigma_n^2\boldsymbol{I}$, $\boldsymbol{A}$ is equal to $\boldsymbol{C}_{\boldsymbol{\Theta}_{\mathrm{T}}} + \frac{\mathrm{tr}(\boldsymbol{R}_{\mathrm{n}})}{E_{\mathrm{tx}}}\boldsymbol{I}$ and

$$\boldsymbol{C}_{\boldsymbol{\Theta}_{\mathrm{T}}} = E[{\boldsymbol{\Theta}_{\mathrm{T}}}^{\mathrm{H}}\boldsymbol{\Theta}_{\mathrm{T}}] = \sum_{i=1}^{K} E[\boldsymbol{\theta}_i^*\boldsymbol{\theta}_i^{\mathrm{T}}] = \sum_{i=1}^{K} \boldsymbol{C}^*_{\mathrm{error},i}. \quad (19)$$

Here, $\boldsymbol{\theta}_i$ is the error vector for the i-th user and $\boldsymbol{C}_{\mathrm{error},i}$ is the corresponding error covariance matrix.

For LP, the *robust transmit Wiener filter* is given by

$$\boldsymbol{P} = g^{-1}\left(\hat{\boldsymbol{H}}^{\mathrm{H}}\hat{\boldsymbol{H}} + \boldsymbol{C}_{\boldsymbol{\Theta}_{\mathrm{T}}} + \frac{\mathrm{tr}(\boldsymbol{R}_{\mathrm{n}})}{E_{\mathrm{tx}}}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{H}}^{\mathrm{H}} \quad (20)$$

with $g^{-1} = \beta$.

Thus, by considering the statistical properties derived for the errors in the previous section, we can compensate in advance the performance degradation due to imperfect CSI at the transmitter.

VI. Simulations

In this section, we present the results of computer simulations that we carried out to validate the proposed system. The results are the mean of 5000 channel realizations and 200 symbols were transmitted per channel realization. The input bits are QPSK modulated and $N_{\mathrm{t}} = K = 8$. We consider an f_{slot} of 1500 Hz at a center frequency of 2 GHz. We have also considered errors due to the feedback delay, being D equal to 1 for all users. The Doppler frequency normalized to the slot period is of 0.037 (v= 30 km/h) which is a relatively fast fading. The *Signal to Noise Ratio* (SNR) is defined as the ratio between the total transmitted energy, E_{tx}, and the noise spectral density.

You can see in [8] how each type of error degrades the system more and more. For the feedback channel we considered, a compression ratio of 12.8 is obtained with a codebook of 1024 entries, $r = 2$ KL coefficients and $L = 32$ bits to encode the real and imaginary value of each channel coefficient. Fig. 2 shows the poor performance obtained with non-robust precoding designs. You can see how the non-robust curves go up for high SNR due to imperfect CSI. The figure plots the improvement in performance when the proposed robust schemes for LP and THP are applied. Thus, we can compensate the channel effects when the mismatch is caused by the different error sources that have been shown in this paper. Note that the effect of imperfect CSI is more pronounced for THP than for LP, but obviously the THP performance goes beyond the LP performance due to its better exploitation of the channel characteristics.

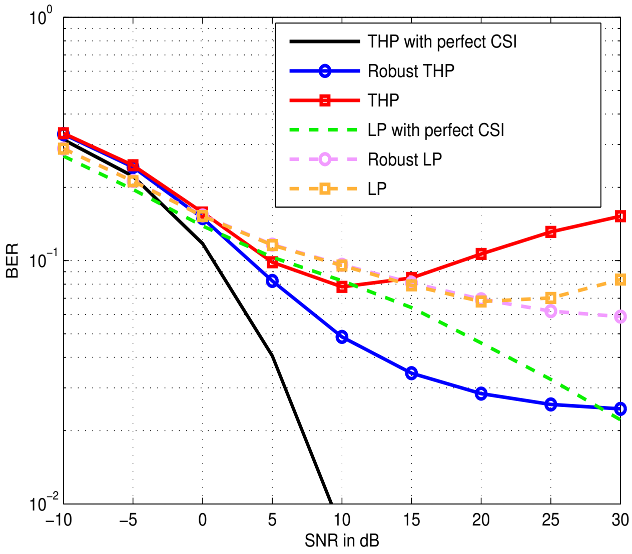

Fig. 2. Comparison between LP and THP: Robust and non-robust schemes

VII. Conclusions

In this paper, we have shown how an adequate modeling of CSI errors due to limited feedback channel can be used to design robust linear and non linear precoders for MU-MISO systems showing a superior performance with respect to conventional precoders. We conclude that robust designs are particularly useful when considering nonlinear precoding since more performance gains are obtained. This is because THP are more sensitive against CSI errors than LP.

Acknowledgments

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References

[1] D. J. Love, R. W. Heath Jr., W. Santipach and M. L. Honig, "What is the value of the limited feedback for MIMO channels?", *IEEE Communications Magazine*, vol. 42, pp. 54-59, Oct. 2004.

[2] K. Kusume, M. Joham, W. Utschick and G. Bauch, "Efficient Tomlinson–Harashima Precoding for Spatial Multiplexing on Flat MIMO Channel", *Proc. ICC 2005*, vol. 3, pp. 2021-2025, May 2005.

[3] T. Zemen, "Time–Variant Channel Estimation Using Discrete Prolate Spheroidal Sequences", *IEEE Trans. on Signal Processing*, vol. 53, No. 9, Sep. 2005.

[4] "3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Spatial channel model for Multiple Input Multiple Output (MIMO) simulations (Release 6)", 2003.

[5] P. M. Castro and L. Castedo, "Adaptive Vector Quantization for precoding using blind channel prediction in frequency selective MIMO mobile channels", *Proc. ITG/IEEE Workshop on Smart Antennas 2005*, Apr. 2005.

[6] A. Gersho and R. Gray. *Vector Quantization and Signal Compression*. Kluwer Academic Publishers, 1992.

[7] F. A. Dietrich and W. Utschick, "Robust Tomlinson-Harashima Precoding", *Proc. of the 16th IEEE Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, pp. 136–140, Berlin, Germany, 2005.

[8] P. M. Castro, M. Joham, L. Castedo and W. Utschick, "Robust Precoding for Multi-User MISO Systems with Limited-Feedback Channels", *Proc. ITG/IEEE Workshop on Smart Antennas 2007*, Feb. 2007.