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An Approach to CrossLayer QoS Management in MISO Systems

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***Abstract*— We study the crosslayer problem of joint optimization of radio parameters in a multiuser MISO system with partial channel state information at the transmitter. The central optimization problem within is formulated as a power minimization subject to a set of QoS constraints. Based upon analytical models for all regarded sublayers and the formulation of the resulting channel outage probability of the MISO system, the mode of operation and the MISO transmit processing are optimized jointly in an fast converging iterative scheme. With these results service requests from different users can be met with a significantly lower amount of resources, resulting in a higher system capacity.**

I. Introduction

The sublayers of the *media access control* (MAC) and the *physical* (PHY) layer of mobile communication systems have been subject to extensive research yielding a large variety of optimum and suboptimum signal processing techniques. Recently, efforts ([1], [2], [3]) have been made to combine the gathered knowledge and optimize two or even multiple of these sublayers jointly. This work introduces a covariance based approach for the joint optimization of radio parameters in the downlink of a mobile multiuser *multiple input single output* (MISO) communication system. Thus only partial *channel state information* (CSI) is required at the transmitter in the given broadcast scenario. The joint objective is to minimize the amount of resources, that are necessary to provide the required *quality of service* (QoS) to the different users.

To explicitly formulate and solve the posed problem, the relevant system components are backed with analytical models. As the lacking reciprocity of the up and downlink channel in *frequency division duplex* (FDD) systems makes full CSI at the transmitter a very demanding assumption the following considerations will all base on partial channel knowledge, which can be obtained from uplink measurements even in FDD systems [4], [5]. The fading MISO channel thus can be completely described by its longterm SINR and the corresponding channel outage probability. This approach allows for the modeling of CDMA processing components via socalled orthogonality factors [6], [7]. Unlike in most publication the performance of the *forward error correcting* (FEC) code is modeled through the cutoffrate theorem as introduced in [8]. Through the linearization of the error exponent the packet error probability can be expressed in terms of the chosen mode parameters and the SINR resulting from downlink MISO processing. This formulation gives way to the stochastic modeling of the *hybrid automatic repeat request* (HARQ) protocol and finally the regarded quality of service parameters throughput R, i.e., mean net data rate and delay τ. The latter is defined in an outage sense as the latency, which in a certain percentage of all cases suffices to transmit a packet error free. With the formulation of the QoS parameters for all users $k = 1, \dots, K$ the central crosslayer optimization problem can be tackled:

$$\{\boldsymbol{M}, \boldsymbol{P}\} = \underset{\boldsymbol{M},\boldsymbol{P}}{\operatorname{argmin}} \|\boldsymbol{P}\|_2^2 \quad \text{s.t.:} \begin{cases} \tau_k \leq \tau_{\mathrm{rq},k}, \forall k \\ R_k \geq R_{\mathrm{rq},k}, \forall k. \end{cases}$$

Within, the columns $\boldsymbol{p}_k$ of the matrix $\boldsymbol{P}$ denote the downlink beamforming weights. The columns $\boldsymbol{m}_k$ of the matrix $\boldsymbol{M}$ contain the mode parameters, i.e., the FEC code rate $R_{\mathrm{c},k}$, the cardinality of the modulation alphabet q_k, and the number of employed CDMA code channels c_k.

II. System Model and Assumptions

A. Channel and Signal Processing

The time variant channel of the regarded Kuser Mantenna MISO system is modeled as a vector valued random variable $\boldsymbol{h}_k \in \mathbb{C}^M$. Introducing its covariance matrix $\boldsymbol{R}_k = \mathrm{E}\left[\boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}}\right]$ with eigenvalues $\lambda_{k,\zeta}$ and eigenvectors $\boldsymbol{u}_{k,\zeta}$ the corresponding KarhunenLoewe transformation yields a representation of the vector valued channel through the M scalar coefficients $\rho_{k,\zeta}$ with complex Gaussian distribution:

$$\boldsymbol{h}_k = \sum_{\zeta=1}^{M} \rho_{k,\zeta} \boldsymbol{u}_{k,\zeta}, \quad \text{with} \quad \rho_{k,\zeta} \sim \mathcal{CN}(0, \lambda_{k,\zeta}). \tag{1}$$

Together with the MISO precoding or beamforming weight $\boldsymbol{p}_k$ (Section I) and the noise variance σ_η^2, we can model the resulting CDMA system as an equivalent *discrete memoryless channel* (DMC) with the instantaneous SINR at the kth mobile unit:

$$\tilde{\gamma}_k = \frac{\frac{\chi}{c_k} \boldsymbol{p}_k^{\mathrm{T}} \boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k^*}{\sum_{\ell=1}^{K} \nu_{\ell,k} \boldsymbol{p}_\ell^{\mathrm{T}} \boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_\ell^* + \sigma_\eta^2}. \tag{2}$$

This compact notation is obtained by the use of socalled orthogonality factors $\nu_{\ell,k}$ [6], [7] that describe the inter symbol and the multipleaccess interference due to lacking orthogonality among the used spreading codes, while the desired signal is weighted with the spreading gain χ, i.e. the length of the spreading sequences. Moreover, all users are assumed to transmit in $c_k$fold code division multiplex, i.e. over c_k code channels. Taking the expected value of the power components in (2) yields the longterm SINR as it can be computed in a base station with partial CSI:

$$\gamma_k = \frac{\frac{\chi}{c_k}\boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{R}_k\boldsymbol{p}_k^*}{\sum_{\ell=1}^{K}\nu_{\ell,k}\boldsymbol{p}_\ell^{\mathrm{T}}\boldsymbol{R}_k\boldsymbol{p}_\ell^* + \sigma_\eta^2}. \quad (3)$$

Performing mode optimization and precoding on the basis of longterm knowledge, i.e. partial CSI, a certain fraction of channel realizations will result in an instantaneous SINR that is smaller than γ_k. These channel realizations are considered outage events and their probability $\pi_{\mathrm{c}} = \Pr[\tilde{\gamma}_k < \gamma_k]$ is derived based upon the chosen mode $\boldsymbol{M}$ and the precoding weights $\boldsymbol{P}$ in Section IIID.

B. FEC Coding

Based on the above DMC model of the CDMA code channel, the cutoff rate theorem [8], [9], [10] allows to formulate an upper bound for the code word error probability of block codes. Through a linearization of the Gallager error exponent, the mode parameterized analytic modeling of the relation between decoder SINR and error probability in coded transmission systems is enabled. Moreover and in contrast to capacity based approaches, it includes the complete mode dependency, i.e. the influence of modulation alphabets with finite cardinality q, the binary code rate $R_{\mathrm{c},k}\mathrm{ld}q$ and the resulting block length B. Aiming for the employment of these very favorable properties within the crosslayer system model, [8] introduces the cutoff rate theorem, that bounds the packet error probability of a block code with block length B and binary code rate $R_{\mathrm{c},k}\mathrm{ld}q < R_0(\gamma_k)$ in bits per CDMA channel use by:

$$P_{\mathrm{PE}}(\gamma_k) = 2^{-B(R_0(\gamma_k) - R_{\mathrm{c},k}\mathrm{ld}q)}. \quad (4)$$

The cutoff rate $R_0(\gamma_k)$ denotes the maximum of the Gallager error exponent and is defined as a function of the conditional probability density of obtaining a channel output given a certain channel input. For DMCs and a modulation alphabet $\mathcal{A} = \{a_1, \ldots, a_q\}$ of cardinality q the derivations in [11], [12] compute the cutoff rate to:

$$R_0(\gamma_k) = \mathrm{ld}\,[q] - \mathrm{ld}\left[1 + \frac{2}{q}\sum_{m=1}^{q-1}\sum_{k=m+1}^{q} \mathrm{e}^{\left(-\frac{1}{4}|a_l - a_m|^2\gamma_k\right)}\right].$$

C. Protocol

Assuming an HARQ protocol in the MAC layer the system is given means to acknowledge the successful transmission of a packet or to demand the retransmission of lost packets. Sparing *incremental redundancy* (IR) methods, the following paragraph focuses on *Chase combining* (CC).[1] As the packets in the different transmissions of the CC mode do not differ and all face independent noise realizations on the channel a soft combining of these packets superimposes noise components incoherently resulting in a cumulative SINR increase and thus an $m$dependent value $\gamma_{k,m}$ for the SINR. Together with the means to quantize this SINR enhancement ([13], [14], [15], [16]), which depends on the employed modulation alphabet, the packet error probability after m transmissions results as:

$$P_{\mathrm{PE}}[m] = P_{\mathrm{PE}}(\gamma_{k,m}) = 2^{-B(R_0(\gamma_{k,m}) - R_{\mathrm{c},k}\mathrm{ld}q)}.$$

This equation allows to formulate the probability $f_m[m]$ that it takes m transmissions to decode a packet error free as the product of the probability of loosing $m-1$ consecutive packets and successfully transmitting the mth. The probability $f_m[m]$ thus is given by:

$$f_m[m] = \left(\prod_{m'=1}^{m-1} P_{\mathrm{PE}}[m']\right)(1 - P_{\mathrm{PE}}[m]). \quad (5)$$

D. QoS Expressions

Through the expression of waiting probabilities in $f_m[m]$ the QoS parameters throughput and outage delay can be formulated. Given the number of HARQ transmissions m the probability that it takes exactly n transmission attempts results from the channel outage probability π_{c} as:

$$f_{n|m}[n] = \pi_{\mathrm{c}}^{(n-m)}(1-\pi_{\mathrm{c}})^m \binom{n}{m}, \quad n > m. \quad (6)$$

The expression contains the probabilities of $n - m$ outage events, m nonoutage events and the number of possible permutations of these different events. The overall probability of waiting exactly n transmission attempts until a packet is received errorfree thus follows from marginalizing (6) with respect to m:

$$f_n[n] = \sum_{m=1}^{\infty} f_m[m] f_{n|m}[n]. \quad (7)$$

Due to the waterfall like behavior of (4) $f_m[m]$ in a very good approximation can be written as $f_m[m] = \delta[m - m^*]$ resulting in:

$$f_n[n] = f_{n|m^*}[n]. \quad (8)$$

The throughput, which is defined as the mean net data rate, thus results directly from the expected value of n as:

$$R_k = \frac{c_k R_{\mathrm{c},k}\mathrm{ld}q_k}{\mathrm{E}[n]}, \quad (9)$$

where $\mathrm{E}[n]$ is given through (7) or (8). Due to the retrans missions additional delay or latency times are introduced. The outage delay is defined as the time, such that a fraction of $1 - \pi_\tau$ of all transmission attempts are expected to succeed with a delay not larger than τ. Introducing the transmisison time interval T, i.e. the time it takes to transmit a packet once, the outage delay is given as:

$$\tau = n^* T, \quad (10)$$

$$n^* = \operatorname*{argmin}_n n \quad \text{s.t.:} \quad \sum_{n'=1}^{n} f_n[n'] \geq (1 - \pi_\tau).$$

[1]Note that publications like [13] and [14] allow to extend the proposed model to IR modes as well.

III. Cross Layer Optimization

We propose an iterative proceeding to determine the mode $\boldsymbol{M}$ (i.e. the code rate, the modulation alphabet, and the number of CDMA code channels) and the downlink beamforming $\boldsymbol{P}$ as the solution of the following problem:

$$\{\boldsymbol{M}, \boldsymbol{P}\} = \underset{\boldsymbol{M},\boldsymbol{P}}{\operatorname{argmin}} \|\boldsymbol{P}\|_2^2 \quad \text{s.t.:} \left\{ \begin{array}{c} \tau_k \leq \tau_{\mathrm{rq},k}, \forall k \\ R_k \geq R_{\mathrm{rq},k}, \forall k. \end{array} \right. \tag{11}$$

Based upon a preliminary assumption on the channel outage probability $\hat{\pi}_\mathrm{c}[i]$ every iteration step i will derive a set of K decoupled equivalent problems, which are independent of the channel realization and mutually decoupled among the users. The solution to these problems is derived in the upcoming paragraphs. Once the optimum mode and the optimum pre coding weights have been determined, the resulting channel outage probability $\pi_\mathrm{c}[i]$ can be computed (cf. Section IIID). This defacto value will be used as an apriori assumption for the next iteration step: $\hat{\pi}_\mathrm{c}[i+1] = \pi_\mathrm{c}[i]$. As the mapping $\pi_\mathrm{c}[i] \mapsto \pi_\mathrm{c}[i+1]$ for $\hat{\pi}_\mathrm{c}[1] = 0$ can be proven to be monotonic, the proof of convergence for the iterative scheme is provided by the bounded nature of the channel outage probability π_c.

A. Equivalent Decoupled Problems

The central approach in this paragraph is the gedankenex periment of posing equivalent requirements $\tilde{R}$ and $\tilde{\tau}$, which if fulfilled in the zerooutage case inherently will fulfill the orig inal requirements when facing the assumed outage probability $\hat{\pi}_\mathrm{c}[i]$. While the derivation of these equivalent requirements is a rather complex task in the general case of (7) the solutions for the well justified special case underlying (8) can be obtained[2] as:

$$\tilde{R}_k^{(\mathrm{rq})} = \frac{1}{(1-\pi_\mathrm{c})^2} R_k^{(\mathrm{rq})}, \tag{12}$$

$$\tilde{\tau}_k^{(\mathrm{rq})} = T\tilde{m}, \tag{13}$$

$$\text{with} \quad \tilde{m} = \underset{m}{\operatorname{argmax}}\, m, \quad \text{s.t.:} \sum_{n'=1}^{\left\lfloor \tau_k^{(\mathrm{rq})}/T \right\rfloor} f_{n|m}[n'] \geq \pi_\tau.$$

With this definition of $\tilde{R}^{(\mathrm{rq})}$ and $\tilde{\tau}^{(\mathrm{rq})}$ let $\boldsymbol{M}^*$ be an optimizer to the problems:

$$\left\{\boldsymbol{m}_k^*, \gamma_k^{(\mathrm{rq})}\right\} = \underset{\boldsymbol{m}_k,\gamma_k}{\operatorname{argmin}} \gamma_k \quad \text{s.t.:} \left\{ \begin{array}{c} \tilde{\tau}_k \leq \tilde{\tau}_k^{(\mathrm{rq})}, \\ \tilde{R}_k \geq \tilde{R}_k^{(\mathrm{rq})}. \end{array} \right. \tag{14}$$

Then $\boldsymbol{M}^*$ is also an optimizer to (11). The corresponding proof is provided by the mode independent nature of the SINRs γ_k and the strict monotonicity of γ_k with respect to all transmit powers. As proven for example in [17] or by the derivatives of (3) the SINR γ_k is monotonically increasing with $\|\boldsymbol{p}_k\|_2$ and monotonically decreasing with all $\|\boldsymbol{p}_\ell\|_2$, $\ell \neq k$.

[2]The expectation of the binomial distribution involved in the computation of $\mathrm{E}[n]$ can be obtained through Gaussian hypergeometric functions.

B. Mode Optimization

As shown in the above Subsection the preliminary as sumption $\hat{\pi}_\mathrm{c}[i]$ on the channel outage probability allows the solution of (11) with respect to $\boldsymbol{M}$ through K equivalent problems that are mutually decoupled among the users and channel independent. Their solution is uniquely determined by the equivalent requirements $\tilde{R}^{(\mathrm{rq})}$ and $\tilde{\tau}^{(\mathrm{rq})}$. Explicitly these problems are time invariant even in highly time variant settings for which they can be solved offline. Storing these offline computed solutions in a 2dimensional lookuptable for a sufficiently close grid of equivalent requirements provides an extremely efficient solution to the core of the posed optimization problem. In general the analytical inversions of

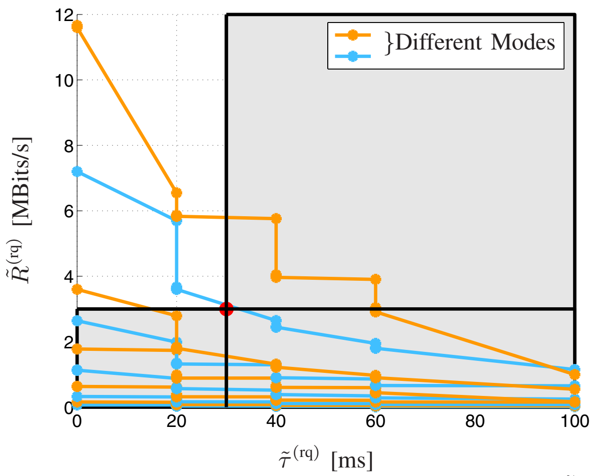

Fig. 1. Feasibility Regions for $\tilde{\tau}^{(\mathrm{rq})} = 30$ ms and $\tilde{R}^{(\mathrm{rq})} = 3$ MBits/s

the relevant system components do not exist, e.g. (4). Still the solutions can be obtained from sampling the $\tilde{R}$ $\tilde{\tau}$ plane with $\boldsymbol{m}_k$ and γ_k, i.e. computing the resulting throughput and delay values for a suitable grid in $\boldsymbol{m}_k$ and γ_k. Searching the resulting database in the feasibility region defined by the posed requirements $\tilde{R}^{(\mathrm{rq})}$ and $\tilde{\tau}^{(\mathrm{rq})}$ yields the optimum mode of operation $\boldsymbol{m}_k^*$. Fig. 1 visualizes this process. Every line represent the points of operation achievable by a certain mode and is parameterized by the underlying SINR. For the equivalent requirements $\tilde{\tau}^{(\mathrm{rq})} = 30$ ms and $\tilde{R}^{(\mathrm{rq})} = 3$ MBits/s Fig. 1 shades the feasibility region to which the search for the point of operation with minimum γ_k is constrained.

C. Resulting Downlink Problem

With the above considerations the optimization with re spect to $\boldsymbol{M}$ is solved through the intermediate objectives of minimizing γ_k. The resulting problem for downlink signal processing thus reads:

$$\boldsymbol{P}^* = \underset{\boldsymbol{P}}{\operatorname{argmin}} \|\boldsymbol{P}\|_2^2, \quad \text{s.t.:} \quad \gamma_k \geq \gamma_k^{(\mathrm{rq})}, \quad \forall k \tag{15}$$

This problem has been subject to several publications, e.g., [17], [18] for which we consider it solved. A short sketch of the optimum solution in [17] known as *SINR Balancing* as well as an efficient suboptimum algorithm from the framework of *linear precoding* in [19] can be found in the Appendix.

D. Channel Outage Probability

For these optimum choices of $\boldsymbol{P}$ and $\boldsymbol{M}$, the resulting channel outage probability π_c can be formulated through the distribution of the channel coefficients. To this end let us reformulate the instantaneous SINR from (2):

$$\tilde{\gamma}_k = \frac{\frac{\chi}{c}\boldsymbol{h}_k^\mathrm{H}\boldsymbol{P}_k\boldsymbol{h}_k}{\sum_{\ell=1}^{K}\nu_{\ell,k}\boldsymbol{h}_k^\mathrm{H}\boldsymbol{P}_\ell\boldsymbol{h}_k+\sigma_\eta^2}, \tag{16}$$

where we introduced $\boldsymbol{P}_k = \boldsymbol{p}_k^*\boldsymbol{p}_k^\mathrm{T}$. Based upon this formulation and the definition:

$$\boldsymbol{P}_k' = \boldsymbol{P}_k - \sum_{\ell=1}^{K}\nu_{\ell,k}\gamma_k^{(\mathrm{rq})}\boldsymbol{P}_\ell, \tag{17}$$

the outage probability for user k can be written as:

$$\pi_\mathrm{c} = \Pr\left(\boldsymbol{h}_k^\mathrm{H}\boldsymbol{P}_k'\boldsymbol{h}_k \leq \gamma_k^{(\mathrm{rq})}\sigma_\eta^2\right). \tag{18}$$

Employing a set of transformations proposed in [20] this problem can be solved by determining the density of a quadratic form in unitvariance chisquared random variables. Thus, Lemma 43b.1 in [21] does apply providing the solution to (18) as:

$$\pi_\mathrm{c} = 1 - \sum_{\zeta=1}^{\tilde{M}}\frac{\mu_{k,\zeta}^{M-1}}{\prod_{\substack{\xi=1\\ \xi\neq\zeta}}^{\tilde{M}}(\mu_{k,\zeta}-\mu_{k,\xi})}\exp\left(-\gamma_k^{(\mathrm{rq})}\frac{\sigma_\eta^2}{\mu_{k,\zeta}}\right). \tag{19}$$

Within, $\mu_{k,\zeta}\in\mathbb{R}$ are the eigenvalues of the Hermitian matrix $\boldsymbol{R}_k^{\frac{1}{2}}\boldsymbol{P}'\boldsymbol{R}_k^{\frac{1}{2}}$ sorted in descending order and $\tilde{M}$ is the number positive eigenvalues, i.e., $\mu_{k,\zeta}>0$, for $\zeta=1,\ldots,\tilde{M}$. Note, that (19) can not be applied in the unlikely case of nondistinct eigenvalues $\mu_{k,\zeta}$. In these cases one has to return to Lemma 4.3b.3 in [21] for a more general expression of the solution.

The result in (19) completes the iteration step i of the proposed crosslayer optimization. If the resulting outage probability $\pi_\mathrm{c}[i]$ differs from the apriori assumption $\hat{\pi}_\mathrm{c}[i]$ the obtained solution is not valid yet and the iteration has to continue until a valid solution to the problem in (11) is found. Table IIID summarizes the proposed iterative procedure.

- Initialize: $i=0$, $\hat{\pi}_\mathrm{c}[0]=1, \pi_\mathrm{c}[0]=0\ \forall k$
- Repeat
  - $i=i+1$, $\hat{\pi}_\mathrm{c}[i]=\pi_\mathrm{c}[i-1]$
  - Given $\hat{\pi}_\mathrm{c}[i]$, compute $\tilde{R}$ and $\tilde{\tau}$ along (12), (13)
  - Determine $\boldsymbol{M}$ by K table lookups
  - Solve (15) for $\boldsymbol{P}$
  - Compute $\pi_\mathrm{c}[i]$ along (19)
- until $|\pi_\mathrm{c}[i]-\hat{\pi}_\mathrm{c}[i]|<\epsilon\ \forall k$

TABLE I
ITERATIVE CROSS LAYER OPTIMIZATION

IV. Evaluation

The following paragraphs demonstrate the potential of the proposed technique and evaluate the algorithms in an HSDPA like setting. Namely, the simulation based upon the assumption of a circular cell with radius 1.5km. Users are distributed uniformly in space. From the resulting distances d [km], the Hata model [22], [23] gives the reciprocal of the occurring pathloss, i.e. the path powers for an urban environment as:

$$\sum_{\zeta=1}^{M}\lambda_{k,\zeta} = \begin{cases} -(133.3+33.8\log_{10}(d)+23)\,\mathrm{dB} & \text{indoor,}\\ -(133.3+33.8\log_{10}(d)+8)\,\mathrm{dB} & \text{outdoor,}\end{cases}$$

where indoor and outdoor scenarios occur with equal probability. In combination with a Laplacian model for the occurring angular spread of 2°, this setting determines the resulting covariance matrices, which are considered valid for the length of one longterm realization. For this longterm realization the crosslayer mode optimization is performed and the resulting mode is applied to 1000 shortterm realizations. Within, a fully implemented HARQ protocol determines the resulting throughput and the occurring latency times. Thereupon a longterm setting is marked infeasible, if either no suitable mode is found that can be served with the available transmit power of 15W[3], or if the posed QoS requirements have not been met by the scheme over the total of 1000 shortterm channel realizations.

The key metric for the evaluation will be the probability, that a certain traffic situation, i.e. user number and corresponding QoS requirements, can be fulfilled by different versions of the investigated crosslayer scheme. To this end, Fig. 2 plots

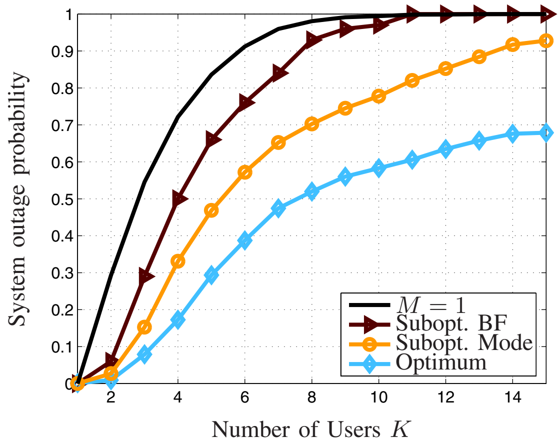

Fig. 2. Probability of infeasible settings for different number of users.

this probability for a system where all users have identical QoS requirements of $R_k^{(\mathrm{rq})}$ = 200kbps and $\tau_k^{(\mathrm{rq})}$ = 20ms over the number of users, that are granted access to the

[3]Moreover, the simulations assumed an additional fixed antenna gain of 16dBi and a noise floor of −95dBm at all mobile stations.

channel simultaneously. The figure shows the significant enhancement in terms of system outage probability obtained by the presented crosslayer optimization. Deviations from the optimum solution result in severe degradation of the system performance. As sketched in Fig. 2, a mode mismatch of 1 already poses significantly stronger SINR demands to the link, for which even the optimum link processing, cf. IA, degrades in performance. Even more severe is a the deterioration, when the suboptimum MISO processing scheme is chosen. In this case only little of the MISO potential is achieved.

A second graph evaluates the above versions of cross layer optimization for a fixed number of $K = 3$ users and shows, how the system outage probability increases with increasing throughput requirement of on user. For the given

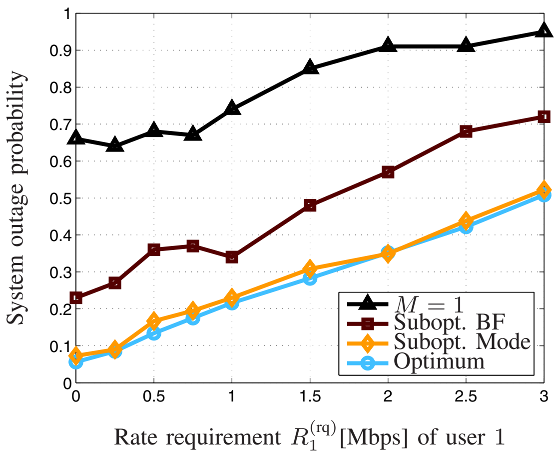

Fig. 3. Probability of infeasible settings for different rate requirements.

link budget of 15W transmit power the crosslayer optimum processing provides the means to drastically reduce the system outage probability, or equivalently enhance the servable system load. Again, the sensitivity with respect to a mode mismatch is not as dramatic as the deterioration when employing a suboptimum beamforming approach.

V. Conclusion and Outlook

We have presented an approach to jointly optimize all radio parameters up to the link scheduling unit, including the MAC protocol, the FEC coding and the linklevel signal processing of a MISO downlink. Within, QoS requirements of the different users are achieved with a minimum amount of resources. Within, an iterative proceeding in terms of the channel outage probability has decomposed the problem into a decoupled set of singleuser constant channel problems for the mode optimization and a well known form of the joint downlink beamforming problem. The provided solutions to both subproblems give a very efficient way to crosslayer optimum MISO processing.

Appendix I
Solutions to the Downlink Beamforming Problem

This section outlines two possible solutions for the downlink beamforming problem. While the first one in Subsection IA provides the optimizer to (15), the second one (cf. Subsection IB) follows a very prominent heuristic to obtain a low complexity suboptimum solution. The following subsections aim at resketching the resulting techniques and adapt them to the background given in this publication. For an insight in the underlying derivation we refer to the cited publications.

A. SINR Balancing

The following ideas base upon the duality of up and down link beamforming problems, i.e. the fact that the achievable SINR regions in up and downlink are identical and that points within those regions can be achieved by the same beamforming vectors.[4] In this context the term beamforming vectors refers to the normalized version of $\boldsymbol{p}_k$ defined as $\tilde{\boldsymbol{p}}_k = \frac{1}{\|\boldsymbol{p}_k\|_2}\boldsymbol{p}_k$. With this background, the considerations in e.g. [17], [24] derive the optimum solution to the problem:

$$\boldsymbol{P}^* = \underset{\boldsymbol{P}}{\operatorname{argmin}} \|\boldsymbol{P}\|_2^2, \quad \text{s.t.:} \quad \gamma_k \geq \gamma_k^{(\mathrm{rq})}. \ \forall k \tag{20}$$

To this end the coupling matrix $\boldsymbol{\Psi}$ is introduced to quantize the interference in the system through the elements:

$$[\boldsymbol{\Psi}]_{\ell,k} = \nu_{\ell,k}\tilde{\boldsymbol{p}}_k^{\mathrm{H}}\boldsymbol{R}_\ell\tilde{\boldsymbol{p}}_k. \tag{21}$$

This coupling matrix can be extended by defining the diagonal matrix $\boldsymbol{D} = \operatorname{diag}_{k=1}^{K}\left(\frac{\gamma_k}{\tilde{\boldsymbol{p}}_k^{\mathrm{H}}\boldsymbol{R}_k\tilde{\boldsymbol{p}}_k}\right)$ and by stacking the noise variances of all mobiles in a vector $\boldsymbol{\sigma}_\eta$:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{D}\boldsymbol{\Psi}^{\mathrm{T}} & \boldsymbol{D}\boldsymbol{\sigma}_\eta \\ \frac{1}{P_{\max}}\mathbf{1}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{\Psi}^{\mathrm{T}} & \frac{1}{P_{\max}}\mathbf{1}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{\sigma}_\eta \end{bmatrix}, \tag{22}$$

which yields the optimal power assignment, i.e. the powers $P_k = \|\boldsymbol{p}_k\|_2$, as first M components of the eigenvector corresponding to the largest eigenvalue of $\boldsymbol{\Lambda}$ normalized by the $M+1$st component. Note, that the corresponding maximum eigenvalue given the reciprocal level of balanced SINRs . The analysis of the dual virtual uplink problem furthermore provides the optimal solutions for the beamforming vectors $\tilde{\boldsymbol{p}}_k$. Let the matrix $\boldsymbol{Q}$ be defined as:

$$\boldsymbol{Q} = \sum_{k=1}^{K} P_k\tilde{\boldsymbol{R}}_k + \boldsymbol{I}, \tag{23}$$

with the normalized covariance matrices $\tilde{\boldsymbol{R}}_k = \boldsymbol{R}_k/\sigma_\eta^2$. Then, the optimal beamforming vectors can be obtained as the maximum generalized eigenvector of the matrix pair $\tilde{\boldsymbol{R}}_k, \boldsymbol{Q}$. As the optimum power allocation as well as the optimum beamforming vectors depend upon each other, the solution is provided by an iterative proceeding, alternating both steps until convergence. A short overview of the algorithmic solution is given in Table IA whereas a detailed convergence analysis and the underlying derivations can be found in [17], [24].

[4]The strict duality only holds for equal receiver noise variances. The concept can be extended to the general case by introducing the dual virtual uplink problem, cf. [17], [24].

- initialize: $i=0,\ P_k=0\ \forall k$
- $\tilde{\boldsymbol{R}}_k = \boldsymbol{R}_k/\sigma_\eta^2$
- $\sigma_\eta^2 = 1$
- repeat
  - $i = i+1$
  - $\tilde{\boldsymbol{p}}_k \Leftarrow$ max. eigenvalue of $\tilde{\boldsymbol{R}}_k, \boldsymbol{Q}\ \forall k$
  - $\tilde{\boldsymbol{p}}_k = \tilde{\boldsymbol{p}}_k/\|\tilde{\boldsymbol{p}}_k\|_2\ \forall k$
  - $[P_1,\ldots,P_K,1]^{\mathrm{T}} \Leftarrow$ max. eigenvector ofΛ
  - $\frac{1}{C[i]} \Leftarrow$ max. eigenvalue of Λ
- until $C[i]-C[i-1]<\epsilon$
- $[P_1,\ldots,P_K,1]^{\mathrm{T}} \Leftarrow$ max. eigenvector of Υ

TABLE II
ITERATIVE DOWNLINK BEAMFORMING

B. Covariance Based Linear Precoding

Unlike the above presented method, linear precoding tech niques can find closed analytic expressions for the downlink beamforming weight. Moreover, these techniques have already been proposed for frequency selective scenarios and systems with common pilot channel estimation, which might be of a certain relevance for the extension of this work. Though, these solutions base the derivation of $\boldsymbol{p}_k$ on the optimization of certain heuristics, which in the given setting only can serve as a suboptimal intermediate goal. The following subsection will explicitely derive the solution for the transmit matched filter that maximizes the desired signal power. In the chosen frequency selective situation and without regard of the receive filter structure, the transmit matched filter maximizes the following objective:

$$\boldsymbol{P} = \underset{\boldsymbol{P}}{\operatorname{argmax}}\ \boldsymbol{p}_k^{\mathrm{H}}\boldsymbol{R}_k\boldsymbol{p}_k, \quad \text{s.t.:}\|\boldsymbol{P}\|_2^2 \leq P_{\max} \tag{24}$$

This formulation allows us to split the determination of the beamforming weights into K separate subproblems that can be solved through the eigendecomposition of the covariance matrix. The vectors $\boldsymbol{p}_k$ thus result as the eigenvector corre sponding to the largest eigenvalue of $\boldsymbol{R}_k$ for which the scheme is also known as longterm eigenbeamforming:

$$\boldsymbol{P} = [\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K] = [\boldsymbol{u}_{1,1},\ldots,\boldsymbol{u}_{K,1}]. \tag{25}$$

The eigenvectors $\boldsymbol{u}_{k,\zeta}$ are defined in (1). The optimum donwlink power allocation thereupon can be found through the maximum eigenvector of the extended downlink coupling matrix $\boldsymbol{\Upsilon}$ as defined in the above paragraph IA.

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