FILTER STRUCTURES FOR DECIMATION: A COMPARISON

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ABSTRACT

In this paper three different digital filter structures for decimation by a factor of 64 are compared. They are based on a commercially available decimator AD7722 by Analog Devices. The AD 7722 is a complete low power $\Sigma\Delta$ analog-to-digital converter with the application in digital audio processing. The applied filter is a two-stage decimator which decreases the sample rate by 32 and 2 respectively. This structure is then compared to a state-of-the-art approach of a FIR decimator with six stages, each of them performing decimation by a factor of 2, and to two different approaches to an almost linear phase IIR decimator based on an very efficient class of lattice wave digital filters with also six down-samplers. The advantages and disadvantages of the selected structures are described.

1. INTRODUCTION

In today's advanced technologies power consumption has become the main limiting factor for many applications. Starting with processors for personal computing through terminals for mobile communications systems, down to applications like bionic ear. In many of those applications digital filtering is a very important issue and represents one of the most power consuming subsystems. Therefore it is important to compare different digital filter architectures to find the one that best fits the requirements for low-power design while maintaining all properties of a good filtering operation. The AD7722 analog-to-digital converter (ADC) [3] employs a $\Sigma\Delta$ conversion technique that converts the analog input into a digital pulse train. Due to high oversampling rate which spreads the quantization noise from 0 to $f_{CLKIN}/2$, the noise energy contained in the band of interest is reduced. The digital filter that follows removes the out of band quantization noise and reduces the data rate from f_{CLKIN} at the input of the filter to $f_{CLKIN}/64$ at the output. The output data rate is a little over twice the signal bandwidth, which guarantees that there is no loss of data in the signal band. The following filter specifications are required:

Input Sampling Frequency: 12.5 MHz
Output Sampling Frequency: 195.3 kHz
Passband(0kHz-90.625kHz) Ripple: 0.005
Stop-band(104.6875kHz-12.395Mhz) Attenuation: 90 dB

In the following different realizations of this digital filter are presented. In Section 2 a two-stage Finite Impulse Response (FIR) structure based on original solution by Analog Devices is given. In Section 3 a more efficient realization based on six half-band FIR filters is described. Section 4 shows two six-stage almost linear phase Infinite Impulse Response (IIR) realizations based on bireciprocal lattice Wave Digital Filters (WDF). The experimental results of Section 5 are followed by conclusions.

2. TWO-STAGE FIR DECIMATOR

The original solution employs two FIR filters in a cascade. The first filter is a 384-tap filter that samples the output of the modulator at f_{CLKIN}. The second filter is a 151-tap half-band filter that samples the output of the first filter at $f_{CLKIN}/32$ and decimates by 2. Half-band filters mark a special case of filters for which

$$\delta_s = \delta_p = \delta$$

$$\omega_s = \pi - \omega_p$$

where δ_p and δ_s are passband and stop-band ripples, ω_p and ω_s are passband and stop-band frequencies, respectively. Then the resulting equiripple optimal solution to the approximation problem has the property that

$$H(e^{j\omega}) = 1 - H(e^{j(\pi-\omega)})$$

which means that the frequency response is symmetric around $\omega = 1/2$ and we get

$$H(e^{j\pi/2}) = 0.5$$

It can be shown, that for such a filter every other impulse response coefficient is exactly 0. Thus, an additional factor of two reduction in computation is obtained [2].

The implementation of the original solution results in a filter with a group delay of 81 sampling periods. The filters are of course linear phase and thus symmetric, which reduces the number of multiplications per sample by a factor of two. Moreover, half of the coefficients of the half-band filter equal 0 which again reduces the number of multiplications. Nevertheless the great disadvantage of this structure is that the first filter runs at a very high sampling rate (f_{CLKIN}). Thus the computational effort per output sample is rather high.

3. SIX-STAGE HALF-BAND FILTER

To improve the computational efficiency of the decimator a cascade of six half-band FIR filters, as in the Fig. 2, can be applied.

The resulting filter is linear phase with a group delay of 81.5 and the passband behaviour is even better than in case of the two-stage filter. There are obvious advantages. The total filter order

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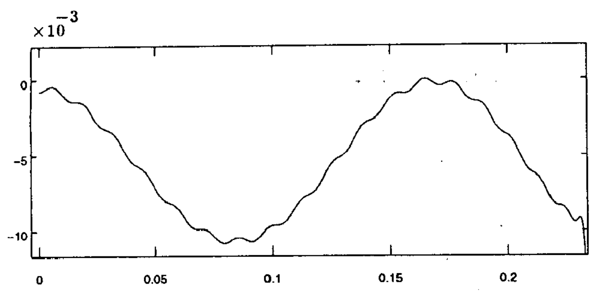

Figure 1: Passband ripple(dB) of the two-stage FIR decimator.

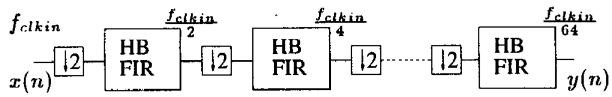

Figure 2: Decimator based on six half-band filters.

could be lowered from 535 to 196.0 All filters in the cascade are half-band, which allows them to run at the lower output sample rate. Thus, even the first filter (order 6) performs computations at $f_{CLKIN}/2$. There's, of course, no impact on the performance of the filter.

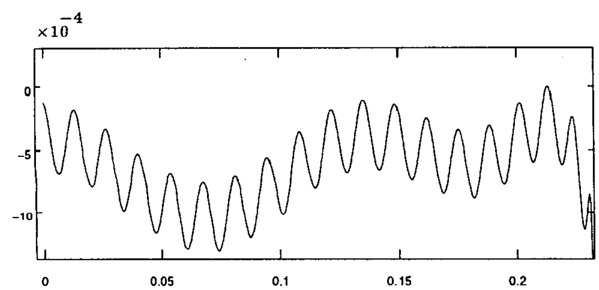

Figure 3: Passband ripple(dB) of the six-stage FIR half-band decimator.

4. SIX-STAGE WAVE DIGITAL FILTER

Wave digital filters (WDFs) are known to have many advantageous properties. They have low coefficient sensitivity, good dynamic range, and especially, good stability properties under quantization effects. Out of all wave digital filters the lattice wave digital filter is the most attractive one. Each WDF has a corresponding filter in a reference domain. The design can therefore be carried out in the analog domain using classical filter approximations. Then a transformation from analog to digital domain can be performed. For lattice WDF explicit formulae are given in [7]. However there exist no closed form solutions for filters satisfying given requirements on both magnitude and phase response.

A lattice WDF is a two-branch structure where each branch realizes an all-pass filter [5]. Out of several ways of realizing them [4] the most attractive one is to use cascaded first-order and second-order sections. They are realized using symmetric two-port adaptors. A bireciprocal (half-band) lattice WDF is a special case of lattice WDF. In this case every other coefficient of the filter becomes 0. This results in a structure shown in Fig. 4. Moreover, when the application is in a decimator by a factor of 2, the filter can run at the output sampling rate [6].

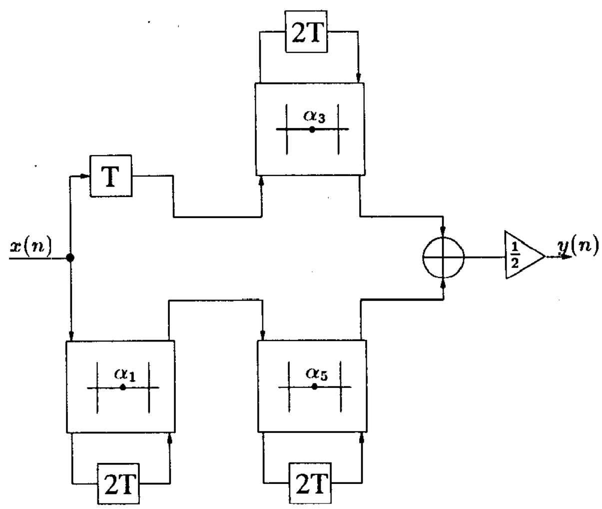

Figure 4: A 7th-order bireciprocal lattice wave digital filter.

The transfer function of a bireciprocal lattice WDF can be written as

$$H(z) = \frac{1}{2}(H_0(z^2) + z^{-1}H_1(z^2))$$

where the transfer function $H_0(z^2)$ corresponds to the lower branch in Fig. 4. The transfer function of the filter and its complementary transfer function are power complementary. For bireciprocal lattice WDFs we therefore have

$$|H(e^{j\omega T})|^2 + |H(e^{j\omega T - \pi})|^2 = 1$$

which means that the passband and stop-band edges are related by $\omega_p T + \omega_s T = \pi$, with T being the sampling time normalized to 2π. The consequence is that the passband ripple will be extremely small for practical requirements on the stop-band attenuation. Thus the bireciprocal WDFs have the efficiency of a FIR half-band filter in terms of reduced computational effort, while preserving the main advantages of IIR filters over FIR, which are sharp transitions for low order filters. Moreover, as will be discussed later, the six-stage WDF solution is more efficient than its FIR counterpart. The plots of magnitude, phase and group delay responses of the not equalized filter are shown in Fig. 5 and 6.

The main drawback of lattice WDFs is the non-linear phase response. However, many methods for obtaining almost linear phase of IIR filters have been presented in the past [9][11][1]. The first method used here is based on all-pass approximation presented in [10].

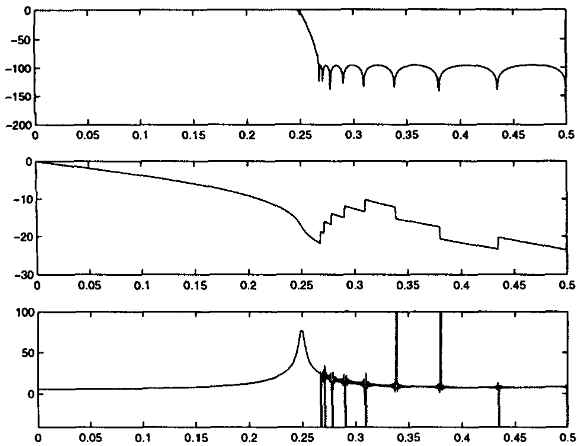

Figure 5: Magnitude(dB), phase(radians) and group delay(sampling intervals) of the six-stage bireciprocal lattice WDF decimator

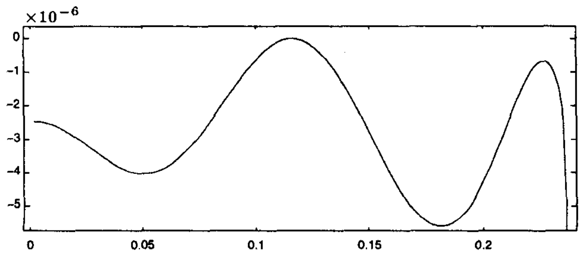

Figure 6: Passband ripple(dB) of the six-stage bireciprocal lattice WDF decimator.

As can be seen in Fig. 7 even with a low order all-pass-equalizer a very good approximation of phase linearity can be obtained. For an order 8 all-pass phase error is less than 0.13 radians, and for order 22 - below $4 \cdot 10^{-3}$ radians. Going as high as for order 40 the equalizer still offers savings of more than 20% accompanied by a meaningless phase error of $4.78 \cdot 10^{-5}$ radians. For the realization of the equalizer we again propose to apply wave digital filters. Also for this purpose they are very efficient and only one multiplier per equalizer order is required. The further advantage of the filter is a low group delay which ranges from 35 with an order 8 equalizer to 55 with order 22 equalizer. Even with an equalizer of order 40 the group delay is only slightly higher than in the case of the FIR.

In this paper we concentrate on bireciprocal lattice wave digital filters since they represent the most efficient, in terms of computational effort, family of IIR filters and are therefore of great interest. It is therefore very important to take a look at the methods dedicated to the design of linear phase bireciprocal lattice WDFs to be able to compare this solution to all-pass equalization. It is possible to obtain a bireciprocal lattice WDF with approximately linear phase by letting one of the branches in Fig. 4 consist of pure

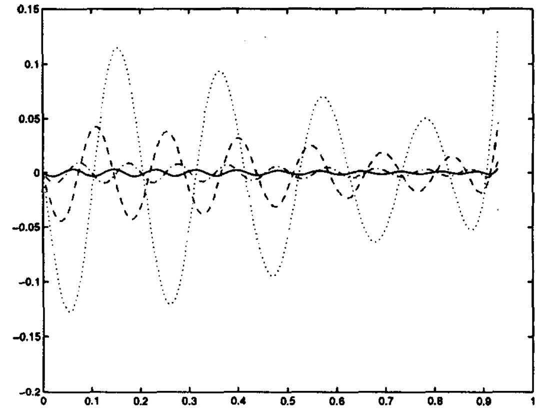

Figure 7: Phase error(radians) in the passband of the equalized bireciprocal lattice WDF decimator for different orders of the equalization all-pass (dotted - order 8, dashed - order 12, dash-dotted - order 18, solid - order 22).

delays [9][11][8]. The other branch is a general all-pass function in z^2, which can be realized using cascaded first and second orders sections (Fig. 8). Even if the order of such designed filter is

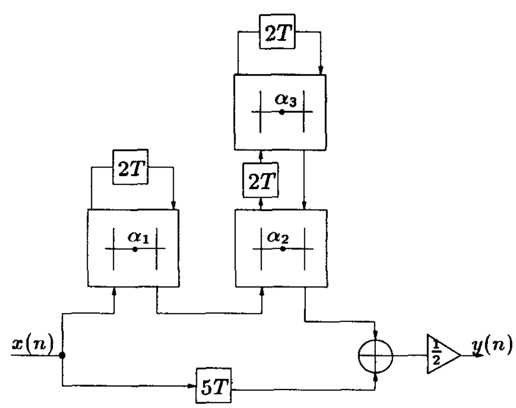

Figure 8: Structure of an 11th order almost linear phase bireciprocal lattice WDF.

much higher than that of the corresponding minimum phase solution, one has to take into account that the number of multipliers in this structure is only $(order + 1)/4$. These filters are thus as efficient, when comparing effort per filter order, as FIR half-band filters and a comparison to FIR solution is straightforward. And of course no additional equalizer is required.

Moreover, it is a well known fact, that wave digital filters have very low coefficient sensitivity. Thus it is possible to represent filter coefficients utilizing only a few bits. This could allow for decreasing the size of applied multipliers or even replacing them by shift and add operations.

5. EXPERIMENTAL RESULTS

The comparison of the computational effort of the described filter architectures is given in Table 1. For each filter, order, number of multiplications per output sample, phase error and group delay are given. Also shown is the reduction in computational complexity compared to FIR half-band approach. Since the FIR half-band solution is the state-of-the-art approach and its phase is linear, it has been taken as a benchmark. It is no surprise that the minimum phase WDF is much more efficient than FIR. But even with an excellent phase linearity given with an order 22 equalizer (phase error of only $4 \cdot 10^{-3}$ radians) the number of multiplications per output sample could be considerably reduced (by 30 %). Even better phase linearity (phase error in the range of 10^{-5} radians) has been achieved with the almost linear phase WDF of Fig. 8 with no loss in computational efficiency. However, only the all-pass equalization gives more degrees of freedom allowing the designer to influence the complexity by choosing how good or bad the approximation of the linear phase will be. Also only one equalizer is needed for all filters in the cascade. In many practical cases the requirements on the phase error will be orders magnitude lower than 10^{-5} and applying an equalizer could lead to an advantageous solution.

Architecture	Filter Order Per Stage	MPS (SAV)	Phase Error	Group Delay
6-Stage HB FIR	6/6/6/10/ 18/150	235 (0%)	0	81.5
6-Stage WDF (no equalizer)	5/5/5/7/ 9/17	141 (40%)	1.97	24
6-Stage WDF (order 8 eq) phase equalizer)	5/5/5/7/ 9/25	149 (36%)	0.129	31
6-Stage WDF (order 14 eq) phase equalizer)	5/5/5/7/ 9/31	155 (34%)	$2 \cdot 10^{-2}$	39
6-Stage WDF (order 22 eq) phase equalizer)	5/5/5/7/ 9/39	163 (30%)	$4 \cdot 10^{-3}$	55
6-Stage WDF (order 40 eq) phase equalizer)	5/5/5/7/ 9/57	181 (23%)	$4 \cdot 10^{-5}$	92
6-Stage almost linear phase WDF	8/8/8/8/ 16/120	158 (33%)	$8 \cdot 10^{-5}$	78

Table 1: Comparison of the three architectures (Phase error in radians, group delay in sampling intervals). MPS - Multiplications Per (Output) Sample, SAV - Savings in percent compared to half-band FIR.

6. CONCLUSION

In this paper a comparison of different decimator architectures has been presented. The results show significant differences in computational effort for their realization. Despite its non-linear phase response the proposed realization based on lattice wave digital filters seems to be advantageous. The computational effort can be varied depending on requirements on phase linearity. For strong linearity contraints almost linear phase WDF makes a better choice, otherwise phase equalization should be preferred.

It should be noted, that this comparison takes into account only the number of multiplications per sample. No attempt has been made here to introduce and compare general methods for low power design like numbers representation, retiming or transistor sizing. Most of these methods are independent of the chosen filter family and can be applied to FIR as well as IIR solutions. The WDF implementation presented here seems to gain on importance from the point of view of low-power design and almost linear phase IIR filters may represent a good alternative to today's standards. Of course the applications are not limited to filters for $\Sigma\Delta$ audio ADC like the one presented here. In fact, it could be applied in any kind of portable devices ranging from MP3-Players to terminals for mobile communication systems, where computational efficiency is extremely important and power consumption the main limiting factor. Especially in mobile communications, where external interferers like multi-path propagation do not allow for perfect symbol synchronization, strict phase linearity may not be a required feature.

7. REFERENCES

[1] M. Abo-Zahhad, M. Yaseen, and T. Henk. Design of Lattice Wave Digital Filters with Prescribed Loss and Phase Specifications. *ECCTD'95 Istanbul, Turkey*, 1:761–764, 1995.

[2] R. E. Crochiere and L. E. Rabiner. Multirate Digital Signal Processing. *Prentice Hall*, 1983.

[3] Analog Devices. AD7722 16-bit, 195kSPS CMOS, Sigma-Delta ADC. *Technical Specification*, 1996.

[4] A. Fettweis. Wave Digital Filters: Theory and Practice. *Proc. IEEE*, 74(2):270–327, February 1986.

[5] A. Fettweis, H. Levin, and A. Sedlmeyer. Wave Digital Lattice Filters. *International Journal on Circuit Theory and Applications*, 2:203–211, June 1974.

[6] A. Fettweis, J. A. Nossek, and K. Meerkötter. Reconstruction of Signals after Filtering and Sampling Rate Reduction. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-33(4):893–902, August 1985.

[7] L. Gazsi. Explicit Formulas for Lattice Wave Digital Filters. *IEEE Transactions on Circuits and Systems*, CAS-32(1):68–87, January 1985.

[8] H. Johansson and L. Wanhammar. Design of Bireciprocal Linear-Phase Lattice Wave Digital Filters. *Report LiTH-ISY-R-1877*, August 1996.

[9] I. Kunold. Linear Phase Realization of Wave Digital Filters. *IEEE Transactions on Acoustics, Speech and Signal Processing*, pages 1455–1458, 1988.

[10] T. Q. Nguyen, T. I. Laakso, and R. D. Koilpillai. Eigenfilter Approach for the Design of Allpass Filters Approximating a Given Phase Response. *IEEE Transactions on Signal Processing*, 42(9):2257–2263, September 1994.

[11] M. Renfors and T. Saramäki. Recursive Nth-Band Digital Filters Part I: Design and Properties. *IEEE Transactions on Circuits and Systems*, CAS-34(1):24–39, January 1987.