# Opportunistic Eigenbeamforming: Exploiting Multiuser Diversity and Channel Correlations

Mario Castañeda, Michael Joham and Josef A. Nossek

*Abstract* **Multiuser diversity is an inherent form of diversity present in any time-varying system with several users. An opportunistic scheduler has to be used in order to exploit this type of diversity. A scheme that increases the effective dynamic range of the channel by deploying multiple antennas at the transmitter is called opportunistic beamforming. Opportunistic beamforming increases the degree of multiuser diversity in several scenarios, including correlated channels. Nevertheless, multiuser diversity can also be combined with other transmit schemes that have proven to be effective in correlated channels, such as eigenbeamforming. Eigenbeamforming is a point-to-point link transmit technique that could easily be combined with an opportunistic scheduler to extract multiuser diversity. We refer to the joint use of eigenbeamforming with an opportunistic scheduler as *opportunistic eigenbeamforming*. In this work we show that the available multiuser diversity with opportunistic eigenbeamforming is larger than the one achieved when opportunistic beamforming is employed using the proportional fair scheduler under different degrees of correlation in the channel. In the present work we have considered a single cell scenario.**

*Keywords* **eigenbeamforming, MIMO systems, mobile communication, opportunistic beamforming, proportional fair scheduling**

## 1. Introduction

In third generation wireless systems such as CDMA2000 and WCDMA, the ever increasing demand for high data rate in the downlink has been addressed by including a high-speed shared channel through the *High Data Rate* (HDR) [1] mode and the *High Speed Downlink Packet Access* (HS-DPA) [2], respectively. In these multiuser systems, the spectral efficiency is improved by exploiting a novel form of diversity called *multiuser diversity*. Traditionally, schemes that employ link diversity view fading as a nuisance and mitigate the fading by averaging it out to resemble more closely a pure *additive white Gaussian noise* (AWGN) channel (e.g. [3]). On the other hand, multiuser diversity does not treat fading as a non-desired element of the system but actually takes advantage of the channel variations.

The multiuser diversity concept is best motivated by the informatic-theoretic result presented in [4]. There, Knopp and Humblet showed that to maximize the sum throughput of the users in the uplink of a multiuser system, a *time division multiple access* (TDMA) scheme must be used, serving at each time slot the user with the largest channel gain. Similar results have been obtained by Tse for the downlink in [5]. Multiuser diversity is inherent in the downlink of a system, which actually represents a point-to-multipoint link. By serving the best user at each time instant, multiuser diversity makes use of the channel gains of all users as efficiently as possible. As the number of users in the system increases, the gain in spectral efficiency achieved by multiuser diversity also increases. However, for the transmitter to be able to serve the best user, feedback of the *signal to noise ratio* (SNR) or partial *channel state information* (CSI) from each user is required. In addition, multiuser diversity can only be exploited by a proper scheduler.

A simple scheduler for a multiuser system is the deterministic *round robin scheduler* (RRS). The round robin scheduler is fair in the sense that it allocates the resources (time slots) equally among all users with a fixed delay. However, it produces a constant sum throughput of the system regardless of the number of users in the system and does not exploit multiuser diversity. On the other hand, an *opportunistic scheduler* serves the users taking into account their CSI, thus being able to exploit multiuser diversity and achieving a higher throughput compared to the round robin scheme. However, this gain comes at the expense of unfairness. Various opportunistic scheduling schemes have been summarized in [6]. There Liu et al. expressed that an opportunistic scheduler basically has two performance measures: throughput and fairness. In addition, a framework for opportunistic schedulers has been presented in [7] considering a minimum-performance requirement and two long-term fairness requirements: temporal fairness and utilitarian fairness. Temporal fairness means that each user obtains a certain part of the resources (time slots); meanwhile, utilitarian fairness means that each user obtains a certain portion of the overall system performance, e.g. spectral efficiency. A scheduler that achieves a good tradeoff between the aforementioned types of fairness and the throughput performance measure is the *proportional fair scheduler* (PFS) [8].

In addition to the number of users, the degree of multiuser diversity depends on other factors such as the dynamic range of the channel fluctuations. An approach that increases the dynamic range with the use of multiple antennas at the transmitter is called *opportunistic beamforming* [9]. Opportunistic beamforming can be considered as random beamforming and as an extension of the phase sweeping antennas presented in [10]. In [9], Viswanath et al. considered a point-to-multi-point link such as the downlink of a system with multiple anntenas at the transmitter and a single antenna at each receiver.

It has been shown in [11, 12, 13] that combining transmit diversity schemes, traditionally designed for point-to-point links, with an opportunistic scheduler under partial CSI feedback reduces the degree of available multiuser di-

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versity compared to a system with no point-to-point link diversity at all. However, as it has been stated in [14], proper use of spatial diversity does not really reduce the available multiuser diversity. Moreover, when high mobility is present among the users, multiuser diversity suffers due to the use of outdated feedback in the opportunistic scheduler [15]. The previous results motivate us to consider combining point-to-point link transmitting schemes with an opportunistic scheduler in a point-to-multi-point link in order to exploit multiuser diversity.

Opportunistic beamforming produces gain in several scenarios but it has been shown that this scheme achieves a higher gain in correlated channels [9, 16]. However, there is a point-to-point link scheme termed eigenbeamforming [17, 18] that has proven to be effective in correlated channels as well. For eigenbeamforming, the transmitter needs to know the principal eigenvector of the correlation matrix of the channel seen by the receiver. To this end, the receiver feeds back the principal eigenvector to the transmitter and not the whole correlation matrix. Note that the transmitter actually only requires partial CSI. Furthermore, in [19] it was shown how eigenbeamforming outperforms opportunistic beamforming in correlated channels for different degrees of spatial correlation. In this work, we investigate how eigenbeamforming combined with multiuser diversity can exploit not only spatial correlations in a channel but also the correlation that exists between time slots. We refer to the scheme that uses eigenbeamforming to exploit multiuser diversity as *opportunistic eigenbeamforming*. In the work at hand, it is shown not only that opportunistic eigenbeamforming is able to make better use of the spatial correlations but also is more robust to outdated feedback. We focus on the downlink of a multiuser system, i.e. a point-to-multi-point link.

In Section 2, an overview of multiuser diversity and the proportional fair scheduler is presented. Section 3 describes the channel model that will be utilized in this work. The concept of opportunistic beamforming is discussed in Section 4. Meanwhile, Section 5 defines the opportunistic eigenbeamforming approach by explaining how it can be combined with multiuser diversity. The results and analysis of our work are given in Section 6. Finally, the conclusions of this papers are presented in Section 7.

## 2. Preliminaries

### 2.1 Multiuser Diversity

Let us consider a point-to-multi-point link where a base station serves several users in the downlink. In order for such a link to attain multiuser diversity two general conditions are required: one at the transmitter and the other at the receivers. To exploit the multiuser diversity, the base station requires the channel gain or equivalent SNR from all of the users before scheduling a user. This information is gathered at the transmitter through fast feedback of the partial CSI from all the users to the base station. To this end, each user must be able to track and estimate his channel magnitude through a common downlink pilot and then feed it back to the base station. Upon feedback of all partial CSI of all the users, the base station decides to which user to transmit with a constant power. Once the transmitter has the partial CSI from all the user, we now come across the other requirement to extract the multiuser diversity: the base station must have the ability to schedule transmission among the users as well as to adapt the data rate to the fed back partial CSI. Nevertheless, the above mentioned requirements are present in the designs of many third generation systems such as IS-856 [1]. However, besides these requirements the actual degree of multiuser diversity depends on additional factors.

With a larger number of users and a larger dynamic range, there is more multiuser diversity available. In addition, a time-varying channel is required in order to exploit multiuser diversity. As the speed of the channel fluctuations increases so does the available multiuser diversity. However there is a limit on how fast the speed of the fading should be, such that the fedback CSI is not outdated. If the time-variance of the channels is incured by random beamforming, which is the same in the pilot- and subsequent payload phase, no outdating of CSI will occur.

Summing up, we can state that for a point-to-point link, a non-fading channel is the most reliable and desirable, but for a point-to-multi-point link with time varying independent fluctuations this statement is no longer true. Instead a system with several users with independent time-varying fading channel with high probability of large channel magnitudes and fast fluctuations is better in order to extract multiuser diversity.

### 2.2 Proportional Fair Scheduler

Multiuser diversity can only be exploited through the use of an opportunistic scheduler, for which we will consider the *proportional fair scheduler* [8] (PFS). Let us define the supported data rate for user $k$ at time slot $n$ as $R_k[n]$. When the PFS is employed, the base station transmits to the user with the largest current supported data rate compared to its own average rate, i.e. the user $k^*$

$$k^*[n] = \underset{k}{\text{argmax}} \frac{R_k[n]}{T_k[n]}, \quad (1)$$

where $T_k[n]$ is the average throughput of user $k$ at time slot $n$. Through this scheduling principle, the statistically weaker users will not suffer at the expense of the stronger user as they do not have to wait to have the best channel or largest supported data rate $R_k[n]$ to be served. In this sense, the user with the best *relative channel* is served. Moreover, the average throughput $T_k$ is updated as follows:

$$T_k[t+1] = \begin{cases} (1-\frac{1}{t_c})T_k[n] + \frac{1}{t_c}R_k[n] & k = k^*[n], \\ (1-\frac{1}{t_c})T_k[n] & k \neq k^*[n], \end{cases} \quad (2)$$

where $k^*[n]$ refers to the user served in time slot $n$ and $t_c$ is a time constant.

The proportional fair scheduler can be tuned to achieve different fairness and delay performances. To this end, let us define the forgetting factor $f$ as the inverse of the time constant $t_c$ ($f = \frac{1}{t_c}$). Then, the forgetting factor ranges from 0 to 1 and it represents the percentage of how much weight the served data rate $R_{k^*}[n]$ for time slot $n$ has on

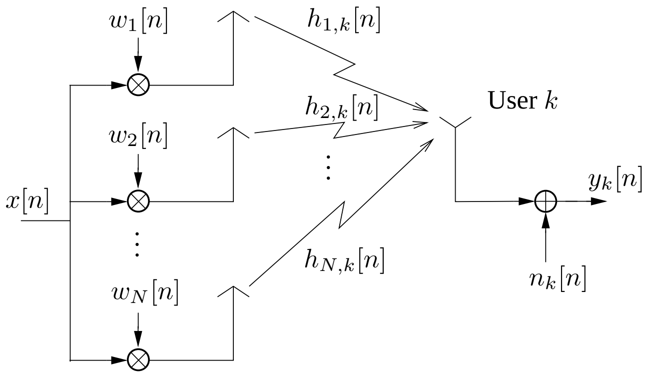


Fig. 1. MISO Channel Model for user $k$

the average throughput $T_{k^*}[n]$ for user $k^*[n]$. The PFS achieves the best delay performance when the forgetting factor approaches 1. In this case, the PFS approaches the *round robin scheduler* and no multiuser diversity can be exploited with this setting. Meanwhile, when the forgetting factor in the PFS approaches 0, the PFS now approaches the the *greedy scheduler* (GS), thus achieving the maximum multiuser diversity of the system but at the expense of increased delay on the weaker users. Hence, the degree of multiuser diversity that can be exploited from the system can be tuned with the forgetting factor $f$ in the PFS.

## 3. Channel Model and Correlations

Before presenting the concepts of opportunistic beamforming and eigenbeamforming let us first introduce the channel model that will be employed. We will consider a flat fading downlink of a multiuser system with $K$ users, i.e. a point-to-multi-point link. $N$ antennas are deployed at the base station while the receiver at each user has only one antenna, thus we have a *multiuser mutiple-input single-output* (MU-MISO) system as shown in Fig. 1 for user $k$. Let us define $x[n] \in \mathbb{C}$ as the transmitted symbol for time slot $n$, $h_{m,k}[n] \in \mathbb{C}$ as the complex channel gain from antenna $m$ to the $k$th user for time slot $n$, $n_k[n] \in \mathbb{C}$ as the additive white noise at the receiver $k$ for time slot $n$, and $y_k[n] \in \mathbb{C}$ as the received signal at user $k$ for time slot $n$. In our model, we assume that $h_{n,k}[n]$ are complex Guassian distributed random variables with unit variance, i.e. Rayleigh fading.

Futhermore, let us assume that the base station has a *uniform linear array* (ULA) with $N$ identical transmit antennas. Even though we have made the assumption of frequency non-selective channels we considered that each channel $h_{m,k}[n]$ is composed of $B$ unresolvable subpaths. We assume that the directions of departure of each of the $B$ subpaths for each user are distributed over a given angle spread $\delta$ with a certain mean angle of departure $\theta_k$ per user $k$. This mean angle $\theta_k$ per user is taken to be uniformly distributed from $[0, 2\pi]$. Furthermore, a far field assumption is made so that the narrow band signals delay caused by the geometry of ULA can be expressed as a phase shift. Therefore, the $m$th element of the steering vector of the antenna array is given by $\mathrm{e}^{-\mathrm{j}(m-1)2\pi d \sin\theta_{k,b}}$, where $d$ and $\theta_{k,b}$ are the distance between antennas given in wavelengths of the signal, and the angle of departure of the $b$th subpath of the $k$th user, respectively. Notice, that $\theta_{k,b} \in [\theta_k - \delta, \theta_k + \delta]$. Let us now denote the channel vector for user $k$ as $\mathbf{h}_k[n] = [h_{1,k}[n], h_{1,k}[n], \ldots, h_{N,k}[n]]^{\mathrm{T}}$, where $(\bullet)^{\mathrm{T}}$ represents the transpose operator. Assuming a distance between antennas of $d = 1/2$ and based on the geometry of the ULA we can model the channel vector $\mathbf{h}_k[n]$, for user $k$ as follows:

$$\mathbf{h}_k[n] = \mathbf{A}_{\mathrm{Tx},k} \cdot \boldsymbol{\phi}_k[n], \tag{3}$$

where $\boldsymbol{\phi}_k[n] \in \mathbb{C}^B$ whose elements are zero mean independent complex Gaussian random variables with variance equal to $1/B$ in order to have $\mathrm{E}\left\{|h_{m,k}|^2\right\} = 1$. Furthermore, we have that $\mathbf{A}_{\mathrm{Tx},k}$ is the transmit array steering matrix given by the Vandermonde matrix:

$$\mathbf{A}_{\mathrm{Tx},k} = \begin{pmatrix} 1 & \cdots & 1 \\ \mathrm{e}^{-\mathrm{j}\pi\sin\theta_{k,1}} & \cdots & \mathrm{e}^{-\mathrm{j}\pi\sin\theta_{k,B}} \\ \vdots & \ddots & \vdots \\ \mathrm{e}^{-\mathrm{j}\pi(M-1)\sin\theta_{k,1}} & \cdots & \mathrm{e}^{-\mathrm{j}\pi(M-1)\sin\theta_{k,B}} \end{pmatrix}, \tag{4}$$

where $\mathbf{A}_{\mathrm{Tx},k} \in \mathbb{C}^{N\times B}$.

If $\mathbf{h}_k$ is generated as shown in (3), then the resulting elements of $h_{m,k}[n]$ are still complex Gaussian random variables with zero mean and unit variance. This comes from the fact that each of the $h_{m,k}[n]$ results from a summation of $B$ complex Gaussian random variables, through the steering matrix $\mathbf{A}_{\mathrm{Tx},k}$, which in turn generate a complex Gaussian random variable with a variance equal to that of the sum of the variances which in this case is $\sum_{b=1}^{B} 1/B = 1$. Therefore, if the $h_{m,k}$, $m = 1, \ldots, N$, are Rayleigh distributed with unit variance and some correlations among them for each user $k$.

Moreover, we have that the spatial transmit correlation matrix of the channel vector of each user $k$ is given by:

$$\mathbf{C}_k = \mathrm{E}\left\{\mathbf{h}_k \cdot \mathbf{h}_k^{\mathrm{H}}\right\} = \frac{1}{B} \cdot \mathbf{A}_{\mathrm{Tx},k}\mathbf{A}_{\mathrm{Tx},k}^{\mathrm{H}} \in \mathbb{C}^{N\times N}, \tag{5}$$

where $(\bullet)^{\mathrm{H}}$ denotes the conjugate transpose or Hermitian operator $(\bullet)^{*,\mathrm{T}}$. This spatial correlation matrix $\mathbf{C}_k$ depends specially on the angle spread $\delta$ of the path to user $k$ among where the $B$ unresolvable paths are located. For a small angle spread $\delta$ ($\delta \approx \sin\delta$) and with a large number of scatterers located on a ring around each user terminal, the spatial correlation between antennas $m$ and $p$, i.e. the elements of the matrix $\mathbf{C}_k$, i.e. $\mathrm{E}\left\{h_{m,k}h_{p,k}^*\right\}$, can then be approximated by [20]:

$$\mathrm{J}_0\left(2\pi(p-m)\, d\, \delta \cos(\theta_k)\right) \mathrm{e}^{-\mathrm{j}2\pi(p-m)d\sin(\theta_k)}, \tag{6}$$

where $\mathrm{J}_0(\bullet)$ denotes the Bessel function of the first kind of order zero.

Furthermore, we assume the channel to have a temporally correlated block fading, which means that $h_{m,k}[n]$ remains constant for time slot $n$. As for the temporal correlation, we assume a Jakes power density spectrum, which results in a temporal auto-correlation function of $h_{m,k}[n]$ for antenna $m$, $m = 1, \ldots, N$, and user $k$ that reads as follows [21]:

$$\mathrm{E}\left\{h_{m,k}[n] \cdot h^*_{m,k}[n+\Delta t]\right\} = \sigma_k^2 \cdot \mathrm{J}_0(2\pi f_\mathrm{n} \Delta t), \quad (7)$$

$f_\mathrm{n}$ and $\Delta t$ denote the normalized Doppler frequency, and the difference in number of time slots, respectively. The normalized Doppler frequency is given by $f_\mathrm{n} = \frac{f_\mathrm{carrier} \cdot v}{f_\mathrm{slot} \cdot \mathrm{c}} \cos\beta$, where $f_\mathrm{carrier}$, $v$, $f_\mathrm{slot}$, c, and $\beta$ are the carrier frequency, the speed of the user, the frequency of the slots, the speed of light, and the angle between the direction of the user and the path to the antenna $m$, respectively [1]. We assume that $\beta = 0$ for every $k$.

The multiple antennas at the base station shown in Fig. 1 will be used for beamforming rather than transmit diversity. In that case, the corresponding MISO system for each user can be described by an equivalent SISO system. However, when considering the rest of the users we now have a multi-point-to-point link. Let us denote the beamforming vector applied at the base station, as shown in Fig. 1, as $\mathbf{w}[n] = [w_1[n], w_2[n], \ldots, w_N[n]]^\mathrm{T} \in \mathbb{C}^N$, where $|w_m[n]| \in [0,1]$ and $\arg(w_m[n]) \in [0, 2\pi]$, for $m = 1, \ldots, N$, are the power allocation and phase allocation on each antenna $m$, respectively. In order to preserve the transmit power, we must satisfy $\sum_{m=1}^N |w_m[n]|^2 = 1$, i.e. the vector $\mathbf{w}[n]$ has unit norm. Therefore, we then have that the received signal $\mathbf{y}_k[n]$ for user $k$, shown in Fig. 1, reads as follows:

$$y_k[n] = \mathbf{w}^\mathrm{T}[n] \cdot \mathbf{h}_k[n] \cdot x[n] + n_k[n] \quad (8)$$

$$= h_k[n] \cdot x[n] + n_k[n], \quad (9)$$

where $h_k[n] = \mathbf{w}^\mathrm{T}[n] \cdot \mathbf{h}_k[n]$ is the equivalent channel seen by user $k$.

## 4. Opportunistic Beamforming

When applying a random vector $\mathbf{w}_\mathrm{ob}[n] \in \mathbb{C}^N$ through opportunistic beamforming in correlated channels, the resulting equivalent channel $h_k[n]$ for each user $k$ from (9) will have a larger fluctuations than the original channels $h_{m,k}[n]$, $m = 1, \ldots, N$, for user $k$. This will increase artificially the degree of multiuser diversity in the system. Furthermore, just as in the case of a single antenna at the base station, the users must track their equivalent channel $h_k[n]$ and feed back to the base station their supported data rate $R_k[n]$ resulting from the power and phase allocation done at the transmitter given by $\mathbf{w}_\mathrm{ob}[n]$. Then, the base station decides which user to transmit to based on the scheduling policy. If the PFS is used, the base station transmits to the best relative user applying $\mathbf{w}_\mathrm{ob}[n]$ at the transmit antennas. With opportunistic beamforming not only does the equivalent channel resembles actually a SISO channel for each user $k$, but also the use of the multiple antennas is completely transparent at each receiving user.

[1] Note that $f_\mathrm{slot}$ appears in the denominator of the normalized Doppler frequency $f_\mathrm{n}$, because $\Delta t$ in (7) is given in number of time slots and not in seconds.

For opportunistic beamforming to be effectively employed in a correlated channel, the generated opportunistic beams $\mathbf{w}_\mathrm{ob}[n]$ must have the same distribution as that of $\mathbf{z}/\sqrt{\mathbf{z}^\mathrm{H}\mathbf{z}}$, where $\mathbf{z} \in \mathbb{C}^N$ is a vector whose elements have the same distribution as that of the $h_{m,k}$ for $m = 1, \ldots, N$ [9]. Furthermore, since the $h_{m,k}$ are complex Gaussian random variables, with independent real and imaginary parts, then the magnitude and phase of each $h_{m,k}$ are independent [22]. Hence, the magnitude and phase of the elements of $\mathbf{w}_\mathrm{ob}[n]$ can be generated seperately. Thus, the magnitudes $|w_{\mathrm{ob},m}|$, $m = 1, \ldots, N$ of the vector $\mathbf{w}_\mathrm{ob}[n]$ are taken form the magnitudes of the elements of an isotropically distributed vector.

As for the distribution of the angles $\theta_m = \arg(w_{\mathrm{ob},m})$ of the elements of $\mathbf{w}_\mathrm{ob}[n]$, one could expect that to match the distribution of the correlated Rayleigh channels $h_{m,k}$, the angles $\theta_m$, $m = 1, \ldots, N$, should be uniformly distributed over $[0, 2\pi]$, i.e. $\mathbf{w}_\mathrm{ob}[n]$ would be an isotropically distributed vector. However, when considering the spatial correlations one needs only to transmit over the strongest beam to user $k$. Therefore, only one angle of departure $\theta[n]$ is required, to transmit over one beam to each user, instead of $N$ independent angles [9, 23]. This can also be observed by looking at the approximation given in (6) of the elements of $\mathbf{C}_k$ for small angle spread. Assuming that the distance between adjacent antennas given in wavelengths is $d = \frac{1}{2}$, then the allocated phase $\theta_m[n]$ would be given by:

$$\theta_m[n] = (m-1)\pi \sin(\theta[n]), \quad (10)$$

for each antenna $m$, $m = 1, \ldots, N$. Notice that assuming that the angle of depature $\theta[n]$ is uniformly distributed over $[0, 2\pi]$ does not lead to a uniform distribution of the angle $\theta_m[n]$, for $m = 1, \ldots, N$. The fact that only one angle needs to be varied can explain why does opportunistic beamforming performs better under correlated fading than under uncorrelated fading. In uncorrelated channel, opportunistic beamforming needs to select appropiately $N$ angles $\theta_m[n]$ in order to coherently beamform a user. However, in a correlated channel it is easier to achieve the maximum rate through coherent beamforming since only one angle instead of $N$ needs to be selected appropiately [24].

## 5. Opportunistic Eigenbeamforming

A transmitting scheme that efficiently makes use of the fading correlations in point-to-point links is eigenbeamforming [17, 18]. Eigenbeamforming takes advantage of the spatial correlations present at the base station by tranmitting over the strongest beam to a given user. To this end, eigenbeamforming requires partial CSI at the transmitter, which in this case refers to the principal eigenvector of the spatial correlation matrix $\mathbf{C}_k$ of the channel for each user $k$. However, the receiving user can not exactly calculate $\mathbf{C}_k$ given by (5) and instead a *long-term* correlation matrix $\mathbf{C}_{\mathrm{LT},k}$ is used as an estimate. How this long-term correlation matrix is estimated will be described later. Let us then denote the sorted eigenvalue decomposition of the correlation matrix $\mathbf{C}_{\text{LT},k}$ as follows:

$$\mathbf{C}_{\text{LT},k} = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^{\text{H}} = \sum_{i=1}^{N} \lambda_{i,k} \mathbf{v}_{i,k} \mathbf{v}_{i,k}^{\text{H}}, \tag{11}$$

where $\mathbf{v}_{1,k}$ is the principal eigenvector of $\mathbf{C}_{\text{LT},k}$, i.e. the eigenvector belonging to the largest eigenvalue $\lambda_{1,k}$ of $\mathbf{C}_{\text{LT},k}$. Under eigenbeamforming, the beam vector $\mathbf{w}_{\text{eb},k}[n]$ applied at the transmitting base station for user $k$ would then be $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$. Contrary to opportunistic beamforming, in opportunistic eigenbeamforming there is a beamforming vector for every user, since each user has his own distinct principal eigenvector. By applying this power and phase allocation at the base station, the data for user $k$ is transmitted over the strongest beam available in the channel to user $k$. This in average increases the throughput of the point-to-point link under the correlations present in the channel [17, 18].

In [19], it was shown how eigenbeamforming can be combined with multiuser diversity. We refer to this combination as *opportunistic eigenbeamforming*. In opportunistic eigenbeamforming the users must feed back their principal eigenvector to the base station. This can be done over several time slots with a given feedback rate. For the users to calculate this principal eigenvector, they first require to track and estimate their channels $h_{m,k}[n]$, $m = 1, \ldots, N$, for user $k$. To this end, the base station must send separate pilot signals on each antenna $m$ for $m = 1, \ldots, N$. Once the receiving users have estimated his channel they proceed to calculate a *short-term* correlation matrix $\mathbf{C}_{\text{ST},k}$ with the current channel conditions:

$$\mathbf{C}_{\text{ST},k}[n] = \mathbf{h}_k[n] \cdot \mathbf{h}_k^{\text{H}}[n], \tag{12}$$

for each user $k$. This short-term correlation matrix is used to update the long-term correlation matrix $\mathbf{C}_{\text{LT},k}$ at time slot $T$ as follows:

$$\mathbf{C}_{\text{LT},k}[n] = \frac{1}{T} \cdot \sum_{n=1}^{T} \mathbf{C}_{\text{ST},k}[n]. \tag{13}$$

Let us now assume that the base station has the principal eigenvector $\mathbf{v}_{1,k}$ for each user $k$. When combining eigenbeamforming with multiuser diversity the base station must decide to which user to transmit based on some fed back partial CSI. Even though, that for opportunistic eigenbeamforming the individual links $h_{m,k}$, for $m = 1, \ldots, N$, are required for updating $\mathbf{C}_{\text{LT}}$, a good estimate of the individual links is not required at each time slot for choosing the best user. At each time slot each user must feedback what would their equivalent channel $h_k$ from (9), if they were served by transmitting over their strongest beam with the beamforming vector $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$ applied at the base station. Based on the Karhunen-Loève expansion [25] we can write the channel vector of user $k$ as follows:

$$\mathbf{h}_k = \sum_{i=1}^{N} \xi_{i,k} \cdot \mathbf{v}_{i,k}, \tag{14}$$

where $\xi_{i,k}$, $i = 1, \ldots, N$ are unit variance complex Gaussian random variables. If the beamforming vector $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$ is applied at the transmitter then the equivalent channel is given from (14) as:

$$h_k = \mathbf{w}_{\text{eb},k}^{\text{T}} \cdot \mathbf{h}_k = \mathbf{v}_{1,k}^{\text{H}} \cdot \mathbf{h}_k = \xi_{1,k}. \tag{15}$$

In order to determine $h_k$, the receivers do not need to measure the individual links $h_{m,k}$, for $m = 1, \ldots, N$. Instead, they just need to measure $\xi_{1,k}$ which represents the equivalent channel $h_k$ seen by user $k$ when applying $\mathbf{w}_{\text{eb},k}[n]$ at the base station. The equivalent channel $h_k$ is still just one complex number as in the case of opportunistic beamforming. Moreover, the users feed back the magnitude of $h_k$ or the supported data rate $R_k[n]$, described in Section 2.2, for this channel $h_k$. Upon reception of all the supported data rates from all the users, the base station decides to which user to transmit by employing an opportunistic scheduler. In case the proportional fair scheduler is employed, the base station transmits to the best relative user.

Therefore, in opportunistic eigenbeamforming the channel is tracked, through the aid of the pilot signals transmitted from the base station, for two purposes. On the one hand, these pilots are used to estimate the channels $h_{m,k}[n]$, for $m = 1, \ldots, N$. These individual links are required by the eigenbeamforming scheme in order to calculate the short-term correlation matrix which is then used to update the long-term correlation matrix from where their current principal eigenvector for each user is estimated. On the other hand, the channel is also tracked in order to estimate the equivalent channel $h_k = \xi_{1,k}$ for each user under the assumption that the base station transmits over their strongest current beam.

### 5.1 Feedback of Principal Eigenvectors

In order to schedule the users and exploit multiuser diversity in both schemes, opportunistic beamforming and opportunistic eigenbeamforming, the feedback of the SNR is required. However, opportunistic eigenbeamforming needs additional feedback so that the principal eigenvector of each user is available at the base station.

In order to provide a quantitative comparison of the additional feedback required by opportunistic eigenbeamforming, let us consider the following example. Let us assume that the SNR feedback is constrained to $500$ Hz and that the feedback is quantized with $B \geq 1$ bits. That means that in one second each user needs to feedback $500B$ bits to the base station. Now, let us assume that the long-term properties of the channel remain constant for one second. For a speed of $36$ kmph, that means that the we assume that the *long-term* correlation matrix $\mathbf{C}_{\text{LT},k}$ and therefore the principal eigenvector of user $k$ remain constant over a distance of $10$ meters, which is a reasonable assumption.

In addition, let us assume that the users feedback their principal eigenvector by using $P \geq 1$ quantization bits per real and imaginary part of the eigenvector. Furthermore, let us recall that the beamforming vector is constrained to have unit norm and that only the relative difference between the phases of the eigenvector elements are of importance. To show this, let us assume that user $k$, instead of feeding back $\mathbf{v}_{i,k}$, feeds back as its principal eigenvector

$\mathbf{v}'_{i,k}$, where

$$\mathbf{v}'_{i,k} = \mathrm{e}^{\mathrm{j}\psi} \cdot \mathbf{v}_{i,k}, \tag{16}$$

where $\psi \in [0, 2\pi]$. Thus, from (15) we have that the equivalent channel for user $k$ with $\mathbf{v}'_{i,k}$ is

$$h'_k = \mathrm{e}^{-\mathrm{j}\psi} \cdot \xi_{1,k}, \tag{17}$$

which still has the same maginitude as $h_k$ from (15). Therefore, we only need $N-1$ complex numbers to characterize the principal eigenvector. The magnitude of the $N$-th element of the eigenvector is determined from the unit norm constraint while the phase of the $N$-th element can be determinely set to zero. Hence, for $N$ antennas each user needs to feedback $2(N-1)P$ bits in order to send the principal eigenvector to the base station.[2] The ratio of the additional feedback of the principal eigenvector to the SNR feedback in one second per user is then:

$$\frac{2(N-1)P}{500B} = \frac{(N-1)P}{250B}. \tag{18}$$

If we consider $N = 4$ antennas with $B = 4$ bits for the SNR feedback and $P = 4$ bits for the quantization of the real/imaginary part per eigenvector element, we have that the additional feedback required by opportunistic eigenbeamforming is just $1.2\%$ of the required SNR feedback. Hence, the additional feedback of the principal eigenvector is negligible compared to the SNR feedback required to exploit multiuser diversity. The $2(N-1)P$ bits which represent the principal eigenvector do not need to be fed back at once and can be distributed over several feedback slots. Additionally, the principal eigenvector feedback can be further reduced by using tracking algorithms such as the approach for tracking the signal subspace presented in [26].

Finally, note that $N$ pilots are required to estimate the principal eigenvector for each user, whereas only one pilot is required in the opportunistic beamforming scheme. In the opportunistic eigenbeamforming scheme, the $N$ pilots are used to estimate for each user $k$, the $h_{m,k}$ for $m = 1, \ldots, N$, with which the correlation matrix $\mathbf{C}_{\text{LT},k}$ can be estimated and in turn the principal eigenvector $\mathbf{v}_{1,k}$ for each user $k$.

# 6. Comparison: Opportunistic Beamforming vrs. Opportunistic Eigenbeamforming

## 6.1 Simulation Setup

To evaluate the performance of opportunistic beamforming and opportunistic eigenbeamforming in correlated channels with outdated feedback, let us consider the downlink of a single cell with a base station with a ULA constituted of $N = 4$ transmit antennas with a distance $d = \frac{1}{2}$ wavelengths between antennas and with only one antenna at each receiver. Thus, each point-to-point link constitutes a MISO system as depicted in Fig 1. Furthermore, we have the overall downlink system represented as a point-to-multi-point link where we assume there are a maximum of $K = 64$ users with the same normalized Doppler frequency $f_\text{n}$ and angle spread $\delta$. The carrier frecuency is $f_\text{c} = 2$ GHz. We assume that the channels $h_{m,k}$, for $m =, 1, \ldots, N$ for user $k$ are block correlated Rayleigh flat fading with unit variance as described in Section 3. Moreover, the average SNR at the receiver is $0$ dB and there are $1500$ time slots transmitted per second.

The effect of the outdated feedback is represented as follows. We consider the existence of a training phase at time slot $n$ where the magnitude of the equivalent channel $h_k[n]$ given by (9) or (15) is measured by user $k$ for opportunistic beamforming and opportunistic eigenbeamforming, respectively. The users are served through the proportional fair scheduler with different forgetting factors $f$. We assume no processing delay and consider that the feedback required to exploit the multiuser diversity by the PFS is fed back during time slot $n+1$, while the actual transmission to the best relative user is done in time slot $n+2$. Therefore, the equivalent channel $h_k[n]$ that is measured is based on the $h_{m,k}[n]$, while the actual channels when the selected user $k$ is served are $h_{m,k}[n+2]$, for $m = 1, \ldots, N$, as shown in Fig. 2. Hence, the resulting *loop delay* is $2$ slots.

| Training Slot | Feedback Slot | Transmission Slot |
|---|---|---|
| $h_{m,k}[n]$ | $h_{m,k}[n+1]$ | $h_{m,k}[n+2]$ |

Fig. 2. Outdated Feedback Model

In addition, the correlation matrix among the transmit antennas is given by (5) but we use the approximation that of each of the elements of this matrix is given by (6). This approximation is valid since we consider small angle spreads such that $\delta \approx \sin\delta$, then the random beam used for opportunistic beamforming will be directed only over one beam by randomly varying a single angle $\theta$ as explained at the end of Section 4. Moreover, the autocorrelation among the time slots is given by a Jakes model described in (7).

Furthermore, when considering opportunistic eigenbeamforming we assume that the long-term correlation matrix $\mathbf{C}_{\text{LT},k}$ has been estimated over a large number of time slots $T$ as given by (13). In addition, we assume that the base station has available the principal eigenvector $\mathbf{v}_{1,k}$ of the long-term correlation matrix for each user $k = 1, \ldots, K$. This is done through some feedback depending on how fast the channel changes. However, if we assume that $\mathbf{C}_{\text{LT},k}[n] = \mathbf{C}_{\text{ST},k}[n]$ at each time slot $n$ and that the users can feedback their principal eigenvector at each time slot, then the base station has available instantaneous channel state information. If this is the case the base station can perform coherent beamforming to the best relative user. With such a theoretic case the maximum rate

[2] Thus, we have $2^{2(N-1)P}$ possible vectors for characterizing the principal eigenvector. If we have $N = 4$ antennas and use $P = 4$ bits, we have more than $1.6 \times 10^7$ possible eigenvectors. If we use $P = 5$ bits, then we have more than $1 \times 10^9$ possible eigenvectors.

can be achieved and it serves as an upper bound for opportunistic eigenbeamforming. We will refer to this scheme in the following as *opportunistic coherent beamforming*.

To depict the corresponding delay performance for different degrees of multiuser diversity achieved through distinct forgetting factors in the proportional fair scheduler, let us define the *outage delay* $D_{\text{out}}$ which is related to a probability $p_{\text{out}}$ as follows:

$$\text{Prob}\{D < D_{\text{out}}\} = 1 - p_{\text{out}}, \tag{19}$$

where $p_{\text{out}}$ is the *outage probability* that a given delay $D$ is larger than $D_{\text{out}}$. The delay $D$ is given in number of time slots. In the simulation we set $p_{\text{out}} = 2\%$. Each forgetting factor in the PFS corresponds to a certain delay performance represented through the outage delay $D_{\text{out}}$.

Regarding the degrees of correlation in the channel, we will consider angle spreads up to $40°$. As for the normalized Doppler frequency the maximum speed treated is $80$ km per hour.

## 6.2 Analysis and Results

In the following, the figure of merit that will be used to asset the degree of multiuser diversity will be the average sum throughput of the system. Furthermore, we assume that the supported data rate or throughput for user $k$ is given by the Shannon equation $R_k[n] = \log_2(1 + \text{SNR}[n])$, where $\text{SNR}[n] = |h_k[n]|/\sigma_k^2$ with $\sigma_k^2$ as the variance of the noise at the receiving user $k$ for which we have assumed is equal to unity for every user. As mentioned in Section 2.1, one of the determining factors in the degree of multiuser diversity is the number of users in the system. In order to observe this performance, we have plotted in Fig. 3 the average sum throughput as a function of the number of users for the three opportunstic schemes detailed in the previous section: opportunistic beamforming (OB), opportunistic eigenbeamforming (OEB) and opportunistic coherent beamforming (OCB). These results correspond for a speed of $35$ kmph with several angle spreads. In addition, the users are served through the PFS with a forgetting factor of $0.001$. The gain resulting from multiuser diversity can clearly be seen from this figure, for which as the number of users increases all of the treated schemes increase their performance represented by the average sum throughput of the system.

To evaluate the peformance for different settings of the proportional fair scheduler, Fig. 4 depicts the average sum throughput for a set of $K = 64$ users as a function of the forgetting factor. As stated in Section 2.2, the degree of multiuser diversity that can be exploited in a system increases as the forgetting factor is reduced which can be clearly seen in this figure. However, this increase in multiuser diversity comes at the expense of delay. To observe the tradeoff between the multiuser diversity and the delay, Fig. 5 shows the average sum throughput but now as a function of the outage delay $D_{\text{out}}$ with a outage probability set to $p_{\text{out}} = 2\%$. Every forgetting factor from Fig. 4 translates into an outage delay in Fig. 5.

Moreover, in each of the previous figures, Figs. 3-5, it can also be seen how opportunistic eigenbeamforming outperforms opportunistic beamforming for different degrees of correlation in the channel. The different degrees of correlation are represented by the different angle spreads. As the angle spread decreases the degree of correlation increases and the performance of opportunistic eigenbeamforming basically matches the one of opportunistic coherent beamforming. For each case, the maximum possible achieved performance is obtained through opportunistic coherent beamforming and is represented as an upper bound on the average sum throughput. The opportunistic eigenbeamforming scheme still outperforms opportunistic beamforming also for different values of the forgetting factor. When the delay performance is considered, it can be seen that the for a given outage delay, the average sum throughput achieved with opportunistic eigenbeamforming is higher than compared to opportunistic beamforming. These results agree with the ones presented in [19]. Nevertheless, we will now proceed to eval-

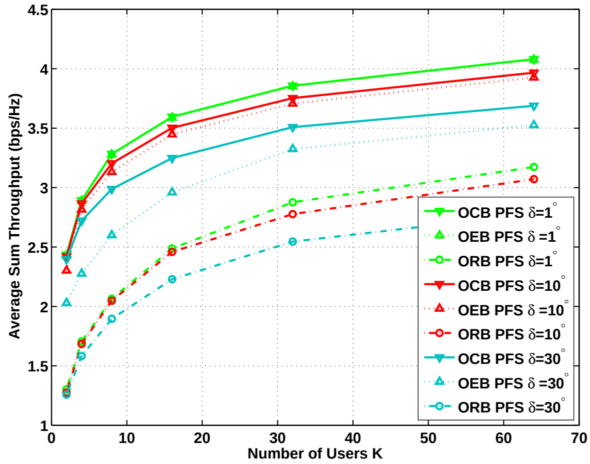


Fig. 3. Multiuser Diversity Gain

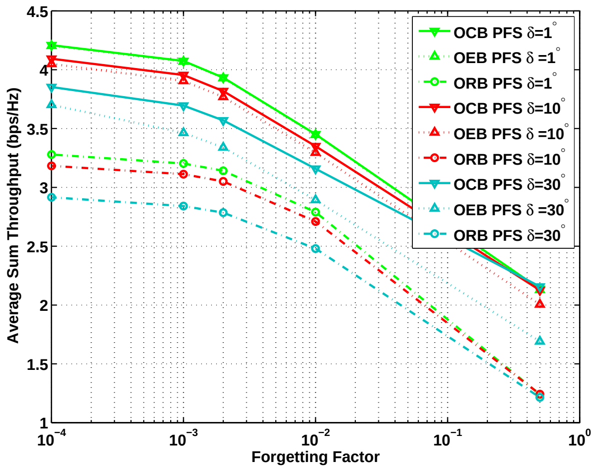


Fig. 4. Multiuser Diversity Gain Tradeoff: Forgetting Factors

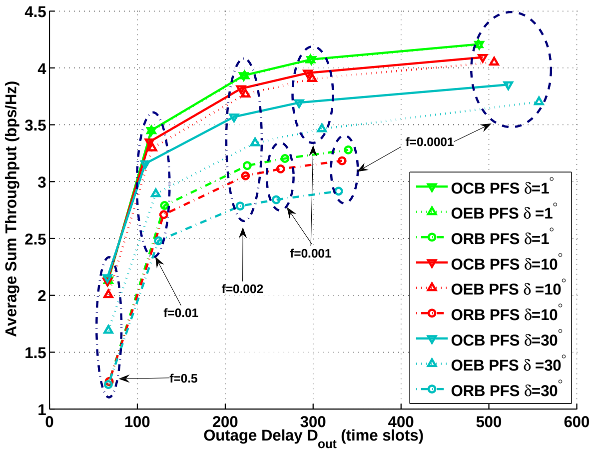


Fig. 5. Multiuser Diversity Gain Tradeoff: Outage Delay

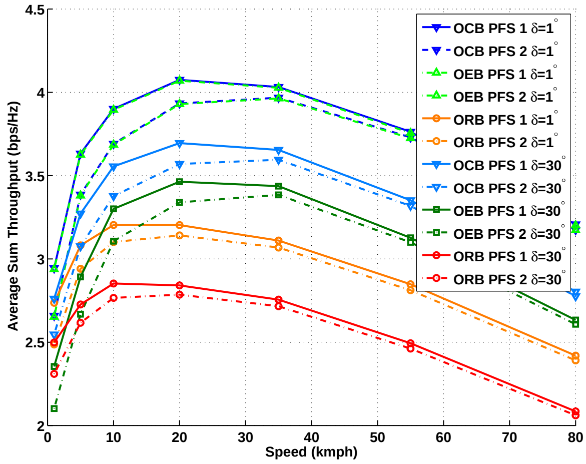


Fig. 6. Outdated Feedback: Average Sum Throughput vrs Speed of the Users

uate the impact of the temporal correlations in the channel and the effect of the outdated feedback on the proportional fair scheduler for different user's velocities under different degrees of correlation.

When different speeds for the users are take into account, one must consider the effect of the outdated feedback, since the channel that was tracked is no longer the same at the moment a user is served. Then, it might turn out that the selected user is no longer the best user. In Fig. 6, the effect of the outdated feedback can be observed for the different opportunistic schemes treated so far. The results presented in this figure correspond to angle spread $\delta = 1^\circ$ and $\delta = 30^\circ$. In addition, PFS 1 stands refers to the proportional fair scheduler with a forgetting factor $f = 0.001$, meanwhile PFS 2 corresponds to the proportional fair scheduler with a forgetting factor $f = 0.002$. One can see that for low speeds, the degree of multiuser diverstiy increases up to a maximum value as the speed of the users increases. This can be explained from the fact that there is a larger degree of multiuser diversity when the channel fluctuations are fast. When there is fast fading, the dynamic range of the channel fluctuations over the latency time scale $t_c$ increases, thus increasing the available multiuser diversity. Notice also that this increase is relatively larger for OCB and OEB as compared to OB, since opportunistic beamforming is already inducing faster channel fluctuations through the use of the random beam at the transmitter. After reaching maximum sum throughput, the degree of multiuser diversity decreases as the speed of the users increases for all of the schemes since they suffer from the effect of the outdated feedback and in fact now it incurs in a loss. Moreover, we have that PFS 1 outperforms PFS 2 since PFS 1 has a smaller forgetting factor.

In order to evaluate the degree of multiuser diversity as a function of the degree of correlation, Fig. 7 depicts the average sum throughput as a function of the angle spread. These results correspond to a speed of 35 kmph with the PFS using two forgetting factors, $f = 0.001$ and $f = 0.002$. A small angle spread indicates a large degree of correlation as all of the subpaths to one user practically have the same angle of departure. As the angle spread increases, the degree of correlation decreases and so the multiuser diversity available in the system. When there is a fully correlated channel, all the power of the channel is allocated over only one eigenmode of the channel. However, as the angle spread increases, i.e. the spatial correlation in the channel decreases, the condition of the spatial correlation matrix decreases since the power of the channel is distributed over all the eigenmodes. This means that the throughput achieved through coherent beamforming of a user with full correlation would be in average larger than the throughput achieved through coherent beamforming of a user with a less correlated channel. This would explain the decrease in performance as the angle spread increases for opportunistic eigenbeamforming, since the eigenvalue corresponding to the principal eigenvector is now smaller as compared to when the angle spread is smaller. In addition, we have that opportunistic beamforming is outperformed by opportunistic eigenbeamforming because OB does not always transmit on the strongest eigenmode of the channel as OEB does. In the limit, when we have a fully uncorrelated channel, where the condition of the spatial correlation matrix of the channel is equal to 1, we would have that the performance of OB is the same as that of OEB.

Furthermore, one can also analyze the performance of opportunistic eigenbeamforming relative to opportunistic beamforming. To this end, let us define the following ratio:

$$\eta(\delta, K) = \frac{S_{\mathrm{OEB}}(\delta, K)}{S_{\mathrm{OB}}(\delta, K)} \tag{20}$$

where $S_{\mathrm{OEB}}$ and $S_{\mathrm{OB}}$ are the sum throughput achieved with the PFS ($f = 0.001$) for opportunistic eigenbeamforming and opportunistic beamforming, respectively. This relative gain $\eta$ is a function of the number of users, speed of the users and of the angle spread. For a speed of 35 kmph, Fig. 8 depicts this ratio as function of the angle spreads for different number of users. It can be seen from this figure

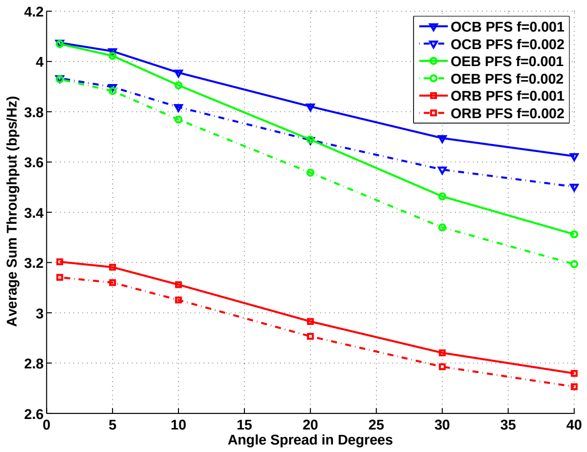


Fig. 7. Degree of Correlation: Average Sum Throughput vrs Angle Spread

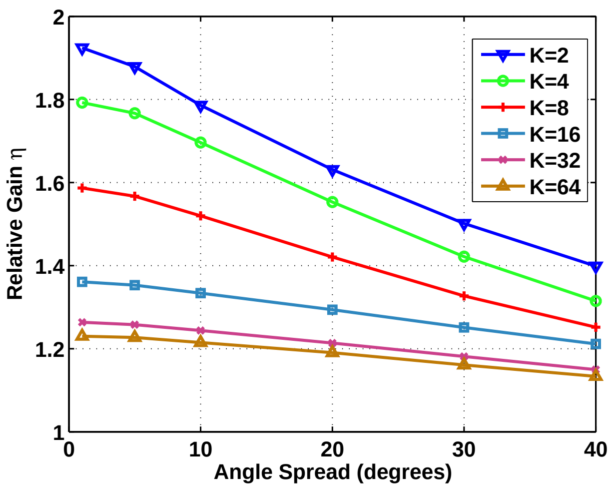


Fig. 8. Gain of OEB over OB: Different Number of Users and Angle Spreads

that as the number of users increase the gain of OEB over OB decreases. This can be explained as follows. As the number of users increases the probability that the random beam generated by OB actually matches the complex conjugate of the eigenbeam of a certian user increases. In the limit, when $K \to \infty$, one can expect that the performance of OEB is the same as that of OB. The multiuser diversity gain is further reduced as the correlation available in the channel decreases, i.e. the angle spread increases.

## 7. Conclusion

Opportunistic schedulers exploit the multiuser diversity inherent in a multiuser system. Through the use of opportunistic beamforming the degree of multiuser diversity is increased in correlated channels. Nevertheless, an efficient transmit schemes for point-to-point correlated links can be employed to achieve an even greater gain. We have shown that combining eigenbeamforming with an opportunistic scheduler, such as the proportional fair scheduler, increases the degree of multiuser diversity. This concept, which we term opportunistic eigenbeamforming, not only outperforms opportunistic beamforming for different degrees of spatial correlations in a channel, but also at different speeds of the users. Opportunistic eigenbeamforming is more robust to outdated feedback that results from the speed of the users. The larger achievable sum throughput of opportunistic eigenbeamforming over opportunistic beamforming is a result of having more partial CSI of each user at the base station. This partial CSI corresponds to the largest eigenvector of each user which must be fedback from each user. However, the feedback of this eigenbeam is not comparable with the SNR feedback required to exploit multiuser diversity in a TDMA system. This additional partial CSI can be fed back a much slower rate than the SNR feedback required by an opportunistic scheduler to serve a user at each time slot.

In addition, the existing tradeoff between the multiuser diversity gain and the delay performance provided through different settings of the forgetting factor in the proportional fair scheduler was also shown. For all the forgetting factors and the corresponding values of the outage delays, opportunistic eigenbeamforming achieves a higher average sum throughput as compared to opporuntistic beamforming. Furthermore it was shown how opportunistic eigenbeamformer comes close to the upper bound of the average sum throughput, achieved through opportunistic coherent beamforming, when the angle spread is very small. As the angle spread increases the power of the channel is distributed over all the eigenmodes of the channel, thus decreasing the multiuser diversity gain that can be extracted with OCB, OEB and OB. However, for any angle spread OEB still outperforms OB. Meanwhile, as the number of users increases and the angle spread increases, the perfromance of the opportunistic beamforming and opportunistic eigenbeamforming converge.

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## References

[1] Bender, P.; Black, P.; Grob, M.; Padovani, R.; Sindhushayana, N.; Viterbi, A.: CDMA/HDR: A Bandwidth Efficient High Speed Wireless Data Service for Nomadic Users. IEEE Commun. Mag. **38** (July 2000), 70–78.

[2] Holma, H.; Toskala, A.: WCDMA for UMTS, radio access for third generation mobile communications. Second edition ed. Chichester, UK: John Wiley and Sons, 2002.

[3] Bauch, G.; Hagenauer, J.: Anaytical Evaluation of Space-Time Transmit Diversity with FEC-Coding. IEEE Global Telecommunications Conference GLOBECOM **1** (November 2001), 435–439.

[4] Knopp, R.; P., H.: Information Capacity and Power Control in Single Cell Multiuser Communications. Proc. IEEE Int. Conf. in Commmun. ICC, Seattle, WA, November 1995. 306–311.

[5] Tse, D. N. C.: Optimal Power Allocation Over Parallel Gaussian Channels. Proc. Int. Symp. Information Theory, Ulm, Germany, June 1997. 27.

[6] Liu, X.; Chong, E.; Shroff, N.: Optimal Opportunistic Scheduling in Wireless Networks. IEEE Vehicular Technology Conference Fall VTC **3** (October 2003), 1417–1421.

[7] Liu, X.; Chong, E.; Shroff, N.: A Framework for Opportunistic Scheduling in Wireless Networks. Computer Networks **41** (March 2003), 451–474.

[8] Jalali, A.; Padovani, R.; Pankaj, R.: Data Throughput of CDMA-HDR a High Efficiency-High Data Rate Personal Communications Wireless System. Proceedings of IEEE Vehicular Technology Conference Spring, Tokyo, May 2000. 1854 – 1858.

[9] Viswanath, P.; Tse, D.; Laroia, R.: Opportunistic Beamforming Using Dumb Antennas. IEEE Trans. Inform. Theory **48** (2002), 1277–1294.

[10] Hiroike, A.; Adachi, F.; Nakajima, N.: Combined Effects of Phase Sweeping Transmitter Diversity and Channel Coding. IEEE Transactions on Vehicular Technology **41** (May 1992), 170–176.

[11] Gozali, R.; Buehrer, R.; Woerner, B.: The Impact of Multiuser Diversity on Space-Time Block Coding. Proc. of the IEEE Vehicular Technology Conference VTC-Fall, September 2002. 420–424.

[12] Mehta, N.; Kostic, Z.; Win, M.: Interaction Between Fast Scheduling Diversity and RAKE Receivers. IEEE Vehicular Technology Conference **2** (April 2003), 1464–1468.

[13] Viswanathan, H.; Venkatesan, S.: The Impact of Antenna Diversity in Packet Data Systems with Scheduling. IEEE Transactions on Communications **52** (April 2004), 546–549.

[14] Larsson, E.: On the Combination of Spatial Diversity and Multiuser Diversity. IEEE Communications Letters **8** (August 2004), 517–519.

[15] Piazza, D.; Milstein, L.: Multiuser Diversity-Mobility Tradeoff: Modeling and Performance Analysis of a Proportional Fair Scheduling. IEEE Global Telecommunications Conference GLOBECOM **1** (November 2002), 906–910.

[16] Rhee, W.; Cioffi, J.: On the Capacity of Multiuser Wireless Channels with Multiple Antennas. IEEE Transaction on Information Theory **49** (October 2003), 2580–2595.

[17] Brunner, C.; Utschick, W.; Nossek, J.: Exploiting the Short-Term and Long-Term Properties in Space and Time: Eigenbeamforming Concepts for the BS in WCDMA. European Transactions in Communications **12** (August 2001), 365–378.

[18] Ivrlac, M.; Kurpjuhn, T.; Brunner, C.; Utschick, W.: Efficient Use of Fading Correlations in MIMO Systems. Proc. of IEEE Vehicular Technology Conf. **4** (October 2001), 2763–2767.

[19] Castaneda, M.; Joham, M.; Ivrlac, M.; Nossek, J.: Combing Multi-User Diversity with Eigenbeamforming in Correlated Channel. Proc. European Wireles, April 2005. 138–144.

[20] Fulghum, T.; Molnar, K.: The Jakes Fading Model Incorporating Angular Spread for a Disk of Scatterers. IEEE Vehicular Technology Conference VTC Spring **1** (May 1998), 489–493.

[21] Jakes, W. C.: Microwave mobile communications. Wiley, 1974.

[22] Papoulis, A.; Pillai, A. U.: Probability, random variables and stochastic processes. Fourth edition ed. New York, NY: McGraw-Hill, 2002.

[23] Senst, A.; Schulz-Rittich, P.; Krause, U. Ascheid, G.; Meyr, H.: Random Beamforming in Correlated MISO Channels for Multiuser Systems. IEEE International Conference on Communications **5** (June 2004), 2909–2913.

[24] Sharma, N.; Ozarow, L.: A Study of Opportunism for Multiple-Antenna Systems. IEEE Transactions on Information Theory **51** (May 2005), 1804–1814.

[25] Scharf, L. L.: Statistical signal processing, detection, estimation, and time series analysis. Reading MA: Addison-Wesley, 1991.

[26] Utschick, W.: Tracking of signal subspace projectors. IEEE Transaction on Signal Processing **50** (April 2002), pp. 769–778.