MAXIMUM LIKELIHOOD DETECTION FOR QUANTIZED MIMO SYSTEMS

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ABSTRACT

In this work, Maximum Likelihood-detection (ML-detection) for MIMO systems operating on quantized data is considered. It turns out that the optimal decision rule is generally intractable. Therefore, we propose a suboptimal solution with lower complexity. Assuming a Rayleigh fading MIMO environment, an upper bound on the error probability is derived for the case where the number of transmit and receive antennas are equal. Furthermore, we introduce a new performance measure that relates the outage properties to the bit resolution in this context.

1. INTRODUCTION

The use of multiple antennas at both sides of the communication link (MIMO) is known to improve the power as well as the bandwidth efficiency of communication systems. A common assumption that is made when designing receiver structures for these links is to assume that the outputs of the channel are continuous. In practice, however, an analog-to-digital (A/D) conversion is applied to the received signal in order to be processed in the digital domain. In this case, the performance of MIMO systems can be limited by the resolution of the analog-to-digital converter (ADC) and some performance measures utilized in the literature can become useless in this context. Especially in high speed application, the reliance on the resolution of the quantizer becomes, from the practical point of view, unjustified due to the high power consumption and the circuit complexity of such components [1]. In this case, quantization can dramatically affect the performance of the proposed designs.

The following work is a complementary work to [2, 3, 4], where we have modified the conventional linear and nonlinear receiver designs, namely the *minimum mean square error* (MMSE) linear receiver and the MMSE-Decision feedback receiver, to take into account the presence of the quantizer in a non-empirical way. Here, we will study the performance of Maximum Likelihood detection (ML-detection) in the context of quantized MIMO system.

The ML-detector based on quantized data is actually much more complex than the conventional ML-detector in the unquantized case, since it requires the computation of the cumulative distribution function of multivariate Gaussian distribution. Therefore, we propose a "naive" ML approach to come up with lower complexity while having almost the same performance. Besides, we revise the definition of the well-established performance measure of *diversity* for Rayleigh fading MIMO channels affected by the quantization process, and derive a relation between the bit resolution and the outage probability at asymptotically high *Signal-to-Noise Ratio* (SNR) in this context.

Our paper is organized as follows. Section 2 describes the system model and notational issues. In Section 3, we discuss the properties of the chosen quantizer. Then we derive the optimal detector in Section 4 and its "naive", computationally less complex, approximation in Section 5. Next, we present some simulation results in Section 6. Finally, in Section 7, we deal with the relationship between quantizer resolution and the performance in terms of *error rate* and *outage probability*.

In our model we assume perfect channel state information (CSI) at the receiver, which can be obtained even with coarse quantization [5].

2. SYSTEM MODEL

We consider a point-to-point MIMO Gaussian channel where the transmitter employs M antennas and the receiver has N antennas. Fig. 1 shows the general form of a quantized MIMO system, where $\boldsymbol{H} \in \mathbb{C}^{N\times M}$ is the channel matrix. The vector $\boldsymbol{x} \in \mathbb{C}^{M}$ comprises the M transmitted symbols from a discrete modulation alphabet $\mathcal{X}$. The vector $\boldsymbol{\eta}$ refers to zero-mean complex circular Gaussian noise with covariance $\boldsymbol{R}_{\eta\eta} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}]$. $\boldsymbol{y} \in \mathbb{C}^{N}$ is the unquantized channel output

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}. \tag{1}$$

In our system, the real parts $y_{i,R}$ and the imaginary parts $y_{i,I}$ of the receive signals y_i, $1 \leq i \leq N$, are each quantized by a b-bit resolution uniform/non-uniform scalar quantizer. Thus, the resulting quantized signals read as with $l \in \{R, I\}$

$$r_{i,l} = Q(y_{i,l}), \tag{2}$$

where $Q(\cdot)$ denotes the quantization operation. For the case that we use a uniform symmetric mid-riser type quantizer [6],

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the quantized receive alphabet for each dimension is given by

$$r_{i,l} \in \{(-\frac{2^b}{2} - \frac{1}{2} + k)\Delta_i;\ k = 1, \cdots, 2^b\}, \tag{3}$$

where Δ_i is the quantizer step-size (the same for the real and imaginary dimensions) of each antenna and b the number of quantizer bits, which is set the same for all the quantizers.

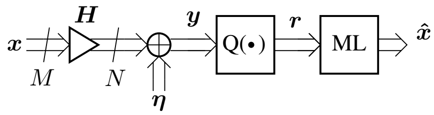

Fig. 1. Quantized MIMO System.

In the following, we aim to compute the ML-estimate $\hat{\boldsymbol{x}}$ of the transmitted uncoded symbol $\boldsymbol{x}$. Throughout this paper, $r_{\alpha\beta}$ denotes $\mathrm{E}[\alpha\beta^*]$, a_i symbolizes the i-th element of the vector $\boldsymbol{a}$ and $[\boldsymbol{a}]_{i,l} = a_{i,l}$ with $l \in \{R, I\}$ is the real or imaginary part of a_i.

3. QUANTIZER DESCRIPTION

Each quantization process is given a distortion factor $\rho_q^{(i,l)}$ to indicate the relative amount of quantization noise generated, which is defined as follows

$$\rho_q^{(i,l)} = \frac{\mathrm{E}[q_{i,l}^2]}{r_{y_{i,l}y_{i,l}}}, \tag{4}$$

where $r_{y_{i,l}y_{i,l}} = \mathrm{E}[y_{i,l}^2]$ is the variance of $y_{i,l}$ and the distortion factor $\rho_q^{(i,l)}$ depends on the number of quantization bits b, the quantizer type (uniform or non-uniform) and the probability density function of $y_{i,l}$. In our system, the uniform/non-uniform quantizer design is based on minimizing the *mean square error* (distortion) between the input $y_{i,l}$ and the output $r_{i,l}$ of each quantizer. In other words, the *signal-to-quantization-noise ratio* (SQNR) values that have an inverse relationship to the distortion factors are maximized. Under this optimal design of the scalar finite resolution quantizer, whether uniform or not, the following equations hold for all $0 \leq i \leq N$, $l \in \{R, I\}$ [6, 7, 8]

$$\mathrm{E}[r_{i,l}q_{i,l}] = 0 \tag{5}$$

$$\mathrm{E}[r_{i,l}r_{i,l}] = (1 - \rho_q^{(i,l)})r_{y_{i,l}y_{i,l}}. \tag{6}$$

Obviously, Eq. (6) follows from Eqs (4) and (5).
Under multipath propagation conditions and for large number of antennas the quantizer input signals $y_{i,l}$ are approximately Gaussian distributed and thus, they undergo nearly the same distortion factor ρ_q, i.e., $\rho_q^{(i,l)} = \rho_q\ \forall i \forall l$. Furthermore, the optimal parameters of the uniform as well as the non-uniform quantizer and the resulting distortion factor ρ_q for Gaussian distributed signal are computed in [8, 9, 10], for different bit resolutions b.
If a uniform quantizer is used, the step size Δ_i for each antenna i is computed as

$$\Delta_i = \sqrt{\frac{r_{y_i y_i}}{2}} \cdot \Delta = \sqrt{\frac{r_{y_i y_i}}{2}} \cdot 2c_q 2^{-b}, \tag{7}$$

where Δ and c_q are the optimal step size and clip level, respectively, for a Gaussian source with unit variance. In the high resolution case, it was shown in [10], that the optimal quantization step Δ for a Gaussian source decreases as $\sqrt{b}2^{-b}$ and that ρ_q is asymptotically well approximated by $\frac{\Delta^2}{12}$.
In practice, the ADC has a fixed step size Δ'. This implicates the necessity of an Automatic Gain Control (AGC) as shown in Fig. 2. The AGC of each antenna i scales its input signal by a factor $\beta_{\mathrm{AGC},i}$ such as the ADC output has a variance given by

$$r_{r'r'} = (\frac{\Delta'}{\Delta})^2(1 - \rho_q). \tag{8}$$

We assume that the system has an instantaneous AGC. Thus, the AGC calibration will not be considered.
In the rest of this paper, we concentrate on the case of uniform quantization.

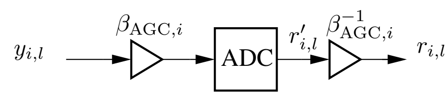

Fig. 2. Automatic Gain Control for the ADC.

4. ML-DETECTION BASED ON QUANTIZED DATA

Since we have a finite number of transmit and receive signal points, the channel model can be referred to as a Discrete Memoryless Channel (DMC) and characterized by a transition probability matrix $\boldsymbol{P} = [P(\boldsymbol{r}|\boldsymbol{x})]$. The ML-detector maximizes the conditioned probability $P(\boldsymbol{r}|\boldsymbol{x})$ for a given received quantized vector $\boldsymbol{r}$

$$\max_{\boldsymbol{x} \in \mathcal{X}^M} P(\boldsymbol{r}|\boldsymbol{x}) = \max_{\boldsymbol{x} \in \mathcal{X}^M} P(\boldsymbol{y} \in D(\boldsymbol{r})|\boldsymbol{x}) \tag{9}$$

$$= \max_{\boldsymbol{x} \in \mathcal{X}^M} \int_{D(\boldsymbol{r})} p(\boldsymbol{y}|\boldsymbol{x}) d\boldsymbol{y}, \tag{10}$$

where

$$p(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{\pi^N |\boldsymbol{R}_{\eta\eta}|} \exp\left(-(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^{\mathrm{H}} \boldsymbol{R}_{\eta\eta}^{-1} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})\right), \tag{11}$$

and $D(\boldsymbol{r})$ represents the quantized hyper-rectangular region in the $(2N)$-dimensional real space corresponding to the construction vector (centroid) $\boldsymbol{r}$, i.e.,

$$D(\boldsymbol{r}) = \left\{\boldsymbol{y} \in \mathbb{C}^N | b^{\mathrm{low}}(r_{i,l}) \leq y_{i,l} \leq b^{\mathrm{up}}(r_{i,l}); \forall i \in \{1, \ldots, N\}, \text{ and } l \in \{R, I\}\right\}, \tag{12}$$

where the lower and upper boundaries of the quantization region read as

$$b^{\mathrm{low}}(r_{i,l}) = \begin{cases} r_{i,l} - \frac{\Delta_i}{2} & \text{for} \quad r_{i,l} \geq -\frac{\Delta_i}{2}(2^b - 2) \\ -\infty & \text{otherwise,} \end{cases}$$

and

$$b^{\mathrm{up}}(r_{i,l}) = \begin{cases} r_{i,l} + \frac{\Delta_i}{2} & \text{for} \quad r_{i,l} \leq \frac{\Delta_i}{2}(2^b - 2) \\ +\infty & \text{otherwise.} \end{cases}$$

The integral in expression (10) is generally intractable and we have to resort to numerical and stochastic methods. Only if the noise is uncorrelated, we can express it as the product of the conditional probabilities on each receiver dimension using the one-dimensional cumulative Gaussian distribution function. For the case $\boldsymbol{R}_{\eta\eta} = \sigma_\eta^2 \boldsymbol{I}$, Eq. (10) becomes then

$$\begin{aligned} P(\boldsymbol{r}|\boldsymbol{x}) &= \prod_{i,l} \int_{b^{\mathrm{low}}(r_{i,l})}^{b^{\mathrm{up}}(r_{i,l})} \frac{1}{\sqrt{\pi}\sigma_\eta} \exp\left(-\left(\frac{y_{i,l} - (\boldsymbol{Hx})_{i,l}}{\sigma_\eta} \right)^2 \right) \mathrm{d}y_{i,l} \\ &= \prod_{i,l} \left\{ \Phi\left(\frac{b^{\mathrm{up}}(r_{i,l}) - (\boldsymbol{Hx})_{i,l}}{\sigma_\eta/\sqrt{2}} \right) - \right. \\ &\qquad \left. \Phi\left(\frac{b^{\mathrm{low}}(r_{i,l}) - (\boldsymbol{Hx})_{i,l}}{\sigma_\eta/\sqrt{2}} \right) \right\}, \end{aligned} \tag{13}$$

where $\Phi(x)$ represents the cumulative Gaussian distribution and reads as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-t^2) \mathrm{dt}. \tag{14}$$

5. "NAIVE" ML-DETECTION BASED ON QUANTIZED DATA

The ML-detector based on quantized data is actually much more complex than the conventional ML-detector in the unquatized case, since it requires the computation of the cumulative distribution function of a multivariate Gaussian distribution. Therefore we propose a "naive" ML approach based on a distance measure with $\boldsymbol{R}_{\eta\eta}^{-1}$ as metric to come up with lower arithmetic complexity. The idea is the following: As the SNR goes to infinity, the integral in expression (10) is characterized by the distance of the noiseless receive vector $\boldsymbol{Hx}$ to the border $\partial D(\boldsymbol{r})$, since this distance determines predominately the probability of a jump from or to the quantized region $D(\boldsymbol{r})$. This can be also justified by the following properties of the Gaussian cumulative distribution function

$$\begin{cases} \Phi(x) \leq \frac{1}{2} e^{\frac{x^2}{2}} & \text{for} \quad x \leq 0 \\ \Phi(x) \geq 1 - \frac{1}{2} e^{-\frac{x^2}{2}} & \text{for} \quad x \geq 0. \end{cases}$$

Obviously, we have to distinguish the cases whether the noiseless received vector $\boldsymbol{Hx}$ is within $D(\boldsymbol{r})$ or not. Fig. 3 shows an example with 2 receive antennas (only real transmission). For the case that all possible transmitted symbols are outside the quantization region $D(\boldsymbol{r})$, we consider the projection according to the minimal distance of each symbol on the border $\partial D(\boldsymbol{r})$. The symbol which has been most probably transmitted is then the one with the minimal distance to the region $D(\boldsymbol{r})$ among all the considered symbols. In Fig. 3, the symbol x_4 will be thus picked out as "naive" ML-estimate.

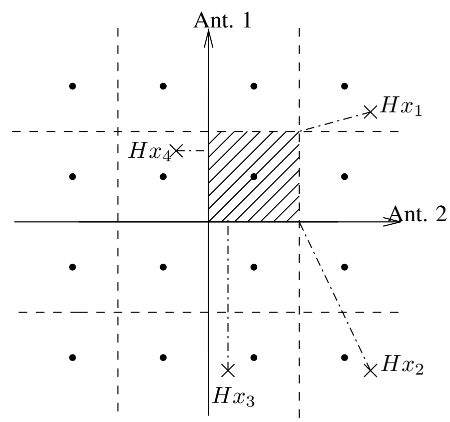

Fig. 3. The "naive" ML approach with possible transmitted symbol lying outside of $D(\boldsymbol{r})$, 2-bit quantizer and two receive antenna.

Fig. 4 illustrates the case of having some possible transmitted symbols within $D(\boldsymbol{r})$. In this case we consider only those symbols; and the one with the maximum distance to the border $\partial D(\boldsymbol{r})$ is the symbol, that has been most likely transmitted. In this example, the symbol x_3 is chosen as estimate. Mathematically, the naive ML-detection for quantized data can be described as follows

$$\hat{\boldsymbol{x}}_{\mathrm{naive-ML}} = \underset{\boldsymbol{x} \in \mathcal{X}^M}{\mathrm{argmin}}\, \mathrm{dist}(\boldsymbol{Hx}, \partial D(\boldsymbol{r})) \cdot \overline{\mathrm{inc}}(\boldsymbol{Hx}, D(\boldsymbol{r})), \tag{15}$$

where

$$\mathrm{dist}(\boldsymbol{Hx}, \partial D(\boldsymbol{r})) = \min_{\boldsymbol{y} \in \partial D(\boldsymbol{r})} \sqrt{(\boldsymbol{Hx} - \boldsymbol{y})^{\mathrm{H}} \boldsymbol{R}_{\eta\eta}^{-1} (\boldsymbol{Hx} - \boldsymbol{y})}, \tag{16}$$

and

$$\overline{\mathrm{inc}}(\boldsymbol{y}, D(\boldsymbol{r})) = \begin{cases} 1 & \text{if} \quad \boldsymbol{y} \notin D(\boldsymbol{r}) \\ -1 & \text{if} \quad \boldsymbol{y} \in D(\boldsymbol{r}). \end{cases}$$

The distance (16) can be easily calculated using quadratic optimization techniques with linear inequality constraints, also called quadratic programming (QP)

$$\min_{\boldsymbol{y}} \|\boldsymbol{Hx} - \boldsymbol{y}\|_{\boldsymbol{R}_{\eta\eta}^{-1}}^2 \text{ s.t. } \overline{\mathrm{inc}}(\boldsymbol{y}, D(\boldsymbol{r})) = -\overline{\mathrm{inc}}(\boldsymbol{Hx}, D(\boldsymbol{r})). \tag{17}$$

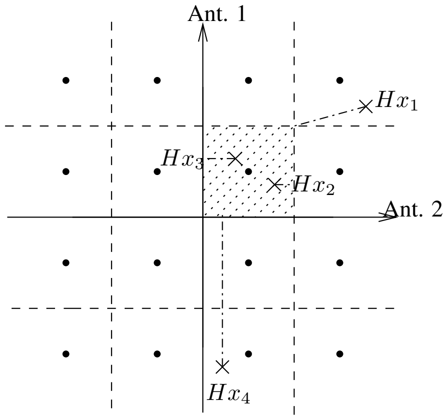

Fig. 4. The "naive" ML approach with possible transmitted symbol lying within $D(\boldsymbol{r})$, 2-bit quantizer and two receive antenna.

For the case of i.i.d. noise, we get the following closed form solution

$$\operatorname{dist}(\boldsymbol{Hx}, \partial D(\boldsymbol{r})) = \begin{cases} \min_{i,l} \{\min\{[\boldsymbol{Hx}]_{i,l} - b^{\text{low}}(\boldsymbol{r}_{i,l}), \\ \quad -[\boldsymbol{Hx}]_{i,l} + b^{\text{up}}(\boldsymbol{r}_{i,l})\}\} & \text{if } \boldsymbol{Hx} \in D(\boldsymbol{r}) \\ \Bigg[\sum_{\substack{i,l \\ [\boldsymbol{Hx}]_{i,l} - b^{\text{low}}(\boldsymbol{r}_{i,l}) \le 0}} \left([\boldsymbol{Hx}]_{i,l} - b^{\text{low}}(\boldsymbol{r}_{i,l})\right)^2 + \\ \quad \sum_{\substack{i,l \\ -[\boldsymbol{Hx}]_{i,l} + b^{\text{up}}(\boldsymbol{r}_{i,l}) \le 0}} \left(-[\boldsymbol{Hx}]_{i,l} + b^{\text{up}}(\boldsymbol{r}_{i,l})\right)^2 \Bigg]^{1/2} & \text{otherwise.} \end{cases}$$

Although this simplified detector based on the Euclidean measure has still exponential complexity, since it requires an exhaustive search over the set $\mathcal{X}^M$, it can be used as a basis for further developments with efficient search techniques.

6. SIMULATION RESULTS

The performance of the ML-decoder operating on quantized channel measurements (MLQ) and its approximation ("naive" MLQ) for a 3-bit quantized 4×4 MIMO systems (QPSK), in terms of *bit error rate* (BER) averaged over 90000 Rayleigh fading channel realizations, is shown in Fig. 5, compared with the conventional ML-decoder (minimizing the euclidean distance $||\boldsymbol{Hx} - \boldsymbol{z}||_2^2$). Clearly, the exact ML-decoder and its "naive" version outperform the conventional ML-decoder when operating on quantized data especially at high SNR. This is because the effect of quantization error is more pronounced at higher SNR values when compared to the additive Gaussian noise variance. Moreover, we see that the performance of the "naive" approach is very close to the exact one in spite of the complexity reduction. The performance of our modified MMSE receiver (WFQ) from [2] and the modified MMSE-DFE (DFEQ) from [4] is also plotted, for comparison. Since these receivers are not consistent, they perform inherently worse than the ML-decoder at high SNR. Obviously, the DFEQ equalizer leads to a considerable performance gain compared to the linear MMSE filter - while retaining a comparable complexity - but is still quite far from the ML performance. The dashed line corresponds to the BER curve for the ML-decoder if no quantization is applied.

Fig. 6 shows the uncoded BER achieved by the discussed ML- and "naive" ML-detectors and the conventional ML-detector for a 1-bit quantized MIMO-system. Due to the low resolution, a clear improvement in terms of BER performance can be achieved by considering the quantization process. We observe, moreover, that the "naive" ML approaches closely the performance of the the exact solution at high SNR values. This confirms the usefulness of the naive MLQ approach even under low resolution quantization.

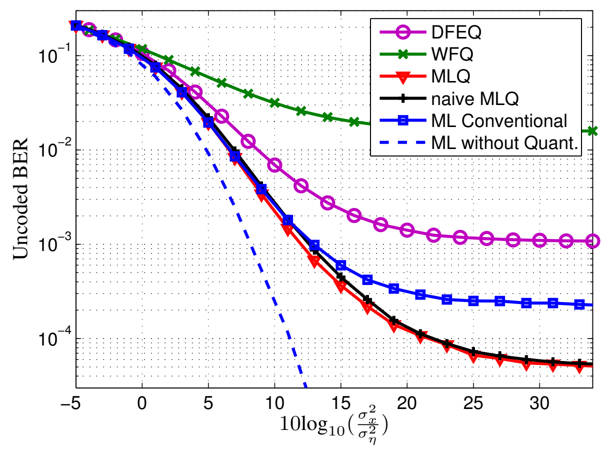

Fig. 5. MLQ "Naive" MLQ vs. ML conventional, ML without quantization, DFEQ and WFQ, QPSK modulation with $M = 4$, $N = 4$ and 3−bit uniform quantizer.

7. DIVERSITY AND OUTAGE OF THE ML-DETECTOR BASED ON QUANTIZED DATA: A NEW DEFINITION

Let us examine in more details the performance of quantized MIMO system in the context of Rayleigh fading channels having independent complex Gaussian distributed coefficients with zero mean and unit variance. For the unquantized MIMO systems, a common used performance measure is the achievable diversity order, which indicate the asymptotic slope of the average probability of error $P_e(\text{SNR})$ as a function of the

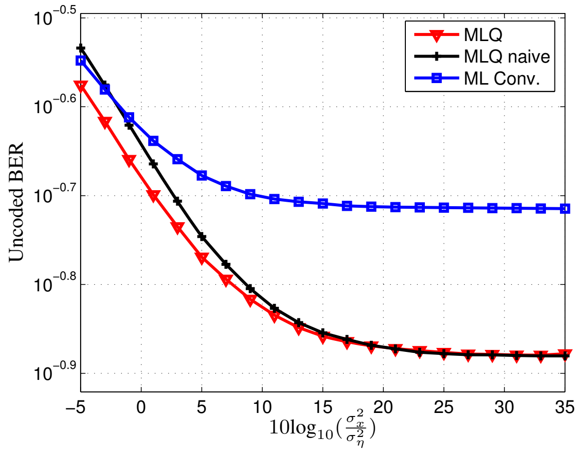

Fig. 6. Naive MLQ and MLQ vs. ML conventional, QPSK modulation with $M = 4$, $N = 4$ and 1−bit uniform quantizer.

average SNR

$$D = \lim_{\text{SNR}\to\infty} -\frac{\log P_e(\text{SNR})}{\log \text{SNR}}. \quad (18)$$

ML-detection achieves a diversity of order N when there is no joint processing among the streams [11], [12].
However, for a b-bit quantized $M \times N$ MIMO system and for asymptotically high SNR, we reach an error floor region. Here the bit error ratio is mainly characterized by bad channel realizations, where the quantizer resolution is not sufficient to distinguish between all transmitted symbols affected by the channel. The conventional definition of the diversity does not make a sense in our context since it leads to a diversity order of zero independently of the number of antennas. Therefore, we suggest the following new definition of diversity in the context of quantized MIMO systems

$$D_{\text{Q}} = \lim_{b\to\infty} -\frac{\log_2 P_e(b)}{b}, \quad (19)$$

where we consider the error rate $P_e(b)$ in the noiseless case ($\boldsymbol{R}_{\eta\eta} = \mathbf{0}$). D_{Q} describes the advantage in terms of the error exponent from increasing the resolution b. In the error floor region of quantized MIMO systems, the BER is dominated by the fact that the channel matrix $\boldsymbol{H}$ is atypically ill-conditioned, also known as "bad channel event" (outage event $\mathcal{O}$) [11], which occurs when the bit resolution is not enough to support the rate of each sub-channel.
First, we aim to study the error floor region by deriving an upper bound for the error probability as a function of b, so that we can determine the minimal quantizer resolution to assure a certain error probability. In the case of a bad channel event, the receiver can not distinguish between two symbols $\boldsymbol{H}\boldsymbol{x}_i$ and $\boldsymbol{H}\boldsymbol{x}_j$ $(i \neq j)$, which are close to each other. In order to characterize this outage phenomena, let us define d_H as the minimum distance between two different constellation points affected by the channel matrix $\boldsymbol{H}$

$$d_H = \min_{\boldsymbol{x}_1,\boldsymbol{x}_2\in\mathcal{A}^M} \|\boldsymbol{H}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\|_2. \quad (20)$$

Now, if $d_H \leq \sqrt{2\sum_i \Delta_i^2}$, where $\Delta_i \sim c_q \cdot 2^{-b+1}\sqrt{\frac{r_{y_iy_i}}{2}}$ (see (7)) is the optimal step size for the antenna i (in real and imaginary part), then there may exist two noiseless received points that lie inside the same $2N$-dimensional quantization region, whose $2N$ edges are given by Δ_i [1]. Therefore, we can upper bound the BER (or also the Symbol Error Ratio (SER)), dominated by the outage probability in the noiseless case ($\boldsymbol{R}_{\eta\eta} = \mathbf{0}$), as

$$\begin{aligned} P_e \sim P(\mathcal{O}) &\leq P\left(d_H \leq \sqrt{2\sum_i \Delta_i^2}\right) && (21)\\ &= P\left(d_H \leq c_q 2^{-b+1}\sqrt{\sum_i r_{y_iy_i}}\right) && (22)\\ &= P\left(d_H \leq c_q 2^{-b+1}\sqrt{\text{tr}\left[\boldsymbol{H}\boldsymbol{H}^{\text{H}}\right]}\right) && (23)\\ &= P\left(d_H \leq c_q 2^{-b+1}\sqrt{\sum_{i=1}^{N}\lambda_i^2}\right), && (24) \end{aligned}$$

where the λ_is are the singular values of the channel matrix $\boldsymbol{H}$.
On the other hand, we have

$$d_H = \min_{\boldsymbol{x}_1,\boldsymbol{x}_2\in\mathcal{A}} \|\boldsymbol{H}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\|_2 \geq \lambda_{\min} d_{\min}, \quad (25)$$

where $\lambda_{\min} = \min_i \lambda_i$ is the minimum singular value of the matrix $\boldsymbol{H}$ and $d_{\min}$ is the minimum distance between two points of the used constellation. Assuming that each stream carries a square constellation of L points. Each (real/imaginary) component has $\sqrt{L}$ levels, equally spaced and separated by d, with total power equal to 1/2 (as the total power per stream is normalized to unity, $\boldsymbol{R}_{xx} = \mathbf{I}$). Thus

$$d_{\min} = \sqrt{\frac{6}{L-1}}. \quad (26)$$

[1] Here, we assume that all received noiseless signal points lie on the granularity region of the quantizer, which holds in the high resolution case.

Therefore, Eq. (24) becomes

$$P_e \leq P(d_H \leq c_q 2^{-b+1}\sqrt{\sum_i \lambda_i^2}) \quad (27)$$

$$\leq P(\lambda_{\min}\sqrt{\frac{6}{L-1}} \leq c_q 2^{-b+1}\sqrt{\sum_i \lambda_i^2}) \quad (28)$$

$$= P\left(\frac{\sqrt{\sum_i \lambda_i^2}}{\lambda_{\min}} \geq \frac{2^{b-1}\sqrt{6}}{c_q\sqrt{L-1}}\right). \quad (29)$$

For $M = N$, in other words for square channel matrices, $\frac{\sqrt{\sum_i \lambda_i^2}}{\lambda_{\min}} = \kappa_D$ is known as the *Demmel condition number* of $\boldsymbol{H}$. For a matrix with i.i.d. normal random complex entries, following holds [13]

$$P(\kappa_D \geq t) = 1 - \left(1 - \frac{M}{t^2}\right)^{M^2-1} \quad \text{for} \quad t \geq \sqrt{M}. \quad (30)$$

Therefore, Eq.(29) becomes

$$P_e \leq 1 - \left(1 - \frac{c_q^2 \cdot M \cdot (L-1)}{3 \cdot 2^{2b-1}}\right)^{M^2-1}. \quad (31)$$

Using this expression, we can determine the bit resolution to guarantee a certain error probability. Note that this upper bound is not tight in general, and holds only for the case $M = N$. Thus we resort to simulations to evaluate the influence of the number of bits and antennas on the error exponent and give a simulative approximation of the diversity order in terms of quantization bits as defined in (19). Fig.7 and 8 show the error exponent as function of the bit resolution for different number of receive antennas N and number of bits b, for $M = 3$ and $M = 4$, respectively. It can be observed that the diversity order D_{Q} as defined in (19), is bounded between N and $2N$. Furthermore, we see from these figures that, for given bit resolution the error exponent increases linearly with the number of receive antennas, with a slope of approximately b. All in all, the slope of error exponent is almost proportional to the product $N \cdot b$ for high b or N, which means that lower BER can be achieved by increasing either the number of bits or the number of receive antennas.
On the other hand, Fig. 9 shows the error exponent achieved by the linear MMSE receiver for quantized MIMO derived in [2]. In this case, only a slope nearly given by $N - M - 1$ with respect to the resolution b is reached. This simulative results seems to be in accordance with the results known for the (conventional) diversity measure in (18) [11], but an analytic approach for the quantized case is still missing.

8. CONCLUTION

We discussed optimal ML-detection for quantized MIMO systems in a Rayleigh fading environment. Since the solution turns to be computationally intractable, we presented a nearly optimal ("naive") ML-detector, which shows almost the same performance for high SNR in terms of BER as the optimal solution, and thus outperforms the conventional ML receiver. We also examined the effect of quantization on the BER for asymptotic high SNR (error floor region) and derived a relation between the outage probability and the number of bits. Hereby, we introduced a new definition of diversity in this context. Interesting topics for the future are exploring further developments for the "naive" ML-detector, in order to overcome the exponential complexity, and considering scenarios with only Partial Channel State Information (PCSI) at the receiver side.

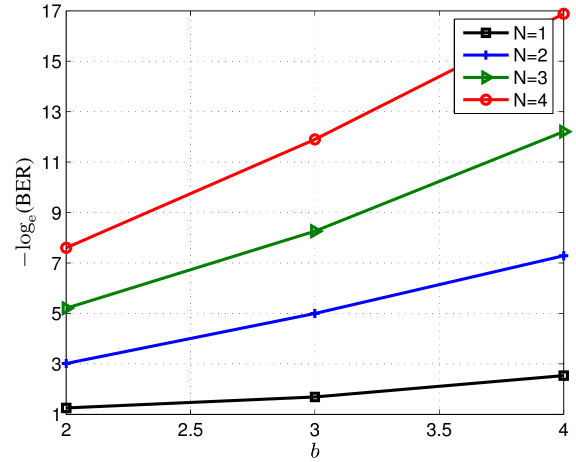

Fig. 7. Error exponent of a MIMO system with an ML-detector operating on quantized data, QPSK modulation, $M = 3$, $N \in \{1, 2, 3, 4\}$ and $b \in \{2, 3, 4\}$.

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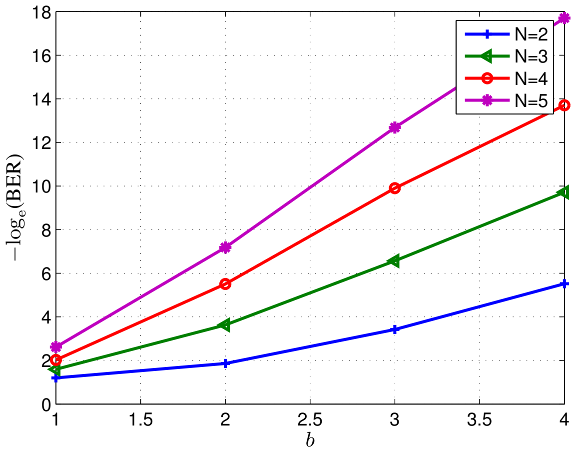

Fig. 8. Error exponent of a MIMO system with an ML-detector operating on quantized data, QPSK modulation, $M = 4$, $N \in \{2, 3, 4, 5\}$ and $b \in \{1, 2, 3, 4\}$.

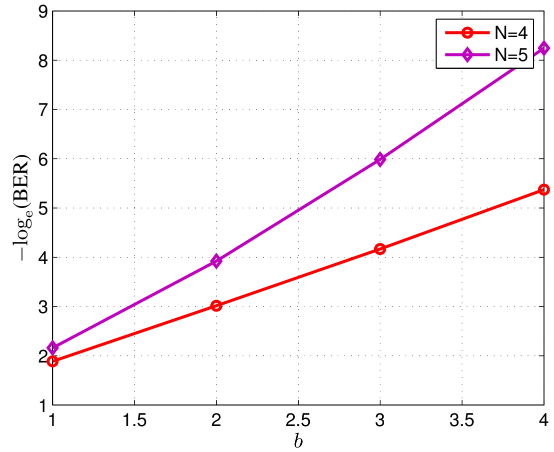

Fig. 9. Error exponent of a MIMO system with the linear MMSE detector opating on quantized data from [2], QPSK modulation, $M = 4$, $N \in \{4, 5\}$ and $b \in \{1, 2, 3, 4\}$.

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