TIME-SHARING SOLUTIONS IN MIMO BROADCAST CHANNEL UTILITY MAXIMIZATION

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ABSTRACT

The problem of maximizing a utility function over the set of achievable rate vectors in a MIMO broadcast channel is considered. If the optimum rate vector lies in a time-sharing region, it is necessary to identify a set of corner points of the time-sharing region such that the optimum rate vector is a convex combination of these corner points. In a K user MIMO BC, the maximum number of corner points is $K!$, thus enumerating all corner points is only feasible for small K. In this work, an efficient algorithm for identifying a subset of relevant corner points is proposed. Simulation results show that in a scenario where the time-sharing region has $K!$ corner points, out of which K are required to construct the optimum rate, the proposed algorithm on average computes less than $K + 1$ corner points until convergence.

1. INTRODUCTION

The *Gaussian MIMO broadcast channel* (MIMO BC) [1] has received wide attention in recent years. Based on the fundamental duality results [2], several works have considered the optimization of the MIMO BC parameters transmit covariance matrices and encoding order under performance criteria such as sum rate [2] and weighted sum rate [3], and highly efficient algorithms have been proposed for both problems [4]. Recently, the problem of determining the optimum rate vector with respect to a generic utility model of the upper layers was considered in [5] and [6].

The capacity region of the MIMO BC contains rate vectors that are only achievable by time sharing. In the case of (weighted) sum rate maximization, it is always possible to find a maximizing rate vector that does not require time sharing. In contrast, for a more general performance metric, the optimum solution may be located within a time sharing region, thus time sharing is required to construct the optimum rate vector. A time sharing solution is always a convex combination of some corner points of the time sharing region. If the set of corner points is known, it is straightforward to find the optimum solution, as discussed in [6]. For a K user MIMO BC, however, the number of corner points of the time sharing region where all K users time share is $K!$, i.e., the number of corner points grows rapidly with the number of users, and a solution that relies on an enumeration of all corner points becomes quickly infeasible.

For capacity regions that correspond to a polytope, an efficient algorithm for finding the desired subset of corner points was proposed in [7]. The capacity region of the MIMO BC is not a polytope. Still, the algorithm from [7] can be extended to the MIMO BC case. This extension is the subject of this work.

The results presented in this work are applicable to all problems where optimum solutions may lie in a time sharing region, such as problems with minimum rate constraints [8] and projections onto the boundary of the capacity region [6].

2. PROBLEM SETUP

A K user MIMO BC with sum power constraint is considered. Let $\mathcal{R}$ denote the set of rate vectors achievable by feasible choices of transmit covariance matrices and encoding order. The capacity region $\mathcal{C}$ is given by the convex hull of $\mathcal{R}$ [2]. Some $\boldsymbol{r} \in \mathcal{C}$ are only achievable by time-sharing, i.e., by a convex combination of $W > 1$ points $\boldsymbol{r}^{(w)} \in \mathcal{R}$ such that

$$\boldsymbol{r} = \sum_{w=1}^{W} \alpha_w \boldsymbol{r}^{(w)}, \alpha_w \geq 0, \sum_{w=1}^{W} \alpha_w = 1.$$

Given a system utility function $u : \mathbb{R}_+^K \to \mathbb{R}$, which is assumed to be concave and strictly monotonically increasing, the problem of determining the rate vector $\boldsymbol{r}^* \in \mathcal{C}$ that maximizes system utility is considered.

For optimum transmission, it is required to set the parameters of the physical layer such that $\boldsymbol{r}^*$ is achieved. There clearly exist functions u such that $\boldsymbol{r}^*$ lies inside a time-sharing region. As a consequence, a complete solution of the utility maximization problem not only includes $\boldsymbol{r}^*$, but, in case of time-sharing optimality, also the points $\boldsymbol{r}^{(w)}$ and time sharing coefficients needed to construct $\boldsymbol{r}^*$. Also note that without any further assumptions on the utility function u, the performance loss that results from approximating $\boldsymbol{r}^*$ by a point in $\mathcal{R}$ may be arbitrarily large.

3. TIME-SHARING REGIONS

Under the assumption of a monotone utility function, it is sufficient to consider rate vectors that lie on the Pareto efficient boundary $\mathcal{E}$ of the capacity region [6]. Due to the convexity of the capacity region, each point on the boundary $\mathcal{E}$ can be obtained as a maximizer of a weighted sum rate optimization over $\mathcal{C}$:

$$\mathcal{E} = \bigcup_{\boldsymbol{\lambda} > \mathbf{0}} \mathcal{E}(\boldsymbol{\lambda}),$$

with

$$\mathcal{E}(\boldsymbol{\lambda}) = \operatorname*{argmax}_{\boldsymbol{r} \in \mathcal{C}} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{r}.$$

In general, there exist $\boldsymbol{\lambda} > \mathbf{0}$ where $\mathcal{E}(\boldsymbol{\lambda})$ is not a singleton set. This case is of particular interest in this work. Define the set of corner points of $\mathcal{E}(\boldsymbol{\lambda})$ as follows:

$$\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda}) = \mathcal{E}(\boldsymbol{\lambda}) \cap \mathcal{R}.$$

All points in $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda})$ can be achieved by a particular setup of transmit covariance matrices and encoding order (as they are elements of $\mathcal{R}$), while all other points in $\mathcal{E}(\boldsymbol{\lambda})$ are only achievable by time-sharing between points in $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda})$. Based on this observation, a set $\mathcal{E}(\boldsymbol{\lambda})$ that is not a singleton set is denoted as a *time-sharing region*, and the corresponding set $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda})$ is said to contain the corner points of this region.

Each point in $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda})$ corresponds to a choice of transmit covariance matrices and encoding order that maximizes a weighted sum rate for weight $\boldsymbol{\lambda}$. The problem of determining optimum covariance matrices and encoding order for a given weight $\boldsymbol{\lambda}$ is considered in, e.g., [3]. Let $\boldsymbol{\pi} \in \{1, \ldots, K\}^K$ denote an encoding order, such that π_j corresponds to the user that is encoded at j-th position. An optimum encoding order fulfills [3]

$$\lambda_{\pi_1} \geq \ldots \geq \lambda_{\pi_K}.$$

If any two entries of $\boldsymbol{\lambda}$ have the same value, the optimum order is not unique. Accordingly, define the set of optimum orders for a given $\boldsymbol{\lambda}$ as

$$\Pi(\boldsymbol{\lambda}) = \{\boldsymbol{\pi} \in \{1, \ldots, K\}^K : \lambda_{\pi_1} \geq \ldots \geq \lambda_{\pi_K}\}.$$

If all entries of $\boldsymbol{\lambda}$ have the same value, all possible orders are optimum, thus $|\Pi(\boldsymbol{\lambda})| \leq K!$, where $|\mathcal{A}|$ denotes the cardinality of $\mathcal{A}$.

For each $\boldsymbol{\pi} \in \Pi(\boldsymbol{\lambda})$, there exists a unique rate vector $\boldsymbol{r}_{\text{cp}}(\boldsymbol{\pi})$ that maximizes $\boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{r}$ [3]. As a result,

$$\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda}) = \{\boldsymbol{r}_{\text{cp}}(\boldsymbol{\pi}) : \boldsymbol{\pi} \in \Pi(\boldsymbol{\lambda})\}$$

and $|\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda})| \leq K!$.

4. OPTIMUM RATE

As shown in [5, 6], the optimum rate vector can be found by solving the dual problem of the modified problem

$$\max_{\boldsymbol{r}, \boldsymbol{s}} u(\boldsymbol{s}) \quad \text{s.t.} \quad \mathbf{0} \leq \boldsymbol{s} \leq \boldsymbol{r}, \boldsymbol{r} \in \mathcal{C}. \tag{1}$$

Given a set $\mathcal{A} \subseteq \mathcal{C}$, define a function $g_{\mathcal{A}}$ as follows:

$$g_{\mathcal{A}}(\boldsymbol{\lambda}) = \sup_{\boldsymbol{s} \geq \mathbf{0}} u(\boldsymbol{s}) - \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{s} + \sup_{\boldsymbol{r} \in \mathcal{A}} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{r}. \tag{2}$$

Then $g_{\mathcal{C}}$ is the dual function of problem (1) resulting from the dualization of the constraint $\boldsymbol{s} \leq \boldsymbol{r}$. Let $\boldsymbol{\lambda}^*$ denote the optimum dual variable, i.e.,

$$\boldsymbol{\lambda}^* \in \operatorname*{argmin}_{\boldsymbol{\lambda} \geq \mathbf{0}} g_{\mathcal{C}}(\boldsymbol{\lambda}).$$

Strong duality holds[1], thus the primal solution $(\boldsymbol{r}^*, \boldsymbol{s}^*)$ can be recovered from the dual solution, following the procedures discussed in [6]. In case of time-sharing, the optimum rate vector $\boldsymbol{r}^*$ lies in the time-sharing region $\mathcal{E}(\boldsymbol{\lambda}^*)$. Accordingly, the solution procedure has two steps: First, compute $\boldsymbol{\lambda}^*$. Second, if $\boldsymbol{\lambda}^*$ indicates that some users time-share, identify a set of corner points and the corresponding time-sharing coefficients that implement $\boldsymbol{r}^*$. If no time-sharing occurs, $\mathcal{E}(\boldsymbol{\lambda}^*)$ is a singleton set and $\boldsymbol{r}^*$ its only element, i.e., $\boldsymbol{r}^*$ can be directly determined by solving a weighted sum rate maximization.

5. COLUMN GENERATION

Now assume that at least two entries in $\boldsymbol{\lambda}^*$ are equal, i.e., the optimum solution requires time-sharing. If the set of corner points $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda}^*)$ is known, it is straightforward to determine the corner points that implement $\boldsymbol{r}^*$ [6]. As discussed in Section 3, $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda}^*)$ can be obtained by computing $\boldsymbol{r}_{\text{cp}}(\boldsymbol{\pi})$ for each optimum order $\boldsymbol{\pi} \in \Pi(\boldsymbol{\lambda}^*)$. In the worst case, however, for K users, there are $K!$ corner points. As a result, enumerating all corner points is only feasible for small K. In this section, an efficient algorithm for identifying the relevant corner points is provided, which avoids computing the complete set $\mathcal{R}_{\text{cp}}(\boldsymbol{\lambda}^*)$.

For the case of the capacity region being a polytope, an efficient algorithm for identifying relevant corner points is proposed in [7]. The algorithm from [7] can be adapted to the MIMO BC case, as shown in the following.

Assume that the capacity region is a polytope. Let $\mathcal{P}$ denote the set of extremal points of this polytope. The algorithm presented in [7] proceeds as follows: Given a set $\mathcal{K} \subseteq \mathcal{P}$, the problem

$$\max_{\boldsymbol{s}, \boldsymbol{\alpha}} u(\boldsymbol{s}) \quad \text{s.t.} \quad \mathbf{0} \leq \boldsymbol{s} \leq \boldsymbol{R}_{\mathcal{K}} \boldsymbol{\alpha}, \boldsymbol{\alpha} \geq \mathbf{0}, \|\boldsymbol{\alpha}\|_1 = 1$$

[1] Assuming that $\mathcal{C}$ is nontrivial, i.e., $\mathcal{C} \supset \{\mathbf{0}\}$, there exist strictly feasible $(\boldsymbol{r}, \boldsymbol{s})$ such that Slater's condition is satisfied.

is solved, where the columns of $\boldsymbol{R}_{\mathcal{K}}$ are the elements of $\mathcal{K}$. This yields a solution $(\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{\alpha}})$, with $\tilde{\boldsymbol{s}} = \boldsymbol{R}_{\mathcal{K}}\tilde{\boldsymbol{\alpha}}$. Clearly, $u(\tilde{\boldsymbol{s}})$ is a lower bound on the maximum utility value obtained by maximizing over $\mathcal{P}$. Let $\tilde{\boldsymbol{\lambda}}$ denote the optimum Lagrange multiplier for the constraint $\boldsymbol{s} \leq \boldsymbol{R}_{\mathcal{K}}\boldsymbol{\alpha}$. By weak duality, an upper bound on the maximum utility is given by $g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}})$. If $u(\tilde{\boldsymbol{s}}) < g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}})$, the point from $\mathcal{P}$ that solves

$$\max_{\boldsymbol{r}} \tilde{\boldsymbol{\lambda}}^{\mathrm{T}}\boldsymbol{r} \quad \text{s.t.} \quad \boldsymbol{r} \in \mathcal{P} \tag{3}$$

is added to $\mathcal{K}$. The whole procedure is repeated until

$$u(\tilde{\boldsymbol{s}}) = g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}}). \tag{4}$$

In [7], it is shown that the algorithm converges. Moreover, if (4) holds, $\mathcal{K}$ contains the relevant corner points, as in this case $\tilde{\boldsymbol{s}} = \boldsymbol{R}_{\mathcal{K}}\tilde{\boldsymbol{\alpha}}$ maximizes utility over $\mathcal{P}$.

To extend the algorithm to the MIMO BC case, the capacity region $\mathcal{C}$ is approximated by a polytope $\mathrm{Co}\left(\{\mathbf{0}\} \cup \mathcal{P}\right)$, where $\mathrm{Co}\left(\mathcal{A}\right)$ is the convex hull of $\mathcal{A}$. Let $\mathcal{S}$ denote the set of single-user points of $\mathcal{C}$. Then $\mathcal{P}$ is chosen as

$$\mathcal{P} = \mathcal{S} \cup \mathcal{R}_{\mathrm{cp}}(\boldsymbol{\lambda}^*).$$

Applying the algorithm from [7] to the MIMO BC consists of two main steps. First, it is shown that $u(\tilde{\boldsymbol{s}}) = g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}})$ implies $u(\tilde{\boldsymbol{s}}) = g_{\mathcal{C}}(\boldsymbol{\lambda}^*)$. Second, it is shown how to efficiently find the point in $\mathcal{P}$ that solves (3).

We have $\max_{\boldsymbol{r}\in\mathcal{P}} \boldsymbol{\lambda}^{*\mathrm{T}}\boldsymbol{r} = \max_{\boldsymbol{r}\in\mathcal{C}} \boldsymbol{\lambda}^{*\mathrm{T}}\boldsymbol{r}$, thus

$$g_{\mathcal{P}}(\boldsymbol{\lambda}^*) = g_{\mathcal{C}}(\boldsymbol{\lambda}^*). \tag{5}$$

Moreover, due to the fact that the approximation is exact on the time-sharing region, both problems yield the same maximum utility. As a result, $\boldsymbol{\lambda}^* \in \mathrm{argmin}_{\boldsymbol{\lambda}\geq\mathbf{0}}\, g_{\mathcal{P}}(\boldsymbol{\lambda})$. This implies

$$g_{\mathcal{P}}(\boldsymbol{\lambda}^*) \leq g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}}). \tag{6}$$

Thus,

$$u(\tilde{\boldsymbol{s}}) \leq g_{\mathcal{C}}(\boldsymbol{\lambda}^*) = g_{\mathcal{P}}(\boldsymbol{\lambda}^*) \leq g_{\mathcal{P}}(\tilde{\boldsymbol{\lambda}}).$$

As a result, if (4) holds, $u(\tilde{\boldsymbol{s}}) = g_{\mathcal{C}}(\boldsymbol{\lambda}^*)$ and $\mathcal{K}$ contains the set of relevant time-sharing points. Accordingly, the algorithm proceeds as in the case where $\mathcal{C}$ is a polytope: Starting with $\mathcal{K} = \mathcal{S}$, the point from $\mathcal{P}$ that solves (3) is added to $\mathcal{K}$ until (4) holds.

The second step is to efficiently identify the point from $\mathcal{P}$ that solves (3) *without* having to compute all points in $\mathcal{R}_{\mathrm{cp}}(\boldsymbol{\lambda}^*)$ beforehand.

The rate vector that maximizes (3) is always an element of $\mathcal{R}_{\mathrm{cp}}(\boldsymbol{\lambda}^*)$, therefore it suffices to consider how to select the correct point from $\mathcal{R}_{\mathrm{cp}}(\boldsymbol{\lambda}^*)$, given $\tilde{\boldsymbol{\lambda}}$. Each point in $\mathcal{R}_{\mathrm{cp}}(\boldsymbol{\lambda}^*)$ corresponds to an encoding order $\boldsymbol{\pi} \in \mathit{\Pi}(\boldsymbol{\lambda}^*)$. Thus, the problem is solved by determining the encoding order $\tilde{\boldsymbol{\pi}}$ such that

$$\tilde{\boldsymbol{\pi}} = \underset{\boldsymbol{\pi}\in\mathit{\Pi}(\boldsymbol{\lambda}^*)}{\mathrm{argmax}}\, \tilde{\boldsymbol{\lambda}}^{\mathrm{T}}\boldsymbol{r}_{\mathrm{cp}}(\boldsymbol{\pi}).$$

6. OPTIMUM ENCODING ORDER

To determine the optimum encoding order $\tilde{\boldsymbol{\pi}}$, the K users are first partitioned into disjoint subsets $\mathcal{I}_\ell \subseteq \{1,\ldots,K\}$ according to their weights λ_k^*, such that all users in a subset $\mathcal{I}_\ell$ have equal weight μ_ℓ:

$$k \in \mathcal{I}_\ell \Leftrightarrow \lambda_k^* = \mu_\ell.$$

Moreover, the sets $\mathcal{I}_\ell$ are chosen such that $\mu_\ell > \mu_{\ell+1}$. Let L denote the number of sets $\mathcal{I}_\ell$ resulting from this partitioning. As an example, $L = 1$ if all users have equal weights, and $L = K$ if all users have unique weights.

As $\tilde{\boldsymbol{\pi}} \in \mathit{\Pi}(\boldsymbol{\lambda}^*)$, in the optimum encoding order the users are encoded in descending order of their weights λ_k^*, i.e., the users with the largest weights are encoded first. Accordingly, the users in $\mathcal{I}_\ell$ are encoded before the users in $\mathcal{I}_{\ell+1}$. However, the users in $\mathcal{I}_\ell$ all have equal weight μ_ℓ. Thus, the encoding order of users in $\mathcal{I}_\ell$ is not determined by $\boldsymbol{\lambda}^*$. This degree of freedom is used to maximize $\tilde{\boldsymbol{\lambda}}^{\mathrm{T}}\boldsymbol{r}$.

Let $\boldsymbol{\pi}^\ell$ denote the encoding order among users in $\mathcal{I}_\ell$, i.e., among all users in $\mathcal{I}_\ell$, user π_1^ℓ is encoded first, and user π_M^ℓ is encoded last, where $M = |\mathcal{I}_\ell|$. The main result is as follows: The optimum encoding order $\boldsymbol{\pi}$ is given by

$$\tilde{\boldsymbol{\pi}} = \left(\boldsymbol{\pi}^1, \ldots, \boldsymbol{\pi}^L\right),$$

where each $\boldsymbol{\pi}^\ell$ satisfies

$$\tilde{\lambda}_{\pi_1^\ell} \geq \ldots \geq \tilde{\lambda}_{\pi_M^\ell}. \tag{7}$$

The proof proceeds in two steps: First, it is shown that given an encoding order $\boldsymbol{\pi}^\ell$ for group $\mathcal{I}_\ell$, swapping the positions of two consecutive users does not change the rate of any other user. Based on this result, the optimum encoding order is found by swapping positions in all $\boldsymbol{\pi}^\ell$ until $\tilde{\boldsymbol{\lambda}}^{\mathrm{T}}\boldsymbol{r}$ is maximized. It then remains to show that the optimum order within a group corresponds to an descending order of the corresponding entries in $\tilde{\boldsymbol{\lambda}}$, which is the second step.

In the dual MAC, the rate of user k is given by

$$r_k = \log\det(\mathbf{1} + \boldsymbol{B}_k^{-1}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{Q}_k\boldsymbol{H}_k),$$

where $\boldsymbol{H}_k^{\mathrm{H}}$ and $\boldsymbol{Q}_k$ denote the uplink channel and covariance matrix of user k, respectively, and $\boldsymbol{B}_k$ represents the interference experienced by user k in the MAC (see [2] for definitions and details). Assuming that $k \in \mathcal{I}_\ell$, $\boldsymbol{B}_k$ can be written as[2]

$$\boldsymbol{B}_k = \mathbf{1} + \sum_{m=1}^{l-1}\sum_{j\in\mathcal{I}_m} \boldsymbol{H}_j^{\mathrm{H}}\boldsymbol{Q}_j\boldsymbol{H}_j + \sum_{j\in\mathcal{S}_k} \boldsymbol{H}_j^{\mathrm{H}}\boldsymbol{Q}_j\boldsymbol{H}_j,$$

where $\mathcal{S}_k \subset \mathcal{I}_\ell$ contains the indices of the users in $\mathcal{I}_\ell$ that are encoded before user k. Clearly, $\mathcal{S}_k$ depends on $\boldsymbol{\pi}^\ell$.

[2]Note that in [2], $\boldsymbol{B}_k$ is defined using the MAC encoding order, while in this work, the term encoding order always corresponds to the BC encoding order. The BC order is given by the reversed MAC order [2].

Plugging $\lambda_k^* = \mu_\ell, \forall k \in \mathcal{I}_\ell$ into [3], Eq. (8) shows that the optimum uplink covariance matrices $\boldsymbol{Q}_k^*$ for a given $\boldsymbol{\lambda}^*$ are independent of the encoding orders $\boldsymbol{\pi}^\ell$ within the groups $\mathcal{I}_\ell$. As a result, the rate r_k of a user $k \in \mathcal{I}_\ell$ depends only on $\boldsymbol{\pi}^\ell$, but not on $\boldsymbol{\pi}^m, m \neq \ell$. Moreover, consider two users $i, j \in \mathcal{I}_\ell \setminus \{k\}$ such that i is encoded directly after j. Swapping the positions of i, j in the encoding order also does not change r_k, as $\mathcal{S}_k$ remains unchanged.

Based on this result, consider users $k, j \in \mathcal{I}_\ell$ and an encoding order $\boldsymbol{\pi}^{\ell,1}$ such that user j is encoded directly after user k. A second encoding order $\boldsymbol{\pi}^{\ell,2}$ results from swapping the positions of k and j. Switching from $\boldsymbol{\pi}^{\ell,1}$ to $\boldsymbol{\pi}^{\ell,2}$ results in a change in the rates of k and j by Δr_k and Δr_j, respectively, while the rates of all other users remain unchanged. User j is added to $\mathcal{S}_k$, thus the interference experienced by user k in the MAC is increased, resulting in $\Delta r_k \leq 0$.

Moreover, as both encoding orders correspond to corner points that maximize $\boldsymbol{\lambda}^{*\mathrm{T}}\boldsymbol{r}$ (and thus yield the same value $\boldsymbol{\lambda}^{*\mathrm{T}}\boldsymbol{r}$),

$$\lambda_k^* \Delta r_k + \lambda_j^* \Delta r_j = 0.$$

But $\lambda_k^* = \lambda_j^* = \mu_\ell$, thus $\Delta r_k = -\Delta r_j$. Accordingly, the change in the merit function $\tilde{\boldsymbol{\lambda}}^{\mathrm{T}}\boldsymbol{r}$ amounts to

$$\tilde{\lambda}_k \Delta r_k + \tilde{\lambda}_j \Delta r_j = (\tilde{\lambda}_k - \tilde{\lambda}_j)\Delta r_k.$$

Thus, if $\tilde{\lambda}_k > \tilde{\lambda}_j$, it is better to encode k first, otherwise encode j first. Accordingly, the optimum encoding order is found by swapping users' positions until (7) holds.

7. SIMULATION RESULTS

To numerically investigate the efficiency of the proposed algorithm, the utility function was chosen such that the optimum solution lies within the region where all users time-share. The number of corner points generated by the column generation algorithm was averaged over 500 channel realizations, and the number of users K varied from three to ten. Figure 1 shows the average number of corner points that were generated until (4) held, using the column generation approach, for different numbers of users. Interestingly, the simulation results show that the number of generated points is, on average, approximately K – i.e., almost all generated points are needed for constructing the time-sharing solution. Moreover, the observed linear growth in K is significantly smaller than the worst case complexity $K!$.

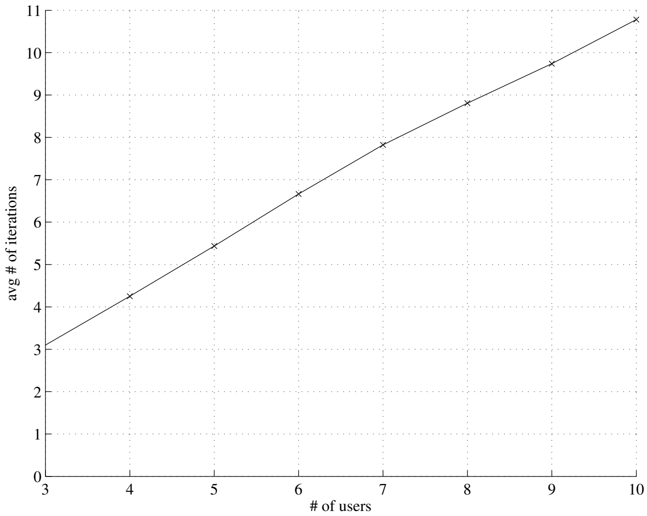

Fig. 1. Average number of generated corner points versus number of users

8. REFERENCES

[1] H. Weingarten, Y. Steinberg, and S. S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Transactions on Information Theory*, vol. 52, pp. 3936–3964, 2006.

[2] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Transactions on Information Theory*, vol. 49, pp. 2658–2668, 2003.

[3] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE Journal on Selected Areas in Communications*, vol. 21, pp. 802–811, 2003.

[4] R. Böhnke and K.D. Kammeyer, "Weighted Sum Rate Maximization for the MIMO-Downlink Using a Projected Conjugate Gradient Algorithm," in *International Workshop on Cross Layer Design (IWCLD)*, Jinan, China, September 2007.

[5] Jia Liu and Y. Thomas Hou, "Cross-layer optimization of MIMO-based mesh networks with Gaussian vector broadcast channels," April 2007, *Submitted to IEEE Transactions on Information Theory. Preprint available on arXiv.*

[6] Johannes Brehmer and Wolfgang Utschick, "Utility maximization strategies in the multi-user MIMO downlink," in *International Workshop on Cross-Layer Design (IWCLD)*, Jinan, China, September 2007.

[7] M. Johansson and L. Xiao, "Cross-layer optimization of wireless networks using nonlinear column generation," *IEEE Transactions on Wireless Communications*, vol. 5, pp. 435–445, 2006.

[8] T. Michel and G. Wunder, "Achieving QoS and Efficiency in the MIMO Downlink with Limited Power," in *ITG/IEEE Int. Workshop on Smart Antennas (WSA)*, Vienna, February 2007.