Pareto-Optimal Beamforming for the MISO Interference Channel With Partial CSI

(Invited Paper)

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***Abstract*—We consider the problem of finding Pareto-optimal (PO) operating points for the multiple-input single-output (MISO) interference channel when the transmitters have statistical (covariance) channel knowledge. We devise a computationally efficient algorithm, based on semidefinite relaxation, to compute the PO rates and the enabling beamforming vectors. We illustrate the effectiveness of our algorithm by a numerical example.**

I. Introduction

The situation when two wireless links operate in the same spectrum, and create mutual interference to one another, is well modeled by a so-called interference channel (IFC). Associated with any IFC there is an achievable rate region, consisting of all pairs of transmission rates R_1 (for link 1) and R_2 (for link 2) that can be achieved, subject to constraints on the powers used by the transmitters. The outer boundary of this region is called the Pareto boundary, and it consists of rate points where increasing R_1 necessarily requires decreasing R_2 and vice versa. It is generally desirable to operate at a rate point that lies on this boundary. In particular, the boundary contains important points such as the maximum-sum-rate point and the Nash bargaining solution [1].

We are specifically concerned with the achievable rate region for a so-called multiple-input single-output (MISO) IFC, consisting of links where the transmitters (called TX_1, TX_2 here) have n antennas and the receivers RX_1, RX_2 have a single antenna [2]. The transmitters can steer power in arbitrary directions by using beamforming. We are especially interested in the case when the transmitters have only partial (statistical) channel state information (CSI), channel covariance knowledge, more precisely.

This paper builds on our previous work [3]–[5], where we provided a set of necessary conditions for beamforming vectors to be Pareto optimal (we treated perfect CSI in [3] and partial CSI in [4], [5]). In the current work, we focus on the computation of the Pareto-optimal (PO) rates and the enabling beamforming vectors for the partial-CSI case. Specifically, we formulate the problem as constrained optimization and use semidefinite relaxation to solve it. The resulting algorithm is computationally efficient and easy to implement.

Notation: $\mathrm{Tr}\{\cdot\}$, $\mathrm{rank}\{\cdot\}$, $\mathrm{eig}\{\cdot\}$, $\mathcal{S}\{\cdot\}$, and $\mathcal{K}\{\cdot\}$ denote the trace, rank, dominant eigenvector, span, and kernel, respectively, of a matrix. $\boldsymbol{\Pi}_{\boldsymbol{Z}} \triangleq \boldsymbol{Z}(\boldsymbol{Z}^H\boldsymbol{Z})^{-1}\boldsymbol{Z}^H$ is the orthogonal projection onto the column space of $\boldsymbol{Z}$. $\dim\{\cdot\}$ denotes the dimension of a subspace and $\mathrm{E}\{\cdot\}$ is the expectation operator.

II. Preliminaries

A. System Model

We assume that transmission consists of scalar coding followed by beamforming[1] and that all propagation channels are frequency-flat. The matched-filtered symbol-sampled complex baseband data received by RX_i is modeled as[2]

$$y_i = \boldsymbol{h}_{ii}^H\boldsymbol{w}_i s_i + \boldsymbol{h}_{ji}^H\boldsymbol{w}_j s_j + e_i \quad j \neq i, \;\; i,j \in \{1,2\}, \tag{1}$$

where $s_i \sim \mathcal{CN}(0,1)$ and $\boldsymbol{w}_i \in \mathbb{C}^n$ are the transmitted symbol and the beamforming vector employed by TX_i, respectively. Also, $e_i \sim \mathcal{CN}(0,\sigma^2)$ models the receiver noise. The (conjugated) channel vector between TX_i and RX_j is modeled as $\boldsymbol{h}_{ij} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{Q}_{ij})$. TX_i knows the channel covariance matrices $\boldsymbol{Q}_{ii}$ and $\boldsymbol{Q}_{ij}$. We denote $r_{ij} \triangleq \mathrm{rank}\{\boldsymbol{Q}_{ij}\}$.

The transmission power is bounded due to regulatory and hardware constraints. Without loss of generality we set this bound to 1. Hence, the set of feasible beamforming vectors is

$$\mathcal{W} \triangleq \{\boldsymbol{w} \in \mathbb{C}^n \mid \|\boldsymbol{w}\|^2 \leq 1\}. \tag{2}$$

Note that the set $\mathcal{W}$ is convex. In what follows, a specific choice of $\boldsymbol{w}_i \in \mathcal{W}$ is denoted as a *transmit strategy* of TX_i.

B. Ergodic Rate Region

For fixed channel vectors and a given pair of beamforming vectors, the following instantaneous rate (in bits/channel use) is achievable

$$I_i(\boldsymbol{w}_i,\boldsymbol{w}_j) = \log_2\left(1 + \frac{|\boldsymbol{h}_{ii}^H\boldsymbol{w}_i|^2}{|\boldsymbol{h}_{ji}^H\boldsymbol{w}_j|^2 + \sigma^2}\right). \tag{3}$$

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[1]This is not optimal, but it is a practical assumption that also simplifies the analysis.

[2]Whenever an expression is valid for both systems, it is denoted once with respect to system i, interpreting system $j \neq i$ as the interfering one.

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The corresponding ergodic rate is obtained by averaging over the channels. From [4], we have

$$R_i(\boldsymbol{w}_i, \boldsymbol{w}_j) \triangleq \mathrm{E}\{I_i(\boldsymbol{w}_i, \boldsymbol{w}_j)\} = \frac{p_{ii}(\boldsymbol{w}_i)}{\ln 2} \frac{f(p_{ii}(\boldsymbol{w}_i)) - f(p_{ji}(\boldsymbol{w}_j))}{p_{ii}(\boldsymbol{w}_i) - p_{ji}(\boldsymbol{w}_j)}, \tag{4}$$

where

$$f(x) \triangleq e^{\sigma^2/x} \int_{\sigma^2/x}^{\infty} \frac{e^{-t}}{t} dt. \tag{5}$$

In (4), $p_{ji}(\boldsymbol{w}_j)$ corresponds to the average power that RX_i receives from TX_j, i.e.,

$$p_{ji}(\boldsymbol{w}_j) \triangleq \mathrm{E}\left\{|\boldsymbol{h}_{ji}^H \boldsymbol{w}_j|^2\right\} = \boldsymbol{w}_j^H \boldsymbol{Q}_{ji} \boldsymbol{w}_j. \tag{6}$$

In [4], we showed that

Lemma 1. *$R_i(\boldsymbol{w}_i, \boldsymbol{w}_j)$ is monotonously increasing with $p_{ii}(\boldsymbol{w}_i)$ for fixed $p_{ji}(\boldsymbol{w}_j)$ and monotonously decreasing with $p_{ji}(\boldsymbol{w}_j)$ for fixed $p_{ii}(\boldsymbol{w}_i)$.*

Lemma 1 reveals the conflict situation associated with the choice of beamforming vectors. A beamforming vector $\boldsymbol{w}_i$ which increases the average signal power $p_{ii}(\boldsymbol{w}_i)$ received at RX_i may also increase the average interference power $p_{ij}(\boldsymbol{w}_i)$ experienced at RX_j. The question that naturally arises is which rates $(R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1))$ are jointly achievable for given channel covariance matrices $\{\boldsymbol{Q}_{ij}\}$. The union of the rate pairs that can be obtained by all feasible beamforming vector pairs defines the achievable rate region

$$\mathcal{R} = \bigcup_{\boldsymbol{w}_1, \boldsymbol{w}_2 \in \mathcal{W}} (R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1)). \tag{7}$$

Note that for fixed $\{\boldsymbol{Q}_{ij}\}$ the region $\mathcal{R}$ is compact, since $\mathcal{W}$ is compact and the mapping of $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ to $(R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1))$ is continuous. However, the region $\mathcal{R}$ is in general nonconvex.

C. Pareto Boundary

Pareto-optimal rate points are rate pairs, for which it is not possible to improve the rate of one link without decreasing the rate of the other link. More precisely:

Definition 1. A rate pair $(R_1^\star, R_2^\star) \in \mathcal{R}$ is PO if there is no other pair $(R_1, R_2) \in \mathcal{R}$ with $(R_1, R_2) \geq (R_1^\star, R_2^\star)$ and $(R_1, R_2) \neq (R_1^\star, R_2^\star)$. (The inequality is componentwise.)

The union of all PO points defines the so-called Pareto boundary of $\mathcal{R}$. We denote as $\underline{R}_i$ and $\overline{R}_i$ the minimum and maximum, respectively, PO rate of link i. The Pareto boundary consists of all PO points on the upper-right boundary of $\mathcal{R}$, between $(\underline{R}_1, \overline{R}_2)$ and $(\overline{R}_1, \underline{R}_2)$ (see Fig. 1).

As evidenced by Lemma 1, the maximum rate $\overline{R}_i$ is achieved when TX_i operates “selfishly”, so that the signal power is maximized and TX_j operates “altruistically”, so that the interference power is zeroed. The selfish operation corresponds to the maximum-ratio (MR) transmit strategy [5]

$$\boldsymbol{w}_i^{\mathrm{MR}} = \arg\max_{w_i \in \mathcal{W}} \; p_{ii}(\boldsymbol{w}_i) = \mathrm{eig}\{\boldsymbol{Q}_{ii}\}. \tag{8}$$

Note that the computation of the MR beamforming vector in (8) requires knowledge only of the covariance matrix for the direct channel. That is, the MR strategy does not take into account the interference that it causes to the other communication link.

Contrarily, the altruistic operation maximizes the signal power while avoids causing interference. This is possible only when the channel covariance matrices are rank deficient and the direct channel has a component that is orthogonal to the coupling channel, i.e., when $\mathcal{S}\{\boldsymbol{Q}_{ii}\} \nsubseteq \mathcal{S}\{\boldsymbol{Q}_{ij}\}$. The so-called zero-forcing (ZF) strategy is determined in [5] to be

$$\boldsymbol{w}_i^{\mathrm{ZF}} = \mathrm{eig}\left\{\boldsymbol{\Pi}_{\mathcal{K}\{\boldsymbol{Q}_{ij}\}} \boldsymbol{Q}_{ii} \boldsymbol{\Pi}_{\mathcal{K}\{\boldsymbol{Q}_{ij}\}}\right\}. \tag{9}$$

When $\mathcal{S}\{\boldsymbol{Q}_{ii}\} \subseteq \mathcal{S}\{\boldsymbol{Q}_{ij}\}$, e.g., when the channel covariance matrices are full-rank, the ZF strategy is to refrain from transmission, i.e., $\boldsymbol{w}_i^{\mathrm{ZF}} = \boldsymbol{0}$.

Hence, the maximum rate depends on the aforementioned selfish and altruistic transmit strategies as

$$\overline{R}_i = R_i(\boldsymbol{w}_i^{\mathrm{MR}}, \boldsymbol{w}_j^{\mathrm{ZF}}). \tag{10}$$

Correspondingly, the minimum PO rate is equal to

$$\underline{R}_i = R_i(\boldsymbol{w}_i^{\mathrm{ZF}}, \boldsymbol{w}_j^{\mathrm{MR}}). \tag{11}$$

Note that $\underline{R}_i > 0$ only when $\boldsymbol{w}_i^{\mathrm{ZF}} \neq \boldsymbol{0}$.

III. Pareto-Optimal Beamforming

As discussed in Sec. II-C, we accurately know the endpoints of the Pareto boundary and the transmit strategies that enable them. In this section, we propose an optimization method to find any PO rate pair $(R_i^\star, R_j^\star)$, along with the corresponding PO beamforming vector pair $(\boldsymbol{w}_i^\star, \boldsymbol{w}_j^\star)$ that achieves it.

By directly interpreting Pareto optimality (see Definition 1), we note that every operating point on the Pareto boundary is uniquely defined by the rate of one communication link. The rate of the other link is implicity defined as the maximum achievable rate. Hence, given the rate $R_j^\star$ of link j, we can find the rate $R_i^\star$ of link i that corresponds to a PO pair by the following optimization problem

$$\max_{\boldsymbol{w}_i, \boldsymbol{w}_j \in \mathcal{W}} \; R_i(\boldsymbol{w}_i, \boldsymbol{w}_j) \tag{12}$$

$$\text{s.t.} \quad R_j(\boldsymbol{w}_j, \boldsymbol{w}_i) = R_j^\star \quad j \neq i. \tag{13}$$

Clearly, the optimization in (12)–(13) is always feasible when the input parameter, i.e., the rate $R_j^\star$, is chosen in $[\underline{R}_j, \overline{R}_j]$. The optimal value of the problem is the other coordinate, i.e., $R_i^\star$, of the sought point on the Pareto boundary. The optimal solution is the pair $(\boldsymbol{w}_i^\star, \boldsymbol{w}_j^\star)$ of transmit strategies that the transmitters have to employ in order to operate at $(R_i^\star, R_j^\star)$. Note that the equality constraint (13) can be equivalently relaxed to a lower-bounded inequality. Due to the concept of Pareto optimality, the bound will be tight at the optimum.

Unfortunately, the optimization problem (12)–(13) is difficult to solve directly. This is because the ergodic rates, which are explicitly expressed by (4)–(6), are involved functions comprising exponential integrals with quadratic terms in their limits. Hence, the constraint (13) cannot be written out in closed form. Moreover, the rates are neither convex nor concave functions of the optimization variables $\boldsymbol{w}_i$ and $\boldsymbol{w}_j$. Therefore, the optimization problem is nonconvex.

In the following, we solve the problem (12)–(13) indirectly, assuming one beamforming vector known and optimizing with respect to the other one. Depending on which beamforming vector we assume known, we define two complementary problems in Sec. III-A and III-B. Either of these problems can be used to efficiently calculate the PO beamforming vectors for each transmitter separately. A simple post-processing step is then required to yield the beamforming vector pairs that correspond to PO rate pairs.

A. Maximizing the Signal Power

Assume that the beamforming vector $\boldsymbol{w}_j^\star$ that corresponds to the sought PO rate pair $(R_i^\star, R_j^\star)$ is known, e.g., given by a genie. Then, the problem (12)–(13) reduces to

$$\max_{\boldsymbol{w}_i \in \mathcal{W}} \; R_i(\boldsymbol{w}_i, \boldsymbol{w}_j^\star) \tag{14}$$

$$\text{s.t.} \quad R_j(\boldsymbol{w}_j^\star, \boldsymbol{w}_i) = R_j^\star, \tag{15}$$

where the optimization is now only with respect to $\boldsymbol{w}_i$. Since $\boldsymbol{w}_j^\star$ is fixed, the terms $p_{ji}(\boldsymbol{w}_j^\star)$ and $p_{jj}(\boldsymbol{w}_j^\star)$ are fixed too. Hence, the rates in (14) and (15) are functions only of $p_{ii}(\boldsymbol{w}_i)$ and $p_{ij}(\boldsymbol{w}_i)$, respectively. Since $R_i(\boldsymbol{w}_i, \boldsymbol{w}_j^\star)$ is monotonously increasing with $p_{ii}(\boldsymbol{w}_i)$, the latter term can be used in lieu of the objective function (14) in the maximization problem. Furthermore, since the rate $R_j(\boldsymbol{w}_j^\star, \boldsymbol{w}_i)$ is monotonously decreasing with $p_{ij}(\boldsymbol{w}_i)$, there exists a unique value, say c_{ij}, of the latter such that the equality (15) is satisfied. Thus, (14)–(15) can be equivalently reformulated as

$$\max_{\boldsymbol{w}_i \in \mathcal{W}} \; p_{ii}(\boldsymbol{w}_i) \tag{16}$$

$$\text{s.t.} \quad p_{ij}(\boldsymbol{w}_i) = c_{ij}. \tag{17}$$

The interpretation of (16)–(17) is that knowing the PO interfering beamforming vector $\boldsymbol{w}_j^\star$, the corresponding PO direct beamforming vector $\boldsymbol{w}_i^\star$ is obtained by maximizing the signal power subject to a specific bound on the caused interference. The input parameter c_{ij} corresponds to the (maximum) level of interference that the sought beamforming vector is allowed to cause in order to achieve a specific point on the Pareto boundary. Every PO beamforming vector is a solution of (16)–(17) for some choice of

$$c_{ij} \in \left[0, p_{ij}\left(\boldsymbol{w}_i^{\text{MR}}\right)\right]. \tag{18}$$

Note that the lower and the maximum value of c_{ij} yield the ZF and MR transmit strategies, respectively. Hence, c_{ij} can be interpreted as a measure of selfishness.

Elaborating (2) and (6), the optimization (16)–(17) is explicitly written as

$$\max_{\boldsymbol{w}_i \in \mathbb{C}^n} \; \boldsymbol{w}_i^H \boldsymbol{Q}_{ii} \boldsymbol{w}_i \tag{19}$$

$$\text{s.t.} \quad \boldsymbol{w}_i^H \boldsymbol{Q}_{ij} \boldsymbol{w}_i = c_{ij}, \tag{20}$$

$$\boldsymbol{w}_i^H \boldsymbol{w}_i \leq 1. \tag{21}$$

The problem (19)–(21) is a quadratically constrained quadratic program (QCQP). All the quadratic terms are homogeneous and convex, since the parameter matrices are positive semidefinite. The equality constraint (20) can be equivalently relaxed to an upper-bounded inequality. Hence, the feasibility set determined by (20)–(21) is convex. However, the objective function (19) yields the optimization nonconvex. Nonconvex QCQPs are in general NP-hard to solve. But, as we elaborate in Sec. IV, the problem (19)–(21) is solved optimally and efficiently with semidefinite relaxation. This is because (19)–(21) belongs to a special class of QCQPs for which the semidefinite relaxation is tight [6, Corollary 3.4].

B. Minimizing the Interference Power

Alternatively to (14)–(15), assuming now $\boldsymbol{w}_i^\star$ known, the problem (12)–(13) reduces to

$$\max_{\boldsymbol{w}_j \in \mathcal{W}} \; R_i(\boldsymbol{w}_i^\star, \boldsymbol{w}_j) \tag{22}$$

$$\text{s.t.} \quad R_j(\boldsymbol{w}_j, \boldsymbol{w}_i^\star) = R_j^\star, \tag{23}$$

where the optimization is only with respect to $\boldsymbol{w}_j$. Since $p_{ii}(\boldsymbol{w}_i^\star)$ and $p_{ij}(\boldsymbol{w}_i^\star)$ are fixed, $R_i(\boldsymbol{w}_i^\star, \boldsymbol{w}_j)$ is monotonously decreasing with $p_{ji}(\boldsymbol{w}_j)$ and the objective function can be replaced by minimization of $p_{ji}(\boldsymbol{w}_j)$. Furthermore, since $R_j(\boldsymbol{w}_j, \boldsymbol{w}_i^\star)$ is monotonously increasing with $p_{jj}(\boldsymbol{w}_j)$, there exists a unique value, say c_{jj}, of the latter such that (23) is satisfied. Thus, (22)–(23) can be equivalently written as

$$\min_{\boldsymbol{w}_j \in \mathcal{W}} \; p_{ji}(\boldsymbol{w}_j) \tag{24}$$

$$\text{s.t.} \quad p_{jj}(\boldsymbol{w}_j) = c_{jj}. \tag{25}$$

The interpretation of (24)–(25) is that knowing the PO direct beamforming vector $\boldsymbol{w}_i^\star$, the corresponding PO interfering beamforming vector $\boldsymbol{w}_j^\star$ is obtained by minimizing the interference power subject to a specific (lower) bound

$$c_{jj} \in \left[p_{jj}(\boldsymbol{w}_j^{\text{ZF}}), p_{jj}(\boldsymbol{w}_j^{\text{MR}})\right] \tag{26}$$

on the signal power.

Optimization (24)–(25) is explicitly written as

$$\min_{\boldsymbol{w}_j \in \mathbb{C}^n} \; \boldsymbol{w}_j^H \boldsymbol{Q}_{ji} \boldsymbol{w}_j \tag{27}$$

$$\text{s.t.} \quad \boldsymbol{w}_j^H \boldsymbol{Q}_{jj} \boldsymbol{w}_j = c_{jj}, \tag{28}$$

$$\boldsymbol{w}_j^H \boldsymbol{w}_j \leq 1. \tag{29}$$

The problem (27)–(29) has a convex objective function, but a nonconvex feasible set due to (28). The equality constraint can be relaxed to a lower-bounded inequality, which however is still nonconvex. However, as with (19)–(21), the semidefinite relaxation of problem (27)–(29) is tight [6, Corollary 3.4].

IV. Semidefinite Relaxation

In this section, we show how the semidefinite relaxation technique can be used to efficiently find the optimal solutions of the complementary optimization problems of *maximizing the signal power for fixed interference* (19)–(21) and *minimizing the interference for fixed signal power* (27)–(29).

First, we exploit the fact that for any PO beamforming vector we have $\boldsymbol{w}_i \in \mathcal{S}\{\boldsymbol{Q}_{ii}, \boldsymbol{Q}_{ij}\}$ [4] and consider the parametrization

$$\boldsymbol{w}_i = \boldsymbol{V}_i \boldsymbol{x}_i, \tag{30}$$

where the columns of $\boldsymbol{V}_i \in \mathbb{C}^{n \times r_i}$ denote an orthonormal basis for $\mathcal{S}\{\boldsymbol{Q}_{ii}, \boldsymbol{Q}_{ij}\}$ and $\boldsymbol{x}_i \in \mathbb{C}^{r_i}$ denotes the parameter vector. Thus, the number of complex variables is

$$r_i \triangleq \dim\{\mathcal{S}\{\boldsymbol{Q}_{ii}, \boldsymbol{Q}_{ij}\}\} \leq r_{ii} + r_{ij}, \tag{31}$$

where $r_i \leq n$. The power constraint now reads $\|\boldsymbol{x}_i\|^2 \leq 1$.

Next, we further change the optimization variables to $\boldsymbol{X}_i \triangleq \boldsymbol{x}_i \boldsymbol{x}_i^H$. Note that

$$\boldsymbol{X}_i = \boldsymbol{x}_i \boldsymbol{x}_i^H \Leftrightarrow \boldsymbol{X}_i \succeq \boldsymbol{0} \text{ and } \operatorname{rank}\{\boldsymbol{X}_i\} = 1. \tag{32}$$

Using (30), (32), and the property that $\operatorname{Tr}\{\boldsymbol{Y}\boldsymbol{Z}\} = \operatorname{Tr}\{\boldsymbol{Z}\boldsymbol{Y}\}$ for matrices $\boldsymbol{Y}$, $\boldsymbol{Z}$ of compatible dimensions, we rewrite the average power term (6) as

$$\begin{aligned} \boldsymbol{w}_j^H \boldsymbol{Q}_{ji} \boldsymbol{w}_j &= \boldsymbol{x}_j^H \boldsymbol{V}_j^H \boldsymbol{Q}_{ji} \boldsymbol{V}_j \boldsymbol{x}_j = \boldsymbol{x}_j^H \boldsymbol{A}_{ji} \boldsymbol{x}_j = \\ &\operatorname{Tr}\{\boldsymbol{x}_j^H \boldsymbol{A}_{ji} \boldsymbol{x}_j\} = \operatorname{Tr}\{\boldsymbol{A}_{ji} \boldsymbol{x}_j \boldsymbol{x}_j^H\} = \operatorname{Tr}\{\boldsymbol{A}_{ji} \boldsymbol{X}_j\}, \end{aligned} \tag{33}$$

where we have defined $\boldsymbol{A}_{ji} \triangleq \boldsymbol{V}_j^H \boldsymbol{Q}_{ji} \boldsymbol{V}_j \in \mathbb{C}^{r_j \times r_j}$.

Due to (30), (32), and (33), we equivalently recast the maximization problem (19)–(21) as

$$\max_{\boldsymbol{X}_i \in \mathbb{C}^{r_i \times r_i}} \quad \operatorname{Tr}\{\boldsymbol{A}_{ii} \boldsymbol{X}_i\} \tag{34}$$

$$\text{s.t.} \quad \operatorname{Tr}\{\boldsymbol{A}_{ij} \boldsymbol{X}_i\} = c_{ij}, \tag{35}$$

$$\operatorname{Tr}\{\boldsymbol{X}_i\} \leq 1, \tag{36}$$

$$\boldsymbol{X}_i \succeq \boldsymbol{0}, \tag{37}$$

$$\operatorname{rank}\{\boldsymbol{X}_i\} = 1. \tag{38}$$

The objective function (34), the equality constraint (35), and the inequality constraint (36) are linear. The cone of positive semidefinite matrices (37) is convex. Only the rank constraint (38) is nonconvex. Dropping this constraint, the problem is relaxed to (34)–(37), which is a semidefinite programming (SDP) problem. Hence, it is solved by means of interior-point methods (IPMs) with polynomial worst-case complexity. Due to the rank relaxation, the IPMs may return a high-rank optimal solution. According to [6], if this SDP problem is feasible, there always exists a rank-1 optimal solution, which can be efficiently found by a post-processing rank-reduction algorithm. We experienced through extensive simulation studies that the IPMs do yield a rank-1 optimal solution.

Similarly, due to (30), (32), and (33), we equivalently recast the minimization problem (27)–(29) as

$$\min_{\boldsymbol{X}_j \in \mathbb{C}^{r_j \times r_j}} \quad \operatorname{Tr}\{\boldsymbol{A}_{ji} \boldsymbol{X}_j\} \tag{39}$$

$$\text{s.t.} \quad \operatorname{Tr}\{\boldsymbol{A}_{jj} \boldsymbol{X}_j\} = c_{jj}, \tag{40}$$

$$\operatorname{Tr}\{\boldsymbol{X}_j\} \leq 1, \tag{41}$$

$$\boldsymbol{X}_j \succeq \boldsymbol{0}, \tag{42}$$

$$\operatorname{rank}\{\boldsymbol{X}_j\} = 1. \tag{43}$$

Dropping the nonconvex rank constraint (43), the problem is relaxed to (39)–(42), which is an SDP problem. In our simulations, the IPMs do yield a rank-1 optimal solution.

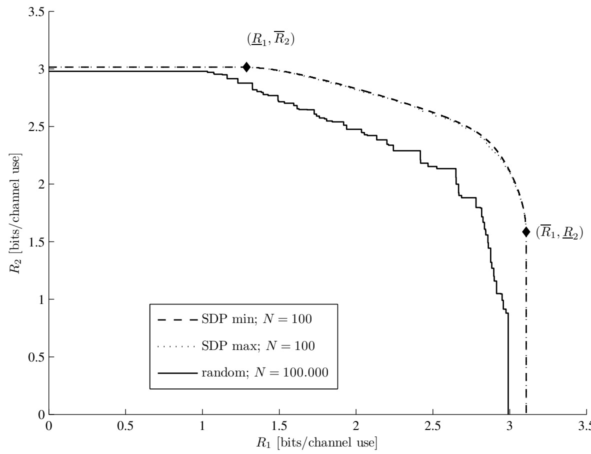

Fig. 1. Example of the ergodic rate region for $n = 8$, $\{r_{ij} = 3\}$.

V. Numerical Result and Conclusion

The efficiency of the advocated optimization method is illustrated in Fig. 1, which shows the computed ergodic rate region when the transmitters have $n = 8$ antennas and all covariance matrices have rank 3. For each transmitter, N beamforming vectors are created, using the SDP optimizations (34)–(37) or (39)–(42), by uniformly sampling the parameters c_{ij} or c_{jj}, respectively. Out of the N^2 possible beamforming vector pairs, the ones that yield rate pairs which are uppermost (or rightmost) in the region are the PO. The corresponding rate pairs are plotted to designate the Pareto boundary.

For comparison purposes, we also plot the resulting boundary by the brute-force method, proposed in [4], of randomly choosing the parameters in the characterization (30). It is seen that the optimization method requires a far smaller number of samples to create a sharper boundary. This is because the characterization (30) is only a necessary but not a sufficient condition for a beamforming vector to be PO. As the number r_i of complex parameters in (30) increases, the superiority of the optimization method is even more prominent.

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